AN

ELEMENTARY TREATISE

ON

THE APPLICATION OF

TRIGONOMETRY

TO

ORTHOGRAPHIC AND STEREOGRAPHIC PROJECTION, DIALLING, MENSURATION OF HEIGHTS AND DISTANCES, NAVIGATION, NAUTICAL ASTRONOMY, SURVEYING AND LEVELLING;

TOGETHER WITH

LOGARITHMIC AND OTHER TABLES;

DESIGNED FOR THE

USE OF THE STUDENTS OF THE UNIVERSITY

AT

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ADVERTISEMENT.

The branches of mathematics comprehended in this volume have usually made a part of the course of instruction at the public seminaries of the United States. But the best treatises upon these subjects are too extended, and of too practical a nature, to be used as a text-book. What is here offered is intended to furnish only those general principles and leading methods, which afford a useful exercise to the learner, and which may be considered as belonging to the pursuit of liberal studies. The works principally used in preparing this treatise are Cagnoli and Bonnycastle's Trigonometry, Delambre's Astronomy, Bèzout's Navigation, Puissant and Malortie's Topography. The tables, except those of meridional parts and astronomical refractions, are from the stereotype plates of Bowditch's Practical Navigator, the correctness of which is too well known to need any recommendation.

An introductory treatise on the Differential and Integral Calculus, nearly ready for the press, will complete this course of mathematics. It is proposed also soon to commence publishing a work on Natural Philosophy, adapted to the same class of students.

JOHN FARRAR,
Professor of Mathematics and Natural Philosophy,
in the University at Cambridge.

Cambridge, May 1822.

AN

ELEMENTARY TREATISE

ON THE

APPLICATION OF GEOMETRY AND TRIGONOMETRY TO THE PROJECTIONS OF THE SPHERE, AND TO THE SOLUTION OF GEODESIC AND NAUTICAL PROBLEMS.

CHAPTER I.

Of Projections and the Construction of Charts.

1. THE position of a point in a plane is determined by the position of any two lines passing through this point, since being in each of these lines it can only be at their intersection.

When there are several points to be designated, the method generally employed consists in taking two lines *AB*, *AC* (*fig.* 1), Fig. 1. perpendicular to each other, to which all the given points are referred. The point *M*, for example, would have its position determined by its distance from each of the lines *AB*, *AC*. Indeed if we take *AQ* equal to the first of these distances, and through *Q* draw *QM* parallel to *AB*, the point proposed will be in this line; it will also be in *PM* parallel to *AC*, and whose distance from *AC* is equal to the distance of the point *M* from this line; the point proposed, therefore, being common to the two lines *QM*, *PM*, will be at their intersection *M*.

2. When the question involves three dimensions, or relates to a body, we adopt a method similar to the preceding; similar, indeed, to that employed by architects; it is that of *plans*, *profiles*, and *elevations*.

When a point is given in space, we can let fall from this point a perpendicular upon an assumed plane, and note the place where it meets the plane; this is called the *projection* of the given point, and the assumed plane is called the *plane of projection*.

Let us suppose, for instance, that all the points of a given figure are referred to a horizontal plane; their projections would

Top. 1

be the intersections of a plumb-line meeting this plane from each point of the given figure, and the lengths of these lines will be the altitudes of the points respectively above their projections.

3. If now we suppose a plane, raised perpendicularly to the horizontal plane, and let fall upon this second plane perpendiculars from each point of the given figure, these perpendiculars will give a second projection of the given figure representing the proper altitude of the several points above the horizontal plane.

Fig. 2. Thus, let *BAC* (*fig.* 2) represent a horizontal plane, *DAB* a vertical plane, raised upon the line *AB*; from the point *M* of a given object let fall the perpendicular *MM'* upon the horizontal plane, and its foot *M'* is the horizontal projection of the given point.

From the point *M* let fall also the perpendicular *MM''* upon the plane *DAB*, and the point *M''* is the vertical projection of the given point.

The two lines *MM'*, *MM''*, are evidently in the same plane, since they cut each other. The line *M'M'''*, drawn in the horizontal plane perpendicularly to the line *AB*, the common intersection of the two planes, will be perpendicular to the vertical plane, and consequently parallel to *MM''*; and these three lines will be in the same plane, which is perpendicular to the vertical and also to the horizontal plane. It is evident that *M''M'''* is equal to *MM'*, that is, the vertical projection *M''* is at the same altitude above the horizontal plane as the point *M*.

By proceeding in a similar manner with the point *P*, we should have its two projections *P'*, *P''*; and it is manifest that, while the vertical projections *M''*, *P''*, give the altitudes of the given points above the horizontal plane, the horizontal projections *M'*, *P'*, will give the distances of the given points from the vertical plane.

In order to represent the different parts of an edifice, an architect assumes a horizontal plane, to which are referred by perpendiculars the leading divisions and remarkable points of the edifice; the design thus formed is called a *plan*; next another plane is taken in a vertical position, upon which are referred such points as are required to be noted, of the altitude at which they are actually placed above the horizontal plane. This with its delineations is called a *section* or *profile*, if it is supposed to pass through the interior, and the *elevation* if it represent the outside of the building.

This method, moreover, is made use of to represent the heights of objects near the earth's surface, as beacons, towers, the tops of mountains, &c.; also the streets of cities, and the boundaries of fields, and generally figures occupying, by their projections, such a portion of the earth's surface, as may without sensible error be considered as a plane.

Of Orthographic Projection.

4. According to the method, here given, which is called *orthographic projection*, a straight line perpendicular to the plane of projection is represented by a point, and a line parallel to the plane of projection by a line of the same length. With respect to a straight line oblique to the plane of projection, it is represented by the distance between the perpendiculars let fall from the extremities upon the plane of projection. Let *AB* (*fig.* 3), for instance, be a Fig. 3.
line inclined at any angle to the plane of projection *PL*. *AC* being drawn parallel to *PL*, the angle *BAC* will be equal to the inclination of the line *AB* to the plane of projection *PL*, and $A'B' = AC$ is the projection of the line *AB*. Now

$$AB : AC :: R : \sin ABC \text{ or } \cos BAC \quad (\textit{Trig. } 30).$$

Thus radius is to the cosine of the inclination, as the line *AB* is to its projection *A'B'*. Therefore if we consider radius unity, *the projection of a line is equal to this line, multiplied by the cosine of its inclination to the plane of projection.* If the line *AB* be considered as unity, its projection *AC* will be the cosine of its inclination simply.

5. This kind of projection is employed in some cases to represent a spherical surface. The sun and moon appear as circles, and the different parts of the hemisphere presented to us have apparently the same relative situation that they would have when projected by means of perpendiculars let fall from each point upon the plane separating the visible from the invisible hemisphere. This is true of any sphere, whose distance compared with its diameter is so great that the rays of light proceeding from it and meeting in the eye may be considered as perpendicular to the plane of projection.

Fig. 4. 6. If we imagine the semicircle *DFH* (*fig.* 4) raised perpendicularly upon the plane of the paper, the diameter *DH* remaining in this plane, and suppose the perpendiculars *FC, IE,* &c., let fall from all the points of the circumference, these perpendiculars would meet the plane of the paper in a series of points forming the diameter *DH.* *C* being the centre, the arc *FH* will be 90°, and its projection *CH* will be equal to radius, that is, to the sine of 90°. In like manner $CE = LI = \sin FI$ will be the projection of the arc *FI.* *When, therefore, an arc has its plane perpendicular to the plane of projection and its origin at the perpendicular which passes through the centre, the orthographic projection of this arc is equal to its sine.*

7. If the plane of the circle instead of being perpendicular is inclined to the plane of projection, the ordinates *FC, IE,* &c., falling upon the diameter, will with their projections *CG, EK,* &c., each make an angle equal to the inclination of the two planes, since the ordinates and their projections are respectively perpendicular to the diameter *DH,* or the common intersection of the two planes (*Geom.* 349). Hence

$$FC : CG :: R : \cos \text{inclination } (4),$$

and $$IE : EK :: R : \cos \text{inclination, \&c.,}$$

consequently $$FC : CG :: IE : EK,$$

$$I'E' : E'K', \text{ \&c.}$$

But the ordinates in a circle are to the corresponding ordinates in an ellipse in a constant ratio, namely, as the semitransverse to the semiconjugate (*Trig.* 114). We infer then that *DK′GH,* the projection of the inclined simicircle *DI′FH* is a semiellipse, of which *DC,* equal to *FC,* is the semitransverse, and *CG* the semiconjugate. The same may be proved by similar reasoning with respect to the other half of the circle. Therefore *the orthographic projection of an inclined circle is an ellipse, of which the transverse is equal to the diameter of the circle, and the conjugate to twice the cosine of its inclination to the plane of projection.*

We have supposed the plane of projection to pass through the centre of the inclined circle. The above theorem, however, will be true, whatever be the distance of the inclined circle from the plane of projection; for we may always suppose a plane parallel to that on which the projection is to be made, and passing through the centre of the inclined circle, and the figures determined by perpendiculars falling upon two parallel planes must evidently be the same.

8. Orthographic projection is little used for geographical maps, because it is liable to great errors when the map is of considerable extent. The difference between a small arc *FI'* and its projection *CE'* is inconsiderable, and in this case the distance between two objects upon the earth's surface may without sensible error be represented on a map by the distance *CE'*. But the more the point *I* approaches towards *D* or *H*, the more will the increase of the arc *FI* exceed the corresponding increase of its projection *CE*, and the more considerable will be the errors in the distances of places thus represented. Suppose $IH = 60° = 2\,FI$; then CE or $LI = \sin 30° = \frac{1}{2}\,CH$ (*Trig.* 18). Therefore $CE = EH$, and consequently the distances *IH*, *FI*, the first of which is double the second upon the globe, will be represented upon the map by equal lines.

The contraction which takes place toward the plane of projection will be evident from fig. 12, which is an orthographic projection of the sphere upon the plane of the equator, the meridians, having their planes pass through the eye, being represented by straight lines, and the parallels of latitude by circles whose radii are equal to the sines of their polar distances respectively, the radius of the equator being unity. Fig. 12.

Notwithstanding the inconvenience above mentioned, astronomers advantageously make use of this projection to represent and predict the circumstance of an eclipse, because, in this case, the question is not about the respective distances of places, but only to describe upon a geographical chart the curves which embrace pretty nearly the countries liable to be eclipsed, or the places from which the same or different phases may be seen.

Of Stereographic Projection.

9. The representation the most convenient for maps, which comprehend a large part of the globe, is one in which the eye is situated in the surface of the sphere, the plane of projection passing through the centre perpendicular to the diameter which is directed to the eye. This is called *stereographic* projection.

In orthographic projection the whole surface of a sphere may be represented upon the plane of a great circle as a base, to which all the visual rays are perpendicular. In stereographic projection only a hemisphere can be represented upon the plane of a great circle, the eye being supposed at the pole of this circle; it is necessary that the plane of projection should be infinitely extended to admit of all the points of the sphere being represented at once. Each hemisphere, however, may be successively represented upon the plane of a great circle, by supposing the eye first in one pole and then in the opposite pole of this circle.

Fig. 5. 10. If *AO* (*fig.* 5) be perpendicular to *BD*, *O* being the place of the eye, *BD* will represent that part of the plane of projection comprehended within a great circle. This is called the *primitive* circle, being that to which all the others are referred. *C*, the *centre of the primitive*, will be the projection of the point *A*, *the pole of the primitive.*

Any circle of the sphere, if we except those whose plane passes through the eye, may be considered as the base of a cone formed by rays proceeding from the circumference of this circle to the eye; and when the circle is parallel to the primitive, its representation upon the plane of projection is evidently a circle; since the cone formed, as above described, is a right cone, and every section of a right cone, made by a plane parallel to the base, is a circle. When, however, the circle is oblique to the plane of projection, the cone formed by the visual rays is also oblique. The circle, for example, which has for its diameter the chord *EF*, will be the base of a cone of which *OEF* is a section through the axis. Moreover, the section *SNT*, made by the plane of projection, is the projection of the circle *EyF*. It is proposed to determine the figure of this section. Through x, the centre of the circle *EyF*, suppose a plane to pass parallel to the plane of projection, meeting the cone in $S'yT'$; the two sections *EyF*, $S'yT'$, being perpendicular each to the plane *OEF*, the common intersection xy will be perpendicular to the plane *OEF*, and consequently to each of the straight lines *EF* and $S'T'$ situated in this plane. Then, since the angle *OTS* is measured by the sum of the arcs *OB*, *DF*,† or half of *ODF*, it is equal to *OEF*, which is also measured by half of *ODF* (*Geom.* 126).

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† Suppose a straight line *Ff* drawn through the point *F* parallel to *DB*, we shall have $Bf = Df$ (*Geom.* 112), and the angle $OFf = OTS$. But the measure of the angle *OFf* is $\frac{1}{2}\,OBf = \frac{1}{2}\,ODF$, therefore this is also the measure of the angle *OTS*.

Also, since $S'T'$ is parallel to ST, the angle $OT'S'$ is equal to OTS, and consequently equal to OEF. Therefore the triangles ExS', $T'xF$, having two angles of the one respectively equal to two angles of the other, are similar; whence

$$Ex : xT' :: S'x : xF,$$

and

$$S'x \times xT' = Ex \times xF.$$

But Ex, xF, xy, being radii of the same circle EyF,

$$Ex \times xF = \overline{xy}^2;$$

consequently

$$S'x \times xT' = \overline{xy}^2;$$

and as this is the case however oblique the two sections EyF and $S'yT'$ are to each other, that is, upon whatever part of $S'T'$ the ordinate xy falls, the section $S'yT'$ must be a circle (*Trig.* 103)

Now suppose a plane $OR'N'$ passing through the axis OR', and making any angle with the plane $OS'T'$, we shall have by similar triangles

$$OR' : OR :: R'S' : RS$$
$$:: R'N' : RN;$$

whence, on account of the common ratio,

$$R'S' : RS :: R'N' : RN$$

or

$$R'S' : R'N' :: RS : RN.$$

But $R'S'$, $R'N'$, are equal, being radii of the same circle; consequently RS, RN, are also equal; and as this is true, whatever angle the plane $OR'N'$ makes with $OS'T'$, the section SNT is a circle. Therefore, *in stereographic projection, every circle of the sphere, whose plane does not pass through the eye, is represented on the plane of projection by a circle.*

11. Let P (*fig.* 6) be the pole of the circle EF. The angles POE, POF, are equal (*Geom.* 126); and the straight line PO, bisecting the angle EOF, will divide the diameter EF, and its projection ST into unequal parts, making SK less than KT (*Geom.* 201). Bisect ST in m, and we shall have Fig. 6

Cm = distance of the centre of the projected circle from the centre of the primitive; and

Sm = to the radius of the projected circle. Call Cm d, and Sm r.

Now, since CS, Cm, CT, are in arithmetical progression (*Alg.* 223), we shall have

$$2\,Cm = CT + CS = \text{tang}\,\tfrac{1}{2}\,AF + \text{tang}\,\tfrac{1}{2}\,AE$$

$$= \frac{\sin\left(\frac{1}{2}AF + \frac{1}{2}AE\right)}{\cos\frac{1}{2}AF\cos\frac{1}{2}AE}\dagger$$

$$= \frac{\sin\left(\frac{1}{2}AF + \frac{1}{2}AE\right.}{\cos\frac{1}{2}(AP+PE)\cos\frac{1}{2}(AP-PE)}$$

$$= \frac{\sin AP\ \ddagger}{\frac{1}{2}(\cos AP + \cos PE)};$$

and

$$Cm \text{ or } d = \frac{\sin AP\ \ddagger}{\cos AP + \cos PE} \qquad (1)$$

In like manner

$$2\,Sm = CT - CS = \text{tang}\,\tfrac{1}{2}\,AF - \text{tang}\,\tfrac{1}{2}\,AE$$

$$= \frac{\sin\left(\frac{1}{2}AF - \frac{1}{2}AE\right)}{\cos\frac{1}{2}AF\cos\frac{1}{2}AE}$$

$$= \frac{\sin\left(\frac{1}{2}AF - \frac{1}{2}AE\right)\dagger\dagger}{\cos\frac{1}{2}(AP+PE)\cos\frac{1}{2}(AP-PE)}$$

$$= \frac{\sin PE}{\frac{1}{2}(\cos AP + \cos PE)}$$

and

$$Sm \text{ or } r = \frac{\sin PE}{\cos AP + \cos PE} \qquad (2)$$

---

† The formula $\text{tang}\,a + \text{tang}\,a = \frac{R^2 \sin(a+b)}{\cos a \cos b}$ (*Trig.* 29),

when $R = OC = 1$, $a = \frac{1}{2}AF$ and $b = \frac{1}{2}AE$, becomes

$$\text{tang}\,\tfrac{1}{2}\,AF + \text{tang}\,\tfrac{1}{2}\,AE = \frac{\sin\left(\frac{1}{2}AF + \frac{1}{2}AE\right)}{\cos\frac{1}{2}AF\cos\frac{1}{2}AE}.$$

‡ AE, AP, AF, being by construction in arithmetical progression,

$$AP = \tfrac{1}{2}(AF + AE) = \tfrac{1}{2}AF + \tfrac{1}{2}AE.$$

Moreover from the formula

$$\cos a + \cos b = \frac{2}{R}\cos\tfrac{1}{2}(a+b)\cos\tfrac{1}{2}(a-b) \quad (\textit{Trig.}\ 29),$$

if we put $R = 1$, $a = AP$ and $b = PE$, and divide both numbers by 2, we shall have

$$\tfrac{1}{2}(\cos AP + \cos PE) = \cos\tfrac{1}{2}(AP + PE)\cos\tfrac{1}{2}(AP - PE.)$$

†† It will be observed that the steps in this investigation do not differ from those just explained, except in the application of the formu-

12. Suppose PE (*fig.* 7) $= 0$, and the formula Fig 7.

$$d = \frac{\sin AP}{\cos AP + \cos PE},$$

since $\cos 0 = 1$, becomes

$$d = \frac{\sin AP}{\cos AP + 1} = \frac{2 \sin \frac{1}{2} AP \cos \frac{1}{2} AP}{2 (\cos \frac{1}{2} AP)^2} \dagger = \frac{\sin \frac{1}{2} AP}{\cos \frac{1}{2} AP}$$

$$= \text{tang} \tfrac{1}{2} AP.$$

The formula (2) in this case becomes

$$r = \frac{\sin PE}{\cos AP + \cos PE} = 0.$$

Thus the circle is reduced to a point, namely, the pole, and the projection of this pole will be the point K, and $CK = \text{tang} \frac{1}{2} AP$.

Therefore *the projection of a point* P *has for its distance from the centre of the primitive the tangent of half its distance on the sphere from the pole of the primitive.*

13. If $PE = 180°$, r is still $= 0$, but

$$d = \frac{\sin AP}{\cos AP + \cos PE} = \frac{\sin AP}{\cos AP + \cos 180°} = \frac{\sin AP}{\cos AP - 1}$$

$$= \frac{2 \sin \frac{1}{2} AP \cos \frac{1}{2} AP}{-2 (\sin \frac{1}{2} AP)^2} \ddagger$$

$$= -\frac{\cos \frac{1}{2} AP}{\sin \frac{1}{2} AP}$$

$$= -\cot \tfrac{1}{2} AP.$$

---

la for the difference of the tangents instead of that for the sum. It is obvious that $AF = AP + PE$, and $AE = AP - PE$, by construction. Also $\frac{1}{2} AF - \frac{1}{2} AE = \frac{1}{2} (AF - AE) = PE$, since AE, AP, AF, being in arithmetical progression, the difference (PE) between AE and AP is half the difference between AE and AF.

† The formula $\sin 2a = \frac{2 \sin a \cos a}{R}$ (*Trig.* 29), when $R = 1$, and $a = \frac{1}{2} AP$, becomes $\sin AP = 2 \sin \frac{1}{2} AP \cos \frac{1}{2} AP$; and the formula $\cos 2a = \frac{2 \cos a^2 - R^2}{R}$, with the same substitutions, becomes $\cos AP = 2 (\cos \frac{1}{2} AP)^2 - 1$, or $\cos AP + 1 = 2 (\cos \frac{1}{2} AP)^2$.

‡ From the formula $\sin a^2 = \frac{1}{2} R (R - \cos 2a)$ (*Trig.* 29), or $2 \sin a^2 = R (R - \cos 2a)$, when $R = 1$ and $a = \frac{1}{2} AP$, we have

$$2 (\sin \tfrac{1}{2} AP)^2 = 1 - \cos AP,$$

or, changing all the signs (*Alg.* 57),

$$-2 (\sin \tfrac{1}{2} AP)^2 = \cos AP - 1.$$

The projection of the opposite pole K' therefore will be upon CB produced to a distance $CK' = - \cot \frac{1}{2} AP$, the sign $-$ signifying that it falls on the other side of C with respect to K.

Fig. 8. 14. Suppose PE (*fig.* 8) $= 90°$, the chord EF will be a diameter, and the circle to be projected will be a great circle; and the formula

$$d = \frac{\sin AP}{\cos AP + \cos PE},$$

since $\cos PE = \cos 90 = 0$, becomes

$$d = \frac{\sin AP}{\cos AP}$$

$$= \text{tang}\, AP.$$

Thus *the distance of the centre of a projected great circle from the centre of the primitive, is equal to the tangent of the distance of its pole from the pole of the primitive, or, which is the same thing, equal to the tangent of the inclination of the great circle to the plane of projection.*

15. The other formula in this case, namely,

$$r = \frac{\sin PE}{\cos AP + \cos PE},$$

becomes

$$r = \frac{1}{\cos AP}$$

$$= \sec AP.$$

Therefore, *the radius of a projected great circle is equal to the secant of the distance of its pole from the pole of the primitive, or the secant of the inclination of the great circle to the plane of projection.*

16. With the radius of the circle to be projected, and the distance of its centre from the centre of the primitive, it is easy to describe the circle upon the plane of projection. But it is to be remarked that *the centre* m *will be in the plane of the great circle which passes through the poles* A, P, *that is, upon the radius* CD *directed to the point* D, *where the perpendicular arc* PD *falls.*

These simple formulas answer for all great circles.

Fig. 9. 17. If AP (*fig.* 9) $= 0$, the pole of the given circle becomes the pole of the primitive, and from the formulas (1) (2) we have

$$d = \frac{\sin AP}{\cos AP + \cos PE} = 0$$

$$r = \frac{\sin PE}{\cos AP + \cos PE} = \frac{\sin PE}{1 + \cos PE} = \text{tang} \tfrac{1}{2} PE†.$$

From these formulas it appears that *all circles parallel to the plane of projection have for their projected centre, the centre of the primitive, and for their radii the tangent of half their distances respectively from the pole of the primitive.*

18. If AP (*fig.* 10) $= 90°$, the general formulas become Fig. 10.

$$d = \frac{\sin AP}{\cos AP + \cos PE} = \frac{1}{\cos PE} = \sec PE,$$

$$r = \frac{\sin PE}{\cos AP + \cos PE} = \frac{\sin PE}{\cos PE} = \text{tang } PE;$$

that is, *circles either great or small, which have their pole in the circumference of the primitive, have each, when projected, for the distance of its centre from the centre of the primitive, the secant of the distance of this circle from its pole, and for its radius the tangent of this same distance.* In the case of great circles, $PE = 90°$, and $\sec PE = \sec 90°$ or infinite; and tang PE is also infinite. Therefore *the centres of great circles, which have their pole in the circumference of the primitive, being at an infinite distance from the centre of the primitive, their projections will be straight lines passing through the centre of the primitive, and cutting each other at angles, which have for their measure the distances of the poles of their circles from each other respectively.*

19. When we have found the two poles K, K' (*fig.* 7), of a Fig. 7. great circle by the methods already given (12, 13), we have two points of each of the circles that intersect each other at these points; the projections of all these circles will have for a common chord the straight line KK', which joins the projections of the two poles; they have all therefore their centres in the straight line VHX, which passes perpendicularly through the middle of KK' (*Geom.* 106).

20. The formulas which we have thus deduced analytically may be obtained by geometrical methods. The formulas of articles 14 and 15, for example, may be demonstrated as follows.

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† The formula $\frac{\sin a}{R + \cos a} = \frac{\text{tang} \frac{1}{2} a}{R}$, (*Trig* 29), when $R = 1$, and $a = PE$, gives the above expression. It may be observed also that the result of art. 13 might have been obtained more concisely by means of the formula $\frac{\sin a}{R - \cos a} = \frac{\cot \frac{1}{2} a}{R}$ in the table above referred to.

Fig. 8. If $PE = 90°$ (*fig.* 8), EF will be a diameter, and the circle will be a great circle, and its projection will have ST for its diameter, and $\frac{1}{2} ST$ or mT for its radius. Join Om; and SOT, having for its measure $\frac{1}{2} EPF$, is equal to 90°. Therefore the circle described upon ST in the plane DBA will pass through the point O (*Geom.* 128); consequently $mO = mT = mS$, and the triangle OmT is isosceles, and $mOT = OTm$; accordingly we have

$$\begin{aligned} OmC = mOT + OTm &= 2\,OTm \\ &= BO - DF\dagger \\ &= DO - DF \\ &= 90° - DF. \end{aligned}$$

But the acute angles of a right-angled triangle being complements of each other,

$$OmC = 90° - COm.$$

Therefore COm is equal to DF, and

$$Cm \text{ the tang } COm = \text{tang } DF = \text{tang } \textit{inclination};$$

and $\quad Om \text{ the sec } COm = \text{sec } DF = \text{sec } \textit{inclination}.$

If we produce Om to I, we shall have

$$AI = 2\,AOI = 2\,DF = 2\,AP,$$

which furnishes this graphical method of projecting great circles. Take AI equal $2\,AP$, or twice the inclination, and draw OmI; m will be the centre, and mO the radius of the circle to be projected.

Again, since $\quad mOC = DF,$

and $\quad OCR = OF,$

we have by addition,

$$\begin{aligned} mOC + OCR &= DF + OF \\ &= 90°; \end{aligned}$$

therefore

$$ORC = 90°.$$

We hence derive this other construction. Draw ORm perpendicular to CF, and we shall have the centre m, and the radius Om of the circle to be projected, as before.

---

† Through the point D suppose a straight line Df drawn parallel to TO. The angle $fDB = OTm$, is measured by

$$\tfrac{1}{2} Bf = \tfrac{1}{2}(BO - fO) = \tfrac{1}{2}(BO - DF).$$

Therefore $\quad 2\,OTm = BO - DF.$

21. The formulas of article 18 are found by a very simple construction. When AP (*fig.* 10) $= 90°$, P coincides with D and the pole of the given circle is in the circumference of the primitive. EF in this case is perpendicular to BD. Bisect the projected diameter ST in m, and join mE, mF, FC. As the projected circle must pass through the points E, F, as well as S, T, it follows that $mF = mT$, and $mFT = mTF$, whence we have Fig. 10.

$$\begin{aligned} FmC = mFT + mTF = 2\,mTF & \\ &= OB - FD \text{ (note to p. 12.)} \\ &= 90° - FCm; \end{aligned}$$

consequently

$$FmC + FCm = 90°,$$

and therefore CFm is a right angle, and Fm, the radius of the projected circle, is the tangent of DF or PE. Also Cm, the distance of the centre of the projected circle from the centre of the primitive, is the secant of PE.

22. It appears from the construction just given, that Sm, the projection of the tangent Em, is equal to Em. Indeed it may be shown generally that, since the angle OEm or

$$SEm = \tfrac{1}{2}\,ODE = \tfrac{1}{2}\,OD + \tfrac{1}{2}\,DE = 45° + \tfrac{1}{2}\,DE$$

and

$$ESm = \quad OSB = \tfrac{1}{2}\,OB + \tfrac{1}{2}\,DE = 45 + \tfrac{1}{2}\,DE \text{ (note to p. 6),}$$

the angles SEm, ESm, are equal; from which we infer that $Sm = Em$. Therefore *the tangent of a great circle terminating in the primitive has for its projection a line equal to this tangent.*

Let Em, Em' (*fig.* 11), be two of these tangents, Sm, Sm', their projections, mm' being joined, the two triangles Smm', Emm', will be equal in all respects, since they have two sides of the one respectively equal to two sides of the other, and one side common. Consequently $mSm' = mEm'$. Therefore, since these tangents make the same angle with each other as the intersecting circles to which they belong (*Geom.* 471), we conclude that *two great circles make by their projections the same angle that they make upon the sphere.* Fig. 11.

23. In the case of an angle formed by two arcs of great circles which do not terminate in the plane of projection, the above theorem will hold true, for we may always suppose the arcs produced to the plane of projection, and then the tangents of these arcs will be equal to their projections, and the arcs and their projections will make the same angle.

In like manner, *arcs of small circles, which have a common intersection and common tangents with those of large circles, will make by their projections the same angles which they make upon the sphere.*

24. By means of the principles above given, we are able to trace the different circles of the sphere and thus to represent the relative positions of objects in the heavens or on the earth. If for example it were required to project the northern hemisphere of the earth, as it would appear to a spectator at the south pole, the earth itself being supposed to be transparent; in this case

Fig. 13. the equator *GEHF* (*fig.* 13) will be the primitive†, and its centre *P* will represent the north pole. $AP = 0$, as in figure 9. We have therefore *P* for the common centre of all the parallels of latitude, and for their radii PF', PF'', &c., the tangent of half their distances respectively from the nearest pole (17). To draw parallels to every ten degrees, for instance, *PF* being taken equal to $\text{tang}\,\frac{90^\circ}{2} = \text{tang}\,45^\circ = \text{radius}$‡, we shall have

$$PF' = \text{tang}\,\frac{80^\circ}{2} = \text{tang}\,40^\circ,\ PF'' = \text{tang}\,\frac{70^\circ}{2} = \text{tang}\,35^\circ,$$

Pf radius of the tropic of cancer $= \text{tang}\,\frac{66^\circ\,32'}{2}$, &c.

Considering these parallels with reference to their distances from the equator we mark the first 10°, the second 20°, &c.

As the meridians have their plane passing through the eye, they will all be represented by straight lines intersecting each other at the centre of the primitive, and dividing the equator into arcs which measure the inclination of these circles to each other respectively (18). Thus *GPH* being the first meridian, 10 *P* 170, 20 *P* 160, &c., will represent the meridians which pass through every tenth degree of longitude. In the same manner we may subdivide the arcs FF', $F'F''$, &c., *G* 10, 10 20, &c., to any degree of minuteness, and thus represent the situation of cities,

† We here consider the paper as the plane of projection, whereas in most of the figures of plate 1 the plane of projection is represented as perpendicular to the paper.

‡ For the method of taking these lines from the sector, see note at the end of this part on the description and use of the *Plane Scale, Sector*, &c.

mountains, the several points of the boundaries of states and kingdoms, rivers, coasts, &c., according to their latitudes and longitudes. We can moreover refer these same lines to the heavens, and project the places of the stars according to their declination and right ascension. This is called a *polar* projection of the sphere. It is more simple than the projection upon the plane of any other circle. It was used by Ptolemy in the construction of his Astrolabe, and is sometimes preferred for the maps of countries situated in high latitudes. But, it is adopted for the most part in celestial rather than terrestrial planispheres, and especially for maps containing half of the heavens.

GEHF being referred to the heavens, we may consider *EF* as the solstitial, and *GH* as the equinoctial colure. Accordingly the ecliptic will pass through the points *G*, *e*, *H*, and to represent this circle we have only to describe a circle which shall pass through these points (*Geom.* 149); or conformably to articles 14, 15, to take for a radius, the secant of 23° 28′, the obliquity of the ecliptic to the equator, and for the distance of the centre from the centre of the primitive, the tangent of this same obliquity. Moreover the direction of this centre will, according to art. 16, be in the line *PF*.

25. If now it were required to project on the same plane secondaries to the ecliptic, we first find the poles of the ecliptic as in figure 7. Thus, *ABOD* being now† the equator, *AeO* the northern half of the ecliptic, and *K*, *K′*, its poles, the centres of all the circles which pass through *K*, *K′*, will be in the line *VHX*. The circle *KXK′V*, drawn with the centre *H* and the radius *HK*, will pass through the equinoxes *O*, *A*. Draw *KG* making the angle *HKG* equal to 10°, and *KG′* making the angle *HKG′* equal to 20°, &c., and *KG*, *KG′* will be the radii of the circles *K′dKb*, *K′d′Kb′*, &c., inclined respectively 10°, 20°,

† In finding the poles *K*, *K′*, we consider the equator *BD* perpendicular to the paper, *A* its pole, and *P*, *P′*, the north and south poles of the ecliptic. Having found *K*, *K′*, in the manner already explained (12, 13,) suppose the plane of the equator to revolve about its diameter *BD* through an arc of 90°, or till it coincides with the paper; the eye will be perpendicularly over the point *C*, and *K*, *K′*, will remain unchanged. By supposing a similar revolution in figure 6, the points *S*, *m*, *T*, will also remain unchanged (26).

&c., to the circle *K'XKV*, or whose longitude is 10°, 20°, &c., respectively (15).

26. Circles parallel to the ecliptic are also easily traced upon
Fig. 6. the plane of projection. If *A* (*fig.* 6) be the pole of the equator, and *P* that of the ecliptic, *EF*, the diameter of one of these circles, will be represented on the plane of projection by *ST*, the centre of which will be the point *m*, the middle of *ST*, and *Sm* will be the radius of the circle to be projected, and *Cm* the distance of its centre from the centre of the primitive.

27. We have given in figure 13 the simplest construction of the sphere upon stereographic principles, but the projection the most used, especially for maps of the world, is that which supposes the eye in the equator, and alternately in the two poles of the first
Fig. 14. meridian. Let the primitive *FEBH* (*fig.* 14), represent the first meridian, the place of the eye being perpendicularly over the point 90. *FB* will be the projection of half of the equator, the points *E*, *H*, will be the poles of the earth, and the straight line joining these will represent the meridian, perpendicular to the first meridian or the meridian of 90°. The other meridians are drawn like the secondaries to the ecliptic figure 7, that is, by taking for radii the secants of their longitude, or the secants of their inclination to the first meridian, and for the distance of their centres, the tangents of these same angles (14, 15). The parallels of latitude have for their radii the tangents of their polar distances respectively, and for the distances of their centres the secants of these same distances (18). Thus 10 *a* 10 for instance is described with a radius equal to the tangent of 80°, and from a centre whose distance from the centre of the primitive is equal to the secant of 80°, and so of the others. This is called an *equatorial* projection.

28. The objection to these projections is that the lines of the sphere are not faithfully represented in the proportions which they actually bear to each other. Of the arcs *FE*, *F'E*, &c., each of which represents a quadrant, only the first is actually 90°.

Fig. 8. The circle described from *m* (*fig.* 8), as a centre with a radius equal to *mO*, will pass through the points *O* and *A*, which are diametrically opposite on the sphere. Thus the arc *OSA* is the projection or representative of an arc of 180°, and *OS*, *SA*, represent 90° each. But

$$OS = OmS = 90^\circ - COm = 90^\circ - \text{the inclination}\ (20).$$

In the same manner it may be shown that the arcs *F'E* (*fig.* 14), Fig. 14.
F''E, &c., which represent quadrants, are each equal to 90° — the longitude respectively, or to the complement of the longitude.

28. With respect to the parallels of latitude, if *EF* (*fig.* 10) Fig. 10.
be considered as a diameter of one of these circles, it will be seen from what has been demonstrated (21), that

$$ES = EmS = FmS = 90 - FCm = 90^\circ - FD.$$

Thus the quadrants 10 *a*, 20 *a'*, (*fig.* 14) of the parallels of latitude Fig. 14.
are each equal respectively to 90° — the polar distance, that is, equal to the latitude of the parallel.

29. It may be observed moreover, that all the lines which compose a hemisphere, taken together, are reduced one half, since a hemisphere, which is equal to two great circles (*Geom.* 536), is represented upon the surface of one. While the parts near the centre are the most contracted, they are at the same time those which, considered among themselves, most faithfully represent the corresponding portions of the sphere. In maps therefore of small extent, whether celestial or terrestrial, we can place the middle of a country or the middle of a constellation at the centre of the projection, and the representation will be sufficiently exact.

30. We have sometimes occasion to measure an arc on the plane of projection, that is, to know the number of degrees which it represents on the sphere. Let *BGD* (*fig.* 15) be the primi- Fig. 15.
tive, lying in the plane of projection *BDT*, *O* the place of the eye, *SIT* the projection of the great circle *FHE*. Since *KT* is in both the planes *BGD*, *BPDO*, it is their common intersection, and will consequently pass through the point *D*. In like manner, because *KI* is the common intersection of the planes *BGD*, *PGO*, it will pass through the point *G*. Hence, the points *H*, *E*, being projected into *I*, *T*, it is plain that the arc *HE* will have for its projection the similar arc *IT*. But since *PE*, *OD*, *PH* and *OG* are each quadrants, if *ED*, *HG*, which are common, be taken away, the remainder or side *PD* will be equal to *OE*, and *PG* to *OH*. And because the opposite angles *DPG*, *EOH*, included by these sides are equal, the base *DG* will be equal to *EH* (*Geom.* 480). Accordingly *HE*, having been shown to be similar to, or the measure of *IT*, its equal *DG* will also be the measure of *IT*.

Again, since *PF* is equal to *OB*, if we add to each of these *FB*, we shall have *PFB* equal *OBF*; and *OH* has been shown to be equal to *PG*. Consequently the two sides *PFB*, *PG*, are equal respectively to the two *OBF*, *OH*, and they contain equal angles. Therefore the third side *BG* is equal to the third side *FH*. But *BG* is that arc of the primitive comprehended between the straight lines drawn from the pole of *SI*, (the projection of *FH*) through its extremities. Accordingly *BG* is the measure of *SI*. Therefore *any projected arc of a great circle is measured by that arc of the primitive which is comprehended by two straight lines drawn from the pole of the given arc through its extremities.*

31. The above theorem furnishes us with a convenient method
Fig. 16. of measuring a projected angle. The angle *APF* (*fig.* 16), for example, formed by the intersection of two great circles *APO*, *FPE*, being measured on the sphere by the arc which joins the poles of these circles, we have only to refer this arc to the primitive by straight lines drawn from its pole through its extremities. Thus *K* being the pole of *APO*, and *k* the pole of *FPE*, *P* will be the pole of *Kk* (*Geom.* 467), and *LM*, intercepted on the primitive by the straight lines *PKL*, *PkM*, will be the measure of the arc on the sphere, of which *Kk* is the projection, that is, of the angle *APF* or *EPO*, the inclination of the given circles to each other. Therefore *an angle formed by the projections of two great circles has for its measure that arc of the primitive which is intercepted by straight lines drawn from the vertex of the given angle through the projected poles of the given circles.*

Other methods of representing a spherical surface upon a plane.

32. In stereographic projection the parts of the sphere, as we have remarked, are most contracted toward the centre of projection; whereas the reverse takes place with respect to orthographic projection (*fig.* 12.) Now in the former case the eye is supposed to be situated in the surface of the sphere, and in the latter at an infinite distance. It is evident therefore that an intermediate situation may be taken, that shall present the different parts of an entire hemisphere taken together more naturally. At the distance of three fourths of radius from the surface, in the plane of the equator for instance, the meridians would appear nearly equidistant, and the parallels of latitude also

nearly equidistant. But these lines would no longer be circular curves. It is moreover obvious that the parts of a spherical surface cannot be perfectly represented upon a plane in all their proportions and bearings. We may indeed divide the equator *FB* (*fig.* 14), and the meridian *EH* into equal parts, and draw the meridians and parallels through these points, instead of the points of unequal division actually employed in this figure. This manner of representing the sphere is exhibited in figure 17. It is called by the constructors of maps a *globular* projection. It shows the parts of the sphere more naturally and more nearly in their true proportions, but it is not strictly speaking a projection. Fig. 14.

33. There are other methods of representing portions of the earth, when they are of small extent, especially in latitude, that are still more just. Let such a portion be comprehended between the meridians *PEP'* (*fig.* 18), *PQP'*, *MN* and *RS* being the extreme parallels of latitude. From *I* and *K* the middle points of the arcs *MR*, *NS*, draw the tangents *IT*, *KT*, meeting the axis *P'P* in the point *T*. The arcs *MR*, *NS*, containing only a small number of degrees, do not sensibly differ from the tangents *IT*, *KT*, and the space *MRSN* may be considered as making a part of the surface of a right cone, which has its vertex in *T*. In order therefore to represent this space developed upon a plane, we take a radius equal to *TI*, and describe an arc *KI*† (*fig.* 19) equal to *KI* (*fig.* 18); and having drawn *TIM*, *TKN*, we set off on each side of *I* and *K*, *IM*, *IR*, and *KN*, *KS*, equal in length each to the arcs *IM*, *IR* of figure 18, or to their chords, which do not sensibly differ from the arcs. Then dividing *MR* and *NS* into as many equal parts as there are degrees in the difference of latitude, we describe through these points from the point *T*, as a centre, arcs representing the arcs of latitude. We divide also the arc *IK* into as many equal parts as there are degrees in the difference of longitude, and draw through these Fig. 18. Fig. 19. Fig. 18

† That the arcs may be equal in length, the number of degrees in *KI* (*fig.* 19) must be as much less than the number of degrees in *KI* (*fig.* 18), as the radius *TI* is greater than the radius *LI* (*Geom.* 288); that is, the number of degrees in *KI* (*fig.* 19) must be to the difference of longitude of the two meridians *P'EP*, *P'QP*, as *IL*, the radius of the middle parallel, is to *TI*.

points and the point T straight lines representing the meridians. This being done, the several places comprehended may be designed according to their latitude and longitude.

34. It will be perceived, that in this kind of construction, as in the preceding, all the meridians tend to meet in the same point. But in orthographic and stereographic projection, the points of the sphere are designed in perspective, and the degrees of the equator and those of the meridians are not represented by equal parts. In this the meridians are represented by straight lines, and the degrees of longitude are equal among themselves, and the degrees of latitude are also equal to each other, although different from the degrees of longitude, which diminish according as the latitude increases. This construction therefore has many advantages over those before given in particular cases. It is not however used at sea to represent the path described or to be described by a ship destined for a particular place. As a ship, sailing upon a given point of the compass, makes the same angle with each meridian that she passes, if these meridians are represented by lines that converge to a point, the route thus described will be indicated by a curve, which would render the operations for finding a ship's place too complicated.

35. To remove this difficulty, through the points I, K, let the
Fig. 19. straight lines AB (*fig.* 19), CD, be drawn parallel to the meridian GT, which passes through the middle of the parallel IK, and we shall have the space before denoted by $MNSR$, now represented by $ACDB$, in which all the parallels are equal to the mean parallel IK, and the meridians MR, NS, become the straight lines AB, CD, parallel to GT; and the point of meeting T being at an infinite distance, the arcs MN, IK, RS, become straight lines perpendicular to GT; we hence derive the following method of constructing a chart.

Having drawn a line QT at pleasure, to represent the meridian that passes through the middle of the chart to be constructed, we divide it into as many equal parts as there are degrees of latitude to be comprehended. Through the middle G, we draw the perpendicular IGK, which will represent the middle parallel, and in order to determine what must be the length of GI, GK, to answer to the required number of degrees of longitude, we must recollect that similar arcs are as their radii, and that consequently arcs of the same number of degrees, taken upon different parallels, are as the cosines of the latitudes of these parallels†. Accordingly with a radius CA (*fig.* 21) equal to the assumed magnitude of a degree of the meridian, which is also that of the equator, we describe the arc AB of a number of degrees equal to that contained in the middle latitude, and let fall upon CA the perpendicular BP, which will give CP, for the magnitude of each degree of the parallel. For in the right-angled triangle CBP we have Fig. 21.

$$CB \text{ or } CA : CP :: R : \sin CBP \text{ or } \cos BCP.$$

Now radius R is by construction equal to the cosine of the latitude of 0°. We apply, therefore, CP from G (*fig.* 20) toward I and toward K, as many times as there are degrees in half the extent which the chart is to have in longitude; then drawing through the points of division of QT lines parallel to IK, and through the divisions of IK lines parallel to QT, we shall have the parallels and meridians, by means of which it will be easy to note down the different places comprehended according to their latitude and longitude. Fig. 20.

36. Charts of this construction are more convenient than the preceding, and may be advantageously used for small distances, and especially between the tropics, where the meridians are nearly parallel. But they become less exact, according as the difference of latitude comprehended is greater, and according also as the middle latitude is greater. They give the degrees of the parallels too small on the one hand and too great on the other. To remedy this defect and retain at the same time the parallelism of the meridians, the following chart, called *Mercator's*, has been contrived.

37. This chart is, properly speaking, only a development of a cylinder, which may be supposed to circumscribe the globe, having

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† Since the circumferences of circles are as their radii, and corresponding parts, or arcs of the same number of degrees, are also as their radii, we shall have $QE : KI$ (*fig* 18) :: $CE : LI$. But CE is radius and LI is the sine of PI or cosine of EI the latitude; therefore QE is to KI as radius to the cosine of the latitude of KI. In the same manner it may be shown that QE is $K'I'$, the corresponding arc of any other parallel, as radius is to the cosine of the latitude of $K'I'$; hence the length of the arc KI is to the length of the arc $K'I'$, as the cosine of the latitude of the former is to the cosine of the latitude of the latter. Fig. 18.

its axis coinciding with that of the earth, and its diameter equal to that of the equator, its length being without limit. It is not therefore a projection, or such a view of the lines represented upon it as would be presented to the eye at any particular point. The object of the construction is simply to render the meridians parallel without changing the ratio of the parts of the meridian to the corresponding parts of the parallel. To effect this, instead of diminishing the length of the degrees of the parallels, according as the latitude increases, we make them throughout of the same magnitude, and equal to the degrees of the equator, which necessarily renders the meridians parallel. But we enlarge at the same time the degrees of any great circle, according to the distance of these degrees from the equator, that is, according to their latitude. Thus, since the magnitude of a degree taken upon any parallel, is to that of a degree taken upon the equator, or upon any great circle, as the cosine of the latitude is to radius (35), or as radius is to the secant of the latitude (*Trig.* 8), if we make the degree of each parallel equal to that of the equator, the degree of any great circle, a meridian, for example, at any given distance from the equator, must have for its value a degree of the equator, augmented in the ratio of radius to the secant of the latitude, that is, multiplied by the secant of the latitude and divided by radius.

In Mercator's chart, therefore, while the degrees of the parallels are all equal to those of the equator, the degrees of the meridian, or of the latitude, must be unequal, that is, they must increase as the latitude increases. We should fall into a mistake,

Fig. 18. however, if, in taking *MN* (*fig.* 18), *RS*, as portions of two parallels, distant a degree from each other, we should conclude from what is said above, that the arc *NS* of a degree which measures the distance of these parallels, is to be expressed upon the chart by a line equal to a degree of the equator multiplied by the secant of the latitude divided by radius. It is very true that at *N*, a degree of a great circle is equal to $\frac{D \times \sec QN}{R}$, *D* being a degree of the equator; and for the same reason, at the point *S*, a degree must be equal to $\frac{D \times \sec QS}{R}$. But these quantities being different, neither the one nor the other can be taken as the measure of the distance between the two parallels *MN*, *RS*. The

former would be too small, and the latter too great. If, therefore, instead of supposing the two parallels distant from each other a degree, we consider them as separated only by an arc of one minute, we shall have for the length of one minute of the meridian at N, $\frac{M \sec QN}{R}$, M being a minute of the equator, and for the length of a minute in S $\frac{M \sec QS}{R}$, quantities that differ but very little from each other, and of which either may consequently be taken as the measure of a minute extending from N to S, or of the space that is to separate these two parallels on Mercator's chart.

38. We see therefore that in order to calculate the augmentations to be allowed to the parts of the meridian relatively to the augmentations given to the parallels in the construction of this chart, we must suppose the meridian divided into very small portions; then one of these portions, multiplied by the secant of the latitude, and divided by radius, will give the corresponding line on the chart, and with greater or less exactness according to the smallness of the portions into which the meridian is divided. We shall attain to all the accuracy necessary for practical purposes, if we divide the meridian into minutes. Thus in order to find the length to be allowed to the meridian intended to mark a particular latitude, it is sufficient to take from the common tables all the secants, from minute to minute, from 0° up to the latitude in question; and the sum of these secants divided by radius will give the number of minutes, which being applied from the equator in the direction of the meridian will determine the degree of the latitude required on the chart. The results thus obtained for the different parallels are called *meridional parts*. The relative lengths of degrees from the equator to the polar circle may be seen in figure 22†.

† There is some intimation of this method of representing the lines of latitude and longitude in the writings of Ptolemy; but the first chart constructed upon this plan was published by Gerard Mercator in 1566. The theory, however, does not appear to have been understood at this time. In the year 1599, Mr. Edward Wright published his *Correction of Errors in Navigation*, in which the principles of the chart are fully explained. Dr. Halley first showed that the arti-

Of Gnomonic Projection, or Dialling.

39. Dialling consists in drawing lines to represent the intersections of the planes of the meridians with any assumed plane or other surface. The time in which the sun apparently completes a revolution about the earth being divided into twenty four parts or hours, his angular motion will be at the rate of $\frac{360°}{24}$ or 15° in an
Fig. 23. hour. If we consider the axis of the earth *PP′* (*fig.* 23) opaque, the earth itself being supposed to be transparent, the shadow of *PP′* will coincide with the plane of the meridian opposite to the sun, and will move at the rate of 15° in an hour. Let *ZPRP′N* be the meridian of any place *Z*; the sun being in this plane at noon, the shadow of *PP′* will coincide with the plane *PRP′*, and will intersect the plane of the horizon *NOR* in *RC*. After one hour, that is, at one o'clock, the shadow will fall in the plane *PIP′*, and after two hours or at two o'clock, it will fall in the plane *PIIP′*, &c. Now these meridians being inclined to each other at angles of 15°, we shall have $RPI = 15°$, $RPII = 30°$, &c. Moreover *PR* is also given, and the angle *PRI* is a right. Whence in the spherical triangle *PRI*, right-angled at *R*, we have two parts given, *PR* and *RPI*, to find a third *RI*, which is the measure of

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ficial meridian line is a scale of logarithmic tangents of half the colatitudes beginning with radius.

Beside the charts above explained there is one in which the degrees of the parallels of latitude are all of the same length and equal to
Fig. 20. those of the meridian. If the divisions of *IK* (*fig.* 20) were made equal to those of *QT*, we should have in figure 20 such a representation of a portion of the earth's surface. This is called a *plane* chart. No allowance is made for the diminution in the length of the degrees of the parallels. Accordingly it can be used for spaces of small extent only at a time, or where there is very little convergence of the meridians. The methods of finding a ship's place &c., founded upon this chart, are comprehended in works on Navigation under the title of *Plain Sailing*; those which depend upon the chart represented in figure 20, as it is actually constructed, are included under the head of *Middle Latitude Sailing*; while those that are derived from the principles of Mercator's chart, fall under the denomination of *Mercator's Sailing*. Examples of each kind will be given in the next chapter.

the plane angle *RCI*, formed by the *XII* and *I* o'clock hour lines. Accordingly, *PR* being the middle part, and *RPI* (complement) and *RI* the adjacent parts, by the first of Napier's rules (*Trig. p.* 64) we have

$$\sin PR = \cot RPI \tan RI = \frac{1}{\tan RPI} \tan RI,$$

whence

$$\tan RI = \sin PR \tan RPI$$

In like manner $\tan RII = \sin PR \tan RPII$.

Therefore *the tangents of the plane angles* XII C I, XII C II, *&c., are equal respectively to the sine of the polar distance multiplied by the tangent of the horary angle.*

40. When *PR* is less than 90° we have *PR* = *EZ* = the latitude of the place *Z*, and the above expressions become

$$\tan RI = \sin \textit{lat.} \tan 15^\circ$$
$$\tan RII = \sin \textit{lat.} \tan 30.$$

If the latitude of the given place, that of Cambridge for instance, be 42° 23′ 28″, we have only to add the logarithmic sine of 42° 23′, 28″ taken from the tables, to the logarithmic tangent of 15°, 30°, &c., successively, and the sums will be the logarithmic tangents respectively of the arcs *RI*, *RII*, &c., or of the rectilineal angles *XII C I*, *XII C II*, &c.

We have considered the plane of the horizon as the *plane of the dial*, and the axis of the earth as the gnomon or *stile*. But if we take at *Z* a plane *n o r* parallel to *NOR*, that is parallel to the horizon, and *c p* parallel to *CP*, the shadows cast by *CP* and *c p* would, on account of the great distance of the sun, be sensibly parallel, and move at the same rate. Consequently the hour angles 12 *c* 1, 12 *c* 2, &c., may be considered as equal respectively to *XII C I*, *XII C II*, &c†. Therefore the formula above given will enable us to draw the hour lines *c* 1, *c* 2, &c. And to subdivide the hours into halves, quarters, &c., we have only to

† We may suppose *c p* equal in length to *CP*, and the plane *n o r* of the same dimensions as *NOR*, and the shadow would obviously revolve upon the one in the same manner as it revolves upon the other, and the hour angles formed at the centres, *C*, *c*, being the same upon both, these are not affected by the lengths of the lines that contain them; we may therefore take *c* 1, *c* 2, &c., as well as *c p*, of any convenient length to suit our purpose.

take the angle P equal to $\frac{15°}{2}, \frac{15°}{4}$, &c. We draw the lines on the other side of c 12 in the same manner, and number them 11, 10, &c. These are to be extended each way toward $c\,n$, so as to comprehend the time during which the sun is above the horizon on the longest day, according to the latitude of the place. $c\,p$ may be represented by a straight rod, or more conveniently, on account of its stability, by the edge of a triangular plate $p\,c$ 12, having its plane perpendicular to the plane of the dial, and its base c 12 resting upon the 12 o'clock hour line, the angle $p\,c$ 12 being equal to the latitude of the place. This is called a *horizontal* dial. Other dials take their name in like manner from the position of the assumed plane, on which the intersections of the planes of the meridians, or hour circles, are traced.

41. Let us suppose the plane NOR to shift it's position so as cut the planes of all the meridians at right angles, that is, to become the plane of the equator EQ. The formula above given will still be applicable; and since $\sin PR = PQ = \sin 90 = 1$, we shall have

$$\begin{aligned} \text{tang}\, RI &= \sin PR \,\text{tang}\, RPI \\ &= \text{tang}\, RPI = \text{tang}\, 15°; \end{aligned}$$

also $$\text{tang}\, RII = \text{tang}\, RPII = \text{tang}\, 30°,$$ &c.;

that is $$RI = 15°,\ RII = 30°,\ \&c.$$

Consequently RI, $I\,II$, &c., are all equal among themselves, and each equal to 15°, and the plane angles $XII\,C\,I$, $I\,C\,II$, &c., are also equal, and the shadow moves at a uniform rate, the stile being perpendicular to the plane of the dial. We may now, as in the former case, suppose a second dial at Z, constructed in the same manner, having its plane and stile, parallel respectively to those of the first, and the same time will be indicated upon both. In other words, we describe a circle of a convenient size upon a plane, divide it by radii drawn from the centre into portions of 15°, for an hour, 7° 30′ for half an hour, &c., insert a rod at the centre perpendicularly to the dial plane, and place the dial so that this rod, representing the axis of the earth, shall be situated like $c\,p$, that is, in the plane of the meridian, and making an angle with the horizon equal to the latitude of the place. This would be an *equinoctial* dial, since its plane is parallel to that of the equator. It would moreover be a horizontal dial at the pole, and a *vertical south* dial at the equator.

42. To construct a vertical south dial for any other latitude Z, let us suppose the plane NOR to change still further till it comes into a vertical position facing the south, as represented in figure 24. At noon the sun being on the meridian of the place, or in the plane of PEP', the shadow of PP' will intersect the plane ZOR in the straight line $CXII$; after one hour the shadow of PP' will be projected in the direction of PIP', intersecting ZOR in the straight line CI, and so on. Thus the hour angles RCI, $RCII$, &c., being angles at the centre of the circle ZOR, will be measured as before by the arcs RI, RII, &c., and these arcs are determined by the formula already employed; for in the spherical triangle PRI, right-angled at R, we have still the angle $RPI = 15°$, and

$$PR = PQ + QR = PQ + EZ = 90° + lat.$$

Whence, since $\sin PR = \sin P'R = \cos QR = \cos EZ$, we have

$$\begin{aligned} \text{tang } RI &= \sin PR \text{ tang } RPI \\ &= \sin P'R \text{ tang } RPI \\ &= \cos lat. \text{ tang } 15°. \end{aligned}$$

Putting for RPI, 30°, 45°, &c., we obtain by the same formula the other hour lines. Also, as in the former case, we may suppose at Z another dial $z\,o\,Z$ having its plane coinciding with ZOR, and provided with a rod $c\,p'$ parallel to CP'; and as the shadow of $c\,p'$ will revolve upon the plane $z\,o\,Z$ sensibly in the same manner and at the same rate as the shadow CP' revolves upon the plane ZOR, the hour angles $12\,c\,1$, $12\,c\,2$, &c., may be considered as equal to $XII\,C\,I$, $XII\,C\,II$, &c., and may be computed by the same formula.

43. It is evident that other planes might be assumed and the angles calculated in a similar manner. But the above will be found sufficient, especially, as clocks and watches have now taken the place of dials, and the latter are rendered of little use except to regulate the former, and afford occasionally an exercise to a speculative mind. Besides, the horizontal dial is the one that is commonly employed. It is the most convenient, because it requires the hour angles to be drawn only on one surface. In the vertical south dial, for instance, at the time of the equinoxes, or when the sun is in the plane of the equator, he rises and sets in the plane of ZOR and when his declination is toward P, the

northern face of the plane *ZOR* is illuminated after sunrise till the sun reaches this plane, and before sunset after passing this plane. So also in the equinoctial dial, its plane being represented by that of the equator, the southern face will be illuminated when the sun is on the south side of the equator, and the northern face when the sun is on the north side of the equator.

44. After the lines are drawn upon a plane intended for a horizontal dial, according to the formula above given, it may be placed in a horizontal position with sufficient accuracy by means of a spirit level, or by adjusting it in such a manner that water put upon it shall not incline more to one side than to another.

45. If the dial after being leveled be turned till the shadow of the stile falls upon the 12 o'clock hour line, when the sun's centre is on the meridian, or at the time of apparent noon by a well regulated clock or watch†, the adjustment of the dial is completed, and this is the most convenient way of determining a meridian line for a dial. But a more common method is to make use of two stars that are in the same horary circle or celestial meridian, and observe the position of a vertical plane that cuts them at the same instant. It is customary to take for this purpose the pole star and the first star in the tail of the Great Bear, called Alioth‡, which are on different sides of the pole almost

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† The error of the clock or watch, in this case, and its rate of going should be carefully ascertained by astronomical observations. The method of doing this will be shown in the chapter on the *Solution of Problems in Spherical Trigonometry.*

‡ The star Alioth or ϵ Ursæ Majoris may be recognized in the heavens from its situation in figure 25, relative to the more remarkable stars in its neighbourhood. The stars thus selected should have the same right ascension, or should differ in their right ascension 180°.

Right ascension of Alioth, beginning of 1820	191°	32′	42″
Pole star	14	12	21
Difference	177	20	21

If the right ascension of the pole star were 2° 39′ 39″ less, the two stars would be on the meridian at the same time. But as the pole star revolves around the pole in a circle of only about a degree and two thirds, an arc of this circle of 2° 39′ 39″ would be seen under an angle of not more than three or four minutes, which would be inconsiderable on a dial. The stars α Ophiuci and β Draconis differ diametrically opposite to each other, and which consequently pass the meridian of any place nearly at the same time. When therefore these two stars are cut by the vertical wire of a telescope†, adjusted to move in a plane perpendicular to the horizon, the axis of the telescope is in the plane of the meridian, and by inclining it toward such objects as are situated in this plane at a proper distance, we can note the points that are necessary for tracing a meridian line at any time.

When a telescope cannot be procured, two plumb lines may be used for this purpose, as represented in figure 26, being so disposed as to admit of the two stars being seen in their plane at the same moment; then the line *AB* joining their horizontal projections will be a meridian line.

46. We proceed in the construction of dials according to the principles of pure geometry, and upon suppositions that are not, strictly speaking, founded in truth. 1. The natural days, or times of the apparent revolution of the sun, not being perfectly equal, the apparent motion of the sun from one hour circle to another is not precisely uniform, but is alternately accelerated and retarded in the course of the year; so that a dial will, when compared with a good clock, be found to loose and gain alternately at the rate of from half a minute to a few seconds in a day, till the difference amounts at a maximum to a little more than a quarter of an hour. But this departure from a uniform measure of time admits of being calculated, and is generally entered upon the dial in a small table entitled "equation of time." To explain the cause of this departure and the method of estimating its amount at different times belongs to astronomy.

2. We suppose the apparent revolution of the sun to take

in right ascension only about 6′ (1820), and they are both stars of the second magnitude, and conveniently situated for an observation of this kind.

† The vertical telescope of a theodolite, hereafter to be described, is well adapted to this purpose. Also where the latitude of the place is not known; by taking with the theodolite the greatest and least altitude of the pole star, that is, the altitude when Alioth is in the same vertical circle below and above the pole, half the sum of these, corrected for refraction, will give the latitude sufficiently accurate for the construction of a dial.

place in circles parallel to the equator, whereas, on account of his continual change of declination, except at the solstices, these daily motions are performed in a kind of spiral curve that is constantly inclined more or less to the meridian, by which the forenoons fall short of the afternoons from the winter to the summer solstice, and exceed them during the rest of the year. But the error arising from this cause is not perceptible in the small arcs which measure the time near to noon, when the dial is most likely to be used.

3. We make no allowance for refraction, or parallax, or the circumstance of the shadow being determined by the sun's limb instead of his centre. Refraction at a mean elevates the sun when in the horizon, about a diameter, and when this takes place at right angles to the hour circles, as at the equator, or when its whole effect is to accelerate or retard the progress of the shadow on the dial, it amounts, at the rate of 15° to an hour, to about two minutes. But this effect diminishes, as the latitude increases, and becomes nothing at the pole. Besides, we seldom make use of a dial, while the sun is less than 8° or 10° above the horizon, when the refraction, and consequently its influence upon the time indicated by the dial, is reduced to a small part of what is above stated. The effect of parallax is altogether insensible†. With regard to the shadow of the stile on the plane of the dial, the extreme part of it or rather of the penumbra, corresponds, in the forenoon for example, to the eastern edge of the sun, and the extreme part of the perfect shadow to the western, and the middle of the penumbra will correspond to the

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† It is true that the plane of the shadow of cp (*fig.* 23) is inclined to that of the shadow of CP, and when the latitude of Z is 0°, the perpendicular distance of these planes is a maximum, and equal to the radius of the earth, and the inclination of the planes is equal to the sun's horizontal parallax (8″). Suppose a plane passing through cp strictly parallel to the shadow of CP, this plane would make an angle of 8″ with the shadow of cp. There being 1296000″ in 360°, 8″ would be $\frac{8}{1296000}$ of a circumference. If the circumference of the dial were 30 inches, for example, an arc of 8″ would be $\frac{8}{1296000}$ of 30 inches, that is $\frac{8}{43200}$ or $\frac{1}{5400}$ of an inch.

sun's centre. This defect therefore may be in a degree corrected; and the others do not exist even in theory at noon, and are, taken together, inconsiderable for some time before and after noon; so that the hours the most favourable to accuracy are those during which we have ordinarily most frequent occasion to make use of this instrument.

Dialling is considered as a projection of the sphere, because the lines *CXII* (*fig.* 23), *CI*, traced upon the assumed plane, are properly projections of the hour circles *PR*, *PI*, &c., or such representations of these circles as, referred to the assumed plane, would be presented to an eye situated at the centre *C*. While therefore in orthographic projection the eye is supposed at an infinite distance, and in stereographic projection at the surface of the sphere, in gnomonic projection the place of the eye is at the centre. Fig. 23.

CHAPTER II.

Of the Solution of Problems in Plane Trigonometry.

47. Rules have been investigated (*Trig.* 32, 35, 38) for the solution of all problems that occur in the calculation of lines near the surface of the earth, where plane triangles only are concerned. In these cases it will be recollected that of the six parts of a triangle three parts are required to be known; and of these one at least must be a side, since the same angles may be common to any number of triangles†.

Mensuration of Heights and Distances.

Fig. 27. 48. The distance *BC* (*fig.* 27), measured in a direct horizontal line from the bottom of a steeple, being 200 feet, and the angle of elevation Abc 47° 30′, the height *AC* of the steeple is easily found.

As the sin $bAc = \cos Abc = \cos 47° 30'$	9,82968
is to $bc = 200$ feet	2,30103
so is sin Abc 47° 30′	9,86763
	12,16866
	9,82968
to the height $Ac = 218, 26$	2,33898

We have here subtracted the logarithm of the first term from the sum of the logarithms of the second and third. But a shorter way would be to take the arithmetical complement (*Alg.* 248) of the first logarithm and add it to the two others; thus,

0,17032
2,30103
9,86763
2,33898

the same as before.

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† Of the manner of measuring the necessary angles and sides and of the instruments that are used for this purpose an account is given in a note subjoined to this part.

In the solution of this problem we have made use of the theorem, *the sines of the angles are to each other as the sides opposite to these angles.* We might also apply the rule given for right-angled triangles (*Trig.* 30), namely, *radius is to the tangent of one of the acute angles, as the side adjacent to this angle is to the side opposite;* thus,

As radius or sine of 90°	10,00000
is to *bc*	2,30103
so is tang *Abc* 47° 30′	10,03795
to the height *Ac* = 218,26	2,33898

if we add to this *bB* or *cC*, the height of the instrument, we shall have the whole height *AC*.

49. It is required to find the perpendicular height *AC* (*fig.* 28) of a hill, the angle of elevation of which *ABC* at the bottom is 46°, and the angle *ADC*, taken 100 yards further off on a level with the bottom 31°. Fig. 28.

Then $ABC - ADC = DAB$,

that is, $46° - 31° = 15°$,

Hence, as sin *DAB* = 15°	9,41300
is to *DB* = 100	2,00000
so is sin *D* = 31°	9,71184
to *AB*.	2,29884

Then,

as radius = sin 90°	10,00000
is to *AB*	2,29884
so is sin *ABC* = 46°	9,85693
to the height *AC* = 143,14	2,15577

50. It is required to find the perpendicular height of a cloud or other object, when its angles of elevation, as taken by two observers at the same time, on the same side of it, and in the same vertical plane, were 64° and 35°, their distance apart being half a mile, or 880 yards.

It is evident from figure 28, that this problem may be solved in the same manner as the last.

51. From the top of a tower *AC* (*fig.* 30) 120 feet high, the angles *CAB*, *CAD*, formed by the perpendicular wall and lines drawn to two objects *B*, *D*, situated in the same plane with *AC* being measured and found to be 33°, and 64° 30′, what is the distance of the two objects *B*, *D*? Fig. 30.

As radius or sin 90°	10,00000
is to $AC = 120$ feet	2,07918
so is tang $BaC = 33°$	9,81252
to $BC = 77{,}93$	1,89170

In like manner DC is found to be $= 251{,}58$. Hence

$$DC - BC = DB = 173{,}65 \text{ feet.}$$

Fig. 31. 52. Given BC (*fig.* 31) $= 100$ yards, the angle $B = 53°$, and the angle $C = 79°\ 12'$, to find the perpendicular distance AD.

Then $$A + B + C - (B + C) = A,$$

that is $$180 - (53° + 79°\ 12') = 47°\ 48' = BAC.$$

Hence,

As sin $BAC = 47°\ 48'$	9,86970
Arith. comp. . . .	0,13020
is to $BC = 100$	2,00000
so is sin $C = 79°\ 12'$	9,99224
to AB	2,12244

And

as sin D = radius	10,00000
is to AB	2,12244
so is sin $B = 53°$	9,90235
to $AD = 105{,}89$	2,02479

Fig. 32. 52. The height of an obelisk (*fig.* 32) standing on a hill being required, we measure from its bottom a distance $CD = 40$ feet, and take the angle $ACD = 41°$, and then measure another distance $CB = 60$ feet in the same direction, and take the angle $ABD = 23°\ 45'$.

Now, since the exterior angle of a triangle is equal to the sum of the interior opposite angles, we have

$$ACD - ABC = BAC,$$

that is $$41° - 23°\ 45' = 17°\ 15'.$$

Hence in the triangle BAC.

As sin $BAC = 17°\ 15'$	9,47209
Arith. comp. .	0,52791
is to the opposite $BC = 60$	1,77815
so is sin $ABC = 23°\ 45'$	9,60503
to the opposite side $AC = 81{,}49$	1,91109

In the triangle ACD.

As $CA + CD = 121,49$	2,08453
	7,91547
is to $CA \backsim CD = 41,49$	1,61792
so is tang $\frac{CAD + ADC}{2}$ = tang 69° 30′ . .	10,42726
to tang $\frac{CAD \backsim ADC}{2}$ = tang 42° 24′½	9,96065

And 69° 30′ — 42° 24′½ = 27° 5′½ = CAD. Lastly, in the same triangle ACD;

As sin CAD = 27° 5′½	9,65840
	0,34160
is to $CD = 40$	1,60206
so is sin ACD = 41°	9,81694
to $AD = 57,624$	1,76060

53. It is required to find the horizontal distance of an object AD (*fig.* 33) that is inaccessible. Angles of elevation being Fig. 33. taken at C and B, situated in a direct line from D, namely, ACD = 58°, and ABD = 32°, and the distance BC being found by measurement = 100 yards,

we have $ACD - ABC = BAC$,

or 58° — 32° = 26°.

Then in the triangle ABC

As sin BAC = 26°	9,64184
is to BC = 100	2,00000
so is sin ABC = 32°	9,72421
to AC	2,08237

And in the triangle ACD

As sin ADC = 90°	10,00000
is to AC	2,08237
so is sin of CAD or cos ACD = cos 58° . .	9,72421
to CD = 64,06 yards	1,80658

54. The height and distance of an object O (*fig.* 34) on the top Fig. 34 of a hill being required, we measure from an assumed station B a base BN of 642 yards up the sloping ground BC, directly from O, the points A, B, N, being in the same vertical plane;

then having set up a staff *BS* of a height equal to that of the theodolite *AN*, we take our station at *N*, and find the angle of elevation *OAH* of the object $O = 3° 59'$, and the angle of depression *HAS* of the point $S = 39'$, the angle of elevation *OPS* at *B* being $5° 52'$; *BR* and *OR* are to be found. We have

$OAH + HAS = OAS$, ASP [or $180 - HAS$] $- OSP = OSA$,

or

$3° 59' + 0° 39' = 4° 38'$, $179° 21' - 5° 52' = 173° 29'$.

Hence

$$180° - (OAS + OSA) = AOS$$
$$180° - 178° 7' = 1° 53'.$$

As sin $AOS = 1° 53'$	8,51673
	1,48327
is to $AS = 642$	2,80754
so is sin OAS 4° 38′	8,90730
to SO	3,19811
As radius or sin $SPO = 90°$	10,00000
is to SO	3,19811
so is sin $OSP = 5° 52'$	9,00951
to $OP = 161,3$	2,20762
As radius or sin 90°	10,00000
is to SO	3,19811
so is cos $OSP = 5° 52'$	9,99772
to $SP = 1569,75$	3,19583

The height of the instrument *SB* or *PR* being added, we shall have the height of the point *O* above the horizontal line *GR*.

55. Let it be required to determine the distance of an inaccessible object *O* (*fig.* 29) from a given point *A* or *B*, when an instrument for measuring angles cannot be procured. We take a point *a* in the direction *OA*, and a point *b* in the direction *OB*, and measure each side of the two triangles *A a B*, *AB b*, namely

Fig. 29.

$AB = 500$ yards

$a A = 100$ $B b = 100$

$a B = 560$ $A b = 550$

Then in the triangle *A aB* to find the angle *a AB* (*Trig.* 38), we add the three sides together, which gives 1160, and from half the sum 580, we subtract each of the sides *A a*, *AB*, successively, and the remainders are 480, and 80, whence

480 log.		2,68124
80 log.		1,90309
500 log.	Arith. comp.	7,30103
100 log.	Arith. comp.	8,00000
Sum		19,88536
half sum or log. sin $\frac{1}{2}$ *a AB*		9,94268

which answers in the tables 61° 12′ 20″; therefore

$$a\,AB = 122° \, 24' \, 40''.$$

This taken from 180° gives the angle $OAB = 57° \, 35' \, 20''$. In the same manner we find the angle $OBA = 64° \, 51'$. The sum of these 122° 26′ 20″, subtracted from 180°, gives the angle

$$O = 57° \, 33' \, 40''$$

from which the sides *AO*, *BO*, are easily found.

56. It will be perceived from the foregoing examples, that *where a right-angled triangle is employed, it is necessary to measure or have given only one of the acute angles and one side.* Thus, if I take a station *D* (*fig.* 30) directly opposite to the object *A*, whose perpendicular distance is required, having measured the side *BD* and the angle *B*, I can determine the side *AD* by one solution thus, Fig. 30.

as sin *BAD* or cos *B* : *BD* : : sin *B* : *AD*; or
as *R* : tang *B* : : *BD* : *AD*.

But *where the case requires an oblique-angled triangle, it is necessary to know either directly or indirectly two angles and a side, or two sides and an angle.* It is common in questions of this kind to measure, besides a side which is always an element, one angle in the triangle containing the side or sides to be determined, together with one of the opposite exterior angles, then the difference of these will be the other interior angle, as in articles 49, 52. An exterior angle is often found by adding the observed angle of elevation or depression to 90°. Thus *HBS* + 90° (*fig.* 35) is equal to the exterior angle of the triangle *SBR*, and *BRS* being subtracted from it we shall have the other interior opposite angle *BSR*. The angle *AGB* (*fig.* 38) is found in a similar manner. Fig. 35. Fig. 38.

In questions respecting heights and distances, although we employ for the most part either vertical or horizontal angles, yet it is obvious that the theorems of trigonometry are equally applicable to triangles whose planes are inclined to the horizon. In figure 40 for instance, having the angles OAC, OCA, and the side AC, we proceed in finding the remaining sides, as if it were horizontal or vertical, that is, without having any regard to the position of its plane. We thus obtain a side of each of the vertical triangles AOB, COB. Care should be taken however in measuring an inclined angle that the plane of the instrument have the same inclination.

What precedes being well understood, the answers to the following questions will be readily found by means of the figures referred to.

Fig. 35. 57. It is required to find the height of the castle BR (*fig.* 35), above the level of the sea AS, and its horizontal distance from a ship S at anchor, the angles of depression HBS, NRS, being given equal respectively to $4° 52'$, and $4° 2'$, the height of the castle being 54 feet.

Answer, $AS = 3690$ feet, and $AB = 314$ feet.

58. It is proposed to find the distance between two objects
Fig. 36. A, B (*fig.* 36), situated on a level, when they cannot be conveniently approached. From two stations C, D. also on a level, we take the angles $ACB = 37°$, $BCD = 58° 20'$, $CDA = 53° 30$, $ADB = 45° 15'$, and measure the distance $CD = 300$ yards.

Answer, $AB = 479, 79$ yards.

Fig. 37. 59. If both the objects A, B (*fig.* 37), can be seen only from one point D, we take a station C where A can be seen, and a station E where B can be seen, and measure $CD = 200$ yards, the angle $ADC = 89°$, $ACD = 50° 30'$; also $DE = 200$ yards, and the angle $BDE = 54° 30'$, $BED = 88° 30'$, the angle ADB being $72° 30'$.

Answer, $AB = 345, 5$.

Fig. 38. 60. From a window A (*fig.* 38) near the bottom of a house supposed to be on a level with the bottom of a church GD, the angle of elevation GAD of the top of the spire being $40°$, and from another window B 18 feet directly above the former, the angle of elevation GBE being $37° 30'$, it is required to find the height of the object GD and its distance AD.

Answer, $GD = 210, 44$ feet, and $AD = 250, 79$ feet.

61. **Being at the station A (*fig.* 39), on a horizontal plane, and wanting to know the height of a tower CD, placed on the top of a hill, we take the angle of elevation DAE of the top of the hill equal to 40°, and of the top of the tower CAE equal to 51°; then measuring in a direct line from the hill 100 yards to B, we take the angle CBE equal to 33° 45′.** Fig. 39

Answer, $CD = 46, 67$ yards.

62. Suppose the object OB (*fig* 40) to stand upon a horizontal plane ABC, and that AC is equal to 250 yards, and that the angles at its extremities are known, namely, $OAC = 56° 46'$, $OCA = 62° 54'$, $OAB = 6° 40'$, $OCB = 7° 6'$. What is the height OB, and the horizontal distances AB, CB? Fig. 40.

Answer, $AB = 254, 989$ yards
$CB = 238, 814$
$OB = 29, 745$.

Navigation.

63. The situation of places on the earth being designated by their latitude and longitude, that is, by their angular distance from the equator, and from some assumed meridian, reckoned on the equator, the situation of a ship with respect to the places she has left, and those to which she is going, is known, when we know her latitude and longitude. The method of finding these that first suggests itself is by astronomical observations. But as this method cannot always be resorted to, and is moreover, particularly in the case of the longitude, attended with labor and difficulty, it becomes an important problem to find the change of latitude and longitude corresponding to any given distance that a ship sails in any given direction†; for, when this

† The *distance*, or the length of the path described by a ship, is estimated by observing the velocity of the ship at stated intervals. The instrument used for this purpose is called the *log*. The direction of the ship's path, or the angle which it makes with the meridian is ascertained by the *compass*, and is technically called the *course*. It will be perceived that, from the unsteadiness of the wind and other causes, neither the distance nor course admits of the accuracy with which lines and angles are measured on land. See note on the description of the compass and log.

is known, by applying it to the latitude and longitude of the place of departure, we learn the latitude and longitude of the place at which the ship has arrived. If the course of the ship be due north or south, she does not change her longitude, and the whole distance in nautical miles is to be considered as so many minutes of a degree, and to be subtracted from the latitude of the place of departure, or added to it, according as her course was to or from the equator. If on the contrary the course of the ship were on a parallel of latitude, the whole distance is to be regarded as a change of longitude, which will be greater or less according to the latitude of the parallel. To reduce any arc of a parallel to degrees or parts of a degree, we find the corresponding arc of the equator by the proportion, cosine of the latitude is to radius, as any portion of a parallel is to that portion of the equator which is comprehended between the same meridians (35). Then, calling the miles minutes, as in the case of an arc of latitude, we have the change of longitude, which being added to the longitude of the place of departure, or subtracted from it, according as the direction has been from or toward the first meridian, we shall have the longitude of the place at which the ship has arrived.

64. But, as a ship sails for the most part neither upon a meridian, nor upon a parallel of latitude, but in a direction oblique to these, she is to be considered as changing both her latitude
Fig. 41. and longitude at the same time. If *AB* (*fig.* 41) represent the line described by a ship, *NS* being the meridian of the place from which she sets out, the angle *CAB* will be the course, *AB* the distance, *AC* her change of latitude, called *difference of latitude*, and *CB* the arc of a parallel of latitude intercepted between the meridian of the place *A* and that of the place *B*, and which corresponds to an arc of the equator that measures the difference of longitude. This is called the *departure*. Hence if we know the course or angle *A* by the compass, and the distance *AB* by the log, *AC* and *CB* are easily calculated. Suppose $AB = 80$, and $CAB =$ two points, or 22° 30′. Then

As radius	10,00000
is to the distance $AB = 80$	1,90309
so is $\sin B = \cos A = \cos 22° 30'$	9,96562
to difference of lat. $AC = 73,91$	1,86871

As radius	. . . .	10,00000
is to the distance $AB = 80$	. . . .	1,90309
so is sin A..*course*...22° 30′	. . . .	9,58284
to departure $CB = 30{,}61$	. . . .	1,48593

In like manner, if in addition to the right angle C, any two parts, of which one is a side, be known, the others are readily found. The latitude AC, for instance, being given $= 73{,}91$ together with the distance AB, the course is determined by the following proportion;

As the distance $AB = 80$	. . . .	1,90309
		8,09691
is to radius	. . . .	10,00000
so is diff. lat. $AC = 73{,}91$	. . . .	1,86871
to sin B = cos A . . cos *course* . . 22° 30′.	. . .	9,96562

65. The above method is not confined to portions of the earth's surface, which, without sensible error, may be considered as planes, but is applicable to routes of any extent, in which the course remains unchanged. The path described by the ship becomes in this case a curved line, since it makes with each successive meridian the same angle. Let A (*fig.* 42), B, be two Fig. 42. points, upon two contiguous meridians. Let AB, BR, be drawn to the northeast, or other point of the compass, to denote the course of the ship at these points. The angles BAP, RBP, are equal. Now the arcs BP, AP, are not parallel; on the contrary they converge and meet at P; therefore the angle PBQ is greater than PAQ, and consequently greater than PBR. Therefore, since BR makes the same angle with the meridian that AB makes, the parts AB, BR, of the path described by the ship are not in the same straight line nor in the same plane†.

66. Let AB (*fig.* 43) be any part of a rhumb line, PBN, Fig. 43. PAM, the extreme meridians, NM the equator, PCK, PEL, two

† A line drawn in such a manner as to make the same oblique angle with each successive meridian is not an arc of a great circle, or of any other circle, but is a peculiar kind of curve that approaches the pole continually in a sort of spiral, without ever reaching it. It is called a *rhumb line* or *loxodromic curve.*

contiguous meridians cutting *AB* in the two points *C*, *E*. If from the pole *P* we draw the arcs *BS*, *CD*, parallel to the equator, it is evident that, if *AB* be the path described by a ship, *AS* will represent the corresponding difference of latitude, and *MN* the difference of longitude, made in passing from *A* to *B*. Accordingly, if *EC* denote the space passed over in a moment on the rhumb line *AB*, *DE* will represent the change of latitude, and *LK* the change of longitude in the same time. Now, as the triangle *CDE*, right-angled at *D*, is indefinitely small, it may be regarded as a plane triangle. If then we suppose the line *AB* to be made up of parts equal respectively to *CE*, and each of these parts to be the hypothenuse of a right angled triangle like *CDE*, it is manifest that these triangles will be equal among themselves, since, beside the right-angle and hypothenuse being the same in all, they have each an angle equal to *CED*. It is readily inferred therefore, that the sum of all the hypothenuses *CE*, or the whole line *AB* is to the sum of all the sides *DE*, or the whole change of latitude, as one hypothenuse *CE* is to the corresponding difference of latitude *DE* (*Geom.* iv). Now since *CDE* is a plane triangle, it is similar to any other plane triangle having the same angles. Accordingly, if we construct a right-angled plane triangle *HGI*, having the angle *G* equal *CED*, this triangle will be similar to the triangle *CDE* (*Geom.* 74), and we shall have

$$GH : GI :: EC : ED.$$

But $$EC : ED :: AB : AS;$$

whence $$GH : GI :: AB : AS.$$

Consequently, if we make *GH* equal to the length of the route, or distance *AB* (*Geom.* 142), *GI* will be the corresponding difference of latitude.

Therefore, notwithstanding the path described by a ship is a curved line, if we construct a right-angled plane triangle having the hypothenuse equal to the distance, and one of the acute angles equal to the course, the side adjacent to this angle will be equal to the difference of latitude.

67. It remains to determine the change of place east or west, answering to any given course and distance. It is evident that this is *CD* for the indefinitely small route *EC*. Now, if we suppose, as above, triangles equal to *CED*, corresponding to the several parts of *AB*, it will be seen, as in the former case, that

the sum of all the hypothenuses *EC*, or the whole distance *AB*, is to the sum of all the sides *CD*, or the whole change of place east or west, as *EC* is to *CD*; or, on account of the similarity of the triangles *CED*, *HGI*, as *GH* is to *HI*.

