A TREATISE ON SURVEYING;

IN WHICH

THE THEORY AND PRACTICE ARE FULLY EXPLAINED.

PRECEDED BY

A SHORT TREATISE ON LOGARITHMS:

AND ALSO BY

A COMPENDIOUS SYSTEM OF PLANE TRIGONOMETRY.

The whole Illustrated by Numerous Examples.

BY

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PREFACE.

In the following treatise on Surveying, the author submits to the public a work which is the result of many years' experience as a teacher of mathematics. While he desires to avoid any unnecessary reference to defects in the works of those authors who have preceded him in this department of science, he yet deems it proper to allude to his belief of the existence of such defects as his inducement to prepare the work.

His aim has been to present the subject, in its practical as well as its theoretical relations, in a manner adapted to the capacity of every student, by presenting the theory plainly and comprehensively, and giving definite and precise directions for practice; and to embrace in the work every thing which an extensive business in land-surveying would be likely to require. How nearly his object has been attained, others must determine: he trusts, however, that the treatise will be found to possess merit sufficient to commend it to the favorable notice of his fellow-teachers. The following brief synopsis of its contents presents the plan and scope of the work.

Chapter I. consists of a short explanation of the nature and use of Logarithms.

Chapter II. contains the geometrical definitions and constructions needed in the subsequent part of the work.

In Chapter III. is presented a treatise on Plane Trigonometry, including a great variety of examples illustrative of the solution of triangles. In this chapter will also be found a full description of the Theodolite and Surveyor's Transit, and directions for their use.

In Chapter IV. the principles of surveying by the Chain are explained. This method is little employed by practical surveyors in this country. Since, however, the measurements require no other instrument than a tape-line, or a cord, or some other means of determining distances, it is of importance to the farmer, who frequently desires to know the contents of particular fields, or of portions of enclosures. The second and third sections of this chapter contain a pretty full treatise on Field Geometry, or the method of performing on the ground, with the chain or measuring line only, those operations which are needed in fixing the positions of points or in locating lines. In Great Britain, Chain Surveying is almost exclusively employed.

Chapter V. is devoted to Compass Surveying. Under this head are included all those methods which require the use of an instrument for determining the bearings of lines, whether that instrument be a Compass, a Transit, or a Theodolite. This chapter contains a full account of the methods to be employed in locating lines by means of such instruments.

The numerous difficulties with which the surveyor will be likely to meet from obstructions on the ground are stated, and the modes of overcoming them explained.

This chapter, with that on Plane Trigonometry, constitutes, in fact, a full treatise on Surveying as practised in this country. In selecting the methods to be employed in overcoming the difficulties both in Compass and in Chain Surveying, care has been taken to adopt such only as may be conveniently employed in the field.

Chapter VI. contains the general principles of Triangular

Surveying. This is the method employed in extensive geodetic operations.

The details of this method are so complex that a *volume*—not a *chapter*—would be required for their development. All that has been attempted is to give some of the more simple principles.

Chapter VII. treats of Laying out and Dividing Land. It is believed that many of the demonstrations in this chapter will be found to be much more simple than those usually given, almost all of them having been reduced to the development of a single principle. On a subject of this kind, which has so long occupied the attention of mathematicians, any thing new could hardly be expected. It has been the aim of the author to select the best methods, not to introduce any thing merely because it was *new*.

Chapter IX. contains a treatise on Practical Astronomy, embracing all that is needed for the surveyor's purposes or is practicable with his instruments. Various methods of running meridian lines, and of determining the latitude and the time of day, are fully explained.

The concluding chapter (X.) is devoted to the subject of the Variation of the Compass. In it will be found information of great value to the practical surveyor. The tables of variation are in all cases drawn from the most recent and authentic sources.

The tables appended to this treatise have been prepared with much care; but the author cannot flatter himself that they are entirely free from errors. He would esteem it a favor if those discovering any, either in the tables or in any other part of the work, would communicate them to him, that they may be corrected in the next edition.

The table on Latitudes and Departures will be found to be more concise than those usually given, and, being extended to four decimal places, will enable the calculator to give greater accuracy to his work. The table of Logarithms of Numbers has been carefully compared with those of Babbage, Hutton, and other standard authors. That on Sines and Tangents was taken from Hutton, and compared with other seven-decimal tables. Besides these, there is a table of Natural Sines and Cosines to every minute, and one of Chords to every five minutes, of the quadrant.

CONTENTS.

CHAPTER I.

ON THE NATURE AND USE OF LOGARITHMS.

CHAPTER II.

PRACTICAL GEOMETRY.

CHAPTER III.

PLANE TRIGONOMETRY.

CHAPTER IV.

CHAIN SURVEYING.

CHAPTER V.

COMPASS SURVEYING.

CHAPTER VIII.

MISCELLANEOUS EXAMPLES.

CHAPTER IX.

MERIDIANS, LATITUDE, AND TIME.

CHAPTER X.

VARIATION OF THE COMPASS.

APPENDIX.

A

TREATISE ON SURVEYING.

CHAPTER I.

ON THE NATURE AND USE OF LOGARITHMS.

SECTION I.

ON THE NATURE OF LOGARITHMS.

1. Definition. EVERY number may be considered as being a power, either integral or fractional, of some other number. Thus, $16 = 4^2$, $8 = 4^{\frac{3}{2}} = 4^{1.5}$, and $32 = 4^{\frac{5}{2}} = 4^{2.5}$.

When natural numbers are all considered as powers of the same root, the indices of those powers are called the logarithms of the numbers, and the root is called the base of the system.

Thus, $2 = 64^{\frac{1}{6}} = 64^{.1666}$, $4 = 64^{\frac{1}{3}} = 64^{.3333}$, $8 = 64^{\frac{1}{2}} = 64^{.5}$, $16 = 64^{\frac{2}{3}} = 64^{.6666}$, $32 = 64^{\frac{5}{6}} = 64^{.8333}$, $128 = 64^{\frac{7}{6}} = 64^{1.165}$, $256 = 64^{\frac{5}{3}} = 64^{1.666}$; and so on.

Therefore, .1666 is the logarithm of 2, to the base 64; and .3333, .5, .6666, .8333, 1.1666, and 1.666 are the logarithms of 4, 8, 16, 32, 128, and 256 respectively, to the same base.

2. It is well known, that, to multiply two powers of a certain root, we add their indices. If, then, all the natural numbers were expressed as powers of some one base, and the indices of those powers were known, all that would be necessary to determine the product of any two or more of them would be to seek out the corresponding indices, add them, and find the number whose index was equal to their sum: this would be the product required. Thus, in the following table, the numbers are regarded as powers of 2, the indices of the powers being set down opposite the number. To multiply any numbers contained in the column of numbers, headed N, take out the corresponding indices, add these, seek their sum in the column of indices, and opposite thereto, in the column of numbers, is the product required.

Suppose, for instance, the product of 32, 1024, and 512 were required: the corresponding indices are 5, 10, and 9. The sum of these is 24; hence, 16777216 is the product required.

TABLE OF POWERS OF 2 AND THE CORRESPONDING INDICES.

N.	I.	N.	I.	N.	I.
2	1	512	9	131072	17
4	2	1024	10	262144	18
8	3	2048	11	524288	19
16	4	4096	12	1048576	20
32	5	8192	13	2097152	21
64	6	16384	14	4194304	22
128	7	32768	15	8388608	23
256	8	65536	16	16777216	24

So likewise division may be performed by means of such a table.

Ex. Required the quotient of 4194304 by 131072.

The indices are 22 and 17. The difference of these is 5. The corresponding number 32 is the quotient required.

3. The table in last article contains only the integral powers of 2. This is sufficient for the purpose of illustration. A complete table contains all the numbers of the natural series, as far as the limits of the table, with the indices, or *logarithms*. These will in most instances be fractions. Thus, the logarithms corresponding to any of the numbers between 4 and 8 would be 2 and some fraction;

of any number between 8 and 16, the logarithm would be 3 and a fraction; and so on.

4. Calculation of Logarithms. Since all numbers are considered as the power of some one base, we will have, if a be the base, and n the number, $a^x = n$. The determination of the logarithm will then consist in solving the above equation so as to find x. This, in general, can only be done by approximation. The details to which it would lead are entirely foreign to the present work. Those who desire to become acquainted with the subject may consult the author's "Treatise on Algebra."

5. Bases. Theoretically, it is of no importance what number is assumed as the base of the system; but practical convenience suggests that 10, the base of our system of notation, should also be the base of the system of logarithms. By the use of this base, it becomes unnecessary to insert in the table of logarithms their integral portions. For, as will be seen hereafter, the *figures* in the decimal portion of the logarithm depend on the *figures* in the number, while the integral portion of the logarithm depends solely on the position of the decimal point in the number.