Accordingly, if we make a right-angled plane triangle having the hypothenuse equal to the distance, and one of the oblique angles equal to the course, the side opposite to this angle will be equal to the departure or the change of place east or west.

68. The two propositions above given hold true, whatever be the distance to be reduced. Although this distance and change of place east or west be curved lines, it is not less rigorously exact to represent them by the sides of a right-angled plane triangle. But we should fall into a mistake if, finding that *GI* is equal to *AS*, we should conclude that *HI* is equal *BS*. *HI* indeed is equal to the sum of all the small arces *CD*, which sum is greater than *BS*, since *CD* is greater than *OQ*. Moreover, if from the pole *P* we describe the arc *AR*, it will be perceived that *HI*, or the sum of the small arcs *CD*, is less than *AR*.

69. When we have determined the change of place north or south, it is easy thence to deduce the difference of latitude; for, since this change takes place in a great circle, we have only to consider every sixty nautical miles as so many degrees, and to call the remainder minutes.

70. As to the difference of longitude we cannot so readily deduce this from the change of place east or west; for, as we have just seen, the sum of the small arcs *CD* is greater than *BS* and less than *AR*. If we knew the latitude of an arc precisely equal to the sum of all the arcs *CD*, it would be easy thence to determine the arc *MN* the difference of longitude (35). Indeed this sum of the arcs *CD* does not materially differ from *TV*, the middle parallel between *AR* and *BS*, except when the distance *AB* is great, or occurs in high latitudes†. When, therefore, we have found the difference of latitude *GI*, if we add the half of this to the less latitude *AM*, the sum will be equal to *MT*, the

† If *AB*, the distance to be reduced, be 600 miles, the greatest error of the above assumption would amount to a little more $1'\frac{1}{4}$ near the parallel of 45°, it would be 4′ towards the parallel of 60°, and $32\frac{2}{3}$ near the parallel of 75°, and these errors vary according to the cube of the distance.

latitude of the middle parallel; we then obtain the difference of longitude by the following proportion; as the cosine of the latitude *MT* of the middle parallel is to radius, so is *TV*, equal nearly to *HI*, the sum of the arcs *CD*, to the number of miles or minutes in the arc *MN*. In other words, we construct a right-angled triangle *BCD* (*fig.* 44), having the angle *CBD* equal to the latitude of the middle parallel, and the side *BC* adjacent to this angle equal to the number of miles in *HI* (*Geom.* 142), and the hypothenuse *BD* is the value of the arc *MN*; for, by the common theorem of trigonometry,

Fig. 44.

$$\sin BDC \text{ or } \cos CBD : R :: BC : BD,$$

and, by what has been shown above,

$$\cos \textit{mid. lat.} : R :: \textit{dep.} : \textit{diff. long.};$$

therefore *BD* is the difference of longitude in miles or minutes.

71. But the difference of longitude may be obtained more correctly and in a manner adapted to all distances and latitudes. By what has already been proved (35,37)

Fig. 43.

$$CD : LK :: \cos LD : R \quad (\textit{fig.}\ 43)$$
$$:: \quad R \quad : \sec LD.$$

Moreover the right angled-triangle *CED* gives

$$ED : CD :: R : \text{tang } CED \ (\textit{Trig.}\ 30);$$

and by taking the product of the corresponding terms, we have

$$ED : LK :: R^2 : \sec LD \text{ tang } CED;$$

whence

$$LK = \frac{ED \times \sec LD \text{ tang } CED}{R^2}$$

$$= \frac{ED \sec LD}{R} \times \frac{\text{tang } CED}{R}.$$

But $\frac{ED \sec LD}{R}$ expresses the magnitude to be allowed to the parts *ED* of the meridian on Mercator's chart (37); by adopting the same reasoning for the arcs *CD* corresponding to the different parts of *AB*, we arrive at the conclusion, that the sum of all the arcs *LK*, or *MN*, is equal to the sum of all the meridional parts of the difference of latitude *AS* multiplied by $\frac{\text{tang } CED}{R}$, that is, multiplied by the ratio of the tangent of the course to radius. We hence derive this simple rule for finding the difference of longitude. *Radius is to the tangent of the course* *as the difference of latitude on Mercator's chart* † *is to the difference of longitude.*

72. If we construct a right-angled triangle $GK'L'$ having the angle G equal to the course, and the side $G'K'$ equal to the difference of latitude on Mercator's chart, or the meridional difference of latitude, the side $K'L'$ will be the difference of longitude; for by trigonometry,

$$R : \text{tang } K'GL' :: GK' : K'L',$$

and, by what is above shown,

$$R : \text{tang } course :: merid.\ diff.\ lat. : diff.\ long.$$

Therefore $K'L'$ is the difference of longitude. This being added to the longitude of the place of departure, or subtracted from it, according as the route has been from or toward the first meridian, will give the longitude of the place at which the ship has arrived.

73. Moreover from the similarity of the triangles GIH, $GK'L'$, we have the proportion

$$GI : IH :: GK' : K'L',$$

that is,

$$diff.\ lat. : departure :: merid.\ diff.\ lat. : diff.\ long.$$

74. Let now the following question be proposed. A ship in latitude 47° 23′ N. and longitude 10° 17′ W. from Greenwich sails 126 miles in a direction S. W. by W. that is, making an angle with the meridian of 56° 15′; what is the latitude and longitude of the place at which she arrives?

We in the first place construct a triangle ABC (*fig.* 45), having the angle A equal to the course or 56° 15′, and the side AB equal to the distance 126; BC will then represent the departure, and AC the difference of latitude. These are found as before (64), in the following manner, Fig. 45.

As sin. C = radius	10,00000
is to AB = 126 miles	2,10037
so is sin A . . 56° 15′	9,91985
to BC = departure = 104 miles	2,02022

† This difference of latitude on Mercator's chart is called *meridional difference of latitude.* It should be observed that, if the two extreme latitudes A, B, be one north and the other south, the *sum* of the latitudes on Mercator's chart is to be taken instead of the *difference.*

As the sine of C = radius	10,00000
is to AB = 126	2,10037
so is sin ABC = cos A . . 56° 15′	9.74474
to AC = diff. lat. = 70 miles = 1° 10′	1,84511

Subtracting 1° 10′ from 47° 23′ we have 46° 13′ for the latitude of the place at which the ship arrives; and by adding half of 1° 10′, or 35′, to the less latitude 46° 13′, we have 46° 48′ for the middle latitude between the two places A and B.

We next draw BD making the angle CBD equal to 46° 48′, the middle latitude. The line BD will then represent the difference of longitude, and is found thus,

As the sin D = cos CBD = cos 46° 48′ . . .	9,83540
	0,16460
is to the departure = BC = 104 miles . . .	2,02022
so is sin C = radius	10.00000
to BD the diff. of long. = 153 = 2° 33′ . . .	2,18482

As the direction in which the ship sailed is from the first meridian, this is to be added to 10° 17′, the longitude of the place of departure, which gives 12° 50′ W. for the longitude of the place at which the ship arrives.

75. The two right-angled triangles ABC, CBD, may be considered as forming one oblique-angled triangle in which AB is the distance, A the course, BD the difference of longitude, and D the complement of the middle latitude. Consequently, any three of these being given, the fourth may be immediately found; thus, in the above example, omitting the proportion for the departure, we might proceed directly to find the difference of latitude, and thence the middle latitude, the complement of which would be the angle D. Knowing, therefore, the angles A and D, and the side AB, we obtain the difference of longitude by the following proportion:

As sin D = cos CBD . . . *mid. lat.* . . . 46° 48′ .	9,83540
	0,16460
is to AB = distance = 126	2,10037
so is sin A . . . course . . . 56° 15′	9,91985
to BD = diff. long. = 153	2,18482

as before.

It will be perceived that in the right-angled triangles *ABC*, *BCD*, any two of the parts of which we have been speaking being known in one, and one part in the other, the right angle excepted, the rest may be found.

76. To perform the above question by the principles of Mercator's chart, we take *AD* (*fig.* 46), equal to the meridional Fig. 46.
difference of latitude†; we then draw *DE* parallel to *BC* and meeting *AB* produced in *E*. *DE* will be the difference of longitude, and is thus found,

As radius	10,00000
is to tang *A* . . . course . . . . 56° 15′ . . .	10,17511
so is *AD* = merid. diff. lat. = 103	2,01284
to *DE* diff. of long. = 154 = 2° 34′	2,18795

When the departure and difference of latitude are known, and not the course, we have the proportion (73),

As *AC* = proper diff. lat. = 70	1,84511
	8,15489
is to *BC* = departure = 104	2,02022
so is *AD* = merid. diff. lat. = 103	2,01284
to *DE* = diff. long. = 2° 34′	2,18795

77. It is scarcely necessary to observe, that in this case, as in that of former solutions, any two parts besides the right angle (provided these two be not the other two angles) being given, the remaining parts may be found; thus, let the latitude and longitude of any two places, either at sea or on land, be known, and their bearing and distance are readily determined. Suppose the two places to be, for instance, the southernmost point of land in England called the Lizard, in latitude 49° 57′ N. longitude 5° 15′ W. and the island St. Mary, one of the Azores in latitude 36° 57′ N. longitude 25° 9′ W.

Lat. Lizard	49° 57′ N.	mer. parts 3470	long. 5° 15′ W.
Lat. St. Mary	36 57 N.	mer. parts 2389	long. 25 9 W.
Diff. lat.	13 0 = 780	mer. diff. lat. 1081	diff. long. 19 54 = 1194

† The meridional difference of latitude may be taken from a table of meridional parts, calculated in the manner described in article 38. See table of *Meridional Parts* at the end of this volume. Or it may be found by means of the line marked *Mer.* on Gunter's Scale.

Fig. 46. Construct the triangle *ADE* (*fig.* 46) making *AD* = 1081, the meridional difference of latitude, and *DE* perpendicular to *AD* = 1194, the difference of longitude, and join *AB*. The angle *DAE* will be the course, or bearing of St. Mary from the Lizard, and is found thus,

As *AD* = 1081	3,03383
	6,96617
is to *DE* = 1194	3,07700
so is radius	10,00000
to the tang *DAE* = bearing . . 47° 51′ . .	10,04317

If now from *A* we set off *AC* = 780, the proper difference of latitude, and through *C* draw *CB* parallel to *DE*, we shall have *AB* equal to the distance of the above places from each other, by the proportion (*Trig.* 30).

As radius	10,00000
is to the sec *course* . . . *A* . . . 47° 15′ . . .	10,17323
so is the diff. lat. = *AC* = 780	2,89209
to the distance = *AB* = 1162	3,06532

78. In resolving the several problems of this chapter we have given only the method by logarithms. Where great accuracy is not required, the operation may be performed very expeditiously by means of the lines on Gunter's Scale. Similar results also might be obtained by geometrical construction. When the given parts of a triangle are such as admit of the others being calculated by the rules of trigonometry, the triangle may be constructed (*Geom.* 141, &c.), and the part sought, if it be a side, is found by taking it in the compasses and applying it to the scale of equal parts, used for the given side or sides. If the required part be an angle, the degrees and minutes contained in it are determined by either of the rules for measuring an angle.
Fig. 46. Thus, in the triangle *ADE* (*fig.* 46), constructed as above described, the angle *A* is found by taking from the scale of chords the chord of 60°, or radius, and describing from the centre *A* an arc, meeting the two sides *AE*, *AD*, produced if necessary, and then applying the chord of the contained arc to the same scale of chords (*Geom.* 136).

79. The foregoing principles and examples being well understood, the learner will be able without difficulty to solve the following questions.

1. A ship in latitude 16° 35′ S. sailed N. E. ¼ N. 540 miles. Required the departure thus made, and the latitude of the place at which she arrives.

Ans. Departure 342 miles, latitude 9° 38′ S.

2. The difference of latitude between two points is 441 miles, and the course S. W. by W. Required their distance asunder, and the departure.

Ans. Distance 793,8 mls. departure 660.

3. A ship from Cape Clear, in latitude 51° 18′ N. and longitude 11° 15′ W. sailed S. E. ¼ S. 480 miles. Required the latitude and longitude of the place at which she arrives.

Ans. Latitude 45° 22′ N. longitude 8° 9′ W.

4. A ship from Bayonne, in latitude 43° 29′ N. and longitude 1° 30′ W. sailed N. W. ½ N. and by observation is found to be in latitude 51° 31′ N. Required the distance sailed, and the longitude of the place at which she arrives.

Ans. Distance 623,5 mls. longitude 11° 17′ W.

5. It is required to find the bearing and distance from Land's End to the island of Bermudas, the latitude of the former being 50° 06′ N. and the longitude 6° 00′ W. and the latitude of the latter place 31° 20′ N. and the longitude 64° 48′ W.

Ans. Bearing S. 66° 55′ W. distance 2872 mls.

6. A ship from Conception in latitude 36° 43′ S. and longitude 72° 40′ W. sailed upon a single course between the north and west till she was found by observation to be in latitude 29° 38′ S. having made 324 miles departure. Required her course, distance, and longitude.

Ans. Course 37° 19′,
Distance 534 miles,
Longitude 79° 07′ W.

Beside the problems in navigation now resolved, there are others of the greatest importance, which depend upon the principles of spherical trigonometry. These will be found in the next chapter.

Miscellaneous Questions to be solved by the rules of Plane Trigonometry.

Fig. 47. 80. 1. Let A (*fig.* 47), C, be two stations on a sloping ground, distant 410 yards, O an object on the top of a hill; the following angles being found by observation, namely, $OCA = 79° 29'$, $OAC = 63° 11'$, and the angles of elevation at A and C 6° 36′ and 5° 22′ respectively, it is required to calculate the height and distance of the object from each station.

Ans. $AG = 660{,}302$, $CB = 600{,}728$.
$OB = 56{,}431$, $OG = 76{,}4$.

2. It is required to find the distance at which the Peak of
Fig. 48. Teneriffe may be seen at sea, its height AB (*fig.* 48) being $2\frac{1}{3}$ statute miles, and the diameter of the earth BE 7916.

Ans. 135 miles†.

3. The top of a mountain being seen in the horizon at the distance of 154 miles, what is the height of the mountain?

Ans. 3 miles†.

4. The bearing of Boston light-house from Harvard Hall in Cambridge, being S. 74° 29′ E. and the distance $12\frac{1}{2}$ miles, required the latitude and longitude of the light-house, the latitude of the above building in Cambridge being 42° 23′ 28″ N. and the longitude 71° 07′ 25″ W. from Greenwich.

Fig. 49. 5. From a ship under sail, an island (*fig.* 49) is observed to bear N. $22\frac{1}{2}°$ E. and after proceeding N. $67\frac{1}{2}°$ W. 20 miles, the bearing of the same island is found to be N. $56\frac{1}{4}°$ E. The distance from each place of observation is required.

Ans. AC 29,93 miles and BC 36 miles.

6. It is required to find the distance and bearing of the city of Washington from Boston, the latitude of the former place being 38° 58′ N. and longitude 77° 2′ W., and the latitude of the latter 42° 22′ N. and the longitude 71° 4′.

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† No allowance is made in these solutions for terrestrial refraction.

CHAPTER III.

Of the Solution of Problems in Spherical Trigonometry.

81. The learner has already been made acquainted with the rules necessary for the solution of the cases that occur in *Nautical Astronomy* (61). The principles also upon which the lines of the celestial sphere are represented have been taught in the chapter on projections. More particular details relating to the construction of spherical triangles may be found in the note on the *Description and Use of the Scale*†.

82. Given the sun's declination equal, for example, to 12° 12′ N. to find his longitude and right ascension, the obliquity of the ecliptic being 23° 27′ 57″.

The solstitial colure *NESQ* (*fig.* 50), being assumed as the primitive, the equator and the ecliptic will have their poles each in the circumference of the primitive, and, being great circles, will be represented by the straight lines *EQ*, ♋♑ passing through the centre of the primitive, and making the angle *A*♈☉ equal to 23° 27′ 57″, the obliquity of the ecliptic (18). Draw the small circle *m n* at the distance of 12° 12′ from *EQ*, or 77° 48′ from *N*, the pole of *EQ* (18), and through ☉, the intersection of ♋♑ and *m n*, draw the celestial meridian *N*☉*S* (*fig.* 14, 15). In the triangle *A*☉♈, right-angled at *A*, we shall have *A*♈☉ equal to 23° 27′ 57,″ the obliquity of the ecliptic, and *A*☉ equal to 12° 12′, the sun's declination, to find ♈☉, the sun's longitude or distance from Aries reckoned on the ecliptic, and ♈*A*, the sun's right ascension or distance from Aries reckoned on the equator. Fig. 50. Fig. 14, 15.

To compute the first of these quantities, the three circular parts in question are *A*☉, co♈☉, co*A*♈☉ (*Trig. page* 63). *A*☉ being separated from the other two, this is the middle part, and the other two opposite parts; whence

† It is supposed in what follows of this chapter, that the student has some knowledge of astronomy, and particularly of the technical language of the science.

R sin *A*⊙ = cos (co ♈⊙) cos (co *A*♈⊙) = sin ♈⊙ sin *A*♈⊙, which gives the proportion†

sin *A*♈⊙ . . obliquity . . . . 23° 27′ 57″	. .	9,60010
Arith. comp.	. . .	0,39990
is to *R*	. .	10,00000
as sin *A*⊙ . . declination . . 12° 12′	. . .	9,32495
is to sin ♈⊙ . . longitude . . 32° 03′ 09″	. .	9,72485

83. To find the right ascension, ♈*A* the three circular parts are co ⊙♈*A*, ♈*A*, and *A*⊙, of which neither being separated from the other two, since the right angle is not considered as disjoining those between which it is placed, ♈*A* is the middle part, and the other two are adjacent parts; whence

R sin ♈*A* = tang (co ⊙♈*A*) tang *A*⊙ = cot ⊙♈*A* tang *A*⊙;

and to find ♈*A* we have the proportion

R	.	10,00000
is to cot ⊙♈*A* . . obliquity . . 23° 27′ 57″	.	10,36241
as tang *A*⊙ . . declination . . . 12° 12′.	. .	9,33487
is to sin ♈*A* . . right ascension 29° 52′ 20″	.	9,69728

If this be converted into time at the rate of 15° to an hour, we shall have for the right ascension in time 1^h 59′ 29″.

84. It will be observed that as the sun's declination is the same four times in the year, namely, at equal distances each way from the equinoctial points, the same declination answers to four different situations of the sun, as to his longitude and right ascension. When the declination is given, therefore, it is necessary to know the time of the year or the part of the ecliptic, at which it takes place, in order thence to determine the sun's longitude and right ascension. Thus, while the sun is moving from ♈ to ♋, or is in the first quadrant of the ecliptic, the longitude is ♈⊙, and the right ascension ♈*A*, as above given. When, however, the sun has passed the solstice ♋, and is descending toward ♎, the computation having reference to the triangle *A*♎⊙, gives ♎⊙, ♎*A*, the distance of the sun from ♎,

† It will be observed that the proportion must begin always with that term of the equation which is multipled by the term sought. The three first terms then become known terms, and the last the term required.

the supplements of which, or what they want of 180°, will be the distance of the sun from ♈, or the longitude and right ascension respectively.

When the sun has passed ♎ and is descending toward ♑, the computation relates to the triangle ♎ *a* ☉, and the quantities ♎ ☉, ♎ *a*, are to be *added* respectively to 180°, to obtain the longitude and right ascension reckoned from ♈.

When the sun has passed ♑ and is ascending toward ♈, we make use of the triangle ♈*a*☉, and the arcs ♈☉, ♈*a*, found as above, are to be taken from 360°, and the remainders will be the longitude and right ascension respectively.

85. It will be perceived that in the triangle *A*♈☉, above mentioned, any two parts beside the right angle being given, the rest may be found. If, for instance, we had the longitude ♈☉ = 32° 03′ 09″, and the obliquity of the ecliptic *A*♈☉ = 23° 27′ 57″, we might proceed to find the declination and right ascension. Thus

To find the declination *A*☉, the three circular parts being co *A*♈☉, co ♈☉ and *A*☉, the middle part is *A*☉, and co *A*♈☉, co ♈☉ are opposite parts; whence

R sin *A*☉ = cos (co *A*♈☉) cos (co ♈☉) = sin *A*♈☉ sin ♈☉,

and

R		10,00000
is to sin *A*♈☉ . . obliquity . .	23° 27′ 57″	9,60010
as sin ♈☉ . . longitude	32° 03′ 09″.	9,72485
is to sin *A*☉ . . declination . .	12° 12′ . . .	9,32495

To find the right ascension ♈*A*, the three circular parts being ♈*A*, co *A*♈☉, co ♈☉, co *A*♈☉ is the middle part, and ♈*A*, co ♈☉ adjacent parts; whence

R sin (co *A*♈☉)

or *R* cos *A*♈☉ = tang ♈*A* tang (co ♈☉) = tang ♈*A* cot ♈☉,

and

cot ♈☉ . . longitude	32° 03′ 09″	10,20333
		9,76667
is to *R*		10,00000
as cos *A*♈☉ . . obliquity	23° 27′ 57″	9,96251
is to tang ♈*A* . . right ascension† . .	29° 52′ 15″	9,75918

† This result differs from the former by 5″. They will be found

86. 1. Given the sun's declination 17° 16′ N. increasing, to find his longitude and right ascension, the obliquity being as above stated 23° 27′ 57″.

Ans. Sun's longitude 47° 35′, and right ascension 45° 07′.

2. Given the sun's right ascension 134° 54′, to find his longitude and declination, the obliquity of the ecliptic being 23° 27′ 57″.

Ans. Sun's longitude 4ˢ 12° 26′, declination 17° 06′ N.

87. The latitude of the place equal, for example, to 42° 23′ 28″ N. and the sun's declination, being given, to find the time of rising and setting of the sun on the 21st of June.

Fig. 51. Taking the meridian *ZHN′O* (*fig.* 51) as the primitive, the equator and horizon will be represented by the straight lines *EQ*, *HO*, passing through the centre of the primitive, and making the angle *E*♈*H* equal to the complement of the latitude, *NS* perpendicular to *EQ* will represent the 6 o'clock hour circle, or the meridian perpendicular to the meridian of the place *ZHNO*. The small circle *n m*, drawn parallel to *EQ* and 23° 27′ 57″ distant from it, will represent the apparent path of the sun on the 21st of June. ⊙ will be the place of the sun at sunrise, and *n* his place at noon. Accordingly the hour angle ⊙*Nn*, or the arc ⊙*n*, or which is the same thing as to the number of degrees, *AE*, converted into time at the rate of 15° to an hour, will give the time of sunrising and sunsetting from noon. But ♈*E* being 90° is equivalent to 6 hours. It is only necessary, therefore, in solving the above problem, to find the arc *A*♈, called *the ascensional difference*. Now in the triangle *A*⊙♈, right-angled at *A*, we have *A*♈⊙ equal *OQ* or *EH*, the complement of the latitude, and *A*⊙ the sun's declination, to find *A*♈. In this case we have *A*♈ for the middle part, and co *A*♈⊙, *A*⊙, for the adjacent parts. Whence

R sin *A*♈ = tang (co *A*♈⊙) tang *A*⊙ = tang *N*♈*O* tang *A*⊙,
and

R	. . . .		10,00000
is to tang *N*♈*O* . . lat.	42° 23′ 28″	. .	9,96040
as tang *A*⊙ . . declin. . . .	23° 27′ 57″	. .	9,63759
is to sin *A*♈ . . ascen. diff. . .	23° 20′ 42″	. .	9,59799

---

to agree, however, when logarithms are employed to seven places of decimals instead of five.

In the preceding solutions, by taking radius into consideration we reduce the operation for the answer to the form of a proportion, analogous to what takes place in most of the cases of plane trigonometry. The same result might be obtained more concisely by following the rule in italics (*Trig. page* 64). Thus in the above problem, for example, we have

$\sin A♈ = \tan (\text{co } A♈⊙) \tan A⊙ = \tan N♈⊙ \tan A⊙,$

or, by taking the logarithms,

$$\log. \sin A♈ = \log. \tan N♈⊙ + \log. \tan A⊙.$$

thus,

tang $N♈⊙$	.. lat.	42° 23′ 28″	9,96040
tang $A⊙$	.. declin. ...	23° 27′ 57″	9,63759
sin $A♈$	.. ascen. diff. ..	23° 20′ 42″	9,59799

The arc 23° 20′ 42″ being converted into time at the rate of 15° to an hour gives 1^h 33′ 23″ nearly for the time the sun rises before and sets after 6 o'clock, on the longest day. Hence

$$6^h - 1^h\ 33'\ 23'' = 4^h\ 26'\ 37'' \text{ time of sunrising,}$$
$$6^h + 1^h\ 33'\ 23'' = 7^h\ 33'\ 23'' \text{ time of sunsetting.}$$

7^h 33′ 23″, the time of sunsetting, being the time from noon, or half the day, if this be doubled it will give 15^h 06′ 46″ for the length of the longest day. Also 4° 26′ 37″, the time of sunrising, being the time from midnight, if this be doubled it will give the length of the shortest night, at any place whose latitude is 42° 23′ 28″.

88. Moreover, if we draw the parallel of declination *r s* 23° 27′ 57″ south of *EQ*, we shall have $B⊙♈$ equal to $A⊙♈$; and $♈B$, equal $♈A$, converted into time, shows how long it is after 6 o'clock before the sun rises, and how long before 6^h the sun sets. The longest night and shortest day, therefore, become equal respectively to the longest day and shortest night, as before found.

It will be perceived from what is above shown, that when the latitude and declination are both north or both south, the sun rises before and sets after 6 o'clock; but when one is north and the other south, the sun rises after and sets before 6.

89. We have seen that *n m* and *r s* represent the apparent path of the sun, when at its greatest declination north and south, and that $m⊙$ measures the time from midnight to sunrise, or half the night, and $r⊙$ the time from sunrise to noon, or half

the day. Consequently, when QO, the complement of the latitude, is less than $\mathcal{A}\odot$ or Qm, the declination circles, $n\,m$ and $r\,s$ will not touch the horizon HO. In such a case, therefore, when the sun is in $n\,m$ it will not set, and when it is in $r\,s$ it will not rise. But this can happen only to those places whose polar distance is less than the sun's greatest declination 23° 27′ 57″.

90. It may be observed, that as O is the north point of the horizon, ♈ is the east point, and ♈⊙ is the sun's *amplitude* or distance from the east at the time of rising, and from the west at the time of setting. This arc of the horizon ♈⊙ belonging to the triangle ♈$\mathcal{A}$⊙, already employed, we have for the circular parts co $\mathcal{A}$♈⊙, co ♈⊙ and $\mathcal{A}$⊙, of which $\mathcal{A}$⊙ is the middle part, being separated from the others by the angle ♈⊙$\mathcal{A}$, and cos (co $\mathcal{A}$♈⊙), cos (co ♈⊙) are opposite parts. Whence sin $\mathcal{A}$⊙ = cos (co $\mathcal{A}$♈⊙) cos (co ♈⊙) = sin $\mathcal{A}$♈⊙ sin ♈⊙. Dividing both sides by sin $\mathcal{A}$♈⊙, we have

$$\sin \Upsilon\odot = \frac{\sin \mathcal{A}\odot}{\sin \mathcal{A}\Upsilon\odot},$$

or, by taking the logarithms,

$$\log.\sin \Upsilon\odot = \log.\sin \mathcal{A}\odot - \log.\sin \mathcal{A}\Upsilon\odot,$$

thus,

sin $\mathcal{A}$⊙	. . . declin. . . .	23° 27′ 57″	9,60010
sin $\mathcal{A}$♈⊙	. . co lat. . .	47° 36′ 32″	9,86839
sin ♈⊙	. . amplitude . .	32° 37′ 35″ from N. .	9,73171

In the triangle ♈$\mathcal{A}$⊙ any two parts beside the right angle being given, the rest may be found. Let the sun's amplitude, for instance, be supposed to be known equal to 32° 37′ 35″ from the north, and his declination equal to 23° 27′ 57″ N. to find the latitude and time of the sun's rising and setting.

Ans. Latitude 42° 23′ 28″ N.
Time of sun's rising 4^h 26′ 37″,
Time of sun's setting 7^h 33′ 23″.

At London in latitude 51° 32′ N. the sun's amplitude being found by observation equal to 29° 48′ from the north, what is the sun's declination and time of rising and setting?

Ans. Sun's declination 23° 27′ 59″ N.
Time of sun's rising 3^h 47′ 32″,
Time of sun's setting 8^h 12′ 28″.

90. Required the azimuth and time of rising and setting of Arcturus at Cambridge, in latitude 42° 23′ 28″ N. August 1, 1820, the declination of the star being 20° 07′ 23″ N. (*fig.* 52), and its right ascension 14ʰ 7′ 28″, and the right ascension of the sun 8ʰ 45′ 59″. Fig. 52.

The learner will proceed in this case to find ♈*A* as before. This reduced to hours, minutes, &c. and added to 6ʰ, will give the time in which the star passes from the horizon to the meridian. But the star crosses the meridian 14ʰ 7′ 28″ — 8ʰ 45′ 59″, or 5ʰ 21′ 29″ after the sun, that is, at 5ʰ 21′ 29″ P. M. Consequently the above time subtracted from this (increased by 12ʰ), will give the time of the star's rising, and added to this, will give the time of the star's setting.

Ans. Star's azimuth 62° 14′ 07″ from the north,
time of rising 10ʰ 03′ 20″ A. M.
time of setting 12ʰ 39′ 38″ P. M.

91. The above method may be employed to find the time of rising and setting of the moon and planets. But when the change of declination is considerable in a short time, as is the case especially with respect to the moon, and with respect to the sun when near the equator, the declination should be ascertained near the time of rising and setting of the body in question.

92. Given the latitude of the place equal to 42° 23′ 28,″ and the sun's declination equal to 23° 27′ 57″ N. to find the sun's altitude and azimuth at 6 o'clock.

The meridian *ESQN* (*fig.* 53), the equator *EQ*, &c. being described as in figure 51, the declination circle *nm* will cut the 6 o'clock hour circle *NS* in ⊙, the sun's place at 6 o'clock. Through ⊙ describe the azimuth circle *Z*⊙*AN*′ cutting the horizon in *A*. Fig. 53.

In the triangle ♈*A*⊙, right-angled at *A*, we shall have *A*♈⊙ = the latitude, ♈⊙ = declination, to find *A*⊙ the altitude. In this case *A*⊙ is the middle part, and co *A*♈⊙, co ♈⊙ opposite parts. Whence

sin *A*⊙ = cos (co *A*♈⊙) cos (co ♈⊙) = sin *A*♈⊙ sin ♈⊙,

and, taking the logarithms,

log. sin *A*⊙ = log. sin *A*♈⊙ + log. sin ♈⊙,

thus

sin $A♈☉$ . . . . lat. . . . 42° 23′ 28″ 9,82878
sin ♈☉ . . . . declin. . . 23° 27′ 57″ 9,60010

sin $A☉$ . . alt. . . . . . . 15° 34′ 22″ 9,42888

It is evident, moreover, from the above construction, that the angle OZA, or which is the same thing, the arc OA, is the sun's *azimuth*, or angular distance from the north. Accordingly, if we find $♈A$ in the above triangle $♈A☉$, and subtract it from 90°, we shall have $OA =$ sun's azimuth at 6 o'clock. The three circular parts in this case are $♈A$, co $A♈☉$, co ♈☉, of which co $A♈☉$ is the middle part, and $♈A$, co ♈☉, adjacent parts. Whence

sin (co $A♈☉$)
or cos $A♈☉$ = tang $♈A$ tang (co ♈☉) = tang $♈A$ cot ♈☉

or $$\text{tang } ♈A = \frac{\cos A♈☉}{\cot ♈☉}$$

and, by logarithms,

log. tang $♈A$ = log. cos $A♈☉$ — log. cot. ♈☉,

thus,

cos $A♈☉$ . . . lat. . . . . . 42° 23′ 28″ . . 9,86839
cot ♈☉ . . . declin. . . . 23° 27′ 57″ . . 10,36241

tang $♈A$ . . co-azimuth . . . 17° 46′ 35″ . . 9,50598

consequently 90° — 17° 46′ 35″ = 72° 13′ 25″ = sun's azimuth at the above place and time.

93. It is obvious to remark that, as the declination ♈☉ increases, the altitude $A☉$ of the sun at 6 o'clock increases also, and the azimuth OA diminishes. While on the contrary, the declination decreases, the reverse takes place, till the declination becomes nothing, when the altitude at 6 o'clock is nothing, and the azimuth 90°; that is, at the equinoxes the sun rises in the east and sets in the west at 6 o'clock.

94. In the above solution the latitude and declination are both of the same kind, namely, north. The same will evidently hold true, when they are both south. When, on the other hand, one is north and the other south, we shall have $a☉$, equal $A☉$, equal to the depression of the sun below the horizon at 6 o'clock; and $a♈$, equal $A♈$, equal to the complement of the sun's azimuth, reckoned from the south point H of the horizon. Thus the sun is as far below the horizon at 6 o'clock on the shortest day, as he is above the horizon at 6 o'clock on the longest day; and the

sun rises as far south of east in the former case, as he rises north of east in the latter.

95. In the above triangle $A♈☉$ any two parts beside the right angle being given, the rest may be found. Let the two given parts be, for instance, the declination $♈☉ = 23° 27' 57''$, and the altitude at 6 o'clock $A☉ = 15° 34' 22''$, to find the latitude of the place $A♈☉$, and the sun's azimuth, the complement of $A♈$.

Ans. Lat. 42° 23′ 28″
Sun's azimuth 72° 13′ 25″.

Given the latitude of the place ON (*fig.* 54) $= 51° 32'$ N. and the declination of a star $♈* = 20° 16'$ N. to find the altitude and azimuth of the star, when on the 6 o'clock hour circle. Fig. 54.

Ans. Altitude 15° 44′
Azimuth 77° 03′.

The time of the star's passing the 6 o'clock hour circle may be found by subtracting 6^h from the difference of the sun and star's right ascension for the given time.

96. Given the latitude of the place 42° 23′ 28″ N. and the sun's declination 23° 27′ 57″ N. to find the time when the sun is east or west, and his altitude at this time.

Figure 51 being constructed as already described (86), ZN' will be the *prime vertical*, or great circle perpendicular to the horizon passing through the east and west points. The point a, where the tropic nm cuts ZN', will be the place of the sun, when seen due east or west, and $♈a$ will be his altitude at this time, and the hour angle $aNZ = BE =$ complement of $♈B$, reduced to time, will give the hours, minutes, &c. from noon, when the sun is in this situation. Accordingly, in the triangle $♈aB$, right-angled at B, we have $a♈B =$ latitude, and $Ba =$ sun's declination, to find $♈a$ and $♈B$. Fig. 51.

1. The three circular parts being Ba, co $♈a$, and co $a♈B$, Ba is the middle part, and the other two are opposite parts. Whence

$$\sin Ba = \cos(\text{co } ♈a)\cos(\text{co } a♈B) = \sin ♈a \sin a♈B,$$

or

$$\sin ♈a = \frac{\sin Ba}{\sin a♈B},$$

and

$$\log.\sin ♈a = \log.\sin Ba - \log.\sin a♈B,$$

thus

sin Ba	. . . declin.	. . . 23° 27′ 57″	. . . .	9,60010
sin $a♈B$	. . lat.	. . . . . 42° 23′ 28″	. . .	9,82878
sin $♈a$	. . . altitude	. . 36° 12′ 09″	. . .	9,77132

2. To find $♈B$, the three circular parts being co $a♈B$, $♈B$ and Ba, $♈B$ is the middle part, and the other two are adjacent parts. Whence

$$\sin ♈B = \text{tang}\,(\text{co}\; a♈B)\ \text{tang}\ Ba = \cot a♈B\ \text{tang}\ Ba,$$

and

$$\log.\sin ♈B = \log.\cot. a♈B + \log.\text{tang}\ Ba,$$

thus

cot $a♈B$	. . . lat.	. . . . 42° 23′ 28″	. .	10,03960
tang Ba	. . . declin.	. . 23° 27′ 57″	. .	9,63759
sin $♈B$	. . hour angle	. . 28° 23′ 42″	. .	9,67719

This angle 28° 23′ 42″ reduced to hours, minutes, &c., gives 1^h 53′ 35″ for the true time after 6 o'clock, on the 21st of June, when the sun is east. Accordingly the actual time is 7^h 53′ 35″. If we subtract this from 12^h we shall have 4^h 6′ 25″ for the time when the sun is on the prime vertical in the afternoon of the same day.

97. In the triangle $♈aB$ any two parts being given beside the right angle, the others are found as before. The sun's declination, for instance Ba, being 23° 27′ 57″ N. and his altitude when on the prime vertical $♈a$ 36° 12′ 09″, it is proposed to find the latitude of the place and the hour of the day.

Ans. Latitude 42° 23′ 28″ N.
Hour of the day 7^h 53′ 35″, A. M.
or 4^h 06′ 25″, P. M.

98. Given the latitude of the place 42° 23′ 28″ N. and the sun's declination 23° 27′ 57″ N. to find the time when twilight begins in the morning and ends in the evening.

Fig. 55. The meridian (*fig.* 55), equator, &c. being described as before, we draw the crepusculum circle rs parallel to the horizon HO and 18° below it, cutting the tropic nm in ⊙. ⊙ will be the place of the sun at the beginning and end of twilight. Through ⊙ draw the vertical circle $Z⊙N'$, and the hour circle $N⊙S$.

In the triangle $Z⊙N$ we have ZN = the co-latitude = 47° 36′ 32″, $Z⊙$ = the zenith distance = 90° + 18° = 108°, $N⊙$

= co-declination or polar distance = 66° 32′ 03″, to find $ZN\odot$ = hour angle from noon (*Trig.* 62).

	ZN = 47° 36′ 32″		
	$Z\odot$ = 108° 00′ 00″		
	$N\odot$ = 66° 32′ 03″		
Sum	222° 08′ 35″		
Half sum	111° 04′ 17″		
	47° 36′ 32″		
1st remainder	63° 27′ 45″ . . . log. sin	. . .	9,95165
2d remainder	44° 39′ 14″ . . . log. sin	. . .	9,84595
ZN	47° 36′ 32″ . . . ar. comp. log. sin		0,13162
$N\odot$	66° 32′ 03″ . . . ar. comp. log. sin		0,03749
			19,96671
	74° 14′ 15″ . . . log. sin ½ $ZN\odot$ =		9,98335
	2		
$ZN\odot$ =	148° 28′ 30″		

This angle reduced to time gives 9^h 53′ 52″ for the hours, minutes, &c. from noon to the commencement and termination of twilight. Thus $12^h - 9^h$ 53′ 52″ = 2^h 06′ 06″ is the time of daybreak, and 9^h 53′ 54″ is the time of the cessation of twilight in the evening on the 21st of June in latitude 42° 23′ 28″ N.

If we subtract the time of daybreak from that of the sun's rising 4^h 26′ 37″ (87), or from the time of the twilight's ceasing that of the sun's setting, we shall have 2^h 20′ 31″ for the duration of twilight at the above time.

99. It will be observed that, when $OQ - Qm$, or the co-latitude — the declination, is less than Os or 18°, the sun does not descend below the crepusculum circle, and the twilight continues all night. Suppose, for instance, the given place to be London in latitude 51° 32′; the co-latitude OQ in this case is 38° 28′. If now from 38° 28′ we subtract the declination Qm = 23° 28′, we shall have Om = 15°, and consequently less than Os. m therefore falls between O and s, and the declination circle nm does not intersect the crepusculum circle rs, and there is no cessation of the twilight during the night. We see, moreover, that if the given place had a less latitude by 3° than that of

London, the crepusculum and declination circles would just touch each other on the longest day, and that the sun would descend only 18° below the horizon at midnight.

1. Given the latitude of the city of Washington 38° 58′ N. to find the duration of twilight on the 21st of March.

Ans. Duration of twilight 1^h 30′.

2. Given the sun's declination 10° S. the latitude of the place being 51° 32′ N. to find the time of daybreak in the morning and end of twilight in the evening.

Ans. Time of daybreak 4^h 54′ 22″,
End of evening twilight 7^h 5′ 38″.

100. The sun appearing at a mean about 33′ above his real place when in the horizon, it is proposed to find how much the day is lengthened on this account at the summer solstice.

Fig. 55. Let the parallel circle *r s* (*fig.* 55) be drawn 33′ below the horizon *HO* instead of 18°, the rest of the figure being constructed as before, and we shall have $Z\odot = 90° \ 33'$, *ZN* and $N\odot$ remaining unchanged, to find $ZN\odot$ the hour angle from noon.

ZN =	47° 36′ 32″		
$Z\odot$ =	90° 33′ 00″		
$N\odot$ =	66° 32′ 03″		
Sum	204° 41′ 35″		
Half sum	102° 20′ 47″		
ZN	47° 36′ 32″		
1st remainder	54° 44′ 15″	. . log. sin.	9,91196
2d remainder	35° 48′ 44″	. . log. sin.	9,76725
ZN	47° 36′ 32″	. . ar. comp. log. sin	0,13161
$N\odot$	66° 32′ 03″	. . ar. comp. log. sin	0,03749
			19,84831
	57° 06′ 53″	. . log. sin $\frac{1}{2}$ $ZN\odot$.	9,92415
	2		
$ZN\odot$ =	114° 13′ 46″		

The angle $ZN\odot$ reduced to time, gives 7^h 36′ 55″. Accordingly, if from this we subtract 7^h 33′ 23″, the computed time of sunsetting where no allowance is made for refraction, we shall have 3′ 32″ for the prolongation of each part of the day on account the apparent elevation of the sun produced by refraction.

101. 1. The latitude of the place remaining the same, it is required to find how much the length of the day is increased on account of refraction at the time of the equinoxes, 21st of March and 23d of September, and at the winter solstice 21st of December.

2. It is required to find how much the day is prolonged on account of refraction at the above times for any other latitude, as that of the city of Washington, for instance.

102. Given the latitude of the place 42° 23′ 28″, the sun's declination 23° 27′ 57″ N. and altitude 46° 20′†, to find the hour of the day.

In figure 56 the meridian, horizon, &c., being constructed as before, draw the declination circle *m n* 23° 27′ 57″ N. and the parallel circle *r s* 46° 20′ above the horizon *HO*, intersecting *m n* in ⊙. Through ⊙ draw the azimuth circle *Z*⊙*N′*, and the hour circle *N*⊙*S*. Fig. 56.

In the oblique-angled triangle *Z*⊙*N* we have *ZN* = co-latitude, *Z*⊙ = co-altitude or zenith distance, and *N*⊙ = co-declination or polar distance, to find *ZN*⊙ the hour-angle from noon.

ZN	= 47° 36′ 32″		
Z⊙	= 43° 40′ 00″		
N⊙	= 66° 32′ 03″		
Sum	157° 48′ 35″		
Half sum	78° 54′ 17″		
ZN	47° 36′ 32″		
1st remainder	31° 17′ 45″	. . log. sin . . .	9,71555
2d remainder	12° 22′ 14″	. . log. sin . . .	9,33089
ZN	47° 36′ 32″	. . ar. comp. log. sin	0,13161
N⊙	66° 32′ 03″	. . ar. comp. log. sin	0,03749
			19,21554
	23° 54′ 34″	. . log. sin ½*ZN*⊙ .	9,60777
	2		
ZN⊙	= 47° 49′ 08″		

The hour angle reduced to time gives 3ʰ 11′ 17″ as the interval from noon. This answers to 12ʰ — 3ʰ 11′ 17″, or 8ʰ 48′ 43″ in

† The altitude is taken with a quadrant, sextant, or other instrument, and corrected for refraction.

the forenoon, or 8^h 11′ 17″ in the afternoon. When the given altitude is furnished by observation, it is of course known to which part of the day it relates.

103. To find the azimuth $NZ\odot$ we proceed according to the above method; thus,

1st rem. as before =	31° 17′ 45″	log sin.	9,71555
½ sum — $Z\odot$ or 2d rem. =	35° 14′ 17″		9,76116
ZN	47° 36′ 32″		0,13161
$Z\odot$	43° 40′ 00″		0,16086
			19,76918
	$\frac{50° 03′ 10″}{2}$	log. sin ½ $NZ\odot$	9,88459

$NZ\odot$ = 100° 06′ 20″

The above method is to be adopted when the three sides only are known. The hour angle $ZN\odot$, for instance, having been found, we should proceed according to the rule, *the sines of the sides are as the sines of the opposite angles* (*Trig.* 47); thus

sin $Z\odot$ . . co-altitude	43° 40′ 00″	9,83914
		0,16086
is to sin $ZN\odot$ . . hour angle	47° 49′ 08″	9,86984
as sin $N\odot$ . . co-declination	66° 32′ 03″	9,96251
is to sin $NZ\odot$ . . azimuth	100° 06′ 20″	9,99321

104. 1. Given the latitude of the place 51° 30′ 54″ N. the sun's declination 19° 39′ N. and the altitude of the sun's centre 38° 19′, to find the azimuth and the hour from noon.

Ans. Azimuth N. 107° 46′ 30″ W.
Hour from noon 3^h 30′.

2. In latitude 51° 32′ N. when the sun has no declination, what is his altitude and azimuth at 3^h 30′ from noon?

Ans. Altitude 22° 15′,
Azimuth S. 59° 01′ E. or W.

3. At the time of the equinox the sun's altitude being found by observation to be 22° 15′, and his azimuth S. 59° E. it is required to find the hour of the day and the latitude of the place.

Ans. Time 8^h 30′,
Latitude 51° 32′ N.

4. In latitude 39° 54′ N. longitude 35° 30′ W. the altitude of the sun's lower limb on the 7th of May 1796, at 5^h 30′ 32″ P. M. per watch was found by observation to be 15° 40′ 57″; how much was the watch too fast or too slow† ?

Ans. Watch too slow 3′ 1″.

105. Given the right ascension and declination of a star, or other heavenly body, to find its latitude and longitude. Let the body be Capella, for example, having a right ascension equal to 75° 51′ 04″, and a declination equal to 45° 48′ 10″ N.

The solstitial colure *NESQ* (*fig.* 57) being taken as the primitive, the equator and ecliptic will have their poles each in the circumference of the primitive, and, being great circles, will be represented by the straight lines *EQ*, ♑ ♋ (18). Draw the oblique circle *NCS*, making an inclination *QNR* equal to the complement of the right ascension. Parallel to *EQ*, and distant from it 45° 48′ 10″, describe a declination circle cutting *NCS* in *C*. Lastly, through the point *C* and the poles of the ecliptic *n*, *m*, project the oblique circle *n C m*; *C* will represent the place of the star; *CV* its latitude, or distance from the ecliptic, and *C n* ♈ its longitude, or angular distance from aries reckoned on the ecliptic. Fig. 57

In the triangle *n NC*, we have *n N* = 23° 27′ 57,″ the obliquity of the ecliptic, *NC* = 90° — 45° 48′ 10″, or 44° 11′ 50,″ the complement of the declination of Capella, and the angle *n Nc* = 90° + 75° 51′ 04″, or 165° 51′ 04″, the right ascension added to 90°, to find *n C*, the complement of the latitude *CV*, and *C n N*, the complement of the longitude ♈ *n V*, or ♈ *V*.

This question is readily solved by the formula for the case where two sides and the contained angle are given (*Trig.* 61). Thus,

$$NC = 44° \ 11' \ 50''$$
$$nN = 23° \ 27' \ 57''$$

$$NC + nN = 67° \ 39' \ \ 4''$$
$$NC - nN = 20° \ 43' \ 53''$$

† In questions of this kind the learner is supposed to have access to the *Nautical Almanac*, or other similar ephemeris for the sun's declination and semidiameter for the given longitude and time. In the present case we should find the sun's declination 16° 58′ 31″ N. and semi diameter 15′ 53″.

$\frac{1}{2}$ (*NC* + *n N*)	=	33° 49′ 53″	cos	9,91943
				0,08057
$\frac{1}{2}$ (*NC* — *n N*)	=	10° 21′ 56″	cos	9,99285
$\frac{1}{2}$ *n NC*	=	82° 55′ 32″	cot	9,09378
$\frac{1}{2}$ (*N n C* + *NC n*)	=	8° 21 38″	tang	9,16720
$\frac{1}{2}$ (*NC* + *n N*)	=	33°.49′ 53″	sin	9,74566
				0,25434
$\frac{1}{2}$ (*NC* — *n N*)	=	10° 21′ 56″	sin	9,25509
$\frac{1}{2}$ *n NC*	=	82° 55′ 32″	cot	9.09378
$\frac{1}{2}$ (*N n C* — *NC n*)	=	2° 17′ 48″	tang	8,60321

Having now the half sum and half difference of the two unknown angles, if we add these together we shall have the greater (*Trig.* p. 81, *note*), which being opposite the greater side (*Geom.* 485), will be *N n C*. Whence

	8° 21′ 38″
	2° 17′ 48″
N n C = *V*♋ = ...	10° 39′ 26″
	90
♈ *n V* = ♈ *V* = long. of *C* = ...	79° 20′ 34″

To find the side *n C*, we make use of the proportion, *the sines of the sides are as the sines of the opposite angles* (Trig. 47.)

As sin *N n C*	10° 39′ 26″ . . .	9,26701
		0,73299
is to sin *NC*	44° 11′ 50″ . . .	9,84331
so is sin *n NC*	165° 51′ 04″ . . .	9,38818
to sin *n C*	67° 08′ 30″ . . .	9,96448

Taking 67° 08′ 30″ from 90°, we have 22° 51′ 30″ equal to *CV*, the latitude of the star *C*.

106. This problem admits of an easy solution by Napier's rules. Suppose a perpendicular *CP* let fall from the point *C* upon *NQ*; we shall have in the triangle *CNP*, right-angled at *P*, the side *CN* and the angle at *N*, by means of which we first find *NP*; thus,

$$\sin(\text{co } CNP) = \text{tang}(\text{co } CN)\ \text{tang } NP,$$

whence $\text{tang } NP = \cos CNP\ \text{tang } CN,$

and by logarithms,

cos *CNP*	14° 08′ 56″	9,98662
tang *CN*	44° 11′ 50″	9,98783
tang *NP*	43° 18′ 55″	9,97445

Adding *NP* to *Nn* we shall have $nP = 66° 46' 52''$, and by considering *NP* and *n P* as middle parts, the third of Napier's rules gives the following proportion :

As sin *NP*	43° 18′ 55″ . .	9,83633
		0,16367
is to tang (co *CNP*) = cot *CNP* .	14° 08′ 56″. .	10,59845
so is sin *n P*	66° 46′ 52″ . .	9,96332
to tang (co *CnN*) = cot *CnN*...	10° 39′ 26″. .	10,72544

Lastly, by the fourth of Napier's rules, co *NC*, co *n c*, being middle parts, we obtain the proportion,

As cos *NP*	43° 18′ 55″ . . .	9,86189
		0,13811
is to sin (co *NC*) = cos *NC* . .	44° 11′ 50″ . . .	9,85549
so is cos *n P*	66° 46′ 52″ . . .	9,59577
to sin (co *n C*) = cos n *C* . .	67° 08′ 24″ . . .	9,58937

We have thus a confirmation of our former results. It is evident, that if the star or other body in question, had been on the other side of ♈, and less than 90° from it, the arc ♈ *V* would show how much the star's longitude wanted of 360°. It would be necessary, therefore, to subtract this arc from an entire circumference, to obtain the distance from ♈, reckoned in the order of the signs. So also if the star were situated more than 90° from ♈, we should consider the figure as representing the opposite portion of the heavens, having ♎ instead of ♈ in its centre. In this case the solution would give the distance from the first of ♎, and it would be necessary to add this distance to 180°, or to subtract it from this quantity, as the case might require, in order to obtain the proper expression for the longitude.

It may be observed, moreover, that if *EQ* be considered as representing the ecliptic, and ♑♋ the equator, the above process would give the declination and right ascension, when the latitude and longitude are known. The two problems may be regarded, therefore, as leading to the same kind of solution.

107. 1. Required the latitude and longitude of Spica Virginis, its right ascension being 198° 34′ 32″, its declination 10° 04′ 31″ S. and the obliquity of the ecliptic 23° 28′.

Ans. Lat. 2° 02′ 23″ S.
Long. 6^s 20° 57′ 10″.

2. The latitude of the moon being 4° 00′ 34″ N., her longitude 7^s 14° 26′ 21″, and the obliquity of the ecliptic 23° 27′ 48″, it is required to find her right ascension and declination.

Ans. Right ascen. 7^s 13° 1[illegible]′ 11″,
Declination 12° 21′ 14″.

3. Required the right ascension in time of the planet Mercury on the 22d of December 1804, its geocentric latitude being 2° 12′ S., and its geocentric longitude 9^s 14° 36′.

Ans. $19^h\ 4\frac{1}{4}'$.

108. Given the right ascension and declination of two stars, or their latitude and longitude, to find their distance asunder. Let the two stars, for example, be Sirius and Procyon. We take from a catalogue of the stars the given quantities; namely,

Right ascen. of Sirius (Jan. 1, 1820),	99° 18′ 12″	dec. 16° 28′ 31″ S.
" " Procyon	112° 28′ 04″	dec. 5° 40′ 48″ N.
Diff. of right ascension	13° 09′ 52″	

We now take the meridian of one of the given stars, Pro-
Fig. 58. cyon for instance, as the primitive (*fig.* 58), *EQ* being the equator, and *S*, *N*, its poles. We draw the oblique circle *SS′N*, making the angle *S′SP* equal to 13° 09′ 52″, the difference of right ascension of the two stars; and we set off upon *SS′N* the arc *SS′* equal to 73° 31′ 29″ the distance of Sirius from the south *SPN* pole, and upon *SPN*, we set off *SP* equal to 90° + 5° 40′ 48″ the distance of Procyon from the same pole.

In the oblique-angled triangle *SS′P*, we have *SS′*, *SP*, and the contained angle *S′SP*, to find the side *S′P*. We proceed according to the formula above referred to (105).

$$SP = 95° 40' 48''$$
$$SS' = 73° 31' 29''$$

$$SP + SS' = 169° 12' 17''$$
$$SP - SS' = 22° 09' 19''$$

$\frac{1}{2}(SP+SS')$	= 84° 36′ 08″..cos...8,97345		.....sin...	9,99807
	1,02655			0,00193
$\frac{1}{2}(SP-SS')$	= 11° 04′ 39″...cos...9,99183		.....sin...	9,28361
$\frac{1}{2}PSS'$	= 6° 34′ 56″...cot...10,93783		.....cot...	10,93783
$\frac{1}{2}(SS'P+SPS')$	= 89° 21′ 59″..tan...11,95621	58° 07′ 30″..tan..		10,22337
$\frac{1}{2}(SS'P-SPS')$	= 59° 07′ 30″			

$SS'P$=148° 29′ 29″

Whence

As	sin $SS'P$....	148° 29′ 29″ ...	9,71819
			0,28181
is to	sin SP	95° 40′ 48″ ...	9,99786
so is	sin PSS' ...	13° 09′ 52″ ...	9,35745
to	sin $S'P$	25° 41′ 53″ ...	9,63712

The distance, therefore, of Sirius and Procyon is 25° 41′ 53″.

109. 1. The same figure will serve also for the case where the latitudes and longitudes of the stars are given, to find their distance. The mean longitude of Sirius for 1820, as put down in the tables, is 3ˢ 11° 38′ 00″, and its latitude 39° 32′ 01″ S.; and the mean longitude of Procyon 3ˢ 23° 19′ 33″, and its latitude 15° 57′ 36″ S. Their distance asunder is required.

Ans. 25° 41′ 21″.

2. Required the distance between Lyra and Arcturus, the declination of the former being 20° 07′ 28″ N. and its right ascension 211° 51′ 45″; and the declination of the latter 38° 37′ 19″ N. and its right ascension 277° 32′ 31″.

Ans. 58° 52′ 38″.

110. The places of two stars being given and their distances from a third star or comet, to find the place of the third object. Suppose the distance of a comet *C* (*fig.* 60), as measured by a sextant, to be 65° 47′ 42″ from Sirius, and 51° 06′ from Procyon, it is proposed to find the latitude and longitude of the comet. Fig. 60.

† It will be observed that figure 58 is adapted to the two problems only in the way of illustrating the process by which they are solved. Two figures would be necessary if any thing depended on the construction.

With the latitude and longitude of Sirius and Procyon, taken from the tables as before (109), we find their distance $SP = 25°$ $41' 21''$ in the manner just explained.

Having the three sides of the triangle SPC, we proceed to calculate the angle CSP, thus,

$CS = 65° 47' 42''$
$CP = 51° 06' 00''$
$SP = 25° 41' 21''$

Sum . . 142° 35′ 03″

Half sum . 71° 17′ 31″

1st remainder	5° 29′ 49″	. . log. sin . . .	8,98133
2d remainder	45° 36′ 10″	. . log. sin . . .	9,85401
CS . . .	65° 47′ 42″	. . ar. comp. log. sin	0,03996
SP . . .	25° 41′ 21″	. . ar. comp. log. sin	0,36302
			19,23832
	24° 35′ 12″		9,61916
	2		

$CSP = 49° 10' 24''$

The next step is to find the angle nSP. This is done by means of the triangle SPn, in which we have $nP = 15° 57' 36'' + 90° = 105' 57' 36''$, $SP = 25° 41' 21''$, and $SnP =$ difference of longitude of S and $P = 11° 41' 33''$; whence

As sin SP	25° 41′ 21″	. . .	9,63698
			0,36302
is to sin SnP . . .	11° 41′ 33″	. . .	9,30677
so is sin nP	105° 57′ 36″	. . .	9,98293
to sin nSP	26° 42′ 39″	. . .	9,65272

If now we take the angle nSP from the angle CSP, we shall have the angle $CSn = 22° 27' 45''$ in the triangle SCn, by means of which and the two containing sides nS, CS, the side nC, or co-latitude of C, and the angle CnS, or difference of longitude of C and S, are immediately determined. Thus, if we suppose a perpendicular CP' let fall from C upon nS, we shall have, by the first Napier's rules, taking co CSP' as the middle part,

$$\sin(\text{co } CSP') = \text{tang}(\text{co } CS) \text{ tang } SP'.$$

Whence $\text{tang } SP' = \cos CSP' \text{ tang } CS,$

and by logarithms,

cos CSP'	. .	22° 27′ 45″ . . .	9,96573
tang CS	. .	65° 47′ 42″ . . .	10,34725
tang SP'	. .	64° 03′ 37″ . . .	10,31298

Sn = 129° 32′ 01″
SP' = 64° 03′ 37″

nP' = 65° 28′ 24″

In the triangles $CP'n$, $CP'S$, by considering co CS, and co Cn, as middle parts, we shall have, by the fourth of Napier's rules, the following proportion,

As cos SP'	64° 03′ 37″ . .	9,64090
		0,35910
is to sin (co CS) = cos CS . .	65° 47′ 42″ . .	9,61279
so is cos nP'	65° 28′ 24″ . .	9,61817
to sin (co nC) = cos nC . .	67° 02′ 45″ . .	9,59006

and 90° — 67° 02′ 45″ = 22° 57′ 15″ = latitude of C.

Lastly,

As sin nC	67° 02′ 45″	9,96417
		0,03583
is to sin CSn . . .	22° 27′ 45″	9,58215
so is sin CS . . .	65° 47′ 42″	9,96003
to sin CnS . . .	22° 14′ 17″	9,57801

Whence, if from the longitude of S = . .	101° 38′ 00″
we take the angle CnS = . .	22° 14′ 17″
we shall have the longitude of C = . .	79° 23′ 43″

By considering ♑ ♋ as representing the equator instead of the ecliptic, we should obtain by the above process, the declination and right ascension of the object C, the places of S and P, as referred to the equator, being known.

111. The distance of a new star was found to be 65° 47′ 42″ from Capella, and 25° 42′ 10″ from Procyon, the latitude of the former at the time of the observation being 22° 51′ 57″ N., and its longitude 78° 57′ 57″, and the latitude of the latter 15° 58′ 14″ S.,

and its longitude 112° 55′ 42″; the latitude and longitude of the new star is required.

Ans. Latitude 39° 34′ 00″,
Longitude 101° 13′ 03″.

112. Given the sun's declination 23° 28′ N., semidiameter 15′ 47″, refraction 33′, and parallax 9″, the latitude of the place being 51° 32′ N., to find the time from noon when the sun's centre appears in the horizon.

The effect of parallax being opposite to that of refraction, we subtract 9″ from 33′, which gives 32′ 51″ for the apparent elevation of the sun above its true place when in the horizon. The upper limb of the sun, therefore, at the time of its first appearance above the horizon is actually 32′ 51″ below it, and the sun's centre at the same moment is 32′ 51″ + 15′ 47″, or 48′ 38″ below the horizon. Accordingly, when the sun's limb first presents

Fig. 60. itself at S (*fig.* 60), the sun's centre is at $\odot$, and $b\odot$ equal to 48′ 38″, added to 90°, is equal to $Z\odot$. Whence, in the triangle $Z\odot N$ we have

$$Z\odot = 90° 48' 38'' = \text{sun's zenith distance}$$
$$ZN = 38° 28' 00'' = \text{co-latitude}$$
$$N\odot = 66° 32' 00'' = \text{sun's co-declination},$$

to find the hour angle $ZN\odot$. Thus prepared, the problem does not differ from others where the three sides are given to find an angle.

Ans. 8h 19′ 20″.

Of Terrestrial Latitude.

113. It will be observed that in the preceding calculations the latitude is a very important element. We have already mentioned one way of determining this (*page* 29, *note*), which may be illustrated by the following example. Suppose the greatest and least altitude of the pole star, as observed at Cambridge, to be, when corrected for refraction, as below.

Greatest altitude	44° 08′ 55″
Least	40° 37′ 45″
Sum	84° 46′ 40″
Half sum or latitude = . . .	42° 23′ 20″

With either of these altitudes and the co-declination of the pole star, taken from the tables, and reduced to the given time, we readily obtain the latitude† ; thus,

Greatest altitude of the pole star	44° 08′ 55″
Co-declination (Jan. 1, 1820)	1° 45′ 35″
Latitude	42° 23′ 20″

This method is not confined to the pole star. It might obviously be applied to any other star in the neighbourhood of the pole. Similar means may be used also with reference to stars or other bodies in other parts of the heavens. *ZE* (*fig.* 51) Fig. 51 being the latitude of a place whose zenith is represented by the point *Z*, if we observe the greatest or meridian altitude *HE*, of any heavenly body situated in the equator, and subtract this from 90°, we shall have *ZE*, the latitude††. Moreover, when the heavenly body, instead of being in the equator, is between the equator and the horizon, by adding this declination, taken from the tables, we still have the altitude of the equator, and consequently the zenith distance or latitude. In case the body in question is between the equator and zenith the declination is manifestly to be subtracted from the altitude in order to arrive at the same result. To deduce the latitude from an observation of the sun, the following are the steps to be taken.

Obs. alt. of sun's upper limb, Oct. 11, 1820,			40° 39′ 05″
Sun's semidiameter	—	16′ 05″	
Refraction	—	1′ 17″	
Parallax	+	6″	
Correction			— 17′ 16″
True alt. of sun's centre			40° 21′ 47″
Sun's declin. south			+ 7° 14′ 45″
Altitude of the equator			47° 36′ 32″
			90°
Latitude			42° 23′ 28″

† Where great accuracy is required, it is necessary to apply a correction for *aberration. nutation*, and *inequality* of the precession of the equinoxes. Also the mean refraction should be corrected for the state of the barometer and thermometer.

†† When the sun, moon, or a planet is employed as above, for obtaining the latitude, beside the correction already mentioned, another is to be applied on account of parallax.

114. It may be impossible on account of the weather or other causes to take a meridian observation. In this case recourse is had to other methods, some of which have been intimated (95, 97). It is very common at sea to make use of two altitudes of the sun and the intervening time. Let the two altitudes, for example, be 38° 19′ and 50° 25′, and the intervening time 1^h 30′, the sun's declination being 19° 39′, and the place being in north latitude.

Fig. 61. Let *A*, *B* (*fig.* 61), represent the two places of the sun, and we shall have *ANB* = 22° 30′, and *NA*, *NB*, each equal to 70° 21′, the sun being supposed not to change his declination during the interval between the observations. Whence, by supposing a perpendicular let fall from *N* to the middle of *AB* (*Geom.* 484) we shall have the proportion,

As	sin	90° 00′	10,00000
is to	sin *AN* or *BN*	70° 21′	9,97394
so is	sin ½ *ANB*	11° 15′	9,29024
to	sin ½ *AB*	10° 35′ 13″	9,26418
		2	
		AB = 21° 10′ 26″	

Again,

As	sin *AB*	21° 10′ 26″	9,55775
			0,44225
is to	sin *ANB*	22° 30′ 00″	9,58284
so is	sin *AN*	70° 21′ 00″	9,97394
to	sin *ABN*	86° 10′ 24″†	9,99903

† The formula for the case where two sides and the included angle are given is applicable to the above question, notwithstanding the equality of the given sides. It will be seen that the factor sin ½ $(b - c)$ (*Trig.* 61), becoming 0, destroys the expression for the difference of the opposite angles, while cos ½ $(b - c)$ is equal to radius; whence we obtain the angles *B*, *C*, as follows:

½ (*AN* + *BN*)	= 70° 21′ 00″	cos	9,52669
			0,47331
½ (*AN* − *BN*)	= 0° 0′ 0″	cos	10,00000
½ *ANB*	= 11° 15′ 00″	cot	10,70134
ABN or *BAN*	= 86° 10′ 24″	tang	11,17465

Now in the triangle *ABZ* we have the side *AZ* = the complement of the 1st altitude = 51° 41′, *BZ* = complement of the 2d altitude = 39° 35′, and *AB* = 21° 10′ 26″, as found above. Whence, by the usual formula,

AZ =	51° 41′ 00″		
BZ =	39° 35′ 00″		
AB =	21° 10′ 26″		
Sum . .	112° 26′ 26″		
Half sum .	56° 13′ 13″		
1st. remainder	16° 38′ 13″	. . sin	9,45683
2d remainder	35° 02′ 47″	. . sin	9,75909
BZ . . .	39° 35′ 00″	. . ar. comp. sin . .	0,19572
AB . . .	21° 10′ 26″	. . ar. comp. sin . .	0,44225
			19.85389
	57° 41′ 29″	. . sin	9,92695
	2		
ABZ =	115° 22′ 58″		
ABN =	86° 10′ 24″		
NBZ =	29° 12′ 34″		

Lastly, in the triangle *BZN* we have the side *NB* = sun's co-declination = 70° 21′, *BZ* = sun's co-altitude = 39° 35′, and the contained angle, to find *ZN*.

By supposing a perpendicular *NP*, let fall upon *BZ* produced, the first of Napier's rules gives

sin (co *NBZ*) = tang (co *NB*) tang *BP*;

whence tang *BP* = cos *NBZ* tang *NB*,

and by logarithms,

cos *NBZ*	29° 12′ 34″	9.94094
tang *NB*	70° 21′ 00″	10,44725
tang *BP*	67° 45′ 05″	10,38819
BZ	39° 35′ 00″	
ZP†	= 28° 10′ 05″	

† By using a perpendicular let fall from *Z* upon *NB*, the process

Accordingly, if we consider co *NB*, co *NZ*, as middle parts, we shall have, by the fourth of Napier's rules,

As	cos *BP*	67° 45′ 05″ . .	9,57821
			0,42179
is to	sin (co *NB*) = cos *NB* . .	70° 21′ 00″ . .	9,52669
so is	cos *ZP*	28° 10′ 05″ . .	9,94526
to	sin (co *NZ*) = cos *NZ* . .	38° 28′ 03″ . .	9,89374
		90° 00′ 00″	
	Latitude =	51° 31′ 57″	

would be nearly the same; and one of these methods may be employed to verify the result obtained by the other.

In questions like the above, the solution by Napier's rules is shorter than that by the formula (*Trig.* 61.), as will be seen by the following application of the formula to the case just solved.

NB = 70° 21′
BZ = 39° 35′

109° 56′

$\frac{1}{2}$(*NB* + *BZ*)	= 54° 58′	cos . . . 9,75895	sin . . .	9,91319
		0,24105		0,08681
$\frac{1}{2}$(*NB* — *BZ*)	= 15° 23′	cos . . . 9,98415	sin	9,42370
$\frac{1}{2}$(*NBZ*)	= 14° 36′ 17″	cot 10,58408	cot	10,58408
$\frac{1}{2}$(*BZN* + *BNZ*)	= 81° 10′ 53″	tang 10,80928	tang	10,09459
$\frac{1}{2}$(*BZN* — *BNZ*)	= 51° 11′ 27″			

BZN = 132° 22′ 20″

As	sin *BZN*	132° 22′ 20″ . . .	9,86852
			0,13148
is to	sin *NB*	70° 21′ 00″ . . .	9,97394
so is	sin *NBZ*	29° 12′ 34″ . . .	9,68842
to	sin *NZ*	38° 28′ 03″ . . .	9,79384

This manner of solving the problem may be preferable, when the time is to be calculated by the same observations of the sun's altitude, for it will be seen that *BNZ*, the hour angle from noon, is immediately deduced from the above operation, since it is equal to $\frac{1}{2}$(*BZN* + *BNZ*) — $\frac{1}{2}$(*BZN* — *BNZ*), or 29° 59′ 26″; that is, 1h 59′ 58″.

115. In the foregoing calculation we have supposed the two altitudes of the sun to be taken at the same place; but as this seldom happens at sea, a correction should be applied to the first altitude for the ship's change of situation.

Let *CE* (*fig.* 43), represent the line described by the ship during the interval between the observations, and *P* the zenith of the place at the 2d observation. It is evident that *ED* will be the correction in question, which is to be added to the 1st altitude, if the track of the ship make an acute angle with the bearing of the sun, and to be subtracted when the angle of direction with respect to the sun is obtuse. The amount of this correction *DE*, is readily found by means of the angle *CED* and the distance *CE*, as already explained (66). Fig. 43.

116. 1. Given the following altitudes of the sun corrected for refraction &c., namely, 18° 30′ and 44° with the intermediate time 3^h, the sun's declination being 20° N, and the place of observation being in north latitude, to find the latitude.

Ans. 54° 01′ N.

2. When the sun's declination was 22° 40′ N. his correct altitude at 10^h 54′ A. M. was 53° 29′, and at 1^h 17′ P. M. it was 52° 48′; required the latitude of the place, it being supposed to be north.

Ans. 57° 08′ 24″ N.

Of Terrestrial Longitude.

117. We have spoken of the difference of longitude of places as deduced from certain data by the principles of plane trigonometry (67 &c). There are other and more correct methods furnished us by astronomy, which remain to be mentioned.

The apparent diurnal motion of the sun round the earth being completed in twenty-four hours, this portion of time is the measure of 360° of longitude, and one hour is the measure of $\frac{360°}{24}$, or 15° of longitude. Now, as time is counted, or the hours, minutes, &c., begin to be reckoned, in different places from the instant of the sun's passage over their respective meridians, the difference of time in any two places is the measure of their difference of longitude, and may be converted into degrees, minutes, &c., by considering one hour as equivalent to 15°, and using the same proportion for a less quantity. A signal, therefore, made from the top of a mountain, or from a balloon, and observed at different places by correct time-keepers, would furnish the means of directly estimating their difference of longitude†.

Celestial phenomena, as eclipses, occultations, &c., are better adapted to this purpose, because they can be seen over a greater portion of the earth's surface. Some of these, as eclipses of the moon, and those of Jupiter's satellites, being an actual obscuration of the body in question, take place at the same point of *absolute* time, and only require to be accurately observed at two different places, in order to obtain their difference of longitude. But with respect to the former, besides their infrequency, the commencement and termination of the phenomenon cannot be precisely noted, on account of the indefiniteness of the earth's shadow. This uncertainty amounts ordinarily to about two minutes, which corresponds to 30 minutes difference of longitude. Eclipses of Jupiter's satellites happen very often, and they admit of great precision as to the time of their occurring; but they cannot be observed without the aid of a telescope, and are therefore of little use to the mariner, on account of the difficulty of using this instrument on board of a ship.

Eclipses of the sun and occultations of stars by the moon, can be observed at sea and accurately noted. But it is to be remarked, that they do not take place at the same point of absolute time, in the different parts of the earth's surface where they are observed. Allowance, however, may be made for this

† It will be perceived, that chronometers would afford the readiest means of ascertaining the difference of longitude of places, if their rate of going could be fully depended upon. Having, for instance, at sea, a watch that accurately shows the time at Greenwich, we have only to find by observation the time for the meridian in which we are situated (102), (that is, how much the watch is too fast or too slow), in order to show the longitude of the place we are in, which will be east or west, according as the time thus found, is later or earlier than that at Greenwich. Chronometers are indeed much used; but, beside the small errors in their rate of going, to which the best are subject, especially in long voyages, they are liable to accidents which cannot be foreseeen, or even known to exist, except by a recourse to other methods of finding the longitude.

lifference, and when this is done, they afford the most accurate means of ascertaining the relative longitude of places.

For the purposes of navigation, these phenomena are liable to the objection of rare occurrence. To supply this defect an ingenious method has been devised, by which any given *distance* of the moon from the sun, or from a star, is substituted for an actual contact, and made use of in the same way. In order, therefore, to apply this method, it is necessary to correct the apparent distance, as actually observed at any particular place, for the effect of parallax and refraction, and thus to reduce it to what it would be, if it were seen from the centre of the earth. Beside the apparent distance and time, the necessary observations are the apparent altitudes of the two bodies in question. With these data, the true distance is found in the following manner.

118. Let *ZM* (*fig.* 62) be the apparent co-altitude or zenith distance of the moon, and *Z m* her true zenith distance, *M m* being the difference between the moon's refraction and her parallax in altitude. In like manner, let *ZS* be the apparent zenith distance of the sun or a star, and *Z s* its true zenith distance, *S s* being the difference between the sun's refraction and parallax, or the refraction simply in the case of a star†. Fig. 62.

There are two cases which present themselves in the solution of this problem; 1, with the three sides *ZS*, *ZM*, *SM*, known by observation, to find the angle *Z*, common to the two triangles *SZM*, *s Z m*; 2, with the two corrected zenith distances *Z s*, *Z m*, and the contained angle *Z*, to find the true distance *s m*. The requisite observations being as below; we proceed according to rules already illustrated.

† Since the observed altitude of any celestial object is affected by refraction and parallax, which always take place in a vertical direction, it is obvious that the observed distance between any two heavenly bodies will be effected by the same causes. In the case of the moon, the effect of parallax always exceeding that of refraction, her true place is above her apparent place, the reverse of which happens with regard to the sun or a star, the parallax of the former never amounting to more than a few seconds, and that of the latter being altogether insensible.

1. Given app. alt. of sun or star S $= 24° 48' 00''$
" " moon M $= 12° 30' 00''$
app. dist. SM $= 51° 28' 35''$

whence, by taking the complements of the altitudes, we have

$ZS = 65° 12' 00''$
$ZM = 77° 30' 00''$
$SM = 51° 28' 35''$

Sum	194° 10′ 35″		
Half sum	97° 05′ 17″		
	77° 30′ 00″		
1st difference	19° 35′ 17″	. log. sin . . .	9,52538
2d difference	31° 53′ 17″	. log. sin . . .	9,722¨9
ZM	77° 30′ 00″	. ar. comp. log. sin	0,01040
ZS	65° 12′ 00″	. . ar. comp. log. sin	0,04202
			19,30059
	26° 33′ 01″	. . log. sin . . .	9,65029
	2		
Z	= 53° 06′ 02″		

2. In the triangle Zsm we have, ZS . . $= 65° 12' 00''$

Refraction + 2′ 02″
Parallax — 0″
Correction . . . . . — 2′ 02″

Zs . . $= 65° 14' 02''$

ZM . . $= 77° 30' 00''$

Parallax —55′ 14″
Refraction + 4′ 32″
Correction . . . . — 50′ 42″

Zm . . $= 76° 39' 18''$

The two sides Zs, Zm, and the contained angle Z, being known, by supposing a perpendicular sP, let fall from s upon Zm, the first of Napier's rules gives

$$\sin(\text{co } Z) = \text{tang}(\text{co } Zs)\ \text{tang } ZP,$$

whence $$\text{tang } ZP = \cos Z \text{ tang } Zs,$$

cos Z	. .	53° 06′ 02″	. . . 9,77845
tang Zs	. .	65° 14′ 02″	. . . 10,33597
tang ZP	. .	52° 27′ 42″	. . . 10,11442
ZM	. .	76° 39′ 18″	
mP	.	= 24° 11′ 36″	

And, by the fourth of Napier's rules, co Zs, co sm, being considered as middle parts, we have the proportion,

as	cos ZP . .	52° 27′ 42″ . .	9,78483
			0,21517
is to	sin (co Zs) = cos Zs . .	65° 14′ 02″ . .	9,62213
so is	cos mP . .	24° 11′ 36″. .	9,96007
to	sin (co sm) = cos sm . . .	51° 09′ 36″ . .	9,79737†

If now we suppose that observations, similar to those above used, are made under another meridian at the same absolute time, and reduced in the same manner, they would evidently give the same true distance of the moon from the sun or a star. The difference of time, therefore, at the two places of observation, as ascertained by well regulated time-keepers, or by calculations founded upon these same observations, would show their difference of longitude; since they would have reference to one common simultaneous occurrence in the heavens††, just as much to be depended upon, and as valuable, as an eclipse of the moon or of one of Jupiter's satellites.

Instead of actual observations at the two given places, it will be seen, that if the true distance and time at one of the places were known by correct tables of the moon's motions, the conclusion would be the same. Hence, by means of a table of the

† There are many ways of abridging the process for obtaining the true from the apparent distance of the moon from the sun or a star. The object of the above is merely to illustrate the essential parts of the operation to the theoretical student.

†† As the moon completes a revolution of 360° in about 30 days, it moves at the rate of 12° in 24 hours, that is, 30′ or its own diameter in an hour. Its change of place, therefore, with respect to the sun, and more especially with respect to stars near its path, will be at a mean about 30″ in a minute of time, a quantity easily distinguished by a good sextant.

true distances of the moon from the sun and certain stars,† *calculated* at sufficiently short intervals, together with the corresponding times for any one particular meridian, as that of Greenwich, for instance, the relative longitude of any other place is readily determined.

119. Let us now suppose, that the foregoing observations were made at a place whose longitude is not known, and that the time of the observations was $10^h\ 11'\ 14''$ in the evening, apparent time; and that the distances of the moon from the same star, approaching the nearest to the above result, as put down in the Nautical Almanac, with the corresponding times, are $51°\ 49'\ 57''$ at 3^h, and $50°\ 21'\ 17''$ at 6^h. Taking the difference of these distances, and the difference between the latter and that in the example, we have the proportion

$$1°\ 28'\ 40'' : 3^h :: 11'\ 41'' : 0^h\ 23'\ 43''$$

Subtracting $0^h\ 23'\ 43''$ from 6^h, we have $5^h\ 36'\ 17''$ for the time at Greenwich, when the distance of the moon from the star was $51°\ 09'\ 36''$, or the time at Greenwich, at the moment when the observation was made, which, as before stated, was $10^h\ 11'\ 14''$. The difference of these times $4^h\ 34'\ 57''$, or, which is the same thing, $68°\ 30'\ 14''$, is therefore the difference of longitude; and we say, moreover, that it is *east*, because the time at the place of observation is later than that at Greenwich; had it been earlier than Greenwich time, the longitude would have been *west*.

1. Given app. alt. of the sun's centre . .	84° 07′ 00″
app. alt. of the moon's centre . .	5° 17′ 00″
app. dist. of sun and moon's centres	90° 21′ 18″
true alt. of sun's centre . . .	84° 06′ 55″
true alt. of moon's centre . . .	6° 09′ 04″

to find the true distance of the sun and moon's centres.

Ans. 89° 29′ 18″

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† The stars, made use of in the Nautical Almanac for this purpose are α *Arietis*, *Aldebaran*, α *Pegasi*, *Pollux*, *Regulus*, *Spica Virginis*, *Antares*, *Fomalhaut*, and α *Aquilæ*. The distances are given for every three hours of apparent time at Greenwich. The time corresponding to any intermediate distance can be found by a simple proportion, since the moon's motion may, without sensible error, be considered as uniform for this space of time.

2. Given app. dist. of the sun and

moon's nearest limbs	38° 14′ 53″
app. alt. of the moon's lower limb .	29° 15′ 59″
app. alt. of the sun's lower limb .	35° 27′ 14″
moon's correct semidiameter . .	15′ 01″
sun's " "	15′ 46″
moon's parallax in alt. . . .	50′ 13″
" refraction	1′ 40″
sun's parallax	07″
" refraction	1′ 19″

It is required to make the reductions and find the true distance of the sun and moon's centres. *Ans.* 38° 28′ 22″.

Let it be supposed that the time of the above observations was 0^h 23′ 32″, or 23′ 32″ past noon, and that the nearest distances put down in the Nautical Almanac for the same day, with the corresponding times, are 37° 50′ 28″ at 3^h, and 39° 34′ 24″ at 6^h. The longitude of the place of observation is required.

Ans. 3^h 42′ 06″ or 55° 30′ 01″ west.

3. Given app. alt. of moon's centre

app. alt. of moon's centre . .	24° 29′ 44″
true alt. " " . . .	25° 17′ 45″
app. alt. of star's centre . . .	45° 09′ 12″
true alt. " " . . .	45° 08′ 15″
app. dist. of moon and star . .	63° 35′ 13″
time at place of observation . .	10^h 15′ 00″

times at Geenwich with the nearest corresponding distances as follows, namely,

at 9^h . . .	dist. . .	62° 49′ 15″
at 12^h . . .	dist. . .	64° 19′ 56″

required the longitude of the place.

Ans. 11° 09′ 30″ E.

Miscellaneous questions to be solved by the rules of Spherical Trigonometry.

120. 1. Given the place of a comet at its first appearance, namely, declination 29° 33′ 12″ N., right ascension 145° 40′ 33″, and also at its last appearance, namely, declination 16° 29′ 08″ N., right ascension 314° 42′ 43″; to find the length of the path described by it while visible, it being supposed to move in the arc of a great circle.

Ans. 132° 44′ 22″.

2. In north latitude at $11^{h.}$ 10′ and at $12^{h.}$ 40′ per watch, the altitude of the sun's lower limb was the same, which being corrected was 26° 55′, and his declination was 5° 17′ S.; required the latitude of the place.

Ans. 57° 09′ N.

3. In north latitude when the sun's declination was 13° 45′ N., his altitude at $8^{h.}$ 39′ 33″ A. M. was 36° 53′. Required the latitude of the place (*fig.* 61).

Ans. 46° 42′.

4. Given the right ascension of Sirius or the *dog star* 99° 18′ 12″, and declination 16° 28′ 21″ S., to find the point of the ecliptic which rises at the same time with the star, in latitude 42° 23′ 28″ N., and thence the time of the year when the star rises *cosmically*, or with the sun†.

5. The diminution of the obliquity of the ecliptic since the time of Eratosthenes having amounted to 23′ 28″, it is proposed to find how much the longest day is diminished and the shortest increased on this account in latitude 42° 23′ 28″.

Ans. 3′ 44″.

6. It is required to find how much the afternoon is increased or diminished at the equinoxes, 21st of March and 23d of September, in latitude 42° 23′ 28″, on account of the sun's change of declination, the amount of this change in $24^{h.}$ according to the Nautical Almanac being 23′ 24″.

Ans. 39″.

7. Given the latitude of the Lizard 49° 57′ N., and its longitude from Greenwich 5° 15′ W., and the latitude of the island St. Mary 36° 57′ N. and its longitude 25° 09′ W.; to find the distance and bearing of the former place from the latter, on the supposition that the earth is a perfect sphere.

Ans. Distance 19° 19′ 21″ = 1159,3 nautical miles,
bearing S. 41° 26′ 45″ W††.

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† An artificial globe will be found of great use in forming the triangles employed in the solution of the above questions.

†† The results of this method will be found to differ somewhat from those obtained by the application of plane trigonometry to Mercator's chart (77), on account of the arc of a great circle differing in its length and position from the loxodromic curve (*note to page* 41).

CHAPTER IV.

Of Surveying and Levelling.

121. SURVEYING consists in several distinct operations; 1, in measuring certain lines and angles in the field to be surveyed; 2, in representing these lines and angles upon paper; 3, in computing the areas or *contents* of the fields or territories thus represented.

The necessary lines and angles are determined in this case like other lines and angles, by means of proper instruments, to which we have had occasion already to refer†. The measures taken in a field are *protracted* or transferred to paper; by the usual problems for the construction of figures (*Geom.* 132, &c). In common surveying the portions of the earth subjected to measurement, being very small, compared with the whole surface, are considered as plane figures. Where the survey extends to large tracts of country, comprehending several degrees of latitude and longitude, allowance is to be made for the curvature of the earth's surface; and the representation should be given according to the laws of projection (24 &c). The plan in this case becomes a map.

Of the contents of fields bounded by straight lines.

122. Let the field, whose content is required, be of the form of a parallelogram, the sides and angles†† being known. In this case the area is equal to the product of the length by the breadth, or base by the altitude (*Geom.* 174).

If the parallelogram be rectangular, as *ABCD* (*fig.* 63), the Fig. 63.

† Beside the instruments employed in the determination of heights and distances, and in navigation, there are others adapted particularly to surveying, the description and use of which will be found in the notes.

†† One of the angles being known, the whole are known, because the opposite angles are equal (*Geom.* 84), and the adjacent ones are supplements of each other (*Geom.* 64). The sines of all the angles therefore are equal to each other.

product of the length by the breadth is the product of any two contiguous sides.

Fig. 64. If the parallelogram be oblique-angled, as $ABCD$ (*fig.* 64), the breadth or perpendicular distance of either two opposite sides, as CP, is equal to the product of the corresponding oblique side CB by the sine of the angle of the parallelogram, radius being unity (*Trig.* 30). Hence, *the area of a parallelogram is equal to the product of any two contiguous sides multiplied by the sine of the contained angle, radius being unity.*

Given $AB = 59$ chains 80 links, or 59,80ch., $AC = 37{,}05$ ch., and $A = 90°$, we have $59{,}8 \times 37{,}05 \times 1 = 2215{,}59$ square chains $= 22155900$ square links. Now, since 10 square chains, or 100000 square links, make an acre, if we divide the area in chains by 10, or the area in links by 100000, and multiply the remainder successively by 4 and by 40, dividing each time by the same number, we shall have the content in the usual denominations employed in surveying; thus,

$$\begin{array}{r} 221{,}559 \\ 4 \\ \hline 2{,}236 \\ 40 \\ \hline 9{,}44 \end{array}$$

Area = 221 acres 2 roods 9,44 perches.

Given $AB = 59{,}80$ ch. $AC = 37{,}05$ ch., the angle $A = 72° \; 10'$, to find the area.

We have $AB \times AC = 2215{,}59$, as before, which, multiplied by 0,95195, the natural sine of 72° 10′, radius being 1, gives 2109,13 = 210a. 3r. 26p.

Or performing the whole by logarithms,

AB . . .	59,80 . . .	log. . . .	1,77670
AC	37,05 . . .	log. . . .	1,56879
A . . .	72° 10′ . . .	log. sin . .	9,97861
	210,91		3,32410

$$\begin{array}{r} 210{,}91 \\ 4 \\ \hline 3{,}64 \\ 40 \\ \hline 25{,}60 \end{array}$$

Area = 210a. 3r. 25,6p.

123. 1. Given the length = 52,25 ch., and the breadth = 38,24 ch., of a rectangular field, to find the area in acres, roods, and perches.

Ans. $199^{a.}\ 3^{r.}\ 8,6^{p.}$

2. Given one side of a parallelogram = 15,36 ch., and its contiguous side = 11,46 ch., the included angle being 47° 30′, to find the area.

Ans. $12^{a.}\ 3^{r.}\ 36^{p.}$

124. Since every parallelogram is divided by its diagonal into two similar and equal triangles (*Geom.* 87), any triangle whatever may be considered as half of a parallelogram. We hence derive the following general rule for those cases where two sides and the included angle are known. *The area of a triangle is equal to half the product of any two of its sides multiplied by the sine of the included angle, radius being unity.*

If the included angle be a right angle, the sine being equal to radius, or 1, the rule will give for the area half the product of the two sides, or, which is the same thing, the product of one side by half the other.

Moreover, since any triangle whatever is equal to a right-angled triangle of the same base and altitude (*Geom.* 170), we can make use of the following simple rule, where the known parts admit of it, as equivalent to the foregoing; namely, *the area of a triangle is equal to the product of the base by half its altitude.*

Given *AB* (*fig.* 65) = 12,38 ch., *AC* = 6,78 ch., and the angle Fig. 65.
A = 46° 24′ to find the area.

12,38	log.	1,09272
6,78	log.	0,83123
40° 24′. . .	log. sin . . .	9,85984
6,0,8986		1,78389
4		
35944		
40		
14,37760		Area = $6^{a.}\ 0^{r.}\ 14^{p.}$

125. 1. Given one side of a triangular field = 18,37 ch., and the perpendicular distance from this side to the opposite angle = 13,44 ch., to find the area.

Ans. $12^{a.}\ 1^{r.}\ 15^{p.}$

2. In a triangular field one side being found by measurement to be 64 perches, and another side 40,5 perches, and the included angle 30°; required the area.

Ans. 4^a. 0^r. 8^p.

3. Required the area of a triangular piece of ground, one side of which measures 19,74 ch., its bearing being N. 82° 30′ W., and another side 17,34 ch., the bearing of this latter from the same station being S. 24° 15′ E.

Ans. 14^a. 2^r. 8^p.

126. When the given parts are a side and the adjacent angles or two sides and an angle opposite to one of them, the side or angle required in order to apply one of the above rules, may be found by trigonometry (*Trig.* 34).

127. It may sometimes happen, either from the want of instruments or the inconvenience of using them in particular situations, that the angles of a triangle are neither of them known. In this case, the three sides being given, we can calculate one of the angles by the rules of trigonometry, or which is preferable, apply the formula

$$A = \sqrt{s(s-b)(s-c)(s-a)} \quad (\textit{Trig. } 75),$$

where a, b, c, are the three sides, s their half, and A the area.

Fig. 66. Given AB (*fig.* 66) $= 49$ ch., $AC = 50{,}25$ ch., $BC = 25{,}69$ ch., to find the area.

	49,00	
	50,25	
	25,69	
Sum . . .	124,94	
Half sum . .	62,47	1,79567
1st remainder	13,47	1,12937
2d remainder	12,22	1,08707
3d remainder	36,78	1,56561
		5,57772
	615,75	2,78886

And 615,75 square chains is equal to 61^a. 2^r. 8^p.

The above formula is one of the most useful in practical geometry, since it enables us to effect the survey of any right-lined field by means of the chain only.

128. 1. Given the three sides a, b, c, of a triangular field; namely, $a = 10{,}64$ ch., $b = 12{,}28$; and $c = 9$, to find the area.

Ans. 4$^{a.}$ 2$^{r.}$ 26$^{p.}$

2. Given the four sides of a quadrilateral field, namely, $AB = 17{,}22$ ch., $AC = 7{,}45$ ch., $CD = 14{,}10$ ch., and $BD = 5{,}25$ ch., together with the diagonal $AD = 15{,}04$ ch., to find the area.

Ans. 8$^{a.}$ 3$^{r.}$ 37,8$^{p.}$

129. Any field bounded by straight lines may be divided into triangles, and the areas of these triangles being computed according to one of the foregoing rules, their sum will be the area of the whole field. Let *ABCDE* (*fig.* 67) be a piece of ground, the sides and angles of which have been measured. By the diagonals *EB*, *EC*, it is decomposed into the triangles *ABE*, *BCE*, *CDE*, which may be computed by one or the other of the rules already given†. Fig. 67.

130. If the given field is bounded in part by a circular curve, as *ABC* (*fig.* 68), this may be separated from the rest of the figure by the radii *AO*, *CO*, or by the chord *AC*. Fig. 68.

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† The diagonals *EB*, *BC*, may be calculated by the rules of trigonometry, and then all the sides of the several triangles being known, their areas are found by article (127). A much shorter method is to take in the compasses the extent of these lines and that of the perpendiculars *AF*, *BG*, *DH*, and apply them to the scale used in protracting the survey, which will give the base and altitude of each of the triangles; then $EB \times \frac{1}{2} AF + EC \times \frac{1}{2} (BG + DH)$ will be the area. It will be perceived, moreover, that instead of measuring the sides and angles of the field, these diagonals and perpendiculars will be sufficient for forming a plan of the field and determining its area.

It may be remarked further, that any four-sided field, as *EBCD*, can be surveyed by measuring either diagonal *EC*, and the perpendiculars let fall from the opposite angles; and the area of the field will be equal to the product of the diagonal by half the sum of the perpendiculars.

If any two sides of a field, or of the portions into which a field is decomposed, be parallel, as *AB*, *EC*, the figure *ABCE* becomes a trapezoid, and its area is found by multiplying half the sum of the parallel sides *AB*, *EC*, by the perpendicular distance *BG* (*Geom.* 178).

Let AO or CO, for example, be 22,50 ch. and the arc ABC, or which is the same thing, the angle AOC equal 23°. Since the sectors of circles are as the number of degrees contained in the arc of the sector, as 360° is to the area of the whole circle or $(22{,}50)^2 \times 3{,}1416$, so is 23° to the area of the sector equal to $\frac{(22{,}50)^2 \times 3{,}1416}{360} \times 23°$ (*Geom.* 290), and by logarithms,

360	log.	2,55630
		7,44370
22,50	2 log.	2,70436
3,1416	log.	0,49714
23	log.	1,36173
Area of $ABCO$ = 1,016		2,00693

$$\begin{array}{r} 1{,}016 \\ 4 \\ \hline 0{,}64 \\ 40 \\ \hline 25{,}60 \end{array}$$

Ans. 10ᵃ· 0ʳ· 25,6ᵖ·

131. A segment of a circle being the difference between the corresponding sector and the triangle contained by the two radii and the chord of the given arc, having found the area of the sector $ABCO$, we obtain that of the segment $ABCD$ by subtracting from the former the area of the triangle AOC.

132. Grounds are sometimes laid out in the form of an ellipse, the area of which is found by multiplying the product of the two axes, or greatest and least diameters by the decimal 0,7854†.

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† Since the ordinates to the transverse axis of an ellipse are to the corresponding ordinates of the circle in a constant ratio, that is, as the semiconjugate to the semitransverse (*Trig.* 114), the sum of the ordinates in the ellipse (or the area of the ellipse) is to the sum of the corresponding ordinates in the circumscribed circle (or the area of the circle) as the conjugate axis is to the transverse.

Again, since the ordinates to the conjugate axis of an ellipse are to the corresponding ordinates of the circle as the transverse axis to the conjugate, the area of the ellipse is to that of the inscribed circle as the transverse axis is to the conjugate. The above proportions may be stated thus, This figure being a mean proportional between its circumscribed and inscribed circles, that is, equal to a circle whose diameter is a mean proportional between the axes of the ellipse, we may consider the product of the two axes of any ellipse is the square of the diameter of a circle of the same area. But the diameter of a circle being squared and multiplied by 0,7854 &c., (the area of a circle whose diameter is one†), the product will be the area. Hence *the area of an ellipse is found by multiplying the product of the two axes or greatest and least diameters by* 0,7854 &c.

133. Sometimes the boundary of a field is irregularly curved, or is made up of straight lines of small extent, as *ABCD* &c. (*fig.* 69). In this case it is usual, especially where great accuracy is not required, to assume a line, as *A′ M′*, from which the perpendicular distances or *offsets A′A, B′B, C′C*, &c., are measured; and in computing the contents the mean of all these distances is taken as equivalent to the average breadth of the part comprehended between *ABCD* &c. and *A′ M′*, and the area is estimated accordingly. Fig. 69

134. The area of a field may be computed by means of the difference of latitude and departure corresponding to the direction and length of the several sides. Thus in the field *ABCD* (*fig.* 70) beginning at the westernmost point *A*, with the bearing of *AB* and its distance, we find the difference of latitude *A b* and departure *b B*, as in navigation†. We proceed in like manner Fig. 70.

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ellipse : circumscribed circle : : conjugate : transverse,
ellipse : inscribed circle : : transverse : conjugate.

If we take the products of the corresponding terms, the two last terms of this new proportion become the same; whence the two first terms are equal, that is, the square of the ellipse is equal to the product of the circumscribed and inscribed circles, or in other words, the ellipse is a mean proportional between the two inches.

† The diameter of a circle being 1, its circumference is 3,14159, &c. (*Geom.* 294). But the area is found by multiplying the circumference by half of the radius or one fourth of the diameter, (*Geom.* 289). Whence $\frac{1}{4}$ of 3,14159 &c., or 0,7854 &c., is the area of a circle whose diameter is 1.

† Instead of calculating each of the latitudes and departures by the rules of trigonometry it is usual to take them from a table of latitudes and departure prepared for problems of this kind.

with each of the other sides, and thus obtain successively $b\,c$, $m\,C$, $n\,D$ or $c\,d$, $C\,n$, $A\,d$, and $D\,d$.

The area of the space $AB\,b$ is found by multiplying half the departure $b\,B$ by the difference of latitude $A\,b$. So also the area of $b\,B\,C\,c$ is found by multiplying half the sum of the departures $b\,B$, $c\,C$, by the difference of latitude $b\,c$, and the area of $A\,d\,D$, is equal to the product of half $d\,D$ by $A\,d$.

But if these several areas be subtracted from the trapezoid $c\,C\,D\,d$, or from $\frac{1}{2}\,(c\,C + d\,D) \times c\,d$, we shall have for the remainder the area of $ABCD$.

It will be seen that all the areas to be subtracted belong to those sides of the field, AB, BC, DA, whose bearing is northerly and whose differences of latitude are all in the same direction; and that the trapezoid $c\,C\,D\,d$ belongs to the side of the field CD, whose bearing or difference of latitude is southerly. Moreover, the departures, 0, $b\,B$, $c\,C$, $d\,D$, 0, are obtained by adding successively the departure corresponding to each side of the field to the preceding, beginning and ending with 0, and regarding the east departures as plus, and the west departures as minus.

The above reasoning is applicable to any rectilinear figure, whatever the number and position of the sides, as will be evident from a slight inspection of figure 71.

When the bearing of one of the sides, as DE, is due east or west, the corresponding difference of latitude being 0, the area of $\frac{1}{2}(d\,D + d\,E) \times 0$ is nothing. Also when one of the sides is due north or south, as FG, the corresponding departure being 0, $\frac{1}{2}(f\,F + g\,G)$ is equal to $f\,F$ or $g\,G$.

Sometimes in the case of a reentering angle like AIH, a portion of the figure AIr is reckoned twice. But this is corrected by the subtractive space $h\,HI\,i$, which includes not only the exterior portion $h\,HI\,A$, but also the whole additive triangle $AI\,i$ belonging to the last side of the figure IA.

When the above method is adopted, it will be found convenient to arrange the several results as in the following table, adapted to figure 70.

Bearings.		Dist	N.	S.	E.	W.	Dep.	Sum †	North areas.	South areas.
AB	N. 23° E.	17	15,66		6,64		6,64	6,64	103,92	
BC	N. 83° E.	11	1,34		10,92		17,56	24,20	32,43	
CD	S. 14° E.	23		22,32	5,56		23,12	40,68		907,98
DA	N. 77° W.	23,66	5,33			23,05		23,12	123,23	
			12,32	22,32	23,12	23,05			259,58	907,98
										259,58
										648,40
									Half	324,20

Area $= 32^{a.}\ 1^{r.}\ 27,2^{p.}$

It may be remarked that, when the several operations are performed with perfect accuracy, the sum of the northings will be equal that of the southings, and the sum of the eastings to that of the westings. This necessarily follows from the circumstance of the surveyor's returning to the place from which he set out; and it affords a means of judging of the correctness of the work. But it is not to be expected that the measurements and calculations in ordinary surveying will strictly bear this test. If there is only a small difference, as in the above example, between the northings and southings, or between the eastings and westings, it may be imputed to slight imperfections in the measurements. If the difference is considerable, the work should be reviewed, and if no error can be discovered, the difference above mentioned ought to be apportioned among the differences of latitude and departure in such a manner as to produce the least possible change in the given numbers. This is done by the following proportions. As the sum of the boundary lines *AB*, *BC*, &c., is to the error in latitude, so is the length of any particular boundary to the correction of its corresponding difference of latitude; and as the sum of the same boundary lines is to the error in the de-departure, so is any particular boundary to the correction of the corresponding departure. The correction in each case is additive or subtractive, according as it belongs to the column whose sum is the least, or to that whose sum is the greatest ††; thus, in the above example,

† Instead of multiplying half the sum of the departures at each step by the difference of latitude, it is more convenient in practice to employ the entire sum of the departures, and then to take half the final result, as in the above example.

†† The learner may find a demonstration of the above rule by Bowditch, and also by professor Adrain, in the Analyst, No. 4, edited by the latter gentleman.

$AB + BC +$ &c. $= 74,66 : 0,07 :: 17 : 0,02$
$:: 11 : 0,01$
$:: 23 : 0,02$
$:: 23,66 : 0,02$

The three first of the above corrections belonging to the column whose sum is the greatest, they are to be subtracted from their respective departures. The fourth correction for the opposite reason is additive. They are applied below according to their signs.

E.	W.	Correction.	Cor. E.	Cor. W.
6,64		— 0,02	6,62	
10,92		— 0,01	10,91	
5,56		— 0,02	5,54	
	23,05	+ 0,02		23,07
		Sum	23,07	23,07

The departures, thus corrected, become equal, and the corrected area is 32a. 1r. 18,1p.

The above will serve as an illustration of the rule. In most cases both the latitudes and departures require correction.

One advantage of the foregoing method of computing the contents of a field is, that it may be directly applied to the original minutes taken of the survey, without any plan being drawn, and without relying in any degree upon the accuracy of a constructed figure.

135. 1. Given the following bearings and distances of the several sides of a field, namely,

1.	N. 58° E.	19ch.
2.	E. 6° S.	20
3.	S. 17° W.	20
4.	W.	20
5.	N. 42° 35′ W.	15,10

to find the area.

Ans. 54a. 3r. 24p.

2. Given the following bearings and distances, namely,

1.	N. 45° E.	40 ch.
2.	S. 30° W.	25
3.	S. 5° E.	36
4.	W.	29,60
5.	N. 20° E.	31

to find the corrected differences of latitude and departure.

Ans.

	N.	S.	E.	W.
1.	28,30		28,30	
2.		21,63		12,49
3.		35,84	3,16	
4.	0,02			29,59
5.	29,15		10,62	
	57,47	57,47	42,08	42,08

Division of Land.

136. There is often occasion after surveying a piece of ground to divide it into portions or *lots*, of certain given dimensions or bearing a certain proportion to the whole.

We have already given general methods adapted to questions of this kind (*Geom.* 255 &c. *Trig.* 80 &c.). But there are particular problems occuring in surveying that admit of very simple solutions.

137. Suppose, for example, that the given field is of a triangular form, and that it is proposed to divide it into two parts that shall be to each other as m to n.

1. If the dividing line is to proceed from one of the angles, as A (*fig.* 72), we divide the opposite side CB into two parts, having the ratio to each other of m to n (*Geom.* 236), and draw the line Ae from the vertex of the angle A to the point of division e. Then since the triangles ACe, AeB have the same altitude, they must be to each other as their bases, that is, as m to n. Fig. 72.

Whatever the number and ratio of the parts into which CB is divided, the partial triangles, found as above, will be to each other as these parts.

2. If the dividing line is required to be parallel to one of the sides of the field as AB (*fig.* 73), the portion cut off being similar to the whole, we shall have (*Geom.* 218), Fig. 73.

$$CAB : Cee' :: \overline{CA}^2 : \overline{Ce}^2,$$

or

$$m+n : m \quad :: \overline{CA}^2 : \overline{Ce}^2;$$

whence

$$\overline{Ce}^2 = \frac{\overline{CA}^2\, m}{m+n}$$

and

$$Ce = \sqrt{\overline{CA}^2 \frac{m}{m+n}} = CA \sqrt{\frac{m}{m+n}}$$

therefore Ce is a mean proportional between CA and $\sqrt{\frac{m}{m+n}}$.

If the side CA be 15ch., and the portion Cee', to be cut off, be one half of the triangle CAB, we shall have

$$CA \sqrt{\frac{m}{m+n}} = 15 \sqrt{\tfrac{1}{2}} = 15 \times 0{,}707 = 10{,}605\text{ch.}$$

Fig. 74. If it were proposed to divide the triangle ABC (*fig.* 74) into three equal parts, by lines parallel to AB, it is evident from what is above shown that we should, have

$$Cd = CA \sqrt{\tfrac{1}{3}}, \text{ and } Cd' = CA \sqrt{\tfrac{2}{3}}.$$

3. The conditions of the problem may require that the divid-
Fig. 75. ing line or lines should proceed from a given point D (*fig.* 75) in one of the sides.

If it were proposed, for instance, to divide the triangle into three equal parts. Having divided the side AC according to this same proportion, through the points of division F, F', we draw eF, $e'F'$ parallel to BD; then joining eD, $e'D$, we shall have AeD, eDe', $e'DC$, equal to each other, or in the ratio of the lines AF, FF', $F'C$. This will be rendered evident by supposing lines drawn from B to F and F', making three triangles of the same altitude and consequently having the same ratio to each other as their bases. But, since eF, $e'F'$, are each parallel to BD, AeD is equal to ABF, and $Ce'D$ to CBF'.

Knowing therefore AB and AD as well as AC, we have only to find a fourth proportional Ae, to the three given lines AD, AB, $\frac{AC}{3}$ or AF (*Geom.* 237). The point e' is found in the same manner.

The above method is evidently applicable to a division of the given triangle into a greater number of parts, and such as bear a different relation to each other.

Fig. 76. 4. The problem may require that the dividing line EF (*fig.* 76) should be perpendicular to one of the sides of the given triangle. In this case, the ratio of the parts being expressed by $m : n$, let $AE = x$, $EF = y$, $AD = a$, $AB = b$, $CD = h$; we have,

$$ABC \text{ or } \frac{bh}{2} : AEF \text{ or } \frac{xy}{2} :: m+n : m.$$

whence

$$\frac{xy}{2} = \frac{\frac{1}{2}bhm}{m+n}.$$

But, since by similar triangles,

$$a : h :: x : y, \quad y = \frac{hx}{a}.$$

Substituting for y this value, we obtain

$$\frac{hx^2}{2a} = \frac{\frac{1}{2}bhm}{m+n} \text{ or } x^2 = \frac{b \cdot am}{m+n}.$$

We find therefore that x or AE is a mean proportional between b and $\frac{am}{m+n}$. If $m = n$, the triangle AFE being one half of ACB, we should have

$$x^2 = \frac{ab}{2},$$

that is, AE is a mean proportional between the base AB and half of the segment AD. AE may chance to be greater than AD, in which case we should designate cB by x, and BD by a.

138. The division to be performed may relate to a field of four or more sides. In the case of a parallelogram, if the dividing line, as EF (*fig.* 77), be required to be parallel to one of the sides AB, the distance at which this line is to be drawn, may be found by dividing the area of the portion to be separated by AB. If the parallelogram be oblique-angled (*fig.* 78), and the conditions of the problem require that the dividing line EF should be perpendicular to one of the sides, and meet at the same time the opposite†, since the figure to be cut off will be a trapezoid, dividing it by EF, supposed to be known, we have for the quotient the distance of EF from I the middle of AD (*Geom.* 179). Fig. 77 Fig. 78.

139. With respect to other quadrilaterals and figures of more than four sides, if we suppose two sides AC, BD (*fig.* 79), produced till they meet, they will form with the intercepted side CD, a triangle, which, as all the parts of given figures are supposed to be known, will be known also. This being added to the part that is to be cut off, the problem reduces itself to one or the other of the cases already considered. Fig. 79.

† If the dividing line do not meet both the sides of the parallelogram, the portion to be cut off is a triangle, and the case refers itself to art. 137,4.

Levelling.

140. Two or more points are said to be on a level when they are equally distant from the centre of the earth†, or from the surface of a tranquil fluid, supposed to be situated immediately above or below them. A level surface, therefore, is one that is every where perpendicular to a plumb-line, or the radius of the earth considered as a sphere. This is called a *true level,* while a straight line or plane that is perpendicular to the radius of the sphere or plumb-line only at one point, is denominated an *ap-*
Fig. 80. *parent level.* Thus AB (*fig.* 80) represents an apparent level, AD a true level, and BD the deviation of the one from the other, or the *difference of level* of the points A, B, referred to a tangent at A.

141. Knowing the tangent AB, we readily find BD by the proportion,

$$BH : AB :: AB : BD \text{ (Geom. 228)},$$

which gives

$$BD = \frac{\overline{AB}^2}{BH} = \frac{\overline{AB}^2}{2\,CD + BD}.$$

But, as BD is always small in comparison with $2\,CD$, the diameter of the earth, it may be neglected in the second member of the above equation†† ; and, in most cases, for the same reason, AD may be considered as equal to AB; whence

$$BD = \frac{\overline{AD}^2}{2\,CD}.$$

In like manner, for another distance AD', we shall have

$$B'D' = \frac{\overline{AD'}^2}{2\,CD};$$

---

† The small errors committed by supposing the earth a sphere instead of a spheroid, are safely neglected in the common operations of levelling.

†† If it were necessary we might find the difference of level without neglecting BD. Thus from the above equation we obtain

$$\overline{BD}^2 + 2\,CD \times BD = \overline{AB}^2,$$

or, calling BD x, CD a, and AB b,

$$x^2 + 2\,a\,x = b^2, \text{ and } x = -a + \sqrt{b^2 + a^2} \text{ (Alg. 109)}.$$

and

$$BD : B'D' :: \frac{\overline{AD}^2}{2\,CD} : \frac{\overline{AD'}^2}{2\,CD}$$
$$:: \overline{AD}^2 : \overline{AD'}^2,$$

that is, *the difference of level for different distances, is as the square of the distance.*

The distance AD being supposed, for example, = 1 statute mile or 5280 feet, and $2\,CD$, the diameter of the earth = 7912 miles, or 7912 × 5280 feet, we have

$$BD = \frac{(5280)^2}{7912 \times 5280},$$

and by logarithms,

5280	. . . 2 log.		7,44521
7912	. . . log. . .	3,89829	
5280	. . . log.	3,72263	
			7,62092
0,6673			— 1,82434
12			

$BD = 8{,}0076$ inches.

Thus the difference between the apparent and true level, answering to a distance of one mile, is 8 inches.

142. For any other distance, as $2\frac{1}{2}$ miles for instance, instead of repeating the above process, we can use the proportion

$$1^2 : (2,5)^2 = 6\tfrac{1}{4} :: 8\text{ in} : 52\text{ in}†.$$

143. The difference of level of two stations is sometimes computed by means of the zenith distance of each station as observed from the other. Let A, B (*fig.* 81), be two stations at which Fig. 81.
the zenith distances are observed; namely, ZAB, VBA, formed

† The difference of level for one mile being in feet $\frac{5280 \times 5280}{7912 \times 5280}$ or $\frac{5280}{7912}$, that is, $\frac{2}{3}$ very nearly, and the difference of level for any other distance being as the square of the distance, we have the following convenient rule for finding the difference of level, namely, *take two thirds of the square of the distance in miles for the difference of level in feet nearly.* Thus in the above example, $\frac{2}{3}\,(2,5)^2$ or $\frac{2}{3}\,6\frac{1}{4} = 4\frac{1}{3}$ feet or 52 inches.

by the vertical lines CZ, CV; and the straight line AB. If through the point A we draw the chord AB', parallel to the terrestrial chord $a\,b$, the points A, B', will be on a level, and BB' will be the height of the point B above the point B'. The arc AB, being supposed to be known, may on account of its smallness be taken for its chord; then, by the common theorem for plane triangles, we have

$$\sin BAB' : \sin ABB' :: BB' : AB',$$

whence

$$BB' = \frac{AB' \times \sin BAB'}{\sin ABB'}.$$

But

$$\begin{aligned} BAB' &= 180° - ZAB - B'AC \\ &= 180° - ZAB - (90° - \tfrac{1}{2}\,C) \\ &= 90° - ZAB + \tfrac{1}{2}\,C; \end{aligned}$$

and

$$ABB' = 180° - VBA.$$

Now from the triangle ABC we have

$$180° = C + (180° - ZAB) + (180° - VBA),$$

or

$$90° = \tfrac{1}{2}\,C + 90° - \tfrac{1}{2}\,ZAB + 90° - \tfrac{1}{2}\,VBA;$$

whence

$$90° = \tfrac{1}{2}\,(ZAB + VBA) - \tfrac{1}{2}\,C.$$

Substituting this value for 90° in the above expressions for BAB', ABB', we obtain

$$\begin{aligned} BAB' &= \tfrac{1}{2}\,(ZAB + VBA) - \tfrac{1}{2}\,C - ZAB + \tfrac{1}{2}\,C \\ &= \tfrac{1}{2}\,(VBA - ZAB) \\ ABB' &= 90° + \tfrac{1}{2}\,(ZAB + VBA) - \tfrac{1}{2}\,C - VBA \\ &= 90° + \tfrac{1}{2}\,(ZAB - VBA - C). \end{aligned}$$

Putting these values for BAB', ABB', in the expression for BB', and designating the chord of the arc AB' by K, we have

$$\begin{aligned} BB' &= \frac{K \sin \tfrac{1}{2}\,(VBA - ZAB)}{\sin\left(90° + \tfrac{1}{2}\,(ZAB - VBA - C)\right)} \\ &= \frac{K \sin \tfrac{1}{2}\,(VBA - ZAB)}{\cos \tfrac{1}{2}\,(VBA - ZAB + C)}. \end{aligned}$$

144. This formula is exact. But in many cases $\tfrac{1}{2}C$ may be neglected; then, since $\frac{\sin}{\cos} = \text{tang}$, the expression becomes

$$BB' = K \text{ tang } \tfrac{1}{2}\,(VBA - ZAB).$$

When VBA exceeds ZAB, BB' is positive, otherwise BB' is negative, ZAB being the observed zenith distance at the place whose elevation $A\,a$ above an assumed level, as that of the sea, is known, and VBA being the observed zenith distance at the place whose elevation is sought.

145. When only one of the zenith distances can be taken, since $VBA = BAC + C = 180° - ZAB + C$, if we substitute this value for VBA in the formula

$$BB' = K \text{ tang } \tfrac{1}{2}\,(VBA - ZAB),$$

we shall have

$$\begin{aligned} BB' &= K \text{ tang } \tfrac{1}{2}\,(180° - ZAB + C - ZAB), \\ &= K \text{ tang } \ (90° - ZAB + \tfrac{1}{2}\,C) \\ &= K \cot \ (ZAB - \tfrac{1}{2}\,C); \end{aligned}$$

and BB' is positive or negative according as ZAB is less or greater than 90°.

146. The above formulas suppose the distance K to be given. The difference of level may, however, be determined without knowing this line. Indeed the formula

$$BB' = K \text{ tang } \tfrac{1}{2}\,(VBA - ZAB)$$

may be made to involve only the zenith distances and radius of the earth. For, since half the chord of any arc is equal to the product of radius by the sine of half this arc (*Trig.* 30), we shall have

$$\begin{aligned} BB' &= 2\,R \sin \tfrac{1}{2}\,C \text{ tang } \tfrac{1}{2}\,(VBA - ZAB) \\ &= -\,2\,R \cos \tfrac{1}{2}\,(ZAB + VBA) \text{ tang } \tfrac{1}{2}\,(VBA - ZAB)\dagger. \end{aligned}$$

The sign of BB', depends upon that of the factors of the second member of the equation.

147. When there is a series of signals, by proceeding in the manner above explained, we determine only their *relative heights*. But it is easy to find the *absolute heights*, or elevations above the same horizon, as that of the sea for example. Let us suppose that the points $B, B', B'', \ldots B^n$, the summits of the signals, are unequally elevated above a common horizon, and that h' represents the elevation of the point B' above B, h'' that of B'' above B', d''' the depression of the point B''' below B'', and so on. By taking H equal to the sum of the elevations $h' + h'' +$ &c., and D equal to the sum of the depressions, we shall have

$$H - D = \text{difference of level};$$

and B^n will be above or below B, according as H is greater or less than D.

---

† From the expression $90° = \tfrac{1}{2}\,(ZAB + VBA) - \tfrac{1}{2}\,C$ (143) we have $\tfrac{1}{2}\,C = \tfrac{1}{2}\,(ZAB + VBA) - 90°$,

and $$\sin \tfrac{1}{2}\,C = -\cos \tfrac{1}{2}\,(ZAB + VBA).$$

If N denote the height of the point B above the level of the sea, $N + H - D$ will be the height of any other point B^n above this same level; and it is obvious that we have only to subtract the length of the signal from the absolute height of its summit, in order to obtain the absolute height of the ground on which it is placed.

148. In the above formula it is supposed that the points A, B, are the summits of the signals employed; but the instrument used for taking the angles can seldom be placed precisely at Fig. 82. these points. In observing B (*fig.* 82) for instance, the instrument for the most part is at some point a below A. At the other station also, the place of observation, instead of being at B, is usually at some lower point b. The zenith distances actually observed therefore will be ZaB, VbA, instead of ZAB, VBA.

In this case, since

$$ZaB + aBA = ZAB \text{ (Geom. 78)},$$

if we add aBA to the zenith distance as observed at a, we shall have the zenith distance such as it would be found to be if the instrument were placed at A. Now AB and the distance Aa being supposed to be known, we obtain the above correction by the following proportion,

$$AB : \sin AaB :: Aa : \sin aBA,$$

which gives

$$\sin aBA = \frac{Aa \sin AaB}{AB}$$

$$= \frac{R'' \, Aa \sin AaB}{AB}$$

in seconds, R'' being the radius in seconds†.

149. In what we have said upon the subject of levelling, we have supposed that light, in coming from an object to the eye,

† The expression for the angle is reduced to seconds by dividing it by 1″, or (on account of the smallness of the difference) by the sine of 1″; or, which amounts to the same thing, by multiplying it by the number of seconds contained in an arc equal in length to radius. But, since the ratio of the diameter to the circumference is as 1 to 3,14159 &c., when the radius is 1 or the diameter 2, the circumference is 2 × 3,14159 &c., and the semi-circumference or 180° is 3,14159 &c.; consequently

$$3{,}14159 \text{ \&c.} : 1 :: 180° : R°$$

whence

$$R = \frac{180°}{3{,}14159 \text{ \&c.}} = 57° \, 17' \, 44{,}8'', \quad R' = 3437{,}75', \quad R'' = 206264{,}8''.$$

The logarithm of R'', which we have frequent occasion for, is 5,31443.

proceeds in a straight line. But it is to be observed that when a ray traverses obliquely the different strata of the atmosphere, it is slightly curved in a vertical plane,† and it is in the direction of a tangent to this curve that the object is actually seen. Thus, if BDA (*fig.* 83) represent the path described by a ray of light in passing from B to A, the object B will be seen at B' in the direction of a tangent at the point A. Fig. 83.

Let C (*fig.* 84) be the centre of the earth, and A, B, two signals, A' the apparent place of A as seen from B, and B' the apparent place of B as seen from A. ZAB', VBA', will be the apparent zenith distances, ZAB, VBA, the true zenith distances, and the difference between the former and the latter respectively will be the refraction sought. Fig. 84.

Since $$ZAB = C + ABC$$
and $$VBA = C + BAC$$
we shall have
$$ZAB + VBA = 2\,C + ABC + BAC = 180° + C;$$
if we subtract from this quantity the apparent zenith distances, we shall have the sum of the two refractions, namely,
$$r + r' = 180 + C - ZAB' - VBA'$$
$$= C - (ZAB' + VBA' - 180°)$$
or, by considering $r = r'$,
$$r = \tfrac{1}{2}\left(C - (ZAB' + VBA' - 180°)\right);$$
that is, *we subtract the sum of the depressions of the two signals below the horizon from their distance asunder, considered as an arc of a great circle of the earth, and take half the difference for the terrestrial refraction.*

150. At 18,6 yards below the top of the signal A, the zenith distance of the upper extremity of the signal B was found by observation to be 90° 13′; and at 16,5 yards below the upper extremity of B the zenith distance of the point A was 89° 56′, the rectilineal distance between the two signals being 31172,8 yards. It is proposed to determine the amount of the refraction.

We first reduce the zenith distances to the tops of the signals by the formula
$$\sin aBA = \frac{R'Aa \sin Aa B}{AB} \quad (148).$$

† There are sometimes lateral deviations, but these are considered as exceptions to the general law.

Thus, to find the reduction in seconds,

AB . .	31172,8 . .	4,49378	AB		4,49378
		5,50622			5,50622
Aa . . .	18,6 . .	1,26951	Bb . .	16,5 . . .	1,21748
sin AaB . .	90° 13′	10,00000	sin AbB . .	89° 56′ . .	10,00000
R''		5,31443	R''		5,31443
$123'' = 2'\,03''$ . .		2,09016	$109'' = 1'\,49''$ . . .		2,03813

Fig. 84. AaB (*fig.* 84) $= 90°\ 13'\ 00''$ $\qquad AbB = 89°\ 56'\ 00''$

$\qquad\qquad\qquad\qquad 2'\ 03'' \qquad\qquad\qquad\qquad 1'\ 49''$

$ZAB' = 90°\ 15'\ 03'' \qquad VBA' = 89°\ 57'\ 49''$

151. Now in order to apply the formula for the terrestrial refraction, since one minute of a degree is equal 6076 feet or 2025,3 yards, 31172,8 yards $= 15'\ 23''$ = the angle C; whence, by the formula

$$r = \tfrac{1}{2}\,(C - (ZAB' + VBA' - 180°)),$$

we have

$$\begin{aligned} r &= \tfrac{1}{2}\,(15'\ 23'' - (90°\ 15'\ 03'' + 89°\ 57'\ 49'' - 180°)),\\ &= \tfrac{1}{2}\,(15'\ 23'' - 12'\ 52'')\\ &= \tfrac{1}{2}\,(\ 2'\ 31'') = \ 1'\ 15{,}5''. \end{aligned}$$

If we divide both members of the equation by C, we shall have

$$\frac{r}{C} = \frac{1'\ 15{,}5''}{15'\ 23''} = \frac{75{,}5''}{923''} = 0{,}08 \text{ or } \tfrac{1}{12} \text{ nearly},$$

whence

$$r = \tfrac{1}{12}\,C \text{ nearly};$$

and for other distances r is found to bear about the same proportion to the angle subtended at the centre, that is, *the terrestrial refraction is ordinarily about one twelfth of the distance between the observer and the object, considered as an arc of a great circle of the earth.* Suppose, for instance, that the top of a ship's mast is seen twelve nautical miles off at sea, this distance, considered as an arc of the circumference of the earth, or as subtending an angle at the centre, amounts to 12′, one twelfth part of which, or 1′, is the angular elevation of the object produced by refraction.

Fig. 83. Thus ADB (*fig.* 83) being supposed equal to 12 nautical miles, B will appear from the point A at B', 1′ minute above its true

place, and BB' may be considered as the natural sine of an arc of one minute, belonging to a circle whose radius is 12 nautical miles. Now the sine of 1′, radius being unity, is 0,00029 (*see table of natural sines*). Accordingly we have

$$BB' = 0{,}00029 \times 12 = 0{,}00348 \text{ of a mile}$$
$$= 0{,}00348 \times 6076 = 21{,}14 \text{ \&c. feet.}$$

It follows therefore that at the distance of 12 nautical miles an object is seen about 20 feet above its true place.

152. It is to be observed, however, that the terrestrial refraction is found to vary considerably in different countries, and at different seasons in the same country. It is estimated at $\frac{1}{10}$ by Dr Maskelyne, and at $\frac{1}{14}$ by Legendre. In France, according to Delambre, it is about 0,075 in summer, 0,08 in spring and autumn, and from 0,09 to 0,1 in winter.

153. The correction for refraction being applied† to the apparent zenith distances as observed at A and B (*fig.* 81), the true difference of level is found by one of the formulas above investigated; thus Fig. 81.

$$BB' = \frac{K \sin \frac{1}{2}(VBA - ZAB)}{\cos \frac{1}{2}(VBA - ZAB + C)}$$
$$= \frac{K \sin \frac{1}{2}(89^\circ\ 57'\ 49'' - 90^\circ\ 15'\ 03'')}{\cos \frac{1}{2}(89^\circ\ 57'\ 49'' - 90^\circ\ 15'\ 03'' + 15'\ 23'')}$$
$$= \frac{31172{,}8 \sin \frac{1}{2}(-17'\ 14'')}{\cos \frac{1}{2}(-17'\ 14'' + 15\ 23'')};$$

or by logarithms,

$\frac{1}{2}(-17'\ 14'' + 15'\ 23'') = -\frac{1}{2}(1'\ 51'')$	55″	log. cos	10,00000
			0,00000
$\frac{1}{2}(-17'\ 14'')$	8′ 37″	log. sin	7,39906
	31172,8		4,49378
$BB' =$	— 78,134 yds.		1,89284

As VBA is less than ZAB, BB' is negative, that is, the distance of B from C is greater than that of A from C.

† It is not necessary to apply the correction for refraction to obtain the difference of level, when we employ the formula involving both the zenith distances, since, by adding the same quantity, 1′ 15″ for instance, to each of the zenith distances VBA, ZBA, the difference remains unaltered.

154. We obtain very nearly the same result by the formula in which $\frac{1}{2}C$ is neglected (144). Thus

$$BB' = K \text{ tang } \tfrac{1}{2}(VBA - ZAB)$$
$$= 31472,8 \text{ tang } 8' 37'';$$

and by logarithms,

	31172,8	. . . .	4,49378
tang . .	8′ 37″	. . . .	7,39907
BB' = . .	— 78,135	. . . . . .	1,89285

BB' is negative for the reason already mentioned (153).

155. If only one of the zenith distances ZAB were known, we should make use of the formula

$$BB' = K \cot ZAB - \tfrac{1}{2}C.$$

Thus, in the present case,

$$BB' = 31172,8 \cot (90^\circ 15' 03' + 1' 15'' - 7' 42'')\dagger,$$

and, by logarithms,

31172,8	4,49378
$90^\circ 15' 03'' + 1' 15''$ or $90^\circ 16' 18'' - 7' 42'' = 90^\circ 08' 36''$ log cot	7,39822
BB' — 77,98	1,89200

156. When K is unknown, both the zenith distances being given, we have recourse to the formula

$$BB' = -2R \cos \tfrac{1}{2}(ZAB + VBA) \text{ tang } \tfrac{1}{2}(VBA - ZAB),$$

which in the above example becomes

$$BB' = -7912 \times 1760 \cos \tfrac{1}{2}(90^\circ 16' 18 + 89^\circ 59' 04'') \text{ tang } \tfrac{1}{2}(89^\circ 59' 04'' - 90^\circ 16' 18'')$$

7912	3,89829
1760	3,24551
$\frac{1}{2}(90^\circ 16' 18'' + 89^\circ 59' 04'') = \frac{1}{2}(180^\circ 15' 22'' = 90^\circ 7' 41''$ log cos	7,34928
$\frac{1}{2}(89^\circ 59' 04'' - 90^\circ 16' 18') = \frac{1}{2}(17' 14'') = 8' 37''$...log tang...	7,39906
BB' — 78,08	—1,89214

Fig. 85. 157. The zenith distance ZAB (*fig.* 85) of the surface of the sea, as observed from the top of a mountain, being given, it is proposed to find the height of the mountain.

We might make use of the formula $BB' = K \cot (ZAB - \frac{1}{2}C)$ (145), since $C = ZBA - 90^\circ$ (*Geom.* 78), and $K = \frac{R \sin C}{\sin ZBA}$. But a more convenient and exact formula may be obtained.

A being a right angle, we have the proportion

† The correction for refraction + 1′ 15″ is applied here, because only one zenith distance is used.

$\sin B = \cos C : CA = R :: \sin A = 1 : CB = R + BB'$,
whence

$$R + BB' = \frac{R}{\cos C}$$

$$BB' = \frac{R}{\cos C} - R = \frac{R - R\cos C}{\cos C} = R\left(\frac{1 - \cos C}{\cos C}\right).$$

But, since $1 - \cos C = \sin C \operatorname{tang} \frac{1}{2} C$†, we obtain

$$BB' = R\left(\frac{\sin C \operatorname{tang} \frac{1}{2} C}{\cos C}\right) = R \operatorname{tang} C \operatorname{tang} \frac{1}{2} C$$

$$= R \operatorname{tang} (VBA - 90^\circ) \operatorname{tang} \tfrac{1}{2} (VBA - 90^\circ),$$

or, since $VBA - 90^\circ$ is very small,

$$BB' = R \operatorname{tang} (VBA - 90^\circ) \tfrac{1}{2} \operatorname{tang} (VBA - 90^\circ)$$

$$= \tfrac{1}{2} R \operatorname{tang} (VBA - 90^\circ)^2.$$

158. In this formula VBA is supposed to be corrected for refraction. If VBA' be the apparent zenith distance, we shall have

$$VBA = VBA' + 0{,}08\ C\ (151)$$

$$= VBA' + 0{,}08\ (VBA' - 90^\circ).$$

This value being substituted for VBA, the above formula becomes

$$BB' = \tfrac{1}{2} R \operatorname{tang} (VBA' - 90^\circ + 0{,}08\ (VBA' - 90^\circ))^2$$

$$= \tfrac{1}{2} R \operatorname{tang} (\ (1 + 0{,}08)\ (VBA' - 90^\circ)\ )^2$$

$$= \tfrac{1}{2} R\ (1 + 0{,}08)^2 \operatorname{tang} (VBA' - 90^\circ)^2 \text{ very nearly.}$$

159. Suppose for example that the angle VBA', as actually observed at the point B, to be 90° 19′ 08″, we readily obtain the height of the mountain in feet; thus

$BB' = \frac{1}{2}\ 3956 \times 5280\ (1 + 0{,}08)^2 \operatorname{tang} (90^\circ\ 19'\ 08'' - 90^\circ)^2$;

and by logarithms

$\frac{1}{2}$ 3956 = 1978	log. . .	3,29623
5280	log. . .	3,72263
1 + 0,08 = 1,08	2 log. . .	0,06685
90° 19′ 08″ — 90°, = 19′ 08″ . .	2 log. tang	15,49102
BB' = 377,34 feet		2,57673

Deducting the allowance for refraction we have 2,57673 — 0,06685 = 2,50988, which gives the height BB' = 325,5 feet.

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† The formula $\frac{\sin a}{R - \cos a} = \frac{\cot \frac{1}{2} a}{R}$ (*Trig.* p. 25), by multiplying by $R - \cos a$ and dividing by $\cot \frac{1}{2} a$, and putting $R = 1$, gives

$$1 - \cos a = \frac{\sin a}{\cot \frac{1}{2} a} = \sin a \operatorname{tang} \tfrac{1}{2} a.$$

Fig. 86. 160. **Reciprocally**, the height of the station BB' (*fig.* 86) being given, it is proposed to find the apparent zenith distance VBA', or the apparent depression of the horizon HBA', called the *dip*.

The formula,

$$BB' = \tfrac{1}{2} R \text{ tang } (VBA - 90°)^2 \quad (157),$$

BB' being known, and VBA the quantity sought, gives

$$\text{tang } (VBA - 90°) = \text{tang } HBA = \sqrt{\frac{BB'}{\frac{1}{2} R}}.$$

But the correction for refraction ABA'

$$= 0{,}08\ B'A\ (151) = 0{,}08\ C = 0{,}08\ HBA;$$

accordingly we have

$$HBA' = HBA - 0{,}08\ HBA = 0{,}92\ HBA$$

$$= 0{,}92 \sqrt{\frac{BB'}{\frac{1}{2} R}} \text{ nearly} (Trig. 17)$$

$$= \frac{0{,}92}{\sin 1'' \sqrt{\frac{1}{2} R}} \sqrt{BB'}$$

in seconds.

$\dfrac{0{,}92}{\sin 1'' \sqrt{\frac{1}{2} R}}$ is constant, and its logarithm, R being estimated in feet, is 1,96379 — 8,19500 = 1,76879. If the height BB' as that of the deck of a ship for example, be 12 feet, we have $\sqrt{BB'} = \frac{1}{2}$ log. 12 = 0,53909. This added to 1,76879 gives 2,30788, which answers to 203″ = 3′ 23″ the dip of the horizon, or quantity to be subtracted from observations of altitude at sea, when the observer's eye is 12 feet above the surface. To find the dip for any other elevation, as a, we have simply to add half the logarithms of a in feet to 0,76879, or, to use the proportion

$$\sqrt{12} : \sqrt{a} :: 3' 23'' : \text{dip required}.$$

161. Where the difference of level is great, as in problems respecting the heights of mountains above their bases or above the sea, the weight of the atmosphere, as determined by the barometer, is generally used in preference to any other method. Suppose

Fig. 87. the whole height of the atmosphere AT (*fig.* 87) to be divided into an indefinite number of strata of the same thickness, and so small that the density of each stratum may be considered as uniform. If we represent the densities of the several sucessive strata beginning at the surface of the earth by a, b, c, &c., since the weight of each is as the density multiplied by the

thickness, the thickness being considered equal to one, we shall have the weights as the densities, that is, as a, b, c, &c. Consequently the weight incumbent upon the 1st stratum will be as $b + c + d +$ &c., that upon the 2d as $c + d + e +$ &c., and so on. But the density is as the incumbent weight†, or compressing force; accordingly we have

$$a : b :: b + c + d + \text{\&c.} : c + d + e + \text{\&c.}$$
$$b : c :: c + d + e + \text{\&c.} : d + e + f + \text{\&c.}$$

and so on; whence

$$a : b :: b : c :: c : d, \text{ and so on††;}$$

that is, when the altitudes above the surface are taken in arithmetical progression, the corresponding densities, and consequently the incumbent weights of the atmosphere at these heights, form a geometrical series; in other words, the heights are the logarithms of the corresponding weights of the atmosphere, according to a particular base, which may be determined by experiment (*Alg.* 254). Distinguishing these logarithms by λ, if we denote any two heights by h, h', and the corresponding weights of the atmosphere, as determined by the barometer, by w, w', we shall have

$$h' - h = \lambda w - \lambda w',$$

or putting $h = 0$†††, $\quad h' = \lambda w - \lambda w'$,

that is, the difference of level, or height of one of the places in question above the other, is expressed by the difference of the

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† This is ascertained by experiment. If we take a portion of air and compress it by a certain weight, upon doubling the weight the air is reduced to half the space. The weight being quadrupled, the air is reduced to one fourth of the space, and so on.

†† If we suppose $a : b :: b : c :: c : d :: d : e :: e : f$ &c., by omitting the first couplet and taking the sum of the antecedents and sum of the consequents (*Geom.* IV), we shall have

$$b : c :: b + c + d + \text{\&c.} : c + d + e + \text{\&c.},$$

and consequently

$$a : b :: b + c + d + \text{\&c.} : c + d + e + \text{\&c.}$$

By omitting the two first couplets we shall have

$$c : d \text{ or } b : c :: e + d + e + \text{\&c.} : d + e + f + \text{\&c.}$$

††† This amounts simply to supposing that the lower station coincides with the common level, the sea, or some assumed level, as the base of a hill, to which the other point is referred.

logarithms of the mercurial columns, these logarithms being constructed upon a particular base adapted to this purpose. Now, since logarithms are changed from one system to another by a constant multiplier (*Alg.* 250), we shall have

$$h' = x\,(\log. w - \log. w'),$$

log. denoting the common logarithm of the quantity before which it is placed. Hence, by taking an object whose elevation has been previously ascertained by other methods, we readily find, once for all, the value of the multiplier x, thus

$$x = \frac{h'}{\log. w - \log. w'}.$$

Let us suppose, for example, that at the bottom and top of a tower, whose height is 200 feet, the mercury stood in the barometer as follows, namely,

at the bottom . . . . 29,96 inches,
at the top 29,74,

the temperature of the air being 48°. We shall have

$$x = \frac{200}{\log. 29{,}96 - \log. 29{,}74} = \frac{200}{1{,}47654 - 1{,}47334} = \frac{200}{0{,}00320} = 62500.$$

162. But this multiplier is constant only when the mean temperature of the air at the two stations is the same; and for a lower temperature the multiplier is less, and for a higher it is greater. *A correction, however, may be applied for any deviation from an assumed temperature, by increasing or diminishing (according as the temperature is higher or lower) the approximate height by its 435th part for each degree of Fahrenheit's thermometer.* We can moreover change the multiplier to a more convenient form by assuming such a temperature as shall reduce this number to 60000 instead of 62500. Now 62500 exceeds 60000 by its 25th part; and, since 1° causes a change of one 435th part, the proportion

$$\tfrac{1}{435} : 1^\circ :: \tfrac{1}{25} : 17^\circ,$$

gives 17° for the reduction to be made in the temperature of the air at the time of the above observations, in order to change the constant multiplier from 62500 to 60000, or to 10000, by calling the height fathoms instead of feet. Thus, the thermometer standing at 48°, we may suppose it to stand at 48° — 17° or 31°; and then we take 10000 as the multiplier, and apply a correction additive for the 17° excess of temperature.

The above observations, for example, being given, to find the height of the tower,

29,96	log.	1,47654
29,74	log.	1,47334
	Diff. of log.	0,00320
	Multiplier	10000
	Product	32

Thus the height of the tower is 32 fathoms, or $32 \times 6 = 192$ feet, on the supposition that the temperature of the air is 31°; but it being 48°, this result is to be increased $\frac{1}{435}$ part for each degree above 31°; that is, by $\frac{17}{435}$ or $\frac{1}{25}$ of the approximate height. By adding $\frac{1}{25}$ of 192, or 8, to 192, we have 200 feet for the height of the tower.

This same method is applicable to other cases, whatever be the temperature of the air at the two stations; that is, *the difference of the logarithms of the two barometric columns, the decimal point being removed four places to the right, is the approximate height or difference of level in fathoms.* A correction being applied for the difference of the mean temperature of the two stations from 31°, according to the rule just given, the true height is obtained.

163. Given the height of the mercury in the barometer at the bottom of a mountain $= 29,37$ in., and at its summit $= 26,59$ in., to find the altitude of the mountain, the mean temperature at the two stations being 26°.

29,37	log.	1,46790
26,59	log.	1,42472
	diff.	0,04318

Approximate height	=	431,8 fathoms,
Correction $\frac{5}{435}$ of 431,8 . .	=	— 4,9
True height		426,9 = 2561,4 feet.

164. We have supposed in the above examples that the temperature of the mercurial columns at the two stations is the same. Where the difference is considerable, the result will evidently be affected by it. If the upper station, for instance, be the coldest, which most frequently happens, the mercurial column will be too short, and will consequently indicate too great a height. The contraction being about one 10000th part for each degree of cold, or 0,0025in. in a column of 25in., it would require

4° difference of temperature to produce an effect amounting to one division on the scale of a common barometer, where the graduation is to hundredths of an inch.

This correction is combined with the foregoing rule in the following formula, in which t, t', represent the temperature of the air, q, q', that of the mercury, at the two stations respectively, 0,0023 being equivalent to $\frac{1}{435}$ nearly.

$$h' = 10000\left(1 + 0{,}0023\left(\frac{t+t'}{2} - 31\right)\right)\log\frac{w}{w' \times (1 + 0{,}0001\,(q - q'))}$$

165. Beside the corrections above considered, regard is sometimes had to the effect of the variation of gravity in different latitudes, and at different elevations above the earth's surface. The latter however is too small to require any notice in an elementary work. The former may be found by multiplying the approximate height by 0,0028371 × cos 2 lat. It is additive, when the latitude is less than 45°, and subtractive when greater. Or it may be taken from the following table.

Latitude.	Correction.
0°	$+\ \frac{1}{352}$ of the app. height.
5°	$+\ \frac{1}{358}$
10°	$+\ \frac{1}{375}$
15°	$+\ \frac{1}{407}$
20°	$+\ \frac{1}{460}$
25°	$+\ \frac{1}{548}$
30°	$+\ \frac{1}{705}$
35°	$+\ \frac{1}{1030}$
40°	$+\ \frac{1}{2030}$
45°	0
50°	$-\ \frac{1}{2030}$
55°	$-\ \frac{1}{1030}$
60°	$-\ \frac{1}{705}$
65°	$-\ \frac{1}{548}$
70°	$-\ \frac{1}{460}$
75°	$-\ \frac{1}{407}$
80°	$-\ \frac{1}{375}$
85°	$-\ \frac{1}{358}$
90°	$-\ \frac{1}{352}$

166. 1. Given the pressure of the atmosphere at the bottom of a mountain equal to 29,68in. of mercury, and that at its summit equal to 25,28in., the mean temperature being 50°, to find the elevation. *Ans.* 727,2 fathoms or 4363,2 feet.

2. The following observations being taken at the foot and summit of a mountain, namely,

at the foot	bar. 29,862	attach. therm 78°	detach. therm†. 71°
at the summit	" 26,137	" 63°	" 55°

to find the elevation.

Ans. 618,9 fathoms, or 3713,4 feet.

3. It is required to find the height of a mountain in latitude 30°, the observations with the barometer and thermometer being as follows; namely,

at the foot	bar. 29,40...	attach. therm. 50°...	detach. therm. 43°
at the summit	" 25,19.	" 46°	" 39°

Ans. 683,27 fathoms, or 4099,62 feet.

Trigonometrical Surveying.

167. THIS term is applied to those operations, which have for their object the more accurate determination of the geographical position of places, and finally that of the magnitude and figure of the earth itself. Where a survey is made to comprehend an extensive country, it is evident that the lines, amounting to many miles in length, and extending over mountains and vallies, cannot actually be measured with any considerable degree of accuracy. It is usual, therefore, in such cases to connect the principal points to be determined, by a series of triangles, as represented in figure 89, the angles of which, together with any one side, being carefully measured, the whole become known, and the distance and bearing of any two points are thence deduced by calculation. Fig. 89.

The principal triangles are made as large as possible, by connecting the most prominent objects that are favourably situated†† for this purpose. The side to be measured, called the *base*, is usually much shorter than those of the principal triangles, and is taken upon a heath or other level piece of ground, that ad-

† The attached thermometer measures the temperature of the mercury in the barometer, and the detached thermometer that of the surrounding air.

†† It evident that the condition the most favourable to accurate results, other things being the same, is when the triangles are equilateral.

mits of its length, as referred to the level of the sea, being ascertained with the greatest precision. It is customary also to measure the side of some other triangle, as *d c*, near the other extremity of the series, called the *base of verification*, and to compare the result of this measurement with the computed length of *d c*, for the purpose of being enabled to judge of the correctness of the whole process†.

Two other operations purely astronomical are now requisite to complete the work. The first is to find the angle comprehended between the meridian and the side of one of the given triangles, as *A b b′*; and the second to determine the latitude of each of the extreme points *A*, *B*, we shall then have the means of computing the perpendicular distance *b b′*, *p p′*, &c., of each station from the meridian, and the distance of each perpendicular *A b′*, *A p′*, &c., from *A*. Knowing therefore the absolute distance *A x*, and the difference of latitude of these two points, we readily find the length of a degree, and thence the length of 360°, or the whole circumference of the earth, considered as a sphere. By performing similar operations in different latitudes, we obtain the length of degrees in those latitudes, and thence the true figure of the earth, on the supposition that it is a spheroid of revolution, or such as would be formed by the revolution of an ellipse about one of its axes.

168. The solution of this important problem belongs properly to astronomy; but it may not be amiss to make the learner acquainted in this place with some of those theorems, that have been investigated for the purpose of facilitating the observations and calculations above referred to.

169. It will be readily seen, that, where a triangle is formed by means of signals placed upon the spires of churches and the highest points of other objects, it will often be impossible to place the instrument at the vertex of the angle to be measured. Suppose, for instance, that the angle *C* (*fig.* 88), in the triangle *ABC*, is required, and that the nearest position in which the instrument can be placed is at *P*. The angle $APB = P$, and $BPC = P'$, be-

Fig. 88.

† In the great French survey, conducted by Mechain and Delambre, the base of verification to a series of triangles extending from Melun to Perpignan, a distance of about 400 miles, differed less than 12 inches from its computed length. Instances of such extreme accuracy occur also in the English survey.

ing observed, and the side CP together with AC and BC, being known, the angle C may be found. Thus,

$$AIB = P + IAP = C + PBC;$$

whence

$$C = P + IAP - PBC.$$

But, by the rules of trigonometry,

$$AC : PC :: \sin CPA : \sin CAP,$$

whence

$$\sin CAP \text{ or } \sin IAP = \frac{PC \sin CPA}{AC} = \frac{PC}{AC} \sin (P + P').$$

Also

$$BC : PC :: \sin BPC : \sin PBC;$$

whence

$$\sin PBC = \frac{PC}{BC} \sin BPC = \frac{PC}{BC} \sin P'.$$

Now, since the angles IAP, PBC, (or the arcs measuring them) are always very small, their sines may be taken as equivalent to the arcs themselves (*Trig.* 17). Accordingly, by this substitution, we shall have

$$C = P + \frac{PC}{AC}\sin(P+P') - \frac{PC}{BC}\sin P' = P + PC\left(\frac{\sin(P+P')}{AC} - \frac{\sin P'}{BC}\right)$$

$$= P + \frac{PC}{\sin 1''}\left(\frac{\sin (P + P')}{AC} - \frac{\sin P'}{BC}\right)$$

in seconds.

The use of this formula cannot in any case be embarrassing, provided the signs of $\sin (P + P')$ and $\sin P'$ be attended to. Thus the second term of the correction will be positive, if the angle $(P + P')$ fall between 0 and 180°, and it will be negative if it exceed 180° (*Trig.* 23). The contrary will take place with respect to the angle P', according as it is less or greater than 180°.

170. When AC or BC becomes infinite with respect to PC, the corresponding term vanishes (*Alg.* 68), and we have, in the first case,

$$C = P - \frac{PC \sin P'}{\sin 1''\, BC},$$

and in the second

$$C = P + \frac{PC \sin (P + P')}{\sin 1''\, AC}.$$

The first of these formulas is applicable when A is a heavenly body, and the second when B is one. When AC, BC, are each infinite with respect to PC, both the terms vanish, and we have

$$C = P.$$

171. But without A and B being at an infinite distance, C may be equal to P in innumerable instances; that is, whenever the centre is placed in the circumference of a circle passing through three points A, B, C; or when the angle BPC is equal to the angle BAC, or to $BAC + 180°$. Whence, though C should be inaccessible, the angle ACB may in many cases be obtained without any calculation.

It may be further observed, that when P falls in the circumference of a circle passing through the three points A, B, C, the angles A, B, C, may be determined solely by measuring the angles APB, BPC. For the opposite angles ABC, APC, of a quadrilateral inscribed in a circle, being equal to 180° (*Geom.* 130), we shall have

$$ABC = 180° - APC,$$

also

$$BAC = 180° - (ABC + ACB)$$
$$= 180° - (ABC + APB).$$

172. If one of the objects viewed from a further station be a vane or staff in the centre of a steeple, it will sometimes be found that such object, when the observer comes near it, is invisible as well as inaccessible. Still there are various methods of finding

Fig. 90. the exact angle at C (*fig.* 90). Suppose, for example, the signal-staff to be in the centre of a circular tower, and that the angle APB was taken at P near its base. In the direction of the tangents PT, PT', let two equal distances Pm, Pm', be taken at pleasure. Bisect mm' at the point n, and the angle nPB will be equal to the angle CPB. Also the distance PS added to the radius CS of the tower, will give PC the distance of the centre of the position P from the centre of the station C.

If the circumference of the tower cannot be measured for the purpose of determining the radius; by measuring the angles BPT, BPT', we shall have

$$BPC = \tfrac{1}{2}(BPT + BPT')$$

and

$$CPT' = BPT - BPT;$$

then the measure of PT will give

$$PC = PT \sec CPT.$$

173. If the base of the tower be a regular polygon, which not

Fig. 91. unfrequently happens, take for the position P (*fig.* 91) the point of

intersection of two of the sides produced, and we shall have, as before, $BPC = \frac{1}{2}(BPT + BPT')$, and PT equal to the distance from P to the middle of one of the sides; and hence PC is found as above. If the figure be a regular hexagon, $P\,m\,C\,m'$ is a rhombus, in which $m\,m'$ is equal to $m\,C$ (*Geom.* 271); accordingly we have

$$1 : \sqrt{3} :: m\,m' : PC \quad (\textit{Geom.}\ 272),$$

and hence

$$PC = m\,m'\sqrt{3}.$$

174. If the instrument used for taking the angles be adapted only to measure angles in the plane passing through the three stations, it becomes necessary to reduce these angles to the horizon. Let BCA (*fig.* 92) be an angle measured in an inclined plane, and $B'CA'$ the corresponding horizontal angle, formed by the projection of CA, CB. The zenith distances $z\,a$, $z\,b$, of the stations A, B, as observed from C, together with the arc $a\,b$, the measure of C, or of ACB, constitute a spherical triangle, in which the angle at z is equal to $A'CB'$ or C', the angle required (*Geom.* 471). Hence, by the common theorem for the case where three sides of a spherical triangle are given to find an angle (*Trig.* 62), we have Fig. 92.

$$\sin\tfrac{1}{2}z = \sin\tfrac{1}{2}C' = \sqrt{\frac{\sin\frac{1}{2}(C + z\,a + z\,b - 2\,z\,a)\sin\frac{1}{2}(C + z\,a + z\,b - 2\,z\,b)}{\sin z\,a \sin z\,b}}$$

$$= \sqrt{\frac{\sin\frac{1}{2}(C + z\,b - z\,a)\sin\frac{1}{2}(C + z\,a - z\,b)}{\sin z\,a \sin z\,b}}.$$

175. It may sometimes be more convenient to take the inclinations ACA', BCB', of A and B, that is, the complements of the zenith distances. If we call these c, c', respectively, by introducing them into the first of the above formulas, we shall have

$$\sin\tfrac{1}{2}C' = \sqrt{\frac{\sin\frac{1}{2}(C + 90^\circ - c + 90^\circ - c' - 2(90^\circ - c))\sin\frac{1}{2}(C + 90^\circ - c + 90^\circ - c' - 2(90^\circ - c'))}{\cos c \cos c'}}$$

$$= \sqrt{\frac{\sin\frac{1}{2}(C + c - c')\sin\frac{1}{2}(C + c' - c)}{\cos c \cos c'}}.$$

176. If c be equal to c', the above formula becomes

$$\sin\tfrac{1}{2}C' = \sqrt{\frac{\sin\frac{1}{2}C \sin\frac{1}{2}C}{\cos c \cos c}} = \frac{\sin\frac{1}{2}C}{\cos c}.$$

177. If the angles c, c', be very small and nearly equal, since the cosines of small angles vary very slowly, we may often use the following formula without sensible error, namely,

$$\sin \tfrac{1}{2} C' = \frac{\sin \frac{1}{2} C}{\cos \frac{1}{2} (c + c')} \text{ very nearly.}$$

178. The observed angles being reduced to the centre of the station, and to the horizon, it is proposed next to find the angle contained by the chords of any two sides of a known spherical triangle.

Fig. 93. The angle A, for example, of the spherical triangle ABC (*fig.* 93), and the two sides b, c, being given, it is required to find the angle A', contained by b', c', the chords of b, c, respectively.

From a known theorem of spherical tigonometry (*Trig.* 53), we have

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

But

$$\begin{aligned} \cos c &= (\cos \tfrac{1}{2} c)^2 - (\sin \tfrac{1}{2} c)^2 \quad (\textit{Trig. } 11), \\ &= 1 - (\sin \tfrac{1}{2} c)^2 - (\sin \tfrac{1}{2} c)^2 \quad (\textit{Trig. } 10), \\ &= 1 - 2 (\sin \tfrac{1}{2} c)^2. \end{aligned}$$

In like manner

$$\cos a = 1 - 2 (\sin \tfrac{1}{2} a)^2,$$

and $$\cos b = 1 - 2 (\sin \tfrac{1}{2} b)^2.$$

Substituting these values in the above equation, we have

$$1 - 2 (\sin \tfrac{1}{2} a)^2 = (1 - 2 (\sin \tfrac{1}{2} b)^2 (1 - 2 (\sin \tfrac{1}{2} c)^2 + \sin b \sin c \cos A,$$

or, since

$$\sin b = 2 \sin \tfrac{1}{2} b \cos \tfrac{1}{2} b, \text{ and } \sin c = 2 \sin \tfrac{1}{2} c \cos \tfrac{1}{2} c \quad (\textit{Trig. } 11),$$

$$1 - 2(\sin \tfrac{1}{2} a)^2 = 1 - 2 (\sin \tfrac{1}{2} b)^2 (1 - 2 (\sin \tfrac{1}{2} c)^2 + 4 \sin \tfrac{1}{2} b \cos \tfrac{1}{2} b \sin \tfrac{1}{2} c \cos \tfrac{1}{2} c \cos A.$$

Now

$$\sin \tfrac{1}{2} a = \tfrac{1}{2} a', \ \sin \tfrac{1}{2} b = \tfrac{1}{2} b', \text{ and } \sin \tfrac{1}{2} c = \tfrac{1}{2} c' \quad (\textit{Trig. } 11),$$

consequently,

$$\begin{aligned} 1 - \tfrac{1}{2} a'^2 &= (1 - \tfrac{1}{2} b'^2)(1 - \tfrac{1}{2} c'^2) + 4 \tfrac{1}{2} b' \cos \tfrac{1}{2} b \tfrac{1}{2} c' \cos \tfrac{1}{2} c \cos A \\ &= 1 - \tfrac{1}{2} b'^2 - \tfrac{1}{2} c'^2 + \tfrac{1}{4} b'^2 c'^2 + b' c' \cos \tfrac{1}{2} b \cos \tfrac{1}{2} c \cos A, \end{aligned}$$

and by transposition,

$$\frac{b'^2 + c'^2 - a'^2}{2} = \tfrac{1}{4} b'^2 c'^2 + b' c' \cos \tfrac{1}{2} b \cos \tfrac{1}{2} c \cos A.$$

But

$$\cos A' = \frac{b'^2 + c'^2 - a'^2}{2 b' c'}, \text{ or } \cos A' b' c' = \frac{b'^2 + c'^2 - a'^2}{2} \quad (\textit{Trig. } 38).$$

whence, by substitution and division, we have

$$\cos A' = \tfrac{1}{2} b' \tfrac{1}{2} c' + \cos \tfrac{1}{2} b \cos \tfrac{1}{2} c \cos A,$$

or, by restoring the values of $\frac{1}{2} b' \frac{1}{2} c'$,

$$\cos A' = \sin \tfrac{1}{2} b \sin \tfrac{1}{2} c + \cos \tfrac{1}{2} b \cos \tfrac{1}{2} c \cos A.$$

179. When the three sides a, b, c, are very small, compared

with the radius of the sphere, the triangle differs but little from a plane triangle; and by considering it as such we can arrive at an approximate solution; but in this case we neglect the excess of the sum of three angles over two right angles (*Geom.* 489). In order to obtain a solution more exact, it is necessary to take account of this excess, which may readily be done by means of a general principle, which we proceed to make known.

Let r be the radius of the sphere on which the proposed triangle is situated. If we suppose a similar triangle, traced upon a sphere whose radius is 1, the sides of this triangle will be

$$\frac{a}{r},\ \frac{b}{r},\ \frac{c}{r}.$$

Then, A being the angle opposite a, we shall have

$$\cos A = \frac{\cos\frac{a}{r} - \cos\frac{b}{r}\cos\frac{c}{r}}{\sin\frac{b}{r}\sin\frac{c}{r}} \quad (\textit{Trig. } 53).$$

But, r being very great with respect to a, b, c,

$$\cos\frac{a}{r} = 1 - \frac{a^2}{2r^2} + \frac{a^4}{2.3.4r^4}, \text{ and } \sin\frac{b}{r} = \frac{b}{r} - \frac{b^3}{2.3r^3};\dagger$$

and $\cos\frac{b}{r}$, $\cos\frac{c}{r}$, $\sin\frac{c}{r}$, admit of similar expressions. Whence, by substituting these values in the above equation, we obtain

$$\cos A = \frac{\left(1 - \frac{a^2}{2r^2} + \frac{a^4}{2.3.4r^4}\right) - \left(1 - \frac{b^2}{2r^2} + \frac{b^4}{2.3.4r^4}\right)\left(1 - \frac{c^2}{2r^2} + \frac{c^4}{2.3.4r^4}\right)}{\left(\frac{b}{r} - \frac{b^3}{2.3r^3}\right)\left(\frac{c}{r} - \frac{c^3}{2.3r^3}\right)}$$

or, by reducing and neglecting all terms exceeding four dimensions, on account of the smallness of the arcs compared with radius,

$$\cos A = \frac{\frac{b^2 + c^2 - a^2}{2r^2} + \frac{a^4 - b^4 - c^4}{24r^4} - \frac{b^2c^2}{4r^4}}{\frac{bc}{r^2} - \frac{b^3c}{6r^4} - \frac{bc^3}{6r^4}}.$$

Multiplying both terms of this fraction by $1 + \frac{b^2 + c^2}{6r^2}$, and neglecting as before all terms of more than four dimensions, the equation becomes

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† For an investigation of these expressions for the sine and cosine of an arc, see note at the end of this part.

$$\cos A = \frac{b^2+c^2-a^2}{2bc} + \frac{a^4-b^4-c^4}{24bcr^2} - \frac{b^2c^2}{4bcr^2} + \frac{b^4+c^4+2b^2c^2-a^2b^2-a^2c^2}{12bcr^2}$$

$$= \frac{b^2+c^2-a^2}{2bc} + \frac{a^4+b^4+c^4-2a^2b^2-2a^2c^2-2b^2c^2}{24bcr^2} \quad (1).$$

If now we suppose A' to be the angle opposite to the side a, in the plane triangle whose sides are equal in length to the arcs a, b, c, we shall have

$$\cos A' = \frac{b^2+c^2-a^2}{2bc} \quad (Trig.\ 38).$$

Also

$$\sin A'^2 = \frac{-a^4-b^4-c^4+2a^2b^2+2a^2c^2+2b^2c^2}{4b^2c^2}\dagger,$$

or, changing all the signs and multiplying both members by $\frac{bc}{6r^2}$,

$$-\frac{bc\sin A'^2}{6r^2} = \frac{a^4+b^4+c^4-2a^2b^2-2a^2c^2-2b^2c^2}{24bcr^2}.$$

These values being substituted in equation (1) give

$$\cos A = \cos A' - \frac{bc\sin A'^2}{6r^2}.$$

Put $A = A' + x$, and we shall have

$$\cos A = \cos A' - x\sin A' \dagger\dagger.$$

But

$$\cos A = \cos A' - \frac{bc\sin A'^2}{6r^2},$$

whence

$$x\sin A' = \frac{bc\sin A'^2}{6r^2},$$

and

$$x = \frac{bc\sin A'}{6r^2}.$$

† $\sin A'^2 = 1 - \cos A'^2$ (*Trig.* 10).

$$= 1 - \frac{(b^2+c^2-a^2)^2}{(2bc)^2}$$

$$= \frac{4b^2c^2-(a^4+b^4+c^4-2a^2b^2-2a^2c^2+2b^2c^2)}{4b^2c^2}$$

$$= \frac{-a^4-b^4-c^4+2a^2b^2+2a^2c^2+2b^2c^2}{4b^2c^2}.$$

†† By the general formula (*Trig.* 11)

$$\cos(A'+x) = \cos A'\cos x - \sin A'\sin x;$$

or, since x is very small, by considering $\cos x = 1$, and $\sin x = x$, we have

$$\cos A = \cos(A'+x) = \cos A' - x\sin A' \text{ very nearly}.$$

Since x is of the second order with respect to $\frac{b}{r}$ and $\frac{c}{r}$, it follows that the above result is exact to quantities of the fourth order; consequently we have

$$A = A' + \frac{b\,c \sin A'}{6\,r^2}.$$

Now $\frac{1}{2}\,b\,c \sin A'$ is the area of a plane triangle (124), whose sides are a, b, c, and which does not sensibly differ from that of the spherical triangle under consideration. Accordingly if we call the area of either the one or the other α, we shall have

$$A = A' + \frac{\alpha}{3\,r^2},$$

or

$$A' = A - \frac{\alpha}{3\,r^2}.$$

In like manner

$$B' = B - \frac{\alpha}{3\,r^2},$$

$$C' = C - \frac{\alpha}{3\,r^2};$$

whence

$$A' + B' + C' \text{ or } 180° = A + B + C - \frac{\alpha}{r^2};$$

that is

$$A + B + C - 180° = \frac{\alpha}{r^2};$$

$\frac{\alpha}{r^2}$ therefore may be considered as the excess of the sum of the three angles of the proposed spherical triangle over two right angles. This being established, we have the following remarkable theorem, by which the resolution of very small spherical triangles is reduced to that of plane triangles.

A spherical triangle being proposed, the sides of which are very small compared with the radius of the sphere, if from each of its angles we subtract a third of the excess of the sum of the three angles over two right angles, the angles so diminished may be taken for the angles of a plane triangle, the sides of which are equal in length to those of the spherical triangle.

In other words—

A spherical triangle, slightly curved, whose angles are **A, B, C,** *and whose opposite sides are* a, b, c, *answers always to a plane triangle, whose sides are of the same length* a, b, c, *and whose opposite angles are* A — $\frac{1}{3}\epsilon$ B — $\frac{1}{3}\epsilon$, C — $\frac{1}{3}\epsilon$, *being the excess of the sum of the angles of the spherical triangle over two right angles*†.

180. The excess ϵ or $\frac{\alpha}{r^2}$, which is proportional to the area of the triangle, may always be calculated *a priori*, by means of what is known in the spherical triangle considered as a plane triangle. If two sides b, c, are given, together with the contained angle A, we have for the area

$$\alpha = \tfrac{1}{2}\, b\, c \sin A \quad (124).$$

If the parts given are a side a and the two adjacent angles B, C, we shall have for the area

$$\alpha = \tfrac{1}{2}\, a^2 \frac{\sin B \sin C}{\sin (B + C)}\dagger\dagger.$$

Moreover we have

$$\epsilon = \frac{\alpha}{r^2} R'',$$

R'' being the number of seconds contained in radius; and thus ϵ will be expressed in seconds, and $\frac{R''}{r^2}$ is a constant quantity.

If the given side or sides be estimated in feet, r is also to be taken in feet, and we shall have in this case

$$\begin{aligned}\log. R'' - \log. r^2 &= \log. 206264{,}8'' - 2 \log. 3956 \times 5280\\ &= 5{,}31443 - 14{,}63978\\ &= \overline{10},67465\end{aligned}$$

We have therefore only to add $\overline{10},67465$ to the logarithm of the area of the triangle, expressed in feet, or to subtract its arithmetical complement 9,32535†††, in order to obtain the spherical excess in seconds.

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† This curious theorem was first announced by Legendre in the Memoirs of the French Academy for 1787.

†† $\text{Sin } A = \sin (B + C) : a :: \sin B : b = \frac{a \sin B}{\sin (B + C)}$,

$\sin A = \sin (B + C) : a :: \sin C : c = \frac{a \sin C}{\sin (B + C)}$.

Hence the area $= \frac{1}{2} b c \sin A = \frac{1}{2} a^2 \frac{\sin B \sin C}{\sin (B + C)}$.

††† The constant subtractive logarithm used by General Roy, in

Suppose, for example, that we had given two sides of a triangle, a, b, and the included angle C, namely $a = 248230$ feet, $b = 212628$ feet, and $C = 103°\ 19'\ 10''$, we should have for the area

$$s = \tfrac{1}{2}\, a\, b \sin\ C, \text{ or } \frac{a\, b \sin C}{2},$$

and, by logarithms,

2	. . . .	log.	0,30107
			9,69897
a	. . 248230 . .	log.	5,39485
b	. . 212628 . .	log.	5,32762
C	. . . 103° 19′ 10″ . .	log. sin . . .	9,98816
Logarithm of the area		. . . .	0,40960
Constant logarithm subtractive		. . .	9,32535
Spherical excess = 12,14″		. . . . . .	1,08425

Thus the spherical excess in this example amounts to a little more than 12″†; and the sum of the three angles ought to exceed 180° by this quantity. We are thus enabled to judge of the accuracy of the observations, and in some degree to apply a correction for slight inevitable errors. It is customary to increase or diminish each of the observed angles by a third of the error, in order to render their sum equal to two right angles plus the spherical excess.

The angles of the several triangles being thus corrected we may proceed to calculate the sides†† directly by the rules of spherical trigonometry, or by reducing the angles to those contained

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the English survey, was 9,3267737, derived from the supposition that $r = 3962 \times 5280$ feet nearly, or 6 miles greater than the above.

† The sides in the example above given are very large, being between 40 and 50 miles. There are few cases in geodesic operations in which the spherical excess exceeds 5″.

†† One or more of the sides are first calculated for the purpose of finding the spherical excess, and by this means, of correcting the observed angles. In the method however of reducing the spherical triangles to those contained by the chords, there is this criterion of the accuracy of the observations, that the sum of the three reduced angles should always be equal to 180°.

by the chords (178), or by deducting one third of the spherical excess from each of the angles, and considering the remaining angles, together with the actual spherical sides, as constituting a plane triangle. The first of these methods was practised by Boscovich, the second by Colonel Mudge and Delambre, and the last by General Roy. Indeed, Delambre informs us, that he computed, by each of the three methods, the whole series of triangles extending from the British channel to the Mediterranean.

Fig. 88. 181. 1. Given AC (*fig.* 88) = 4510 yards, BC = 4730 yards, $APB = 33° 58' 37{,}43''$, $BPC = 232° 55'$, to find the angle ACB, or the reduction to the centre of the station.

Ans. $ACB = 266° 52' 54{,}35''$

Fig. 92. 2. Given the inclined angle ACB (*fig.* 92) and the apparent zenith distances $Z\,a = 91° 25' 51''$, $Z\,b = 91° 32' 45''$, to find the corresponding horizontal angle $A'CB'$.

Ans. $A'CB' = 61° 10' 49{,}18''$.

Fig. 91. 3. Given the arcs b, c (*fig.* 91) equal respectively to 15′ 19″, 7′ 26″, and the angle A equal to 66° 30′ 36,88″, to find the angle A' contained by the chords of a, b.

Ans. $A' = 66° 30' 36{,}68''$

4. Given two sides of a triangle equal to 70290,2, and 18349,6 feet respectively, and the contained angle equal to 55° 43′ 07″, to find the spherical excess.

Ans. 5,022″

5. Given the three observed angles of a triangle, namely, $A = 53° 58' 35{,}75''$, $B = 57° 36' 39{,}5''$, $C = 68° 24' 44'$, and the side c opposite to $C = 79211{,}22$ feet, to find the sum of the errors of observation.