6. Assuming, then, 10 for a base, we have the following series:—

Numbers,	1,	10,	100,	1000,	10000,	100000,	1000000;
Logarithms,	0	1	2	3	4	5	6.

The logarithm of any number between 1 and 10 will be wholly decimal; between 10 and 100, it will be 1 and a decimal; and so on.

If the powers of 10 be continued downwards, we have

the powers	1	.1	.01	.001	.0001	.00001,
and indices	0	—1	—2	—3	—4	—5.

The logarithm of any number between .1 and 1 is therefore —1 + a decimal, of a number between .01 and .1 it is —2 + a decimal, &c.

7. Indices of Logarithms. The integral portion of every logarithm is called the *index*, the decimal portion being sometimes called the *mantissa*. From the above series, it is manifest that, if the number is greater than 1, the index is positive, and one less than the number of integral figures. Thus, 246.75 coming between 100 and 1000, its logarithm will be 2 and a decimal. If the number is less than 1, the index will be negative. For example, the logarithm of .0024675, which comes between .001 and .01, will be —3 + a decimal.

8. Mantissæ. The mantissæ of logarithms to the base 10 depend solely on the figures of the number, without any regard to the position of the decimal point.

Let the logarithm of 31.416 be 1.497151: then, since 314.16 is 10 times 31.416, its logarithm will be 1.497151 + 1 = 2.497151. Similarly, the logarithm of 31416, which is 1000 times 31.416, will be 1.497151 + 3 = 4.497151.

Again, .031416 = 31.416 ÷ 1000: its logarithm is therefore 1.497151 − 3 = —2.497151, in which the sign — is understood to belong solely to the index 2, and not to the mantissa. Since, then, the index can be supplied by attention to the position of the decimal point, the mantissæ alone are inserted in the body of a table of logarithms.

The annexed table will illustrate the above more fully:—

Number.	Logarithm.
64790	4.811508
6479	3.811508
647.9	2.811508
64.79	1.811508
6.479	0.811508
.6479	—1.811508
.06479	—2.811508
.006479	—3.811508.

9. Table of Logarithms. A table of logarithms consists of the series of natural numbers, with their logarithms, or, rather, the mantissæ of their logarithms, so arranged that one can be readily determined from the other. In the table of logarithms appended to this treatise, the mantissæ of the logarithms of all numbers, from 1 to 9999 inclusive, are given. On the first page are found the numbers from 1 to 99, with their logarithms in full. The remaining pages contain only the mantissæ of the logarithms. The first column, headed N, contains the numbers, from 100 to 999; and the second, headed 0, the mantissæ of their logarithms. Thus, the logarithm of the number 897 is 2.952792; the index being 2, because there are three integral figures in the number.

The remaining columns contain the last four figures of the mantissæ of the logarithms of numbers of four figures, the first three of which are found in the first column, and the fourth, at the head. Thus, if the number were 8976, the last four figures 3083 of the mantissa of its logarithm would be found in the column headed 6; the first two, 95, found in the second column, being common to them all. The logarithm of 8976 is, therefore, 3.953083.

10. To denote the point in which the second figure changes, when such change does not take place in the first logarithmic column, the first of the four figures from the change to the end of the line is printed as an index figure; thus, on page 20 of the tables, we have the lines

N.	0	1	2	3	4	5	6	7	8	9
456	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821
457	9916	$^{0}011$	$^{0}106$	$^{0}201$	$^{0}296$	$^{0}391$	$^{0}486$	$^{0}581$	$^{0}676$	$^{0}771$
458	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718

In such cases the first two figures are found in the next line. The logarithm of 4575 is, therefore, 3.660391.

11. To find the Logarithm of a number from the tables. If the number consists of one or two figures only, its logarithm is found on the first page of the table. If the two figures are both integers, the index is given also; but, if the one or both figures be decimal, the decimal part only of the logarithm should be taken out. Thus, the logarithm of 8 is 0.903090; of 59 is 1.770852.

If the number be wholly or part a decimal, the index must be changed in accordance with the principles laid down in Art. 7. Thus, the index must be one less than the number of figures in the integral part of the natural number. But when the natural number is wholly a decimal the index is *negative*, and must be one more than the number of ciphers between the first significant figure and the decimal point. Thus, the logarithm of

.8 is —1.903090; of .059 is —2.770852.

If the number consists of three figures, look for it in the remaining pages of the table, in the column headed N. Opposite to it, in the first column, will be found the decimal portion of the logarithm; the first two figures of the logarithm, being common to all the columns, are printed but once, to save room. Thus, the logarithm of

272 is 2.434569; of 529 is 2.723456;

the index being placed in accordance with the above rule.

If the number consists of four figures, the first three must be found as before; and the fourth, at the top of the table. The last four figures of the logarithm are found opposite to the first three figures of the number, and under the fourth; the first two figures of the logarithm being found in the first logarithmic column. Thus, if the number were 445.8, look for 445 in the column headed N, and opposite thereto, in the column headed 8, the figures 9140 are found; these affixed to 64, found in the first column, give 649140 for the decimal portion of the logarithm; and, as there are three integral figures, the index is 2. Hence, the complete logarithm is 2.649140.

If there are more than four figures in the number, find the logarithm of the first four figures as before. Take the difference between this logarithm and the next greater in the table; multiply this difference by the remaining figures in the number, and from the product separate as many figures from the right hand as are contained in the multiplier; then add the remainder to the logarithm first taken out: the sum will be the required logarithm.

Let the logarithm of 6475.48 be required.

The logarithm of 6475 is	.811240
The next greater is	1307
	67

67 × 48 = 32,16

32 added to 811240 gives .811272;
and the index being 3, the complete logarithm is 3.811272.

Next let the logarithm of .0026579 be required.

The logarithm of 2657 is	.424392
The next greater	4555
Difference	163
	9
	146,7

424392 + 147 = .424539, and the index being —3. the complete logarithm is —3.424539.

Note.—In this last example, the product is 1467: the figure stricken off being 7, which is more than 5, 147 is taken instead of 146.

Examples.

Required the logarithms of the following numbers:—

1. Of 7.5	0.875061	7. Of .0645775	—2.810081
2. Of 876	2.942504	8. Of .004679	—3.670153
3. Of 93.37	1.970207	9. Of 37196.2	4.570499
4. Of .4725	—1.674402	10. Of .14638	—1.165482
5. Of .869427	—1.939233	11. Of 6273.69	3.797523
6. Of .01367	—2.135769	12. Of .037429	—2.573208

12. To find the natural number corresponding to a given Logarithm. If four figures only be needed in the answer, seek in the columns of logarithms for the one nearest to the decimal part of the given logarithm: the first three figures of the natural number will be found in the column marked N; and the fourth, at the top of the column in which the logarithm is found.

When the index is positive, the number of integral figures will be one greater than the number expressed by the index; but, if the index is negative, the number will be wholly decimal, and have one less cipher between the decimal point and the first significant figure than the number expressed by the index. Thus, the natural number corresponding to the logarithm 2.860996 is 726.1; and that corresponding to —2.860996 is .07261.

If the logarithm be found exactly in the tables, and there be not enough figures in the corresponding number, the deficiency must be supplied by ciphers. Thus, the natural number corresponding to 6.891649 is 7792000.

But, if five or six figures be required, find in the table the logarithm next less than the given one, and take out the corresponding number as before; subtract this logarithm from the next greater in the table, and also from the given logarithm; annex one or two ciphers to the latter remainder, according as five or six figures are required, and divide the result by the former. The quotient annexed to the figures first taken out will give the figures required, the decimal point being placed as before.

Required the number corresponding to 2.649378, to six figures

Given logarithm	.649378	
Next less	.649335	cor. num. 4460
Difference	43	
Next greater logarithm	.649432	
Next less	.649335	
Difference	97)4300(44	
	388	
	420	
	388	
	32	

Hence, the number is 446.044.

Examples.

Required the natural numbers corresponding to the following logarithms.

1.	2.467415	Ans. 293.37	5.	4.617392	Ans. 41437.3
2.	—1.396143	.24897	6.	1.947138	88.54
3.	2.041637	110.062	7.	—2.960014	.091204
4.	—3.167149	.0014694	8.	—2.760116	.057559

SECTION II.

ON THE USE OF LOGARITHMS.