Ans. — 1,83″.

6. Suppose that two sides a, b, and the included angle C, of a spherical triangle have been determined as follows, namely,

$$a = 79211{,}22,\ b = 71934{,}2, \text{ and } C = 53° 58' 35{,}75'',$$

it is proposed to find, by the theorem of Legendre, the angles, and the remaining side, of the corresponding plane triangle.

	angles of the spherical triangle.	angles of the plane triangle.	sides.
Ans.	A 68° 24′ 45″, ...	$A' = 68° 24' 44{,}64''$, ..	$a = 79211{,}22$
	B 57° 36′ 40,33″, ..	$B' = 57° 36' 39{,}98''$, ..	$b = 71934{,}2$
	C 53° 58′ 35,75″, ..	$C' = 53° 58' 35{,}39''$, ..	$c = 68896{,}9$

NOTES.

I.

Description and use of the Plane Scale, Sector, &c.

THE *plane scale* is an instrument of wood or metal varying in length from six inches to two feet. It usually contains lines of the following denominations, namely,

1.	Equal Parts,	marked	E. P.†
2.	Chords,		Cho.
3.	Rhumbs,		Rh.
4.	Sines,		Sin.
5.	Tangents,		Tan.
6.	Secants,		Sec.
7.	Semi-Tangents,		S. T.
8.	Longitude,		Long.
9.	Latitude,		Lat.
10.	Hours,		Ho.
11.	Inclination of Meridians,		In. Mer.

1. Lines of equal parts are of two kinds. The first, represented by figure 94, consists simply of a certain number of equal portions of any convenient length, the extreme one on the left being decimally, or duodecimally subdivided, and the rest being numbered 1, 2, 3, &c. There are usually several of these lines adapted to different purposes and distinguished by the numbers 20, 25, &c., showing into how many parts an inch is divided.

These lines are used in laying down any distance, as feet, chains, miles, &c., expressed by a number consisting of two denominations, or two figures. The several divisions may be considered as feet, for example; then the decimal subdivisions would be tenths of a foot, and the duodecimal subdivisions, inches. So also each of the principal divisions

† These letters are often omitted, and the names of the other lines still further abbreviated, as Ch. or C. for Chords, Si. or S. for Sines, Ta. or T. for Tangents, &c.

may be regarded as ten feet, ten miles, &c., and in this case the decimal subdivisions will represent feet, miles, &c., respectively.

The second construction for lines of equal parts is represented by figure 95. It consists, when intended for a decimal scale, of eleven lines, drawn parallel to each other, and at equal distances, the extreme ones being divided in the manner above explained, and the subdividing points being connected by *diagonal* lines, that is, by lines proceeding from the first point on the one side, to the second on the opposite, and so on, as exhibited in the figure. Then, by similar triangles, as $ab : ac :: bo : cd$, that is, cd is one tenth of the subdivisions, or one hundredth of the primary divisions of the scale. In like manner ef may be shown to be two hundredths, and gh three hundredths of $a\,1$. While, therefore, the first scale is limited to two figures, this is adapted to three. If the number proposed were 253, for example, we should take 250 as on the former scale, by extending the compasses from 2 of the principal divisions, considered as containing each ten parts, to five of the subdivisions, then by opening the compasses to the corresponding extent on the third of the parallel lines, we shall obtain the length required.

There are generally two diagonal scales laid down on the same face of the instrument, the unit of the one being double that of the other, and commencing on opposite ends of the scale.

Fig. 96. In order to construct the remaining lines of the plane scale, describe the circle *AEBD* (*fig.* 96) with any convenient radius *AC*, and draw the diameters *AB*, *DE*, at right angles to each other. Continue *BA* at pleasure toward *F*, and through *D* draw *DG* parallel to *BF*. About the circle circumscribe the square *HMN*, having its sides *HM*, *MN*, parallel respectively to the diameters *AB*, *DE*, and draw the chords *BD*, *BE*, *AD*, *AE*.

2. For the line of chords, divide the arc *DA* into equal parts, marking the tenth divisions with the figures 10, 20, 30, &c. With one foot of the compasses in D transfer the several distances *D* 10, *D* 20, &c., to the chord *DA*, which, marked with the corresponding figures of the arc, will become a line of chords. There are usually several of these lines, constructed with different radii, upon different parts of the scale.

3. For the line of rhumbs, divide the arc *BE* into eight equal parts, marking them with the figures 1, 2, 3, &c., and subdivide each of these parts into four quarters; then, with one foot of the compasses on *B*, transfer the several distances *B* 1, *B* 2, &c., to the chord *BE*, which, marked with the corresponding figures of the arc, will be a line of rhumbs.

4. For the line of sines, through each of the divisions of the arc *DA* draw lines parallel to the radius *AC*, and *CD* will be divided into a line of sines, which are to be numbered from *C* to *D* for the right sines, and from *D* to *C* for the versed sines. These may be continued to 180° by applying the divisions of the radius *CD* from *C* to *E*.

5. For the line of tangents, lay a ruler on *C* and the several divisions of the arc *DA*, and the intersections with the line *DG*, being numbered with the corresponding figures of the arc, will become a line of tangents.

6. For the line of secants, with one foot of the compasses in *C* transfer the distances from the centre *C* to the divisions on the line of tangents, namely, *C* 10, *C* 20, &c., to the line *AF*, and these will give the divisions of the line of secants, which is to be numbered with the corresponding figures of the line of tangents.

7. For the line of semi-tangents, lay a ruler on *E* and the several divisions of the arc *AD*, and the points of intersection with the radius *CA*, being numbered with the corresponding figures of the arc *AD*, will be a line of semi-tangents. This line is generally continued as far as the length of the scale will admit. The divisions beyond 90° are found by dividing the arc *AE* like the arc *AD*, and placing a ruler on *E* and these divisions of *AE*, and the line of semi-tangents above 90° will be obtained on *CA* continued.

8. For the line of longitude, divide *AH* into sixty equal parts, and through each of these points draw lines parallel to the radius *AC*, and meeting the arc *AE*. With one foot of the compasses in *E* transfer these divisions to the chord *AE*, and this line will become a line of longitude. If this line be put on a scale close to the line of chords, but inverted so that 60° on the line of longitude shall be against 0° on the line of chords, &c., and any degree of latitude be counted on the line of chords, we shall have opposite to it on the line of longitude, the miles contained in one degree of longitude in that latitude, the measure of one degree at the equator being sixty geographical miles.

9. For the line of latitude, place a ruler on *A* and the several divisions of *CD*, and note the intersections made on the arc *BD*. With one foot of the compasses in *B*, transfer these divisions to the chord *BD*, numbering them with the corresponding divisions of *CD*, and this will be a line of latitude.

10. For the line hours, bisect the quadrantal arcs *BD*, *BE*, in *a*, *b*. Divide the quadrant *a b* into six equal parts, making 15° to an hour, and subdivide each of these into four parts for quarters of an hour. A ruler on *C* and the several divisions of the arc *a b* will intersect the line *MN* in the hour points, which are to be marked as in the figure.

11. For the line of inclination of meridians, bisect the arc *EA* in *c*; divide the quadrant *b c* into ninety equal parts; place a ruler on *C* and the several division of the arc *b c*, and the intersections with the line *HM* will be the divisions of a line of inclination of meridians.

The line of chords is used in protracting and measuring angles. If Fig. 97. it were required, for instance, to draw the lines *AB*, *AC* (*fig.* 97.), making the angle *A* equal to 20°, having drawn one of these lines, as *AB*, we should extend the compasses on the line of chords from 0 to 60°, and with this extent, describe from *A* as a centre, an arc cutting the line *AB* in *m*; then since the chord of 60° is equal to radius (*Geom.* 271), this arc will have the same radius as the circle to which the given scale of chords belongs; accordingly if we take in the compasses on the same scale the extent from 0° to 20°, and apply it from *m* to *n*, and through the point *n* draw the line *AC*, the arc *m n* will be equal in all respects to the arc *D* 20°, that is, it will contain 20°, and being the measure of the angle *A*, this angle will be of the required magnitude.

In like manner, if the lines *AB*, *AC*, were already drawn, and it were proposed to measure the angle contained by them, having described the arc *m n* with the chord of 60°, we take in the compasses the extent from *m* to *n*, and applying it from 0 toward *A* on the same line of chords, and the number against which it falls shows the magnitude of the angle *A*.

The line of rhumbs is also a line of chords. It has reference to the divisions of the Mariner's Compass, in which a right angle, instead of being divided into 90°, is considered as containing 8 points of 11° 15′ each, and each point is subdivided into four quarters. Problems in navigation are frequently solved by estimating the angles in points and quarter points instead of degrees and minutes, but the nature of the solution is evidently not affected by this change in the denomination of angular magnitude.

The lines of sines, tangents, secants, and semi-tangents, as also the line of chords, are used in orthographic and stereographic projection. Fig. 12. Thus, the primitive circle (*fig.* 12), being described with a radius equal to the sine of 90°, the radii *C* 10, *C* 20, &c., of the projected parallels are the sines respectively of the polar distances of these parallels, that is, of 80°, 70°, &c. (7). In the polar stereographic projection Fig. 14. (*fig.* 14) the radii *P* 10, *P* 20, &c., of the parallels of latitude, being the tangents of half the polar distances respectively (17), that is, the semi-tangents of the polar distances, the primitive being described with the semi-tangent of 90°, or radius, for the radii of the other parallels we have simply to take from the same scale, the semi-tangent of 80°, of 70°, &c. So also in the equatorial projection (*fig.* 14), the line of secants gives the radii of the oblique circles *EF'H*, *EF''H*, &c. (15), and on the line of tangents we have the distances of their centres respectively from the centre of the primitive (14). These same lines serve also to find the distances of the centres and the radii of the projected parallels 10 *a* 10, 20 *a'* 20, &c. (18). Fig. 14.

The line of longitude, placed by the side of a line of chords inverted, shows the length of a degree of longitude in different latitudes. Let the two meridians *PEP'*, *PQP'* (*fig.* 18), be inclined to each other one degree, or, in other words, let the arcs *QE*, *KI*, &c., be each one degree of their respective parallels. The lengths of these arcs are to each other as their radii *CE*, *LI*, &c. If, therefore, *CE* be divided into 60 equal parts, the perpendiculars from *M*, *I*, *R*, &c†., upon *CP*, (being taken in the compasses, and applied to *CE*), will show the length of a degree in each of these latitudes, or, which amounts to the same thing, the divisions of *CE*, numbered from *C* to *E*, may be transferred, with the corresponding numbers, to the perpendiculars above mentioned, by means of lines drawn through *M*, *I*, *R*, &c., parallel to *CP*, as in figure 96. The numbers against *M*, *I*, *R*, &c., will then show the length of a degree in the latitude of *M*, *I*, *R*, &c, respectively, and if these numbers be written on a scale of chords, (as that of *EP*,) against the chord of *EM*, *EI*, *ER*, &c., respectively, the latitude of a place being given, we shall have, by inspection, the number of nautical miles contained in one degree of longitude at that distance from the equator. Fig. 18.

The lines of latitude, hours, and inclination of meridians, are employed in the construction of dials. To give an example of the use of these lines, let *c S*, *c' S'* (*fig.* 96) be the meridian, *c c'* being the thickness of the stile, and VI *c c'* VI the six o'clock hour line. From the line of latitudes, take the extent from the beginning of the line to the division corresponding to the latitude of the place for which the dial is to be made, and set it off from c to *m*, and from c' to *m'*. From the points *m*, *m'*, draw the lines *m n*, *m' n'*, each equal to the whole length *M N* of the line of hours, and terminating in the meridian line at *n*, *n'*. Transfer the divisions of the line of hours to the lines *m n*, *m' n'*, numbering them, as in the figure. From the points c, c', draw the lines c I, c II, &c., c' XI, c' X, &c., and these will be the lines of the dial. Fig. 96.

To show the truth of this construction, let the latitude for which the dial is made be equal to the number of degrees in the arc *A p*. Then letting fall the perpendicular *p q*, and drawing through the

† *M*, *R*, &c., are supposed in this case to be in the meridian *EP*.

point q the line $A\,q\,r$, and joining $B\,r$, we shall have the triangle $AB\,r$ equal, in all respects, to $m\,c\,n$, namely, AB or $MN = m\,n$, $B\,r = c\,m$, &c. Whence

$$\text{Radius} : \sin lat. :: AC : C\,q$$
$$:: A\,r : B\,r,$$

consequently

$$\text{Radius} : \sin lat. :: n\,c : c\,m.$$

Let H be the point in which one of the hour lines, as IV P. M., for example, meets $m\,n$. On the VI o'clock hour line take $c\,R$ equal to $c\,n$; join $n\,R$, and through H draw KHL parallel to $c\,R$, meeting the meridian in K, and the line $n\,R$ in L; and join $c\,L$. Now because $R\,n$ and $m\,n$ are similarly divided at L and H (*Geom.* 196), and $m\,H$ $H\,n$, are respectively equal to M IV, IV N, $R\,n$, and MN, are similarly divided at L and IV. But the triangles $R\,c\,n$ and $M\,C\,N$ are manifestly similar; consequently the angle $n\,c\,L$ is equal to the angle $N\,C$ IV, and therefore equal to the angle described by the sun between noon and IV o'clock P. M.

Now LK or $n\,K : HK ::$ tang $K\,c\,L$ or tang $n\,c\,L$: tang $n\,c\,H$.
But $n\,K : HK :: n\,c : c\,m$
$:: \text{rad} : \sin lat.$

Accordingly

$$\text{Rad.} : \sin lat. :: \text{tang } hor.\ ang. : \text{tang } n\,c\,H,$$

whence

$$\text{tang } n\,c\,H = \sin lat. \text{ tang } hor.\ ang.$$

Therefore the angle which the line $H\,c$ or IV c makes with the meridian is of the required magnitude (39). In like manner the other hour lines may be shown to be drawn according to the formula above referred to.

To construct a vertical south dial, we have only to take the complement of the latitude instead of the latitude, and it is evident, that by proceeding in the manner above pointed out, we should have for the result

$$\text{tang } H\,c\,n = \cos lat. \text{ tang } hor.\ ang.$$

which agrees with the formula for a vertical south dial (42).

The line of inclination of meridians, it will be observed, is constructed like the line of hours, except that the angles are expressed in degrees instead of hours and quarters. It indicates the angles made by the intersections of the planes of the meridians with the plane of the dial, and may be used like the line of hours, as will be sufficiently evident from what has been said above.

When a dial has been constructed by means of the lines on a plane scale, the stile is to be made, and the dial is to be placed, according to the directions already given (40 &c.).

Of the Sector.

The sector consists of two arms or radii moveable about a centre. It contains, beside the lines above described†, double sets of lines of the following denominations, namely,

1	. . .	Lines or equal parts, marked	Lin. or	L.
2	. . .	Chords, ...	Cho.	C.
3	. . .	Sines, ...	Sin.	S.
4	. . .	Tangents to 45°, . . .	Tan.	T.
5	. . .	Secants, ...	Sec.	S.
6	. . .	Tangents above 45°, . .	Tan.	T.
7	. . .	Polygons, ...	Pol.	

These lines diverge from the centre of the axis about which the arms of the sector turn. The lines of chords, sines, and tangents to 45°, have the same radius, so that the chord of 60°, the sine of 90°, and the tangent of 45°, are equal to each other, and each equal to ten divisions on the line of lines. This common radius is nearly equal to the length of the instrument when closed. The line of tangents above 45° and that of secants, have each for its radius, one fourth of the above, and are continued to about 76°. The line of polygons is placed upon the inner edge of each arm of the sector, beginning with 4, and extending backward or toward the centre to 12.

The use of the sector depends upon the proportionality of the sides of similar triangles. Thus, let *AB* (*fig.* 98) be equal to *AC*, and *A b* to *A c*, and we shall have (*Geom.* 202) Fig. 98.

$$AB : BC :: A\,b : b\,c.$$

Therefore whatever part *A b* is of *AB*, the same is *b c* of *BC*. If *A b* be a chord, sine, or tangent to *AB* as radius, *b c* will be the same to *BC* as radius. The use of this instrument will be best understood by a few examples.

1. To divide a given line into any number of equal parts, as nine for instance. Take the length of the given line in the compasses, and open the sector till the distance between 9 and 9 on the two lines of equal parts is equal to the above extent. Then the transverse†† distance between 1 and 1 will be one ninth of the given line.

2. It is proposed to represent a field 140 poles in length, on a plan that shall be just 6 inches in its greatest extent. If we make the

† The small sectors of six inches do not contain all of the above lines.

†† An extent from one point to another point on the same line is called a *lateral* distance, and the extent between two corresponding points of lines of the same kind is called a *transverse* distance.

transverse distance on the lines of equal parts between 7 and 7, or 70 and 70, equal to half of 6 inches, we shall have in the transverse distances between corresponding numbers on these lines, a scale of equal parts of the required magnitude.

It is evident, moreover, from the nature of similar triangles, that many other questions may be solved by the above lines, as the finding of third proportionals, fourth proportionals, mean proportionals, &c. (*Geom.* 237, &c.)

By means of the lines of chords, sines, tangents, and secants we may form scales adapted to any radius less than the length of the sector when open. If it were required, for example, to measure the Fig. 21. arc of a circle already drawn, as AB (*fig.* 21), or to lay off any number of degrees on this arc; taking in the compasses, the radius of this arc $= AC$, and opening the sector till the length of this line extends from 60 to 60 on the line of chords, or from 90 to 90 on the line of sines, or from 45 to 45 on the line of tangents, (each requiring the same opening), we shall have a scale of each of these lines adapted to the radius of the given arc, and the number of degrees in AB is found, by applying the chord of AB to the line of chords, or the sine of $AB = BP$, to the line of sines, or the tangent of $AB = AB'$, to the line of tangents; also any number of degrees may be laid off on AB by means of the same lines.

In using the line of chords, if the arc to be measured or to be laid off, exceed 60°, we may first measure or lay off 60°, and then the remainder; if it exceed 120°, we may take 60° twice, and proceed with the remainder as with an original arc of this magnitude.

If we have occasion to employ a tangent of more than 45°, we make use of the second line of tangents, the radius of which is one fourth of that of the first, and equal to that of the line of secants.

Suppose it were required to make a stereographic projection of the sphere upon the plane of the meridian, similar to figure 14, the radius of the primitive being given equal to two inches. We open the sector till the transverse distance between 0 and 0 on the line of secants, or between 45 and 45 on the second line of tangents, is equal to two inches. We then take for the radii of the several meridians to be projected, the secants of their inclinations respectively, and for the distance of their centres from the centre of the primitive, the tangents of these same inclinations, and in projecting the parallels of latitude, we take for their radii the tangents of their polar distances, and for the distance of their centres, the secants of these same distances.

The line of polygons is a line of chords of $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, &c., part of 360°. It is used to inscribe a regular polygon in a circle. Let it be proposed, for example, to inscribe in a given circle a regular polygon of 8 sides. Open the sector till the transverse distance between 6 and 6, (answering to the chord of 60° on the two lines), is equal to the given radius, then the transverse distance between 8 and 8 will be the chord of the 8th part of the given circle, or the side of an inscribed octagon. In like manner a polygon of any other number of sides not exceeding 12, may be inscribed in a circle whose radius is known.

So also when the polygon to be constructed has one side given, by reversing the above process, we can find the radius of the circle in which this line can be inscribed the given number of times. Take the given line in the compasses, and open the sector till the transverse distance between 5 and 5, for example, if the required figure be a pentagon, be equal to this extent; then the distance between 6 and 6 will be the radius of a circle, in which the given line will make one side of a regular inscribed pentagon.

Of Gunter's Scale.

Gunter's scale, commonly of two feet in length, contains on one side the lines of the plane scale, already described, and on the other corresponding logarithmic lines†.

The line of numbers, marked *Num.* or *N.*, on which most of the others depend, is constructed thus, Let a line, equal to half the length of the proposed scale, be divided into 1000 equal parts; then, since the logarithm of 1 is 0, the distance of 1 from the beginning of the line is 0, that is, 1 stands at the beginning of the line. And, because the logarithm of 2 is 0,301, when the logarithm of 10 is 1, or, which is the same thing, the logarithm of 2 is 301, when the logarithm of 10 is 1000. Therefore the distance between 1 and 2 is 301 equal parts of the above scale. For the logarithm of 3 we take 477 of the same parts, and set it from 1 to 3; for the logarithm of 4, 602 parts, and so on, the numbers being taken from a common table of logarithms, but extending only to three places instead of five or seven. The primary divisions being thus formed, the intermediate divisions are obtained in a similar manner, by taking the logarithms of the intermediate numbers. Thus the logarithm of 1,1 is 41, the logarithm of 1,2 is 79, and so on. These numbers being set off in order from 1 will divide the primary division into ten parts. The other primary divisions are subdivided in a similar manner.

The line of sines is constructed by taking from the same scale of equal parts, the arithmetical complements of the logarithmic sines, as

† There are usually several of these lines on the small scales of six inches, namely, the line of numbers, and those of sines and tangents.

found in the common table, and setting them off from 90 backward or toward the left hand. A similar method is observed in the construction of the other lines.

There is moreover a line of ***meridional parts***, marked ***Mer.*** or ***M.***, placed directly over a line of equal parts with which it is used. The line of equal parts is numbered 0, 10, 20, &c., from the right hand to the left. Each of these large divisions represents 10 degrees of the equator or 600 nautical miles. The first of these divisions is sometimes divided into 40 equal parts, each representing 15 miles or 15′.

This line is constructed thus. Take the meridional parts corresponding to the several degrees of latitude from a table of meridional parts, and reduce them to degrees by dividing by 60. Take the quotients thus obtained, from the scale of equal parts, connected with the line to be constructed, and set them on this line from the right hand toward the left.

The extent from the brass pin on the line of meridional parts to any division on this line, applied to the line of equal parts, will give in degrees, the meridional parts answering to the latitude of that division. The extent from one division to another on the line of meridional parts, applied to the line of equal parts, will give the meridional difference of latitude between the two places denoted by the two divisions. This meridional difference is reduced to leagues by multiplying by 20, or to miles by multiplying by 60.

Problems in trigonometry may be solved by the double lines of the
Fig. 28. sector. Thus, in the question of art. 49 (*fig.* 28), having the angle $D = 31°$, and $DAB = 15°$, and the side $DB = 100$ yards, we take half† of 100 on the line of lines, and open the sector till this extent shall reach from 15° to 15° on the lines of sines, then the sine of 31°, taken as a transverse distance and applied to the same scale of lines, will give half of $AB = 99\frac{1}{2}$. Open the sector till the distance $99\frac{1}{2}$ shall extend from 90° to 90° on the lines of sines, and the sine of 46°, taken as a transverse distance and applied to the line of lines, will give $71\frac{1}{2}$, the double of which, 143, is the height required AC.

It will be readily seen that the above operation amounts to a geometrical construction, in which one of the sides is made the sine of its opposite angle.

A similar result may be obtained by logarithms, taken from the tables (49), or by the logarithmic lines on Gunter's scale.

† There are many cases in which it is convenient to take a certain part of the given side or sides, which need occasion no embarrassment, since all the sides may be considered as diminished in the same proportion without altering the angles (*Geom.* 205).

Corresponding to the geometrical proportion, or proportion by quotients,

sin 15° : sin 31° : : 100 : 199,

we have the arithmetical proportion, or equidifference,

log. sin 15° . log. sin 31° : : log. 100 . log. 199.

Accordingly, if we apply the compasses on the line of logarithmic sines from 15° to 31°, this extent will reach on the line of numbers from 100 to the term sought 199.

In like manner for the second solution of the article above referred to, the extent from 90° to 46° on the line of logarithmic sines, will reach from 199 to 143 on the line of numbers.

It is hardly necessary to observe, that when the first extent is taken progressively, or from a less to a greater, the second is also to be taken progressively, and vice versa†. It is equally obvious, that, when the proportion happens to present itself in such a form, that the first two terms are of a different nature, that is, one a side and the other an angle, and also the last two, the two middle terms may always be made to change places. Such a change is supposed in the second of the above examples.

In the case of spherical triangles, the sides being expressed in degrees as well as the angles, the four terms may all be considered as of the same kind, even though some are expressed by their tangents and others by their sines, it being observed always, where a cosine or cotangent occurs, to take the sine and tangent of the complement respectively.

In the first problem of the chapter on nautical astronomy, if we open the compasses from 23° 58′ to 90°, or the extremity of radius, on the line of logarithmic sines, this extent will reach on the same line, and in the same direction, from 12° 12′ to 32° 03′. So also in the second problem, the portion of the line of tangents from 45° to the tangent of the complement of 23° 58′, will extend from 12° 12′ on the same line to a point opposite to 29° 52′ on the line of sines.

We have indicated a method of solving plane triangles by geometrical construction *(78)*. Spherical triangles, in like manner, admit of being represented and of having their unknown parts determined independently of calculation.

From the manner in which *A* ♈ ☉ *(fig.* 50*)* is constructed, it Fig. 50.
will be observed, that the three given parts of *A* ♈ ☉, are such as they would appear to be to an eye situated in the surface of the sphere

† Regard however is to be paid to the circumstance of the tangent beyond 45° being continued back on the same line.

on which they are supposed to be delineated. Moreover the parts required ♈⊙, ♈*A*, are faithfully represented according to the same method of projection. Consequently, if from the point *N* or *S*, the poles of *E Q*, we draw straight lines through *A* and ♈, meeting the primitive, the arc of the primitive, thus intercepted, will be the true measure of *A*♈ (30). The number of degrees in ♈⊙ is found in a similar manner.

The same rule will apply where the required side is a portion of Fig. 57. an oblique circle, as *N C* (*fig.* 57), that is, we first find the pole *P*, of the oblique circle *n C m*†, and from this point we draw lines through the extremities of the required arc to the primitive, and the number of degrees, thus intercepted, will be the measure sought.

If the required part be an angle, we have only to draw from the vertex of this angle, through the poles of the containing sides, two straight lines meeting the primitive, and the arc thus intercepted will be the measure of the proposed angle.

II.

Instruments for measuring lengths.

1. The instrument most commonly used, for determining distances, especially in surveying, is the *chain*. It is ordinarily four rods, or twenty two yards in length, and is divided into one hundred links. Each link therefore is $\frac{22}{100}$ of a yard, $\frac{66}{100}$ of a foot, or 7,92 inches.

The manner of using the chain is very obvious. Ten small *arrows* being provided, two persons. called the *leader* and the *follower*, apply the chain successively along the line to be measured, the leader putting down an arrow at the termination of each chain's length, which is taken up by the follower, being employed both as a mark for placing the chain, and as a tally to show the number of times it is

† The projected pole *P* of any oblique circle *n C m* is always in ♑ ♋ drawn at right angles to *n m*, and its distance ♈ *P* from ♈, the centre of the primitive, is equal to the tangent of half the distance of its pole from the pole of the primitive, or, which is the same thing, half the inclination of the oblique circle to the primitive (12); hence, if a straight line *n V*, be drawn meeting the primitive in *L*, and from *L* we take *L M* equal to 90°, the line joining *n*, *M*, will pass through *P*, the pole of *n C m*.

in any required distance. One or more pickets or station-staves are set up as a guide to the chain-men to prevent any lateral deviation from a direct course. In surveying, an allowance is made for the oblique position of lines to the horizon, especially where the inclination is considerable.

Where great accuracy is required, distances are often measured in yards, or in feet and inches. In this case graduated rods, or poles, or measuring tapes, are used. In levelling, two staves (*fig.* 99), are employed, divided into inches and tenths, each staff consisting of two pieces that slide the one upon the other, so as to rise to the height of ten or twelve feet when extended. A signal or *target*, having a white stripe upon a black ground, to render it conspicuous at a distance, is attached to each staff in such a manner as to adhere to any part of its length, and thus to point out the elevation of the horizontal line, or line of sight of the levelling instrument, above the ground. Fig. 99

Extensive routes are often measured by means of a wheel (*fig.* 100) one half of a rod in circumference, so connected with a dial by mechanism, that the number of miles, furlongs, &c., are shown by indexes. It is sometimes attached to carriages, and is very convenient for measuring roads and lines of great extent, but is not generally so accurate as the chain. It is called a *perambulator* or *way wiser*.

Distances in navigation are estimated by the *log* (*fig.* 101). This instrument consists of a line attached to a thin sectoral piece of wood of about four or five inches radius. By means of a strip of lead fastened to the arc, it is made to float in a vertical position, about two thirds being immersed in the water. Upon being thrown into the sea, therefore, while the ship is under weigh, it will remain nearly stationary, and the quantity of line drawn freely from a reel on which it is wound, will show how far the ship has sailed during the time employed in this experiment. It is usual to take half a minute, as measured by a half minute glass. The length of line run off in this time, multiplied by 120, will accordingly give the rate per hour, on the supposition that the motion has been uniform. But instead of proceeding thus, the practice of seamen is to divide the line into portions, called *knots*, that bear the same proportion to a nautical mile that a half minute bears to an hour. Then the number of these knots run off in a half minute will show directly the rate of the ship's sailing per hour. Fig 101

III.

Of the levelling instrument and instruments for measuring angles.

1. The essential part of the most approved instruments employed in levelling, is a glass tube, filled with ether or spirits of wine, except a small portion containing air. The bore of the tube being straight or very slightly curved upward, it is obvious that the bubble of air, on account of its tendency to the highest point, will remain in the middle only when the two ends of the tube are on a level. If therefore a pair of sights, or small holes in a brass plate, be made to range on a line parallel to the two ends of a spirit-level, constructed as above described, it is evident that these sights will be horizontal, when the air-bubble occupies the middle of the tube. Instead of plain sights, a small telescope, a foot and a half or two feet in length, is often employed. This enables the observer to see to a greater distance and with more distinctness.

The use of the levelling instrument is very simple. Being adjusted, and the levelling staves being placed, one on each side by two assistants, in the direction of the route where the difference of level
Fig. 102 is to be found (*fig.*102), the instrument is successively turned to each of the levelling staves, and the distance of the line of apparent level, from the ground in each direction noted in two columns under the title of *fore* and *back* observations. The instrument with the back staff is now moved forward, the fore staff remaining, and the same process is repeated successively, till the extreme parts of the proposed route are connected together. Then the difference between the sum of the fore observations and that of the back observations will be the difference of level required nearly.

It will be perceived that, in the foregoing method, if the instrument be nearly in the middle between the two levelling staves at each station, no correction is necessary for refraction, or for the curvature of the earth's surface, since the error in one direction compensates for the error in the other. Where great accuracy is required, or where there is a great difference on the whole between the fore and back distances, from the instrument to the levelling staves, it becomes necessary to measure these distances and to apply a correction for the inequality.

Fig. 103 2. The *mariner's compass* (*fig.* 103) is used to trace a route through a wood, and to find the bearings of roads, the boundaries of fields, &c.; but it is particularly important in navigation, as it serves to indicate the course of a ship in the readiest and most convenient manner without the aid of the sun or stars. It consists principally of a magnetic needle attached to a circular card, the circumference of which is divided into 32 equal parts called *points*, and each point is subdivided into four parts called *quarters*. The line on which a ship sails, as indicated by the compass, is called a rhumb line, and its position is denoted by the angle which it makes with the meridian, expressed ordinarily in points and quarter points, but which may also be expressed in degrees and minutes, by allowing 11° 15′ to a point, and using the same proportion for a smaller quantity.

The denominations for the several points of the compass, in an abbreviated form, may be seen in the figure.

It is to be carefully observed that the magnetic needle does not point exactly north and south, except in certain particular places. Allowance therefore is to be made in almost all cases for this deviation, called the *variation* or *declination* of the magnetic needle, and this allowance is different in different places, and at different times in the same place. It has been ascertained by observation in the most frequented parts of the earth, and put down in charts for the use of seamen, and the change from year to year is for the most part not so great as to require to be attended to, till after the lapse of a considerable period.

3. A *theodolite* (*fig.* 104), contains, beside a compass, a horizontal Fig. 104
circle and a vertical arc, each divided so as to measure degrees and minutes. It is also provided with telescopic sights and a spirit level for the proper adjustment of the above graduated limbs, and is thus capable of being used as a levelling instrument. It is particularly adapted to measure the angles used in surveying, and in the mensuration of heights and distances, and is occasionally employed for astronomical purposes.

If we suppose an instrument, supported like the above, and provided only with a magnetic needle, a pair of plain sights, and a horizontal arch of 180°, this would be a *semicircle*. It is often used in surveying.

If the instrument have only a pair of sights and a compass box, divided into degrees, as well as into points, &c., it will still answer the purpose of measuring angles in a field. We have only in this case to determine the bearing of each of the sides containing the required angle, and to subtract the less from the greater. The difference will obviously be the angle sought. An instrument so constructed is called a *circumferentor*.

Fig. 106 Sometimes a simple table, (*fig.* 105), covered with a sheet of paper, is placed successively at the several corners of a field, and by means of a rule supporting two sights, the actual angles are laid down upon the paper, and a plan of the field is drawn on the spot. This is called a *plain table*. It is usually provided with a magnetic needle and a scale of equal parts.

4. The *quadrant of reflection*, commonly called *Hadley's quadrant*, represented in figure 106, is fitted to measure not only horizontal and vertical angles, but such as have their planes inclined in any manner whatever to the horizon. It is particularly useful at sea, where the motion of the ship prevents the use of instruments in which the plumb-line, or spirit-level are employed. The angular distance between two objects, as between the sun's limb and the horizon, between two stars, or between two station-staves on the surface of the earth is determined in the following manner. The instrument is so constructed, by means of two mirrors *a*, *b*, one of which is attached to the moveable index *I*, as to admit of the observer's seeing one of the given objects, the sun's lower limb for instance, directly and by reflection, at the same time. If now, the image and object thus coinciding, the index, carrying one of the mirrors, be moved forward, till the image of the sun's limb be brought to the horizon, or surface of the sea, the arc described by the index, according to a well known principle in optics, will be just half the arc described by the reflected image. We have only therefore to double the above arc, described by the index, or, which comes to the same thing, in the graduation of the arc *A B*, to call half degrees degrees, &c., and then the angle may be read off in the usual way. An eighth part of a circle is thus made to measure ninety degrees, and where there is occasion to measure a larger angle, as in taking the distance of the moon from the sun or a star, for the purpose of finding the longitude, the graduated limb is extended to sixty degrees, and is accordingly adapted to the measurement of angles of one hundred and twenty degrees. So constructed, the instrument is called a *sextant*. It is usually made with more care than the quadrant, and furnished with telescopic sights, and with magnifiers for reading off the divisions.

The sextant is sometimes made with a radius of only two or three inches, to be used in surveying and engineering, instead of the less portable instruments above described. It is usually enclosed in a box, and called a *box* or *pocket sextant*.

Finally, the graduated arc is enlarged to an entire circumference, and the mirrors so disposed as to admit of the measurement of the angle being repeated continually; then the sum of all the angles being divided by the number of observations, we obtain a result more to be relied on, than a single observation, since it is in a degree freed from certain errors in the construction and adjustment of the instrument that have hitherto been found unavoidable.

We have attempted to give the learner only some general notion of the leading properties of the foregoing instruments. More may be learned by a few minutes' actual inspection, than by the most extended and laboured description. The more minute details relating to the history of these inventions, their construction, adjustment, and use, are left to the teacher, who, with the aid of the instrument itself instead of a drawing, will be much better able to give the necessary information.

IV.

Investigation of the expressions for the sine and cosine of an arc made use of in article 179.

Radius being supposed equal to 1, we have the equation $\cos A^2 + \sin A^2 = 1$, the first member of which may be regarded as the product of the two imaginary factors $\cos A + \sqrt{-1}\sin A$ and $\cos A - \sqrt{-1}\sin A$. If we multiply together the two similar factors, $\cos A + \sqrt{-1}\sin A$, $\cos B + \sqrt{-1}\sin B$, the product will be

$$\cos A\cos B - \sin A\sin B + (\sin A\cos B + \sin B\cos A)\sqrt{-1}$$

This reduces itself to the form

$$\cos(A+B) + \sqrt{-1}\sin(A+B)$$

which is similar to each of the factors. We have, therefore, as a general result

$$(\cos A + \sqrt{-1}\sin A)(\cos B + \sqrt{-1}\sin B) = \cos(A+B) + \sqrt{-1}\sin(A+B);$$

and it is remarkable that quantities of this kind are multiplied together by simply adding the arcs, which is a property analogous to that of logarithms. Whence

$$(\cos A + \sqrt{-1}\sin A)(\cos A + \sqrt{-1}\sin A) = \cos 2A + \sqrt{-1}\sin 2A$$

$$(\cos A + \sqrt{-1}\sin A)(\cos 2A + \sqrt{-1}\sin 2A) = \cos 3A + \sqrt{-1}\sin 3A$$

$$(\cos A + \sqrt{-1}\sin A)(\cos 3A + \sqrt{-1}\sin 3A) = \cos 4A + \sqrt{-1}\sin 4A$$

&c.

The first product is equal to $(\cos A + \sqrt{-1}\sin A)^2$, the second to $(\cos A + \sqrt{-1}\sin A)^3$, and so on. Therefore, in general, n being equal to any entire number whatever, we have

$$(\cos A + \sqrt{-1}\sin A)^n = \cos nA + \sqrt{-1}\sin nA$$

from which is derived by changing the sign of $\sqrt{-1}$,

$$(\cos A - \sqrt{-1} \sin A)^n = \cos n A - \sqrt{-1} \sin n A.$$

From these two equations, which are a consequence the one of the other, we deduce the separate values of sin nA and cos nA; thus,

$$\cos n A = (\cos A + \sqrt{-1} \sin A)^n - \sqrt{-1} \sin n A,$$
$$\cos n A = (\cos A - \sqrt{-1} \sin A)^n + \sqrt{-1} \sin n A,$$

Whence

$$2 \cos n A = (\cos A + \sqrt{-1} \sin A)^n + (\cos A - \sqrt{-1} \sin A)^n,$$

or

$$\cos n A = \tfrac{1}{2} (\cos A + \sqrt{-1} \sin A)^n + \tfrac{1}{2} (\cos A - \sqrt{-1} \sin A)^n.$$

In like manner we obtain

$$2\sqrt{-1} \sin n A = (\cos A + \sqrt{-1} \sin A)^n - (\cos A - \sqrt{-1} \sin A)^n$$

or

$$\sin n A = \frac{1}{2\sqrt{-1}} (\cos A + \sqrt{-1} \sin A)^n - \frac{1}{2\sqrt{-1}} (\cos A - \sqrt{-1} \sin A)^n.$$

In order to express these same quantities in a series, it is necessary to develope, by the binomial formula, $(\cos A + \sqrt{-1} \sin A)^n$, which will give

$$\cos A^n + \frac{n}{1} \cos A^{n-1} \sin A \sqrt{-1} - \frac{n \cdot n-1}{1 \cdot 2} (\cos A^{n-2} \sin A^2$$
$$- \frac{n \cdot n-1 \cdot n-2}{1 \cdot 2 \cdot 3} \cos A^{n-3} \sin A^3 \sqrt{-1}$$
$$+ \frac{n \cdot n-1 \cdot n-2 \cdot n-3}{1 \cdot 2 \cdot 3 \cdot 4} \cos A^{n-4} \sin A^4 + \&c.$$

This quantity being the value of $\cos n A + \sqrt{-1} \sin n A$, if we put the real part equal to cos nA, and the imaginary part to $\sqrt{-1} \sin n A$, we shall have cos nA =

$$\cos A^n - \frac{n \cdot n-1}{1 \cdot 2} \cos A^{n-2} \sin A^2 + \frac{n \cdot n-1 \cdot n-2 \cdot n-3}{1 \cdot 2 \cdot 3 \cdot 4} \cos A^{n-4} \sin A^4$$

— &c. Also by multiplying the same two equal expressions by $\sqrt{-1}$, and putting their real part equal to sin nA, and the imaginary part to $\sqrt{-1} \cos n A$, we find

$$\sin n A = \frac{n}{1} \cos A^{n-1} \sin A - \frac{n \cdot n-1 \cdot n-2}{1 \cdot 2 \cdot 3} \cos A^{n-3} \sin A^3 + \&c.$$

By means of these series, the law of which will be easily perceived, the sine and cosine of an arc, the multiple of A, may be obtained in a manner more expeditious than by the method heretofore given. (*Trig*. 11, &c.)

These series admit of the form exhibited below, it being recollected that $\sin A = \cos A \operatorname{tang} A$ (*Trig*. 8.)

$$\cos n A = \cos A^n - \frac{n \cdot n-1}{1 \cdot 2} \cos A^n \operatorname{tang} A^2$$

$$+\frac{n.n-1.n-2.n-3}{1.2.3.4}\cos A^n \text{ tang } A^4 - \&c.$$

$$=\cos A^n\left(1-\frac{n.n-1}{1.2}\text{tang } A^2+\frac{n.n-1.n-2.n-3}{1.2.3.4}\text{tang } A^4-\&c.\right)$$

$$\sin nA=\cos A^n\left(\frac{n}{1}\text{tang } A-\frac{n.n-1.n-2}{1.2.3}\text{tang } A^3+\&c.\right)$$

Let $n=\frac{x}{A}$. Then, by substituting this value, still retaining the factor $\cos A^n$, we shall have $\cos x =$

$$\cos A^n\left(1-\frac{x.x-A}{1.2}.\frac{\text{tang } A^2}{A^2}+\frac{x.x-A.x-2A.x-3A}{1.2.3.4}.\frac{\text{tang } A^4}{A^4}-\&c.\right)$$

$$\sin x=\cos A^n\left(\frac{x}{1}.\frac{\text{tang } A}{A}-\frac{x.x-A.x-2A}{1.2.3}.\frac{\text{tang } A^3}{A^3}+\&c.\right)$$

In these formulas A may be taken of any magnitude we please. Suppose A very small, and we shall have $\frac{\text{tang } A}{A}$ but little different from unity, since the tangent of a very small arc is nearly equal to this arc. Still, while the arc is greater than 0, $\text{tang } A > A$* or $\frac{\text{tang } A}{A} < 1$; we have at the same time $A > \sin A$**; therefore $\frac{\text{tang } A}{A} < \frac{\text{tang } A}{\sin A}$, or $\frac{\text{tang } A}{A} < \frac{1}{\cos A}$. Whence it will be seen that the ratio $\frac{\text{tang } A}{A}$ is always comprehended between the limits 1 and $\frac{1}{\cos A}$. Let $A=0$, and we have $\cos A = 1$. Therefore, since $\frac{\text{tang } A}{A}$ is comprehended between 1 and $\frac{1}{\cos A}$, it follows that we must have precisely $\frac{\text{tang } A}{A}=1$. Hence, by making $A=0$, the above formulas become

$$\cos x=\cos A^n\left(1-\frac{x^2}{1.2}+\frac{x^4}{1.2.3.4}-\frac{x^6}{1.2.3.4.5.6}+\&c.\right)$$

$$\sin x=\cos A^n\left(x-\frac{x^3}{1.2.3}+\frac{x^5}{1.2.3.4.5}-\&c.\right)$$

It remains to see what is the value of $\cos A^n$, when A diminishes more and more, till it finally become zero. Now we have

---

* AB' (*fig.* 21) is greater than AB, because the triangle is to the sector ACB :: $AB' \times \frac{1}{2}AC$: $AB \times \frac{1}{2}AC$ :: AB' : AB (*Geom.* 290.)

** AB is greater than BP because the half of any arc is greater than half its chord (*Geom.* 283.)

$$\frac{1}{\cos \mathcal{A}^2} = \sec \mathcal{A}^2 \ (Trig.\ 29) = 1 + \text{tang}\, \mathcal{A}^2,$$

whence

$$\cos \mathcal{A} = \frac{1}{(1 + \text{tang}\, \mathcal{A}^2)^{\frac{1}{2}}},$$

and accordingly

$$\cos \mathcal{A}^n = \frac{1}{(1 + \text{tang}\, \mathcal{A}^2)^{\frac{n}{2}}} = 1 - \tfrac{n}{2} \text{tang}\, \mathcal{A}^2 + \frac{n \, . \, n + 2}{2 \, . \, 4} \text{tang}\, \mathcal{A}^4 - \&c.$$

Substituting for n its value $\frac{x}{\mathcal{A}}$ we have

$$\cos \mathcal{A}^n = 1 - \frac{x}{2} \mathcal{A} . \frac{\text{tang}\, \mathcal{A}^2}{\mathcal{A}^2} + \frac{x \, . \, x + 2\mathcal{A}}{2 \, . \, 4} \mathcal{A}^2 \frac{\text{tang}\, \mathcal{A}^4}{\mathcal{A}^4} - \&c.$$

If now we suppose $\mathcal{A}$ to diminish more and more, x remaining the same, the value of $\cos \mathcal{A}^n$ will approach nearer and nearer to unity, till upon making $\mathcal{A} = 0$, and $\frac{\text{tang}\, \mathcal{A}}{\mathcal{A}} = 1$, we shall obtain exactly $\cos \mathcal{A}^2 = 1$. We have therefore the following formulas;

$$\cos x = 1 - \frac{x^2}{1 \, . \, 2} + \frac{x^4}{1 \, . \, 2 \, . \, 3 \, . \, 4} - \frac{x^6}{1 \, . \, 2 \, . \, 3 \, . \, 4 \, . \, 5 \, . \, 6} + \&c.,$$

$$\sin x = x - \frac{x^3}{1 \, . \, 2 \, . \, 3} + \frac{x^5}{1 \, . \, 2 \, . \, 3 \, . \, 4 \, . \, 5} - \&c.,$$

which, when $x = \frac{a}{r}$, become

$$\cos \frac{a}{r} = 1 - \frac{a^2}{2 r^2} + \frac{a^4}{2 \, . \, 3 \, . \, 4 \, r^4} - \&c.,$$

$$\sin \frac{a}{r} = \frac{a}{r} - \frac{a^3}{2 \, . \, 3 \, r^3} + \&c.$$

See Legendre's Trig. art. xxxii.

APPENDIX

CONTAINING LOGARITHMIC AND OTHER TABLES.

Table of Meridional parts.

The construction and use of this table have been already explained (38, 71). It is only necessary to observe, therefore, that the degrees and minutes of any given latitude being found in the two first columes, we shall have on the same line in the column marked *Leng.* the length of the corresponding line on Mercator's chart.

Table of Astronomical Refractions.

This table contains the mean astronomical refractions for every degree of altitude or zenith distance, with the corresponding variations for $\frac{1}{10}$ of an inch of the barometer, and 20° of Fahrenheit's thermometer. The following example will sufficiently illustrate the use of this table, it being recollected that the refraction is increased by cold and by greater density, and diminished by heat and by greater rarity. Let the refraction be required, when the zenith distance or complement of the altitude is 25° 20′, and the barometer at 29,6, and Fahrenheit's thermometer at 60°. The refraction corresponding to 25° 20′, is 27″,0. The variation for $\frac{1}{10}$ of an inch of the barometer, is 0″,8; and therefore the variation for $\frac{4}{10}$ will be —0″,4, with the sign —, since the barometer is lower than in the Table. The variation for 20° of Fahrenheit, is 1″,3; and therefore the variation for 5° is + 0″,3, with the sign +, since the thermometer is higher than in the Table. The refraction consequently is 27″.0—0″4+0″3=26″9.

Table of Natural Sines.

This table contains the natural sine and cosine of every minute of the quadrant, constructed according to the methods furnished by trigonometry (*Trig.* 20), radius being 100000.

It is evident that these numbers may be readily adapted to any other radius. By supposing, for instance, a radius equal to 1, or one hundred thousandth part of the radius of the tables, we have only to reduce the numbers expressing the sines and cosines in the same proportion, that is, to divide them by 100000, or to separate the five right hand figures for decimals, and we have the value of the sines and cosines belonging to a circle whose radius is unity.

It will be seen that the same column is marked at one extremity *N. sine* and at the other *N. cos.* This is done for the sake of abbreviating the table. Every number, which expresses the sine of an arc, denoting also the cosine of its complement, it would be superfluous to repeat these numbers for the sole purpose of keeping the sines and cosines distinct. It will be observed therefore that the denominations at the tops of the columns correspond to the degrees at the top, taken in connexion with the minutes on the left; and that the denominations at the bottoms of the columns correspond to the degrees at the bottom, taken in connexion with the minutes on the right.

Table of the Logarithms of numbers.

1. The theory of logarithms and the method of obtaining them have been the subject of consideration (*Alg.* 238). It only remains therefore to point out the practical application of them to the solution of questions.

The tables here employed contain the logarithms of numbers from one to ten thousand. On the first page of the table the column of numbers, marked *N*, extends from 1 to 100, and against them in the same line on the right, are the entire logarithms, marked *Log*. Throughout the rest of the table only the fractional part of the logarithm is put down, as the integral part, or *characteristic*, may be readily supplied, it being recollected that it always contains as many units wanting one as there are figures in the given number (*Alg.* 245).

2. If the number whose logarithm is sought be between 100 and 1000, it is to be looked for in the first column of the table intitled *No.*, and the fractional part of the corresponding logarithm will be found on the same line in the second column. Of numbers between 1000 and 10000 the three first figures are to be sought in the first column, and the fourth figure in the upper line but one, and the corresponding logarithm will be found on the line of the three first figures and directly under the fourth.

3. If the given number exceed 10000, consider the first four figures on the left as a whole number, and the remaining figures as decimals. Find the logarithm of the number so reduced, by using a proportion for the decimal part, and then restore the original value

of the given number, by adding to the characteristic as many units as there are figures in the part cut off for decimals. Thus, to find the logarithim of 21598, for instance, I separate by a comma the four first figures on the left, which gives 2159,8. The log. of this number will consequently fall between the log. of 2159 or 3,33425, and that of 2160 or 3,33445. Now the difference between the logarithms of these two numbers is 0,00020. Consequently,

$$1 : 0{,}00020 :: 0{,}8 : 0{,}00016.$$

Accordingly, if we add 0,00016 to 3,33425, the log. of 2159, we shall have the log. of 2159,8 equal to 3,33441. But the given number is 21598 or 2159,8 × 10; we have therefore

$$\text{log. } 21598 = \text{log. } (2159{,}8 \times 10) = 3{,}33441 + 1 = 4{,}33441;$$

Whence the reason of the rule is evident.

4. To obtain the log. of a fractional number greater than unity, subtract the log. of the denominator from that of the numerator, and the remainder will express the log. required. Thus,

$$\text{log. } \tfrac{3549}{25} = 3{,}55011 - 1{,}39794 = 2{,}15217.$$

$$\text{log. } 7\tfrac{3}{11} = \text{log. } \tfrac{81}{11} = 1{,}90849 - 1{,}04139 = 0{,}86710.$$

5. The log. of a fraction less than unity is susceptible of two different forms. If it is desired that the log. should be entirely negative, subtract the log. of the numerator from that of the denominator, and the remainder, affected with the sign —, will be the logarithm sought. Accordingly we have

$$\text{log. } \tfrac{25}{3549} = -(3{,}55011 - 1{,}39794) = -2{,}15217.$$

Indeed the fraction $\tfrac{25}{3549}$ may be considered as the quotient arising from the division of 1 by $\tfrac{3549}{25}$; therefore, since the log. of a quotient is equal to the log. of the dividend minus the log. of the divisor, we have

$$\begin{aligned}\text{log. } \tfrac{25}{3549} &= \text{log. } 1 - \text{log. } \tfrac{3549}{25}\\ &= 0 - \text{log. } \tfrac{3549}{25}\\ &= -\text{log. } \tfrac{3549}{25} = -(3{,}55011 - 1{,}39794) = -2{,}15217.\end{aligned}$$

6. If the characteristic only is required to be negative, add as many units to the log. of the numerator, as will suffice for subtracting the log. of the denominator from it; perform this subtraction, and the decimal part of the remainder with a negative characteristic prefixed, equal to the difference between the units of the remainder and the units added to render the subtraction possible, will be the log. sought. If we add, in the above example, 7 units to the log. of 25 or 1,39794, we shall have 8,39794. Subtracting from this the log. of 3549 or 3,55011 we obtain for a remainder 4,84783. The decimal part of which 0,84783, with a characteristic 3, equal to the difference between the four units of the remainder and the 7 units added to make the subtraction possible, will be the required log. the characteristic of

which only is negative. Logarithms of this kind are distinguished by placing the sign — over the figure to be affected by it, thus $\bar{3},84783$.

The reason of the process here pursued will be easily perceived. Since the log. of a fraction is equal to the log. of the numerator minus the log. of the denominator, if we add a number of units to the log. of the numerator, the remainder will be just so much too great, and is accordingly to be diminished by the number of units added; that is, in the above example, 4 is to be diminished by 7; but we can actually take away only 4, and we indicate the remaining deduction by the expression — 3, according to the ordinary use of the sign minus. We should evidently arrive at the same result by adding any other number of units to the log. of the numerator.

7. The log. of a decimal number, either greater or less than unity, might be obtained by finding the log. of its equivalent vulgar fraction. But it is more convenient to operate directly with the decimal number, according to the following rules.

In the case of a decimal number greater than unity, suppose the decimal point removed, and proceed to find the log. of the entire expression considered as a whole number. Then diminish the characteristic by as many units, as the proposed number contained decimal figures, and the result will be the log. required. Thus,

$$\text{Log. } 21{,}598 = \text{log. } \frac{21598}{1000} = 4{,}33441 - 3 = 1{,}33441.$$

This is agreeable to what has just been shown. *See also Alg.* 246.

8. The log. of a decimal number less than unity admits of two forms. If it is required to be entirely negative, the decimal point being suppressed, find the logarithm of the given number, considered as a whole number, and subtract it from as many units as there are figures in the given decimal. Thus,

$$\text{log. } 0{,}000456 = \text{log. } \frac{456}{1000000} = 2{,}65896 - 6 = -3{,}34104.$$

9. If it were proposed that the characteristic only should be negative, find the log. of the given number, considered as a whole number, and the decimal part of this log. with a negative characteristic prefixed, equal to the number of ciphers which precede the first significant figure of the given decimal, will be the log. sought. Thus,

$$\text{log. } 0{,}000456 = \text{log. } \frac{456}{1000000} = 2{,}65896 - 6 = \bar{4}{,}65896.$$

10. Having pointed out the method of obtaining from the table the log. of any given number, we proceed to show how to find a number answering to any given log.

If the given log. is in the table, in which case the characteristic is 0, 1, 2, or 3, the corresponding number, the characteristic being less han 3, will be found on the same line in the column marked *No.* If the characteristic exceed 3, the three first figures of the corresponding number will be in the column marked *No*, and the fourth in the upper line but one directly over the given log. Thus the number belonging to the log. 3,56573 is 3679.

11. If the decimal part of the given log., the characteristic for instance being 3, cannot be found in the table, take the two logarithms, which are next greater and next less, and we shall have the proportion, as the difference of these two logarithms is to the difference of the corresponding numbers, so is the difference between the given log. and that which is nearest to it in the table, to the corresponding numerical difference. This numerical difference being added to the number belonging to the above nearest log. or subtracted from it, according as the nearest log. is greater or less than the given log., we shall obtain the number sought.

The given log. we instance being 3,33441, the next greater log. found in the table is 3,33445, and the next less 3,33425, the difference of which is 0,00020; the difference of the corresponding numbers is 1, and the difference between the given log. 3,33441 and the one in the table nearest to it in value, 3,33445, is 0,00004. Whence

$$0{,}00020 : 1 :: 0{,}00004 : 0{,}2.$$

Accordingly, the nearest log. being greater than the given log., if we subtract 0,2 from the number 2160, belonging to the nearest log. 3,33445, we shall have 2159,8 for the number answering to the given log. 3,33441.

12. If the characteristic of the given log. be more than 3, subtract from it its excess above 3, and find, by one of the above rules, the number answering to the remainder; if this number be entire, annex as many ciphers as there were units subtracted from the characteristic; if the number be decimal, remove the decimal point as many figures to the right as there were units subtracted from the characteristic; the result in each case will be the number answering to the given log.

Let the given log. be 7,56573; subtratting 4 from the characteristic we have for the remainder 3,56573, the number corresponding to which is 3679. Four ciphers being annexed to this gives 36790000 for the number belonging to the log. 7,56573. The reason may be briefly shown, thus,

$$36790000 = 3679 \times 10000,$$

$$\log.\ 36790000 = \log.\ 3679 + \log.\ 10000$$

$$= 3{,}56573 + 4 = 7{,}56573.$$

Let the given log. be 5,33441; 2 being subtracted from the characteristic leaves 3,33441, corresponding to which we have the number 2159,8; the decimal point being removed two places gives 215580 for the number answering to the log. 5,33441.

13. When the given log. is wholly negative, subtract it from a number of units greater than the characteristic, and the number answering to this remainder, with as many ciphers prefixed as there are units in the characteristic of the given log., will be the decimal fraction to which the given log. belongs.

Let the given log. be $-5,34104$; subtracting this from 8 for instance, we shall have for a remainder 2,65896, which answer to the number 456; prefixing 5 ciphers we obtain 0,00000456 as the number corresponding to the log. $-5,34104$.

The reason may be shown thus,

$$0,00000456 = 456 \times 0,00000001$$
$$\text{and log. } (456 \times 0,00000001) = 2,65896 + \text{log. } 0,00000001$$
$$= 2,65896 - 8$$
$$= -5,34104.$$

14. If the characteristic only of the given log. be negative, add a number of units greater than this characteristic, and the number belonging to the log. thus obtained with as many ciphers, wanting one, as there are units in the negative characteristic, will be the decimal fraction answering to the given log.

Let the given log. be $\bar{6},65896$. Adding 8 to this, we have for the sum 2,65896, to which the corresponding number is 456. Five ciphers being prefixed, gives 0,00000456 for the number required, appertaining to the log. $\bar{6},65896$.

The reason of the above process will appear from what is said above; since $2,65896 - 8$ becomes $\bar{6},65896$, instead of $-5,34104$, when the fractional part is considered as positive, and the characteristic only is required to be negative.

15. It will be remarked that where a log. either wholly or in part negative is changed to one that is positive by the addition of a larger positive characteristic the resulting log. so obtained, may be made to have for its characteristic 0, 1, 2, 3, &c. at pleasure, and it is not always indifferent which of these be employed.

If it were proposed, for example, to find the product of 2,745 multiplied by 20,01, we should take the sum of the log. of these two factors, thus,

$$\text{log. } 2,745 + \text{log. } 20,01 = 0,43854 + 1,30125 = 1,73979.$$

If we seek directly the number to which this log. belongs, we shall obtain for the required product 54,92848. But the true product is

54,92745. The error, therefore, in this case is 103 hundred thousandths. If now we add two to the characteristic of the above log., we shall find for the corresponding number 5492,750, which, being one hundred times too great on account of the above addition, will give for the required product 54,92750. The error, therefore, is reduced, by employing a larger characteristic, from 103 hundred thousandths to 5 hundred thousandths. By always employing the characteristic 3, which need occasion no perplexity, we shall arrive at the most correct results of which these tables are susceptible. We may always rely upon the exactnesss of the four first figures on the left. When this degree of approximation is not sufficient, we must have recourse to more extended tables.

Of the Table of Log. Sines, Tangents, and Secants†.

1. To obtain the logarithmic sine, tangent, or secant corresponding to any number of degrees and minutes, find the given degrees at the top of the page, except this number fall between 45° and 135°, in which case they are to be sought at the bottom, the minutes being found in the column marked *M*, which stands on the side of the page on which the degrees are marked. Thus, if the degrees are less than 45, the minutes are to be found in the left hand column, and it must be noted, that if the degrees are found at the top, the names of hour, sine, cosine, tangent &c., must also be found at the top. If the degrees are found at the bottom, the names, sine, cosine &c. must also be found at the bottom. Then opposite to the number of minutes will be found the log. sine, log. secant, &c. in the column marked sine, secant, &c. respectively.

If the log. sine of 28° 37′, for example, were required, we should find 28° at the top of the page, and directly below it in the left hand column 37′, against which, in the column marked sine, is 9,68029 the log. sought.

The logarithms secant of 126° 20′ being required, we find 126° at the bottom of the page, and directly above it in the left hand column 20′, against which, in the column marked secant, is 10,22732 the log. sought.

2. To obtain the log. sine, cosine, &c. for degrees, minutes, and seconds, we find the log. corresponding to the even minutes next

† It will be observed, that if a table of natural sines, cosines, &c., be computed to a radius of 10000000000, and the logarithms of these numbers be calculated like the logarithms of any other numbers, they would form a table like that above referred to, in which the log. of radius is 10,00000.

above and below the given degrees and minutes, and take their difference. Then as 1′ or 60″ is to the given seconds, so is the above difference to the log. of the given seconds, which is to be added to the log. corresponding to the less number of degrees and minutes, or subtracted from it, according as this log. is less or greater than the other.

The log. sine of 24° 16′ 48″, for example, being required, we take the following two logarithms, namely, log. sine of 24° 16′ = 9, 61382, and log. sine of 24° 17′ = 9, 61411, the difference of which is 0,00029; whence

60″ : 48″ :: 0,00029 : 0,00023,

which, added to 9,61382, the log. sine of 24° 16′, gives 9,61405 for the log. sine of 24° 16′ 48″.

To find the log. secant of 105° 20′ 16″, we take the log. secant of 105° 20′ = 10,57768, and the log. secant of 105° 21′ = 10,57772, the difference of which is 46; whence

60″ : 16″ :: 0,00046 : 0,00012,

which being subtracted from the log. corresponding to the least number of degrees and minutes, (since this is greater than the other) gives 10,57756 for the log. secant of 105° 20′ 16″.

If the given seconds be ½, ⅓, ¼, or any other even parts of a minute, the like part may be taken of the difference of the logarithms and added or subtracted, according to the above rule. This may frequently be done by inspection.

3. To obtain the degrees, minutes, and secouds corresponding to any given log. sine, cosine, &c. we find the two nearest numbers to the given log. sine, cosine, &c., in the column marked sine, cosine, &c., respectively, one being greater and the other less, and take their difference; we take also the difference between the given log. and the log. corresponding to the least number of degrees and minutes. Then the first of the above differences, is to the second, as 60″ is to the number of seconds corresponding to the second difference, which being annexed to the smaller number of degrees and minutes, before found, will give the quantity sought.

Thus to find the degrees, minutes, and seconds (less than 90°), corresponding to the log. sine 9,61405, we take the two nearest logarithms with the corresponding degrees and minutes, namely,

Next less log.	9,61382	24° 16′
Next greater log.	9,61411	24° 17′
Difference	0,00029	1′

We also take the difference between the given log. and the log.

belonging to the least number of degrees, and minutes, namely, 0,00021. Then from the proportion

0,00029 : 0,00028 :: 60″ : 48″

we have 48″ as the quantity to be annexed to 24° 16′ to make the entire number of degrees &c., answering to the given log. sine 9,61405.

Preceding the table of the logs. of numbers will be found a table containing the log. sines, tangents, and secants, to every point and quarter point of the compass. This differs from the table last explained only in having the angles expressed in points and quarters of a point instead of degrees and minutes.

M.	D.	Leng.	D.	Leng.	D.	Leng.	D.	Leng.	D.	Leng.	D.	Leng.	D.	Leng.
0	0	0	7	421	14	848	21	1289	28	1751	35	2244	42	2782
10		10		431		859		1300		1762		2256		2795
20		20		441		869		1311		1774		2269		2809
30		30		451		879		1321		1785		2281		2822
40		40		461		890		1332		1797		2293		2836
50		50		471		900		1343		1808		2306		2849
0	1	60	8	482	15	910	22	1354	29	1819	36	2318	43	2863
10		70		492		921		1364		1831		2330		2877
20		80		502		931		1375		1842		2343		2890
30		90		512		941		1386		1854		2355		2904
40		100		522		952		1397		1865		2368		2918
50		110		532		962		1408		1877		2380		2932
0	2	120	9	542	16	973	23	1419	30	1888	37	2393	44	2946
10		130		552		983		1429		1900		2405		2960
20		140		562		993		1440		1911		2418		2974
30		150		573		1004		1451		1923		2430		2988
40		160		583		1014		1462		1935		2443		3002
50		170		593		1025		1473		1946		2456		3016
0	3	180	10	603	17	1035	24	1484	31	1958	38	2468	45	3030
10		190		613		1046		1495		1970		2481		3044
20		200		623		1056		1506		1981		2494		3058
30		210		634		1067		1517		1993		2506		3072
40		220		644		1077		1528		2005		2519		3087
50		230		654		1088		1539		2017		2532		3101
0	4	240	11	664	18	1098	25	1550	32	2028	39	2545	46	3116
10		250		674		1109		1561		2040		2558		3130
20		260		684		1119		1572		2052		2571		3144
30		270		695		1130		1583		2064		2584		3159
40		280		705		1140		1594		2076		2597		3173
50		290		715		1151		1605		2088		2610		3188
0	5	300	12	725	19	1161	26	1616	33	2099	40	2623	47	3203
10		310		735		1172		1628		2111		2636		3217
20		320		746		1183		1639		2123		2649		3232
30		330		756		1193		1650		2135		2662		3247
40		340		766		1204		1661		2147		2675		3262
50		350		776		1214		1672		2159		2688		3276
0	6	360	13	787	20	1225	27	1684	34	2171	41	2702	48	3291
10		370		797		1236		1695		2184		2715		3306
20		380		807		1246		1706		2196		2728		3321
30		390		818		1257		1717		2208		2741		3337
40		400		828		1268		1729		2220		2755		3352
50		410		838		1278		1740		2232		2768		3367

Meridional Parts.

M.	D.	Leng.	D.	Leng.	D.	Leng.	D.	Leng.	D.	Leng.	D.	Leng.
0	49	3382	56	4074	63	4905	70	5966	77	7467	84	10137
10		3397		4092		4927		5995		7512		10234
20		3412		4110		4949		6025		7557		10334
30		3428		4128		4972		6055		7603		10437
40		3443		4146		4994		6085		7650		10543
50		3459		4164		5017		6115		7697		10652
0	50	3474	57	4183	64	5039	71	6146	78	7745	85	10765
10		3490		4201		5062		6177		7793		10881
20		3506		4219		5085		6208		7842		11002
30		3521		4238		5108		6240		7892		11127
40		3537		4257		5132		6271		7942		11257
50		3553		4275		5155		6303		7994		11392
0	51	3569	58	4294	65	5179	72	6335	79	8046	86	11533
10		3585		4313		5202		6367		8099		11679
20		3601		4332		5226		6400		8152		11832
30		3617		4351		5250		6433		8207		11992
40		3633		4370		5275		6467		8262		12160
50		3649		4389		5299		6500		8318		12334
0	52	3655	59	4409	66	5323	73	6534	80	8375	87	12522
10		3681		4429		5348		6569		8433		12719
20		3698		4448		5373		6603		8492		12927
30		3714		4468		5398		6638		8552		13149
40		3731		4488		5423		6674		8614		13387
50		3747		4507		5448		6710		8676		13641
0	53	3764	60	4527	67	5474	74	6746	81	8739	88	13917
10		3780		4547		5500		6782		8803		14216
20		3797		4568		5526		6819		8869		14543
30		3814		4588		5552		6856		8936		14906
40		3831		4608		5578		6894		9004		15311
50		3848		4629		5604		6932		9074		15770
0	54	3865	61	4649	68	5631	75	6970	82	9145	89	16300
10		3882		4670		5658		7009		9218		16926
20		3899		4691		5685		7048		9292		17694
30		3916		4712		5712		7088		9368		18682
40		3933		4733		5739		7128		9446		20075
50		3950		4754		5767		7169		9525		22458
0	55	3967	62	4775	69	5794	76	7210	83	9606	90	Infinite.
10		3985		4796		5822		7251		9689		
20		4003		4818		5851		7293		9774		
30		4021		4839		5879		7336		9861		
40		4038		4861		5908		7379		9951		
50		4056		4883		5937		7423		10043		

Astronomical Refractions, when the barometer is at 30,0 English inches, and Fahrenheit's thermometer at 55°, or when the barometer is at 29,6, and Fahrenheit's Thermometer at 50°.

Altitude.	Zenith Distance.	Refraction for Barom. 30,0, Therm. 55°.	Variation for 9-10ths of an inch of Barom.	Variation for 20° of Fahrenheit's Therm.	Altitude.	Zenith Distance.	Refraction for Barom. 30,0, Therm. 55°.	Variation for 9-10ths of an inch of Barom.	Variation for 20° of Fahrenheit's Therm.	Altitude.	Zenith Distance.	Refraction for Barom. 30,0, Therm. 55°.	Variation for 9-10ths of an inch of Barom.	Variation for 20° of Fahrenheit's Therm.
°	°	′ ″	″	″	°	°	′ ″	″	″	°	°	′ ″	″	″
90	0	0 0.0	0.0	0.0	60	30	0 33.1	1.0	1.5	30	60	1 39.0	2.9	4.5
89	1	0 1.0	0.0	0.0	59	31	0 34.4	1.0	1.5	29	61	1 43.2	3.0	4.7
88	2	0 2.0	0.1	0.1	58	32	0 35.8	1.1	1.6	28	62	1 47.6	3.1	4.9
87	3	0 3.0	0.1	0.1	57	33	0 37.2	1.1	1.7	27	63	1 52.3	3.2	5.1
86	4	0 4.0	0.1	0.2	56	34	0 38.7	1.2	1.7	26	64	1 57.2	3.4	5.3
85	5	0 5.0	0.2	0.2	55	35	0 40.2	1.2	1.8	25	65	2 2.4	3.5	5.6
84	6	0 6.1	0.2	0.2	54	36	0 41.7	1.2	1.9	24	66	2 8.0	3.7	5.9
83	7	0 7.1	0.2	0.3	53	37	0 43.3	1.3	2.0	23	67	2 14.2	4.0	6.3
82	8	0 8.1	0.2	0.3	52	38	0 44.9	1.3	2.0	22	68	2 20.9	4.2	6.6
81	9	0 9.2	0.3	0.4	51	39	0 46.5	1.4	2.1	21	69	2 28.3	4.4	6.9
80	10	0 10.2	0.3	0.4	50	40	0 48.1	1.4	2.2	20	70	2 36.3	4.7	7.3
79	11	0 11.2	0.3	0.5	49	41	0 49.8	1.5	2.2	19	71	2 45.1	5.0	7.7
78	12	0 12.3	0.4	0.5	48	42	0 51.6	1.5	2.3	18	72	2 54.7	5.3	8.2
77	13	0 13.3	0.4	0.6	47	43	0 53.4	1.6	2.4	17	73	3 5.5	5.6	8.7
76	14	0 14.4	0.4	0.6	46	44	0 55.3	1.6	2.5	16	74	3 17.5	5.9	9.3
75	15	0 15.4	0.5	0.7	45	45	0 57.3	1.7	2.6	15	75	3 31.0	6.3	9.9
74	16	0 16.5	0.5	0.7	44	46	0 59.3	1.8	2.7	14	76	3 46.4	6.8	10.6
73	17	0 17.6	0.5	0.8	43	47	1 1.4	1.9	2.8	13	77	4 3.8	7.3	11.5
72	18	0 18.7	0.6	0.8	42	48	1 3.6	2.0	2.9	12	78	4 24.0	7.9	12.5
71	19	0 19.8	0.6	0.9	41	49	1 5.9	2.0	3.0	11	79	4 46.6	8.6	13.7
70	20	0 20.9	0.6	0.9	40	50	1 8.2	2.1	3.1	10	80	5 15.6	9.4	14.9
69	21	0 22.0	0.7	1.0	39	51	1 10.6	2.2	3.2	9	81	5 49.0	10.4	16.5
68	22	0 23.2	0.7	1.0	38	52	1 13.2	2.3	3.4	8	82	6 29.7	11.6	18.5
67	23	0 24.3	0.7	1.1	37	53	1 15.9	2.3	3.5	7	83	7 20.3	13.1	21.3
66	24	0 25.5	0.8	1.2	36	54	1 18.7	2.4	3.7	6	84	8 24.7	15.0	24.9
65	25	0 26.7	0.8	1.2	35	55	1 21.6	2.5	3.8	5	85	9 48.8	17.5	29.6
64	26	0 28.0	0.8	1.3	34	56	1 24.7	2.5	3.9	4	86	11 41.8		
63	27	0 29.2	0.9	1.3	33	57	1 28.0	2.6	4.1	3	87	14 18.4		
62	28	0 30.5	0.9	1.4	32	58	1 31.5	2.7	4.2	2	88	18 1.3		
61	29	0 31.8	1.0	1.4	31	59	1 35.2	2.8	4.3	1	89	23 21.4		
60	30	0 33.1	1.0	1.5	30	60	1 39.0	2.9	4.5	0	90	30 50.8	55.0	129.2

M.	0° N.sine.	0° N. cos.	1° N.sine.	1° N. cos.	2° N.sine.	2° N. cos.	3° N.sine.	3° N. cos.	4° N.sine.	4° N. cos.	
0	00000	100000	01745	99985	03490	99939	05234	99863	06976	99756	60
1	00029	100000	01774	99984	03519	99938	05263	99861	07005	99754	59
2	00058	100000	01803	99984	03548	99937	05292	99860	07034	99752	58
3	00087	100000	01832	99983	03577	99936	05321	99858	07063	99750	57
4	00116	100000	01862	99983	03606	99935	05350	99857	07092	99748	56
5	00145	100000	01891	99982	03635	99934	05379	99855	07121	99746	55
6	00175	100000	01920	99982	03664	99933	05408	99854	07150	99744	54
7	00204	100000	01949	99981	03693	99932	05437	99852	07179	99742	53
8	00233	100000	01978	99980	03723	99931	05466	99851	07208	99740	52
9	00262	100000	02007	99980	03752	99930	05495	99849	07237	99738	51
10	00291	100000	02036	99979	03781	99929	05524	99847	07266	99736	50
11	00320	99999	02065	99979	03810	99927	05553	99846	07295	99734	49
12	00349	99999	02094	99978	03839	99926	05582	99844	07324	99731	48
13	00378	99999	02123	99977	03868	99925	05611	99842	07353	99729	47
14	00407	99999	02152	99977	03897	99924	05640	99841	07382	99727	46
15	00436	99999	02181	99976	03926	99923	05669	99839	07411	99725	45
16	00465	99999	02211	99976	03955	99922	05698	99838	07440	99723	44
17	00495	99999	02240	99975	03984	99921	05727	99836	07469	99721	43
18	00524	99999	02269	99974	04013	99919	05756	99834	07498	99719	42
19	00553	99998	02298	99974	04042	99918	05785	99833	07527	99716	41
20	00582	99998	02327	99973	[illegible]	99917	05814	99831	07556	99714	40
21	00611	99998	02356	99972	[illegible]	99916	05844	99829	07585	99712	39
22	00640	99998	02385	99972	04129	99915	05873	99827	07614	99710	38
23	00669	99998	02414	99971	04159	99913	05902	[illegible]	07643	99708	37
24	00698	99998	02443	99970	04188	99912	05931	99824	07672	99705	36
25	00727	99997	02472	99969	04217	99911	05960	99822	07701	99703	35
26	00756	99997	02501	99969	04246	99910	05989	99821	07730	99701	34
27	00785	99997	02530	99968	04275	99909	06018	99819	07759	99699	33
28	00814	99997	02560	99967	04304	99907	06047	99817	07788	99696	32
29	00844	99996	02589	99966	04333	99906	06076	99815	07817	99694	31
30	00873	99996	02618	99966	04362	99905	06105	99813	07846	99692	30
31	00902	99996	02647	99965	04391	99904	06134	99812	07875	99689	29
32	00931	99996	02676	99964	04420	99902	06163	99810	07904	99687	28
33	00960	99995	02705	99963	04449	99901	06192	99808	07933	99685	27
34	00989	99995	02734	99963	04478	99900	06221	99806	07962	99683	26
35	01018	99995	02763	99962	04507	99898	06250	99804	07991	99680	25
36	01047	99995	02792	99961	04536	99897	06279	99803	08020	99678	24
37	01076	99994	02821	99960	04565	99896	06308	99801	08049	99676	23
38	01105	99994	02850	99959	04594	99894	06337	99799	08078	99673	22
39	01134	99994	02879	99959	04623	99893	06366	99797	08107	99671	21
40	01164	99993	02908	99958	04653	99892	06395	99795	08136	99668	20
41	01193	99993	02938	99957	04682	99890	06424	99793	08165	99666	19
42	01222	99993	02967	99956	04711	99889	06453	99792	08194	99664	18
43	01251	99992	02996	99955	04740	99888	06482	99790	08223	99661	17
44	01280	99992	03025	99954	04769	99886	06511	99788	08252	99659	16
45	01309	99991	03054	99953	04798	99885	06540	99786	08281	99657	15
46	01338	99991	03083	99952	04827	99883	06569	99784	08310	99654	14
47	01367	99991	03112	99952	04856	99882	06598	99782	08339	99652	13
48	01396	99990	03141	99951	04885	99881	06627	99780	08368	99649	12
49	01425	99990	03170	99950	04914	99879	06656	99778	08397	99647	11
50	01454	99989	03199	99949	04943	99878	06685	99776	08426	99644	10
51	01483	99989	03228	99948	04972	99876	06714	99774	08455	99642	9
52	01513	99989	03257	99947	05001	99875	06743	99772	08484	99639	8
53	01542	99988	03286	99946	05030	99873	06773	99770	08513	99637	7
54	01571	99988	03316	99945	05059	99872	06802	99768	08542	99635	6
55	01600	99987	03345	99944	05088	99870	06831	99766	08571	99632	5
56	01629	99987	03374	99943	05117	99869	06860	99764	08600	99630	4
57	01658	99986	03403	99942	05146	99867	06889	99762	08629	99627	3
58	01687	99986	03432	99941	05175	99866	06918	99760	08658	99625	2
59	01716	99985	03461	99940	05205	99864	06947	99758	08687	99622	1
60	01745	99985	03490	99939	05234	99863	06976	99756	08716	99619	0
	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	M.
	89°		88°		87°		86°		85°		

	5°		6°		7°		8°		9°		
M.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	
0	08716	99619	10453	99452	12187	99255	13917	99027	15643	98769	60
1	08745	99617	10482	99449	12216	99251	13946	99023	15672	98764	59
2	08774	99614	10511	99446	12245	99248	13975	99019	15701	98760	58
3	08803	99612	10540	99443	12274	99244	14004	99015	15730	98755	57
4	08831	99609	10569	99440	12302	99240	14033	99011	15758	98751	56
5	08860	99607	10597	99437	12331	99237	14061	99006	15787	98746	55
6	08889	99604	10626	99434	12360	99233	14090	99002	15816	98741	54
7	08918	99602	10655	99431	12389	99230	14119	98998	15845	98737	53
8	08947	99599	10684	99428	12418	99226	14148	98994	15873	98732	52
9	08976	99596	10713	99424	12447	99222	14177	98990	15902	98728	51
10	09005	99594	10742	99421	12476	99219	14205	98986	15931	98723	50
11	09034	99591	10771	99418	12504	99215	14234	98982	15959	98718	49
12	09063	99588	10800	99415	12533	99211	14263	98978	15988	98714	48
13	09092	99586	10829	99412	12562	99208	14292	98973	16017	98709	47
14	09121	99583	10858	99409	12591	99204	14320	98969	16046	98704	46
15	09150	99580	10887	99406	12620	99200	14349	98965	16074	98700	45
16	09179	99578	10916	99402	12649	99197	14378	98961	16103	98695	44
17	09208	99575	10945	99399	12678	99193	14407	98957	16132	98690	43
18	09237	99572	10973	99396	12706	99189	14436	98953	16160	98686	42
19	09266	99570	11002	99393	12735	99186	14464	98948	16189	98681	41
20	09295	99567	11031	99390	12764	99182	14493	98944	16218	98676	40
21	09324	99564	11060	99386	12793	99178	14522	98940	16246	98671	39
22	09353	99562	11089	99383	12822	99175	14551	98936	16275	98667	38
23	09382	99559	11118	99380	12851	99171	14580	98931	16304	98662	37
24	09411	99556	11147	99377	12880	99167	14608	98927	16333	98657	36
25	09440	99553	11176	99374	12908	99163	14637	98923	16361	98652	35
26	09469	99551	11205	99370	12937	99160	14666	98919	16390	98648	34
27	09498	99548	11234	99367	12966	99156	14695	98914	16419	98643	33
28	09527	99545	11263	99364	12995	99152	14723	98910	16447	98638	32
29	09556	99542	11291	99360	13024	99148	14752	98906	16476	98633	31
30	09585	99540	11320	99357	13053	99144	14781	98902	16505	98629	30
31	09614	99537	11349	99354	13081	99141	14810	98897	16533	98624	29
32	09642	99534	11378	99351	13110	99137	14838	98893	16562	98619	28
33	09671	99531	11407	99347	13139	99133	14867	98889	16591	98614	27
34	09700	99528	11436	99344	13168	99129	14896	98884	16620	98609	26
35	09729	99526	11465	99341	13197	99125	14925	98880	16648	98604	25
36	09758	99523	11494	99337	13226	99122	14954	98876	16677	98600	24
37	09787	99520	11523	99334	13254	99118	14982	98871	16706	98595	23
38	09816	99517	11552	99331	13283	99114	15011	98867	16734	98590	22
39	09845	99514	11580	99327	13312	99110	15040	98863	16763	98585	21
40	09874	99511	11609	99324	13341	99106	15069	98858	16792	98580	20
41	09903	99508	11638	99320	13370	99102	15097	98854	16820	98575	19
42	09932	99506	11667	99317	13399	99098	15126	98849	16849	98570	18
43	09961	99503	11696	99314	13427	99094	15155	98845	16878	98565	17
44	09990	99500	11725	99310	13456	99091	15184	98841	16906	98561	16
45	10019	99497	11754	99307	13485	99087	15212	98836	16935	98556	15
46	10048	99494	11783	99303	13514	99083	15241	98832	16964	98551	14
47	10077	99491	11812	99300	13543	99079	15270	98827	16992	98546	13
48	10106	99488	11840	99297	13572	99075	15299	98823	17021	98541	12
49	10135	99485	11869	99293	13600	99071	15327	98818	17050	98536	11
50	10164	99482	11898	99290	13629	99067	15356	98814	17078	98531	10
51	10192	99479	11927	99286	13658	99063	15385	98809	17107	98526	9
52	10221	99476	11956	99283	13687	99059	15414	98805	17136	98521	8
53	10250	99473	11985	99279	13716	99055	15442	98800	17164	98516	7
54	10279	99470	12014	99276	13744	99051	15471	98796	17193	98511	6
55	10308	99467	12043	99272	13773	99047	15500	98791	17222	98506	5
56	10337	99464	12071	99269	13802	99043	15529	98787	17250	98501	4
57	10366	99461	12100	99265	13831	99039	15557	98782	17279	98496	3
58	10395	99458	12129	99262	13860	99035	15586	98778	17308	98491	2
59	10424	99455	12158	99258	13889	99031	15615	98773	17336	98486	1
60	10453	99452	12187	99255	13917	99027	15643	98769	17365	98481	0
	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	M.
	84°		83°		82°		81°		80°		

	10°		11°		12°		13°		14°		
M.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	
0	17365	98481	19081	98163	20791	97815	22495	97437	24192	97030	60
1	17393	98476	19109	98157	20820	97809	22523	97430	24220	97023	59
2	17422	98471	19138	98152	20848	97803	22552	97424	24249	97015	58
3	17451	98466	19167	98146	20877	97797	22580	97417	24277	97008	57
4	17479	98461	19195	98140	20905	97791	22608	97411	24305	97001	56
5	17508	98455	19224	98135	20933	97784	22637	97404	24333	96994	55
6	17537	98450	19252	98129	20962	97778	22665	97398	24362	96987	54
7	17565	98445	19281	98124	20990	97772	22693	97391	24390	96980	53
8	17594	98440	19309	98118	21019	97766	22722	97384	24418	96973	52
9	17623	98435	19338	98112	21047	97760	22750	97378	24446	96966	51
10	17651	98430	19366	98107	21076	97754	22778	97371	24474	96959	50
11	17680	98425	19395	98101	21104	97748	22807	97365	24503	96952	49
12	17708	98420	19423	98096	21132	97742	22835	97358	24531	96945	48
13	17737	98414	19452	98090	21161	97735	22863	97351	24559	96937	47
14	17766	98409	19481	98084	21189	97729	22892	97345	24587	96930	46
15	17794	98404	19509	98079	21218	97723	22920	97338	24615	96923	45
16	17823	98399	19538	98073	21246	97717	22948	97331	24644	96916	44
17	17852	98394	19566	98067	21275	97711	22977	97325	24672	96909	43
18	17880	98389	19595	98061	21303	97705	23005	97318	24700	96902	42
19	17909	98383	19623	98056	21331	97698	23033	97311	24728	96894	41
20	17937	98378	19652	98050	21360	97692	23062	97304	24756	96887	40
21	17966	98373	19680	98044	21388	97686	23090	97298	24784	96880	39
22	17995	98368	19709	98039	21417	97680	23118	97291	24813	96873	38
23	18023	98362	19737	98033	21445	97673	23146	97284	24841	96866	37
24	18052	98357	19766	98027	21474	97667	23175	97278	24869	96858	36
25	18081	98352	19794	98021	21502	97661	23203	97271	24897	96851	35
26	18109	98347	19823	98016	21530	97655	23231	97264	24925	96844	34
27	18138	98341	19851	98010	21559	97648	23260	97257	24954	96837	33
28	18166	98336	19880	98004	21587	97642	23288	97251	24982	96829	32
29	18195	98331	19908	97998	21616	97636	23316	97244	25010	96822	31
30	18224	98325	19937	97992	21644	97630	23345	97237	25038	96815	30
31	18252	98320	19965	97987	21672	97623	23373	97230	25066	96807	29
32	18281	98315	19994	97981	21701	97617	23401	97223	25094	96800	28
33	18309	98310	20022	97975	21729	97611	23429	97217	25122	96793	27
34	18338	98304	20051	97969	21758	97604	23458	97210	25151	96786	26
35	18367	98299	20079	97963	21786	97598	23486	97203	25179	96778	25
36	18395	98294	20108	97958	21814	97592	23514	97196	25207	96771	24
37	18424	98288	20136	97952	21843	97585	23542	97189	25235	96764	23
38	18452	98283	20165	97946	21871	97579	23571	97182	25263	96756	22
39	18481	98277	20193	97940	21899	97573	23599	97176	25291	96749	21
40	18509	98272	20222	97934	21928	97566	23627	97169	25320	96742	20
41	18538	98267	20250	97928	21956	97560	23656	97162	25348	96734	19
42	18567	98261	20279	97922	21985	97553	23684	97155	25376	96727	18
43	18595	98256	20307	97916	22013	97547	23712	97148	25404	96719	17
44	18624	98250	20336	97910	22041	97541	23740	97141	25432	96712	16
45	18652	98245	20364	97905	22070	97534	23769	97134	25460	96705	15
46	18681	98240	20393	97899	22098	97528	23797	97127	25488	96697	14
47	18710	98234	20421	97893	22126	97521	23825	97120	25516	96690	13
48	18738	98229	20450	97887	22155	97515	23853	97113	25545	96682	12
49	18767	98223	20478	97881	22183	97508	23882	97106	25573	96675	11
50	18795	98218	20507	97875	22212	97502	23910	97100	25601	96667	10
51	18824	98212	20535	97869	22240	97496	23938	97093	25629	96660	9
52	18852	98207	20563	97863	22268	97489	23966	97086	25657	96653	8
53	18881	98201	20592	97857	22297	97483	23995	97079	25685	96645	7
54	18910	98196	20620	97851	22325	97476	24023	97072	25713	96638	6
55	18938	98190	20649	97845	22353	97470	24051	97065	25741	96630	5
56	18967	98185	20677	97839	22382	97463	24079	97058	25769	96623	4
57	18995	98179	20706	97833	22410	97457	24108	97051	25798	96615	3
58	19024	98174	20734	97827	22438	97450	24136	97044	25826	96608	2
59	19052	98168	20763	97821	22467	97444	24164	97037	25854	96600	1
60	19081	98163	20791	97815	22495	97437	24192	97030	25882	96593	0
	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	M.
	79°		78°		77°		76°		75°		

	15°		16°		17°		18°		19°		
M.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	
0	25882	96593	27564	96126	29237	95630	30902	95106	32557	94552	60
1	25910	96585	27592	96118	29265	95622	30929	95097	32584	94542	59
2	25938	96578	27620	96110	29293	95613	30957	95088	32612	94533	58
3	25966	96570	27648	96102	29321	95605	30985	95079	32639	94523	57
4	25994	96562	27676	96094	29348	95596	31012	95070	32667	94514	56
5	26022	96555	27704	96086	29376	95588	31040	95061	32694	94504	55
6	26050	96547	27731	96078	29404	95579	31068	95052	32722	94495	54
7	26079	96540	27759	96070	29432	95571	31095	95043	32749	94485	53
8	26107	96532	27787	96062	29460	95562	31123	95033	32777	94476	52
9	26135	96524	27815	96054	29487	95554	31151	95024	32804	94466	51
10	26163	96517	27843	96046	29515	95545	31178	95015	32832	94457	50
11	26191	96509	27871	96037	29543	95536	31206	95006	32859	94447	49
12	26219	96502	27899	96029	29571	95528	31233	94997	32887	94438	48
13	26247	96494	27927	96021	29599	95519	31261	94988	32914	94428	47
14	26275	96486	27955	96013	29626	95511	31289	94979	32942	94418	46
15	26303	96479	27983	96005	29654	95502	31316	94970	32969	94409	45
16	26331	96471	28011	95997	29682	95493	31344	94961	32997	94399	44
17	26359	96463	28039	95989	29710	95485	31372	94952	33024	94390	43
18	26387	96456	28067	95981	29737	95476	31399	94943	33051	94380	42
19	26415	96448	28095	95972	29765	95467	31427	94933	33079	94370	41
20	26443	96440	28123	95964	29793	95459	31454	94924	33106	94361	40
21	26471	96433	28150	95956	29821	95450	31482	94915	33134	94351	39
22	26500	96425	28178	95948	29849	95441	31510	94906	33161	94342	38
23	26528	96417	28206	95940	29876	95433	31537	94897	33189	94332	37
24	26556	96410	28234	95931	29904	95424	31565	94888	33216	94322	36
25	26584	96402	28262	95923	29932	95415	31593	94878	33244	94313	35
26	26612	96394	28290	95915	29960	95407	31620	94869	33271	94303	34
27	26640	96386	28318	95907	29987	95398	31648	94860	33298	94293	33
28	26668	96379	28346	95898	30015	95389	31675	94851	33326	94284	32
29	26696	96371	28374	95890	30043	95380	31703	94842	33353	94274	31
30	26724	96363	28402	95882	30071	95372	31730	94832	33381	94264	30
31	26752	96355	28429	95874	30098	95363	31758	94823	33408	94254	29
32	26780	96347	28457	95865	30126	95354	31786	94814	33436	94245	28
33	26808	96340	28485	95857	30154	95345	31813	94805	33463	94235	27
34	26836	96332	28513	95849	30182	95337	31841	94795	33490	94225	26
35	26864	96324	28541	95841	30209	95328	31868	94786	33518	94215	25
36	26892	96316	28569	95832	30237	95319	31896	94777	33545	94206	24
37	26920	96308	28597	95824	30265	95310	31923	94768	33573	94196	23
38	26948	96301	28625	95816	30292	95301	31951	94758	33600	94186	22
39	26976	96293	28652	95807	30320	95293	31979	94749	33627	94176	21
40	27004	96285	28680	95799	30348	95284	32006	94740	33655	94167	20
41	27032	96277	28708	95791	30376	95275	32034	94730	33682	94157	19
42	27060	96269	28736	95782	30403	95266	32061	94721	33710	94147	18
43	27088	96261	28764	95774	30431	95257	32089	94712	33737	94137	17
44	27116	96253	28792	95766	30459	95248	32116	94702	33764	94127	16
45	27144	96246	28820	95757	30486	95240	32144	94693	33792	94118	15
46	27172	96238	28847	95749	30514	95231	32171	94684	33819	94108	14
47	27200	96230	28875	95740	30542	95222	32199	94674	33846	94098	13
48	27228	96222	28903	95732	30570	95213	32227	94665	33874	94088	12
49	27256	96214	28931	95724	30597	95204	32254	94656	33901	94078	11
50	27284	96206	28959	95715	30625	95195	32282	94646	33929	94068	10
51	27312	96198	28987	95707	30653	95186	32309	94637	33956	94058	9
52	27340	96190	29015	95698	30680	95177	32337	94627	33983	94049	8
53	27368	96182	29042	95690	30708	95168	32364	94618	34011	94039	7
54	27396	96174	29070	95681	30736	95159	32392	94609	34038	94029	6
55	27424	96166	29098	95673	30763	95150	32419	94599	34065	94019	5
56	27452	96158	29126	95664	30791	95142	32447	94590	34093	94009	4
57	27480	96150	29154	95656	30819	95133	32474	94580	34120	93999	3
58	27508	96142	29182	95647	30846	95124	32502	94571	34147	93989	2
59	27536	96134	29209	95639	30874	95115	32529	94561	34175	93979	1
60	27564	96126	29237	95630	30902	95106	32557	94552	34202	93969	0
	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	M.
	74°		73°		72°		71°		70°		

	20°		21°		22°		23°		24°		
M.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N.cos.	
0	34202	93969	35837	93358	37461	92718	39073	92050	40674	91355	60
1	34229	93959	35864	93348	37488	92707	39100	92039	40700	91343	59
2	34257	93949	35891	93337	37515	92697	39127	92028	40727	91331	58
3	34284	93939	35918	93327	37542	92686	39153	92016	40753	91319	57
4	34311	93929	35945	93316	37569	92675	39180	92005	40780	91307	56
5	34339	93919	35973	93306	37595	92664	39207	91994	40806	91295	55
6	34366	93909	36000	93295	37622	92653	39234	91982	40833	91283	54
7	34393	93899	36027	93285	37649	92642	39260	91971	40860	91272	53
8	34421	93889	36054	93274	37676	92631	39287	91959	40886	91260	52
9	34448	93879	36081	93264	37703	92620	39314	91948	40913	91248	51
10	34475	93869	36108	93253	37730	92609	39341	91936	40939	91236	50
11	34503	93859	36135	93243	37757	92598	39367	91925	40966	91224	49
12	34530	93849	36162	93232	37784	92587	39394	91914	40992	91212	48
13	34557	93839	36190	93222	37811	92576	39421	91902	41019	91200	47
14	34584	93829	36217	93211	37838	92565	39448	91891	41045	91188	46
15	34612	93819	36244	93201	37865	92554	39474	91879	41072	91176	45
16	34639	93809	36271	93190	37892	92543	39501	91868	41098	91164	44
17	34666	93799	36298	93180	37919	92532	39528	91856	41125	91152	43
18	34694	93789	36325	93169	37946	92521	39555	91845	41151	91140	42
19	34721	93779	36352	93159	37973	92510	39581	91833	41178	91128	41
20	34748	93769	36379	93148	37999	92499	39608	91822	41204	91116	40
21	34775	93759	36406	93137	38026	92488	39635	91810	41231	91104	39
22	34803	93748	36434	93127	38053	92477	39661	91799	41257	91092	38
23	34830	93738	36461	93116	38080	92466	39688	91787	41284	91080	37
24	34857	93728	36488	93106	38107	92455	39715	91775	41310	91068	36
25	34884	93718	36515	93095	38134	92444	39741	91764	41337	91056	35
26	34912	93708	36542	93084	38161	92432	39768	91752	41363	91044	34
27	34939	93698	36569	93074	38188	92421	39795	91741	41390	91032	33
28	34966	93688	36596	93063	38215	92410	39822	91729	41416	91020	32
29	34993	93677	36623	93052	38241	92399	39848	91718	41443	91008	31
30	35021	93667	36650	93042	38268	92388	39875	91706	41469	90996	30
31	35048	93657	36677	93031	38295	92377	39902	91694	41496	90984	29
32	35075	93647	36704	93020	38322	92366	39928	91683	41522	90972	28
33	35102	93637	36731	93010	38349	92355	39955	91671	41549	90960	27
34	35130	93626	36758	92999	38376	92343	39982	91660	41575	90948	26
35	35157	93616	36785	92988	38403	92332	40008	91648	41602	90936	25
36	35184	93606	36812	92978	38430	92321	40035	91636	41628	90924	24
37	35211	93596	36839	92967	38456	92310	40062	91625	41655	90911	23
38	35239	93585	36867	92956	38483	92299	40088	91613	41681	90899	22
39	35266	93575	36894	92945	38510	92287	40115	91601	41707	90887	21
40	35293	93565	36921	92935	38537	92276	40141	91590	41734	90875	20
41	35320	93555	36948	92924	38564	92265	40168	91578	41760	90863	19
42	35347	93544	36975	92913	38591	92254	40195	91566	41787	90851	18
43	35375	93534	37002	92902	38617	92243	40221	91555	41813	90839	17
44	35402	93524	37029	92892	38644	92231	40248	91543	41840	90826	16
45	35429	93514	37056	92881	38671	92220	40275	91531	41866	90814	15
46	35456	93503	37083	92870	38698	92209	40301	91519	41892	90802	14
47	35484	93493	37110	92859	38725	92198	40328	91508	41919	90790	13
48	35511	93483	37137	92849	38752	92186	40355	91496	41945	90778	12
49	35538	93472	37164	92838	38778	92175	40381	91484	41972	90766	11
50	35565	93462	37191	92827	38805	92164	40408	91472	41998	90753	10
51	35592	93452	37218	92816	38832	92152	40434	91461	42024	90741	9
52	35619	93441	37245	92805	38859	92141	40461	91449	42051	90729	8
53	35647	93431	37272	92794	38886	92130	40488	91437	42077	90717	7
54	35674	93420	37299	92784	38912	92119	40514	91425	42104	90704	6
55	35701	93410	37326	92773	38939	92107	40541	91414	42130	90692	5
56	35728	93400	37353	92762	38966	92096	40567	91402	42156	90680	4
57	35755	93389	37380	92751	38993	92085	40594	91390	42183	90668	3
58	35782	93379	37407	92740	39020	92073	40621	91378	42209	90655	2
59	35810	93368	37434	92729	39046	92062	40647	91366	42235	90643	1
60	35837	93358	37461	92718	39073	92050	40674	91355	42262	90631	0
	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	M.
	69°		68°		67°		66°		65°		

	25°		26°		27°		28°		29°		
M.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	
0	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60
1	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59
2	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58
3	42341	90594	43916	89841	45477	89061	47024	88254	48557	87420	57
4	42367	90582	43942	89828	45503	89048	47050	88240	48583	87406	56
5	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55
6	42420	90557	43994	89803	45554	89021	47101	88213	48634	87377	54
7	42446	90545	44020	89790	45580	89008	47127	88199	48659	87363	53
8	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52
9	42499	90520	44072	89764	45632	88981	47178	88172	48710	87335	51
10	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50
11	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49
12	42578	90483	44151	89726	45710	88942	47255	88130	48786	87292	48
13	42604	90470	44177	89713	45736	88928	47281	88117	48811	87278	47
14	42631	90458	44203	89700	45762	88915	47306	88103	48837	87264	46
15	42657	90446	44229	89687	45787	88902	47332	88089	48862	87250	45
16	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44
17	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43
18	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42
19	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41
20	42788	90383	44359	89623	45917	88835	47460	88020	48989	87178	40
21	42815	90371	44385	89610	45942	88822	47486	88006	49014	87164	39
22	42841	90358	44411	89597	45968	88808	47511	87993	49040	87150	38
23	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37
24	42894	90334	44464	89571	46020	88782	47562	87965	49090	87121	36
25	42920	90321	44490	89558	46046	88768	47588	87951	49116	87107	35
26	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34
27	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33
28	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32
29	43025	90271	44594	89506	46149	88715	47690	87896	49217	87050	31
30	43051	90259	44620	89493	46175	88701	47716	87882	49242	87036	30
31	43077	90246	44646	89480	46201	88688	47741	87868	49268	87021	29
32	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28
33	43130	90221	44698	89454	46252	88661	47793	87840	49318	86993	27
34	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26
35	43182	90196	44750	89428	46304	88634	47844	87812	49369	86964	25
36	43209	90183	44776	89415	46330	88620	47869	87798	49394	86949	24
37	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23
38	43261	90158	44828	89389	46381	88593	47920	87770	49445	86921	22
39	43287	90146	44854	89376	46407	88580	47946	87756	49470	86906	21
40	43313	90133	44880	89363	46433	88566	47971	87743	49495	86892	20
41	43340	90120	44906	89350	46458	88553	47997	87729	49521	86878	19
42	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18
43	43392	90095	44958	89324	46510	88526	48048	87701	49571	86849	17
44	43418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16
45	43445	90070	45010	89298	46561	88499	48099	87673	49622	86820	15
46	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14
47	43497	90045	45062	89272	46613	88472	48150	87645	49672	86791	13
48	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12
49	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11
50	43575	90007	45140	89232	46690	88431	48226	87603	49748	86748	10
51	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9
52	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8
53	43654	89968	45218	89193	46767	88390	48303	87561	49824	86704	7
54	43680	89956	45243	89180	46793	88377	48328	87546	49849	86690	6
55	43706	89943	45269	89167	46819	88363	48354	87532	49874	86675	[illegible]
56	43733	89930	45295	89153	46844	88349	48379	87518	49899	86661	[illegible]
57	43759	89918	45321	89140	46870	88336	48405	87504	49924	86646	3
58	43785	89905	45347	89127	46896	88322	48430	87490	49950	86632	2
59	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1
60	43837	89879	45399	89101	46947	88295	48481	87462	50000	86603	[illegible]
	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	
	64°		63°		62°		61°		60°		

	30°		31°		32°		33°		34°		
M.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	
0	50000	86603	51504	85717	52992	84805	54464	83867	55919	82904	60
1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	59
2	50050	86573	51554	85687	53041	84774	54513	83835	55968	82871	58
3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57
4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	56
5	50126	86530	51628	85642	53115	84728	54586	83788	56040	82822	55
6	50151	86515	51653	85627	53140	84712	54610	83772	56064	82806	54
7	50176	86501	51678	85612	53164	84697	54635	83756	56088	82790	53
8	50201	86486	51703	85597	53189	84681	54659	83740	56112	82773	52
9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	51
10	50252	86457	51753	85567	53238	84650	54708	83708	56160	82741	50
11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	49
12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	48
13	50327	86413	51828	85521	53312	84604	54781	83660	56232	82692	47
14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	46
15	50377	86384	51877	85491	53361	84573	54829	83629	56280	82659	45
16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	44
17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	43
18	50453	86340	51952	85446	53435	84526	54902	83581	56353	82610	42
19	50478	86325	51977	85431	53460	84511	54927	83565	56377	82593	41
20	50503	86310	52002	85416	53484	84495	54951	83549	56401	82577	40
21	50528	86295	52026	85401	53509	84480	54975	83533	56425	82561	39
22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	38
23	50578	86266	52076	85370	53558	84448	55024	83501	56473	82528	37
24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	36
25	50628	86237	52126	85340	53607	84417	55072	83469	56521	82495	35
26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	34
27	50679	86207	52175	85310	53656	84386	55121	83437	56569	82462	33
28	50704	86192	52200	85294	53681	84370	55145	83421	56593	82446	32
29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	31
30	50754	86163	52250	85264	53730	84339	55194	83389	56641	82413	30
31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	29
32	50804	86133	52299	85234	53779	84308	55242	83356	56689	82380	28
33	50829	86119	52324	85218	53804	84292	55266	83340	56713	82363	27
34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	26
35	50879	86089	52374	85188	53853	84261	55315	83308	56760	82330	25
36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	24
37	50929	86059	52423	85157	53902	84230	55363	83276	56808	82297	23
38	50954	86045	52448	85142	53926	84214	55388	83260	56832	82281	22
39	50979	86030	52473	85127	53951	84198	55412	83244	56856	82264	21
40	51004	86015	52498	85112	53975	84182	55436	83228	56880	82248	20
41	51029	86000	52522	85096	54000	84167	55460	83212	56904	82231	19
42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	18
43	51079	85970	52572	85066	54049	84135	55509	83179	56952	82198	17
44	51104	85956	52597	85051	54073	84120	55533	83163	56976	82181	16
45	51129	85941	52621	85035	54097	84104	55557	83147	57000	82165	15
46	51154	85926	52646	85020	54122	84088	55581	83131	57024	82148	14
47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	13
48	51204	85896	52696	84989	54171	84057	55630	83098	57071	82115	12
49	51229	85881	52720	84974	54195	84041	55654	83082	57095	82098	11
50	51254	85866	52745	84959	54220	84025	55678	83066	57119	82082	10
51	51279	85851	52770	84943	54244	84009	55702	83050	57143	82065	9
52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	8
53	51329	85821	52819	84913	54293	83978	55750	83017	57191	82032	7
54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	6
55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	5
56	51404	85777	52893	84866	54366	83930	55823	82969	57262	81982	4
57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	3
58	51454	85747	52943	84836	54415	83899	55871	82936	57310	81949	2
59	51479	85732	52967	84820	54440	83883	55895	82920	57334	81932	1
60	51504	85717	52992	84805	54464	83867	55919	82904	57358	81915	0
	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	M.
	59°		58°		57°		56°		55°		

	35°		36°		37°		38°		39°		
M.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	
0	57358	81915	58779	80902	60182	79864	61566	78801	62932	77715	60
1	57381	81899	58802	80885	60205	79846	61589	78783	62955	77696	59
2	57405	81882	58826	80867	60228	79829	61612	78765	62977	77678	58
3	57429	81865	58849	80850	60251	79811	61635	78747	63000	77660	57
4	57453	81848	58873	80833	60274	79793	61658	78729	63022	77641	56
5	57477	81832	58896	80816	60298	79776	61681	78711	63045	77623	55
6	57501	81815	58920	80799	60321	79758	61704	78694	63068	77605	54
7	57524	81798	58943	80782	60344	79741	61726	78676	63090	77586	53
8	57548	81782	58967	80765	60367	79723	61749	78658	63113	77568	52
9	57572	81765	58990	80748	60390	79706	61772	78640	63135	77550	51
10	57596	81748	59014	80730	60414	79688	61795	78622	63158	77531	50
11	57619	81731	59037	80713	60437	79671	61818	78604	63180	77513	49
12	57643	81714	59061	80696	60460	79653	61841	78586	63203	77494	48
13	57667	81698	59084	80679	60483	79635	61864	78568	63225	77476	47
14	57691	81681	59108	80662	60506	79618	61887	78550	63248	77458	46
15	57715	81664	59131	80644	60529	79600	61909	78532	63271	77439	45
16	57738	81647	59154	80627	60553	79583	61932	78514	63293	77421	44
17	57762	81631	59178	80610	60576	79565	61955	78496	63316	77402	43
18	57786	81614	59201	80593	60599	79547	61978	78478	63338	77384	42
19	57810	81597	59225	80576	60622	79530	62001	78460	63361	77366	41
20	57833	81580	59248	80558	60645	79512	62024	78442	63383	77347	40
21	57857	81563	59272	80541	60668	79494	62046	78424	63406	77329	39
22	57881	81546	59295	80524	60691	79477	62069	78405	63428	77310	38
23	57904	81530	59318	80507	60714	79459	62092	78387	63451	77292	37
24	57928	81513	59342	80489	60738	79441	62115	78369	63473	77273	36
25	57952	81496	59365	80472	60761	79424	62138	78351	63496	77255	35
26	57976	81479	59389	80455	60784	79406	62160	78333	63518	77236	34
27	57999	81462	59412	80438	60807	79388	62183	78315	63540	77218	33
28	58023	81445	59436	80420	60830	79371	62206	78297	63563	77199	32
29	58047	81428	59459	80403	60853	79353	62229	78279	63585	77181	31
30	58070	81412	59482	80386	60876	79335	62251	78261	63608	77162	30
31	58094	81395	59506	80368	60899	79318	62274	78243	63630	77144	29
32	58118	81378	59529	80351	60922	79300	62297	78225	63653	77125	28
33	58141	81361	59552	80334	60945	79282	62320	78206	63675	77107	27
34	58165	81344	59576	80316	60968	79264	62342	78188	63698	77088	26
35	58189	81327	59599	80299	60991	79247	62365	78170	63720	77070	25
36	58212	81310	59622	80282	61015	79229	62388	78152	63742	77051	24
37	58236	81293	59646	80264	61038	79211	62411	78134	63765	77033	23
38	58260	81276	59669	80247	61061	79193	62433	78116	63787	77014	22
39	58283	81259	59693	80230	61084	79176	62456	78098	63810	76996	21
40	58307	81242	59716	80212	61107	79158	62479	78079	63832	76977	20
41	58330	81225	59739	80195	61130	79140	62502	78061	63854	76959	19
42	58354	81208	59763	80178	61153	79122	62524	78043	63877	76940	18
43	58378	81191	59786	80160	61176	79105	62547	78025	63899	76921	17
44	58401	81174	59809	80143	61199	79087	62570	78007	63922	76903	16
45	58425	81157	59832	80125	61222	79069	62592	77988	63944	76884	15
46	58449	81140	59856	80108	61245	79051	62615	77970	63966	76866	14
47	58472	81123	59879	80091	61268	79033	62638	77952	63989	76847	13
48	58496	81106	59902	80073	61291	79016	62660	77934	64011	76828	12
49	58519	81089	59926	80056	61314	78998	62683	77916	64033	76810	11
50	58543	81072	59949	80038	61337	78980	62706	77897	64056	76791	10
51	58567	81055	59972	80021	61360	78962	62728	77879	64078	76772	9
52	58590	81038	59995	80003	61383	78944	62751	77861	64100	76754	8
53	58614	81021	60019	79986	61406	78926	62774	77843	64123	76735	7
54	58637	81004	60042	79968	61429	78908	62796	77824	64145	76717	6
55	58661	80987	60065	79951	61451	78891	62819	77806	64167	76698	5
56	58684	80970	60089	79934	61474	78873	62842	77788	64190	76679	4
57	58708	80953	60112	79916	61497	78855	62864	77769	64212	76661	3
58	58731	80936	60135	79899	61520	78837	62887	77751	64234	76642	2
59	58755	80919	60158	79881	61543	78819	62909	77733	64256	76623	1
60	58779	80902	60182	79864	61566	78801	62932	77715	64279	76604	0
	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	M.
	54°		53°		52°		51°		50°		

	40°		41°		42°		43°		44°		
M.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	
0	64279	76604	65606	75471	66913	74314	68200	73135	69466	71934	60
1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59
2	64323	76567	65650	75433	66956	74276	68242	73096	69508	71894	58
3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57
4	64368	76530	65694	75395	66999	74237	68285	73056	69549	71853	56
5	64390	76511	65716	75375	67021	74217	68306	73036	69570	71833	55
6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54
7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53
8	64457	76455	65781	75318	67086	74159	68370	72976	69633	71772	52
9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51
10	64501	76417	65825	75280	67129	74120	68412	72937	69675	71732	50
11	64524	76398	65847	75261	67151	74100	68434	72917	69696	71711	49
12	64546	76380	65869	75241	67172	74080	68455	72897	69717	71691	48
13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47
14	64590	76342	65913	75203	67215	74041	68497	72857	69758	71650	46
15	64612	76323	65935	75184	67237	74022	68518	72837	69779	71630	45
16	64635	76304	65956	75165	67258	74002	68539	72817	69800	71610	44
17	64657	76286	65978	75146	67280	73983	68561	72797	69821	71590	43
18	64679	76267	66000	75126	67301	73963	68582	72777	69842	71569	42
19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41
20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40
21	64746	76210	66066	75069	67366	73904	68645	72717	69904	71508	39
22	64768	76192	66088	75050	67387	73885	68666	72697	69925	71488	38
23	64790	76173	66109	75030	67409	73865	68688	72677	69946	71468	37
24	64812	76154	66131	75011	67430	73846	68709	72657	69966	71447	36
25	64834	76135	66153	74992	67452	73826	68730	72637	69987	71427	35
26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34
27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33
28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32
29	64923	76059	66240	74915	67538	73747	68814	72557	70070	71345	31
30	64945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30
31	64967	76022	66284	74876	67580	73708	68857	72517	70112	71305	29
32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28
33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27
34	65033	75965	66349	74818	67645	73649	68920	72457	70174	71243	26
35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25
36	65077	75927	66393	74780	67688	73610	68962	72417	70215	71203	24
37	65100	75908	66414	74760	67709	73590	68983	72397	70236	71182	23
38	65122	75889	66436	74741	67730	73570	69004	72377	70257	71162	22
39	65144	75870	66458	74722	67752	73551	69025	72357	70277	71141	21
40	65166	75851	66480	74703	67773	73531	69046	72337	70298	71121	20
41	65188	75832	66501	74683	67795	73511	69067	72317	70319	71100	19
42	65210	75813	66523	74664	67816	73491	69088	72297	70339	71080	18
43	65232	75794	66545	74644	67837	73472	69109	72277	70360	71059	17
44	65254	75775	66566	74625	67859	73452	69130	72257	70381	71039	16
45	65276	75756	66588	74606	67880	73432	69151	72236	70401	71019	15
46	65298	75738	66610	74586	67901	73413	69172	72216	70422	70998	14
47	65320	75719	66632	74567	67923	73393	69193	72196	70443	70978	13
48	65342	75700	66653	74548	67944	73373	69214	72176	70463	70957	12
49	65364	75680	66675	74528	67965	73353	69235	72156	70484	70937	11
50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10
51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9
52	65430	75623	66740	74470	68029	73294	69298	72095	70546	70875	8
53	65452	75604	66762	74451	68051	73274	69319	72075	70567	70855	7
54	65474	75585	66783	74431	68072	73254	69340	72055	70587	70834	6
55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5
56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4
57	65540	75528	66848	74373	68136	73195	69403	71995	70649	70772	3
58	65562	75509	66870	74353	68157	73175	69424	71974	70670	70752	2
59	65584	75490	66891	74334	68179	73155	69445	71954	70690	70731	1
60	65606	75471	66913	74314	68200	73135	69466	71934	70711	70711	0
	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	M.
	49°		48°		47°		46°		45°		

Of Logarithmic Sines, Tangents, and Secants to every Point and Quarter Point of the Compass.

Points.	Sine.	Co. sine.	Tangent.	Co. tang.	Secant.	Co. secant.	
0	Inf. neg.	10.00000	Inf. neg.	Infinite.	10.00000	Infinite.	8
0 ¼	8.69080	9.99948	8.69132	11.30868	10.00052	11.30920	7 ¾
0 ½	8.99130	9.99790	8.99340	11.00660	10.00210	11.00870	7 ½
0 ¾	9.16652	9.99527	9.17125	10.82875	10.00473	10.83348	7 ¼
1	9.29024	9.99157	9.29866	10.70134	10.00843	10.70976	7
1 ¼	9.38557	9.98679	9.39879	10.60121	10.01321	10.61443	6 ¾
1 ½	9.46282	9.98088	9.48194	10.51806	10.01912	10.53718	6 ½
1 ¾	9.52749	9.97384	9.55365	10.44635	10.02616	10.47251	6 ¼
2	9.58284	9.96562	9.61722	10.38278	10.03438	10.41716	6
2 ¼	9.63099	9.95616	9.67483	10.32517	10.04384	10.36901	5 ¾
2 ½	9.67339	9.94543	9.72796	10.27204	10.05457	10.32661	5 ½
2 ¾	9.71105	9.93335	9.77770	10.22230	10.06665	10.28895	5 ¼
3	9.74474	9.91985	9.82489	10.17511	10.08015	10.25526	5
3 ¼	9.77503	9.90483	9.87020	10.12980	10.09517	10.22497	4 ¾
3 ½	9.80236	9.88819	9.91417	10.08583	10.11181	10.19764	4 ½
3 ¾	9.82708	9.86979	9.95729	10.04271	10.13021	10.17292	4 ¼
4	9.84949	9.84949	10.00000	10.00000	10.15051	10.15051	4
	Co. sine.	Sine.	Co. tang.	Tangent.	Co. secant.	Secant.	Points.

LOGARITHMS OF NUMBERS.

No. 1——100. Log. 0.00000——2.00000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.00000	21	1.32222	41	1.61278	61	1.78533	81	1.90849
2	0.30103	22	1.34242	42	1.62325	62	1.79239	82	1.91381
3	0.47712	23	1.36173	43	1.63347	63	1.79934	83	1.91908
4	0.60206	24	1.38021	44	1.64345	64	1.80618	84	1.9242[illegible]
5	0.69897	25	1.39794	45	1.65321	65	1.81291	85	1.9294[illegible]
6	0.77815	26	1.41497	46	1.66276	66	1.81954	86	1.9345[illegible]
7	0.84510	27	1.43136	47	1.67210	67	1.82607	87	1.9395[illegible]
8	0.90309	28	1.44716	48	1.68124	68	1.83251	88	1.9444[illegible]
9	0.95424	29	1.46240	49	1.69020	69	1.83885	89	1.949[illegible]
10	1.00000	30	1.47712	50	1.69897	70	1.84510	90	1.954[illegible]
11	1.04139	31	1.49136	51	1.70757	71	1.85126	91	1.959[illegible]
12	1.07918	32	1.50515	52	1.71600	72	1.85733	92	1.963[illegible]
13	1.11394	33	1.51851	53	1.72428	73	1.86332	93	1.968[illegible]
14	1.14613	34	1.53148	54	1.73239	74	1.86923	94	1.973[illegible]
15	1.17609	35	1.54407	55	1.74036	75	1.87506	95	1.977[illegible]
16	1.20412	36	1.55630	56	1.74819	76	1.88081	96	1.98[illegible]
17	1.23045	37	1.56820	57	1.75587	77	1.88649	97	1.9[illegible]
18	1.25527	38	1.57978	58	1.76343	78	1.89209	98	1.991[illegible]
19	1.27875	39	1.59106	59	1.77085	79	1.89763	99	1.99[illegible]
20	1.30103	40	1.60206	60	1.77815	80	1.90309	100	2.000[illegible]

No. 100——1600. Log. 00000——20412.

No.	0	1	2	3	4	5	6	7	8	9
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389
101	00432	00475	00518	00561	00604	00647	00689	00732	00775	00817
102	00860	00903	00945	00988	01030	01072	01115	01157	01199	01242
103	01284	01326	01368	01410	01452	01494	01536	01578	01620	01662
104	01703	01745	01787	01828	01870	01912	01953	01995	02036	02078
105	02119	02160	02202	02243	02284	02325	02366	02407	02449	02490
106	02531	02572	02612	02653	02694	02735	02776	02816	02857	02898
107	02938	02979	03019	03060	03100	03141	03181	03222	03262	03302
108	03342	03383	03423	03463	03503	03543	03583	03623	03663	03703
109	03743	03782	03822	03862	03902	03941	03981	04021	04060	04100
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493
111	04532	04571	04610	04650	04689	04727	04766	04805	04844	04883
112	04922	04961	04999	05038	05077	05115	05154	05192	05231	05269
113	05308	05346	05385	05423	05461	05500	05538	05576	05614	05652
114	05690	05729	05767	05805	05843	05881	05918	05956	05994	06032
115	06070	06108	06145	06183	06221	06258	06296	06333	06371	06408
116	06446	06483	06521	06558	06595	06633	06670	06707	06744	06781
117	06819	06856	06893	06930	06967	07004	07041	07078	07115	07151
118	07188	07225	07262	07298	07335	07372	07408	07445	07482	07518
119	07555	07591	07628	07664	07700	07737	07773	07809	07846	07882
120	07918	07954	07990	08027	08063	08099	08135	08171	08207	08243
121	08279	08314	08350	08386	08422	08458	08493	08529	08565	08600
122	08636	08672	08707	08743	08778	08814	08849	08884	08920	08955
123	08991	09026	09061	09096	09132	09167	09202	09237	09272	09307
124	09342	09377	09412	09447	09482	09517	09552	09587	09621	09656
125	09691	09726	09760	09795	09830	09864	09899	09934	09968	10003
126	10037	10072	10106	10140	10175	10209	10243	10278	10312	10346
127	10380	10415	10449	10483	10517	10551	10585	10619	10653	10687
128	10721	10755	10789	10823	10857	10890	10924	10958	10992	11025
129	11059	11093	11126	11160	11193	11227	11261	11294	11327	11361
130	11394	11428	11461	11494	11528	11561	11594	11628	11661	11694
131	11727	11760	11793	11826	11860	11893	11926	11959	11992	12024
132	12057	12090	12123	12156	12189	12222	12254	12287	12320	12352
133	12385	12418	12450	12483	12516	12548	12581	12613	12646	12678
134	12710	12743	12775	12808	12840	12872	12905	12937	12969	13001
135	13033	13066	13098	13130	13162	13194	13226	13258	13290	13322
136	13354	13386	13418	13450	13481	13513	13545	13577	13609	13640
137	13672	13704	13735	13767	13799	13830	13862	13893	13925	13956
138	13988	14019	14051	14082	14114	14145	14176	14208	14239	14270
139	14301	14333	14364	14395	14426	14457	14489	14520	14551	14582
140	14613	14644	14675	14706	14737	14768	14799	14829	14860	14891
141	14922	14953	14983	15014	15045	15076	15106	15137	15168	15198
142	15229	15259	15290	15320	15351	15381	15412	15442	15473	15503
143	15534	15564	15594	15625	15655	15685	15715	15746	15776	15806
144	15836	15866	15897	15927	15957	15987	16017	16047	16077	16107
145	16137	16167	16197	16227	16256	16286	16316	16346	16376	16406
146	16435	16465	16495	16524	16554	16584	16613	16643	16673	16702
147	16732	16761	16791	16820	16850	16879	16909	16938	16967	16997
148	17026	17056	17085	17114	17143	17173	17202	17231	17260	17289
149	17319	17348	17377	17406	17435	17464	17493	17522	17551	17580
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869
151	17898	17926	17955	17984	18013	18041	18070	18099	18127	18156
152	18184	18213	18241	18270	18298	18327	18355	18384	18412	18441
153	18469	18498	18526	18554	18583	18611	18639	18667	18696	18724
154	18752	18780	18808	18837	18865	18893	18921	18949	18977	19005
155	19033	19061	19089	19117	19145	19173	19201	19229	19257	19285
156	19312	19340	19368	19396	19424	19451	19479	19507	19535	19562
157	19590	19618	19645	19673	19700	19728	19756	19783	19811	19838
158	19866	19893	19921	19948	19976	20003	20030	20058	20085	20112
159	20140	20167	20194	20222	20249	20276	20303	20330	20358	20385
No.	0	1	2	3	4	5	6	7	8	9

No. 1600——2200. Log. 20412——34242.

No.	0	1	2	3	4	5	6	7	8	9
160	20412	20439	20466	20493	20520	20548	20575	20602	20629	20656
161	20683	20710	20737	20763	20790	20817	20844	20871	20898	20925
162	20952	20978	21005	21032	21059	21085	21112	21139	21165	21192
163	21219	21245	21272	21299	21325	21352	21378	21405	21431	21458
164	21484	21511	21537	21564	21590	21617	21643	21669	21696	21722
165	21748	21775	21801	21827	21854	21880	21906	21932	21958	21985
166	22011	22037	22063	22089	22115	22141	22167	22194	22220	22246
167	22272	22298	22324	22350	22376	22401	22427	22453	22479	22505
168	22531	22557	22583	22608	22634	22660	22686	22712	22737	22763
169	22789	22814	22840	22866	22891	22917	22943	22968	22994	23019
170	23045	23070	23096	23121	23147	23172	23198	23223	23249	23274
171	23300	23325	23350	23376	23401	23426	23452	23477	23502	23528
172	23553	23578	23603	23629	23654	23679	23704	23729	23754	23779
173	23805	23830	23855	23880	23905	23930	23955	23980	24005	24030
174	24055	24080	24105	24130	24155	24180	24204	24229	24254	24279
175	24304	24329	24353	24378	24403	24428	24452	24477	24502	24527
176	24551	24576	24601	24625	24650	24674	24699	24724	24748	24773
177	24797	24822	24846	24871	24895	24920	24944	24969	24993	25018
178	25042	25066	25091	25115	25139	25164	25188	25212	25237	25261
179	25285	25310	25334	25358	25382	25406	25431	25455	25479	25503
180	25527	25551	25575	25600	25624	25648	25672	25696	25720	25744
181	25768	25792	25816	25840	25864	25888	25912	25935	25959	25983
182	26007	26031	26055	26079	26102	26126	26150	26174	26198	26221
183	26245	26269	26293	26316	26340	26364	26387	26411	26435	26458
184	26482	26505	26529	26553	26576	26600	26623	26647	26670	26694
185	26717	26741	26764	26788	26811	26834	26858	26881	26905	26928
186	26951	26975	26998	27021	27045	27068	27091	27114	27138	27161
187	27184	27207	27231	27254	27277	27300	27323	27346	27370	27393
188	27416	27439	27462	27485	27508	27531	27554	27577	27600	27623
189	27646	27669	27692	27715	27738	27761	27784	27807	27830	27852
190	27875	27898	27921	27944	27967	27989	28012	28035	28058	28081
191	28103	28126	28149	28171	28194	28217	28240	28262	28285	28307
192	28330	28353	28375	28398	28421	28443	28466	28488	28511	28533
193	28556	28578	28601	28623	28646	28668	28691	28713	28735	28758
194	28780	28803	28825	28847	28870	28892	28914	28937	28959	28981
195	29003	29026	29048	29070	29092	29115	29137	29159	29181	29203
196	29226	29248	29270	29292	29314	29336	29358	29380	29403	29425
197	29447	29469	29491	29513	29535	29557	29579	29601	29623	29645
198	29667	29688	29710	29732	29754	29776	29798	29820	29842	29863
199	29885	29907	29929	29951	29973	29994	30016	30038	30060	30081
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298
201	30320	30341	30363	30384	30406	30428	30449	30471	30492	30514
202	30535	30557	30578	30600	30621	30643	30664	30685	30707	30728
203	30750	30771	30792	30814	30835	30856	30878	30899	30920	30942
204	30963	30984	31006	31027	31048	31069	31091	31112	31133	31154
205	31175	31197	31218	31239	31260	31281	31302	31323	31345	31366
206	31387	31408	31429	31450	31471	31492	31513	31534	31555	31576
207	31597	31618	31639	31660	31681	31702	31723	31744	31765	31785
208	31806	31827	31848	31869	31890	31911	31931	31952	31973	31994
209	32015	32035	32056	32077	32098	32118	32139	32160	32181	32201
210	32222	32243	32263	32284	32305	32325	32346	32366	32387	32408
211	32428	32449	32469	32490	32510	32531	32552	32572	32593	32613
212	32634	32654	32675	32695	32715	32736	32756	32777	32797	32818
213	32838	32858	32879	32899	32919	32940	32960	32980	33001	33021
214	33041	33062	33082	33102	33122	33143	33163	33183	33203	33224
215	33244	33264	33284	33304	33325	33345	33365	33385	33405	33425
216	33445	33465	33486	33506	33526	33546	33566	33586	33606	33626
217	33646	33666	33686	33706	33726	33746	33766	33786	33806	33826
218	33846	33866	33885	33905	33925	33945	33965	33985	34005	34025
219	34044	34064	34084	34104	34124	34143	34163	34183	34203	34223
No.	0	1	2	3	4	5	6	7	8	9

No. 2200——2800. Log. 34242——44716.

No.	0	1	2	3	4	5	6	7	8	9
220	34242	34262	34282	34301	34321	34341	34361	34380	34400	34420
221	34439	34459	34479	34498	34518	34537	34557	34577	34596	34616
222	34635	34655	34674	34694	34713	34733	34753	34772	34792	34811
223	34830	34850	34869	34889	34908	34928	34947	34967	34986	35005
224	35025	35044	35064	35083	35102	35122	35141	35160	35180	35199
225	35218	35238	35257	35276	35295	35315	35334	35353	35372	35392
226	35411	35430	35449	35468	35488	35507	35526	35545	35564	35583
227	35603	35622	35641	35660	35679	35698	35717	35736	35755	35774
228	35793	35813	35832	35851	35870	35889	35908	35927	35946	35965
229	35984	36003	36021	36040	36059	36078	36097	36116	36135	36154
230	36173	36192	36211	36229	36248	36267	36286	36305	36324	36342
231	36361	36380	36399	36418	36436	36455	36474	36493	36511	36530
232	36549	36568	36586	36605	36624	36642	36661	36680	36698	36717
233	36736	36754	36773	36791	36810	36829	36847	36866	36884	36903
234	36922	36940	36959	36977	36996	37014	37033	37051	37070	37088
235	37107	37125	37144	37162	37181	37199	37218	37236	37254	37273
236	37291	37310	37328	37346	37365	37383	37401	37420	37438	37457
237	37475	37493	37511	37530	37548	37566	37585	37603	37621	37639
238	37658	37676	37694	37712	37731	37749	37767	37785	37803	37822
239	37840	37858	37876	37894	37912	37931	37949	37967	37985	38003
240	38021	38039	38057	38075	38093	38112	38130	38148	38166	38184
241	38202	38220	38238	38256	38274	38292	38310	38328	38346	38364
242	38382	38399	38417	38435	38453	38471	38489	38507	38525	38543
243	38561	38578	38596	38614	38632	38650	38668	38686	38703	38721
244	38739	38757	38775	38792	38810	38828	38846	38863	38881	38899
245	38917	38934	38952	38970	38987	39005	39023	39041	39058	39076
246	39094	39111	39129	39146	39164	39182	39199	39217	39235	39252
247	39270	39287	39305	39322	39340	39358	39375	39393	39410	39428
248	39445	39463	39480	39498	39515	39533	39550	39568	39585	39602
249	39620	39637	39655	39672	39690	39707	39724	39742	39759	39777
250	39794	39811	39829	39846	39863	39881	39898	39915	39933	39950
251	39967	39985	40002	40019	40037	40054	40071	40088	40106	40123
252	40140	40157	40175	40192	40209	40226	40243	40261	40278	40295
253	40312	40329	40346	40364	40381	40398	40415	40432	40449	40466
254	40483	40500	40518	40535	40552	40569	40586	40603	40620	40637
255	40654	40671	40688	40705	40722	40739	40756	40773	40790	40807
256	40824	40841	40858	40875	40892	40909	40926	40943	40960	40976
257	40993	41010	41027	41044	41061	41078	41095	41111	41128	41145
258	41162	41179	41196	41212	41229	41246	41263	41280	41296	41313
259	41330	41347	41363	41380	41397	41414	41430	41447	41464	41481
260	41497	41514	41531	41547	41564	41581	41597	41614	41631	41647
261	41664	41681	41697	41714	41731	41747	41764	41780	41797	41814
262	41830	41847	41863	41880	41896	41913	41929	41946	41963	41979
263	41996	42012	42029	42045	42062	42078	42095	42111	42127	42144
264	42160	42177	42193	42210	42226	42243	42259	42275	42292	42308
265	42325	42341	42357	42374	42390	42406	42423	42439	42455	42472
266	42488	42504	42521	42537	42553	42570	42586	42602	42619	42635
267	42651	42667	42684	42700	42716	42732	42749	42765	42781	42797
268	42813	42830	42846	42862	42878	42894	42911	42927	42943	42959
269	42975	42991	43008	43024	43040	43056	43072	43088	43104	43120
270	43136	43152	43169	43185	43201	43217	43233	43249	43265	43281
271	43297	43313	43329	43345	43361	43377	43393	43409	43425	43441
272	43457	43473	43489	43505	43521	43537	43553	43569	43584	43600
273	43616	43632	43648	43664	43680	43696	43712	43727	43743	43759
274	43775	43791	43807	43823	43838	43854	43870	43886	43902	43917
275	43933	43949	43965	43981	43996	44012	44028	44044	44059	44075
276	44091	44107	44122	44138	44154	44170	44185	44201	44217	44232
277	44248	44264	44279	44295	44311	44326	44342	44358	44373	44389
278	44404	44420	44436	44451	44467	44483	44498	44514	44529	44545
279	44560	44576	44592	44607	44623	44638	44654	44669	44685	44700
No.	0	1	2	3	4	5	6	7	8	9

No. 2800——3400. Log. 44716——53148.

No.	0	1	2	3	4	5	6	7	8	9
280	44716	44731	44747	44762	44778	44793	44809	44824	44840	44855
281	44871	44886	44902	44917	44932	44948	44963	44979	44994	45010
282	45025	45040	45056	45071	45086	45102	45117	45133	45148	45163
283	45179	45194	45209	45225	45240	45255	45271	45286	45301	45317
284	45332	45347	45362	45378	45393	45408	45423	45439	45454	45469
285	45484	45500	45515	45530	45545	45561	45576	45591	45606	45621
286	45637	45652	45667	45682	45697	45712	45728	45743	45758	45773
287	45788	45803	45818	45834	45849	45864	45879	45894	45909	45924
288	45939	45954	45969	45984	46000	46015	46030	46045	46060	46075
289	46090	46105	46120	46135	46150	46165	46180	46195	46210	46225
290	46240	46255	46270	46285	46300	46315	46330	46345	46359	46374
291	46389	46404	46419	46434	46449	46464	46479	46494	46509	46523
292	46538	46553	46568	46583	46598	46613	46627	46642	46657	46672
293	46687	46702	46716	46731	46746	46761	46776	46790	46805	46820
294	46835	46850	46864	46879	46894	46909	46923	46938	46953	46967
295	46982	46997	47012	47026	47041	47056	47070	47085	47100	47114
296	47129	47144	47159	47173	47188	47202	47217	47232	47246	47261
297	47276	47290	47305	47319	47334	47349	47363	47378	47392	47407
298	47422	47436	47451	47465	47480	47494	47509	47524	47538	47553
299	47567	47582	47596	47611	47625	47640	47654	47669	47683	47698
300	47712	47727	47741	47756	47770	47784	47799	47813	47828	47842
301	47857	47871	47885	47900	47914	47929	47943	47958	47972	47986
302	48001	48015	48029	48044	48058	48073	48087	48101	48116	48130
303	48144	48159	48173	48187	48202	48216	48230	48244	48259	48273
304	48287	48302	48316	48330	48344	48359	48373	48387	48401	48416
305	48430	48444	48458	48473	48487	48501	48515	48530	48544	48558
306	48572	48586	48601	48615	48629	48643	48657	48671	48686	48700
307	48714	48728	48742	48756	48770	48785	48799	48813	48827	48841
308	48855	48869	48883	48897	48911	48926	48940	48954	48968	48982
309	48996	49010	49024	49038	49052	49066	49080	49094	49108	49122
310	49136	49150	49164	49178	49192	49206	49220	49234	49248	49262
311	49276	49290	49304	49318	49332	49346	49360	49374	49388	49402
312	49415	49429	49443	49457	49471	49485	49499	49513	49527	49541
313	49554	49568	49582	49596	49610	49624	49638	49651	49665	49679
314	49693	49707	49721	49734	49748	49762	49776	49790	49803	49817
315	49831	49845	49859	49872	49886	49900	49914	49927	49941	49955
316	49969	49982	49996	50010	50024	50037	50051	50065	50079	50092
317	50106	50120	50133	50147	50161	50174	50188	50202	50215	50229
318	50243	50256	50270	50284	50297	50311	50325	50338	50352	50365
319	50379	50393	50406	50420	50433	50447	50461	50474	50488	50501
320	50515	50529	50542	50556	50569	50583	50596	50610	50623	50637
321	50651	50664	50678	50691	50705	50718	50732	50745	50759	50772
322	50786	50799	50813	50826	50840	50853	50866	50880	50893	50907
323	50920	50934	50947	50961	50974	50987	51001	51014	51028	51041
324	51055	51068	51081	51095	51108	51121	51135	51148	51162	51175
325	51188	51202	51215	51228	51242	51255	51268	51282	51295	51308
326	51322	51335	51348	51362	51375	51388	51402	51415	51428	51441
327	51455	51468	51481	51495	51508	51521	51534	51548	51561	51574
328	51587	51601	51614	51627	51640	51654	51667	51680	51693	51706
329	51720	51733	51746	51759	51772	51786	51799	51812	51825	51838
330	51851	51865	51878	51891	51904	51917	51930	51943	51957	51970
331	51983	51996	52009	52022	52035	52048	52061	52075	52088	52101
332	52114	52127	52140	52153	52166	52179	52192	52205	52218	52231
333	52244	52257	52270	52284	52297	52310	52323	52336	52349	52362
334	52375	52388	52401	52414	52427	52440	52453	52466	52479	52492
335	52504	52517	52530	52543	52556	52569	52582	52595	52608	52621
336	52634	52647	52660	52673	52686	52699	52711	52724	52737	52750
337	52763	52776	52789	52802	52815	52827	52840	52853	52866	52879
338	52892	52905	52917	52930	52943	52956	52969	52982	52994	53007
339	53020	53033	53046	53058	53071	53084	53097	53110	53122	53135
No.	0	1	2	3	4	5	6	7	8	9

No. 3400——4000. Log. 53148——60206.

No.	0	1	2	3	4	5	6	7	8	9
340	53148	53161	53173	53186	53199	53212	53224	53237	53250	53263
341	53275	53288	53301	53314	53326	53339	53352	53364	53377	53390
342	53403	53415	53428	53441	53453	53466	53479	53491	53504	53517
343	53529	53542	53555	53567	53580	53593	53605	53618	53631	53643
344	53656	53668	53681	53694	53706	53719	53732	53744	53757	53769
345	53782	53794	53807	53820	53832	53845	53857	53870	53882	53895
346	53908	53920	53933	53945	53958	53970	53983	53995	54008	54020
347	54033	54045	54058	54070	54083	54095	54108	54120	54133	54145
348	54158	54170	54183	54195	54208	54220	54233	54245	54258	54270
349	54283	54295	54307	54320	54332	54345	54357	54370	54382	54394
350	54407	54419	54432	54444	54456	54469	54481	54494	54506	54518
351	54531	54543	54555	54568	54580	54593	54605	54617	54630	54642
352	54654	54667	54679	54691	54704	54716	54728	54741	54753	54765
353	54777	54790	54802	54814	54827	54839	54851	54864	54876	54888
354	54900	54913	54925	54937	54949	54962	54974	54986	54998	55011
355	55023	55035	55047	55060	55072	55084	55096	55108	55121	55133
356	55145	55157	55169	55182	55194	55206	55218	55230	55242	55255
357	55267	55279	55291	55303	55315	55328	55340	55352	55364	55376
358	55388	55400	55413	55425	55437	55449	55461	55473	55485	55497
359	55509	55522	55534	55546	55558	55570	55582	55594	55606	55618
360	55630	55642	55654	55666	55678	55691	55703	55715	55727	55739
361	55751	55763	55775	55787	55799	55811	55823	55835	55847	55859
362	55871	55883	55895	55907	55919	55931	55943	55955	55967	55979
363	55991	56003	56015	56027	56038	56050	56062	56074	56086	56098
364	56110	56122	56134	56146	56158	56170	56182	56194	56205	56217
365	56229	56241	56253	56265	56277	56289	56301	56312	56324	56336
366	56348	56360	56372	56384	56396	56407	56419	56431	56443	56455
367	56467	56478	56490	56502	56514	56526	56538	56549	56561	56573
368	56585	56597	56608	56620	56632	56644	56656	56667	56679	56691
369	56703	56714	56726	56738	56750	56761	56773	56785	56797	56808
370	56820	56832	56844	56855	56867	56879	56891	56902	56914	56926
371	56937	56949	56961	56972	56984	56996	57008	57019	57031	57043
372	57054	57066	57078	57089	57101	57113	57124	57136	57148	57159
373	57171	57183	57194	57206	57217	57229	57241	57252	57264	57276
374	57287	57299	57310	57322	57334	57345	57357	57368	57380	57392
375	57403	57415	57426	57438	57449	57461	57473	57484	57496	57507
376	57519	57530	57542	57553	57565	57576	57588	57600	57611	57623
377	57634	57646	57657	57669	57680	57692	57703	57715	57726	57738
378	57749	57761	57772	57784	57795	57807	57818	57830	57841	57852
379	57864	57875	57887	57898	57910	57921	57933	57944	57955	57967
380	57978	57990	58001	58013	58024	58035	58047	58058	58070	58081
381	58092	58104	58115	58127	58138	58149	58161	58172	58184	58195
382	58206	58218	58229	58240	58252	58263	58274	58286	58297	58309
383	58320	58331	58343	58354	58365	58377	58388	58399	58410	58422
384	58433	58444	58456	58467	58478	58490	58501	58512	58524	58535
385	58546	58557	58569	58580	58591	58602	58614	58625	58636	58647
386	58659	58670	58681	58692	58704	58715	58726	58737	58749	58760
387	58771	58782	58794	58805	58816	58827	58838	58850	58861	58872
388	58883	58894	58906	58917	58928	58939	58950	58961	58973	58984
389	58995	59006	59017	59028	59040	59051	59062	59073	59084	59095
390	59106	59118	59129	59140	59151	59162	59173	59184	59195	59207
391	59218	59229	59240	59251	59262	59273	59284	59295	59306	59318
392	59329	59340	59351	59362	59373	59384	59395	59406	59417	59428
393	59439	59450	59461	59472	59483	59494	59506	59517	59528	59539
394	59550	59561	59572	59583	59594	59605	59616	59627	59638	59649
395	59660	59671	59682	59693	59704	59715	59726	59737	59748	59759
396	59770	59780	59791	59802	59813	59824	59835	59846	59857	59868
397	59879	59890	59901	59912	59923	59934	59945	59956	59966	59977
398	59988	59999	60010	60021	60032	60043	60054	60065	60076	60086
399	60097	60108	60119	60130	60141	60152	60163	60173	60184	60195
No.	0	1	2	3	4	5	6	7	8	9

No. 4000——4600. Log. 60206——66276.

No.	0	1	2	3	4	5	6	7	8	9
400	60206	60217	60228	60239	60249	60260	60271	60282	60293	60304
401	60314	60325	60336	60347	60358	60369	60379	60390	60401	60412
402	60423	60433	60444	60455	60466	60477	60487	60498	60509	60520
403	60531	60541	60552	60563	60574	60584	60595	60606	60617	60627
404	60638	60649	60660	60670	60681	60692	60703	60713	60724	60735
405	60746	60756	60767	60778	60788	60799	60810	60821	60831	60842
406	60853	60863	60874	60885	60895	60906	60917	60927	60938	60949
407	60959	60970	60981	60991	61002	61013	61023	61034	61045	61055
408	61066	61077	61087	61098	61109	61119	61130	61140	61151	61162
409	61172	61183	61194	61204	61215	61225	61236	61247	61257	61268
410	61278	61289	61300	61310	61321	61331	61342	61352	61363	61374
411	61384	61395	61405	61416	61426	61437	61448	61458	61469	61479
412	61490	61500	61511	61521	61532	61542	61553	61563	61574	61584
413	61595	61606	61616	61627	61637	61648	61658	61669	61679	61690
414	61700	61711	61721	61731	61742	61752	61763	61773	61784	61794
415	61805	61815	61826	61836	61847	61857	61868	61878	61888	61899
416	61909	61920	61930	61941	61951	61962	61972	61982	61993	62003
417	62014	62024	62034	62045	62055	62066	62076	62086	62097	62107
418	62118	62128	62138	62149	62159	62170	62180	62190	62201	62211
419	62221	62232	62242	62252	62263	62273	62284	62294	62304	62315
420	62325	62335	62346	62356	62366	62377	62387	62397	62408	62418
421	62428	62439	62449	62459	62469	62480	62490	62500	62511	62521
422	62531	62542	62552	62562	62572	62583	62593	62603	62613	62624
423	62634	62644	62655	62665	62675	62685	62696	62706	62716	62726
424	62737	62747	62757	62767	62778	62788	62798	62808	62818	62829
425	62839	62849	62859	62870	62880	62890	62900	62910	62921	62931
426	62941	62951	62961	62972	62982	62992	63002	63012	63022	63033
427	63043	63053	63063	63073	63083	63094	63104	63114	63124	63134
428	63144	63155	63165	63175	63185	63195	63205	63215	63225	63236
429	63246	63256	63266	63276	63286	63296	63306	63317	63327	63337
430	63347	63357	63367	63377	63387	63397	63407	63417	63428	63438
431	63448	63458	63468	63478	63488	63498	63508	63518	63528	63538
432	63548	63558	63568	63579	63589	63599	63609	63619	63629	63639
433	63649	63659	63669	63679	63689	63699	63709	63719	63729	63739
434	63749	63759	63769	63779	63789	63799	63809	63819	63829	63839
435	63849	63859	63869	63879	63889	63899	63909	63919	63929	63939
436	63949	63959	63969	63979	63988	63998	64008	64018	64028	64038
437	64048	64058	64068	64078	64088	64098	64108	64118	64128	64137
438	64147	64157	64167	64177	64187	64197	64207	64217	64227	64237
439	64246	64256	64266	64276	64286	64296	64306	64316	64326	64335
440	64345	64355	64365	64375	64385	64395	64404	64414	64424	64434
441	64444	64454	64464	64473	64483	64493	64503	64513	64523	64532
442	64542	64552	64562	64572	64582	64591	64601	64611	64621	64631
443	64640	64650	64660	64670	64680	64689	64699	64709	64719	64729
444	64738	64748	64758	64768	64777	64787	64797	64807	64816	64826
445	64836	64846	64856	64865	64875	64885	64895	64904	64914	64924
446	64933	64943	64953	64963	64972	64982	64992	65002	65011	65021
447	65031	65040	65050	65060	65070	65079	65089	65099	65108	65118
448	65128	65137	65147	65157	65167	65176	65186	65196	65205	65215
449	65225	65234	65244	65254	65263	65273	65283	65292	65302	65312
450	65321	65331	65341	65350	65360	65369	65379	65389	65398	65408
451	65418	65427	65437	65447	65456	65466	65475	65485	65495	65504
452	65514	65523	65533	65543	65552	65562	65571	65581	65591	65600
453	65610	65619	65629	65639	65648	65658	65667	65677	65686	65696
454	65706	65715	65725	65734	65744	65753	65763	65772	65782	65792
455	65801	65811	65820	65830	65839	65849	65858	65868	65877	65887
456	65896	65906	65916	65925	65935	65944	65954	65963	65973	65982
457	65992	66001	66011	66020	66030	66039	66049	66058	66068	66077
458	66087	66096	66106	66115	66124	66134	66143	66153	66162	66172
459	66181	66191	66200	66210	66219	66229	66238	66247	66257	66266
No.	0	1	2	3	4	5	6	7	8	9

No. 4600———5200. Log. 66276———71600.

No.	0	1	2	3	4	5	6	7	8	9
460	66276	66285	66295	66304	66314	66323	66332	66342	66351	66361
461	66370	66380	66389	66398	66408	66417	66427	66436	66445	66455
462	66464	66474	66483	66492	66502	66511	66521	66530	66539	66549
463	66558	66567	66577	66586	66596	66605	66614	66624	66633	66642
464	66652	66661	66671	66680	66689	66699	66708	66717	66727	66736
465	66745	66755	66764	66773	66783	66792	66801	66811	66820	66829
466	66839	66848	66857	66867	66876	66885	66894	66904	66913	66922
467	66932	66941	66950	66960	66969	66978	66987	66997	67006	67015
468	67025	67034	67043	67052	67062	67071	67080	67089	67099	67108
469	67117	67127	67136	67145	67154	67164	67173	67182	67191	67201
470	67210	67219	67228	67237	67247	67256	67265	67274	67284	67293
471	67302	67311	67321	67330	67339	67348	67357	67367	67376	67385
472	67394	67403	67413	67422	67431	67440	67449	67459	67468	67477
473	67486	67495	67504	67514	67523	67532	67541	67550	67560	67569
474	67578	67587	67596	67605	67614	67624	67633	67642	67651	67660
475	67669	67679	67688	67697	67706	67715	67724	67733	67742	67752
476	67761	67770	67779	67788	67797	67806	67815	67825	67834	67843
477	67852	67861	67870	67879	67888	67897	67906	67916	67925	67934
478	67943	67952	67961	67970	67979	67988	67997	68006	68015	68024
479	68034	68043	68052	68061	68070	68079	68088	68097	68106	68115
480	68124	68133	68142	68151	68160	68169	68178	68187	68196	68205
481	68215	68224	68233	68242	68251	68260	68269	68278	68287	68296
482	68305	68314	68323	68332	68341	68350	68359	68368	68377	68386
483	68395	68404	68413	68422	68431	68440	68449	68458	68467	68476
484	68485	68494	68502	68511	68520	68529	68538	68547	68556	68565
485	68574	68583	68592	68601	68610	68619	68628	68637	68646	68655
486	68664	68673	68681	68690	68699	68708	68717	68726	68735	68744
487	68753	68762	68771	68780	68789	68797	68806	68815	68824	68833
488	68842	68851	68860	68869	68878	68886	68895	68904	68913	68922
489	68931	68940	68949	68958	68966	68975	68984	68993	69002	69011
490	69020	69028	69037	69046	69055	69064	69073	69082	69090	69099
491	69108	69117	69126	69135	69144	69152	69161	69170	69179	69188
492	69197	69205	69214	69223	69232	69241	69249	69258	69267	69276
493	69285	69294	69302	69311	69320	69329	69338	69346	69355	69364
494	69373	69381	69390	69399	69408	69417	69425	69434	69443	69452
495	69461	69469	69478	69487	69496	69504	69513	69522	69531	69539
496	69548	69557	69566	69574	69583	69592	69601	69609	69618	69627
497	69636	69644	69653	69662	69671	69679	69688	69697	69705	69714
498	69723	69732	69740	69749	69758	69767	69775	69784	69793	69801
499	69810	69819	69827	69836	69845	69854	69862	69871	69880	69888
500	69897	69906	69914	69923	69932	69940	69949	69958	69966	69975
501	69984	69992	70001	70010	70018	70027	70036	70044	70053	70062
502	70070	70079	70088	70096	70105	70114	70122	70131	70140	70148
503	70157	70165	70174	70183	70191	70200	70209	70217	70226	70234
504	70243	70252	70260	70269	70278	70286	70295	70303	70312	70321
505	70329	70338	70346	70355	70364	70372	70381	70389	70398	70406
506	70415	70424	70432	70441	70449	70458	70467	70475	70484	70492
507	70501	70509	70518	70526	70535	70544	70552	70561	70569	70578
508	70586	70595	70603	70612	70621	70629	70638	70646	70655	70663
509	70672	70680	70689	70697	70706	70714	70723	70731	70740	70749
510	70757	70766	70774	70783	70791	70800	70808	70817	70825	70834
511	70842	70851	70859	70868	70876	70885	70893	70902	70910	70919
512	70927	70935	70944	70952	70961	70969	70978	70986	70995	71003
513	71012	71020	71029	71037	71046	71054	71063	71071	71079	71088
514	71096	71105	71113	71122	71130	71139	71147	71155	71164	71172
515	71181	71189	71198	71206	71214	71223	71231	71240	71248	71257
516	71265	71273	71282	71290	71299	71307	71315	71324	71332	71341
517	71349	71357	71366	71374	71383	71391	71399	71408	71416	71425
518	71433	71441	71450	71458	71466	71475	71483	71492	71500	71508
519	71517	71525	71533	71542	71550	71559	71567	71575	71584	71592
No.	0	1	2	3	4	5	6	7	8	9

No. 5200——5800. Log. 71600——76343.

No.	0	1	2	3	4	5	6	7	8	9
520	71600	71609	71617	71625	71634	71642	71650	71659	71667	71675
521	71684	71692	71700	71709	71717	71725	71734	71742	71750	71759
522	71767	71775	71784	71792	71800	71809	71817	71825	71834	71842
523	71850	71858	71867	71875	71883	71892	71900	71908	71917	71925
524	71933	71941	71950	71958	71966	71975	71983	71991	71999	72008
525	72016	72024	72032	72041	72049	72057	72066	72074	72082	72090
526	72099	72107	72115	72123	72132	72140	72148	72156	72165	72173
527	72181	72189	72198	72206	72214	72222	72230	72239	72247	72255
528	72263	72272	72280	72288	72296	72304	72313	72321	72329	72337
529	72346	72354	72362	72370	72378	72387	72395	72403	72411	72419
530	72428	72436	72444	72452	72460	72469	72477	72485	72493	72501
531	72509	72518	72526	72534	72542	72550	72558	72567	72575	72583
532	72591	72599	72607	72616	72624	72632	72640	72648	72656	72665
533	72673	72681	72689	72697	72705	72713	72722	72730	72738	72746
534	72754	72762	72770	72779	72787	72795	72803	72811	72819	72827
535	72835	72843	72852	72860	72868	72876	72884	72892	72900	72908
536	72916	72925	72933	72941	72949	72957	72965	72973	72981	72989
537	72997	73006	73014	73022	73030	73038	73046	73054	73062	73070
538	73078	73086	73094	73102	73111	73119	73127	73135	73143	73151
539	73159	73167	73175	73183	73191	73199	73207	73215	73223	73231
540	73239	73247	73255	73263	73272	73280	73288	73296	73304	73312
541	73320	73328	73336	73344	73352	73360	73368	73376	73384	73392
542	73400	73408	73416	73424	73432	73440	73448	73456	73464	73472
543	73480	73488	73496	73504	73512	73520	73528	73536	73544	73552
544	73560	73568	73576	73584	73592	73600	73608	73616	73624	73632
545	73640	73648	73656	73664	73672	73679	73687	73695	73703	73711
546	73719	73727	73735	73743	73751	73759	73767	73775	73783	73791
547	73799	73807	73815	73823	73830	73838	73846	73854	73862	73870
548	73878	73886	73894	73902	73910	73918	73926	73933	73941	73949
549	73957	73965	73973	73981	73989	73997	74005	74013	74020	74028
550	74036	74044	74052	74060	74068	74076	74084	74092	74099	74107
551	74115	74123	74131	74139	74147	74155	74162	74170	74178	74186
552	74194	74202	74210	74218	74225	74233	74241	74249	74257	74265
553	74273	74280	74288	74296	74304	74312	74320	74327	74335	74343
554	74351	74359	74367	74374	74382	74390	74398	74406	74414	74421
555	74429	74437	74445	74453	74461	74468	74476	74484	74492	74500
556	74507	74515	74523	74531	74539	74547	74554	74562	74570	74578
557	74586	74593	74601	74609	74617	74624	74632	74640	74648	74656
558	74663	74671	74679	74687	74695	74702	74710	74718	74726	74733
559	74741	74749	74757	74764	74772	74780	74788	74796	74803	74811
560	74819	74827	74834	74842	74850	74858	74865	74873	74881	74889
561	74896	74904	74912	74920	74927	74935	74943	74950	74958	74966
562	74974	74981	74989	74997	75005	75012	75020	75028	75035	75043
563	75051	75059	75066	75074	75082	75089	75097	75105	75113	75120
564	75128	75136	75143	75151	75159	75166	75174	75182	75189	75197
565	75205	75213	75220	75228	75236	75243	75251	75259	75266	75274
566	75282	75289	75297	75305	75312	75320	75328	75335	75343	75351
567	75358	75366	75374	75381	75389	75397	75404	75412	75420	75427
568	75435	75442	75450	75458	75465	75473	75481	75488	75496	75504
569	75511	75519	75526	75534	75542	75549	75557	75565	75572	75580
570	75587	75595	75603	75610	75618	75626	75633	75641	75648	75656
571	75664	75671	75679	75686	75694	75702	75709	75717	75724	75732
572	75740	75747	75755	75762	75770	75778	75785	75793	75800	75808
573	75815	75823	75831	75838	75846	75853	75861	75868	75876	75884
574	75891	75899	75906	75914	75921	75929	75937	75944	75952	75959
575	75967	75974	75982	75989	75997	76005	76012	76020	76027	76035
576	76042	76050	76057	76065	76072	76080	76087	76095	76103	76110
577	76118	76125	76133	76140	76148	76155	76163	76170	76178	76185
578	76193	76200	76208	76215	76223	76230	76238	76245	76253	76260
579	76268	76275	76283	76290	76298	76305	76313	76320	76328	76335
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No. 5800——6400. Log. 76343——80613.

No.	0	1	2	3	4	5	6	7	8	9
580	76343	76350	76358	76365	76373	76380	76388	76395	76403	76410
581	76418	76425	76433	76440	76448	76455	76462	76470	76477	76485
582	76492	76500	76507	76515	76522	76530	76537	76545	76552	76559
583	76567	76574	76582	76589	76597	76604	76612	76619	76626	76634
584	76641	76649	76656	76664	76671	76678	76686	76693	76701	76708
585	76716	76723	76730	76738	76745	76753	76760	76768	76775	76782
586	76790	76797	76805	76812	76819	76827	76834	76842	76849	76856
587	76864	76871	76879	76886	76893	76901	76908	76916	76923	76930
588	76938	76945	76953	76960	76967	76975	76982	76989	76997	77004
589	77012	77019	77026	77034	77041	77048	77056	77063	77070	77078
590	77085	77093	77100	77107	77115	77122	77129	77137	77144	77151
591	77159	77166	77173	77181	77188	77195	77203	77210	77217	77225
592	77232	77240	77247	77254	77262	77269	77276	77283	77291	77298
593	77305	77313	77320	77327	77335	77342	77349	77357	77364	77371
594	77379	77386	77393	77401	77408	77415	77422	77430	77437	77444
595	77452	77459	77466	77474	77481	77488	77495	77503	77510	77517
596	77525	77532	77539	77546	77554	77561	77568	77576	77583	77590
597	77597	77605	77612	77619	77627	77634	77641	77648	77656	77663
598	77670	77677	77685	77692	77699	77706	77714	77721	77728	77735
599	77743	77750	77757	77764	77772	77779	77786	77793	77801	77808
600	77815	77822	77830	77837	77844	77851	77859	77866	77873	77880
601	77887	77895	77902	77909	77916	77924	77931	77938	77945	77952
602	77960	77967	77974	77981	77988	77996	78003	78010	78017	78025
603	78032	78039	78046	78053	78061	78068	78075	78082	78089	78097
604	78104	78111	78118	78125	78132	78140	78147	78154	78161	78168
605	78176	78183	78190	78197	78204	78211	78219	78226	78233	78240
606	78247	78254	78262	78269	78276	78283	78290	78297	78305	78312
607	78319	78326	78333	78340	78347	78355	78362	78369	78376	78383
608	78390	78398	78405	78412	78419	78426	78433	78440	78447	78455
609	78462	78469	78476	78483	78490	78497	78504	78512	78519	78526
610	78533	78540	78547	78554	78561	78569	78576	78583	78590	78597
611	78604	78611	78618	78625	78633	78640	78647	78654	78661	78668
612	78675	78682	78689	78696	78704	78711	78718	78725	78732	78739
613	78746	78753	78760	78767	78774	78781	78789	78796	78803	78810
614	78817	78824	78831	78838	78845	78852	78859	78866	78873	78880
615	78888	78895	78902	78909	78916	78923	78930	78937	78944	78951
616	78958	78965	78972	78979	78986	78993	79000	79007	79014	79021
617	79029	79036	79043	79050	79057	79064	79071	79078	79085	79092
618	79099	79106	79113	79120	79127	79134	79141	79148	79155	79162
619	79169	79176	79183	79190	79197	79204	79211	79218	79225	79232
620	79239	79246	79253	79260	79267	79274	79281	79288	79295	79302
621	79309	79316	79323	79330	79337	79344	79351	79358	79365	79372
622	79379	79386	79393	79400	79407	79414	79421	79428	79435	79442
623	79449	79456	79463	79470	79477	79484	79491	79498	79505	79511
624	79518	79525	79532	79539	79546	79553	79560	79567	79574	79581
625	79588	79595	79602	79609	79616	79623	79630	79637	79644	79650
626	79657	79664	79671	79678	79685	79692	79699	79706	79713	79720
627	79727	79734	79741	79748	79754	79761	79768	79775	79782	79789
628	79796	79803	79810	79817	79824	79831	79837	79844	79851	79858
629	79865	79872	79879	79886	79893	79900	79906	79913	79920	79927
630	79934	79941	79948	79955	79962	79969	79975	79982	79989	79996
631	80003	80010	80017	80024	80030	80037	80044	80051	80058	80065
632	80072	80079	80085	80092	80099	80106	80113	80120	80127	80134
633	80140	80147	80154	80161	80168	80175	80182	80188	80195	80202
634	80209	80216	80223	80229	80236	80243	80250	80257	80264	80271
635	80277	80284	80291	80298	80305	80312	80318	80325	80332	80339
636	80346	80353	80359	80366	80373	80380	80387	80393	80400	80407
637	80414	80421	80428	80434	80441	80448	80455	80462	80468	80475
638	80482	80489	80496	80502	80509	80516	80523	80530	80536	80543
639	80550	80557	80564	80570	80577	80584	80591	80598	80604	80611
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No. 6400——7000. Log. 80618——84510.

No.	0	1	2	3	4	5	6	7	8	9
640	80618	80625	80632	80638	80645	80652	80659	80665	80672	80679
641	80686	80693	80699	80706	80713	80720	80726	80733	80740	80747
642	80754	80760	80767	80774	80781	80787	80794	80801	80808	80814
643	80821	80828	80835	80841	80848	80855	80862	80868	80875	80882
644	80889	80895	80902	80909	80916	80922	80929	80936	80943	80949
645	80956	80963	80969	80976	80983	80990	80996	81003	81010	81017
646	81023	81030	81037	81043	81050	81057	81064	81070	81077	81084
647	81090	81097	81104	81111	81117	81124	81131	81137	81144	81151
648	81158	81164	81171	81178	81184	81191	81198	81204	81211	81218
649	81224	81231	81238	81245	81251	81258	81265	81271	81278	81285
650	81291	81298	81305	81311	81318	81325	81331	81338	81345	81351
651	81358	81365	81371	81378	81385	81391	81398	81405	81411	81418
652	81425	81431	81438	81445	81451	81458	81465	81471	81478	81485
653	81491	81498	81505	81511	81518	81525	81531	81538	81544	81551
654	81558	81564	81571	81578	81584	81591	81598	81604	81611	81617
655	81624	81631	81637	81644	81651	81657	81664	81671	81677	81684
656	81690	81697	81704	81710	81717	81723	81730	81737	81743	81750
657	81757	81763	81770	81776	81783	81790	81796	81803	81809	81816
658	81823	81829	81836	81842	81849	81856	81862	81869	81875	81882
659	81889	81895	81902	81908	81915	81921	81928	81935	81941	81948
660	81954	81961	81968	81974	81981	81987	81994	82000	82007	82014
661	82020	82027	82033	82040	82046	82053	82060	82066	82073	82079
662	82086	82092	82099	82105	82112	82119	82125	82132	82138	82145
663	82151	82158	82164	82171	82178	82184	82191	82197	82204	82210
664	82217	82223	82230	82236	82243	82249	82256	82263	82269	82276
665	82282	82289	82295	82302	82308	82315	82321	82328	82334	82341
666	82347	82354	82360	82367	82373	82380	82387	82393	82400	82406
667	82413	82419	82426	82432	82439	82445	82452	82458	82465	82471
668	82478	82484	82491	82497	82504	82510	82517	82523	82530	82536
669	82543	82549	82556	82562	82569	82575	82582	82588	82595	82601
670	82607	82614	82620	82627	82633	82640	82646	82653	82659	82666
671	82672	82679	82685	82692	82698	82705	82711	82718	82724	82730
672	82737	82743	82750	82756	82763	82769	82776	82782	82789	82795
673	82802	82808	82814	82821	82827	82834	82840	82847	82853	82860
674	82866	82872	82879	82885	82892	82898	82905	82911	82918	82924
675	82930	82937	82943	82950	82956	82963	82969	82975	82982	82988
676	82995	83001	83008	83014	83020	83027	83033	83040	83046	83052
677	83059	83065	83072	83078	83085	83091	83097	83104	83110	83117
678	83123	83129	83136	83142	83149	83155	83161	83168	83174	83181
679	83187	83193	83200	83206	83213	83219	83225	83232	83238	83245
680	83251	83257	83264	83270	83276	83283	83289	83296	83302	83308
681	83315	83321	83327	83334	83340	83347	83353	83359	83366	83372
682	83378	83385	83391	83398	83404	83410	83417	83423	83429	83436
683	83442	83448	83455	83461	83467	83474	83480	83487	83493	83499
684	83506	83512	83518	83525	83531	83537	83544	83550	83556	83563
685	83569	83575	83582	83588	83594	83601	83607	83613	83620	83626
686	83632	83639	83645	83651	83658	83664	83670	83677	83683	83689
687	83696	83702	83708	83715	83721	83727	83734	83740	83746	83753
688	83759	83765	83771	83778	83784	83790	83797	83803	83809	83816
689	83822	83828	83835	83841	83847	83853	83860	83866	83872	83879
690	83885	83891	83897	83904	83910	83916	83923	83929	83935	83942
691	83948	83954	83960	83967	83973	83979	83985	83992	83998	84004
692	84011	84017	84023	84029	84036	84042	84048	84055	84061	84067
693	84073	84080	84086	84092	84098	84105	84111	84117	84123	84130
694	84136	84142	84148	84155	84161	84167	84173	84180	84186	84192
695	84198	84205	84211	84217	84223	84230	84236	84242	84248	84255
696	84261	84267	84273	84280	84286	84292	84298	84305	84311	84317
697	84323	84330	84336	84342	84348	84354	84361	84367	84373	84379
698	84386	84392	84398	84404	84410	84417	84423	84429	84435	84442
699	84448	84454	84460	84466	84473	84479	84485	84491	84497	84504
No.	0	1	2	3	4	5	6	7	8	9

No. 7000——7600. Log. 84510——88081.