13. Multiplication. *To multiply numbers by means of logarithms.* Add together the logarithms of the factors, and take out the natural number corresponding to the sum. If any of the indices be negative, the figure to be carried from the sum of the decimal portions must be considered positive, and added to the sum of the positive, or subtracted from the sum of the negative indices. Then collect the affirmative indices into one sum, and the negative into another, take the difference between these sums, and prefix thereto the sign of the greater sum.

EXAMPLES.

Ex. 1. Multiply 47.25 and 397.3.

47.25	log.	1.674402
397.3	"	2.599119
Product, 18772.5		4.273521

Ex. 2. Required the product of 764.3, .8175, .04729, and .00125.

764.3	log.	2.883264
.8175	"	—1.912488
.04729	"	—2.674769
.00125	"	—3.096910
Product, .0369344		—2.567431

Ex. 3. Required the product of 87.5 and 6.7.

Ans. 586.25.

Ex. 4. Required the continued product of .0625, 41.67, .81427, and 2.1463. Ans. 4.5516.

Ex. 5. Multiply 67.594, .8739, and 463.92 together. Ans. 27404.

Ex. 6. Multiply 46.75, .841, .037654, and .5273 together. Ans. .780633.

Ex. 7. Multiply .00314, 16.2587, .32734, .05642, and 1.7638 together. Ans. .001663.

14. Division. *To divide numbers by logarithms.* Subtract the logarithm of the divisor from that of the dividend: the remainder will be the logarithm of the quotient.

If one or both of the indices are negative, subtract the decimal portions of the logarithm as before; and, if there be one to carry from the last figure, add it to the index of the divisor, if this be positive, but subtract if it be negative; then conceive the sign of the result to be changed, and if, when so changed, the two indices have the same sign, add them together; but, if they have different signs, take their difference and prefix the sign of the greater.

EXAMPLES.

Ex. 1. Divide	6740	log.	3.828660
by	87	log.	1.939519
Quotient, 77.471			1.889141
Ex. 2. Divide	86.47	log.	1.936865
by	.0124	log.	—2.093422
Quotient, 6973.4			3.843443
Ex. 3. Divide	.0642	log.	—2.807535
by	87.63	log.	1.942653
Quotient, .00073263 .			—4.864882
Ex. 4. Divide	.0642	log.	—2.807535
by	.008763	log.	—3.942653
Quotient, 7.3263			0.864882

Ex. 5. Divide 407.3 by 27.564. Ans. 14.7765.

Ex. 6. Divide .80743 by 63.87. Ans. .012642.

Ex. 7. Divide 963.7 by .00416. Ans. 231659.

Ex. 8. Divide 86.39 by .09427. Ans. 916.41.

Ex. 9. Divide .006357 by .0574. Ans. .11075.

Ex. 10. Divide 76.342 by .09427. Ans. 809.82.

15. To involve a number to a power. Multiply the logarithm of the number by the index of the power to which it is to be raised.

If the index of the logarithm is negative, and there is any thing to be carried from the product of the decimal part by the multiplier, instead of adding this to the product of the index, subtract it: the difference will be the index of the product, and will always be negative.

Ex. 1. Required the fourth power of 5.5.

5.5	log. 0.740363
	4
915.065	2.961452.

Ex. 2. Required the fifth power of .63.

.63	log. —1.799341
	5
.099244	—2.996705.

Ex. 3. Required the fourth power of 7.639.
Ans. 3405.24.

Ex. 4. Required the third power of .03275.
Ans. .00003513.

Ex. 5. What is the fifteenth power of 1.06?
Ans. 2.3966.

Ex. 6. What is the sixth power of .1362?
Ans. .0000063836.

Ex. 7. What is the tenth power of .9637?
Ans. .69091.

16. To extract a given root of a number. Divide the logarithm of the number by the degree of the root to be extracted: the quotient will be the logarithm of the root.

If the index of the logarithm is negative, and does not contain the divisor an exact number of times, increase it by so many as are necessary to make it do so, and carry the number so borrowed, as so many tens to the first figure of the decimal.

Ex. 1. Extract the fourth root of 56.372.

56.372	log. 4)1.751063
Result, 2.7401	.437766

Ex. 2. Extract the fifth root of .000763.

.000763	log. 5)—4.882525
Result, .23796	—1.376505.

Ex. 3. What is the fifth root of .00417? Ans. .3342.

Ex. 4. Required the fourth root of .419. Ans. .80455.

Ex. 5. Required the tenth root of 8764.5. Ans. 2.479.

Ex. 6. Required the seventh root of .046375. Ans. .6449.

Ex. 7. Required the fifth root of .84392. Ans. .96663.

Ex. 8. Required the sixth root of .0043667. Ans. .40429.

17. Arithmetical Complements. When several numbers are to be added, and others subtracted from the sum, it is often more convenient to perform the operation as though it were a simple case of addition. This may be done by conceiving each subtractive quantity to be taken from a unit of the next higher order than any to be found among the numbers employed; then add the results with the additive numbers, and deduct from the result as many units of the order mentioned as there were subtractive numbers. The difference between any number and a unit of the next higher order than the highest it contains is called the *arithmetical complement* of the number. Thus, the arithmetical complement of 8765 is 1235. It is easily obtained by taking the first significant figure on the right from ten, and each of the others from nine. This may be done mentally, so that the arithmetical complements need not be written down.

Thus, suppose A started out with 375 dollars to collect some bills and to pay sundry debts. From B he received \$104, to D he pays \$215, to E he pays \$75, from F he receives \$437, and, finally, pays to G \$137. How much has he left?

$$\left.\begin{array}{r} 375 \\ 104 \\ -215 \\ -\ 75 \\ 437 \\ -137 \end{array}\right\} \text{which are added as though they were} \left\{\begin{array}{l} 375 \\ 104 \\ 785 \\ 925 \\ 437 \\ 863 \end{array}\right.$$

$$\text{Ans.}\ \ 489 \qquad\qquad\qquad 3489,$$

deducting 3000 from the final result 3489, because there were three subtractive quantities.

The arithmetical complements of logarithms are generally employed where there are more subtractive logarithms than one. To give symmetry to the result, it would be neater to employ them in all cases. To a person who has much facility in calculation, it is most convenient to write down the logarithm as taken from the table, and obtain the arithmetical complement as the work is carried on. Thus, in the example above, the numbers could be written as in the first column; but in the addition, instead of employing the figures as they appear in the subtractive number, the complement of the first significant figure to ten, and of the others to nine, should be employed.

As an example of the use of the arithmetical complements of the logarithms of numbers, let it be required to work by logarithms the proportion as $\frac{27}{55} : \frac{475}{17} :: 125 : x$.

Here, as the first term is a fraction, it will have to be inverted; and the question will be the same as finding the value of $\frac{55 \times 475 \times 125}{27 \times 17}$.

$$\begin{array}{lr} \text{log.} & 27 \\ \text{“} & 17 \\ \text{“} & 55 \\ \text{“} & 475 \\ \text{“} & 125 \end{array} \left.\begin{array}{r} \left\{\begin{array}{r} 1.431364 \\ 1.230449 \end{array}\right. \\ 1.740363 \\ 2.676694 \\ 2.096910 \end{array}\right\} \begin{array}{c} \text{which are} \\ \text{added as} \\ \text{though} \\ \text{they were} \\ \text{written} \end{array} \left\{\begin{array}{r} \text{A. C. } 8.568636 \\ \text{A. C. } 8.769551 \\ 1.740363 \\ 2.676694 \\ 2.096910 \end{array}\right.$$

$$\text{Result, } 7114.66 \quad 3.852154 \qquad\qquad\qquad 3.852154$$

deducting 20, because there were two arithmetical complements employed.

In the examples wrought out in the subsequent part of this work, the arithmetical complements of the logarithms of the first term of every proportion are employed.

CHAPTER II.

PRACTICAL GEOMETRY.

SECTION I.

DEFINITIONS.

18. The practical surveyor will find a good knowledge of Algebra and of the Elements of Geometry an invaluable aid not only in elucidating the principles of the science, but in enabling him to overcome difficulties with which he will be certain to meet. In fact, so completely is Surveying dependent on geometrical principles, that no one can obtain other than a mere *practical* knowledge of it, without first having mastered them; and he who depends solely on his practical experience will be certain to meet with cases which will call for a kind of knowledge which he does not possess, and which he can obtain only from Geometry.

Every student, therefore, who desires to become an intelligent surveyor, should first study Euclid, or some other treatise on Geometry. He will then have a key which will not only unlock the mysteries contained in the ordinary practice, but which will also open the way to the solution of all the more difficult cases which occur. To those who have taken the course above recommended, the problems solved in the present chapter will be familiar. They are inserted for the benefit of those who may not be thus prepared, and also as affording some of the most convenient modes of performing the operations on the ground.