No.	0	1	2	3	4	5	6	7	8	9
700	84510	84516	84522	84528	84535	84541	84547	84553	84559	84566
701	84572	84578	84584	84590	84597	84603	84609	84615	84621	84628
702	84634	84640	84646	84652	84658	84665	84671	84677	84683	84689
703	84696	84702	84708	84714	84720	84726	84733	84739	84745	84751
704	84757	84763	84770	84776	84782	84788	84794	84800	84807	84813
705	84819	84825	84831	84837	84844	84850	84856	84862	84868	84874
706	84880	84887	84893	84899	84905	84911	84917	84924	84930	84936
707	84942	84948	84954	84960	84967	84973	84979	84985	84991	84997
708	85003	85009	85016	85022	85028	85034	85040	85046	85052	85058
709	85065	85071	85077	85083	85089	85095	85101	85107	85114	85120
710	85126	85132	85138	85144	85150	85156	85163	85169	85175	85181
711	85187	85193	85199	85205	85211	85217	85224	85230	85236	85242
712	85248	85254	85260	85266	85272	85278	85285	85291	85297	85303
713	85309	85315	85321	85327	85333	85339	85345	85352	85358	85364
714	85370	85376	85382	85388	85394	85400	85406	85412	85418	85425
715	85431	85437	85443	85449	85455	85461	85467	85473	85479	85485
716	85491	85497	85503	85509	85516	85522	85528	85534	85540	85546
717	85552	85558	85564	85570	85576	85582	85588	85594	85600	85606
718	85612	85618	85625	85631	85637	85643	85649	85655	85661	85667
719	85673	85679	85685	85691	85697	85703	85709	85715	85721	85727
720	85733	85739	85745	85751	85757	85763	85769	85775	85781	85788
721	85794	85800	85806	85812	85818	85824	85830	85836	85842	85848
722	85854	85860	85866	85872	85878	85884	85890	85896	85902	85908
723	85914	85920	85926	85932	85938	85944	85950	85956	85962	85968
724	85974	85980	85986	85992	85998	86004	86010	86016	86022	86028
725	86034	86040	86046	86052	86058	86064	86070	86076	86082	86088
726	86094	86100	86106	86112	86118	86124	86130	86136	86141	86147
727	86153	86159	86165	86171	86177	86183	86189	86195	86201	86207
728	86213	86219	86225	86231	86237	86243	86249	86255	86261	86267
729	86273	86279	86285	86291	86297	86303	86308	86314	86320	86326
730	86332	86338	86344	86350	86356	86362	86368	86374	86380	86386
731	86392	86398	86404	86410	86415	86421	86427	86433	86439	86445
732	86451	86457	86463	86469	86475	86481	86487	86493	86499	86504
733	86510	86516	86522	86528	86534	86540	86546	86552	86558	86564
734	86570	86576	86581	86587	86593	86599	86605	86611	86617	86623
735	86629	86635	86641	86646	86652	86658	86664	86670	86676	86682
736	86688	86694	86700	86705	86711	86717	86723	86729	86735	86741
737	86747	86753	86759	86764	86770	86776	86782	86788	86794	86800
738	86806	86812	86817	86823	86829	86835	86841	86847	86853	86859
739	86864	86870	86876	86882	86888	86894	86900	86906	86911	86917
740	86923	86929	86935	86941	86947	86953	86958	86964	86970	86976
741	86982	86988	86994	86999	87005	87011	87017	87023	87029	87035
742	87040	87046	87052	87058	87064	87070	87075	87081	87087	87093
743	87099	87105	87111	87116	87122	87128	87134	87140	87146	87151
744	87157	87163	87169	87175	87181	87186	87192	87198	87204	87210
745	87216	87221	87227	87233	87239	87245	87251	87256	87262	87268
746	87274	87280	87286	87291	87297	87303	87309	87315	87320	87326
747	87332	87338	87344	87349	87355	87361	87367	87373	87379	87384
748	87390	87396	87402	87408	87413	87419	87425	87431	87437	87442
749	87448	87454	87460	87466	87471	87477	87483	87489	87495	87500
750	87506	87512	87518	87523	87529	87535	87541	87547	87552	87558
751	87564	87570	87576	87581	87587	87593	87599	87604	87610	87616
752	87622	87628	87633	87639	87645	87651	87656	87662	87668	87674
753	87679	87685	87691	87697	87703	87708	87714	87720	87726	87731
754	87737	87743	87749	87754	87760	87766	87772	87777	87783	87789
755	87795	87800	87806	87812	87818	87823	87829	87835	87841	87846
756	87852	87858	87864	87869	87875	87881	87887	87892	87898	87904
757	87910	87915	87921	87927	87933	87938	87944	87950	87955	87961
758	87967	87973	87978	87984	87990	87996	88001	88007	88013	88018
759	88024	88030	88036	88041	88047	88053	88058	88064	88070	88076
No.	0	1	2	3	4	5	6	7	8	9

No. 7600——8200. Log. 88081——91381.

No.	0	1	2	3	4	5	6	7	8	9
760	88081	88087	88093	88098	88104	88110	88116	88121	88127	88133
761	88138	88144	88150	88156	88161	88167	88173	88178	88184	88190
762	88195	88201	88207	88213	88218	88224	88230	88235	88241	88247
763	88252	88258	88264	88270	88275	88281	88287	88292	88298	88304
764	88309	88315	88321	88326	88332	88338	88343	88349	88355	88360
765	88366	88372	88377	88383	88389	88395	88400	88406	88412	88417
766	88423	88429	88434	88440	88446	88451	88457	88463	88468	88474
767	88480	88485	88491	88497	88502	88508	88513	88519	88525	88530
768	88536	88542	88547	88553	88559	88564	88570	88576	88581	88587
769	88593	88598	88604	88610	88615	88621	88627	88632	88638	88643
770	88649	88655	88660	88666	88672	88677	88683	88689	88694	88700
771	88705	88711	88717	88722	88728	88734	88739	88745	88750	88756
772	88762	88767	88773	88779	88784	88790	88795	88801	88807	88812
773	88818	88824	88829	88835	88840	88846	88852	88857	88863	88868
774	88874	88880	88885	88891	88897	88902	88908	88913	88919	88925
775	88930	88936	88941	88947	88953	88958	88964	88969	88975	88981
776	88986	88992	88997	89003	89009	89014	89020	89025	89031	89037
777	89042	89048	89053	89059	89064	89070	89076	89081	89087	89092
778	89098	89104	89109	89115	89120	89126	89131	89137	89143	89148
779	89154	89159	89165	89170	89176	89182	89187	89193	89198	89204
780	89209	89215	89221	89226	89232	89237	89243	89248	89254	89260
781	89265	89271	89276	89282	89287	89293	89298	89304	89310	89315
782	89321	89326	89332	89337	89343	89348	89354	89360	89365	89371
783	89376	89382	89387	89393	89398	89404	89409	89415	89421	89426
784	89432	89437	89443	89448	89454	89459	89465	89470	89476	89481
785	89487	89492	89498	89504	89509	89515	89520	89526	89531	89537
786	89542	89548	89553	89559	89564	89570	89575	89581	89586	89592
787	89597	89603	89609	89614	89620	89625	89631	89636	89642	89647
788	89653	89658	89664	89669	89675	89680	89686	89691	89697	89702
789	89708	89713	89719	89724	89730	89735	89741	89746	89752	89757
790	89763	89768	89774	89779	89785	89790	89796	89801	89807	89812
791	89818	89823	89829	89834	89840	89845	89851	89856	89862	89867
792	89873	89878	89883	89889	89894	89900	89905	89911	89916	89922
793	89927	89933	89938	89944	89949	89955	89960	89966	89971	89977
794	89982	89988	89993	89998	90004	90009	90015	90020	90026	90031
795	90037	90042	90048	90053	90059	90064	90069	90075	90080	90086
796	90091	90097	90102	90108	90113	90119	90124	90129	90135	90140
797	90146	90151	90157	90162	90168	90173	90179	90184	90189	90195
798	90200	90206	90211	90217	90222	90227	90233	90238	90244	90249
799	90255	90260	90266	90271	90276	90282	90287	90293	90298	90304
800	90309	90314	90320	90325	90331	90336	90342	90347	90352	90358
801	90363	90369	90374	90380	90385	90390	90396	90401	90407	90412
802	90417	90423	90428	90434	90439	90445	90450	90455	90461	90466
803	90472	90477	90482	90488	90493	90499	90504	90509	90515	90520
804	90526	90531	90536	90542	90547	90553	90558	90563	90569	90574
805	90580	90585	90590	90596	90601	90607	90612	90617	90623	90628
806	90634	90639	90644	90650	90655	90660	90666	90671	90677	90682
807	90687	90693	90698	90703	90709	90714	90720	90725	90730	90736
808	90741	90747	90752	90757	90763	90768	90773	90779	90784	90789
809	90795	90800	90806	90811	90816	90822	90827	90832	90838	90843
810	90849	90854	90859	90865	90870	90875	90881	90886	90891	90897
811	90902	90907	90913	90918	90924	90929	90934	90940	90945	90950
812	90956	90961	90966	90972	90977	90982	90988	90993	90998	91004
813	91009	91014	91020	91025	91030	91036	91041	91046	91052	91057
814	91062	91068	91073	91078	91084	91089	91094	91100	91105	91110
815	91116	91121	91126	91132	91137	91142	91148	91153	91158	91164
816	91169	91174	91180	91185	91190	91196	91201	91206	91212	91217
817	91222	91228	91233	91238	91243	91249	91254	91259	91265	91270
818	91275	91281	91286	91291	91297	91302	91307	91312	91318	91323
819	91328	91334	91339	91344	91350	91355	91360	91365	91371	91376
No.	0	1	2	3	4	5	6	7	8	9

8200——8800. Log. 91381——94448.

No.	0	1	2	3	4	5	6	7	8	9
20	91381	91387	91392	91397	91403	91408	91413	91418	91424	91429
21	91434	91440	91445	91450	91455	91461	91466	91471	91477	91482
22	91487	91492	91498	91503	91508	91514	91519	91524	91529	91535
23	91540	91545	91551	91556	91561	91566	91572	91577	91582	91587
24	91593	91598	91603	91609	91614	91619	91624	91630	91635	91640
25	91645	91651	91656	91661	91666	91672	91677	91682	91687	91693
26	91698	91703	91709	91714	91719	91724	91730	91735	91740	91745
27	91751	91756	91761	91766	91772	91777	91782	91787	91793	91798
28	91803	91808	91814	91819	91824	91829	91834	91840	91845	91850
29	91855	91861	91866	91871	91876	91882	91887	91892	91897	91903
30	91908	91913	91918	91924	91929	91934	91939	91944	91950	91955
31	91960	91965	91971	91976	91981	91986	91991	91997	92002	92007
32	92012	92018	92023	92028	92033	92038	92044	92049	92054	92059
33	92065	92070	92075	92080	92085	92091	92096	92101	92106	92111
34	92117	92122	92127	92132	92137	92143	92148	92153	92158	92163
35	92169	92174	92179	92184	92189	92195	92200	92205	92210	92215
36	92221	92226	92231	92236	92241	92247	92252	92257	92262	92267
37	92273	92278	92283	92288	92293	92298	92304	92309	92314	92319
38	92324	92330	92335	92340	92345	92350	92355	92361	92366	92371
39	92376	92381	92387	92392	92397	92402	92407	92412	92418	92423
40	92428	92433	92438	92443	92449	92454	92459	92464	92469	92474
41	92480	92485	92490	92495	92500	92505	92511	92516	92521	92526
42	92531	92536	92542	92547	92552	92557	92562	92567	92572	92578
43	92583	92588	92593	92598	92603	92609	92614	92619	92624	92629
44	92634	92639	92645	92650	92655	92660	92665	92670	92675	92681
845	92686	92691	92696	92701	92706	92711	92716	92722	92727	92732
846	92737	92742	92747	92752	92758	92763	92768	92773	92778	92783
847	92788	92793	92799	92804	92809	92814	92819	92824	92829	92834
848	92840	92845	92850	92855	92860	92865	92870	92875	92881	92886
849	92891	92896	92901	92906	92911	92916	92921	92927	92932	92937
850	92942	92947	92952	92957	92962	92967	92973	92978	92983	92988
851	92993	92998	93003	93008	93013	93018	93024	93029	93034	93039
852	93044	93049	93054	93059	93064	93069	93075	93080	93085	93090
853	93095	93100	93105	93110	93115	93120	93125	93131	93136	93141
854	93146	93151	93156	93161	93166	93171	93176	93181	93186	93192
855	93197	93202	93207	93212	93217	93222	93227	93232	93237	93242
856	93247	93252	93258	93263	93268	93273	93278	93283	93288	93293
857	93298	93303	93308	93313	93318	93323	93328	93334	93339	93344
858	93349	93354	93359	93364	93369	93374	93379	93384	93389	93394
859	93399	93404	93409	93414	93420	93425	93430	93435	93440	93445
860	93450	93455	93460	93465	93470	93475	93480	93485	93490	93495
861	93500	93505	93510	93515	93520	93526	93531	93536	93541	93546
862	93551	93556	93561	93566	93571	93576	93581	93586	93591	93596
863	93601	93606	93611	93616	93621	93626	93631	93636	93641	93646
864	93651	93656	93661	93666	93671	93676	93682	93687	93692	93697
865	93702	93707	93712	93717	93722	93727	93732	93737	93742	93747
866	93752	93757	93762	93767	93772	93777	93782	93787	93792	93797
867	93802	93807	93812	93817	93822	93827	93832	93837	93842	93847
868	93852	93857	93862	93867	93872	93877	93882	93887	93892	93897
869	93902	93907	93912	93917	93922	93927	93932	93937	93942	93947
870	93952	93957	93962	93967	93972	93977	93982	93987	93992	93997
871	94002	94007	94012	94017	94022	94027	94032	94037	94042	94047
872	94052	94057	94062	94067	94072	94077	94082	94086	94091	94096
873	94101	94106	94111	94116	94121	94126	94131	94136	94141	94146
874	94151	94156	94161	94166	94171	94176	94181	94186	94191	94196
875	94201	94206	94211	94216	94221	94226	94231	94236	94240	94245
876	94250	94255	94260	94265	94270	94275	94280	94285	94290	94295
877	94300	94305	94310	94315	94320	94325	94330	94335	94340	94345
878	94349	94354	94359	94364	94369	94374	94379	94384	94389	94394
879	94399	94404	94409	94414	94419	94424	94429	94433	94438	94443
No.	0	1	2	3	4	5	6	7	8	9

No. 8800——9400. Log. 94448——97313.

No.	0	1	2	3	4	5	6	7	8	9
880	94448	94453	94458	94463	94468	94473	94478	94483	94488	94493
881	94498	94503	94507	94512	94517	94522	94527	94532	94537	94542
882	94547	94552	94557	94562	94567	94571	94576	94581	94586	94591
883	94596	94601	94606	94611	94616	94621	94626	94630	94635	94640
884	94645	94650	94655	94660	94665	94670	94675	94680	94685	94689
885	94694	94699	94704	94709	94714	94719	94724	94729	94734	94738
886	94743	94748	94753	94758	94763	94768	94773	94778	94783	94787
887	94792	94797	94802	94807	94812	94817	94822	94827	94832	94836
888	94841	94846	94851	94856	94861	94866	94871	94876	94880	94885
889	94890	94895	94900	94905	94910	94915	94919	94924	94929	94934
890	94939	94944	94949	94954	94959	94963	94968	94973	94978	94983
891	94988	94993	94998	95002	95007	95012	95017	95022	95027	95032
892	95036	95041	95046	95051	95056	95061	95066	95071	95075	95080
893	95085	95090	95095	95100	95105	95109	95114	95119	95124	95129
894	95134	95139	95143	95148	95153	95158	95163	95168	95173	95177
895	95182	95187	95192	95197	95202	95207	95211	95216	95221	95226
896	95231	95236	95240	95245	95250	95255	95260	95265	95270	95274
897	95279	95284	95289	95294	95299	95303	95308	95313	95318	95323
898	95328	95332	95337	95342	95347	95352	95357	95361	95366	95371
899	95376	95381	95386	95390	95395	95400	95405	95410	95415	95419
900	95424	95429	95434	95439	95444	95448	95453	95458	95463	95468
901	95472	95477	95482	95487	95492	95497	95501	95506	95511	95516
902	95521	95525	95530	95535	95540	95545	95550	95554	95559	95564
903	95569	95574	95578	95583	95588	95593	95598	95602	95607	95612
904	95617	95622	95626	95631	95636	95641	95646	95650	95655	95660
905	95665	95670	95674	95679	95684	95689	95694	95698	95703	95708
906	95713	95718	95722	95727	95732	95737	95742	95746	95751	95756
907	95761	95766	95770	95775	95780	95785	95789	95794	95799	95804
908	95809	95813	95818	95823	95828	95832	95837	95842	95847	95852
909	95856	95861	95866	95871	95875	95880	95885	95890	95895	95899
910	95904	95909	95914	95918	95923	95928	95933	95938	95942	95947
911	95952	95957	95961	95966	95971	95976	95980	95985	95990	95995
912	95999	96004	96009	96014	96019	96023	96028	96033	96038	96042
913	96047	96052	96057	96061	96066	96071	96076	96080	96085	96090
914	96095	96099	96104	96109	96114	96118	96123	96128	96133	96137
915	96142	96147	96152	96156	96161	96166	96171	96175	96180	96185
916	96190	96194	96199	96204	96209	96213	96218	96223	96227	96232
917	96237	96242	96246	96251	96256	96261	96265	96270	96275	96280
918	96284	96289	96294	96298	96303	96308	96313	96317	96322	96327
919	96332	96336	96341	96346	96350	96355	96360	96365	96369	96374
920	96379	96384	96388	96393	96398	96402	96407	96412	96417	96421
921	96426	96431	96435	96440	96445	96450	96454	96459	96464	96468
922	96473	96478	96483	96487	96492	96497	96501	96506	96511	96515
923	96520	96525	96530	96534	96539	96544	96548	96553	96558	96562
924	96567	96572	96577	96581	96586	96591	96595	96600	96605	96609
925	96614	96619	96624	96628	96633	96638	96642	96647	96652	96656
926	96661	96666	96670	96675	96680	96685	96689	96694	96699	96703
927	96708	96713	96717	96722	96727	96731	96736	96741	96745	96750
928	96755	96759	96764	96769	96774	96778	96783	96788	96792	96797
929	96802	96806	96811	96816	96820	96825	96830	96834	96839	96844
930	96848	96853	96858	96862	96867	96872	96876	96881	96886	96890
931	96895	96900	96904	96909	96914	96918	96923	96928	96932	96937
932	96942	96946	96951	96956	96960	96965	96970	96974	96979	96984
933	96988	96993	96997	97002	97007	97011	97016	97021	97025	97030
934	97035	97039	97044	97049	97053	97058	97063	97067	97072	97077
935	97081	97086	97090	97095	97100	97104	97109	97114	97118	97123
936	97128	97132	97137	97142	97146	97151	97155	97160	97165	97169
937	97174	97179	97183	97188	97192	97197	97202	97206	97211	97216
938	97220	97225	97230	97234	97239	97243	97248	97253	97257	97262
939	97267	97271	97276	97280	97285	97290	97294	97299	97304	97308
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No. 9400——10000. Log. 97313——99996.

No.	0	1	2	3	4	5	6	7	8	9
940	97313	97317	97322	97327	97331	97336	97340	97345	97350	97354
941	97359	97364	97368	97373	97377	97382	97387	97391	97396	97400
942	97405	97410	97414	97419	97424	97428	97433	97437	97442	97447
943	97451	97456	97460	97465	97470	97474	97479	97483	97488	97493
944	97497	97502	97506	97511	97516	97520	97525	97529	97534	97539
945	97543	97548	97552	97557	97562	97566	97571	97575	97580	97585
946	97589	97594	97598	97603	97607	97612	97617	97621	97626	97630
947	97635	97640	97644	97649	97653	97658	97663	97667	97672	97676
948	97681	97685	97690	97695	97699	97704	97708	97713	97717	97722
949	97727	97731	97736	97740	97745	97749	97754	97759	97763	97768
950	97772	97777	97782	97786	97791	97795	97800	97804	97809	97813
951	97818	97823	97827	97832	97836	97841	97845	97850	97855	97859
952	97864	97868	97873	97877	97882	97886	97891	97896	97900	97905
953	97909	97914	97918	97923	97928	97932	97937	97941	97946	97950
954	97955	97950	97964	97968	97973	97978	97982	97987	97991	97996
955	98000	98005	98009	98014	98019	98023	98028	98032	98037	98041
956	98046	98050	98055	98059	98064	98068	98073	98078	98082	98087
957	98091	98096	98100	98105	98109	98114	98118	98123	98127	98132
958	98137	98141	98146	98150	98155	98159	98164	98168	98173	98177
959	98182	98186	98191	98195	98200	98204	98209	98214	98218	98223
960	98227	98232	98236	98241	98245	98250	98254	98259	98263	98268
961	98272	98277	98281	98286	98290	98295	98299	98304	98308	98313
962	98318	98322	98327	98331	98336	98340	98345	98349	98354	98358
963	98363	98367	98372	98376	98381	98385	98390	98394	98399	98403
964	98408	98412	98417	98421	98426	98430	98435	98439	98444	98448
965	98453	98457	98462	98466	98471	98475	98480	98484	98489	98493
966	98498	98502	98507	98511	98516	98520	98525	98529	98534	98538
967	98543	98547	98552	98556	98561	98565	98570	98574	98579	98583
968	98588	98592	98597	98601	98605	98610	98614	98619	98623	98628
969	98632	98637	98641	98646	98650	98655	98659	98664	98668	98673
970	98677	98682	98686	98691	98695	98700	98704	98709	98713	98717
971	98722	98726	98731	98735	98740	98744	98749	98753	98758	98762
972	98767	98771	98776	98780	98784	98789	98793	98798	98802	98807
973	98811	98816	98820	98825	98829	98834	98838	98843	98847	98851
974	98856	98860	98865	98869	98874	98878	98883	98887	98892	98896
975	98900	98905	98909	98914	98918	98923	98927	98932	98936	98941
976	98945	98949	98954	98958	98963	98967	98972	98976	98981	98985
977	98989	98994	98998	99003	99007	99012	99016	99021	99025	99029
978	99034	99038	99043	99047	99052	99056	99061	99065	99069	99074
979	99078	99083	99087	99092	99096	99100	99105	99109	99114	99118
980	99123	99127	99131	99136	99140	99145	99149	99154	99158	99162
981	99167	99171	99176	99180	99185	99189	99193	99198	99202	99207
982	99211	99216	99220	99224	99229	99233	99238	99242	99247	99251
983	99255	99260	99264	99269	99273	99277	99282	99286	99291	99295
984	99300	99304	99308	99313	99317	99322	99326	99330	99335	99339
985	99344	99348	99352	99357	99361	99366	99370	99374	99379	99383
986	99388	99392	99396	99401	99405	99410	99414	99419	99423	99427
987	99432	99436	99441	99445	99449	99454	99458	99463	99467	99471
988	99476	99480	99484	99489	99493	99498	99502	99506	99511	99515
989	99520	99524	99528	99533	99537	99542	99546	99550	99555	99559
990	99564	99568	99572	99577	99581	99585	99590	99594	99599	99603
991	99607	99612	99616	99621	99625	99629	99634	99638	99642	99647
992	99651	99656	99660	99664	99669	99673	99677	99682	99686	99691
993	99695	99699	99704	99708	99712	99717	99721	99726	99730	99734
994	99739	99743	99747	99752	99756	99760	99765	99760	99774	99778
995	99782	99787	99791	99795	99800	99804	99808	99813	99817	99822
996	99826	99830	99835	99839	99843	99848	99852	99856	99861	99865
997	99870	99874	99878	99883	99887	99891	99896	99900	99904	99909
998	99913	99917	99922	99926	99930	99935	99939	99944	99948	99952
999	99957	99961	99965	99970	99974	99978	99983	99987	99991	99996
No.	0	1	2	3	4	5	6	7	8	9

0 Deg. Degs. 179.

M.	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M.
0	12 0 0	0 0 0	Inf. Neg.	10.00000	Inf. Neg.	Infinite.	10.00000	Infinite.	60
1	11 59 52	0 8	6.46373	00000	6.46373	13.53627	00000	13.53627	59
2	59 44	0 16	76476	00000	76476	23524	00000	23524	58
3	59 36	0 24	94085	00000	94085	05915	00000	05915	57
4	59 28	0 32	7.06579	00000	7.06579	12.93421	00000	12.93421	56
5	11 59 20	0 0 40	7.16270	10.00000	7.16270	12.83730	10.00000	12.83730	55
6	59 12	0 48	24188	00000	24188	75812	00000	75812	54
7	59 4	0 56	30682	00000	30882	69118	00000	69118	53
8	58 56	1 4	36682	00000	36682	63318	00000	63318	52
9	58 48	1 12	41797	00000	41797	58203	00000	58203	51
10	11 58 40	0 1 20	7.46373	10.00000	7.46373	12.53627	10.00000	12.53627	50
11	58 32	1 28	50512	00000	50512	49488	00000	49488	49
12	58 24	1 36	54291	00000	54291	45709	00000	45709	48
13	58 16	1 44	57767	00000	57767	42233	00000	42233	47
14	58 8	1 52	60985	00000	60986	39014	00000	39015	46
15	11 58 0	0 2 0	7.63982	10.00000	7.63982	12.36018	10.00000	12.36018	45
16	57 52	2 8	66784	00000	66785	33215	00000	33216	44
17	57 44	2 16	69417	9.99999	69418	30582	00001	30583	43
18	57 36	2 24	71900	99999	71900	28100	00001	28100	42
19	57 28	2 32	74248	99999	74248	25752	00001	25752	41
20	11 57 20	0 2 40	7.76475	9.99999	7.76476	12.23524	10.00001	12.23525	40
21	57 12	2 48	78594	99999	78595	21405	00001	21406	39
22	57 4	2 56	80615	99999	80615	19385	00001	19385	38
23	56 56	3 4	82545	99999	82546	17454	00001	17455	37
24	56 48	3 12	84393	99999	84394	15606	00001	15607	36
25	11 56 40	0 3 20	7.86166	9.99999	7.86167	12.13833	10.00001	12.13834	35
26	56 32	3 28	87870	99999	87871	12129	00001	12130	34
27	56 24	3 36	89509	99999	89510	10490	00001	10491	33
28	56 16	3 44	91088	99999	91089	08911	00001	08912	32
29	56 8	3 52	92612	99998	92613	07387	00002	07388	31
30	11 56 0	0 4 0	7.94084	9.99998	7.94086	12.05914	10.00002	12.05916	30
31	55 52	4 8	95508	99998	95510	04490	00002	04492	29
32	55 44	4 16	96887	99998	96889	03111	00002	03113	28
33	55 36	4 24	98223	99998	98225	01775	00002	01777	27
34	55 28	4 32	99520	99998	99522	00478	00002	00480	26
35	11 55 20	0 4 40	8.00779	9.99998	8.00781	11.99219	10.00002	11.99221	25
36	55 12	4 48	02002	99998	02004	97996	00002	97998	24
37	55 4	4 56	03192	99997	03194	96806	00003	96808	23
38	54 56	5 4	04350	99997	04353	95647	00003	95650	22
39	54 48	5 12	05478	99997	05481	94519	00003	94522	21
40	11 54 40	0 5 20	8.06578	9.99997	8.06581	11.93419	10.00003	11.93422	20
41	54 32	5 28	07650	99997	07653	92347	00003	92350	19
42	54 24	5 36	08696	99997	08700	91300	00003	91304	18
43	54 16	5 44	09718	99997	09722	90278	00003	90282	17
44	54 8	5 52	10717	99996	10720	89280	00004	89283	16
45	11 54 0	0 6 0	8.11693	9.99996	8.11696	11.88304	10.00004	11.88307	15
46	53 52	6 8	12647	99996	12651	87349	00004	87353	14
47	53 44	6 16	13581	99996	13585	86415	00004	86419	13
48	53 36	6 24	14495	99996	14500	85500	00004	85505	12
49	53 28	6 32	15391	99996	15395	84605	00004	84609	11
50	11 53 20	0 6 40	8.16268	9.99995	8.16273	11.83727	10.00005	11.83732	10
51	53 12	6 48	17128	99995	17133	82867	00005	82872	9
52	53 4	6 56	17971	99995	17976	82024	00005	82029	8
53	52 56	7 4	18798	99995	18804	81196	00005	81202	7
54	52 48	7 12	19610	99995	19616	80384	00005	80390	6
55	11 52 40	0 7 20	8.20407	9.99994	8.20413	11.79587	10.00006	11.79593	5
56	52 32	7 28	21189	99994	21195	78805	00006	78811	4
57	52 24	7 36	21958	99994	21964	78036	00006	78042	3
58	52 16	7 44	22713	99994	22720	77280	00006	77287	2
59	52 8	7 52	23456	99994	23462	76538	00006	76544	1
60	52 0	8 0	24186	99993	24192	75808	00007	75814	0
M.	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M.

90 Degs. Degs. 89.

Log. Sines, Tangents and Secants.

1 Deg. Degs. 178.

M.	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M.
0	11 52 0	0 8 0	8.24186	9.99993	8.24192	11.75808	10.00007	11.75814	60
1	51 52	8 8	24903	99993	24910	75090	00007	75097	59
2	51 44	8 16	25609	99993	25616	74384	00007	74391	58
3	51 36	8 24	26304	99993	26312	73688	00007	73696	57
4	51 28	8 32	26988	99992	26996	73004	00008	73012	56
5	11 51 20	0 8 40	8.27661	9.99992	8.27669	11.72331	10.00008	11.72339	55
6	51 12	8 48	28324	99992	28332	71668	00008	71676	54
7	51 4	8 56	28977	99992	28986	71014	00008	71023	53
8	50 56	9 4	29621	99992	29629	70371	00008	70379	52
9	50 48	9 12	30255	99991	30263	69737	00009	69745	51
10	11 50 40	0 9 20	8.30879	9.99991	8.30888	11.69112	10.00009	11.69121	50
11	50 32	9 28	31495	99991	31505	68495	00009	68505	49
12	50 24	9 36	32103	99990	32112	67888	00010	67897	48
13	50 16	9 44	32702	99990	32711	67289	00010	67298	47
14	50 8	9 52	33292	99990	33302	66698	00010	66708	46
15	11 50 0	0 10 0	8.33875	9.99990	8.33886	11.66114	10.00010	11.66125	45
16	49 52	10 8	34450	99989	34461	65539	00011	65550	44
17	49 44	10 16	35018	99989	35029	64971	00011	64982	43
18	49 36	10 24	35578	99989	35590	64410	00011	64422	42
19	49 28	10 32	36131	99989	36143	63857	00011	63369	41
20	11 49 20	0 10 40	8.36678	9.99988	8.36689	11.63311	10.00012	11.63322	40
21	49 12	10 48	37217	99988	37229	62771	00012	62783	39
22	49 4	10 56	37750	99988	37762	62238	00012	62250	38
23	48 56	11 4	38276	99987	38289	61711	00013	61724	37
24	48 48	11 12	38796	99987	38809	61191	00013	61204	36
25	11 48 40	0 11 20	8.39310	9.99987	8.39323	11.60677	10.00013	11.60690	35
26	48 32	11 28	39818	99986	39832	60168	00014	60182	34
27	48 24	11 36	40320	99986	40334	59666	00014	59680	33
28	48 16	11 44	40816	99986	40830	59170	00014	59184	32
29	48 8	11 52	41307	99985	41321	58679	00015	58693	31
30	11 48 0	0 12 0	8.41792	9.99935	8.41807	11.58193	10.00015	11.58208	30
31	47 52	12 8	42272	99985	42287	57713	00015	57728	29
32	47 44	12 16	42746	99984	42762	57238	00016	57254	28
33	47 36	12 24	43216	99984	43232	56768	00016	56784	27
34	47 28	12 32	43680	99984	43696	56304	00016	56320	26
35	11 47 20	0 12 40	8.44139	9.99983	8.44156	11.55844	10.00017	11.55861	25
36	47 12	12 48	44594	99983	44611	55389	00017	55406	24
37	47 4	12 56	45044	99983	45061	54939	00017	54956	23
38	46 56	13 4	45489	99982	45507	54493	00018	54511	22
39	46 48	13 12	45930	99982	45948	54052	00018	54070	21
40	11 46 40	0 13 20	8.46366	9.99982	8.46385	11.53615	10.00018	11.53634	20
41	46 32	13 28	46799	99981	46817	53183	00019	53201	19
42	46 24	13 36	47226	99981	47245	52755	00019	52774	18
43	46 16	13 44	47650	99981	47669	52331	00019	52350	17
44	46 8	13 52	48069	99980	48089	51911	00020	51931	16
45	11 46 0	0 14 0	8.48485	9.99980	8.48505	11.51495	10.00020	11.51515	15
46	45 52	14 8	48896	99979	48917	51083	00021	51104	14
47	45 44	14 16	49304	99979	49325	50675	00021	50696	13
48	45 36	14 24	49708	99979	49729	50271	00021	50292	12
49	45 28	14 32	50108	99978	50130	49870	00022	49892	11
50	11 45 20	0 14 40	8.50504	9.99978	8.50527	11.49473	10.00022	11.49496	10
51	45 12	14 48	50897	99977	50920	49080	00023	49103	9
52	45 4	14 56	51287	99977	51310	48690	00023	48713	8
53	44 56	15 4	51673	99977	51696	48304	00023	48327	7
54	44 48	15 12	52055	99976	52079	47921	00024	47945	6
55	11 44 40	0 15 20	8.52434	9.99976	8.52459	11.47541	10.00024	11.47566	5
56	44 32	15 28	52810	99975	52835	47165	00025	47190	4
57	44 24	15 36	53183	99975	53208	46792	00025	46817	3
58	44 16	15 44	53552	99974	53578	46422	00026	46448	2
59	44 8	15 52	53919	99974	53945	46055	00026	46081	1
60	44 0	16 0	54282	99974	54308	45692	00026	45718	0
M.	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant	M.

91 Degs. Degs. 88.

M.	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M.
0	11 44 0	0 16 0	8.54282	9.99974	8.54308	11.45692	10.00026	11.45713	60
1	43 52	16 8	54642	99973	54669	45331	00027	45358	59
2	43 44	16 16	54999	99973	55027	44973	00027	45001	58
3	43 36	16 24	55354	99972	55382	44618	00028	44646	57
4	43 28	16 32	55705	99972	55734	44266	00028	44295	56
5	11 43 20	0 16 40	8.56054	9.99971	8.56083	11.43917	10.00029	11.43946	55
6	43 12	16 48	56400	99971	56429	43571	00029	43600	54
7	43 4	16 56	56743	99970	56773	43227	00030	43257	53
8	42 56	17 4	57084	99970	57114	42886	00030	42916	52
9	42 48	17 12	57421	99969	57452	42548	00031	42579	51
10	11 42 40	0 17 20	8.57757	9.99969	8.57788	11.42212	10.00031	11.42243	50
11	42 32	17 28	58089	99968	58121	41879	00032	41911	49
12	42 24	17 36	58419	99968	58451	41549	00032	41581	48
13	42 16	17 44	58747	99967	58779	41221	00033	41253	47
14	42 8	17 52	59072	99967	59105	40895	00033	40928	46
15	11 42 0	0 18 0	8.59395	9.99967	8.59428	11.40572	10.00033	11.40605	45
16	41 52	18 8	59715	99966	59749	40251	00034	40285	44
17	41 44	18 16	60033	99966	60068	39932	00034	39967	43
18	41 36	18 24	60349	99965	60384	39616	00035	39651	42
19	41 28	18 32	60662	99964	60698	39302	00036	39338	41
20	11 41 20	0 18 40	8.60973	9.99964	8.61009	11.38991	10.00036	11.39027	40
21	41 12	18 48	61282	99963	61319	38681	00037	38718	39
22	41 4	18 56	61589	99963	61626	38374	00037	38411	38
23	40 56	19 4	61894	99962	61931	38069	00038	38106	37
24	40 48	19 12	62196	99962	62234	37766	00038	37804	36
25	11 40 40	0 19 20	8.62497	9.99961	8.62535	11.37465	10.00039	11.37503	35
26	40 32	19 28	62795	99961	62834	37166	00039	37205	34
27	40 24	19 36	63091	99960	63131	36869	00040	36909	33
28	40 16	19 44	63385	99960	63426	36574	00040	36615	32
29	40 8	19 52	63678	99959	63718	36282	00041	36322	31
30	11 40 0	0 20 0	8.63968	9.99959	8.64009	11.35991	10.00041	11.36032	30
31	39 52	20 8	64256	99958	64298	35702	00042	35744	29
32	39 44	20 16	64543	99958	64585	35415	00042	35457	28
33	39 36	20 24	64827	99957	64870	35130	00043	35173	27
34	39 28	20 32	65110	99956	65154	34846	00044	34890	26
35	11 39 20	0 20 40	8.65391	9.99956	8.65435	11.34565	10.00044	11.34609	25
36	39 12	20 48	65670	99955	65715	34285	00045	34330	24
37	39 4	20 56	65947	99955	65993	34007	00045	34053	23
38	38 56	21 4	66223	99954	66269	33731	00046	33777	22
39	38 48	21 12	66497	99954	66543	33457	00046	33503	21
40	11 38 40	0 21 20	8.66769	9.99953	8.66816	11.33184	10.00047	11.33231	20
41	38 32	21 28	67039	99952	67087	32913	00048	32961	19
42	38 24	21 36	67308	99952	67356	32644	00048	32692	18
43	38 16	21 44	67575	99951	67624	32376	00049	32425	17
44	38 8	21 52	67841	99951	67890	32110	00049	32159	16
45	11 38 0	0 22 0	8.68104	9.99950	8.68154	11.31846	10.00050	11.31896	15
46	37 52	22 8	68367	99949	68417	31583	00051	31633	14
47	37 44	22 16	68627	99949	68678	31322	00051	31373	13
48	37 36	22 24	68886	99948	68938	31062	00052	31114	12
49	37 28	22 32	69144	99948	69196	30804	00052	30856	11
50	11 37 20	0 22 40	8.69400	9.99947	8.69453	11.30547	10.00053	11.30600	10
51	37 12	22 48	69654	99946	69708	30292	00054	30346	9
52	37 4	22 56	69907	99946	69962	30038	00054	30093	8
53	36 56	23 4	70159	99945	70214	29786	00055	29841	7
54	36 48	23 12	70409	99944	70465	29535	00056	29591	6
55	11 36 40	0 23 20	8.70658	9.99944	8.70714	11.29286	10.00056	11.29342	5
56	36 32	23 28	70905	99943	70962	29038	00057	29095	4
57	36 24	23 36	71151	99942	71208	28792	00058	28849	3
58	36 16	23 44	71395	99942	71453	28547	00058	28605	2
59	36 8	23 52	71638	99941	71697	28303	00059	28362	1
60	36 0	24 0	71880	99940	71940	28060	00060	28120	0
M.	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M.

3 Degs. Degs. 176.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	11 36 0	0 24 0	8.71880	9.99940	8.71940	11.28060	10.00060	11.28120	60
1	35 52	24 8	72120	99940	72181	27819	00060	27880	59
2	35 44	24 16	72359	99939	72420	27580	00061	27641	58
3	35 36	24 24	72597	99938	72659	27341	00062	27403	57
4	35 28	24 32	72834	99938	72896	27104	00062	27166	56
5	11 35 20	0 24 40	8.73069	9.99937	8.73132	11.26868	10.00063	11.26931	55
6	35 12	24 48	73303	99936	73366	26634	00064	26697	54
7	35 4	24 56	73535	99936	73600	26400	00064	26465	53
8	34 56	25 4	73767	99935	73832	26168	00065	26233	52
9	34 48	25 12	73997	99934	74063	25937	00066	26003	51
10	11 34 40	0 25 20	8.74226	9.99934	8.74292	11.25708	10.00066	11.25774	50
11	34 32	25 28	74454	99933	74521	25479	00067	25546	49
12	34 24	25 36	74680	99932	74748	25252	00068	25320	48
13	34 16	25 44	74906	99932	74974	25026	00068	25094	47
14	34 8	25 52	75130	99931	75199	24801	00069	24870	46
15	11 34 0	0 26 0	8.75353	9.99930	8.75423	11.24577	10.00070	11.24647	45
16	33 52	26 8	75575	99929	75645	24355	00071	24425	44
17	33 44	26 16	75795	99929	75867	24133	00071	24205	43
18	33 36	26 24	76015	99928	76087	23913	00072	23985	42
19	33 28	26 32	76234	99927	76306	23694	00073	23766	41
20	11 33 20	0 26 40	8.76451	9.99926	8.76525	11.23475	10.00074	11.23549	40
21	33 12	26 48	76667	99926	76742	23258	00074	23333	39
22	33 4	26 56	76883	99925	76958	23042	00075	23117	38
23	32 56	27 4	77097	99924	77173	22827	00076	22903	37
24	32 48	27 12	77310	99923	77387	22613	00077	22690	36
25	11 32 40	0 27 20	8.77522	9.99923	8.77600	11.22400	10.00077	11.22478	35
26	32 32	27 28	77733	99922	77811	22189	00078	22267	34
27	32 24	27 36	77943	99921	78022	21978	00079	22057	33
28	32 16	27 44	78152	99920	78232	21768	00080	21848	32
29	32 8	27 52	78360	99920	78441	21559	00080	21640	31
30	11 32 0	0 28 0	8.78568	9.99919	8.78649	11.21351	10.00081	11.21432	30
31	31 52	28 8	78774	99918	78855	21145	00082	21226	29
32	31 44	28 16	78979	99917	79061	20939	00083	21021	28
33	31 36	28 24	79183	99917	79266	20734	00083	20817	27
34	31 28	28 32	79386	99916	79470	20530	00084	20614	26
35	11 31 20	0 28 40	8.79588	9.99915	8.79673	11.20327	10.00085	11.20412	25
36	31 12	28 48	79789	99914	79875	20125	00086	20211	24
37	31 4	28 56	79990	99913	80076	19924	00087	20010	23
38	30 56	29 4	80189	99913	80277	19723	00087	19811	22
39	30 48	29 12	80388	99912	80476	19524	00088	19612	21
40	11 30 40	0 29 20	8.80585	9.99911	8.80674	11.19326	10.00089	11.19415	20
41	30 32	29 28	80782	99910	80872	19128	00090	19218	19
42	30 24	29 36	80978	99909	81068	18932	00091	19022	18
43	30 16	29 44	81173	99909	81264	18736	00091	18827	17
44	30 8	29 52	81367	99908	81459	18541	00092	18633	16
45	11 30 0	0 30 0	8.81560	9.99907	8.81653	11.18347	10.00093	11.18440	15
46	29 52	30 8	81752	99906	81846	18154	00094	18248	14
47	29 44	30 16	81944	99905	82038	17962	00095	18056	13
48	29 36	30 24	82134	99904	82230	17770	00096	17866	12
49	29 28	30 32	82324	99904	82420	17580	00096	17676	11
50	11 29 20	0 30 40	8.82513	9.99903	8.82610	11.17390	10.00097	11.17487	10
51	29 12	30 48	82701	99902	82799	17201	00098	17299	9
52	29 4	30 56	82888	99901	82987	17013	00099	17112	8
53	28 56	31 4	83075	99900	83175	16825	00100	16925	7
54	28 48	31 12	83261	99899	83361	16639	00101	16739	6
55	11 28 40	0 31 20	8.83446	9.99898	8.83547	11.16453	10.00102	11.16554	5
56	28 32	31 28	83630	99898	83732	16268	00102	16370	4
57	28 24	31 36	83813	99897	83916	16084	00103	16187	3
58	28 16	31 44	83996	99896	84100	15900	00104	16004	2
59	28 8	31 52	84177	99895	84282	15718	00105	15823	1
60	28 0	32 0	84358	99894	84464	15536	00106	15642	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant.	Secant.	M

93 Degs. Degs. 86

Log. Sines, Tangents and Secants.

4 Degs. Degs. 17

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant.	M
0	11 28 0	0 32 0	8.84358	9.99894	8.84464	11.15536	10.00106	11.15642	60
1	27 52	32 8	84539	99893	84646	15354	00107	15461	59
2	27 44	32 16	84718	99892	85826	15174	00108	15282	58
3	27 36	32 24	84897	99891	85006	14994	00109	15103	57
4	27 28	32 32	85075	99891	85185	14815	00109	14925	56
5	11 27 20	0 32 40	8.85252	9.99890	8.85363	11.14637	10.00110	11.14748	55
6	27 12	32 48	85429	99889	85540	14460	00111	14571	54
7	27 4	32 56	85605	99888	85717	14283	00112	14395	53
8	26 56	33 4	85780	99887	85893	14107	00113	14220	52
9	26 48	33 12	85955	99886	86069	13931	00114	14045	51
10	11 26 40	0 33 20	8.86128	9.99885	8.86243	11.13757	10.00115	11.13872	50
11	26 32	33 28	86301	99884	86417	13583	00116	13699	49
12	26 24	33 36	86474	99883	86591	13409	00117	13526	48
13	26 16	33 44	86645	99882	86763	13237	00118	13355	47
14	26 8	33 52	86816	99881	86935	13065	00119	13184	46
15	11 26 0	0 34 0	8.86987	9.99880	8.87106	11.12894	10.00120	11.13013	45
16	25 52	34 8	87156	99879	87277	12723	00121	12844	44
17	25 44	34 16	87325	99879	87447	12553	00121	12675	43
18	25 36	34 24	87494	99878	87616	12384	00122	12506	42
19	25 28	34 32	87661	99877	87785	12215	00123	12339	41
20	11 25 20	0 34 40	8.87829	9.99876	8.87953	11.12047	10.00124	11.12171	40
21	25 12	34 48	87995	99875	88120	11880	00125	12005	39
22	25 4	34 56	88161	99874	88287	11713	00126	11839	38
23	24 56	35 4	88326	99873	88453	11547	00127	11674	37
24	24 48	35 12	88490	99872	88618	11382	00128	11510	36
25	11 24 40	0 35 20	8.88654	9.99871	8.88783	11.11217	10.00129	11.11346	35
26	24 32	35 28	88817	99870	88948	11052	00130	11183	34
27	24 24	35 36	88980	99869	89111	10889	00131	11020	33
28	24 16	35 44	89142	99868	89274	10726	00132	10858	32
29	24 8	35 52	89304	99867	89437	10563	00133	10696	31
30	11 24 0	0 36 0	8.89464	9.99866	8.89598	11.10402	10.00134	11.10536	30
31	23 52	36 8	89625	99865	89760	10240	00135	10375	29
32	23 44	36 16	89784	99864	89920	10080	00136	10216	28
33	23 36	36 24	89943	99863	90080	09920	00137	10057	27
34	23 28	36 32	90102	99862	90240	09760	00138	09898	26
35	11 23 20	0 36 40	8.90260	9.99861	8.90399	11.09601	10.00139	11.09740	25
36	23 12	36 48	90417	99860	90557	09443	00140	09583	24
37	23 4	36 56	90574	99859	90715	09285	00141	09426	23
38	22 56	37 4	90730	99858	90872	09128	00142	09270	22
39	22 48	37 12	90885	99857	91029	08971	00143	09115	21
40	11 22 40	0 37 20	8.91040	9.99856	8.91185	11.08815	10.00144	11.08960	20
41	22 32	37 28	91195	99855	91340	08660	00145	08805	19
42	22 24	37 36	91349	99854	91495	08505	00146	08651	18
43	22 16	37 44	91502	99853	91650	08350	00147	08498	17
44	22 8	37 52	91655	99852	91803	08197	00148	08345	16
45	11 22 0	0 38 0	8.91807	9.99851	8.91957	11.08043	10.00149	11.08193	15
46	21 52	38 8	91959	99850	92110	07890	00150	08041	14
47	21 44	38 16	92110	99848	92262	07738	00152	07890	13
48	21 36	38 24	92261	99847	92414	07586	00153	07739	12
49	21 28	38 32	92411	99846	92565	07435	00154	07589	11
50	11 21 20	0 38 40	8.92561	9.99845	8.92716	11.07284	10.00155	11.07439	10
51	21 12	38 48	92710	99844	92866	07134	00156	07290	9
52	21 4	38 56	92859	99843	93016	06984	00157	07141	8
53	20 56	39 4	93007	99842	93165	06835	00158	06993	7
54	20 48	39 12	93154	99841	93313	06687	00159	06846	6
55	11 20 40	0 39 20	8.93301	9.99840	8.93462	11.06538	10.00160	11.06699	5
56	20 32	39 28	93448	99839	93609	06391	00161	06552	4
57	20 24	39 36	93594	99838	93756	06244	00162	06406	3
58	20 16	39 44	93740	99837	93903	06097	00163	06260	2
59	20 8	39 52	93885	99836	94049	05951	00164	06115	1
60	20 0	40 0	94030	99834	94195	05805	00166	05970	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant.	Secant.	M

94 Degs. Degs.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant.	M
0	11 20 0	0 40 0	8.94030	9.99834	8.94195	11.05805	10.00166	11.05970	60
1	19 52	40 8	94174	99833	94340	05660	00167	05826	59
2	19 44	40 16	94317	99832	94485	05515	00168	05683	58
3	19 36	40 24	94461	99831	94630	05370	00169	05539	57
4	19 28	40 32	94603	99830	94773	05227	00170	05397	56
5	11 19 20	0 40 40	8.94746	9.99829	8.94917	11.05083	10.00171	11.05254	55
6	19 12	40 48	94887	99828	95060	04940	00172	05113	54
7	19 4	40 56	95029	99827	95202	04798	00173	04971	53
8	18 56	41 4	95170	99825	95344	04656	00175	04830	52
9	18 48	41 12	95310	99824	95486	04514	00176	04690	51
10	11 18 40	0 41 20	8.95450	9.99823	8.95627	11.04373	10.00177	11.04550	50
11	18 32	41 28	95589	99822	95767	04233	00178	04411	49
12	18 24	41 36	95728	99821	95908	04092	00179	04272	48
13	18 16	41 44	95867	99820	96047	03953	00180	04133	47
14	18 8	41 52	96005	99819	96187	03813	00181	03995	46
15	11 18 0	0 42 0	8.96143	9.99817	8.96325	11.03675	10.00183	11.03857	45
16	17 52	42 8	96280	99816	96464	03536	00184	03720	44
17	17 44	42 16	96417	99815	96602	03398	00185	03583	43
18	17 36	42 24	96553	99814	96739	03261	00186	03447	42
19	17 28	42 32	96689	99813	96877	03123	00187	03311	41
20	11 17 20	0 42 40	8.96825	9.99812	8.97013	11.02987	10.00188	11.03175	40
21	17 12	42 48	96960	99810	97150	02850	00190	03040	39
22	17 4	42 56	97095	99809	97285	02715	00191	02905	38
23	16 56	43 4	97229	99808	97421	02579	00192	02771	37
24	16 48	43 12	97363	99807	97556	02444	00193	02637	36
25	11 16 40	0 43 20	8.97496	9.99806	8.97691	11.02309	10.00194	11.02504	35
26	16 32	43 28	97629	99804	97825	02175	00196	02371	34
27	16 24	43 36	97762	99803	97959	02041	00197	02238	33
28	16 16	43 44	97894	99802	98092	01908	00198	02106	32
29	16 8	43 52	98026	99801	98225	01775	00199	01974	31
30	11 16 0	0 44 0	8.98157	9.99800	8.98358	11.01642	10.00200	11.01843	30
31	15 52	44 8	98288	99798	98490	01510	00202	01712	29
32	15 44	44 16	98419	99797	98622	01378	00203	01581	28
33	15 36	44 24	98549	99796	98753	01247	00204	01451	27
34	15 28	44 32	98679	99795	98884	01116	00205	01321	26
35	11 15 20	0 44 40	8.98808	9.99793	8.99015	11.00985	10.00207	11.01192	25
36	15 12	44 48	98937	99792	99145	00855	00208	01063	24
37	15 4	44 56	99066	99791	99275	00725	00209	00934	23
38	14 56	45 4	99194	99790	99405	00595	00210	00806	22
39	14 48	45 12	99322	99788	99534	00466	00212	00678	21
40	11 14 40	0 45 20	8.99450	9.99787	8.99662	11.00338	10.00213	11.00550	20
41	14 32	45 28	99577	99786	99791	00209	00214	00423	19
42	14 24	45 36	99704	99785	99919	00081	00215	00296	18
43	14 16	45 44	99830	99783	9.00046	10.99954	00217	00170	17
44	14 8	45 52	99956	99782	00174	99826	00218	00044	16
45	11 14 0	0 46 0	9.00082	9.99781	9.00301	10.99699	10.00219	10.99918	15
46	13 52	46 8	00207	99780	00427	99573	00220	99793	14
47	13 44	46 16	00332	99778	00553	99447	00222	99668	13
48	13 36	46 24	00456	99777	00679	99321	00223	99544	12
49	13 28	46 32	00581	99776	00805	99195	00224	99419	11
50	11 13 20	0 46 40	9.00704	9.99775	9.00930	10.99070	10.00225	10.99296	10
51	13 12	46 48	00828	99773	01055	98945	00227	99172	9
52	13 4	46 56	00951	99772	01179	98821	00228	99049	8
53	12 56	47 4	01074	99771	01303	98697	00229	98926	7
54	12 48	47 12	01196	99769	01427	98573	00231	98804	6
55	11 12 40	0 47 20	9.01318	9.99768	9.01550	10.98450	10.00232	10.98682	5
56	12 32	47 28	01440	99767	01673	98327	00233	98560	4
57	12 24	47 36	01561	99765	01796	98204	00235	98439	3
58	12 16	47 44	01682	99764	01918	98082	00236	98318	2
59	12 8	47 52	01803	99763	02040	97960	00237	98197	1
60	12 0	48 0	01923	99761	02162	97838	00239	98077	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

6 Degs. | Degs. 173.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant.	M
0	11 12 0	0 48 0	9.01923	9.99761	9.02162	10.97838	10.00239	10.98077	60
1	11 52	48 8	02043	99760	02283	97717	00240	97957	59
2	11 44	48 16	02163	99759	02404	97596	00241	97837	58
3	11 36	48 24	02283	99757	02525	97475	00243	97717	57
4	11 28	48 32	02402	99756	02645	97355	00244	97598	56
5	11 11 20	0 48 40	9.02520	9.99755	9.02766	10.97234	10.00245	10.97480	55
6	11 12	48 48	02639	99753	02885	97115	00247	97361	54
7	11 4	48 56	02757	99752	03005	96995	00248	97243	53
8	10 56	49 4	02874	99751	03124	96876	00249	97126	52
9	10 48	49 12	02992	99749	03242	96758	00251	97008	51
10	11 10 40	0 49 20	9.03109	9.99748	9.03361	10.96639	10.00252	10.96891	50
11	10 32	49 28	03226	99747	03479	96521	00253	96774	49
12	10 24	49 36	03342	99745	03597	96403	00255	96658	48
13	10 16	49 44	03458	99744	03714	96286	00256	96542	47
14	10 8	49 52	03574	99742	03832	96168	00258	96426	46
15	11 10 0	0 50 0	9.03690	9.99741	9.03948	10.96052	10.00259	10.96310	45
16	9 52	50 8	03805	99740	04065	95935	00260	96195	44
17	9 44	50 16	03920	99738	04181	95819	00262	96080	43
18	9 36	50 24	04034	99737	04297	95703	00263	95966	42
19	9 28	50 32	04149	99736	04413	95587	00264	95851	41
20	11 9 20	0 50 40	9.04262	9.99734	9.04528	10.95472	10.00266	10.95738	40
21	9 12	50 48	04376	99733	04643	95357	00267	95624	39
22	9 4	50 56	04490	99731	04758	95242	00269	95510	38
23	8 56	51 4	04603	99730	04873	95127	00270	95397	37
24	8 48	51 12	04715	99728	04987	95013	00272	95285	36
25	11 8 40	0 51 20	9.04828	9.99727	9.05101	10.94899	10.00273	10.95172	35
26	8 32	51 28	04940	99726	05214	94786	00274	95060	34
27	8 24	51 36	05052	99724	05328	94672	00276	94948	33
28	8 16	51 44	05164	99723	05441	94559	00277	94836	32
29	8 8	51 52	05275	99721	05553	94447	00279	94725	31
30	11 8 0	0 52 0	9.05386	9.99720	9.05666	10.94334	10.00280	10.94614	30
31	7 52	52 8	05497	99718	05778	94222	00282	94503	29
32	7 44	52 16	05607	99717	05890	94110	00283	94393	28
33	7 36	52 24	05717	99716	06002	93998	00284	94283	27
34	7 28	52 32	05827	99714	06113	93887	00286	94173	26
35	11 7 20	0 52 40	9.05937	9.99713	9.06224	10.93776	10.00287	10.94063	25
36	7 12	52 48	06046	99711	06335	93665	00289	93954	24
37	7 4	52 56	06155	99710	06445	93555	00290	93845	23
38	6 56	53 4	06264	99708	06556	93444	00292	93736	22
39	6 48	53 12	06372	99707	06666	93334	00293	93628	21
40	11 6 40	0 53 20	9.06481	9.99705	9.06775	10.93225	10.00295	10.93519	20
41	6 32	53 28	06589	99704	06885	93115	00296	93411	19
42	6 24	53 36	06696	99702	06994	93006	00298	93304	18
43	6 16	53 44	06804	99701	07103	92897	00299	93196	17
44	6 8	53 52	06911	99699	07211	92789	00301	93089	16
45	11 6 0	0 54 0	9.07018	9.99698	9.07320	10.92680	10.00302	10.92982	15
46	5 52	54 8	07124	99696	07428	92572	00304	92876	14
47	5 44	54 16	07231	99695	07536	92464	00305	92769	13
48	5 36	54 24	07337	99693	07643	92357	00307	92663	12
49	5 28	54 32	07442	99692	07751	92249	00308	92558	11
50	11 5 20	0 54 40	9.07548	9.99690	9.07858	10.92142	10.00310	10.92452	10
51	5 12	54 48	07653	99689	07964	92036	00311	92347	9
52	5 4	54 56	07758	99687	08071	91929	00313	92242	8
53	4 56	55 4	07863	99686	08177	91823	00314	92137	7
54	4 48	55 12	07968	99684	08283	91717	00316	92032	6
55	11 4 40	0 55 20	9.08072	9.99683	9.08389	10.91611	10.00317	10.91928	5
56	4 32	55 28	08176	99681	08495	91505	00319	91824	4
57	4 24	55 36	08280	99680	08600	91400	00320	91720	3
58	4 16	55 44	08383	99678	08705	91295	00322	91617	2
59	4 8	55 52	08486	99677	08810	91190	00323	91514	1
60	4 0	56 0	08589	99675	08914	91086	00325	91411	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant.	Secant.	M

96 Degs. | Degs. 83.

7 Degs. Degs. 172.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	11 4 0	0 56 0	9.08589	9.99675	9.08914	10.91086	10.00325	10.91411	60
1	3 52	56 8	08692	99674	09019	90981	00326	91308	59
2	3 44	56 16	08795	99672	09123	90877	00328	91205	58
3	3 36	56 24	08897	99670	09227	90773	00330	91103	57
4	3 28	56 32	08999	99669	09330	90670	00331	91001	56
5	11 3 20	0 56 40	9.09101	9.99667	9.09434	10.90566	10.00333	10.90899	55
6	3 12	56 48	09202	99666	09537	90463	00334	90798	54
7	3 4	56 56	09304	99664	09640	90360	00336	90696	53
8	2 56	57 4	09405	99663	09742	90258	00337	90595	52
9	2 48	57 12	09506	99661	09845	90155	00339	90494	51
10	11 2 40	0 57 20	9.09606	9.99659	9.09947	10.90053	10.00341	10.90394	50
11	2 32	57 28	09707	99658	10049	89951	00342	90293	49
12	2 24	57 36	09807	99656	10150	89850	00344	90193	48
13	2 16	57 44	09907	99655	10252	89748	00345	90093	47
14	2 8	57 52	10006	99653	10353	89647	00347	89994	46
15	11 2 0	0 58 0	9.10106	9.99651	9.10454	10.89546	10.00349	10.89894	45
16	1 52	58 8	10205	99650	10555	89445	00350	89795	44
17	1 44	58 16	10304	99648	10656	89344	00352	89696	43
18	1 36	58 24	10402	99647	10756	89244	00353	89598	42
19	1 28	58 32	10501	99645	10856	89144	00355	89499	41
20	11 1 20	0 58 40	9.10599	9.99643	9.10956	10.89044	10.00357	10.89401	40
21	1 12	58 48	10697	99642	11056	88944	00358	89303	39
22	1 4	58 56	10795	99640	11155	88845	00360	89205	38
23	0 56	59 4	10893	99638	11254	88746	00362	89107	37
24	0 48	59 12	10990	99637	11353	88647	00363	89010	36
25	11 0 40	0 59 20	9.11087	9.99635	9.11452	10.88548	10.00365	10.88913	35
26	0 32	59 28	11184	99633	11551	88449	00367	88816	34
27	0 24	59 36	11281	99632	11649	88351	00368	88719	33
28	0 16	59 44	11377	99630	11747	88253	00370	88623	32
29	0 8	59 52	11474	99629	11845	88155	00371	88526	31
30	11 0 0	1 0 0	9.11570	9.99627	9.11943	10.88057	10.00373	10.88430	30
31	10 59 52	0 8	11666	99625	12040	87960	00375	88334	29
32	59 44	0 16	11761	99624	12138	87862	00376	88239	28
33	59 36	0 24	11857	99622	12235	87765	00378	88143	27
34	59 28	0 32	11952	99620	12332	87668	00380	88048	26
35	10 59 20	1 0 40	9.12047	9.99618	9.12428	10.87572	10.00382	10.87953	25
36	59 12	0 48	12142	99617	12525	87475	00383	87858	24
37	59 4	0 56	12236	99615	12621	87379	00385	87764	23
38	58 56	1 4	12331	99613	12717	87283	00387	87669	22
39	58 48	1 12	12425	99612	12813	87187	00388	87575	21
40	10 58 40	1 1 20	9.12519	9.99610	9.12909	10.87091	10.00390	10.87481	20
41	58 32	1 28	12612	99608	13004	86996	00392	87388	19
42	58 24	1 36	12706	99607	13099	86901	00393	87294	18
43	58 16	1 44	12799	99605	13194	86806	00395	87201	17
44	58 8	1 52	12892	99603	13289	86711	00397	87108	16
45	10 58 0	1 2 0	9.12985	9.99601	9.13384	10.86616	10.00399	10.87015	15
46	57 52	2 8	13078	99600	13478	86522	00400	86922	14
47	57 44	2 16	13171	99598	13573	86427	00402	86829	13
48	57 36	2 24	13263	99596	13667	86333	00404	86737	12
49	57 28	2 32	13355	99595	13761	86239	00405	86645	11
50	10 57 20	1 2 40	9.13447	9.99593	9.13854	10.86146	10.00407	10.86553	10
51	57 12	2 48	13539	99591	13948	86052	00409	86461	9
52	57 4	2 56	13630	99589	14041	85959	00411	86370	8
53	56 56	3 4	13722	99588	14134	85866	00412	86278	7
54	56 48	3 12	13813	99586	14227	85773	00414	86187	6
55	10 56 40	1 3 20	9.13904	9.99584	9.14320	10.85680	10.00416	10.86096	5
56	56 32	3 28	13994	99582	14412	85588	00418	86006	4
57	56 24	3 36	14085	99581	14504	85496	00419	85915	3
58	56 16	3 44	14175	99579	14597	85403	00421	85825	2
59	56 8	3 52	14266	99577	14688	85312	00423	85734	1
60	56 0	4 0	14356	99575	14780	85220	00425	85644	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

97 Degs. Degs. 82.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	10 56 0	1 4 0	9.14356	9.99575	9.14780	10.85220	10.00425	10.85644	60
1	55 52	4 8	14445	99574	14872	85128	00426	85555	59
2	55 44	4 16	14535	99572	14963	85037	00428	85465	58
3	55 36	4 24	14624	99570	15054	84946	00430	85376	57
4	55 28	4 32	14714	99568	15145	84855	00432	85286	56
5	10 55 20	1 4 40	9.14803	9.99566	9.15236	10.84764	10.00434	10.85197	55
6	55 12	4 48	14891	99565	15327	84673	00435	85109	54
7	55 4	4 56	14980	99563	15417	84583	00437	85020	53
8	54 56	5 4	15069	99561	15508	84492	00439	84931	52
9	54 48	5 12	15157	99559	15598	84402	00441	84843	51
10	10 54 40	1 5 20	9.15245	9.99557	9.15688	10.84312	10.00443	10.84755	50
11	54 32	5 28	15333	99556	15777	84223	00444	84667	49
12	54 24	5 36	15421	99554	15867	84133	00446	84579	48
13	54 16	5 44	15508	99552	15956	84044	00448	84492	47
14	54 8	5 52	15596	99550	16046	83954	00450	84404	46
15	10 54 0	1 6 0	9.15683	9.99548	9.16135	10.83865	10.00452	10.84317	45
16	53 52	6 8	15770	99546	16224	83776	00454	84230	44
17	53 44	6 16	15857	99545	16312	83688	00455	84143	43
18	53 36	6 24	15944	99543	16401	83599	00457	84056	42
19	53 28	6 32	16030	99541	16489	83511	00459	83970	41
20	10 53 20	1 6 40	9.16116	9.99539	9.16577	10.83423	10.00461	10.83884	40
21	53 12	6 48	16203	99537	16665	83335	00463	83797	39
22	53 4	6 56	16289	99535	16753	83247	00465	83711	38
23	52 56	7 4	16374	99533	16841	83159	00467	83626	37
24	52 48	7 12	16460	99532	16928	83072	00468	83540	36
25	10 52 40	1 7 20	9.16545	9.99530	9.17016	10.82984	10.00470	10.83455	35
26	52 32	7 23	16631	99528	17103	82897	00472	83369	34
27	52 24	7 36	16716	99526	17190	82810	00474	83284	33
28	52 16	7 44	16801	99524	17277	82723	00476	83199	32
29	52 8	7 52	16886	99522	17363	82637	00478	83114	31
30	10 52 0	1 8 0	9.16970	9.99520	9.17450	10.82550	10.00480	10.83030	30
31	51 52	8 8	17055	99518	17536	82464	00482	82945	29
32	51 44	8 16	17139	99517	17622	82378	00483	82861	28
33	51 36	8 24	17223	99515	17708	82292	00485	82777	27
34	51 28	8 32	17307	99513	17794	82206	00487	82693	26
35	10 51 20	1 8 40	9.17391	9.99511	9.17880	10.82120	10.00489	10.82609	25
36	51 12	8 48	17474	99509	17965	82035	00491	82526	24
37	51 4	8 56	17558	99507	18051	81949	00493	82442	23
38	50 56	9 4	17641	99505	18136	81864	00495	82359	22
39	50 48	9 12	17724	99503	18221	81779	00497	82276	21
40	10 50 40	1 9 20	9.17807	9.99501	9.18306	10.81694	10.00499	10.82193	20
41	50 32	9 28	17890	99499	18391	81609	00501	82110	19
42	50 24	9 36	17973	99497	18475	81525	00503	82027	18
43	50 16	9 44	18055	99495	18560	81440	00505	81945	17
44	50 8	9 52	18137	99494	18644	81356	00506	81863	16
45	10 50 0	1 10 0	9.18220	9.99492	9.18728	10.81272	10.00508	10.81780	15
46	49 52	10 8	18302	99490	18812	81188	00510	81698	14
47	49 44	10 16	18383	99488	18896	81104	00512	81617	13
48	49 36	10 24	18465	99486	18979	81021	00514	81535	12
49	49 28	10 32	18547	99484	19063	80937	00516	81453	11
50	10 49 20	1 10 40	9.18628	9.99482	9.19146	10.80854	10.00518	10.81372	10
51	49 12	10 48	18709	99480	19229	80771	00520	81291	9
52	49 4	10 56	18790	99478	19312	80688	00522	81210	8
53	48 56	11 4	18871	99476	19395	80605	00524	81129	7
54	48 48	11 12	18952	99474	19478	80522	00526	81048	6
55	10 48 40	1 11 20	9.19033	9.99472	9.19561	10.80439	10.00528	10.80967	5
56	48 32	11 28	19113	99470	19643	80357	00530	80887	4
57	48 24	11 36	19193	99468	19725	80275	00532	80807	3
58	48 16	11 44	19273	99466	19807	80193	00534	80727	2
59	48 8	11 52	19353	99464	19889	80111	00536	80647	1
60	48 0	12 0	19433	99462	19971	80029	00538	80567	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	10 48 0	1 12 0	9.19433	9.99462	9.19971	10.80029	10.00538	10.80567	60
1	47 52	12 8	19513	99460	20053	79947	00540	80487	59
2	47 44	12 16	19592	99458	20134	79866	00542	80408	58
3	47 36	12 24	19672	99456	20216	79784	00544	80328	57
4	47 28	12 32	19751	99454	20297	79703	00546	80249	56
5	10 47 20	1 12 40	9.19830	9.99452	9.20378	10.79622	10.00548	10.80170	55
6	47 12	12 48	19909	99450	20459	79541	00550	80091	54
7	47 4	12 56	19988	99448	20540	79460	00552	80012	53
8	46 56	13 4	20067	99446	20621	79379	00554	79933	52
9	46 48	13 12	20145	99444	20701	79299	00556	79855	51
10	10 46 40	1 13 20	9.20223	9.99442	9.20782	10.79218	10.00558	10.79777	50
11	46 32	13 28	20302	99440	20862	79138	00560	79698	49
12	46 24	13 36	20380	99438	20942	79058	00562	79620	48
13	46 16	13 44	20458	99436	21022	78978	00564	79542	47
14	46 8	13 52	20535	99434	21102	78898	00566	79465	46
15	10 46 0	1 14 0	9.20613	9.99432	9.21182	10.78818	10.00568	10.79387	45
16	45 52	14 8	20691	99429	21261	78739	00571	79309	44
17	45 44	14 16	20768	99427	21341	78659	00573	79232	43
18	45 36	14 24	20845	99425	21420	78580	00575	79155	42
19	45 28	14 32	20922	99423	21499	78501	00577	79078	41
20	10 45 20	1 14 40	9.20999	9.99421	9.21578	10.78422	10.00579	10.79001	40
21	45 12	14 48	21076	99419	21657	78343	00581	78924	39
22	45 4	14 56	21153	99417	21736	78264	00583	78847	38
23	44 56	15 4	21229	99415	21814	78186	00585	78771	37
24	44 48	15 12	21306	99413	21893	78107	00587	78694	36
25	10 44 40	1 15 20	9.21382	9.99411	9.21971	10.78029	10.00589	10.78618	35
26	44 32	15 28	21458	99409	22049	77951	00591	78542	34
27	44 24	15 36	21534	99407	22127	77873	00593	78466	33
28	44 16	15 44	21610	99404	22205	77795	00596	78390	32
29	44 8	15 52	21685	99402	22283	77717	00598	78315	31
30	10 44 0	1 16 0	9.21761	9.99400	9.22361	10.77639	10.00600	10.78239	30
31	43 52	16 8	21836	99398	22438	77562	00602	78164	29
32	43 44	16 16	21912	99396	22516	77484	00604	78088	28
33	43 36	16 24	21987	99394	22593	77407	00606	78013	27
34	43 28	16 32	22062	99392	22670	77330	00608	77938	26
35	10 43 20	1 16 40	9.22137	9.99390	9.22747	10.77253	10.00610	10.77863	25
36	43 12	16 48	22211	99388	22824	77176	00612	77789	24
37	43 4	16 56	22286	99385	22901	77099	00615	77714	23
38	42 56	17 4	22361	99383	22977	77023	00617	77639	22
39	42 48	17 12	22435	99381	23054	76946	00619	77565	21
40	10 42 40	1 17 20	9.22509	9.99379	9.23130	10.76870	10.00621	10.77491	20
41	42 32	17 28	22583	99377	23206	76794	00623	77417	19
42	42 24	17 36	22657	99375	23283	76717	00625	77343	18
43	42 16	17 44	22731	99372	23359	76641	00628	77269	17
44	42 8	17 52	22805	99370	23435	76565	00630	77195	16
45	10 42 0	1 18 0	9.22878	9.99368	9.23510	10.76490	10.00632	10.77122	15
46	41 52	18 8	22952	99366	23586	76414	00634	77048	14
47	41 44	18 16	23025	99364	23661	76339	00636	76975	13
48	41 36	18 24	23098	99362	23737	76263	00638	76902	12
49	41 28	18 32	23171	99359	23812	76188	00641	76829	11
50	10 41 20	1 18 40	9.23244	9.99357	9.23887	10.76113	10.00643	10.76756	10
51	41 12	18 48	23317	99355	23962	76038	00645	76683	9
52	41 4	18 56	23390	99353	24037	75963	00647	76610	8
53	40 56	19 4	23462	99351	24112	75888	00649	76538	7
54	40 48	19 12	23535	99348	24186	75814	00652	76465	6
55	10 40 40	1 19 20	9.23607	9.99346	9.24261	10.75739	10.00654	10.76393	5
56	40 32	19 28	23679	99344	24335	75665	00656	76321	4
57	40 24	19 36	23752	99342	24410	75590	00658	76248	3
58	40 16	19 44	23823	99340	24484	75516	00660	76177	2
59	40 8	19 52	23895	99337	24558	75442	00663	76105	1
60	40 0	20 0	23967	99335	24632	75368	00665	76033	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

10 Degs. | Degs. 169.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant.	M
0	10 40 0	1 20 0	9.23967	9.99335	9.24632	10.75368	10.00665	10.76033	60
1	39 52	20 8	24039	99333	24706	75294	00667	75961	59
2	39 44	20 16	24110	99331	24779	75221	00669	75890	58
3	39 36	20 24	24181	99328	24853	75147	00672	75819	57
4	39 28	20 32	24253	99326	24926	75074	00674	75747	56
5	10 39 20	1 20 40	9.24324	9.99324	9.25000	10.75000	10.00676	10.75676	55
6	39 12	20 48	24395	99322	25073	74927	00678	75605	54
7	39 4	20 56	24466	99319	25146	74854	00681	75534	53
8	38 56	21 4	24536	99317	25219	74781	00683	75464	52
9	38 48	21 12	24607	99315	25292	74708	00685	75393	51
10	10 38 40	1 21 20	9.24677	9.99313	9.25365	10.74635	10.00687	10.75323	50
11	38 32	21 28	24748	99310	25437	74563	00690	75252	49
12	38 24	21 36	24818	99308	25510	74490	00692	75182	48
13	38 16	21 44	24888	99306	25582	74418	00694	75112	47
14	38 8	21 52	24958	99304	25655	74345	00696	75042	46
15	10 38 0	1 22 0	9.25028	9.99301	9.25727	10.74273	10.00699	10.74972	45
16	37 52	22 8	25098	99299	25799	74201	00701	74902	44
17	37 44	22 16	25168	99297	25871	74129	00703	74832	43
18	37 36	22 24	25237	99294	25943	74057	00706	74763	42
19	37 28	22 32	25307	99292	26015	73985	00708	74693	41
20	10 37 20	1 22 40	9.25376	9.99290	9.26086	10.73914	10.00710	10.74624	40
21	37 12	22 48	25445	99288	26158	73842	00712	74555	39
22	37 4	22 56	25514	99285	26229	73771	00715	74486	38
23	36 56	23 4	25583	99283	26301	73699	00717	74417	37
24	36 48	23 12	25652	99281	26372	73628	00719	74348	36
25	10 36 40	1 23 20	9.25721	9.99278	9.26443	10.73557	10.00722	10.74279	35
26	36 32	23 28	25790	99276	26514	73486	00724	74210	34
27	36 24	23 36	25858	99274	26585	73415	00726	74142	33
28	36 16	23 44	25927	99271	26655	73345	00729	74073	32
29	36 8	23 52	25995	99269	26726	73274	00731	74005	31
30	10 36 0	1 24 0	9.26063	9.99267	9.26797	10.73203	10.00733	10.73937	30
31	35 52	24 8	26131	99264	26867	73133	00736	73869	29
32	35 44	24 16	26199	99262	26937	73063	00738	73801	28
33	35 36	24 24	26267	99260	27008	72992	00740	73733	27
34	35 28	24 32	26335	99257	27078	72922	00743	73665	26
35	10 35 20	1 24 40	9.26403	9.99255	9.27148	10.72852	10.00745	10.73597	25
36	35 12	24 48	26470	99252	27218	72782	00748	73530	24
37	35 4	24 56	26538	99250	27288	72712	00750	73462	23
38	34 56	25 4	26605	99248	27357	72643	00752	73395	22
39	34 48	25 12	26672	99245	27427	72573	00755	73328	21
40	10 34 40	1 25 20	9.26739	9.99243	9.27496	10.72504	10.00757	10.73261	20
41	34 32	25 28	26806	99241	27566	72434	00759	73194	19
42	34 24	25 36	26873	99238	27635	72365	00762	73127	18
43	34 16	25 44	26940	99236	27704	72296	00764	73060	17
44	34 8	25 52	27007	99233	27773	72227	00767	72993	16
45	10 34 0	1 26 0	9.27073	9.99231	9.27842	10.72158	10.00769	10.72927	15
46	33 52	26 8	27140	99229	27911	72089	00771	72860	14
47	33 44	26 16	27206	99226	27980	72020	00774	72794	13
48	33 36	26 24	27273	99224	28049	71951	00776	72727	12
49	33 28	26 32	27339	99221	28117	71883	00779	72661	11
50	10 33 20	1 26 40	9.27405	9.99219	9.28186	10.71814	10.00781	10.72595	10
51	33 12	26 48	27471	99217	28254	71746	00783	72529	9
52	33 4	26 56	27537	99214	28323	71677	00786	72463	8
53	32 56	27 4	27602	99212	28391	71609	00788	72398	7
54	32 48	27 12	27668	99209	28459	71541	00791	72332	6
55	10 32 40	1 27 20	9.27734	9.99207	9.28527	10.71473	10.00793	10.72266	5
56	32 32	27 28	27799	99204	28595	71405	00796	72201	4
57	32 24	27 36	27864	99202	28662	71338	00798	72136	3
58	32 16	27 44	27930	99200	28730	71270	00800	72070	2
59	32 8	27 52	27995	99197	28798	71202	00803	72005	1
60	32 0	28 0	28060	99195	28865	71135	00805	71940	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant.	Secant.	M

100 Degs. | Degs. 79.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	10 32 0	1 28 0	9.28060	9.99195	9.28865	10.71135	10.00805	10.71940	60
1	31 52	28 8	28125	99192	28933	71067	00808	71875	59
2	31 44	28 16	28190	99190	29000	71000	00810	71810	58
3	31 36	28 24	28254	99187	29067	70933	00813	71746	57
4	31 28	28 32	28319	99185	29134	70866	00815	71681	56
5	10 31 20	1 28 40	9.28384	9.99182	9.29201	10.70799	10.00818	10.71616	55
6	31 12	28 48	28448	99180	29268	70732	00820	71552	54
7	31 4	28 56	28512	99177	29335	70665	00823	71488	53
8	30 56	29 4	28577	99175	29402	70598	00825	71423	52
9	30 48	29 12	28641	99172	29468	70532	00828	71359	51
10	10 30 40	1 29 20	9.28705	9.99170	9.29535	10.70465	10.00830	10.71295	50
11	30 32	29 28	28769	99167	29601	70399	00833	71231	49
12	30 24	29 36	28833	99165	29668	70332	00835	71167	48
13	30 16	29 44	28896	99162	29734	70266	00838	71104	47
14	30 8	29 52	28960	99160	29800	70200	00840	71040	46
15	10 30 0	1 30 0	9.29024	9.99157	9.29866	10.70134	10.00843	10.70976	45
16	29 52	30 8	29087	99155	29932	70068	00845	70913	44
17	29 44	30 16	29150	99152	29998	70002	00848	70850	43
18	29 36	30 24	29214	99150	30064	69936	00850	70786	42
19	29 28	30 32	29277	99147	30130	69870	00853	70723	41
20	10 29 20	1 30 40	9.29340	9.99145	9.30195	10.69805	10.00855	10.70660	40
21	29 12	30 48	29403	99142	30261	69739	00858	70597	39
22	29 4	30 56	29466	99140	30326	69674	00860	70534	38
23	28 56	31 4	29529	99137	30391	69609	00863	70471	37
24	28 48	31 12	29591	99135	30457	69543	00865	70409	36
25	10 28 40	1 31 20	9.29654	9.99132	9.30522	10.69478	10.00868	10.70346	35
26	28 32	31 28	29716	99130	30587	69413	00870	70284	34
27	28 24	31 36	29779	99127	30652	69348	00873	70221	33
28	28 16	31 44	29841	99124	30717	69283	00876	70159	32
29	28 8	31 52	29903	99122	30782	69218	00878	70097	31
30	10 28 0	1 32 0	9.29966	9.99119	9.30846	10.69154	10.00881	10.70034	30
31	27 52	32 8	30028	99117	30911	69089	00883	69972	29
32	27 44	32 16	30090	99114	30975	69025	00886	69910	28
33	27 36	32 24	30151	99112	31040	63960	00888	69849	27
34	27 28	32 32	30213	99109	31104	68896	00891	69787	26
35	10 27 20	1 32 40	9.30275	9.99106	9.31168	10.68832	10.00894	10.69725	25
36	27 12	32 48	30336	99104	31233	68767	00896	69664	24
37	27 4	32 56	30398	99101	31297	68703	00899	69602	23
38	26 56	33 4	30459	99099	31361	68639	00901	69541	22
39	26 48	33 12	30521	99096	31425	68575	00904	69479	21
40	10 26 40	1 33 20	9.30582	9.99093	9.31489	10.68511	10.00907	10.69418	20
41	26 32	33 28	30643	99091	31552	68448	00909	69357	19
42	26 24	33 36	30704	99088	31616	68384	00912	69296	18
43	26 16	33 44	30765	99086	31679	68321	00914	69235	17
44	26 8	33 52	30826	99083	31743	68257	00917	69174	16
45	10 26 0	1 34 0	9.30887	9.99080	9.31806	10.68194	10.00920	10.69113	15
46	25 52	34 8	30947	99078	31870	68130	00922	69053	14
47	25 44	34 16	31008	99075	31933	68067	00925	68992	13
48	25 36	34 24	31068	99072	31996	68004	00928	68932	12
49	25 28	34 32	31129	99070	32059	67941	00930	68871	11
50	10 25 20	1 34 40	9.31189	9.99067	9.32122	10.67878	10.00933	10.68811	10
51	25 12	34 48	31250	99064	32185	67815	00936	68750	9
52	25 4	34 56	31310	99062	32248	67752	00938	68690	8
53	24 56	35 4	31370	99059	32311	67689	00941	68630	7
54	24 48	35 12	31430	99056	32373	67627	00944	68570	6
55	10 24 40	1 35 20	9.31490	9.99054	9.32436	10.67564	10.00946	10.68510	5
56	24 32	35 28	31549	99051	32498	67502	00949	68451	4
57	24 24	35 36	31609	99048	32561	67439	00952	68391	3
58	24 16	35 44	31669	99046	32623	67377	00954	68331	2
59	24 8	35 52	31728	99043	32685	67315	00957	68272	1
60	24 0	36 0	31788	99040	32747	67253	00960	68212	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

12 Degs. Degs. 167.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	10 24 0	1 36 0	9.31788	9.99040	9.32747	10.67253	10.00960	10.68212	60
1	23 52	36 8	31847	99038	32810	67190	00962	68153	59
2	23 44	36 16	31907	99035	32872	67128	00965	68093	58
3	23 36	36 24	31966	99032	32933	67067	00968	68034	57
4	23 28	36 32	32025	99030	32995	67005	00970	67975	56
5	10 23 20	1 36 40	9.32084	9.99027	9.33057	10.66943	10.00973	10.67916	55
6	23 12	36 48	32143	99024	33119	66881	00976	67857	54
7	23 4	36 56	32202	99022	33180	66820	00978	67798	53
8	22 56	37 4	32261	99019	33242	66758	00981	67739	52
9	22 48	37 12	32319	99016	33303	66697	00984	67681	51
10	10 22 40	1 37 20	9.32378	9.99013	9.33365	10.66635	10.00987	10.67622	50
11	22 32	37 28	32437	99011	33426	66574	00989	67563	49
12	22 24	37 36	32495	99008	33487	66513	00992	67505	48
13	22 16	37 44	32553	99005	33548	66452	00995	67447	47
14	22 8	37 52	32612	99002	33609	66391	00998	67388	46
15	10 22 0	1 38 0	9.32670	9.99000	9.33670	10.66330	10.01000	10.67330	45
16	21 52	38 8	32728	98997	33731	66269	01003	67272	44
17	21 44	38 16	32786	98994	33792	66208	01006	67214	43
18	21 36	38 24	32844	98991	33853	66147	01009	67156	42
19	21 28	38 32	32902	98989	33913	66087	01011	67098	41
20	10 21 20	1 38 40	9.32960	9.98986	9.33974	10.66026	10.01014	10.67040	40
21	21 12	38 48	33018	98983	34034	65966	01017	66982	39
22	21 4	38 56	33075	98980	34095	65905	01020	66925	38
23	20 56	39 4	33133	98978	34155	65845	01022	66867	37
24	20 48	39 12	33190	98975	34215	65785	01025	66810	36
25	10 20 40	1 39 20	9.33248	9.98972	9.34276	10.65724	10.01028	10.66752	35
26	20 32	39 28	33305	98969	34336	65664	01031	66695	34
27	20 24	39 36	33362	98967	34396	65604	01033	66638	33
28	20 16	39 44	33420	98964	34456	65544	01036	665[illegible]0	32
29	20 8	39 52	33477	98961	34516	65484	01039	66523	31
30	10 20 0	1 40 0	9.33534	9.98958	9.34576	10.65424	10.01042	10.66466	30
31	19 52	40 8	33591	98955	34635	65365	01045	66409	29
32	19 44	40 16	33647	98953	34695	65305	01047	66353	28
33	19 36	40 24	33704	98950	34755	65245	01050	66296	27
34	19 28	40 32	33761	98947	34814	65186	01053	66239	26
35	10 19 20	1 40 40	9.33818	9.98944	9.34874	10.65126	10.01056	10.66182	25
36	19 12	40 48	33874	98941	34933	65067	01059	66126	24
37	19 4	40 56	33931	98938	34992	65008	01062	66069	23
38	18 56	41 4	33987	98936	35051	64949	01064	66013	22
39	18 48	41 12	34043	98933	35111	64889	01067	65957	21
40	10 18 40	1 41 20	9.34100	9.98930	9.35170	10.64830	10.01070	10.65900	20
41	18 32	41 28	34156	98927	35229	64771	01073	65844	19
42	18 24	41 36	34212	98924	35288	64712	01076	65788	18
43	18 16	41 44	34268	98921	35347	64653	01079	65732	17
44	18 8	41 52	34324	98919	35405	64595	01081	65676	16
45	10 18 0	1 42 0	9.34380	9.98916	9.35464	10.64536	10.01084	10.65620	15
46	17 52	42 8	34436	98913	35523	64477	01087	65564	14
47	17 44	42 16	34491	98910	35581	64419	01090	65509	13
48	17 36	42 24	34547	98907	35640	64360	01093	65453	12
49	17 28	42 32	34602	98904	35698	64302	01096	65398	11
50	10 17 20	1 42 40	9.34658	9.98901	9.35757	10.64243	10.01099	10.65342	10
51	17 12	42 48	34713	98898	35815	64185	01102	65287	9
52	17 4	42 56	34769	98896	35873	64127	01104	65231	8
53	16 56	43 4	34824	98893	35931	64069	01107	65176	7
54	16 48	43 12	34879	98890	35989	64011	01110	65121	6
55	10 16 40	1 43 20	9.34934	9.98887	9.36047	10.63953	10.01113	10.65066	5
56	16 32	43 28	34989	98884	36105	63895	01116	65011	4
57	16 24	43 36	35044	98881	36163	63837	01119	64956	3
58	16 16	43 44	35099	98878	36221	63779	01122	64901	2
59	16 8	43 52	35154	98875	36279	63721	01125	64846	1
60	16 0	44 0	35209	98872	36336	63664	01128	64791	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

102 Degs. Degs. 77

Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
16 0	1 44 0	9.35209	9.98872	9.36336	10.63664	10.01128	10.64791	60
15 52	44 8	35263	98869	36394	63606	01131	64737	59
15 44	44 16	35318	98867	36452	63548	01133	64682	58
15 36	44 24	35373	98864	36509	63491	01136	64627	57
15 28	44 32	35427	98861	36566	63434	01139	64573	56
15 20	1 44 40	9.35481	9.98858	9.36624	10.63376	10.01142	10.64519	55
15 12	44 48	35536	98855	36681	63319	01145	64464	54
15 4	44 56	35590	98852	36738	63262	01148	64410	53
14 56	45 4	35644	98849	36795	63205	01151	64356	52
14 48	45 12	35698	98846	36852	63148	01154	64302	51
14 40	1 45 20	9.35752	9.98843	9.36909	10.63091	10.01157	10.64248	50
14 32	45 28	35806	98840	36966	63034	01160	64194	49
14 24	45 36	35860	98837	37023	62977	01163	64140	48
14 16	45 44	35914	98834	37080	62920	01166	64086	47
14 8	45 52	35968	98831	37137	62863	01169	64032	46
14 0	1 46 0	9.36022	9.98828	9.37193	10.62807	10.01172	10.63978	45
13 52	46 8	36075	98825	37250	62750	01175	63925	44
13 44	46 16	36129	98822	37306	62694	01178	63871	43
13 36	46 24	36182	98819	37363	62637	01181	63818	42
13 28	46 32	36236	98816	37419	62581	01184	63764	41
10 13 20	1 46 40	9.36289	9.98813	9.37476	10.62524	10.01187	10.63711	40
13 12	46 48	36342	98810	37532	62468	01190	63658	39
13 4	46 56	36395	98807	37588	62412	01193	63605	38
12 56	47 4	36449	98804	37644	62356	01196	63551	37
12 48	47 12	36502	98801	37700	62300	01199	63498	36
10 12 40	1 47 20	9.36555	9.98798	9.37756	10.62244	10.01202	10.63445	35
12 32	47 28	36608	98795	37812	62188	01205	63392	34
12 24	47 36	36660	98792	37868	62132	01208	63340	33
12 16	47 44	36713	98789	37924	62076	01211	63287	32
12 8	47 52	36766	98786	37980	62020	01214	63234	31
10 12 0	1 48 0	9.36819	9.98783	9.38035	10.61965	10.01217	10.63181	30
11 52	48 8	36871	98780	38091	61909	01220	63129	29
11 44	48 16	36924	98777	38147	61853	01223	63076	28
11 36	48 24	36976	98774	38202	61798	01226	63024	27
11 28	48 32	37028	98771	38257	61743	01229	62972	26
10 11 20	1 48 40	9.37081	9.98768	9.38313	10.61687	10.01232	10.62919	25
11 12	48 48	37133	98765	38368	61632	01235	62867	24
11 4	48 56	37185	98762	38423	61577	01238	62815	23
10 56	49 4	37237	98759	38479	61521	01241	62763	22
10 48	49 12	37289	98756	38534	61466	01244	62711	21
10 10 40	1 49 20	9.37341	9.98753	9.38589	10.61411	10.01247	10.62659	20
10 32	49 28	37393	98750	38644	61356	01250	62607	19
10 24	49 36	37445	98746	38699	61301	01254	62555	18
10 16	49 44	37497	98743	38754	61246	01257	62503	17
10 8	49 52	37549	98740	38808	61192	01260	62451	16
10 10 0	1 50 0	9.37600	9.98737	9.38863	10.61137	10.01263	10.62400	15
9 52	50 8	37652	98734	38918	61082	01266	62348	14
9 44	50 16	37703	98731	38972	61028	01269	62297	13
9 36	50 24	37755	98728	39027	60973	01272	62245	12
9 28	50 32	37806	98725	39082	60918	01275	62194	11
10 9 20	1 50 40	9.37858	9.98722	9.39136	10.60864	10.01278	10.62142	10
9 12	50 48	37909	98719	39190	60810	01281	62091	9
9 4	50 56	37960	98715	39245	60755	01285	62040	8
8 56	51 4	38011	98712	39299	60701	01288	61989	7
8 48	51 12	38062	98709	39353	60647	01291	61938	6
10 8 40	1 51 20	9.38113	9.98706	9.39407	10.60593	10.01294	10.61887	5
8 32	51 28	38164	98703	39461	60539	01297	61836	4
8 24	51 36	38215	98700	39515	60485	01300	61785	3
8 16	51 44	38266	98697	39569	60431	01303	61734	2
8 8	51 52	38317	98694	39623	60377	01306	61683	1
8 0	52 0	38368	98690	39677	60323	01310	61632	0
Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

Log. Sines, Tangents and Secants.