19. Geometry is the science of magnitude and position.

20. A solid is a magnitude having length, breadth, and thickness.

All material bodies are *solids*, and so are all portions of space, whether they are occupied with material substances or not. Geometry, treating only of dimension and position, has no reference to the physical properties of matter.

21. The surfaces of solids are *superficies*. A superficies has, therefore, only length and breadth.

22. The boundaries of superficies, and the intersection of superficies, are *lines*. Hence, a line has length only.

23. The extremities of lines, and the intersections of lines, are *points*. A point has, therefore, neither length, breadth, or thickness.

24. A *point*, therefore, may be defined as that which has *position*, but not *magnitude*.

25. A *line* is that which has length only.

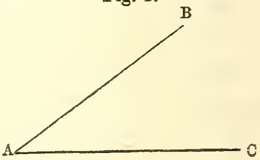
26. A *straight line* is one the direction of which does not change. It is the shortest line that can be drawn between two points.

27. A *superficies* has length and breadth only.

28. A *plane superficies*, generally called simply a *plane*, is one with which a *straight line* may be made to coincide in any direction.

29. A *plane rectilineal angle*, or simply an *angle*, is the inclination of two lines which meet each other. (Fig. 1.)

Fig. 1.

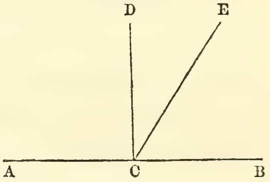


30. An angle may be read either by the single letter at the intersection of the lines, or by three letters, of which that at the intersection must always occupy the middle. Thus, (Fig. 1,) the angle between BA and AC may be read simply A or BAC.

31. The *magnitude* of an angle has no reference to the space included between the lines, nor to their length, but solely to their *inclination*.

32. Where one straight line stands on another so as to make the adjacent angles equal, each of these angles is called a *right* angle; and the lines are said to be *perpendicular* to each other. Thus, (Fig. 2,) if ACD = BCD, each is a right angle, and CD is perpendicular to AB.

Fig. 2.



33. An angle less than a right angle is called an *acute angle*. Thus, BCE or ECD (Fig. 2) is an acute angle.

34. An angle greater than a right angle is called an *obtuse angle*. ACE (Fig. 2) is an obtuse angle.

35. The *distance* of a point from a straight line is the length of the perpendicular from that point to the line.

36. *Parallel* straight lines are those of which all points in the one are equidistant from the other.

37. A *figure* is an enclosed space.

38. A *triangle* is a figure bounded by three straight lines.

39. An *equilateral triangle* is one the three sides of which are equal.

40. An *isosceles triangle* is one of which two of the sides are equal. The third side is called the base.

41. A *scalene triangle* has three unequal sides.

42. A *right-angled triangle* has one of its angles a right angle.

43. The side opposite the right angle is called the *hypothenuse*, and the other sides, the *legs*.

44. An *obtuse-angled triangle* has one of its angles obtuse.

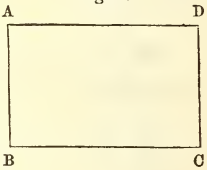
45. A *quadrilateral figure* is bounded by four sides.

Fig. 3.

46. A *parallelogram* (Fig. 3) is a quadrilateral, the opposite sides of which are parallel.

47. A *rectangle* (Fig. 4) is a parallelogram, the adjacent sides of which are perpendicular to each other. Thus, ABCD is a rectangle. A rectangle is read either by naming the letters around it in their order, or by naming two of the sides adjacent to any angle. Thus, the rectangle ABCD is read the rectangle AB.BC.

Fig. 4.



Whenever the rectangle of two lines, such as DE.EF, is spoken of, a rectangular parallelogram, the adjacent sides of which are equal to the lines DE and EF, is meant.

48. A *square* is a rectangle, all the sides of which are equal.

49. A *rhombus* is an oblique parallelogram, the sides of which are equal.

50. A *rhomboid* is an oblique parallelogram, the adjacent sides of which are unequal.

51. All quadrilaterals that are not parallelograms are called *trapeziums*.

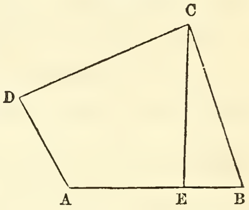
52. A *trapezoid* is a trapezium, having two of its sides parallel.

53. Figures of any number of sides are called *polygons*, though this term is generally restricted to those having more than four sides.

54. The *diagonal* of a figure is a line joining any two opposite angles.

55. The *base* of any figure is the side on which it may be supposed to stand. Thus, AB (Fig. 5) is the base of ABCD.

Fig. 5.



56. The *altitude* of a figure is the distance of the highest point from the line of the base. CE (Fig. 5) is the altitude of ABCD.

57. The *diameter* of a circle is a straight line through the centre, terminating in the circumference.

58. The *radius* of a circle is a straight line drawn from the centre to the circumference.

59. A *segment* of a circle is any part cut off by a straight line. Thus, ABCD is a segment.

Fig. 6.

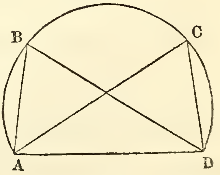
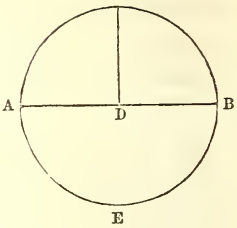


Fig. 7.



60. A *semicircle* is a segment cut off by the diameter. ABC and AEB (Fig. 7) are semicircles.

61. A *quadrant* is a portion of a circle included between two radii at right angles to each other. ADC and BDC (Fig. 7) are quadrants.

62. The *angle in a segment* is the angle contained between two straight lines drawn from any point in the arc of a segment to the extremities of that arc. Thus, ABD and ACD (Fig. 6) are angles in the segment ABCD.

63. Similar rectilineal figures have their angles equal, and the sides about the equal angles proportionals.

64. Similar segments of a circle are those which contain equal angles.

SECTION II.

GEOMETRICAL PROPERTIES AND PROBLEMS.

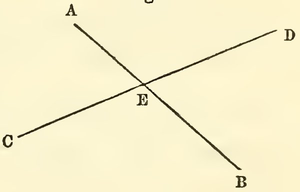
A.—GEOMETRICAL PROPERTIES.

65. ALL right angles are equal to each other.

66. The angles which one straight line makes with another on one side of it are together equal to two right angles. Thus, ACE and ECB (Fig. 2) are together equal to two right angles. (13.1.)

67. If a number of straight lines are drawn from a point in another straight line, all the successive angles are together equal to two right angles. Thus, ACD + DCE + ECB (Fig. 2) make two right angles.

Fig. 8.



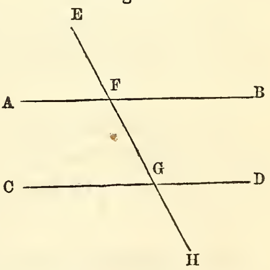
68. If two straight lines intersect each other, the angles vertically opposite are equal. Thus, AEC (Fig. 8) = BED, and AED = BEC. (15.1.)

69. Triangles which have two sides and the included angle of one respectively equal to the two sides and the included angle of the other, are equal in all respects. (4.1.)

70. Triangles which have two angles and the interjacent side of one respectively equal to two angles and the interjacent side of the other, are equal in all respects. (26.1.)

71. Triangles which have two angles of the one respectively equal to two angles of the other, and which have also the sides opposite to two equal angles equal to each other, are equal in all respects. (26.1.)

Fig. 9.



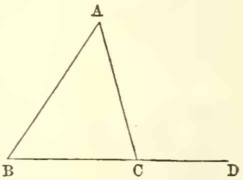
72. If a straight line cuts two parallel lines, the angles *similarly* situated in respect to these lines, and also those *alternately* situated, will be equal to each (29.1) other. Thus, (Fig. 9,) EFB = FGD, BFG = DGH, AFE = CGF, and AFG = CGH, being similarly situated; and AFE = DGH, EFB = CGH, AFG = FGD, and BFG = FGC, being alternately situated.

73. If a straight line cuts two parallel straight lines, the two exterior angles on the same side of the cutting line, and also the two interior angles, are equal to two right angles. Thus, (Fig. 9,) EFB and DGH are equal to two right angles, as are also AFE and CGH. So also the pairs of interior angles AFG and FGC, BFG and FGD, are each equal to two right angles. (29.1.)

74. The angles at the base of an isosceles triangle are equal to each other. (5.1.)

75. If one side of a triangle be produced, the exterior angle so formed will be equal to the two angles adjacent to the opposite side, and the three interior angles are equal to two right angles. Thus, (Fig. 10,) ACD = ABC + BAC, and ABC + BAC + ACB = two right angles. (32.1.)

Fig. 10.



76. The interior angles of any rectilineal figure are equal to twice as many right angles as the figure has sides, diminished by four right angles. The interior angles of a quadrilateral are therefore equal to four right angles. (Cor. 1, 32.1.)

77. The opposite sides and angles of a parallelogram are equal to each other. (34.1.)

78. Conversely, any quadrilateral of which the opposite sides or the opposite angles are equal is a parallelogram.

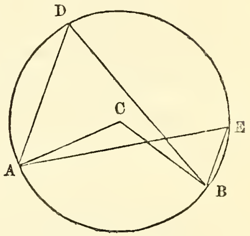
79. Parallelograms having equal bases and altitudes, and also triangles having equal bases and altitudes, are equal to each other. (35–38.1.)

80. A parallelogram is double a triangle having the same base and altitude. (41.1.)

81. The square on the hypothenuse of a right-angled triangle is equal to the sum of the squares of the legs. (47.1.)

82. Any figure described on the hypothenuse of a right-angled triangle is equal to the sum of the similar figures similarly described on the sides. (31.6.)

Fig. 11.



83. The angle at the centre of a circle is double the angle at the circumference on the same base. Thus, the angle at C (Fig. 11) is double either D or E. (20.3.)

84. Angles in the same segment of a circle are equal. Thus, D and E (Fig. 11) are equal.

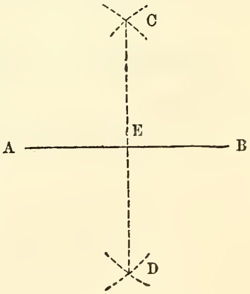
85. The angle in a semicircle is a right angle; the angle in a segment greater than a semicircle is acute; and that in a segment less than a semicircle is obtuse.

86. The sides about the equal angles of equiangular triangles are proportional. (4.6.)

B.—GEOMETRICAL PROBLEMS.

Under this head are given those methods of construction which are applicable to paper drawings. The methods to be used in field operations will be given in a subsequent chapter.

Fig. 12.

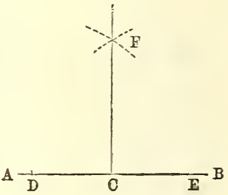


87. Problem 1.—*To bisect a given straight line.* Let AB (Fig. 12) be the given line. With the centres A and B, and radius greater than half AB, describe arcs cutting in C and D. Join CD cutting AB in E, and the thing is done. (10.1.)

Problem 2. *To draw a perpendicular to a straight line from a given point in it.*

a. When the point is not near the end.

Fig. 13.

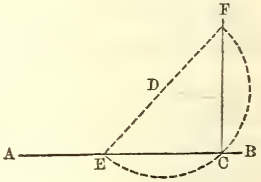


88. Let AB (Fig. 13) be the line and C the given point. Lay off CD = CE, and with D and E as centres, and any radius greater than DC, describe arcs cutting in F. Draw CF, and the thing is done. (11.1)

b. When the point is near the end of the line.

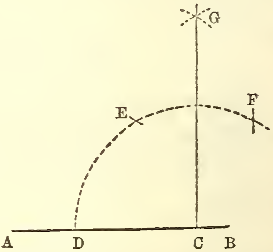
89. *First Method.*—Take any point D (Fig. 14) not in the line, and with the centre D and radius DC describe the circle ECF, cutting AB in E. Join ED and produce it to F. Then will CF be the perpendicular. For ECF, being an angle in a semicircle, is a right angle. (85.)

Fig. 14.



90. *Second Method.*—With C (Fig. 15) and any radius describe DEF; with D and the same radius cross the circle in E; and with E as a centre, and the same radius, cross it in F. With E and F as centres, and any radius, describe arcs cutting in G. Then will CG be the perpendicular.

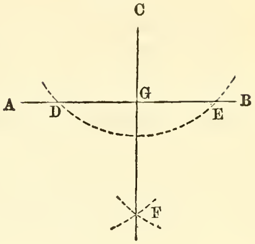
Fig. 15.



Problem 3.—*To let fall a perpendicular to a line from a point without it.*

a. When the point is not nearly opposite the end of the line.

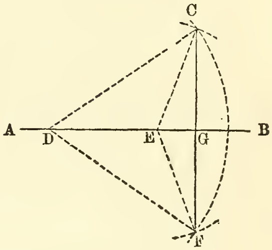
Fig. 16.



91. Let AB (Fig. 16) be the line and C the given point. With the centre C describe an arc cutting AB in D and E. With the centres D and E and any radius describe arcs cutting in F. Join CF, and the thing is done. (12.1.)

b. When the point is nearly opposite the end of the line.

Fig. 17.

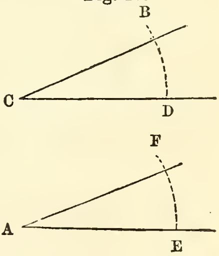


92. *First Method.*—With D and E as centres, and radii DC and EC, describe arcs cutting in F: then will CF be the perpendicular. For, the triangles CDE and FDE being equal, (8.1,) DGC and FGD will be equal. (4.1.)

93. *Second Method.*—Let F (Fig. 14) be the point. From F to any point E in the line AB draw FE. On it describe a semicircle cutting AB in C. Join F and C, and FC will be the perpendicular (85.)

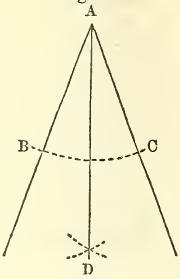
Problem 4.—*At a given point in a given straight line to make an angle equal to a given angle.*

Fig. 18.



94. Let BCD (Fig. 18) be the given angle, and A the given point in AE. With the centre C and any radius describe BD, cutting the sides of the angle in B and D. With A as a centre and the same radius describe EF; make EF = DB; draw AF, and the thing is done.

Fig. 19.

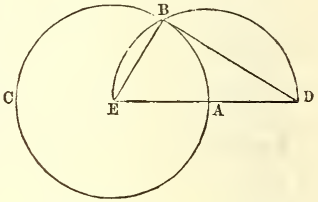


Problem 5.—*To bisect a given angle.*

95. Let BAC (Fig. 19) be the given angle. With the centre A and any radius describe an arc cutting the sides in B and C. With the centres B and C, and the same or any other radius, describe arcs cutting in D. Join AD, and the thing is done. (9.1.)

Problem 6.—*To draw a straight line touching a circle from a given point without it.*

Fig. 20.



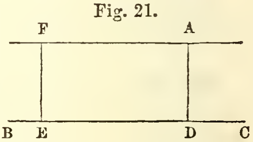
96. Let ABC be the given circle, and D the given point. Join D and the centre E. On DE describe a semicircle cutting the circumference in B. Join DB, and it will be the tangent required.

For DBE, being an angle in a semicircle, is a right angle, (31.3;) therefore, DB touches the circle, (16.3.)

If the point were in the circumference at B. Join EB, and draw BD perpendicular to it. BD will be the tangent.

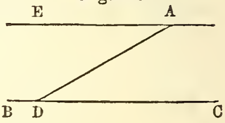
Problem 7.—*Through a given point to draw a line parallel to a given straight line.*

Fig. 21.



97. *First Method.*—Let A (Fig. 21) be the given point, and BC the given line. From A to BC let fall a perpendicular AD; and at any other point E in BC erect a perpendicular EF equal to AD. Through A and F draw AF, which will be the parallel required.

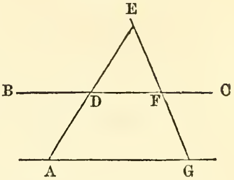
Fig. 22.



98. *Second Method.*—From A (Fig. 22) to D, any point in BC, draw AD. Make DAE = ADC, and AE will be parallel to BC. (27.1.)

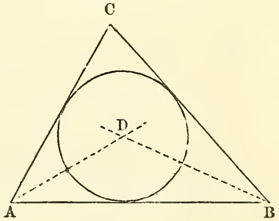
99. *Third Method.*—Through A draw ADE, cutting BC in D. Make DE = AD. Through E draw any other line EFG, cutting BC in F. Make FG = EF: then AG will be parallel to BC. (2.6.)

Fig. 23.



Problem 8.—*To inscribe a circle in a given triangle.*

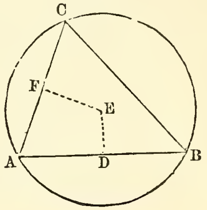
Fig. 24.