14 Degs. — Deg. 165.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	10 8 0	1 52 0	9.38368	9.98690	9.39677	10.60323	10.01310	10.61632	60
1	7 52	52 8	38418	98687	39731	60269	01313	61582	59
2	7 44	52 16	38469	98684	39785	60215	01316	61531	58
3	7 36	52 24	38519	98681	39838	60162	01319	61481	57
4	7 28	52 32	38570	93678	39892	60108	01322	61430	56
5	10 7 20	1 52 40	9.38620	9.98675	9.39945	10.60055	10.01325	10.61380	55
6	7 12	52 48	38670	98671	39999	60001	01329	61330	54
7	7 4	52 56	38721	98668	40052	59948	01332	61279	53
8	6 56	53 4	38771	98665	40106	59894	01335	61229	52
9	6 48	53 12	38821	98662	40159	59841	01338	61179	51
10	10 6 40	1 53 20	9.38871	9.98659	9.40212	10.59788	10.01341	10.61129	50
11	6 32	53 28	38921	98656	40266	59734	01344	61079	49
12	6 24	53 36	38971	98652	40319	59681	01348	61029	48
13	6 16	53 44	39021	93649	40372	59628	01351	60979	47
14	6 8	53 52	39071	98646	40425	59575	01354	60929	46
15	10 6 0	1 54 0	9.39121	9.98643	9.40478	10.59522	10.01357	10.60879	45
16	5 52	54 8	39170	98640	40531	59469	01360	60830	44
17	5 44	54 16	39220	98636	40584	59416	01364	60780	43
18	5 36	54 24	39270	98633	40636	59364	01367	60730	42
19	5 28	54 32	39319	98630	40689	59311	01370	60681	41
20	10 5 20	1 54 40	9.39369	9.98627	9.40742	10.59258	10.01373	10.60631	40
21	5 12	54 48	39418	98623	40795	59205	01377	60582	39
22	5 4	54 56	39467	98620	40847	59153	01380	60533	38
23	4 56	55 4	39517	98617	40900	59100	01383	60433	37
24	4 48	55 12	39566	98614	40952	59048	01386	60434	36
25	10 4 40	1 55 20	9.39615	9.98610	9.41005	10.58995	10.01390	10.60385	35
26	4 32	55 28	39664	98607	41057	58943	01393	60336	34
27	4 24	55 36	39713	98604	41109	58891	01396	60287	33
28	4 16	55 44	39762	98601	41161	58839	01399	60238	32
29	4 8	55 52	39811	98597	41214	58786	01403	60189	31
30	10 4 0	1 56 0	9.39860	9.98594	9.41266	10.58734	10.01406	10.60140	30
31	3 52	56 8	39909	98591	41318	58682	01409	60091	29
32	3 44	56 16	39958	98588	41370	58630	01412	60042	28
33	3 36	56 24	40006	98584	41422	58578	01416	59994	27
34	3 28	56 32	40055	98581	41474	58526	01419	59945	26
35	10 3 20	1 56 40	9.40103	9.98578	9.41526	10.58474	10.01422	10.59897	25
36	3 12	56 48	40152	98574	41578	58422	01426	59848	24
37	3 4	56 56	40200	98571	41629	58371	01429	59800	23
38	2 56	57 4	40249	98568	41681	58319	01432	59751	22
39	2 48	57 12	40297	98565	41733	58267	01435	59703	21
40	10 2 40	1 57 20	9.40346	9.98561	9.41784	10.58216	10.01439	10.59654	20
41	2 32	57 28	40394	98558	41836	58164	01442	59606	19
42	2 24	57 36	40442	98555	41887	58113	01445	59558	18
43	2 16	57 44	40490	98551	41939	58061	01449	59510	17
44	2 8	57 52	40538	98548	41990	58010	01452	59462	16
45	10 2 0	1 58 0	9.40586	9.98545	9.42041	10.57959	10.01455	10.59414	15
46	1 52	58 8	40634	98541	42093	57907	01459	59366	14
47	1 44	58 16	40682	98538	42144	57856	01462	59318	13
48	1 36	58 24	40730	98535	42195	57805	01465	59270	12
49	1 28	58 32	40778	98531	42246	57754	01469	59222	11
50	10 1 20	1 58 40	9.40825	9.98528	9.42297	10.57703	10.01472	10.59175	10
51	1 12	58 48	40873	98525	42348	57652	01475	59127	9
52	1 4	58 56	40921	98521	42399	57601	01479	59079	8
53	0 56	59 4	40968	98518	42450	57550	01482	59032	7
54	0 48	59 12	41016	98515	42501	57499	01485	58984	6
55	10 0 40	1 59 20	9.41063	9.98511	9.42552	10.57448	10.01489	10.58937	5
56	0 32	59 28	41111	98508	42603	57397	01492	58889	4
57	0 24	59 36	41158	98505	42653	57347	01495	58842	3
58	0 16	59 44	41205	98501	42704	57296	01499	58795	2
59	0 8	59 52	41252	98498	42755	57245	01502	58748	1
60	0 0	2 0 0	41300	98494	42805	57195	01506	58700	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

104 Degs. — Degs. 75.

15 Degs. Degs. 164.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	10. 0 0	2 0 0	9.41300	9.98494	9.42805	10.57195	10.01506	10.58700	60
1	9.59 52	0 8	41347	98491	42856	57144	01509	58653	59
2	59 44	0 16	41394	98488	42906	57094	01512	58606	58
3	59 36	0 24	41441	98484	42957	57043	01516	58559	57
4	59 28	0 32	41488	98481	43007	56993	01519	58512	56
5	9.59 20	2 0 40	9.41535	9.98477	9.43057	10.56943	10.01523	10.58465	55
6	59 12	0 48	41582	98474	43108	56892	01526	58418	54
7	59 4	0 56	41628	98471	43158	56842	01529	58372	53
8	58 56	1 4	41675	98467	43208	56792	01533	58325	52
9	58 48	1 12	41722	98464	43258	56742	01536	58278	51
10	9.58 40	2 1 20	9.41768	9.98460	9.43308	10.56692	10.01540	10.58232	50
11	58 32	1 28	41815	98457	43358	56642	01543	58185	49
12	58 24	1 36	41861	98453	43408	56592	01547	58139	48
13	58 16	1 44	41908	98450	43458	56542	01550	58092	47
14	58 8	1 52	41954	98447	43508	56492	01553	58046	46
15	9.58 0	2 2 0	9.42001	9.98443	9.43558	10.56442	10.01557	10.57999	45
16	57 52	2 8	42047	98440	43607	56393	01560	57953	44
17	57 44	2 16	42093	98436	43657	56343	01564	57907	43
18	57 36	2 24	42140	98433	43707	56293	01567	57860	42
19	57 28	2 32	42186	98429	43756	56244	01571	57814	41
20	9.57 20	2 2 40	9.42232	9.98426	9.43806	10.56194	10.01574	10.57768	40
21	57 12	2 48	42278	98422	43855	56145	01578	57722	39
22	57 4	2 56	42324	98419	43905	56095	01581	57676	38
23	56 56	3 4	42370	98415	43954	56046	01585	57630	37
24	56 48	3 12	42416	98412	44004	55996	01588	57584	36
25	9.56 40	2 3 20	9.42461	9.98409	9.44053	10.55947	10.01591	10.57539	35
26	56 32	3 28	42507	98405	44102	55898	01595	57493	34
27	56 24	3 36	42553	98402	44151	55849	01598	57447	33
28	56 16	3 44	42599	98398	44201	55799	01602	57401	32
29	56 8	3 52	42644	93395	44250	55750	01605	57356	31
30	9.56 0	2 4 0	9.42690	9.98391	9.44299	10.55701	10.01609	10.57310	30
31	55 52	4 8	42735	98388	44348	55652	01612	57265	29
32	.55 44	4 16	42781	98384	44397	55603	01616	57219	28
33	55 36	4 24	42826	98381	44446	55554	01619	57174	27
34	55 28	4 32	42872	98377	44495	55505	01623	57128	26
35	9.55 20	2 4 40	9.42917	9.98373	9.44544	10.55456	10.01627	10.57083	25
36	55 12	4 48	42962	98370	44592	55408	01630	57038	24
37	55 4	4 56	43008	98366	44641	55359	01634	56992	23
38	54 56	5 4	43053	98363	44690	55310	01637	56947	22
39	54 48	5 12	43098	98359	44738	55262	01641	56902	21
40	9.54 40	2 5 20	9.43143	9.98356	9.44787	10.55213	10.01644	10.56857	20
41	54 32	5 28	43188	98352	44836	55164	01648	56812	19
42	54 24	5 36	43233	98349	44884	55116	01651	56767	18
43	54 16	5 44	43278	98345	44933	55067	01655	56722	17
44	54 8	5 52	43323	98342	44981	55019	01658	56677	16
45	9.54 0	2 6 0	9.43367	9.98338	9.45029	10.54971	10.01662	10.56633	15
46	53 52	6 8	43412	98334	45078	54922	01666	56588	14
47	53 44	6 16	43457	98331	45126	54874	01669	56543	13
48	53 36	6 24	43502	98327	45174	54826	01673	56498	12
49	53 28	6 32	43546	98324	45222	54778	01676	56454	11
50	9.53 20	2 6 40	9.43591	9.98320	9.45271	10.54729	10.01680	10.56409	10
51	53 12	6 48	43635	98317	45319	54681	01683	56365	9
52	53 4	6 56	43680	98313	45367	54633	01687	56320	8
53	52 56	7 4	43724	98309	45415	54585	01691	56276	7
54	52 48	7 12	43769	98306	45463	54537	01694	56231	6
55	9.52 40	2 7 20	9.43813	9.98302	9.45511	10.54489	10.01698	10.56187	5
56	52 32	7 28	43857	98299	45559	54441	01701	56143	4
57	52 24	7 36	43901	98295	45606	54394	01705	56099	3
58	52 16	7 44	43946	98291	45654	54346	01709	56054	2
59	52 8	7 52	43990	98288	45702	54298	01712	56010	1
60	52 0	8 0	44034	98284	45750	54250	01716	55966	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

105 Degs. Degs. 74.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant.	M
0	9 52 0	2 8 0	9.44034	9.98284	9.45750	10.54250	10.01716	10.55966	60
1	51 52	8 8	44078	98281	45797	54203	01719	55922	59
2	51 44	8 16	44122	98277	45845	54155	01723	55878	58
3	51 36	8 24	44166	98273	45892	54108	01727	55834	57
4	51 28	8 32	44210	98270	45940	54060	01730	55790	56
5	9 51 20	2 8 40	9.44253	9.98266	9.45987	10.54013	10.01734	10.55747	55
6	51 12	8 48	44297	98262	46035	53965	01738	55703	54
7	51 4	8 56	44341	98259	46082	53918	01741	55659	53
8	50 56	9 4	44385	98255	46130	53870	01745	55615	52
9	50 48	9 12	44428	98251	46177	53823	01749	55572	51
10	9 50 40	2 9 20	9.44472	9.98248	9.46224	10.53776	10.01752	10.55528	50
11	50 32	9 28	44516	98244	46271	53729	01756	55484	49
12	50 24	9 36	44559	98240	46319	53681	01760	55441	48
13	50 16	9 44	44602	98237	46366	53634	01763	55398	47
14	50 8	9 52	44646	98233	46413	53587	01767	55354	46
15	9 50 0	2 10 0	9.44689	9.98229	9.46460	10.53540	10.01771	10.55311	45
16	49 52	10 8	44733	98226	46507	53493	01774	55267	44
17	49 44	10 16	44776	98222	46554	53446	01778	55224	43
18	49 36	10 24	44819	98218	46601	53399	01782	55181	42
19	49 28	10 32	44862	98215	46648	53352	01785	55138	41
20	9 49 20	2 10 40	9.44905	9.98211	9.46694	10.53306	10.01789	10.55095	40
21	49 12	10 48	44948	98207	46741	53259	01793	55052	39
22	49 4	10 56	44992	98204	46788	53212	01796	55008	38
23	48 56	11 4	45035	98200	46835	53165	01800	54965	37
24	48 48	11 12	45077	98196	46881	53119	01804	54923	36
25	9 48 40	2 11 20	9.45120	9.98192	9.46928	10.53072	10.01808	10.54880	35
26	48 32	11 28	45163	98189	46975	53025	01811	54837	34
27	48 24	11 36	45206	98185	47021	52979	01815	54794	33
28	48 16	11 44	45249	98181	47068	52932	01819	54751	32
29	48 8	11 52	45292	98177	47114	52886	01823	54708	31
30	9 48 0	2 12 0	9.45334	9.98174	9.47160	10.52840	10.01826	10.54666	30
31	47 52	12 8	45377	98170	47207	52793	01830	54623	29
32	47 44	12 16	45419	98166	47253	52747	01834	54581	28
33	47 36	12 24	45462	98162	47299	52701	01838	54538	27
34	47 28	12 32	45504	98159	47346	52654	01841	54496	26
35	9 47 20	2 12 40	9.45547	9.98155	9.47392	10.52608	10.01845	10.54453	25
36	47 12	12 48	45589	98151	47438	52562	01849	54411	24
37	47 4	12 56	45632	98147	47484	52516	01853	54368	23
38	46 56	13 4	45674	98144	47530	52470	01856	54326	22
39	46 48	13 12	45716	98140	47576	52424	01860	54284	21
40	9 46 40	2 13 20	9.45758	9.98136	9.47622	10.52378	10.01864	10.54242	20
41	46 32	13 28	45801	98132	47668	52332	01868	54199	19
42	46 24	13 36	45843	98129	47714	52286	01871	54157	18
43	46 16	13 44	45885	98125	47760	52240	01875	54115	17
44	46 8	13 52	45927	98121	47806	52194	01879	54073	16
45	9 46 0	2 14 0	9.45969	9.98117	9.47852	10.52148	10.01883	10.54031	15
46	45 52	14 8	46011	98113	47897	52103	01887	53989	14
47	45 44	14 16	46053	98110	47943	52057	01890	53947	13
48	45 36	14 24	46095	98106	47989	52011	01894	53905	12
49	45 28	14 32	46136	98102	48035	51965	01898	53864	11
50	9 45 20	2 14 40	9.46178	9.98098	9.48080	10.51920	10.01902	10.53822	10
51	45 12	14 48	46220	98094	48126	51874	01906	53780	9
52	45 4	14 56	46262	98090	48171	51829	01910	53738	8
53	44 56	15 4	46303	98087	48217	51783	01913	53697	7
54	44 48	15 12	46345	98083	48262	51738	01917	53655	6
55	9 44 40	2 15 20	9.46386	9.98079	9.48307	10.51693	10.01921	10.53614	5
56	44 32	15 28	46428	98075	48353	51647	01925	53572	4
57	44 24	15 36	46469	98071	48398	51602	01929	53531	3
58	44 16	15 44	46511	98067	48443	51557	01933	53489	2
59	44 8	15 52	46552	98063	48489	51511	01937	53448	1
60	44 0	16 0	46594	98060	48534	51466	01940	53406	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant.	Secant.	M

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent	Co-tang.	Secant.	Co-secant	M
0	9 44 0	2 16 0	9.46594	9.98060	9.48534	10.51466	10.01940	10.53406	60
1	43 52	16 8	46635	98056	48579	51421	01944	53365	59
2	43 44	16 16	46676	98052	48624	51376	01948	53324	58
3	43 36	16 24	46717	98048	48669	51331	01952	53283	57
4	43 28	16 32	46758	98044	48714	51286	01956	53242	56
5	9 43 20	2 16 40	9.46800	9.98040	9.48759	10.51241	10.01960	10.53200	55
6	43 12	16 48	46841	98036	48804	51196	01964	53159	54
7	43 4	16 56	46882	98032	48849	51151	01968	53118	53
8	42 56	17 4	46923	98029	48894	51106	01971	53077	52
9	42 48	17 12	46964	98025	48939	51061	01975	53036	51
10	9 42 40	2 17 20	9.47005	9.98021	9.48984	10.51016	10.01979	10.52995	50
11	42 32	17 28	47045	98017	49029	50971	01983	52955	49
12	42 24	17 36	47086	98013	49073	50927	01987	52914	48
13	42 16	17 44	47127	98009	49118	50882	01991	52873	47
14	42 8	17 52	47168	98005	49163	50837	01995	52832	46
15	9 42 0	2 18 0	9.47209	9.98001	9.49207	10.50793	10.01999	10.52791	45
16	41 52	18 8	47249	97997	49252	50748	02003	52751	44
17	41 44	18 16	47290	97993	49296	50704	02007	52710	43
18	41 36	18 24	47330	97989	49341	50659	02011	52670	42
19	41 28	18 32	47371	97986	49385	50615	02014	52629	41
20	9 41 20	2 18 40	9.47411	9.97982	9.49430	10.50570	10.02018	10.52589	40
21	41 12	18 48	47452	97978	49474	50526	02022	52548	39
22	41 4	18 56	47492	97974	49519	50481	02026	52508	38
23	40 56	19 4	47533	97970	49563	50437	02030	52467	37
24	40 48	19 12	47573	97966	49607	50393	02034	52427	36
25	9 40 40	2 19 20	9.47613	9.97962	9.49652	10.50348	10.02038	10.52387	35
26	40 32	19 28	47654	97958	49696	50304	02042	52346	34
27	40 24	19 36	47694	97954	49740	50260	02046	52306	33
28	40 16	19 44	47734	97950	49784	50216	02050	52266	32
29	40 8	19 52	47774	97946	49828	50172	02054	52226	31
30	9 40 0	2 20 0	9.47814	9.97942	9.49872	10.50128	10.02058	10.52186	30
31	39 52	20 8	47854	97938	49916	50084	02062	52146	29
32	39 44	20 16	47894	97934	49960	50040	02066	52106	28
33	39 36	20 24	47934	97930	50004	49996	02070	52066	27
34	39 28	20 32	47974	97926	50048	49952	02074	52026	26
35	9 39 20	2 20 40	9.48014	9.97922	9.50092	10.49908	10.02078	10.51986	25
36	39 12	20 48	48054	97918	50136	49864	02082	51946	24
37	39 4	20 56	48094	97914	50180	49820	02086	51906	23
38	38 56	21 4	48133	97910	50223	49777	02090	51867	22
39	38 48	21 12	48173	97906	50267	49733	02094	51827	21
40	9 38 40	2 21 20	9.48213	9.97902	9.50311	10.49689	10.02098	10.51787	20
41	38 32	21 28	48252	97898	50355	49645	02102	51748	19
42	38 24	21 36	48292	97894	50398	49602	02106	51708	18
43	38 16	21 44	48332	97890	50442	49558	02110	51668	17
44	38 8	21 52	48371	97886	50485	49515	02114	51629	16
45	9 38 0	2 22 0	9.48411	9.97882	9.50529	10.49471	10.02118	10.51589	15
46	37 52	22 8	48450	97878	50572	49428	02122	51550	14
47	37 44	22 16	48490	97874	50616	49384	02126	51510	13
48	37 36	22 24	48529	97870	50659	49341	02130	51471	12
49	37 28	22 32	48568	97866	50703	49297	02134	51432	11
50	9 37 20	2 22 40	9.48607	9.97861	9.50746	10.49254	10.02139	10.51393	10
51	37 12	22 48	48647	97857	50789	49211	02143	51353	9
52	37 4	22 56	48686	97853	50833	49167	02147	51314	8
53	36 56	23 4	48725	97849	50876	49124	02151	51275	7
54	36 48	23 12	48764	97845	50919	49081	02155	51236	6
55	9 36 40	2 23 20	9.48803	9.97841	9.50962	10.49038	10.02159	10.51197	5
56	36 32	23 28	48842	97837	51005	48995	02163	51158	4
57	36 24	23 36	48881	97833	51048	48952	02167	51119	3
58	36 16	23 44	48920	97829	51092	48908	02171	51080	2
59	36 8	23 52	48959	97825	51135	48865	02175	51041	1
60	36 0	24 0	48998	97821	51178	48822	02179	51002	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Cotang.	Tangent.	Co-secant	Secant.	M

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant.	M
0	9 36 0	2 24 0	9.48998	9.97821	9.51178	10.48822	10.02179	10.51002	60
1	35 52	24 8	49037	97817	51221	48779	02183	50963	59
2	35 44	24 16	49076	97812	51264	48736	02188	50924	58
3	35 36	24 24	49115	97808	51306	48694	02192	50885	57
4	35 28	24 32	49153	97804	51349	48651	02196	50847	56
5	9 35 20	2 24 40	9.49192	9.97800	9.51392	10.48608	10.02200	10.50808	55
6	35 12	24 48	49231	97796	51435	48565	02204	50769	54
7	35 4	24 56	49269	97792	51478	48522	02208	50731	53
8	34 56	25 4	49308	97788	51520	48480	02212	50692	52
9	34 48	25 12	49347	97784	51563	48437	02216	50653	51
10	9 34 40	2 25 20	9.49385	9.97779	9.51606	10.48394	10.02221	10.50615	50
11	34 32	25 28	49424	97775	51648	48352	02225	50576	49
12	34 24	25 36	49462	97771	51691	48309	02229	50538	48
13	34 16	25 44	49500	97767	51734	48266	02233	50500	47
14	34 8	25 52	49539	97763	51776	48224	02237	50461	46
15	9 34 0	2 26 0	9.49577	9.97759	9.51819	10.48181	10.02241	10.50423	45
16	33 52	26 8	49615	97754	51861	48139	02246	50385	44
17	33 44	26 16	49654	97750	51903	48097	02250	50346	43
18	33 36	26 24	49692	97746	51946	48054	02254	50308	42
19	33 28	26 32	49730	97742	51988	48012	02258	50270	41
20	9 33 20	2 26 40	9.49768	9.97738	9.52031	10.47969	10.02262	10.50232	40
21	33 12	26 48	49806	97734	52073	47927	02266	50194	39
22	33 4	26 56	49844	97729	52115	47885	02271	50156	38
23	32 56	27 4	49882	97725	52157	47843	02275	50118	37
24	32 48	27 12	49920	97721	52200	47800	02279	50080	36
25	9 32 40	2 27 20	9.49958	9.97717	9.52242	10.47758	10.02283	10.50042	35
26	32 32	27 28	49996	97713	52284	47716	02287	50004	34
27	32 24	27 36	50034	97708	52326	47674	02292	49966	33
28	32 16	27 44	50072	97704	52368	47632	02296	49928	32
29	32 8	27 52	50110	97700	52410	47590	02300	49890	31
30	9 32 0	2 28 0	9.50148	9.97696	9.52452	10.47548	10.02304	10.49852	30
31	31 52	28 8	50185	97691	52494	47506	02309	49815	29
32	31 44	28 16	50223	97687	52536	47464	02313	49777	28
33	31 36	28 24	50261	97683	52578	47422	02317	49739	27
34	31 28	28 32	50298	97679	52620	47380	02321	49702	26
35	9 31 20	2 28 40	9.50336	9.97674	9.52661	10.47339	10.02326	10.49664	25
36	31 12	28 48	50374	97670	52703	47297	02330	49626	24
37	31 4	28 56	50411	97666	52745	47255	02334	49589	23
38	30 56	29 4	50449	97662	52787	47213	02338	49551	22
39	30 48	29 12	50486	97657	52829	47171	02343	49514	21
40	9 30 40	2 29 20	9.50523	9.97653	9.52870	10.47130	10.02347	10.49477	20
41	30 32	29 28	50561	97649	52912	47088	02351	49439	19
42	30 24	29 36	50598	97645	52953	47047	02355	49402	18
43	30 16	29 44	50635	97640	52995	47005	02360	49365	17
44	30 8	29 52	50673	97636	53037	46963	02364	49327	16
45	9 30 0	2 30 0	9.50710	9.97632	9.53078	10.46922	10.02368	10.49290	15
46	29 52	30 8	50747	97628	53120	46880	02372	49253	14
47	29 44	30 16	50784	97623	53161	46839	02377	49216	13
48	29 36	30 24	50821	97619	53202	46798	02381	49179	12
49	29 28	30 32	50858	97615	53244	46756	02385	49142	11
50	9 29 20	2 30 40	9.50896	9.97610	9.53285	10.46715	10.02390	10.49104	10
51	29 12	30 48	50933	97606	53327	46673	02394	49067	9
52	29 4	30 56	50970	97602	53368	46632	02398	49030	8
53	28 56	31 4	51007	97597	53409	46591	02403	48993	7
54	28 48	31 12	51043	97593	53450	46550	02407	48957	6
55	9 28 40	2 31 20	9.51080	9.97589	9.53492	10.46508	10.02411	10.48920	5
56	28 32	31 28	51117	97584	53533	46467	02416	48883	4
57	28 24	31 36	51154	97580	53574	46426	02420	48846	3
58	28 16	31 44	51191	97576	53615	46385	02424	48809	2
59	28 8	31 52	51227	97571	53656	46344	02429	48773	1
60	28 0	32 0	51264	97567	53697	46303	02433	48736	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant.	Secant.	M

Long. Sines, Tangents and Secants.

19 Degs.									Degs. 160.
M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	9 28 0	2 32 0	9.51264	9.97567	9.53697	10.46303	10.02433	10.48736	60
1	27 52	32 8	51301	97563	53738	46262	02437	48699	59
2	27 44	32 16	51338	97558	53779	46221	02442	48662	58
3	27 36	32 24	51374	97554	53820	46180	02446	48626	57
4	27 28	32 32	51411	97550	53861	46139	02450	48589	56
5	9 27 20	2 32 40	9.51447	9.97545	9.53902	10.46098	10.02455	10.48553	55
6	27 12	32 48	51484	97541	53943	46057	02459	48516	54
7	27 4	32 56	51520	97536	53984	46016	02464	48480	53
8	26 56	33 4	51557	97532	54025	45975	02468	48443	52
9	26 48	33 12	51593	97528	54065	45935	02472	48407	51
10	9 26 40	2 33 20	9.51629	9.97523	9.54106	10.45894	10.02477	10.48371	50
11	26 32	33 28	51666	97519	54147	45853	02481	48334	49
12	26 24	33 36	51702	97515	54187	45813	02485	48298	48
13	26 16	33 44	51738	97510	54228	45772	02490	48262	47
14	26 8	33 52	51774	97506	54269	45731	02494	48226	46
15	9 26 0	2 34 0	9.51811	9.97501	9.54309	10.45691	10.02499	10.48189	45
16	25 52	34 8	51847	97497	54350	45650	02503	48153	44
17	25 44	34 16	51883	97492	54390	45610	02508	48117	43
18	25 36	34 24	51919	97488	54431	45569	02512	48081	42
19	25 28	34 32	51955	97484	54471	45529	02516	48045	41
20	9 25 20	2 34 40	9.51991	9.97479	9.54512	10.45488	10.02521	10.48009	40
21	25 12	34 48	52027	97475	54552	45448	02525	47973	39
22	25 4	34 56	52063	97470	54593	45407	02530	47937	38
23	24 56	35 4	52099	97466	54633	45367	02534	47901	37
24	24 48	35 12	52135	97461	54673	45327	02539	47865	36
25	9 24 40	2 35 20	9.52171	9.97457	9.54714	10.45286	10.02543	10.47829	35
26	24 32	35 28	52207	97453	54754	45246	02547	47793	34
27	24 24	35 36	52242	97448	54794	45206	02552	47758	33
28	24 16	35 44	52278	97444	54835	45165	02556	47722	32
29	24 8	35 52	52314	97439	54875	45125	02561	47686	31
30	9 24 0	2 36 0	9.52350	9.97435	9.54915	10.45085	10.02565	10.47650	30
31	23 52	36 8	52385	97430	54955	45045	02570	47615	29
32	23 44	36 16	52421	97426	54995	45005	02574	47579	28
33	23 36	36 24	52456	97421	55035	44965	02579	47544	27
34	23 28	36 32	52492	97417	55075	44925	02583	47508	26
35	9 23 20	2 36 40	9.52527	9.97412	9.55115	10.44885	10.02588	10.47473	25
36	23 12	36 48	52563	97408	55155	44845	02592	47437	24
37	23 4	36 56	52598	97403	55195	44805	02597	47402	23
38	22 56	37 4	52634	97399	55235	44765	02601	47366	22
39	22 48	37 12	52669	97394	55275	44725	02606	47331	21
40	9 22 40	2 37 20	9.52705	9.97390	9.55315	10.44685	10.02610	10.47295	20
41	22 32	37 28	52740	97385	55355	44645	02615	47260	19
42	22 24	37 36	52775	97381	55395	44605	02619	47225	18
43	22 16	37 44	52811	97376	55434	44566	02624	47189	17
44	22 8	37 52	52846	97372	55474	44526	02628	47154	16
45	9 22 0	2 38 0	9.52881	9.97367	9.55514	10.44486	10.02633	10.47119	15
46	21 52	38 8	52916	97363	55554	44446	02637	47084	14
47	21 44	38 16	52951	97358	55593	44407	02642	47049	13
48	21 36	38 24	52986	97353	55633	44367	02647	47014	12
49	21 28	38 32	53021	97349	55673	44327	02651	46979	11
50	9 21 20	2 38 40	9.53056	9.97344	9.55712	10.44288	10.02656	10.46944	10
51	21 12	38 48	53092	97340	55752	44248	02660	46908	9
52	21 4	38 56	53126	97335	55791	44209	02665	46874	8
53	20 56	39 4	53161	97331	55831	44169	02669	46839	7
54	20 48	39 12	53196	97326	55870	44130	02674	46804	6
55	9 20 40	2 39 20	9.53231	9.97322	9.55910	10.44090	10.02678	10.46769	5
56	20 32	39 28	53266	97317	55949	44051	02683	46734	4
57	20 24	39 36	53301	97312	55989	44011	02688	46699	3
58	20 16	39 44	53336	97308	56028	43972	02692	46664	2
59	20 8	39 52	53370	97303	56067	43933	02697	46630	1
60	20 0	40 0	53405	97299	56107	43893	02701	46595	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M
109 Degs.									Degs. 70.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	9 20 0	2 40 0	9.53405	9.97299	9.56107	10.43893	10.02701	10.46595	60
1	19 52	40 8	53440	97294	56146	43854	02706	46560	59
2	19 44	40 16	53475	97289	56185	43815	02711	46525	58
3	19 36	40 24	53509	97285	56224	43776	02715	46491	57
4	19 28	40 32	53544	97280	56264	43736	02720	46456	56
5	9 19 20	2 40 40	9.53578	9.97276	9.56303	10.43697	10.02724	10.46422	55
6	19 12	40 48	53613	97271	56342	43658	02729	46387	54
7	19 4	40 56	53647	97266	56381	43619	02734	46353	53
8	18 56	41 4	53682	97262	56420	43580	02738	46318	52
9	18 48	41 12	53716	97257	56459	43541	02743	46284	51
10	9 18 40	2 41 20	9.53751	9.97252	9.56498	10.43502	10.02748	10.46249	50
11	18 32	41 28	53785	97248	56537	43463	02752	46215	49
12	18 24	41 36	53819	97243	56576	43424	02757	46181	48
13	18 16	41 44	53854	97238	56615	43385	02762	46146	47
14	18 8	41 52	53888	97234	56654	43346	02766	46112	46
15	9 18 0	2 42 0	9.53922	9.97229	9.56693	10.43307	10.02771	10.46078	45
16	17 52	42 8	53957	97224	56732	43268	02776	46043	44
17	17 44	42 16	53991	97220	56771	43229	02780	46009	43
18	17 36	42 24	54025	97215	56810	43190	02785	45975	42
19	17 28	42 32	54059	97210	56849	43151	02790	45941	41
20	9 17 20	2 42 40	9.54093	9.97206	9.56887	10.43113	10.02794	10.45907	40
21	17 12	42 48	54127	97201	56926	43074	02799	45873	39
22	17 4	42 56	54161	97196	56965	43035	02804	45839	38
23	16 56	43 4	54195	97192	57004	42996	02808	45805	37
24	16 48	43 12	54229	97187	57042	42958	02813	45771	36
25	9 16 40	2 43 20	9.54263	9.97182	9.57081	10.42919	10.02818	10.45737	35
26	16 32	43 28	54297	97178	57120	42880	02822	45703	34
27	16 24	43 36	54331	97173	57158	42842	02827	45669	33
28	16 16	43 44	54365	97168	57197	42803	02832	45635	32
29	16 8	43 52	54399	97163	57235	42765	02837	45601	31
30	9 16 0	2 44 0	9.54433	9.97159	9.57274	10.42726	10.02841	10.45567	30
31	15 52	44 8	54466	97154	57312	42688	02846	45534	29
32	15 44	44 16	54500	97149	57351	42649	02851	45500	28
33	15 36	44 24	54534	97145	57389	42611	02855	45466	27
34	15 28	44 32	54567	97140	57428	42572	02860	45433	26
35	9 15 20	2 44 40	9.54601	9.97135	9.57466	10.42534	10.02865	10.45399	25
36	15 12	44 48	54635	97130	57504	42496	02870	45365	24
37	15 4	44 56	54668	97126	57543	42457	02874	45332	23
38	14 56	45 4	54702	97121	57581	42419	02879	45298	22
39	14 48	45 12	54735	97116	57619	42381	02884	45265	21
40	9 14 40	2 45 20	9.54769	9.97111	9.57658	10.42342	10.02889	10.45231	20
41	14 32	45 28	54802	97107	57696	42304	02893	45198	19
42	14 24	45 36	54836	97102	57734	42266	02898	45164	18
43	14 16	45 44	54869	97097	57772	42228	02903	45131	17
44	14 8	45 52	54903	97092	57810	42190	02908	45097	16
45	9 14 0	2 46 0	9.54936	9.97087	9.57849	10.42151	10.02913	10.45064	15
46	13 52	46 8	54969	97083	57887	42113	02917	45031	14
47	13 44	46 16	55003	97078	57925	42075	02922	44997	13
48	13 36	46 24	55036	97073	57963	42037	02927	44964	12
49	13 28	46 32	55069	97068	58001	41999	02932	44931	11
50	9 13 20	2 46 40	9.55102	9.97063	9.58039	10.41961	10.02937	10.44898	10
51	13 12	46 48	55136	97059	58077	41923	02941	44864	9
52	13 4	46 56	55169	97054	58115	41885	02946	44831	8
53	12 56	47 4	55202	97049	58153	41847	02951	44798	7
54	12 48	47 12	55235	97044	58191	41809	02956	44765	6
55	9 12 40	2 47 20	9.55268	9.97039	9.58229	10.41771	10.02961	10.44732	5
56	12 32	47 28	55301	97035	58267	41733	02965	44699	4
57	12 24	47 36	55334	97030	58304	41696	02970	44666	3
58	12 16	47 44	55367	97025	58342	41658	02975	44633	2
59	12 8	47 52	55400	97020	58380	41620	02980	44600	1
60	12 0	48 0	55433	97015	58418	41582	02985	44567	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant.	M
9 12 0	2 48 0	9.55433	9.97015	9.58418	10.41582	10.02985	10.44567	60
11 52	48 8	55466	97010	58455	41545	02990	44534	59
11 44	48 16	55499	97005	58493	41507	02995	44501	58
11 36	48 24	55532	97001	58531	41469	02999	44468	57
11 28	48 32	55564	96996	58569	41431	03004	44436	56
9 11 20	2 48 40	9.55597	9.96991	9.58606	10.41394	10.03009	10.44403	55
11 12	48 48	55630	96986	58644	41356	03014	44370	54
11 4	48 56	55663	96981	58681	41319	03019	44337	53
10 56	49 4	55695	96976	58719	41281	03024	44305	52
10 48	49 12	55728	96971	58757	41243	03029	44272	51
9 10 40	2 49 20	9.55761	9.96966	9.58794	10.41206	10.03034	10.44239	50
10 32	49 28	55793	96962	58832	41168	03038	44207	49
10 24	49 36	55826	96957	58869	41131	03043	44174	48
10 16	49 44	55858	96952	58907	41093	03048	44142	47
10 8	49 52	55891	96947	58944	41056	03053	44109	46
9 10 0	2 50 0	9.55923	9.96942	9.58981	10.41019	10.03058	10.44077	45
9 52	50 8	55956	96937	59019	40981	03063	44044	44
9 44	50 16	55988	96932	59056	40944	03068	44012	43
9 36	50 24	56021	96927	59094	40906	03073	43979	42
9 28	50 32	56053	96922	59131	40869	03078	43947	41
9 9 20	2 50 40	9.56085	9.96917	9.59168	10.40832	10.03083	10.43915	40
9 12	50 48	56118	96912	59205	40795	03088	43882	39
9 4	50 56	56150	96907	59243	40757	03093	43850	38
8 56	51 4	56182	96903	59280	40720	03097	43818	37
8 48	51 12	56215	96898	59317	40683	03102	43785	36
9 8 40	2 51 20	9.56247	9.96893	9.59354	10.40646	10.03107	10.43753	35
8 32	51 28	56279	96888	59391	40609	03112	43721	34
8 24	51 36	56311	96883	59429	40571	03117	43689	33
8 16	51 44	56343	96878	59466	40534	03122	43657	32
8 8	51 52	56375	96873	59503	40497	03127	43625	31
9 8 0	2 52 0	9.56408	9.96868	9.59540	10.40460	10.03132	10.43592	30
7 52	52 8	56440	96863	59577	40423	03137	43560	29
7 44	52 16	56472	96858	59614	40386	03142	43528	28
7 36	52 24	56504	96853	59651	40349	03147	43496	27
7 28	52 32	56536	96848	59688	40312	03152	43464	26
9 7 20	2 52 40	9.56568	9.96843	9.59725	10.40275	10.03157	10.43432	25
7 12	52 48	56599	96838	59762	40238	03162	43401	24
7 4	52 56	56631	96833	59799	40201	03167	43369	23
6 56	53 4	56663	96828	59835	40165	03172	43337	22
6 48	53 12	56695	96823	59872	40128	03177	43305	21
9 6 40	2 53 20	9.56727	9.96818	9.59909	10.40091	10.03182	10.43273	20
6 32	53 28	56759	96813	59946	40054	03187	43241	19
6 24	53 36	56790	96808	59983	40017	03192	43210	18
6 16	53 44	56822	96803	60019	39981	03197	43178	17
6 8	53 52	56854	96798	60056	39944	03202	43146	16
9 6 0	2 54 0	9.56886	9.96793	9.60093	10.39907	10.03207	10.43114	15
5 52	54 8	56917	96788	60130	39870	03212	43083	14
5 44	54 16	56949	96783	60166	39834	03217	43051	13
5 36	54 24	56980	96773	60203	39797	03222	43020	12
5 28	54 32	57012	96772	60240	39760	03228	42988	11
9 5 20	2 54 40	9.57044	9.96767	9.60276	10.39724	10.03233	10.42956	10
5 12	54 48	57075	96762	60313	39687	03238	42925	9
5 4	54 56	57107	96757	60349	39651	03243	42893	8
4 56	55 4	57138	96752	60386	39614	03248	42862	7
4 48	55 12	57169	96747	60422	39578	03253	42831	6
9 4 40	2 55 20	9.57201	9.96742	9.60459	10.39541	10.03258	10.42799	5
4 32	55 28	57232	96737	60495	39505	03263	42768	4
4 24	55 36	57264	96732	60532	39468	03268	42736	3
4 16	55 44	57295	96727	60568	39432	03273	42705	2
4 8	55 52	57326	96722	60605	39395	03278	42674	1
4 0	56 0	57358	96717	60641	39359	03283	42642	0
Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant.	Secant.	M

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	9 4 0	2 56 0	9.57358	9.96717	9.60641	10.39359	10.03283	10.42642	60
1	3 52	56 8	57389	96711	60677	39323	03289	42611	59
2	3 44	56 16	57420	96706	60714	39286	03294	42580	58
3	3 36	56 24	57451	96701	60750	39250	03299	42549	57
4	3 28	56 32	57482	96696	60786	39214	03304	42518	56
5	9 3 20	2 56 40	9.57514	9.96691	9.60823	10.39177	10.03309	10.42486	55
6	3 12	56 48	57545	96686	60859	39141	03314	42455	54
7	3 4	56 56	57576	96681	60895	39105	03319	42424	53
8	2 56	57 4	57607	96676	60931	39069	03324	42393	52
9	2 48	57 12	57638	96670	60967	39033	03330	42362	51
10	9 2 40	2 57 20	9.57669	9.96665	9.61004	10.38996	10.03335	10.42331	50
11	2 32	57 28	57700	96660	61040	38960	03340	42300	49
12	2 24	57 36	57731	96655	61076	38924	03345	42269	48
13	2 16	57 44	57762	96650	61112	38888	03350	42238	47
14	2 8	57 52	57793	96645	61148	38852	03355	42207	46
15	9 2 0	2 58 0	9.57824	9.96640	9.61184	10.38816	10.03360	10.42176	45
16	1 52	58 8	57855	96634	61220	38780	03366	42145	44
17	1 44	58 16	57885	96629	61256	38744	03371	42115	43
18	1 36	58 24	57916	96624	61292	38708	03376	42084	42
19	1 28	58 32	57947	96619	61328	38672	03381	42053	41
20	9 1 20	2 58 40	9.57978	9.96614	9.61364	10.38636	10.03386	10.42022	40
21	1 12	58 48	58008	96608	61400	38600	03392	41992	39
22	1 4	58 56	58039	96603	61436	38564	03397	41961	38
23	0 56	59 4	58070	96598	61472	38528	03402	41930	37
24	0 48	59 12	58101	96593	61508	38492	03407	41899	36
25	9 0 40	2 59 20	9.58131	9.96588	9.61544	10.38456	10.03412	10.41869	35
26	0 32	59 28	58162	96582	61579	38421	03418	41838	34
27	0 24	59 36	58192	96577	61615	38385	03423	41808	33
28	0 16	59 44	58223	96572	61651	38349	03428	41777	32
29	0 8	59 52	58253	96567	61687	38313	03433	41747	31
30	9 0 0	3 0 0	9.58284	9.96562	9.61722	10.38278	10.03438	10.41716	30
31	8 59 52	0 8	58314	96556	61758	38242	03444	41686	29
32	59 44	0 16	58345	96551	61794	38206	03449	41655	28
33	59 36	0 24	58375	96546	61830	38170	03454	41625	27
34	59 28	0 32	58406	96541	61865	38135	03459	41594	26
35	8 59 20	3 0 40	9.58436	9.96535	9.61901	10.38099	10.03465	10.41564	25
36	59 12	0 48	58467	96530	61936	38064	03470	41533	24
37	59 4	0 56	58497	96525	61972	38028	03475	41503	23
38	58 56	1 4	58527	96520	62008	37992	03480	41473	22
39	58 48	1 12	58557	96514	62043	37957	03486	41443	21
40	8 58 40	3 1 20	9.58588	9.96509	9.62079	10.37921	10.03491	10.41412	20
41	58 32	1 28	58618	96504	62114	37886	03496	41382	19
42	58 24	1 36	58648	96498	62150	37850	03502	41352	18
43	58 16	1 44	58678	96493	62185	37815	03507	41322	17
44	58 8	1 52	58709	96488	62221	37779	03512	41291	16
45	8 58 0	3 2 0	9.58739	9.96483	9.62256	10.37744	10.03517	10.41261	15
46	57 52	2 8	58769	96477	62292	37708	03523	41231	14
47	57 44	2 16	58799	96472	62327	37673	03528	41201	13
48	57 36	2 24	58829	96467	62362	37638	03533	41171	12
49	57 28	2 32	58859	96461	62398	37602	03539	41141	11
50	8 57 20	3 2 40	9.58889	9.96456	9.62433	10.37567	10.03544	10.41111	10
51	57 12	2 48	58919	96451	62468	37532	03549	41081	9
52	57 4	2 56	58949	96445	62504	37496	03555	41051	8
53	56 56	3 4	58979	96440	62539	37461	03560	41021	7
54	56 48	3 12	59009	96435	62574	37426	03565	40991	6
55	8 56 40	3 3 20	9.59039	9.96429	9.62609	10.37391	10.03571	10.40961	5
56	56 32	3 28	59069	96424	62645	37355	03576	40931	4
57	56 24	3 36	59098	96419	62680	37320	03581	40902	3
58	56 16	3 44	59128	96413	62715	37285	03587	40872	2
59	56 8	3 52	59158	96408	62750	37250	03592	40842	1
60	56 0	4 0	59188	96403	62785	37215	03597	40812	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Cotang.	Tangent.	Co-secant	Secant.	M

Degs. | | | | | | | | | Degs. 156.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	8 56 0	3 4 0	9.59188	9.96403	9.62785	10.37215	10.03597	10.40812	60
1	55 52	4 8	59218	96397	62820	37180	03603	40782	59
2	55 44	4 16	59247	96392	62855	37145	03608	40753	58
3	55 36	4 24	59277	96387	62890	37110	03613	40723	57
4	55 28	4 32	59307	96381	62926	37074	03619	40693	56
5	8 55 20	3 4 40	9.59336	9.96376	9.62961	10.37039	10.03624	10.40664	55
6	55 12	4 48	59366	96370	62996	37004	03630	40634	54
7	55 4	4 56	59396	96365	63031	36969	03635	40604	53
8	54 56	5 4	59425	96360	63066	36934	03640	40575	52
9	54 48	5 12	59455	96354	63101	36899	03646	40545	51
0	8 54 40	3 5 20	9.59484	9.96349	9.63135	10.36865	10.03651	10.40516	50
1	54 32	5 28	59514	96343	63170	36830	03657	40486	49
2	54 24	5 36	59543	96338	63205	36795	03662	40457	48
3	54 16	5 44	59573	96333	63240	36760	03667	40427	47
4	54 8	5 52	59602	96327	63275	36725	03673	40398	46
5	8 54 0	3 6 0	9.59632	9.96322	9.63310	10.36690	10.03678	10.40368	45
6	53 52	6 8	59661	96316	63345	36655	03684	40339	44
7	53 44	6 16	59690	96311	63379	36621	03689	40310	43
8	53 36	6 24	59720	96305	63414	36586	03695	40280	42
9	53 28	6 32	59749	96300	63449	36551	03700	40251	41
0	8 53 20	3 6 40	9.59778	9.96294	9.63484	10.36516	10.03706	10.40222	40
1	53 12	6 48	59808	96289	63519	36481	03711	40192	39
2	53 4	6 56	59837	96284	63553	36447	03716	40163	38
3	52 56	7 4	59866	96278	63588	36412	03722	40134	37
4	52 48	7 12	59895	96273	63623	36377	03727	40105	36
5	8 52 40	3 7 20	9.59924	9.96267	9.63657	10.36343	10.03733	10.40076	35
6	52 32	7 28	59954	96262	63692	36308	03738	40046	34
7	52 24	7 36	59983	96256	63726	36274	03744	40017	33
8	52 16	7 44	60012	96251	63761	36239	03749	39988	32
9	52 8	7 52	60041	96245	63796	36204	03755	39959	31
0	8 52 0	3 8 0	9.60070	9.96240	9.63830	10.36170	10.03760	10.39930	30
1	51 52	8 8	60099	96234	63865	36135	03766	39901	29
2	51 44	8 16	60128	96229	63899	36101	03771	39872	28
3	51 36	8 24	60157	96223	63934	36066	03777	39843	27
4	51 28	8 32	60186	96218	63968	36032	03782	39814	26
5	8 51 20	3 8 40	9.60215	9.96212	9.64003	10.35997	10.03788	10.39785	25
6	51 12	8 48	60244	96207	64037	35963	03793	39756	24
7	51 4	8 56	60273	96201	64072	35928	03799	39727	23
8	50 56	9 4	60302	96196	64106	35894	03804	39698	22
9	50 48	9 12	60331	96190	64140	35860	03810	39669	21
0	8 50 40	3 9 20	9.60359	9.96185	9.64175	10.35825	10.03815	10.39641	20
1	50 32	9 28	60388	96179	64209	35791	03821	39612	19
2	50 24	9 36	60417	96174	64243	35757	03826	39583	18
3	50 16	9 44	60446	96168	64278	35722	03832	39554	17
4	50 8	9 52	60474	96162	64312	35688	03838	39526	16
5	8 50 0	3 10 0	9.60503	9.96157	9.64346	10.35654	10.03843	10.39497	15
6	49 52	10 8	60532	96151	64381	35619	03849	39468	14
7	49 44	10 16	60561	96146	64415	35585	03854	39439	13
8	49 36	10 24	60589	96140	64449	35551	03860	39411	12
9	49 28	10 32	60618	96135	64483	35517	03865	39382	11
0	8 49 20	3 10 40	9.60646	9.96129	9.64517	10.35483	10.03871	10.39354	10
1	49 12	10 48	60675	96123	64552	35448	03877	39325	9
2	49 4	10 56	60704	96118	64586	35414	03882	39296	8
3	48 56	11 4	60732	96112	64620	35380	03888	39268	7
4	48 48	11 12	60761	96107	64654	35346	03893	39239	6
5	8 48 40	3 11 20	9.60789	9.96101	9.64688	10.35312	10.03899	10.39211	5
6	48 32	11 28	60818	96095	64722	35278	03905	39182	4
7	48 24	11 36	60846	96090	64756	35244	03910	39154	3
8	48 16	11 44	60875	96084	64790	35210	03916	39125	2
9	48 8	11 52	60903	96079	64824	35176	03921	39097	1
0	48 0	12 0	60931	96073	64858	35142	03927	39069	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

Degs. | | | | | | | | | Degs. 66.

24 Degs. Degs. 155.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	8 48 0	3 12 0	9.60931	9.96073	9.64858	10.35142	10.03927	10.39069	60
1	47 52	12 8	60960	96067	64892	35108	03933	39040	59
2	47 44	12 16	60988	96062	64926	35074	03938	39012	58
3	47 36	12 24	61016	96056	64960	35040	03944	38984	57
4	47 28	12 32	61045	96050	64994	35006	03950	38955	56
5	8 47 20	3 12 40	9.61073	9.96045	9.65028	10.34972	10.03955	10.38927	55
6	47 12	12 48	61101	96039	65062	34938	03961	38899	54
7	47 4	12 56	61129	96034	65096	34904	03966	38871	53
8	46 56	13 4	61158	96028	65130	34870	03972	38842	52
9	46 48	13 12	61186	96022	65164	34836	03978	38814	51
10	8 46 40	3 13 20	9.61214	9.96017	9.65197	10.34803	10.03983	10.38786	50
11	46 32	13 28	61242	96011	65231	34769	03989	38758	49
12	46 24	13 36	61270	96005	65265	34735	03995	38730	48
13	46 16	13 44	61298	96000	65299	34701	04000	38702	47
14	46 8	13 52	61326	95994	65333	34667	04006	38674	46
15	8 46 0	3 14 0	9.61354	9.95988	9.65366	10.34634	10.04012	10.38646	45
16	45 52	14 8	61382	95982	65400	34600	04018	38618	44
17	45 44	14 16	61411	95977	65434	34566	04023	38589	43
18	45 36	14 24	61438	95971	65467	34533	04029	38562	42
19	45 28	14 32	61466	95965	65501	34499	04035	38534	41
20	8 45 20	3 14 40	9.61494	9.95960	9.65535	10.34465	10.04040	10.38506	40
21	45 12	14 48	61522	95954	65568	34432	04046	38478	39
22	45 4	14 56	61550	95948	65602	34398	04052	38450	38
23	44 56	15 4	61578	95942	65636	34364	04058	38422	37
24	44 48	15 12	61606	95937	65669	34331	04063	38394	36
25	8 44 40	3 15 20	9.61634	9.95931	9.65703	10.34297	10.04069	10.38366	35
26	44 32	15 28	61662	95925	65736	34264	04075	38338	34
27	44 24	15 36	61689	95920	65770	34230	04080	38311	33
28	44 16	15 44	61717	95914	65803	34197	04086	38283	32
29	44 8	15 52	61745	95908	65837	34163	04092	38255	31
30	8 44 0	3 16 0	9.61773	9.95902	9.65870	10.34130	10.04098	10.38227	30
31	43 52	16 8	61800	95897	65904	34096	04103	38200	29
32	43 44	16 16	61828	95891	65937	34063	04109	38172	28
33	43 36	16 24	61856	95885	65971	34029	04115	38144	27
34	43 28	16 32	61883	95879	66004	33996	04121	38117	26
35	8 43 20	3 16 40	9.61911	9.95873	9.66038	10.33962	10.04127	10.38089	25
36	43 12	16 48	61939	95868	66071	33929	04132	38061	24
37	43 4	16 56	61966	95862	66104	33896	04138	38034	23
38	42 56	17 4	61994	95856	66138	33862	04144	38006	22
39	42 48	17 12	62021	95850	66171	33829	04150	37979	21
40	8 42 40	3 17 20	9.62049	9.95844	9.66204	10.33796	10.04156	10.37951	20
41	42 32	17 28	62076	95839	66238	33762	04161	37924	19
42	42 24	17 36	62104	95833	66271	33729	04167	37896	18
43	42 16	17 44	62131	95827	66304	33696	04173	37869	17
44	42 8	17 52	62159	95821	66337	33663	04179	37841	16
45	8 42 0	3 18 0	9.62186	9.95815	9.66371	10.33629	10.04185	10.37814	15
46	41 52	18 8	62214	95810	66404	33596	04190	37786	14
47	41 44	18 16	62241	95804	66437	33563	04196	37759	13
48	41 36	18 24	62268	95798	66470	33530	04202	37732	12
49	41 28	18 32	62296	95792	66503	33497	04208	37704	11
50	8 41 20	3 18 40	9.62323	9.95786	9.66537	10.33463	10.04214	10.37677	10
51	41 12	18 48	62350	95780	66570	33430	04220	37650	9
52	41 4	18 56	62377	95775	66603	33397	04225	37623	8
53	40 56	19 [illegible]	62405	95769	66636	33364	04231	37595	7
54	40 48	19 12	62432	95763	66669	33331	04237	37568	6
55	8 40 40	3 19 20	9.62459	9.95757	9.66702	10.33298	10.04243	10.37541	5
56	40 32	19 28	62486	95751	66735	33265	04249	37514	4
57	40 24	19 36	62513	95745	66768	33232	04255	37487	3
58	40 16	19 44	62541	95739	66801	33199	04261	37459	2
59	40 8	19 52	62568	95733	66834	33166	04267	37432	1
60	40 0	20 0	62595	95728	66867	33133	04272	37405	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

114 Degs. Degs. 65

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant.	M
0	8 40 0	3 20 0	9.62595	9.95728	9.66867	10.33133	10.04272	10.37405	60
1	39 52	20 8	62622	95722	66900	33100	04278	37378	59
2	39 44	20 16	62649	95716	66933	33067	04284	37351	58
3	39 36	20 24	62676	95710	66966	33034	04290	37324	57
4	39 28	20 32	62703	95704	66999	33001	04296	37297	56
5	8 39 20	3 20 40	9.62730	9.95698	9.67032	10.32968	10.04302	10.37270	55
6	39 12	20 48	62757	95692	67065	32935	04308	37243	54
7	39 4	20 56	62784	95686	67098	32902	04314	37216	53
8	38 56	21 4	62811	95680	67131	32869	04320	37189	52
9	38 48	21 12	62838	95674	67163	32837	04326	37162	51
10	8 38 40	3 21 20	9.62865	9.95668	9.67196	10.32804	10.04332	10.37135	50
11	38 32	21 28	62892	95663	67229	32771	04337	37108	49
12	38 24	21 36	62918	95657	67262	32738	04343	37082	48
13	38 16	21 44	62945	95651	67295	32705	04349	37055	47
14	38 8	21 52	62972	95645	67327	32673	04355	37028	46
15	8 38 0	3 22 0	9.62999	9.95639	9.67360	10.32640	10.04361	10.37001	45
16	37 52	22 8	63026	95633	67393	32607	04367	36974	44
17	37 44	22 16	63052	95627	67426	32574	04373	36948	43
18	37 36	22 24	63079	95621	67458	32542	04379	36921	42
19	37 28	22 32	63106	95615	67491	32509	04385	36894	41
20	8 37 20	3 22 40	9.63133	9.95609	9.67524	10.32476	10.04391	10.36867	40
21	37 12	22 48	63159	95603	67556	32444	04397	36841	39
22	37 4	22 56	63186	95597	67589	32411	04403	36814	38
23	36 56	23 4	63213	95591	67622	32378	04409	36787	37
24	36 48	23 12	63239	95585	67654	32346	04415	36761	36
25	8 36 40	3 23 20	9.63266	9.95579	9.67687	10.32313	10.04421	10.36734	35
26	36 32	23 28	63292	95573	67719	32281	04427	36708	34
27	36 24	23 36	63319	95567	67752	32248	04433	36681	33
28	36 16	23 44	63345	95561	67785	32215	04439	36655	32
29	36 8	23 52	63372	95555	67817	32183	04445	36628	31
30	8 36 0	3 24 0	9.63398	9.95549	9.67850	10.32150	10.04451	10.36602	30
31	35 52	24 8	63425	95543	67882	32118	04457	36575	29
32	35 44	24 16	63451	95537	67915	32085	04463	36549	28
33	35 36	24 24	63478	95531	67947	32053	04469	36522	27
34	35 28	24 32	63504	95525	67980	32020	04475	36496	26
35	8 35 20	3 24 40	9.63531	9.95519	9.68012	10.31988	10.04481	10.36469	25
36	35 12	24 48	63557	95513	68044	31956	04487	36443	24
37	35 4	24 56	63583	95507	68077	31923	04493	36417	23
38	34 56	25 4	63610	95500	68109	31891	04500	36390	22
39	34 48	25 12	63636	95494	68142	31858	04506	36364	21
40	8 34 40	3 25 20	9.63662	9.95488	9.68174	10.31826	10.04512	10.36338	20
41	34 32	25 28	63689	95482	68206	31794	04518	36311	19
42	34 24	25 36	63715	95476	68239	31761	04524	36285	18
43	34 16	25 44	63741	95470	68271	31729	04530	36259	17
44	34 8	25 52	63767	95464	68303	31697	04536	36233	16
45	8 34 0	3 26 0	9.63794	9.95458	9.68336	10.31664	10.04542	10.36206	15
46	33 52	26 8	63820	95452	68368	31632	04548	36180	14
47	33 44	26 16	63846	95446	68400	31600	04554	36154	13
48	33 36	26 24	63872	95440	68432	31568	04560	36128	12
49	33 28	26 32	63898	95434	68465	31535	04566	36102	11
50	8 33 20	3 26 40	9.63924	9.95427	9.68497	10.31503	10.04573	10.36076	10
51	33 12	26 48	63950	95421	68529	31471	04579	36050	9
52	33 4	26 56	63976	95415	68561	31439	04585	36024	8
53	32 56	27 4	64002	95409	68593	31407	04591	35998	7
54	32 48	27 12	64028	95403	68626	31374	04597	35972	6
55	8 32 40	3 27 20	9.64054	9.95397	9.68658	10.31342	10.04603	10.35946	5
56	32 32	27 28	64080	95391	68690	31310	04609	35920	4
57	32 24	27 36	64106	95384	68722	31278	04616	35894	3
58	32 16	27 44	64132	95378	68754	31246	04622	35868	2
59	32 8	27 52	64158	95372	68786	31214	04628	35842	1
60	32 0	28 0	64184	95366	68818	31182	04634	35816	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant.	Secant.	M

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	8 32 0	3 28 0	9.64184	9.95366	9.68818	10.31182	10.04634	10.35816	60
1	31 52	28 8	64210	95360	68850	31150	04640	35790	59
2	31 44	28 16	64236	95354	68882	31118	04646	35764	58
3	31 36	28 24	64262	95348	68914	31086	04652	35738	57
4	31 28	28 32	64288	95341	68946	31054	04659	35712	56
5	8 31 20	3 28 40	9.64313	9.95335	9.68978	10.31022	10.04665	10.35687	55
6	31 12	28 48	64339	95329	69010	30990	04671	35661	54
7	31 4	28 56	64365	95323	69042	30958	04677	35635	53
8	30 56	29 4	64391	95317	69074	30926	04683	35609	52
9	30 48	29 12	64417	95310	69106	30894	04690	35583	51
10	8 30 40	3 29 20	9.64442	9.95304	9.69138	10.30862	10.04696	10.35558	50
11	30 32	29 28	64468	95298	69170	30830	04702	35532	49
12	30 24	29 36	64494	95292	69202	30798	04708	35506	48
13	30 16	29 44	64519	95286	69234	30766	04714	35481	47
14	30 8	29 52	64545	95279	69266	30734	04721	35455	46
15	8 30 0	3 30 0	9.64571	9.95273	9.69298	10.30702	10.04727	10.35429	45
16	29 52	30 8	64596	95267	69329	30671	04733	35404	44
17	29 44	30 16	64622	95261	69361	30639	04739	35378	43
18	29 36	30 24	64647	95254	69393	30607	04746	35353	42
19	29 28	30 32	64673	95248	69425	30575	04752	35327	41
20	8 29 20	3 30 40	9.64698	9.95242	9.69457	10.30543	10.04758	10.35302	40
21	29 12	30 48	64724	95236	69488	30512	04764	35276	39
22	29 4	30 56	64749	95229	69520	30480	04771	35251	38
23	28 56	31 4	64775	95223	69552	30448	04777	35225	37
24	28 48	31 12	64800	95217	69584	30416	04783	35200	36
25	8 28 40	3 31 20	9.64826	9.95211	9.69615	10.30385	10.04789	10.35174	35
26	28 32	31 28	64851	95204	69647	30353	04796	35149	34
27	28 24	31 36	64877	95198	69679	30321	04802	35123	33
28	28 16	31 44	64902	95192	69710	30290	04808	35098	32
29	28 8	31 52	64927	95185	69742	30258	04815	35073	31
30	8 28 0	3 32 0	9.64953	9.95179	9.69774	10.30226	10.04821	10.35047	30
31	27 52	32 8	64978	95173	69805	30195	04827	35022	29
32	27 44	32 16	65003	95167	69837	30163	04833	34997	28
33	27 36	32 24	65029	95160	69868	30132	04840	34971	27
34	27 28	32 32	65054	95154	69900	30100	04846	34946	26
35	8 27 20	3 32 40	9.65079	9.95148	9.69932	10.30068	10.04852	10.34921	25
36	27 12	32 48	65104	95141	69963	30037	04859	34896	24
37	27 4	32 56	65130	95135	69995	30005	04865	34870	23
38	26 56	33 4	65155	95129	70026	29974	04871	34845	22
39	26 48	33 12	65180	95122	70058	29942	04878	34820	21
40	8 26 40	3 33 20	9.65205	9.95116	9.70089	10.29911	10.04884	10.34795	20
41	26 32	33 28	65230	95110	70121	29879	04890	34770	19
42	26 24	33 36	65255	95103	70152	29848	04897	34745	18
43	26 16	33 44	65281	95097	70184	29816	04903	34719	17
44	26 8	33 52	65306	95090	70215	29785	04910	34694	16
45	8 26 0	3 34 0	9.65331	9.95084	9.70247	10.29753	10.04916	10.34669	15
46	25 52	34 8	65356	95078	70278	29722	04922	34644	14
47	25 44	34 16	65381	95071	70309	29691	04929	34619	13
48	25 36	34 24	65406	95065	70341	29659	04935	34594	12
49	25 28	34 32	65431	95059	70372	29628	04941	34569	11
50	8 25 20	3 34 40	9.65456	9.95052	9.70404	10.29596	10.04948	10.34544	10
51	25 12	34 48	65481	95046	70435	29565	04954	34519	9
52	25 4	34 56	65506	95039	70466	29534	04961	34494	8
53	24 56	35 4	65531	95033	70498	29502	04967	34469	7
54	24 48	35 12	65556	95027	70529	29471	04973	34444	6
55	8 24 40	3 35 20	9.65580	9.95020	9.70560	10.29440	10.04980	10.34420	5
56	24 32	35 28	65605	95014	70592	29408	04986	34395	4
57	24 24	35 36	65630	95007	70623	29377	04993	34370	3
58	24 16	35 44	65655	95001	70654	29346	04999	34345	2
59	24 8	35 52	65680	94995	70685	29315	05005	34320	1
60	24 0	36 0	65705	94988	70717	29283	05012	34295	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	8 24 0	3 36 0	9.65705	9.94988	9.70717	10.29283	10.05012	10.34295	60
1	23 52	36 8	65729	94982	70748	29252	05018	34271	59
2	23 44	36 16	65754	94975	70779	29221	05025	34246	58
3	23 36	36 24	65779	94969	70810	29190	05031	34221	57
4	23 28	36 32	65804	94962	70841	29159	05038	34196	56
5	8 23 20	3 36 40	9.65828	9.94956	9.70873	10.29127	10.05044	10.34172	55
6	23 12	36 48	65853	94949	70904	29096	05051	34147	54
7	23 4	36 56	65878	94943	70935	29065	05057	34122	53
8	22 56	37 4	65902	94936	70966	29034	05064	34098	52
9	22 48	37 12	65927	94930	70997	29003	05070	34073	51
10	8 22 40	3 37 20	9.65952	9.94923	9.71028	10.28972	10.05077	10.34048	50
11	22 32	37 28	65976	94917	71059	28941	05083	34024	49
12	22 24	37 36	66001	94911	71090	28910	05089	33999	48
13	22 16	37 44	66025	94904	71121	28879	05096	33975	47
14	22 8	37 52	66050	94898	71153	28847	05102	33950	46
15	8 22 0	3 38 0	9.66075	9.94891	9.71184	10.28816	10.05109	10.33925	45
16	21 52	38 8	66099	94885	71215	28785	05115	33901	44
17	21 44	38 16	66124	94878	71246	28754	05122	33876	43
18	21 36	38 24	66148	94871	71277	28723	05129	33852	42
19	21 28	38 32	66173	94865	71308	28692	05135	33827	41
20	8 21 20	3 38 40	9.66197	9.94858	9.71339	10.28661	10.05142	10.33803	40
21	21 12	38 48	66221	94852	71370	28630	05148	33779	39
22	21 4	38 56	66246	94845	71401	28599	05155	33754	38
23	20 56	39 4	66270	94839	71431	28569	05161	33730	37
24	20 48	39 12	66295	94832	71462	28538	05168	33705	36
25	8 20 40	3 39 20	9.66319	9.94826	9.71493	10.28507	10.05174	10.33681	35
26	20 32	39 28	66343	94819	71524	28476	05181	33657	34
27	20 24	39 36	66368	94813	71555	28445	05187	33632	33
28	20 16	39 44	66392	94806	71586	28414	05194	33608	32
29	20 8	39 52	66416	94799	71617	28383	05201	33584	31
30	8 20 0	3 40 0	9.66441	9.94793	9.71648	10.28352	10.05207	10.33559	30
31	19 52	40 8	66465	94786	71679	28321	05214	33535	29
32	19 44	40 16	66489	94780	71709	28291	05220	33511	28
33	19 36	40 24	66513	94773	71740	28260	05227	33487	27
34	19 28	40 32	66537	94767	71771	28229	05233	33463	26
35	8 19 20	3 40 40	9.66562	9.94760	9.71802	10.28198	10.05240	10.33438	25
36	19 12	40 48	66586	94753	71833	28167	05247	33414	24
37	19 4	40 56	66610	94747	71863	28137	05253	33390	23
38	18 56	41 4	66634	94740	71894	28106	05260	33366	22
39	18 48	41 12	66658	94734	71925	28075	05266	33342	21
40	8 18 40	3 41 20	9.66682	9.94727	9.71955	10.28045	10.05273	10.33318	20
41	18 32	41 28	66706	94720	71986	28014	05280	33294	19
42	18 24	41 36	66731	94714	72017	27983	05286	33269	18
43	18 16	41 44	66755	94707	72048	27952	05293	33245	17
44	18 8	41 52	66779	94700	72078	27922	05300	33221	16
45	8 18 0	3 42 0	9.66803	9.94694	9.72109	10.27891	10.05306	10.33197	15
46	17 52	42 8	66827	94687	72140	27860	05313	33173	14
47	17 44	42 16	66851	94680	72170	27830	05320	33149	13
48	17 36	42 24	66875	94674	72201	27799	05326	33125	12
49	17 28	42 32	66899	94667	72231	27769	05333	33101	11
50	8 17 20	3 42 40	9.66922	9.94660	9.72262	10.27738	10.05340	10.33078	10
51	17 12	42 48	66946	94654	72293	27707	05346	33054	9
52	17 4	42 56	66970	94647	72323	27677	05353	33030	8
53	16 56	43 4	66994	94640	72354	27646	05360	33006	7
54	16 48	43 12	67018	94634	72384	27616	05366	32982	6
55	8 16 40	3 43 20	9.67042	9.94627	9.72415	10.27585	10.05373	10 32958	5
56	16 32	43 28	67066	94620	72445	27555	05380	32934	4
57	16 24	43 36	67090	94614	72476	27524	05386	32910	3
58	16 16	43 44	67113	94607	72506	27494	05393	32887	2
59	16 8	43 52	67137	94600	72537	27463	05400	32863	1
60	16 0	44 0	67161	94593	72567	27433	05407	32839	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

Log. Sines, Tangents and Secants.