100. Let ABC (Fig. 24) be the given triangle. Bisect two of its angles A and B by the lines AD, BD, cutting in D. Then will D be the centre. (4.4.)

Problem 9.—*To describe a circle about a given triangle.*

Fig. 25.

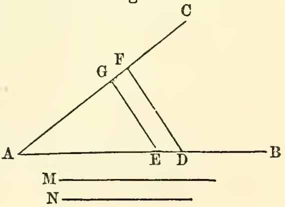


101. Bisect two of the sides, as AC and AB, (Fig. 25,) by the perpendiculars FE and DE, cutting in E. Then will E be the centre of the required circle.

Problem 10.—*To find a third proportional to two straight lines.*

102. Let M and N (Fig. 26) be the given lines. Draw two lines AB and AC, making any angle at A. Lay off AD = M, and AE and AF each equal to N. Join DF, and draw EG parallel to it. AG will be the third proportional required. (11.6.)

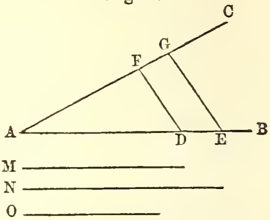
Fig. 26.



Problem 11.—*To find a fourth proportional to three given straight lines.*

103. Let M, N, and O (Fig. 27) be the three lines. Draw any two lines AB and AC, meeting at A. Lay off AD = M, AE = N, and AF = O. Join DF, and draw EG parallel to it: then AG is the fourth proportional required. (12.6.)

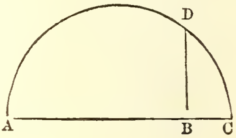
Fig. 27.



Problem 12.—*To find a mean proportional between two straight lines.*

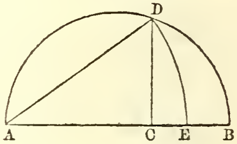
104. *First Method.*—Place the lines AB and BC (Fig. 28) in the same straight line. On AC describe a semicircle cutting the perpendicular through B in D. BD will be the mean proportional required. (13.6.)

Fig. 28.



105. *Second Method.*—Let AB and AC (Fig. 29) be the given lines. On AB describe a semicircle cutting the perpendicular at C in D. Join AD. AD is the mean proportional required. (Cor. 8.6.) Make AE = AD.

Fig. 29.

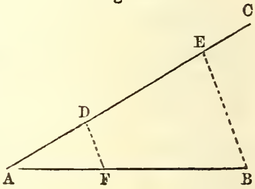


Note.—This is a very convenient construction, and is often employed in the Division of Land.

Problem 13.—*To divide a given line into parts having the same ratio as two given numbers M and N.*

106. Let AB (Fig. 30) be the given line. Draw AC making any angle with AB. Lay off AD = M, taken from any scale of equal parts, and DE = N, taken from the same scale. Join BE, and draw DF parallel to it, and the thing is done. (2.6.)

Fig. 30.



CHAPTER III.

PLANE TRIGONOMETRY.

SECTION I.

DEFINITIONS.

107. Plane Trigonometry is the science which treats of the relations between the sides and angles of plane triangles; which develops the principles by which, when any three of the six parts of a triangle,—viz.: the three angles and the three sides,—except the three angles, are given, the others may be found. It likewise treats of the properties of the trigonometrical functions.

108. Measure of Angles. An angle is the inclination between two straight lines: it is measured by the intercepted arc of a circle described about the angular point as a centre.

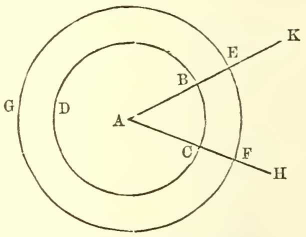
In the measurement of angles, it is not the absolute length of the arc that is needed, but the ratio which that length bears to the whole circumference.

For the purpose of expressing this ratio readily, the circumference is supposed to be divided into 360 parts, called degrees, each degree into 60 parts, called minutes, and each minute into 60 seconds. Degrees are marked with a cipher ° over them, minutes with one accent ′, and seconds with two ″. Thus, 37 degrees, 45 minutes, and 30 seconds, would be written $37° \; 45' \; 30''$.

When we speak of an arc of 35°, we mean an arc which is $\frac{35}{360}$ of the circumference. An arc of 180° is half the circumference, one of 90° is a quadrant, and of 45° the half of a quadrant.

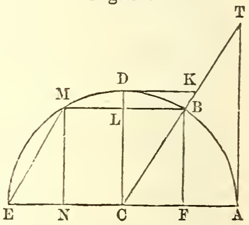
It is evident that, if several circles be described about the same point, the arcs intercepted between two lines drawn from the centre will bear the same ratio to the circumferences of which they are portions. Thus, if around the point A (Fig. 31) two circles BCD and EFG be described, cutting AK and AH in B, E, C, F, the arc BC will have to the circumference BCD the same ratio as EF has to the circumference EFG. In the measurement of angles, it is a matter of indifference, therefore, what radius is assumed as that of the circle of reference. The radius which is generally adopted is unity. This value of the radius makes it unnecessary to write it down in the formulæ.

Fig. 31.



The radius adopted in the construction of the Table of Logarithmic Sines and Tangents, to be described hereafter, is 10,000,000,000.

Fig. 32.



109. The *complement* of an arc or angle is what it differs from a quadrant, or 90°. Thus, DB (Fig. 32) is the *complement* of AB, and MD of AM.

110. The *supplement* of an arc or angle is what it wants of 180°. Thus, BE (Fig. 32) is the *supplement* of AB, and ME of AM.

111. Trigonometrical Functions. The trigonometrical functions are lines having definite geometrical relations to the arc to which they belong. Those most in use are the sine, the cosine, the tangent, the cotangent, the secant, and the cosecant.

The *chord* of an arc is the right line joining the extremities of that arc. Thus, EM (Fig. 32) is the chord of the arc EM.

The *sine* of an arc is the line drawn from one extremity of the arc, perpendicular to the diameter through the other extremity. BF (Fig. 32) is the sine of AB or of EB, and BL of BD.

Note.—The sine of an arc is equal to the sine of its supplement.

The *cosine* of an arc is the line intercepted between the foot of the sine and the centre. CF is the cosine of AB or of BE.

Since CF = BL, it is manifest that the cosine of an arc is equal to the sine of its complement.

The *tangent* of an arc is a line touching the arc at one extremity and produced till it meets the radius through the other extremity. Thus, AT is the tangent of AB, and DK of DB.

The *cotangent* of an arc is the tangent of its complement. Thus, DK (Fig. 32) is the cotangent of AB.

The *secant* of an arc is the line intercepted between the centre and the extremity of the tangent. Thus, CT (Fig. 32) is the secant of AB.

The *cosecant* of an arc is the secant of the complement of that arc. Thus, CK (Fig. 32) is the cosecant of AB.

The sine, cosine, &c. of an arc are also called the sine, cosine, &c. of the angle measured by that arc. Thus, BF and CF (Fig. 32) are the sine and cosine of the angle ACB.

Note.—The tangent, cotangent, secant, or cosecant of an arc is equal to the tangent, cotangent, secant, or cosecant of its supplement.

112. Properties of the Sines, Tangents, &c. of an arc or angle.

The sine of 90°, the cosine of 0°, the tangent of 45°, the cotangent of 45°, the secant of 0°, and the cosecant of 90°, is each equal to radius.

The square of the sine + the square of the cosine of any arc is equal to the square of radius. ($\text{Sin.}^2 a + \cos.^2 a = R^2$.) This is evident from the right-angled triangle CFB, (Fig. 32.) (47.1.)

The square of the tangent + the square of radius is equal to the square of the secant. $\text{Tan.}^2 a + R^2 = \text{sec.}^2 a$. (47.1.)

Tan. a : R : : R : cotan. a, or tan. a. cot. $a = R^2$. This is evident from the similarity of the triangles ACT and DKC, (Fig. 32.) which give (4.6) AT : AC : : CD : DK.

The sine of 30° and the cosine of 60° is each equal to half radius.

113. Geometrical properties most employed in Plane Trigonometry.

The angles at the base of an isosceles triangle are equal; and conversely, if two angles of a triangle are equal, the sides which subtend them are equal. (5 and 6.1.)

The external angle of a triangle is equal to the two opposite internal ones. (32.1.)

The three interior angles of a triangle are equal to two right angles or 180°. (32.1.)

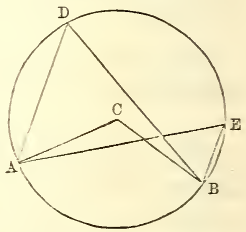
Hence, if the sum of two angles be subtracted from 180°, the remainder will be the third angle.