28 Degs. Degs. 151.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	8 16 0	3 44 0	9.67161	9.94593	9.72567	10.27433	10.05407	10.32839	60
1	15 52	44 8	67185	94587	72598	27402	05413	32815	59
2	15 44	44 16	67208	94580	72628	27372	05420	32792	58
3	15 36	44 24	67232	94573	72659	27341	05427	32768	57
4	15 28	44 32	67256	94567	72689	27311	05433	32744	56
5	8 15 20	3 44 40	9.67280	9.94560	9.72720	10.27280	10.05440	10.32720	55
6	15 12	44 48	67303	94553	72750	27250	05447	32697	54
7	15 4	44 56	67327	94546	72780	27220	05454	32673	53
8	14 56	45 4	67350	94540	72811	27189	05460	32650	52
9	14 48	45 12	67374	94533	72841	27159	05467	32626	51
10	8 14 40	3 45 20	9.67398	9.94526	9.72872	10.27128	10.05474	10.32602	50
11	14 32	45 28	67421	94519	72902	27098	05481	32579	49
12	14 24	45 36	67445	94513	72932	27068	05487	32555	48
13	14 16	45 44	67468	94506	72963	27037	05494	32532	47
14	14 8	45 52	67492	94499	72993	27007	05501	32508	46
15	8 14 0	3 46 0	9.67515	9.94492	9.73023	10.26977	10.05508	10.32485	45
16	13 52	46 8	67539	94485	73054	26946	05515	32461	44
17	13 44	46 16	67562	94479	73084	26916	05521	32438	43
18	13 36	46 24	67586	94472	73114	26886	05528	32414	42
19	13 28	46 32	67609	94465	73144	26856	05535	32391	41
20	8 13 20	3 46 40	9.67633	9.94458	9.73175	10.26825	10.05542	10.32367	40
21	13 12	46 48	67656	94451	73205	26795	05549	32344	39
22	13 4	46 56	67680	94445	73235	26765	05555	32320	38
23	12 56	47 4	67703	94438	73265	26735	05562	32297	37
24	12 48	47 12	67726	94431	73295	26705	05569	32274	36
25	8 12 40	3 47 20	9.67750	9.94424	9.73326	10.26674	10.05576	10.32250	35
26	12 32	47 28	67773	94417	73356	26644	05583	32227	34
27	12 24	47 36	67796	94410	73386	26614	05590	32204	33
28	12 16	47 44	67820	94404	73416	26584	05596	32180	32
29	12 8	47 52	67843	94397	73446	26554	05603	32157	31
30	8 12 0	3 48 0	9.67866	9.94390	9.73476	10.26524	10.05610	10.32134	30
31	11 52	48 8	67890	94383	73507	26493	05617	32110	29
32	11 44	48 16	67913	94376	73537	26463	05624	32087	28
33	11 36	48 24	67936	94369	73567	26433	05631	32064	27
34	11 28	48 32	67959	94362	73597	26403	05638	32041	26
35	8 11 20	3 48 40	9.67982	9.94355	9.73627	10.26373	10.05645	10.32018	25
36	11 12	48 48	68006	94349	73657	26343	05651	31994	24
37	11 4	48 56	68029	94342	73687	26313	05658	31971	23
38	10 56	49 4	68052	94335	73717	26283	05665	31948	22
39	10 48	49 12	68075	94328	73747	26253	05672	31925	21
40	8 10 40	3 49 20	9.68098	9.94321	9.73777	10.26223	10.05679	10.31902	20
41	10 32	49 28	68121	94314	73807	26193	05686	31879	19
42	10 24	49 36	68144	94307	73837	26163	05693	31856	18
43	10 16	49 44	68167	94300	73867	26133	05700	31833	17
44	10 8	49 52	68190	94293	73897	26103	05707	31810	16
45	8 10 0	3 50 0	9.68213	9.94286	9.73927	10.26073	10.05714	10.31787	15
46	9 52	50 8	68237	94279	73957	26043	05721	31763	14
47	9 44	50 16	68260	94273	73987	26013	05727	31740	13
48	9 36	50 24	68283	94266	74017	25983	05734	31717	12
49	9 28	50 32	68305	94259	74047	25953	05741	31695	11
50	8 9 20	3 50 40	9.68328	9.94252	9.74077	10.25923	10.05748	10.31672	10
51	9 12	50 48	68351	94245	74107	25893	05755	31649	9
52	9 4	50 56	68374	94238	74137	25863	05762	31626	8
53	8 56	51 4	68397	94231	74166	25834	05769	31603	7
54	8 48	51 12	68420	94224	74196	25804	05776	31580	6
55	8 8 40	3 51 20	9.68443	9.94217	9.74226	10.25774	10.05783	10.31557	5
56	8 32	51 28	68466	94210	74256	25744	05790	31534	4
57	8 24	51 36	68489	94203	74286	25714	05797	31511	3
58	8 16	51 44	68512	94196	74316	25684	05804	31488	2
59	8 8	51 52	68534	94189	74345	25655	05811	31466	1
60	8 0	52 0	68557	94182	74375	25625	05818	31443	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

118 Degs. Degs. 61.

29 Degs. Degs. 150.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	8 8 0	3 52 0	9.68557	9.94182	9.74375	10.25625	10.05818	10.31443	60
1	7 52	52 8	68580	94175	74405	25595	05825	31420	59
2	7 44	52 16	68603	94168	74435	25565	05832	31397	58
3	7 36	52 24	68625	94161	74465	25535	05839	31375	57
4	7 28	52 32	68648	94154	74494	25506	05846	31352	56
5	8 7 20	3 52 40	9.68671	9.94147	9.74524	10.25476	10.05853	10.31329	55
6	7 12	52 48	68694	94140	74554	25446	05860	31306	54
7	7 4	52 56	68716	94133	74583	25417	05867	31284	53
8	6 56	53 4	68739	94126	74613	25387	05874	31261	52
9	6 48	53 12	68762	94119	74643	25357	05881	31238	51
10	8 6 40	3 53 20	9.68784	9.94112	9.74673	10.25327	10.05888	10.31216	50
11	6 32	53 28	68807	94105	74702	25298	05895	31193	49
12	6 24	53 36	68829	94098	74732	25268	05902	31171	48
13	6 16	53 44	68852	94090	74762	25238	05910	31148	47
14	6 8	53 52	68875	94083	74791	25209	05917	31125	46
15	8 6 0	3 54 0	9.68897	9.94076	9.74821	10.25179	10.05924	10.31103	45
16	5 52	54 8	68920	94069	74851	25149	05931	31080	44
17	5 44	54 16	68942	94062	74880	25120	05938	31058	43
18	5 36	54 24	68965	94055	74910	25090	05945	31035	42
19	5 28	54 32	68987	94048	74939	25061	05952	31013	41
20	8 5 20	3 54 40	9.69010	9.94041	9.74969	10.25031	10.05959	10.30990	40
21	5 12	54 48	69032	94034	74998	25002	05966	30968	39
22	5 4	54 56	69055	94027	75028	24972	05973	30945	38
23	4 56	55 4	69077	94020	75058	24942	05980	30923	37
24	4 48	55 12	69100	94012	75087	24913	05988	30900	36
25	8 4 40	3 55 20	9.69122	9.94005	9.75117	10.24883	10.05995	10.30878	35
26	4 32	55 28	69144	93998	75146	24854	06002	30856	34
27	4 24	55 36	69167	93991	75176	24824	06009	30833	33
28	4 16	55 44	69189	93984	75205	24795	06016	30811	32
29	4 8	55 52	69212	93977	75235	24765	06023	30788	31
30	8 4 0	3 56 0	9.69234	9.93970	9.75264	10.24736	10.06030	10.30766	30
31	3 52	56 8	69256	93963	75294	24706	06037	30744	29
32	3 44	56 16	69279	93955	75323	24677	06045	30721	28
33	3 36	56 24	69301	93948	75353	24647	06052	30699	27
34	3 28	56 32	69323	93941	75382	24618	06059	30677	26
35	8 3 20	3 56 40	9.69345	9.93934	9.75411	10.24589	10.06066	10.30655	25
36	3 12	56 48	69368	93927	75441	24559	06073	30632	24
37	3 4	56 56	69390	93920	75470	24530	06080	30610	23
38	2 56	57 4	69412	93912	75500	24500	06088	30588	22
39	2 48	57 12	69434	93905	75529	24471	06095	30566	21
40	8 2 40	3 57 20	9.69456	9.93898	9.75558	10.24442	10.06102	10.30544	20
41	2 32	57 28	69479	93891	75588	24412	06109	30521	19
42	2 24	57 36	69501	93884	75617	24383	06116	30499	18
43	2 16	57 44	69523	93876	75647	24353	06124	30477	17
44	2 8	57 52	69545	93869	75676	24324	06131	30455	16
45	8 2 0	3 58 0	9.69567	9.93862	9.75705	10.24295	10.06138	10.30433	15
46	1 52	58 8	69589	93855	75735	24265	06145	30411	14
47	1 44	58 16	69611	93847	75764	24236	06153	30389	13
48	1 36	58 24	69633	93840	75793	24207	06160	30367	12
49	1 28	58 32	69655	93833	75822	24178	06167	30345	11
50	8 1 20	3 58 40	9.69677	9.93826	9.75852	10.24148	10.06174	10.30323	10
51	1 12	58 48	69699	93819	75881	24119	06181	30301	9
52	1 4	58 56	69721	93811	75910	24090	06189	30279	8
53	0 56	59 4	69743	93804	75939	24061	06196	30257	7
54	0 48	59 12	69765	93797	75969	24031	06203	30235	6
55	8 0 40	3 59 20	9.69787	9.93789	9.75998	10.24002	10.06211	10.30213	5
56	0 32	59 28	69809	93782	76027	23973	06218	30191	4
57	0 24	59 36	69831	93775	76056	23944	06225	30169	3
58	0 16	59 44	69853	93768	76086	23914	06232	30147	2
59	0 8	59 52	69875	93760	76115	23885	06240	30125	1
60	0 0	4 0 0	69897	93753	76144	23856	06247	30103	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

119 Degs. Degs. 60.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant.	M
0	8 0 0	4 0 0	9.69897	9.93753	9.76144	10.23856	10.06247	10.30103	60
1	7 59 52	0 8	69919	93746	76173	23827	06254	30081	59
2	59 44	0 16	69941	93738	76202	23798	06262	30059	58
3	59 36	0 24	69963	93731	76231	23769	06269	30037	57
4	59 28	0 32	69984	93724	76261	23739	06276	30016	56
5	7 59 20	4 0 40	9.70006	9.93717	9.76290	10.23710	10.06283	10.29994	55
6	59 12	0 48	70028	93709	76319	23681	06291	29972	54
7	59 4	0 56	70050	93702	76348	23652	06298	29950	53
8	58 56	1 4	70072	93695	76377	23623	06305	29928	52
9	58 48	1 12	70093	93687	76406	23594	06313	29907	51
10	7 58 40	4 1 20	9.70115	9.93680	9.76435	10.23565	10.06320	10.29885	50
11	58 32	1 28	70137	93673	76464	23536	06327	29863	49
12	58 24	1 36	70159	93665	76493	23507	06335	29841	48
13	58 16	1 44	70180	93658	76522	23478	06342	29820	47
14	58 8	1 52	70202	93650	76551	23449	06350	29798	46
15	7 58 0	4 2 0	9.70224	9.93643	9.76580	10.23420	10.06357	10.29776	45
16	57 52	2 8	70245	93636	76609	23391	06364	29755	44
17	57 44	2 16	70267	93628	76639	23361	06372	29733	43
18	57 36	2 24	70288	93621	76668	23332	06379	29712	42
19	57 28	2 32	70310	93614	76697	23303	06386	29690	41
20	7 57 20	4 2 40	9.70332	9.93606	9.76725	10.23275	10.06394	10.29668	40
21	57 12	2 48	70353	93599	76754	23246	06401	29647	39
22	57 4	2 56	70375	93591	76783	23217	06409	29625	38
23	56 56	3 4	70396	93584	76812	23188	06416	29604	37
24	56 48	3 12	70418	93577	76841	23159	06423	29582	36
25	7 56 40	4 3 20	9.70439	9.93569	9.76870	10.23130	10.06431	10.29561	35
26	56 32	3 28	70461	93562	76899	23101	06438	29539	34
27	56 24	3 36	70482	93554	76928	23072	06446	29518	33
28	56 16	3 44	70504	93547	76957	23043	06453	29496	32
29	56 8	3 52	70525	93539	76986	23014	06461	29475	31
30	7 56 0	4 4 0	9.70547	9.93532	9.77015	10.22985	10.06468	10.29453	30
31	55 52	4 8	70568	93525	77044	22956	06475	29432	29
32	55 44	4 16	70590	93517	77073	22927	06483	29410	28
33	55 36	4 24	70611	93510	77101	22899	06490	29389	27
34	55 28	4 32	70633	93502	77130	22870	06498	29367	26
35	7 55 20	4 4 40	9.70654	9.93495	9.77159	10.22841	10.06505	10.29346	25
36	55 12	4 48	70675	93487	77188	22812	06513	29325	24
37	55 4	4 56	70697	93480	77217	22783	06520	29303	23
38	54 56	5 4	70718	93472	77246	22754	06528	29282	22
39	54 48	5 12	70739	93465	77274	22726	06535	29261	21
40	7 54 40	4 5 20	9.70761	9.93457	9.77303	10.22697	10.06543	10.29239	20
41	54 32	5 28	70782	93450	77332	22668	06550	29218	19
42	54 24	5 36	70803	93442	77361	22639	06558	29197	18
43	54 16	5 44	70824	93435	77390	22610	06565	29176	17
44	54 8	5 52	70846	93427	77418	22582	06573	29154	16
45	7 54 0	4 6 0	9.70867	9.93420	9.77447	10.22553	10.06580	10.29133	15
46	53 52	6 8	70888	93412	77476	22524	06588	29112	14
47	53 44	6 16	70909	93405	77505	22495	06595	29091	13
48	53 36	6 24	70931	93397	77533	22467	06603	29069	12
49	53 28	6 32	70952	93390	77562	22438	06610	29048	11
50	7 53 20	4 6 40	9.70973	9.93382	9.77591	10.22409	10.06618	10.29027	10
51	53 12	6 48	70994	93375	77619	22381	06625	29006	9
52	53 4	6 56	71015	93367	77648	22352	06633	28985	8
53	52 56	7 4	71036	93360	77677	22323	06640	28964	7
54	52 48	7 12	71058	93352	77706	22294	06648	28942	6
55	7 52 40	4 7 20	9.71079	9.93344	9.77734	10.22266	10.06656	10.28921	5
56	52 32	7 28	71100	93337	77763	22237	06663	28900	4
57	52 24	7 36	71121	93329	77791	22209	06671	28879	3
58	52 16	7 44	71142	93322	77820	22180	06678	28858	2
59	52 8	7 52	71163	93314	77849	22151	06686	28837	1
60	52 0	8 0	71184	93307	77877	22123	06693	28816	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	7 52 0	4 8 0	9.71184	9.93307	9.77877	10.22123	10.06693	10.28816	60
1	51 52	8 8	71205	93299	77906	22094	06701	28795	59
2	51 44	8 16	71226	93291	77935	22065	06709	28774	58
3	51 36	8 24	71247	93284	77963	22037	06716	28753	57
4	51 28	8 32	71268	93276	77992	22008	06724	28732	56
5	7 51 20	4 8 40	9.71289	9.93269	9.78020	10.21980	10.06731	10.28711	55
6	51 12	8 48	71310	93261	78049	21951	06739	28690	54
7	51 4	8 56	71331	93253	78077	21923	06747	28669	53
8	50 56	9 4	71352	93246	78106	21894	06754	28648	52
9	50 48	9 12	71373	93238	78135	21865	06762	28627	51
10	7 50 40	4 9 20	9.71393	9.93230	9.78163	10.21837	10.06770	10.28607	50
11	50 32	9 28	71414	93223	78192	21808	06777	28586	49
12	50 24	9 36	71435	93215	78220	21780	06785	28565	48
13	50 16	9 44	71456	93207	78249	21751	06793	28544	47
14	50 8	9 52	71477	93200	78277	21723	06800	28523	46
15	7 50 0	4 10 0	9.71498	9.93192	9.78306	10.21694	10.06808	10.28502	45
16	49 52	10 8	71519	93184	78334	21666	06816	28481	44
17	49 44	10 16	71539	93177	78363	21637	06823	28461	43
18	49 36	10 24	71560	93169	78391	21609	06831	28440	42
19	49 28	10 32	71581	93161	78419	21581	06839	28419	41
20	7 49 20	4 10 40	9.71602	9.93154	9.73448	10.21552	10.06846	10.28398	40
21	49 12	10 48	71622	93146	78476	21524	06854	28378	39
22	49 4	10 56	71643	93138	78505	21495	06862	28357	38
23	48 56	11 4	71664	93131	78533	21467	06869	28336	37
24	48 48	11 12	71685	93123	78562	21438	06877	28315	36
25	7 48 40	4 11 20	9.71705	9.93115	9.78590	10.21410	10.06885	10.28295	35
26	48 32	11 28	71726	93108	78618	21382	06892	28274	34
27	48 24	11 36	71747	93100	78647	21353	06900	28253	33
28	48 16	11 44	71767	93092	78675	21325	06908	28233	32
29	48 8	11 52	71788	93084	78704	21296	06916	28212	31
30	7 48 0	4 12 0	9.71809	9.93077	9.78732	10.21268	10.06923	10.28191	30
31	47 52	12 8	71829	93069	78760	21240	06931	28171	29
32	47 44	12 16	71850	93061	78789	21211	06939	28150	28
33	47 36	12 24	71870	93053	78817	21183	06947	28130	27
34	47 28	12 32	71891	93046	78845	21155	06954	28109	26
35	7 47 20	4 12 40	9.71911	9.93038	9.78874	10.21126	10.06962	10.28089	25
36	47 12	12 48	71932	93030	78902	21098	06970	28068	24
37	47 4	12 56	71952	93022	78930	21070	06978	28048	23
38	46 56	13 4	71973	93014	78959	21041	06986	28027	22
39	46 48	13 12	71994	93007	78987	21018	06993	28006	21
40	7 46 40	4 13 20	9.72014	9.92999	9.79015	10.20985	10.07001	10.27986	20
41	46 32	13 28	72034	92991	79043	20957	07009	27966	19
42	46 24	13 36	72055	92983	79072	20928	07017	27945	18
43	46 16	13 44	72075	92976	79100	20900	07024	27925	17
44	46 8	13 52	72096	92968	79128	20872	07032	27904	16
45	7 46 0	4 14 0	9.72116	9.92960	9.79156	10.20844	10.07040	10.27884	15
46	45 52	14 8	72137	92952	79185	20815	07048	27863	14
47	45 44	14 16	72157	92944	79213	20787	07056	27843	13
48	45 36	14 24	72177	92936	79241	20759	07064	27823	12
49	45 28	14 32	72198	92929	79269	20781	07071	27802	11
50	7 45 20	4 14 40	9.72218	9.92921	9.79297	10.20703	10.07079	10.27782	10
51	45 12	14 48	72238	92913	79326	20674	07087	27762	9
52	45 4	14 56	72259	92905	79354	20646	07095	27741	8
53	44 56	15 4	72279	92897	79382	20618	07103	27721	7
54	44 48	15 12	72299	92889	79410	20590	07111	27701	6
55	7 44 40	4 15 20	9.72320	9.92881	9.79438	10.20562	10.07119	10.27680	5
56	44 32	15 28	72340	92874	79466	20534	07126	27660	4
57	44 24	15 36	72360	92866	79495	20505	07134	27640	3
58	44 16	15 44	72381	92858	79523	20477	07142	27619	2
59	44 8	15 52	72401	92850	79551	20449	07150	27599	1
60	44 0	16 0	72421	92842	79579	20421	07158	27579	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

32 Degs. Degs. 147.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	7 44 0	4 16 0	9.72421	9.92842	9.79579	10.20421	10.07158	10.27579	60
1	43 52	16 8	72441	92834	79607	20393	07166	27559	59
2	43 44	16 16	72461	92826	79635	20365	07174	27539	58
3	43 36	16 24	72482	92818	79663	20337	07182	27518	57
4	43 28	16 32	72502	92810	79691	20309	07190	27498	56
5	7 43 20	4 16 40	9.72522	9.92803	9.79719	10.20281	10.07197	10.27478	55
6	43 12	16 48	72542	92795	79747	20253	07205	27458	54
7	43 4	16 56	72562	92787	79776	20224	07213	27438	53
8	42 56	17 4	72582	92779	79804	20196	07221	27418	52
9	42 48	17 12	72602	92771	79832	20168	07229	27398	51
10	7 42 40	4 17 20	9.72622	9.92763	9.79860	10.20140	10.07237	10.27378	50
11	42 32	17 28	72643	92755	79888	20112	07245	27357	49
12	42 24	17 36	72663	92747	79916	20084	07253	27337	48
13	42 16	17 44	72683	92739	79944	20056	07261	27317	47
14	42 8	17 52	72703	92731	79972	20028	07269	27297	46
15	7 42 0	4 18 0	9.72723	9.92723	9.80000	10.20000	10.07277	10.27277	45
16	41 52	18 8	72743	92715	80028	19972	07285	27257	44
17	41 44	18 16	72763	92707	80056	19944	07293	27237	43
18	41 36	18 24	72783	92699	80084	19916	07301	27217	42
19	41 28	18 32	72803	92691	80112	19888	07309	27197	41
20	7 41 20	4 18 40	9.72823	9.92683	9.80140	10.19860	10.07317	10.27177	40
21	41 12	18 48	72843	92675	80168	19832	07325	27157	39
22	41 4	18 56	72863	92667	80195	19805	07333	27137	38
23	40 56	19 4	72883	92659	80223	19777	07341	27117	37
24	40 48	19 12	72902	92651	80251	19749	07349	27098	36
25	7 40 40	4 19 20	9.72922	9.92643	9.80279	10.19721	10.07357	10.27078	35
26	40 32	19 28	72942	92635	80307	19693	07365	27058	34
27	40 24	19 36	72962	92627	80335	19665	07373	27038	33
28	40 16	19 44	72982	92619	80363	19637	07381	27018	32
29	40 8	19 52	73002	92611	80391	19609	07389	26998	31
30	7 40 0	4 20 0	9.73022	9.92603	9.80419	10.19581	10.07397	10.26978	30
31	39 52	20 8	73041	92595	80447	19553	07405	26959	29
32	39 44	20 16	73061	92587	80474	19526	07413	26939	28
33	39 36	20 24	73081	92579	80502	19498	07421	26919	27
34	39 28	20 32	73101	92571	80530	19470	07429	26899	26
35	7 39 20	4 20 40	9.73121	9.92563	9.80558	10.19442	10.07437	10.26879	25
36	39 12	20 48	73140	92555	80586	19414	07445	26860	24
37	39 4	20 56	73160	92546	80614	19386	07454	26840	23
38	38 56	21 4	73180	92538	80642	19358	07462	26820	22
39	38 48	21 12	73200	92530	80669	19331	07470	26800	21
40	7 38 40	4 21 20	9.73219	9.92522	9.80697	10.19303	10.07478	10.26781	20
41	38 32	21 28	73239	92514	80725	19275	07486	26761	19
42	38 24	21 36	73259	92506	80753	19247	07494	26741	18
43	38 16	21 44	73278	92498	80781	19219	07502	26722	17
44	38 8	21 52	73298	92490	80808	19192	07510	26702	16
45	7 38 0	4 22 0	9.73318	9.92482	9.80836	10.19164	10.07518	10.26682	15
46	37 52	22 8	73337	92473	80864	19136	07527	26663	14
47	37 44	22 16	73357	92465	80892	19108	07535	26643	13
48	37 36	22 24	73377	92457	80919	19081	07543	26623	12
49	37 28	22 32	73396	92449	80947	19053	07551	26604	11
50	7 37 20	4 22 40	9.73416	9.92441	9.80975	10.19025	10.07559	10.26584	10
51	37 12	22 48	73435	92433	81003	18997	07567	26565	9
52	37 4	22 56	73455	92425	81030	18970	07575	26545	8
53	36 56	23 4	73474	92416	81058	18942	07584	26526	7
54	36 48	23 12	73494	92408	81086	18914	07592	26506	6
55	7 36 40	4 23 20	9.73513	9.92400	9.81113	10.18887	10.07600	10.26487	5
56	36 32	23 28	73533	92392	81141	18859	07608	26467	4
57	36 24	23 36	73552	92384	81169	18831	07616	26448	3
58	36 16	23 44	73572	92376	81196	18804	07624	26428	2
59	36 8	23 52	73591	92367	81224	18776	07633	26409	1
60	36 0	24 0	73611	92359	81252	18748	07641	26389	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

122 Degs. Degs. 57.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant.	M
0	7 36 0	4 24 0	9.73611	9.92359	9.81252	10.18748	10.07641	10.26389	60
1	35 52	24 8	73630	92351	81279	18721	07649	26370	59
2	35 44	24 16	73650	92343	81307	18693	07657	26350	58
3	35 36	24 24	73669	92335	81335	18665	07665	26331	57
4	35 28	24 32	73689	92326	81362	18638	07674	26311	56
5	7 35 20	4 24 40	9.73708	9.92318	9.81390	10.18610	10.07682	10.26292	55
6	35 12	24 48	73727	92310	81418	18582	07690	26273	54
7	35 4	24 56	73747	92302	81445	18555	07698	26253	53
8	34 56	25 4	73766	92293	81473	18527	07707	26234	52
9	34 48	25 12	73785	92285	81500	18500	07715	26215	51
10	7 34 40	4 25 20	9.73805	9.92277	9.81528	10.18472	10.07723	10.26195	50
11	34 32	25 28	73824	92269	81556	18444	07731	26176	49
12	34 24	25 36	73843	92260	81583	18417	07740	26157	48
13	34 16	25 44	73863	92252	81611	18389	07748	26137	47
14	34 8	25 52	73882	92244	81638	18362	07756	26118	46
15	7 34 0	4 26 0	9.73901	9.92235	9.81666	10.18334	10.07765	10.26099	45
16	33 52	26 8	73921	92227	81693	18307	07773	26079	44
17	33 44	26 16	73940	92219	81721	18279	07781	26060	43
18	33 36	26 24	73959	92211	81748	18252	07789	26041	42
19	33 28	26 32	73978	92202	81776	18224	07798	26022	41
20	7 33 20	4 26 40	9.73997	9.92194	9.81803	10.18197	10.07806	10.26003	40
21	33 12	26 48	74017	92186	81831	18169	07814	25983	39
22	33 4	26 56	74036	92177	81858	18142	07823	25964	38
23	32 56	27 4	74055	92169	81886	18114	07831	25945	37
24	32 48	27 12	74074	92161	81913	18087	07839	25926	36
25	7 32 40	4 27 20	9.74093	9.92152	9.81941	10.18059	10.07848	10.25907	35
26	32 32	27 28	74113	92144	81968	18032	07856	25887	34
27	32 24	27 36	74132	92136	81996	18004	07864	25868	33
28	32 16	27 44	74151	92127	82023	17977	07873	25849	32
29	32 8	27 52	74170	92119	82051	17949	07881	25830	31
30	7 32 0	4 28 0	9.74189	9.92111	9.82078	10.17922	10.07889	10.25811	30
31	31 52	28 8	74208	92102	82106	17894	07898	25792	29
32	31 44	28 16	74227	92094	82133	17867	07906	25773	28
33	31 36	28 24	74246	92086	82161	17839	07914	25754	27
34	31 28	28 32	74265	92077	82188	17812	07923	25735	26
35	7 31 20	4 28 40	9.74284	9.92069	9.82215	10.17785	10.07931	10.25716	25
36	31 12	28 48	74303	92060	82243	17757	07940	25697	24
37	31 4	28 56	74322	92052	82270	17730	07948	25678	23
38	30 56	29 4	74341	92044	82298	17702	07956	25659	22
39	30 48	29 12	74360	92035	82325	17675	07965	25640	21
40	7 30 40	4 29 20	9.74379	9.92027	9.82352	10.17648	10.07973	10.25621	20
41	30 32	29 28	74398	92018	82380	17620	07982	25602	19
42	30 24	29 36	74417	92010	82407	17593	07990	25583	18
43	30 16	29 44	74436	92002	82435	17565	07998	25564	17
44	30 8	29 52	74455	91993	82462	17538	08007	25545	16
45	7 30 0	4 30 0	9.74474	9.91985	9.82489	10.17511	10.08015	10.25526	15
46	29 52	30 8	74493	91976	82517	17483	08024	25507	14
47	29 44	30 16	74512	91968	82544	17456	08032	25488	13
48	29 36	30 24	74531	91959	82571	17429	08041	25469	12
49	29 28	30 32	74549	91951	82599	17401	08049	25451	11
50	7 29 20	4 30 40	9.74568	9.91942	9.82626	10.17374	10.08058	10.25432	10
51	29 12	30 48	74587	91934	82653	17347	08066	25413	9
52	29 4	30 56	74606	91925	82681	17319	08075	25394	8
53	28 56	31 4	74625	91917	82708	17292	08083	25375	7
54	28 48	31 12	74644	91908	82735	17265	08092	25356	6
55	7 28 40	4 31 20	9.74662	9.91900	9.82762	10.17238	10.08100	10.25338	5
56	28 32	31 28	74681	91891	82790	17210	08109	25319	4
57	28 24	31 36	74700	91883	82817	17183	08117	25300	3
58	28 16	31 44	74719	91874	82844	17156	08126	25281	2
59	28 8	31 52	74737	91866	82871	17129	08134	25263	1
60	28 0	32 0	74756	91857	82899	17101	08143	25244	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant.	Secant.	M

Log. Sines, Tangents and Secants.

34 Degs. | Degs. 145.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	7 28 0	4 32 0	9.74756	9.91857	9.82899	10.17101	10.08143	10.25244	60
1	27 52	32 8	74775	91849	82926	17074	08151	25225	59
2	27 44	32 16	74794	91840	82953	17047	08160	25206	58
3	27 36	32 24	74812	91832	82980	17020	08168	25188	57
4	27 28	32 32	74831	91823	83008	16992	08177	25169	56
5	7 27 20	4 32 40	9.74850	9.91815	9.83035	10.16965	10.08185	10.25150	55
6	27 12	32 48	74868	91806	83062	16938	08194	25132	54
7	27 4	32 56	74887	91798	83089	16911	08202	25113	53
8	26 56	33 4	74906	91789	83117	16883	08211	25094	52
9	26 48	33 12	74924	91781	83144	16856	08219	25076	51
10	7 26 40	4 33 20	9.74943	9.91772	9.83171	10.16829	10.08228	10.25057	50
11	26 32	33 28	74961	91763	83198	16802	08237	25039	49
12	26 24	33 36	74980	91755	83225	16775	08245	25020	48
13	26 16	33 44	74999	91746	83252	16748	08254	25001	47
14	26 8	33 52	75017	91738	83280	16720	08262	24983	46
15	7 26 0	4 34 0	9.75036	9.91729	9.83307	10.16693	10.08271	10.24964	45
16	25 52	34 8	75054	91720	83334	16666	08280	24946	44
17	25 44	34 16	75073	91712	83361	16639	08288	24927	43
18	25 36	34 24	75091	91703	83388	16612	08297	24909	42
19	25 28	34 32	75110	91695	83415	16585	08305	24890	41
20	7 25 20	4 34 40	9.75128	9.91686	9.83442	10.16558	10.08314	10.24872	40
21	25 12	34 48	75147	91677	83470	16530	08323	24853	39
22	25 4	34 56	75165	91669	83497	16503	08331	24835	38
23	24 56	35 4	75184	91660	83524	16476	08340	24816	37
24	24 48	35 12	75202	91651	83551	16449	08349	24798	36
25	7 24 40	4 35 20	9.75221	9.91643	9.83578	10.16422	10.08357	10.24779	35
26	24 32	35 28	75239	91634	83605	16395	08366	24761	34
27	24 24	35 36	75258	91625	83632	16368	08375	24742	33
28	24 16	35 44	75276	91617	83659	16341	08383	24724	32
29	24 8	35 52	75294	91608	83686	16314	08392	24706	31
30	7 24 0	4 36 0	9.75313	9.91599	9.83713	10.16287	10.08401	10.24687	30
31	23 52	36 8	75331	91591	83740	16260	08409	24669	29
32	23 44	36 16	75350	91582	83768	16232	08418	24650	28
33	23 36	36 24	75368	91573	83795	16205	08427	24632	27
34	23 28	36 32	75386	91565	83822	16178	08435	24614	26
35	7 23 20	4 36 40	9.75405	9.91556	9.83849	10.16151	10.08444	10.24595	25
36	23 12	36 48	75423	91547	83876	16124	08453	24577	24
37	23 4	36 56	75441	91538	83903	16097	08462	24559	23
38	22 56	37 4	75459	91530	83930	16070	08470	24541	22
39	22 48	37 12	75478	91521	83957	16043	08479	24522	21
40	7 22 40	4 37 20	9.75496	9.91512	9.83984	10.16016	10.08488	10.24504	20
41	22 32	37 28	75514	91504	84011	15989	08496	24486	19
42	22 24	37 36	75533	91495	84038	15962	08505	24467	18
43	22 16	37 44	75551	91486	84065	15935	08514	24449	17
44	22 8	37 52	75569	91477	84092	15908	08523	24431	16
45	7 22 0	4 38 0	9.75587	9.91469	9.84119	10.15881	10.08531	10.24413	15
46	21 52	38 8	75605	91460	84146	15854	08540	24395	14
47	21 44	38 16	75624	91451	84173	15827	08549	24376	13
48	21 36	38 24	75642	91442	84200	15800	08558	24358	12
49	21 28	38 32	75660	91433	84227	15773	08567	24340	11
50	7 21 20	4 38 40	9.75678	9.91425	9.84254	10.15746	10.08575	10.24322	10
51	21 12	38 48	75696	91416	84280	15720	08584	24304	9
52	21 4	38 56	75714	91407	84307	15693	08593	24286	8
53	20 56	39 4	75733	91398	84334	15666	08602	24267	7
54	20 48	39 12	75751	91389	84361	15639	08611	24249	6
55	7 20 40	4 39 20	9.75769	9.91381	9.84388	10.15612	10.08619	10.24231	5
56	20 32	39 28	75787	91372	84415	15585	08628	24213	4
57	20 24	39 36	75805	91363	84442	15558	08637	24195	3
58	20 16	39 44	75823	91354	84469	15531	08646	24177	2
59	20 8	39 52	75841	91345	84496	15504	08655	24159	1
60	20 0	40 0	75859	91336	84523	15477	08664	24141	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

124 Degs. | Degs. 55.

35 Degs. Degs. 144.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	7 20 0	4 40 0	9.75859	9.91336	9.84523	10.15477	10.08664	10.24141	60
1	19 52	40 8	75877	91328	84550	15450	08672	24123	59
2	19 44	40 16	75895	91319	84576	15424	08681	24105	58
3	19 36	40 24	75913	91310	84603	15397	08690	24087	57
4	19 28	40 32	75931	91301	84630	15370	08699	24069	56
5	7 19 20	4 40 40	9.75949	9.91292	9.84657	10.15343	10.08708	10.24051	55
6	19 12	40 48	75967	91283	84684	15316	08717	24033	54
7	19 4	40 56	75985	91274	84711	15289	08726	24015	53
8	18 56	41 4	76003	91266	84738	15262	08734	23997	52
9	18 48	41 12	76021	91257	84764	15236	08743	23979	51
10	7 18 40	4 41 20	9.76039	9.91248	9.84791	10.15209	10.08752	10.23961	50
11	18 32	41 28	76057	91239	84818	15182	08761	23943	49
12	18 24	41 36	76075	91230	84845	15155	08770	23925	48
13	18 16	41 44	76093	91221	84872	15128	08779	23907	47
14	18 8	41 52	76111	91212	84899	15101	08788	23889	46
15	7 18 0	4 42 0	9.76129	9.91203	9.84925	10.15075	10.08797	10.23871	45
16	17 52	42 8	76146	91194	84952	15048	08806	23854	44
17	17 44	42 16	76164	91185	84979	15021	08815	23836	43
18	17 36	42 24	76182	91176	85006	14994	08824	23818	42
19	17 28	42 32	76200	91167	85033	14967	08833	23800	41
20	7 17 20	4 42 40	9.76218	9.91158	9.85059	10.14941	10.08842	10.23782	40
21	17 12	42 48	76236	91149	85086	14914	08851	23764	39
22	17 4	42 56	76253	91141	85113	14887	08859	23747	38
23	16 56	43 4	76271	91132	85140	14860	08868	23729	37
24	16 48	43 12	76289	91123	85166	14834	08877	23711	36
25	7 16 40	4 43 20	9.76307	9.91114	9.85193	10.14807	10.08886	10.23693	35
26	16 32	43 28	76324	91105	85220	14780	08895	23676	34
27	16 24	43 36	76342	91096	85247	14753	08904	23658	33
28	16 16	43 44	76360	91087	85273	14727	08913	23640	32
29	16 8	43 52	76378	91078	85300	14700	08922	23622	31
30	7 16 0	4 44 0	9.76395	9.91069	9.85327	10.14673	10.08931	10.23605	30
31	15 52	44 8	76413	91060	85354	14646	08940	23587	29
32	15 44	44 16	76431	91051	85380	14620	08949	23569	28
33	15 36	44 24	76448	91042	85407	14593	08958	23552	27
34	15 28	44 32	76466	91033	85434	14566	08967	23534	26
35	7 15 20	4 44 40	9.76484	9.91023	9.85460	10.14540	10.08977	10.23516	25
36	15 12	44 48	76501	91014	85487	14513	08986	23499	24
37	15 4	44 56	76519	91005	85514	14486	08995	23481	23
38	14 56	45 4	76537	90996	85540	14460	09004	23463	22
39	14 48	45 12	76554	90987	85567	14433	09013	23446	21
40	7 14 40	4 45 20	9.76572	9.90978	9.85594	10.14406	10.09022	10.23428	20
41	14 32	45 28	76590	90969	85620	14380	09031	23410	19
42	14 24	45 36	76607	90960	85647	14353	09040	23393	18
43	14 16	45 44	76625	90951	85674	14326	09049	23375	17
44	14 8	45 52	76642	90942	85700	14300	09058	23358	16
45	7 14 0	4 46 0	9.76660	9.90933	9.85727	10.14273	10.09067	10.23340	15
46	13 52	46 8	76677	90924	85754	14246	09076	23323	14
47	13 44	46 16	76695	90915	85780	14220	09085	23305	13
48	13 36	46 24	76712	90906	85807	14193	09094	23288	12
49	13 28	46 32	76730	90896	85834	14166	09104	23270	11
50	7 13 20	4 46 40	9.76747	9.90887	9.85860	10.14140	10.09113	10.23253	10
51	13 12	46 48	76765	90878	85887	14113	09122	23235	9
52	13 4	46 56	76782	90869	85913	14087	09131	23218	8
53	12 56	47 4	76800	90860	85940	14060	09140	23200	7
54	12 48	47 12	76817	90851	85967	14033	09149	23183	6
55	7 12 40	4 47 20	9.76835	9.90842	9.85993	10.14007	10.09158	10.23165	5
56	12 32	47 28	76852	90832	86020	13980	09168	23148	4
57	12 24	47 36	76870	90823	86046	13954	09177	23130	3
58	12 16	47 44	76887	90814	86073	13927	09186	23113	2
59	12 8	47 52	76904	90805	86100	13900	09195	23096	1
60	12 0	48 0	76922	90796	86126	13874	09204	23078	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

125 Degs. Degs. 54.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	7 12 0	4 48 0	9.76922	9.90796	9.86126	10.13874	10.09204	10.23078	60
1	11 52	48 8	76939	90787	86153	13847	09213	23061	59
2	11 44	48 16	76957	90777	86179	13821	09223	23043	58
3	11 36	48 24	76974	90768	86206	13794	09232	23026	57
4	11 28	48 32	76991	90759	86232	13768	09241	23009	56
5	7 11 20	4 48 40	9.77009	9.90750	9.86259	10.13741	10.09250	10.22991	55
6	11 12	48 48	77026	90741	86285	13715	09259	22974	54
7	11 4	48 56	77043	90731	86312	13688	09269	22957	53
8	10 56	49 4	77061	90722	86338	13662	09278	22939	52
9	10 48	49 12	77078	90713	86365	13635	09287	22922	51
10	7 10 40	4 49 20	9.77095	9.90704	9.86392	10.13608	10.09296	10.22905	50
11	10 32	49 28	77112	90694	86418	13582	09306	22888	49
12	10 24	49 36	77130	90685	86445	13555	09315	22870	48
13	10 16	49 44	77147	90676	86471	13529	09324	22853	47
14	10 8	49 52	77164	90667	86498	13502	09333	22836	46
15	7 10 0	4 50 0	9.77181	9.90657	9.86524	10.13476	10.09343	10.22819	45
16	9 52	50 8	77199	90648	86551	13449	09352	22801	44
17	9 44	50 16	77216	90639	86577	13423	09361	22784	43
18	9 36	50 24	77233	90630	86603	13397	09370	22767	42
19	9 28	50 32	77250	90620	86630	13370	09380	22750	41
20	7 9 20	4 50 40	9.77268	9.90611	9.86656	10.13344	10.09389	10.22732	40
21	9 12	50 48	77285	90602	86683	13317	09398	22715	39
22	9 4	50 56	77302	90592	86709	13291	09408	22698	38
23	8 56	51 4	77319	90583	86736	13264	09417	22681	37
24	8 48	51 12	77336	90574	86762	13238	09426	22664	36
25	7 8 40	4 51 20	9.77353	9.90565	9.86789	10.13211	10.09435	10.22647	35
26	8 32	51 28	77370	90555	86815	13185	09445	22630	34
27	8 24	51 36	77387	90546	86842	13158	09454	22613	33
28	8 16	51 44	77405	90537	86868	13132	09463	22595	32
29	8 8	51 52	77422	90527	86894	13106	09473	22578	31
30	7 8 0	4 52 0	9.77439	9.90518	9.86921	10.13079	10.09482	10.22561	30
31	7 52	52 8	77456	90509	86947	13053	09491	22544	29
32	7 44	52 16	77473	90499	86974	13026	09501	22527	28
33	7 36	52 24	77490	90490	87000	13000	09510	22510	27
34	7 28	52 32	77507	90480	87027	12973	09520	22493	26
35	7 7 20	4 52 40	9.77524	9.90471	9.87053	10.12947	10.09529	10.22476	25
36	7 12	52 48	77541	90462	87079	12921	09538	22459	24
37	7 4	52 56	77558	90452	87106	12894	09548	22442	23
38	6 56	53 4	77575	90443	87132	12868	09557	22425	22
39	6 48	53 12	77592	90434	87158	12842	09566	22408	21
40	7 6 40	4 53 20	9.77609	9.90424	9.87185	10.12815	10.09576	10.22391	20
41	6 32	53 28	77626	90415	87211	12789	09585	22374	19
42	6 24	53 36	77643	90405	87238	12762	09595	22357	18
43	6 16	53 44	77660	90396	87264	12736	09604	22340	17
44	6 8	53 52	77677	90386	87290	12710	09614	22323	16
45	7 6 0	4 54 0	9.77694	9.90377	9.87317	10.12683	10.09623	10.22306	15
46	5 52	54 8	77711	90368	87343	12657	09632	22289	14
47	5 44	54 16	77728	90358	87369	12631	09642	22272	13
48	5 36	54 24	77744	90349	87396	12604	09651	22256	12
49	5 28	54 32	77761	90339	87422	12578	09661	22239	11
50	7 5 20	4 54 40	9.77778	9.90330	9.87448	10.12552	10.09670	10.22222	10
51	5 12	54 48	77795	90320	87475	12525	09680	22205	9
52	5 4	54 56	77812	90311	87501	12499	09689	22188	8
53	4 56	55 4	77829	90301	87527	12473	09699	22171	7
54	4 48	55 12	77846	90292	87554	12446	09708	22154	6
55	7 4 40	4 55 20	9.77862	9.90282	9.87580	10.12420	10.09718	10.22138	5
56	4 32	55 28	77879	90273	87606	12394	09727	22121	4
57	4 24	55 36	77896	90263	87633	12367	09737	22104	3
58	4 16	55 44	77913	90254	87659	12341	09746	22087	2
59	4 8	55 52	77930	90244	87685	12315	09756	22070	1
60	4 0	56 0	77946	90235	87711	12289	09765	22054	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	7 4 0	4 56 0	9.77946	9.90235	9.87711	10.12289	10.09765	10.22054	60
1	3 52	56 8	77963	90225	87738	12262	09775	22037	59
2	3 44	56 16	77980	90216	87764	12236	09784	22020	58
3	3 36	56 24	77997	90206	87790	12210	09794	22003	57
4	3 28	56 32	78013	90197	87817	12183	09803	21987	56
5	7 3 20	4 56 40	9.78030	9.90187	9.87843	10.12157	10.09813	10.21970	55
6	3 12	56 48	78047	90178	87869	12131	09822	21953	54
7	3 4	56 56	78063	90168	87895	12105	09832	21937	53
8	2 56	57 4	78080	90159	87922	12078	09841	21920	52
9	2 48	57 12	78097	90149	87948	12052	09851	21903	51
10	7 2 40	4 57 20	9.78113	9.90139	9.87974	10.12026	10.09861	10.21887	50
11	2 32	57 28	78130	9013[illegible]	88000	12000	09870	21870	49
12	2 24	57 36	78147	90120	88027	11973	09880	21853	48
13	2 16	57 44	78163	90111	88053	11947	09889	21837	47
14	2 8	57 52	78180	90101	88079	11921	09899	21820	46
15	7 2 0	4 58 0	9.78197	9.90091	9.88105	10.11895	10.09909	10.21803	45
16	1 52	58 8	78213	90082	88131	11869	09918	21787	44
17	1 44	58 16	78230	90072	88158	11842	09928	21770	43
18	1 36	58 24	78246	90063	88184	11816	09937	21754	42
19	1 28	58 32	78263	90053	88210	11790	09947	21737	41
20	7 1 20	4 58 40	9.78280	9.90043	9.88236	10.11764	10.09957	10.21720	40
21	1 12	58 48	78296	90034	88262	11738	09966	21704	39
22	1 4	58 56	78313	90024	88289	11711	09976	21687	38
23	0 56	59 4	78329	90014	88315	11685	09986	21671	37
24	0 48	59 12	78346	90005	88341	11659	09995	21654	36
25	7 0 40	4 59 20	9.78362	9.89995	9.88367	10.11633	10.10005	10.21638	35
26	0 32	59 28	78379	89985	88393	11607	10015	21621	34
27	0 24	59 36	78395	89976	88420	11580	10024	21605	33
28	0 16	59 44	78412	89966	88446	11554	10034	21588	32
29	0 8	59 52	78428	89956	88472	11528	10044	21572	31
30	7 0 0	5 0 0	9.78445	9.89947	9.88498	10.11502	10.10053	10.21555	30
31	6 59 52	0 8	78461	89937	88524	11476	10063	21539	29
32	59 44	0 16	78478	89927	88550	11450	10073	21522	28
33	59 36	0 24	78494	89918	88577	11423	10082	21506	27
34	59 28	0 32	78510	89908	88603	11397	10092	21490	26
35	6 59 20	5 0 40	9.78527	9.89898	9.88629	10.11371	10.10102	10.21473	25
36	59 12	0 48	78543	89888	88655	11345	10112	21457	24
37	59 4	0 56	78560	89879	88681	11319	10121	21440	23
38	58 56	1 4	78576	89869	88707	11293	10131	21424	22
39	58 48	1 12	78592	89859	88733	11267	10141	21408	21
40	6 58 40	5 1 20	9.78609	9.89849	9.88759	10.11241	10.10151	10.21391	20
41	58 32	1 28	78625	89840	88786	11214	10160	21375	19
42	58 24	1 36	78642	89830	88812	11188	10170	21358	18
43	58 16	1 44	78658	89820	88838	11162	10180	21342	17
44	58 8	1 52	78674	89810	88864	11136	10190	21326	16
45	6 58 0	5 2 0	9.78691	9.89801	9.88890	10.11110	10.10199	10.21309	15
46	57 52	2 8	78707	89791	88916	11084	10209	21293	14
47	57 44	2 16	78723	89781	88942	11058	10219	21277	13
48	57 36	2 24	78739	89771	88968	11032	10229	21261	12
49	57 28	2 32	78756	89761	88994	11006	10239	21244	11
50	6 57 20	5 2 40	9.78772	9.89752	9.89020	10.10980	10.10248	10.21228	10
51	57 12	2 48	78788	89742	89046	10954	10258	21212	9
52	57 4	2 56	78805	89732	89073	10927	10268	21195	8
53	56 56	3 4	78821	89722	89099	10901	10278	21179	7
54	56 48	3 12	78837	89712	89125	10875	10288	21163	6
55	6 56 40	5 3 20	9.78853	9.89702	9.89151	10.10849	10.10298	10.21147	5
56	56 32	3 28	78869	89693	89177	10823	10307	21131	4
57	56 24	3 36	78886	89683	89203	10797	10317	21114	3
58	56 16	3 44	78902	89673	89229	10771	10327	21098	2
59	56 8	3 52	78918	89663	89255	10745	10337	21082	1
60	56 0	4 0	78934	89653	89281	10719	10347	21066	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	6 56 0	5 4 0	9.78934	9.89653	9.89281	10.10719	10.10347	10.21066	60
1	55 52	4 8	78950	89643	89307	10693	10357	21050	59
2	55 44	4 16	78967	89633	89333	10667	10367	21033	58
3	55 36	4 24	78983	89624	89359	10641	10376	21017	57
4	55 28	4 32	78999	89614	89385	10615	10386	21001	56
5	6 55 20	5 4 40	9.79015	9.89604	9.89411	10.10589	10.10396	10.20985	55
6	55 12	4 48	79031	89594	89437	10563	10406	20969	54
7	55 4	4 56	79047	89584	89463	10537	10416	20953	53
8	54 56	5 4	79063	89574	89489	10511	10426	20937	52
9	54 48	5 12	79079	89564	89515	10485	10436	20921	51
10	6 54 40	5 5 20	9.79095	9.89554	9.89541	10.10459	10.10446	10.20905	50
11	54 32	5 28	79111	89544	89567	10433	10456	20889	49
12	54 24	5 36	79128	89534	89593	10407	10466	20872	48
13	54 16	5 44	79144	89524	89619	10381	10476	20856	47
14	54 8	5 52	79160	89514	89645	10355	10486	20840	46
15	6 54 0	5 6 0	9.79176	9.89504	9.89671	10.10329	10.10496	10.20824	45
16	53 52	6 8	79192	89495	89697	10303	10505	20808	44
17	53 44	6 16	79208	89485	89723	10277	10515	20792	43
18	53 36	6 24	79224	89475	89749	10251	10525	20776	42
19	53 28	6 32	79240	89465	89775	10225	10535	20760	41
20	6 53 20	5 6 40	9.79256	9.89455	9.89801	10.10199	10.10545	10.20744	40
21	53 12	6 48	79272	89445	89827	10173	10555	20728	39
22	53 4	6 56	79288	89435	89853	10147	10565	20712	38
23	52 56	7 4	79304	89425	89879	10121	10575	20696	37
24	52 48	7 12	79319	89415	89905	10095	10585	20681	36
25	6 52 40	5 7 20	9.79335	9.89405	9.89931	10.10069	10.10595	10.20665	35
26	52 32	7 28	79351	89395	89957	10043	10605	20649	34
27	52 24	7 36	79367	89385	89983	10017	10615	20633	33
28	52 16	7 44	79383	89375	90009	09991	10625	20617	32
29	52 8	7 52	79399	89364	90035	09965	10636	20601	31
30	6 52 0	5 8 0	9.79415	9.89354	9.90061	10.09939	10.10646	10.20585	30
31	51 52	8 8	79431	89344	90086	09914	10656	20569	29
32	51 44	8 16	79447	89334	90112	09888	10666	20553	28
33	51 36	8 24	79463	89324	90138	09862	10676	20537	27
34	51 28	8 32	79478	89314	90164	09836	10686	20522	26
35	6 51 20	5 8 40	9.79494	9.89304	9.90190	10.09810	10.10696	10.20506	25
36	51 12	8 48	79510	89294	90216	09784	10706	20490	24
37	51 4	8 56	79526	89284	90242	09758	10716	20474	23
38	50 56	9 4	79542	89274	90268	09732	10726	20458	22
39	50 48	9 12	79558	89264	90294	09706	10736	20442	21
40	6 50 40	5 9 20	9.79573	9.89254	9.90320	10.09680	10.10746	10.20427	20
41	50 32	9 28	79589	89244	90346	09654	10756	20411	19
42	50 24	9 36	79605	89233	90371	09629	10767	20395	18
43	50 16	9 44	79621	89223	90397	09603	10777	20379	17
44	50 8	9 52	79636	89213	90423	09577	10787	20364	16
45	6 50 0	5 10 0	9.79652	9.89203	9.90449	10.09551	10.10797	10.20348	15
46	49 52	10 8	79668	89193	90475	09525	10807	20332	14
47	49 44	10 16	79684	89183	90501	09499	10817	20316	13
48	49 36	10 24	79699	89173	90527	09473	10827	20301	12
49	49 28	10 32	79715	89162	90553	09447	10838	20285	11
50	6 49 20	5 10 40	9.79731	9.89152	9.90578	10.09422	10.10848	10.20269	10
51	49 12	10 48	79746	89142	90604	09396	10858	20254	9
52	49 4	10 56	79762	89132	90630	09370	10868	20238	8
53	48 56	11 4	79778	89122	90656	09344	10878	20222	7
54	48 48	11 12	79793	89112	90682	09318	10888	20207	6
55	6 48 40	5 11 20	9.79809	9.89101	9.90708	10.09292	10.10899	10.20191	5
56	48 32	11 28	79825	89091	90734	09266	10909	20175	4
57	48 24	11 36	79840	89081	90759	09241	10919	20160	3
58	48 16	11 44	79856	89071	90785	09215	10929	20144	2
59	48 8	11 52	79872	89060	90811	09189	10940	20128	1
60	48 0	12 0	79887	89050	90837	09163	10950	20113	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

39 Degs. | Deg. 140.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	6 48 0	5 12 0	9.79887	9.89050	9.90837	10.09163	10.10950	10.20113	60
1	47 52	12 8	79903	89040	90863	09137	10960	20097	59
2	47 44	12 16	79918	89030	90889	09111	10970	20082	58
3	47 36	12 24	79934	89020	90914	09086	10980	20066	57
4	47 28	12 32	79950	89009	90940	09060	10991	20050	56
5	6 47 20	5 12 40	9.79965	9.88999	9.90966	10.09034	10.11001	10.20035	55
6	47 12	12 48	79981	88989	90992	09008	11011	20019	54
7	47 4	12 56	79996	88978	91018	08982	11022	20004	53
8	46 56	13 4	80012	88968	91043	08957	11032	19988	52
9	46 48	13 12	80027	88958	91069	08931	11042	19973	51
10	6 46 40	5 13 20	9.80043	9.88948	9.91095	10.08905	10.11052	10.19957	50
11	46 32	13 28	80058	88937	91121	08879	11063	19942	49
12	46 24	13 36	80074	88927	91147	08853	11073	19926	48
13	46 16	13 44	80089	88917	91172	08828	11083	19911	47
14	46 8	13 52	80105	88906	91198	08802	11094	19895	46
15	6 46 0	5 14 0	9.80120	9.88896	9.91224	10.08776	10.11104	10.19880	45
16	45 52	14 8	80136	88886	91250	08750	11114	19864	44
17	45 44	14 16	80151	88875	91276	08724	11125	19849	43
18	45 36	14 24	80166	88865	91301	08699	11135	19834	42
19	45 28	14 32	80182	88855	91327	08673	11145	19818	41
20	6 45 20	5 14 40	9.80197	9.88844	9.91353	10.08647	10.11156	10.19803	40
21	45 12	14 48	80213	88834	91379	08621	11166	19787	39
22	45 4	14 56	80228	88824	91404	08596	11176	19772	38
23	44 56	15 4	80244	88813	91430	08570	11187	19756	37
24	44 48	15 12	80259	88803	91456	08544	11197	19741	36
25	6 44 40	5 15 20	9.80274	9.88793	9.91482	10.08518	10.11207	10.19726	35
26	44 32	15 28	80290	88782	91507	08493	11218	19710	34
27	44 24	15 36	80305	88772	91533	08467	11228	19695	33
28	44 16	15 44	80320	88761	91559	08441	11239	19680	32
29	44 8	15 52	80336	88751	91585	08415	11249	19664	31
30	6 44 0	5 16 0	9.80351	9.88741	9.91610	10.08390	10.11259	10.19649	30
31	43 52	16 8	80366	88730	91636	08364	11270	19634	29
32	43 44	16 16	80382	88720	91662	08338	11280	19618	28
33	43 36	16 24	80397	88709	91688	08312	11291	19603	27
34	43 28	16 32	80412	88699	91713	08287	11301	19588	26
35	6 43 20	5 16 40	9.80428	9 88688	9.91739	10.08261	10.11312	10.19572	25
36	43 12	16 48	80443	88678	91765	08235	11322	19557	24
37	43 4	16 56	80458	88668	91791	08209	11332	19542	23
38	42 56	17 4	80473	88657	91816	08184	11343	19527	22
39	42 48	17 12	80489	88647	91842	08158	11353	19511	21
40	6 42 40	5 17 20	9.80504	9.88636	9.91868	10.08132	10.11364	10.19496	20
41	42 32	17 28	80519	88626	91893	08107	11374	19481	19
42	42 24	17 36	80534	88615	91919	08081	11385	19466	18
43	42 16	17 44	80550	88605	91945	08055	11395	19450	17
44	42 8	17 52	80565	88594	91971	08029	11406	19435	16
45	6 42 0	5 18 0	9.80580	9.88584	9.91996	10.08004	10.11416	10.19420	15
46	41 52	18 8	80595	88573	92022	07978	11427	19405	14
47	41 44	18 16	80610	88563	92048	07952	11437	19390	13
48	41 36	18 24	80625	88552	92073	07927	11448	19375	12
49	41 28	18 32	80641	88542	92099	07901	11458	19359	11
50	6 41 20	5 18 40	9.80656	9.88531	9.92125	10.07875	10.11469	10.19344	10
51	41 12	18 48	80671	88521	92150	07850	11479	19329	9
52	41 4	18 56	80686	88510	92176	07824	11490	19314	8
53	40 56	19 4	80701	88499	92202	07798	11501	19299	7
54	40 48	19 12	80716	88489	92227	07773	11511	19284	6
55	6 40 40	5 19 20	9.80731	9.88478	9.92253	10.07747	10.11522	10.19269	5
56	40 32	19 28	80746	88468	92279	07721	11532	19254	4
57	40 24	19 36	80762	88457	92304	07696	11543	19238	3
58	40 16	19 44	80777	88447	92330	07670	11553	19223	2
59	40 8	19 52	80792	88436	92356	07644	11564	19208	1
60	40 0	20 0	80807	88425	92381	07619	11575	19193	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

129 Degs. | Degs. 50.

40 Degs. Degs. 139.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	6 40 0	5 20 0	9.80807	9.88425	9.92381	10.07619	10.11575	10.19193	60
1	39 52	20 8	80822	88415	92407	07593	11585	19178	59
2	39 44	20 16	80837	88404	92433	07567	11596	19163	58
3	39 36	20 24	80852	88394	92458	07542	11606	19148	57
4	39 28	20 32	80867	88383	92484	07516	11617	19133	56
5	6 39 20	5 20 40	9.80882	9.88372	9.92510	10.07490	10.11628	10.19118	55
6	39 12	20 48	80897	88362	92535	07465	11638	19103	54
7	39 4	20 56	80912	88351	92561	07439	11649	19088	53
8	38 56	21 4	80927	88340	92587	07413	11660	19073	52
9	38 48	21 12	80942	88330	92612	07388	11670	19058	51
10	6 38 40	5 21 20	9.80957	9.88319	9.92638	10.07362	10.11681	10.19043	50
11	38 32	21 28	80972	88308	92663	07337	11692	19028	49
12	38 24	21 36	80987	88298	92689	07311	11702	19013	48
13	38 16	21 44	81002	88287	92715	07285	11713	18998	47
14	38 8	21 52	81017	88276	92740	07260	11724	18983	46
15	6 38 0	5 22 0	9.81032	9.88266	9.92766	10.07234	10.11734	10.18968	45
16	37 52	22 8	81047	88255	92792	07208	11745	18953	44
17	37 44	22 16	81061	88244	92817	07183	11756	18939	43
18	37 36	22 24	81076	88234	92843	07157	11766	18924	42
19	37 28	22 32	81091	88223	92868	07132	11777	18909	41
20	6 37 20	5 22 40	9.81106	9.88212	9.92894	10.07106	10.11788	10.18894	40
21	37 12	22 48	81121	88201	92920	07080	11799	18879	39
22	37 4	22 56	81136	88191	92945	07055	11809	18864	38
23	36 56	23 4	81151	88180	92971	07029	11820	18849	37
24	36 48	23 12	81166	88169	92996	07004	11831	18834	36
25	6 36 40	5 23 20	9.81180	9.88158	9.93022	10.06978	10.11842	10.18820	35
26	36 32	23 28	81195	88148	93048	06952	11852	18805	34
27	36 24	23 36	81210	88137	93073	06927	11863	18790	33
28	36 16	23 44	81225	88126	93099	06901	11874	18775	32
29	36 8	23 52	81240	88115	93124	06876	11885	18760	31
30	6 36 0	5 24 0	9.81254	9.88105	9.93150	10.06850	10.11895	10.18746	30
31	35 52	24 8	81269	88094	93175	06825	11906	18731	29
32	35 44	24 16	81284	88083	93201	06799	11917	18716	28
33	35 36	24 24	81299	88072	93227	06773	11928	18701	27
34	35 28	24 32	81314	88061	93252	06748	11939	18686	26
35	6 35 20	5 24 40	9.81328	9.88051	9.93278	10.06722	10.11949	10.18672	25
36	35 12	24 48	81343	88040	93303	06697	11960	18657	24
37	35 4	24 56	81358	88029	93329	06671	11971	18642	23
38	34 56	25 4	81372	88018	93354	06646	11982	18628	22
39	34 48	25 12	81387	88007	93380	06620	11993	18613	21
40	6 34 40	5 25 20	9.81402	9.87996	9.93406	10.06594	10.12004	10.18598	20
41	34 32	25 28	81417	87985	93431	06569	12015	18583	19
42	34 24	25 36	81431	87975	93457	06543	12025	18569	18
43	34 16	25 44	81446	87964	93482	06518	12036	18554	17
44	34 8	25 52	81461	87953	93508	06492	12047	18539	16
45	6 34 0	5 26 0	9.81475	9.87942	9.93533	10.06467	10.12058	10.18525	15
46	33 52	26 8	81490	87931	93559	06441	12069	18510	14
47	33 44	26 16	81505	87920	93584	06416	12080	18495	13
48	33 36	26 24	81519	87909	93610	06390	12091	18481	12
49	33 28	26 32	81534	87898	93636	06364	12102	18466	11
50	6 33 20	5 26 40	9.81549	9.87887	9.93661	10.06339	10.12113	10.18451	10
51	33 12	26 48	81563	87877	93687	06313	12123	18437	9
52	33 4	26 56	81578	87866	93712	06288	12134	18422	8
53	32 56	27 4	81592	87855	93738	06262	12145	18408	7
54	32 48	27 12	81607	87844	93763	06237	12156	18393	6
55	6 32 40	5 27 20	9.81622	9.87833	9.93789	10.06211	10.12167	10.18378	5
56	32 32	27 28	81636	87822	93814	06186	12178	18364	4
57	32 24	27 36	81651	87811	93840	06160	12189	18349	3
58	32 16	27 44	81665	87800	93865	06135	12200	18335	2
59	32 8	27 52	81680	87789	93891	06109	12211	18320	1
60	32 0	28 0	81694	87778	93916	06084	12222	18306	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

130 Degs. Degs. 49.

Log. Sines, Tangents and Secants.

41 Degs. Degs. 138.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	6 32 0	5 28 0	9.81694	9.87778	9.93916	10.06084	10.12222	10.18306	60
1	31 52	28 8	81709	87767	93942	06058	12233	18291	59
2	31 44	28 16	81723	87756	93967	06033	12244	18277	58
3	31 36	28 24	81738	87745	93993	06007	12255	18262	57
4	31 28	28 32	81752	87734	94018	05982	12266	18248	56
5	6 31 20	5 28 40	9.81767	9.87723	9.94044	10.05956	10.12277	10.18233	55
6	31 12	28 48	81781	87712	94069	05931	12288	18219	54
7	31 4	28 56	81796	87701	94095	05905	12299	18204	53
8	30 56	29 4	81810	87690	94120	05880	12310	18190	52
9	30 48	29 12	81825	87679	94146	05854	12321	18175	51
10	6 30 40	5 29 20	9.81839	9.87668	9.94171	10.05829	10.12332	10.18161	50
11	30 32	29 28	81854	87657	94197	05803	12343	18146	49
12	30 24	29 36	81868	87646	94222	05778	12354	18132	48
13	30 16	29 44	81882	87635	94248	05752	12365	18118	47
14	30 8	29 52	81897	87624	94273	05727	12376	18103	46
15	6 30 0	5 30 0	9.81911	9.87613	9.94299	10.05701	10.12387	10.18089	45
16	29 52	30 8	81926	87601	94324	05676	12399	18074	44
17	29 44	30 16	81940	87590	94350	05650	12410	18060	43
18	29 36	30 24	81955	87579	94375	05625	12421	18045	42
19	29 28	30 32	81969	87568	94401	05599	12432	18031	41
20	6 29 20	5 30 40	9.81983	9.87557	9.94426	10.05574	10.12443	10.18017	40
21	29 12	30 48	81998	87546	94452	05548	12454	18002	39
22	29 4	30 56	82012	87535	94477	05523	12465	17988	38
23	28 56	31 4	82026	87524	94503	05497	12476	17974	37
24	28 48	31 12	82041	87513	94528	05472	12487	17959	36
25	6 28 40	5 31 20	9.82055	9.87501	9.94554	10.05446	10.12499	10.17945	35
26	28 32	31 28	82069	87490	94579	05421	12510	17931	34
27	28 24	31 36	82084	87479	94604	05396	12521	17916	33
28	28 16	31 44	82098	87468	94630	05370	12532	17902	32
29	28 8	31 52	82112	87457	94655	05345	12543	17888	31
30	6 28 0	5 32 0	9.82126	9.87446	9.94681	10.05319	10.12554	10.17874	30
31	27 52	32 8	82141	87434	94706	05294	12566	17859	29
32	27 44	32 16	82155	87423	94732	05268	12577	17845	28
33	27 36	32 24	82169	87412	94757	05243	12588	17831	27
34	27 28	32 32	82184	87401	94783	05217	12599	17816	26
35	6 27 20	5 32 40	9.82198	9.87390	9.94808	10.05192	10.12610	10.17802	25
36	27 12	32 48	82212	87378	94834	05166	12622	17788	24
37	27 4	32 56	82226	87367	94859	05141	12633	17774	23
38	26 56	33 4	82240	87356	94884	05116	12644	17760	22
39	26 48	33 12	82255	87345	94910	05090	12655	17745	21
40	6 26 40	5 33 20	9.82269	9.87334	9.94935	10.05065	10.12666	10.17731	20
41	26 32	33 28	82283	87322	94961	05039	12678	17717	19
42	26 24	33 36	82297	87311	94986	05014	12689	17703	18
43	26 16	33 44	82311	87300	95012	04988	12700	17689	17
44	26 8	33 52	82326	87288	95037	04963	12712	17674	16
45	6 26 0	5 34 0	9.82340	9.87277	9.95062	10.04938	10.12723	10.17660	15
46	25 52	34 8	82354	87266	95088	04912	12734	17646	14
47	25 44	34 16	82368	87255	95113	04887	12745	17632	13
48	25 36	34 24	82382	87243	95139	04861	12757	17618	12
49	25 28	34 32	82396	87232	95164	04836	12768	17604	11
50	6 25 20	5 34 40	9.82410	9.87221	9.95190	10.04810	10.12779	10.17590	10
51	25 12	34 48	82424	87209	95215	04785	12791	17576	9
52	25 4	34 56	82439	87198	95240	04760	12802	17561	8
53	24 56	35 4	82453	87187	95266	04734	12813	17547	7
54	24 48	35 12	82467	87175	95291	04709	12825	17533	6
55	6 24 40	5 35 20	9.82481	9.87164	9.95317	10.04683	10.12836	10.17519	5
56	24 32	35 28	82495	87153	95342	04658	12847	17505	4
57	24 24	35 36	82509	87141	95368	04632	12859	17491	3
58	24 16	35 44	82523	87130	95393	04607	12870	17477	2
59	24 8	35 52	82537	87119	95418	04582	12881	17463	1
60	24 0	36 0	82551	87107	95444	04556	12893	17449	0
M	Hour P.M.	Hour A.M.	Co-sine.	Sine.	Co-tang.	Tangent.	Co-secant	Secant.	M

131 Degs. Degs. 48.

Log. Sines, Tangents and Secants.

M	Hour A.M.	Hour P.M.	Sine.	Co-sine.	Tangent.	Co-tang.	Secant.	Co-secant	M
0	6 8 0	5 52 0	9.84177	9.85693	9.98484	10.01516	10.14307	10.15823	60
1	7 52	52 8	84190	85681	98509	01491	14319	15810	59
2	7 44	52 16	84203	85669	98534	01466	14331	15797	58
3	7 36	52 24	84216	85657	98560	01440	14343	15784	57
4	7 28	52 32	84229	85645	98585	01415	14355	15771	56
5	6 7 20	5 52 40	9.84242	9.85632	9.98610	10.01390	10.14368	10.15758	55
6	7 12	52 48	84255	85620	98635	01365	14380	15745	54
7	7 4	52 56	84269	85608	98661	01339	14392	15731	53
8	6 56	53 4	84282	85596	98686	01314	14404	15718	52
9	6 48	53 12	84295	85583	98711	01289	14417	15705	51
10	6 6 40	5 53 20	9.84308	9.85571	9.98737	10.01263	10.14429	10.15692	50
11	6 32	53 28	84321	85559	98762	01238	14441	15679	49
12	6 24	53 36	84334	85547	98787	01213	14453	15666	48
13	6 16	53 44	84347	85534	98812	01188	14466	15653	47
14	6 8	53 52	84360	85522	98838	01162	14478	15640	46
15	6 6 0	5 54 0	9.84373	9.85510	9.98863	10.01137	10.14490	10.15627	45
16	5 52	54 8	84385	85497	98888	01112	14503	15615	44
17	5 44	54 16	84398	85485	98913	01087	14515	15602	43
18	5 36	54 24	84411	85473	98939	01061	14527	15589	42
19	5 28	54 32	84424	85460	98964	01036	14540	15576	41
20	6 5 20	5 54 40	9.84437	9.85148	9.98989	10.01011	10.14552	10.15563	40
21	5 12	54 48	84450	85436	99015	00985	14564	15550	39
22	5 4	54 56	84463	85423	99040	00960	14577	15537	38
23	4 56	55 4	84476	85411	99065	00935	14589	15524	37
24	4 48	55 12	84489	85399	99090	00910	14601	15511	36
25	6 4 40	5 55 20	9.84502	9.85386	9.99116	10.00884	10.14614	10.15498	35
26	4 32	55 28	84515	85374	99141	00859	14626	15485	34
27	4 24	55 36	84528	85361	99166	00834	14639	15472	33
28	4 16	55 44	84540	85349	99191	00809	14651	15460	32
29	4 8	55 52	84553	85337	99217	00783	14663	15447	31
30	6 4 0	5 56 0	9.84566	9.85324	9.99242	10.00758	10.14676	10.15434	30
31	3 52	56 8	84579	85312	99267	00733	14688	15421	29
32	3 44	56 16	84592	85299	99293	00707	14701	15408	28
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35	6 3 20	5 56 40	9.84630	9.85262	9.99368	10.00632	10.14738	10.15370	25
36	3 12	56 48	84643	85250	99394	00606	14750	15357	24
37	3 4	56 56	84656	85237	99419	00581	14763	15344	23
38	2 56	57 4	84669	85225	99444	00556	14775	15331	22
39	2 48	57 12	84682	85212	99469	00531	14788	15318	21
40	6 2 40	5 57 20	9.84694	9.85200	9.99495	10.00505	10.14800	10.15306	20
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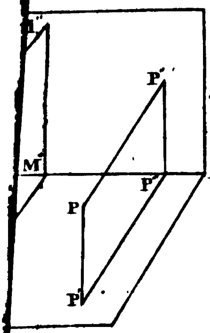
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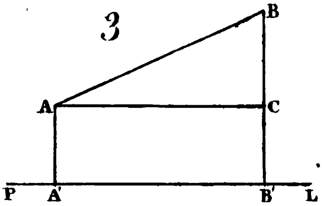
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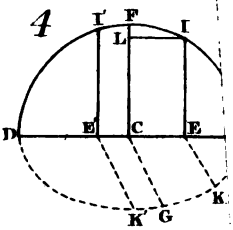
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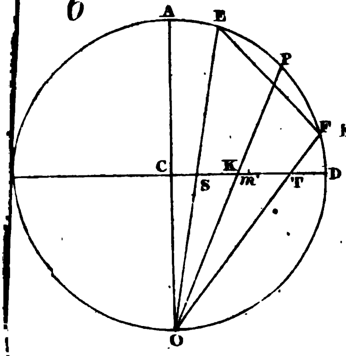
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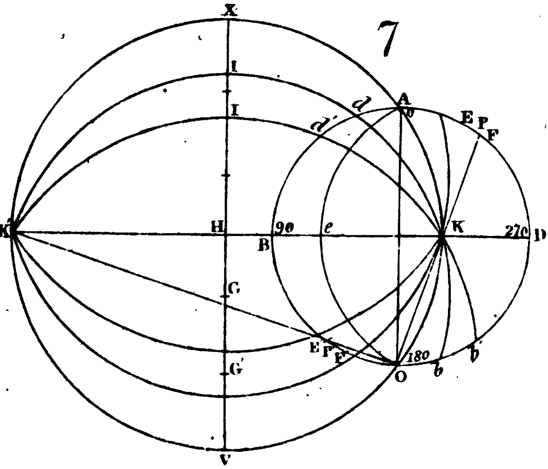
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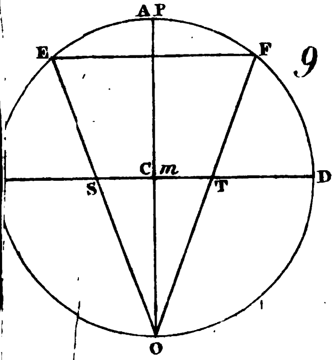
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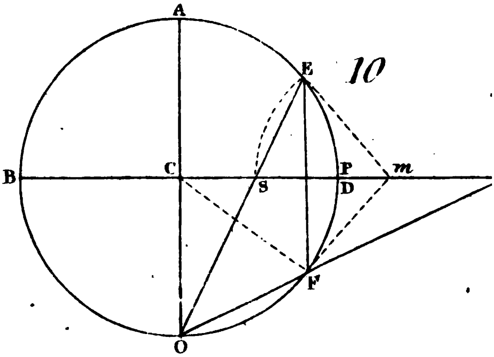
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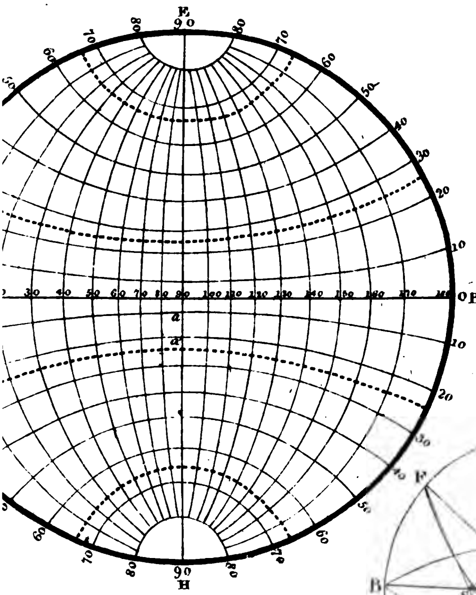

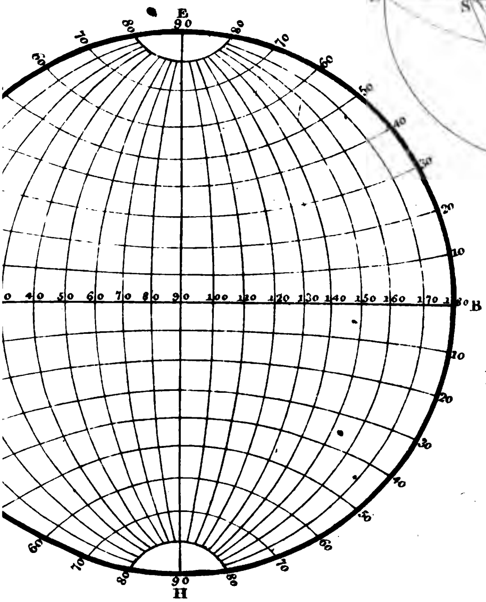

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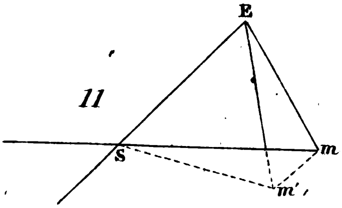

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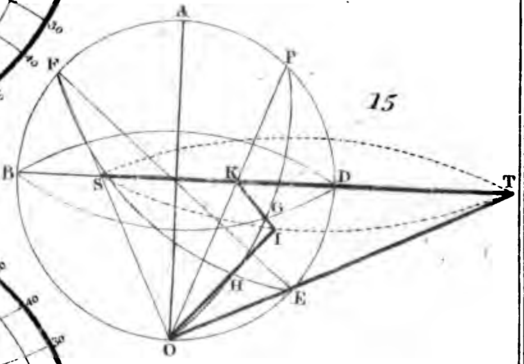

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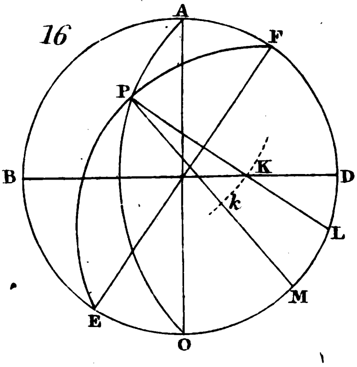

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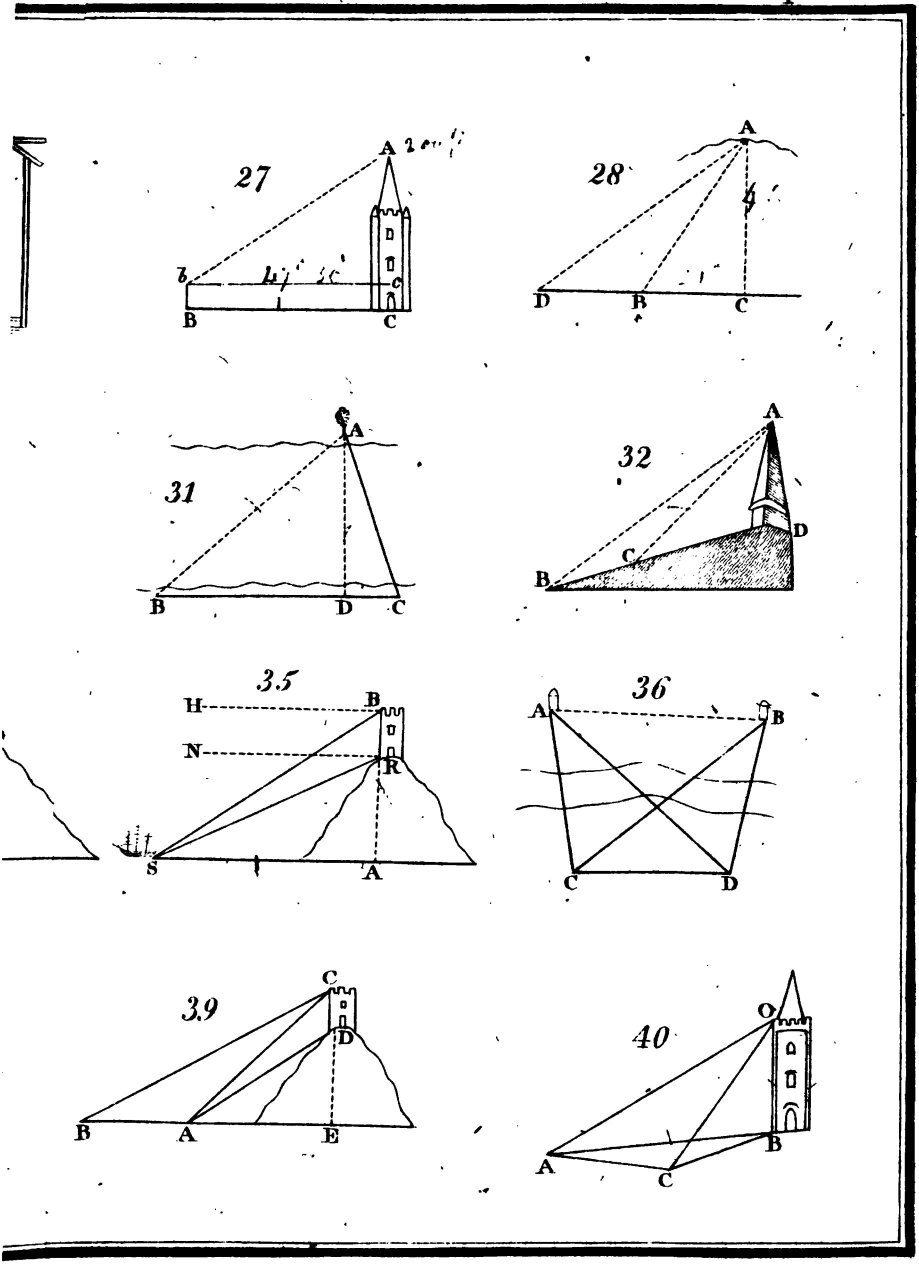
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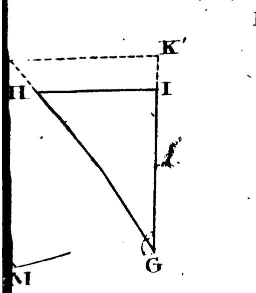

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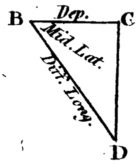

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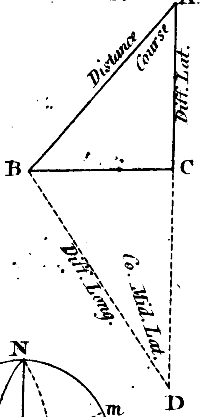

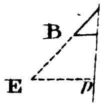

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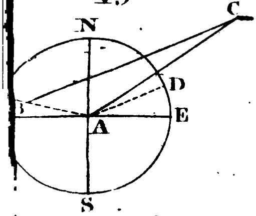

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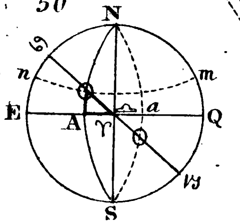

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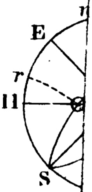

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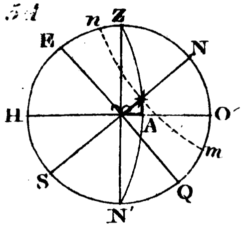

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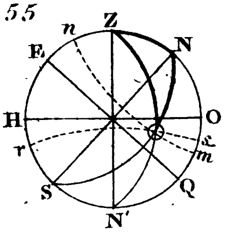

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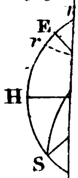

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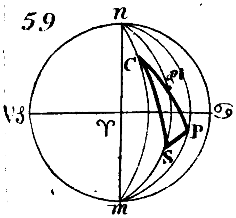

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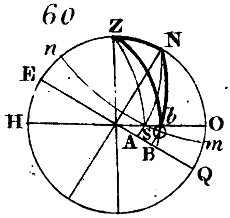

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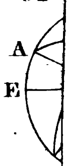

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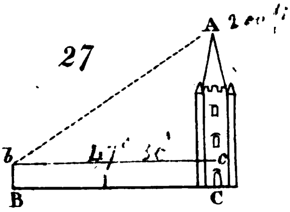

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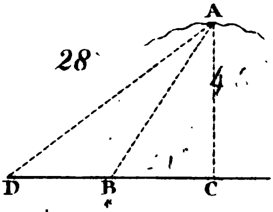

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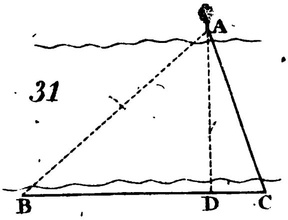

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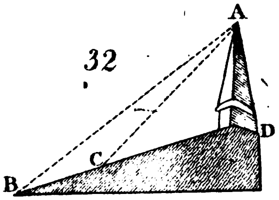

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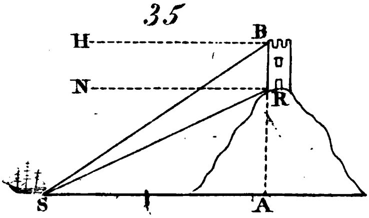

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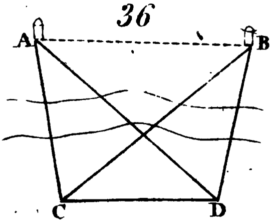

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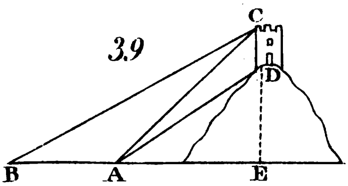

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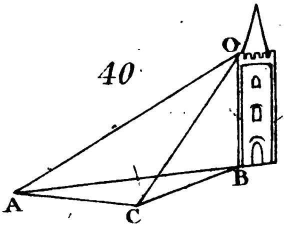

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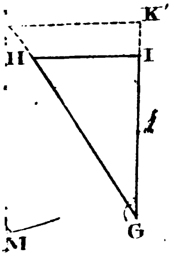

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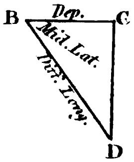

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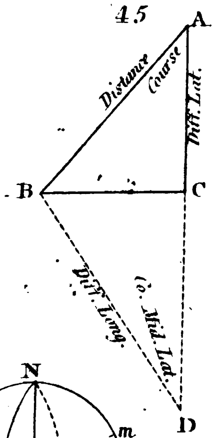

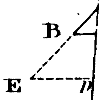

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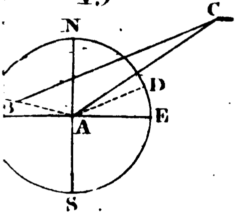

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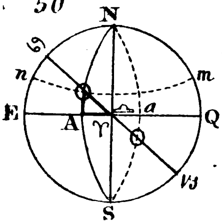

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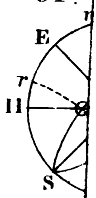

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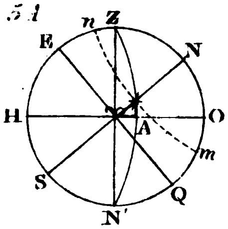

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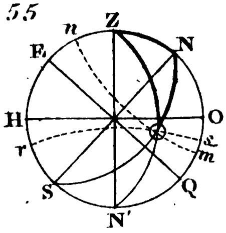

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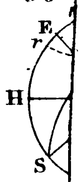

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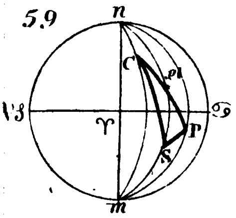

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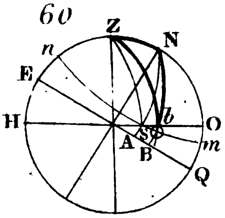

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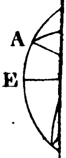

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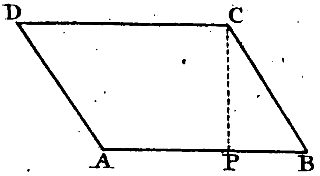

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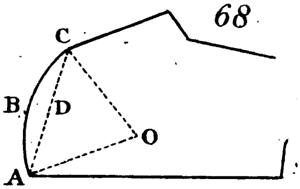

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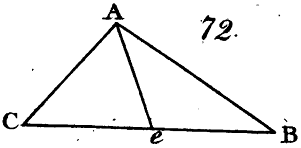

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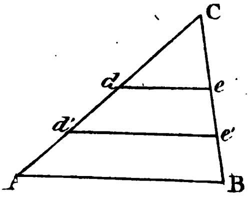

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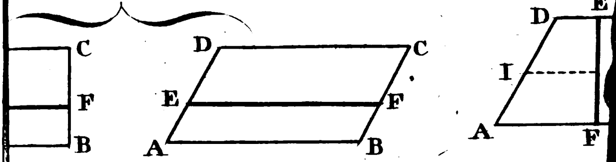

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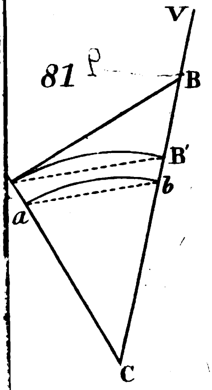

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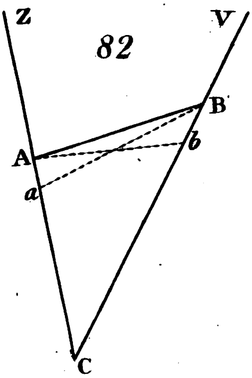

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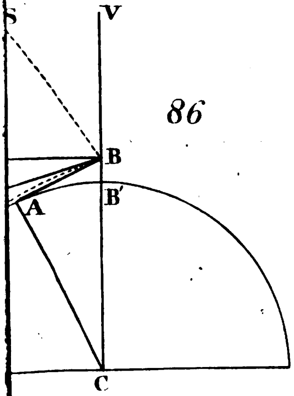

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