If one angle be subtracted from 180°, the remainder is the sum of the other angles.

If one oblique angle of a right-angled triangle be subtracted from 90°, the remainder is the other angle.

The sum of the squares of the legs of a right-angled triangle is equal to the square of the hypothenuse. (47.1.)

Fig. 11.



The angle at the centre of a circle is double the angle at the circumference upon the same arc; or, in other words, the angle at the circumference of a circle is measured by half the arc intercepted by its sides. (20.3.) Thus, the angle ADB is half ACB; and is, therefore, measured by one-half of the arc AB.

The sides about the equal angles of equiangular triangles are proportionals. (4.6.)

SECTION II.

DRAFTING OR PLATTING.

114. DRAFTING is making a correct drawing of the parts of an object. *Platting* is drawing the lines of a tract of land so as correctly to represent its boundaries, divisions, and the various circumstances needful to be recorded. It is, in fact, making a map of the tract. It is of great importance to a surveyor to be able to make a correct and neat plat of his surveys. The facility of doing so can only be acquired by practice; the student should, therefore, be required to make a neat and accurate draft of every problem in Trigonometry he is required to solve, and of every survey he is required to calculate. It is not sufficient that he should draw a figure, as he does in his demonstrations in Geometry, that will serve to demonstrate his principles or afford him a diagram to refer to, but he should be obliged to make all parts in the exact proportion given by the data, so that he can, if needful, determine the length of any line, or the magnitude of any angle, by measurement.

115. Straight lines. Straight lines are generally drawn with a straight-edged ruler. If a very long straight line is needed, a fine silk thread may be stretched between the points that are to be joined, and points pricked in the paper at convenient distances; these may then be joined by a ruler.

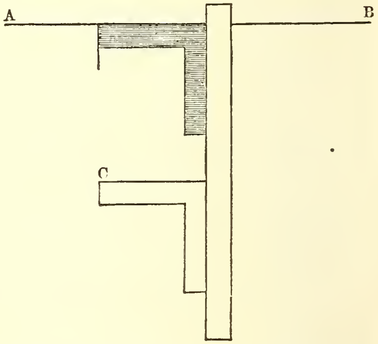
In drawing straight lines, care should be taken to avoid determining a long line by producing a short one, as any variation from the true direction will become more manifest the farther the line is produced. When it is *necessary* to produce a line, the ruler is fixed with most ease and certainty by putting the points of the compasses into the line to be produced, and bringing the ruler against them.

116. Parallels. Parallels may be drawn as described in

Arts. 97, 98. Practically, however, it is better to draw them by some instrument specially adapted to the purpose.

The square and ruler are very convenient instruments for this purpose. The square consists of two arms, which should be made at right angles to each other, to facilitate the erection of perpendiculars. Let AB (Fig. 33) be the line to which a parallel is to be drawn through C. Adjust one edge of the square to the line AB, and bring a ruler firmly against the other leg; move the square along the ruler until the edge coincides with C: this edge will then be parallel to the given line.

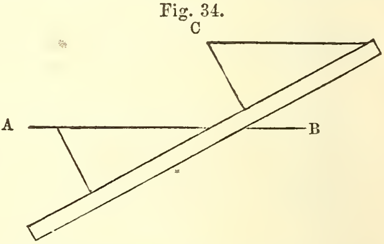
Fig. 33.



If a T square be substituted for a simple right angle, it may be held more firmly against the ruler.

Instead of a square, a right-angled triangle is frequently used. The legs should be made accurately at right angles, that it may be used for drawing perpendiculars. Let AB (Fig. 34) be the line, and C the point through which it is required to draw a parallel. Bring one edge of the triangle accurately to the line, and then place a ruler against one of the other sides. Slide the triangle along the ruler until the point C is in the side which before coincided with the line: this side is then parallel to the given line.

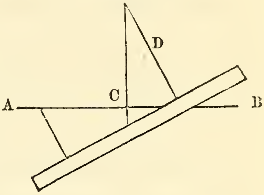
Fig. 34.



The parallel rulers which accompany most cases of instruments are *theoretically* accurate. They are, however, generally made with so little care that they cannot be depended on where correctness is required; and, even if made true, they are liable to become inaccurate in consequence of wear of the joints.

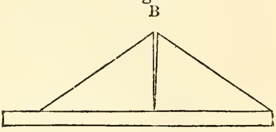
117. Perpendiculars. Perpendiculars may be drawn as directed, (Art. 88, *et seq.*) A more ready means is to place one leg of the square (Fig. 33) upon the line: the other will then be perpendicular to that line. The triangle is another very convenient instrument for this purpose. Let AB (Fig. 35) be the line to which a perpendicular is to be drawn. Place the hypothenuse of the triangle coincident with AB, and bring the ruler against one of the other sides. Remove the triangle and place it with the third side against the ruler, as at D: then the hypothenuse will be perpendicular to AB.

Fig. 35.



This method requires the angle of the triangle to be precisely a right angle. To test whether it is so, bring one leg against a ruler, as at A, (Fig. 36,) and scribe the other leg. Reverse the triangle, and bring the right angle to the same point A, and again scribe the leg. If the angle is a right angle, the two scribes will exactly coincide. If they do not coincide, the triangle requires rectification.

Fig. 36.



118. Circles and Arcs. These are generally drawn with the compasses, which should have one leg movable, so that a pen or a pencil may be inserted instead of a point. When circles of long radii are required, the beam compasses should be used.

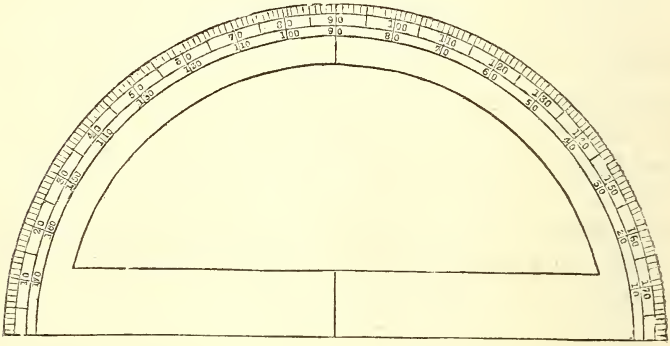
These consist of a bar of wood or metal, dressed to a uniform size, and having two slides furnished with points. These slides can be adjusted to any part of the beam, and clamped, by means of screws adapted to the purpose. The point connected with one of the slides is movable, so that a pencil or drawing pen may be substituted.

When the beam compasses are not at hand, a strip of drawing paper or pasteboard may be substituted: a pin through one point will serve as a centre; the pencil point can be passed through a hole at the required distance.

119. Angles. Angles may be laid off by a protractor. This is usually a semicircle of metal, the arc of which is divided into degrees. To use it, place it with the centre at the point at which the angle is to be made, and the straight edge coincident with the given line; then with a fine point prick off the number of degrees required, and join the point thus determined to the centre.

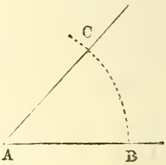
The figures on the protractor should begin at each end of the arc, as represented in Fig. 37.

Fig. 37.



120. By the Scale of Chords. The scale of chords, which is engraved on the ivory scales contained in a box of instruments, may also be used for making angles. For this purpose take from the scale the chord of 60° for a radius. With point A, at which the angle is to be made, as a centre, and that radius, describe an arc. Take off from the scale the chord of the required number of degrees and lay it on the arc from the given line, join the extremity of the arc thus laid off to the centre, and the thing is done.

Fig. 38.



Thus, if at the point A (Fig. 38) it were required to make an angle BAC of 47°, With the centre A and radius equal to the chord of 60° describe the arc BC. Then, taking the chord of 47° from the scale, lay it off from B to C. Join AC, and BAC will be the required angle.

If an angle of more than 90° is required: first lay off 90°, and from the extremity of that arc lay off the remainder.

121. By the Table of Chords. The table of chords (page 97 of the tables) affords a much more accurate means of laying off angles.

Take for a radius the distance 10 from any scale of equal parts,—to be described hereafter,—and describe the arc BC, (Fig. 38.) Then, finding the chord of the required angle by the table, multiply it by 10, and, taking the product from the same scale, lay it off from B to C as before. Join AC, and the thing is done.

If the angle is much over 60° it is best to lay off the 60° first. This is done by using the radius as a chord. The remainder can then be laid off from the extremity of the arc of 60° thus determined.

122. Distances. Every line on a draft should be drawn of such a length as correctly to represent the distance of the points connected, in due relation to the other parts of the drawing. In *perspective* drawing, the parts are delineated so as to present to the eye the same relations that the natural object does when viewed from a particular point. To produce this effect the figure must be distorted. Right angles are represented as right, obtuse, or acute, according to the position of the lines; and the lengths of lines are proportionally increased or diminished according to their position. In *drafting*, on the contrary, every part must be represented as it *is*. The angles should be of the same magnitude as they are in reality, and the lines should bear to each other the exact ratio that those which they are intended to represent do. The plat should, in fact, be a miniature representation of the figure.

123. Drawing to a Scale. In order that the due pro portion should exist in the parts of the figure, every line should be made some *definite* part of the length of that which it is intended to represent. This is called drawing to a *scale*. The scale to be used depends on the size of the map or draft that is required, and the purposes for which it is to be used. Carpenters often use the scale of an inch to a foot: the lines will then be the twelfth part of their real length. In plats of surveys, or maps of larger tracts of country, a greater diminution is necessary. The scale should, however, in all cases, be adapted to the purpose intended and to the number of objects to be represented. Where the purpose is merely to give a correct representation of the plat, without filling up the details, the main object will be to make the map of a *convenient* size; but where many details are to be represented the scale should be proportionally larger.

Thus, for example, in delineating a harbor where there are few obstructions to navigation, a map on a small scale may be drawn; but where the rocks and shoals are numerous, the scale should be so large that every part may be perfectly distinct.

The scales on which the drawing is made should always be mentioned on the map. They may be expressed by naming the lengths which are used as equivalents, thus,—"Scale, 10 feet to an inch, 1 mile to an inch, 3 chains to a foot;" or better fractionally, thus,—1 : 100, 1 : 250, 1 : 10,000, &c.

124. Surveys of Farms. Where the farm is small, 1 chain* to an inch, (1 : 792,) or 2 chains to the inch, (1 : 1584,) may be used; but if the tract be large, as this would make a plat of a very inconvenient size, a smaller scale must be adopted. When, however, any calculations are to be based on measurements taken from the plat, a smaller scale than 3 chains to the inch (1 : 2376) should not be employed.

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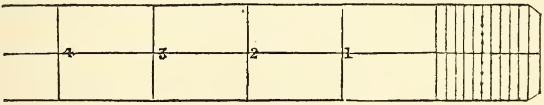
* The surveyor's chain—commonly called Gunter's Chain—is 4 poles, or 66 feet, in length, and is divided into one hundred links, each of which is therefore .66 feet, or 7.92 inches in length.

125. Scales. Scales are generally made of ivory or boxwood, having a feather-edge, on which the divisions are marked. The distances can then be laid off by placing the ruler on the line, and pricking the paper or marking it with a fine pointed pencil; or the length of a line may be read off without any difficulty. Boxwood scales, if the wood is clear from knots, are to be preferred to ivory. They are less liable to warp, and suffer less expansion and contraction from changes in the hygrometric condition of the atmosphere.

Paper scales are often employed. These may be procured with divisions to suit almost any purpose, or the surveyor may make them himself. Take a piece of drawing-paper, and cut a slip about an inch in width; draw a line along its middle, and divide it as desired, either into inches or tenths of a foot. The end division should be subdivided into ten parts, and perpendiculars drawn through all the divisions, as represented in the figure, (Fig. 39.) Each of these parts may then represent a chain, ten chains, &c.

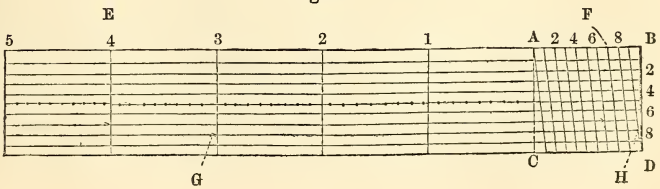
Fig. 39.



Paper scales, being subject to nearly the same expansion and contraction as the paper on which the map is drawn, are, on this account, preferable to those made of wood or ivory. They cannot, however, be divided with the same accuracy.

126. The plane diagonal scale (Fig. 40) consists of eleven

Fig. 40.



lines drawn parallel and equidistant. These are crossed at right angles by lines 1, 2, 3, drawn usually at intervals of half an inch. The first division, on the upper and lower lines, is subdivided into ten equal parts: diagonal lines are then drawn, as in the figure, from each division of the top to the next on the bottom,—the first, from A to the first division on the bottom line; the second, from the first on the top to the second on the bottom; and so on.

It is evident that, whatever distance the primary division from A to 1, or 1 to 2, &c. represents, the parts of the line AB will represent tenth parts of that distance. If then it were required to take off the distance of 47 feet on a scale of half an inch to 10 feet, the compasses should be extended from E to F.

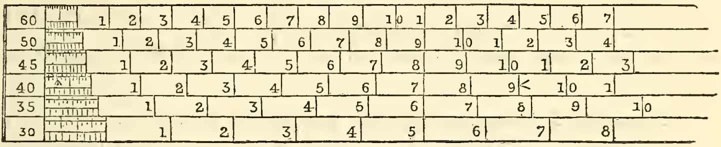
The diagonal lines serve to subdivide each of the smaller divisions into tenths, thus:—The first diagonal, extending from A to the first division on the bottom line and crossing ten equal spaces, will have advanced $\frac{1}{10}$ of one of those divisions at the first intermediate line, $\frac{2}{10}$ at the second, $\frac{3}{10}$ at the third, and so on. All the other diagonals will advance in the same manner.

If then the distance were taken from the line AC along the horizontal line marked 6 to the fourth diagonal, the distance would be .46, the division AB being a unit, or 4.6 if AB were 10. To take off, then, 39.8 feet on a scale of half an inch to 10 feet, the compasses should be extended to the points marked by the arrow heads G and H: similarly, 46.7, on the same scale, would extend from one of the arrow heads on the seventh line to the other.

In using the diagonal scale the primary divisions should always be made to represent 1, 10, 100, or 1000. When any other scale is required,—say 1 : 300,—it is better to divide or multiply all the distances and then take off the results. Thus, if 83.7 were required to be taken off on a scale of $\frac{1}{2}$ inch to 30 feet, first divide 83.7 by 3, giving 27.9, and then take off the quotient on a scale of $\frac{1}{2}$ inch to 10 feet. The other lines must all be reduced in the same proportion. The above method requires less calculation, and involves less liability to error, than that of determining the value of each division on the reduced scale.

127. Proportional Scale. On most of the rulers furnished with cases of instruments there is another set of scales, divided as below, (Fig. 41.)

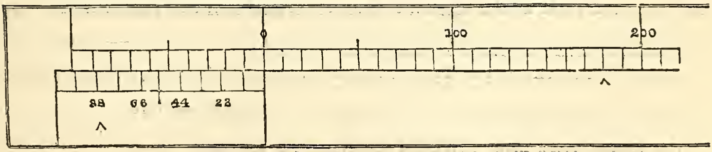
Fig. 41.



The figures on the left express the number of divisions to the inch. To lay off 97 feet on a scale of 40 feet to the inch, the compasses would be extended between the arrowheads on the line 40. Scales of this kind are very convenient in altering the size of a drawing. Suppose, for example, it is desired to reduce a drawing in the ratio of 5 to 3: the lengths of the lines should be determined on the scale marked 30, and the same number of divisions on the scale 50 will give a line of the desired length.

128. Vernier Scale. Make a scale (Fig. 42) with inches divided into tenths, and mark the end of the first inch 0, of the second 100, and so on. From the zero point, backwards, lay off a space equal to eleven tenths of an inch, and divide it into ten equal parts, numbering the parts backwards, as represented in the figure. This smaller scale

Fig. 42.



is a vernier. Now, since the ten divisions of the vernier are equal to eleven of the scale, each of the vernier divisions is equal to $\frac{11}{10}$ of $\frac{1}{10} = \frac{11}{100}$ of an inch. From the zero point, therefore, to the second division of the vernier is .22 inch, to the third .33, and so on.

To measure any line by the scale, take the distance in the compasses, and move them along the scale until you find that they exactly extend from some division on the vernier to a division on the scale. Add the number on the scale to the number on the vernier for the distance required. Thus, suppose the compasses extended from 66 on the vernier to 110 on the scale, the length is 176.

To lay off a distance by the scale, for example 175, take 55 from 175, and 120 is left: extend the compass from 120 on the scale to 55 on the vernier. To lay off $268 = 180 + 88$, extend the compasses from 180 on the scale to 88 on the vernier, as marked by the arrow heads.

The vernier scale is equally accurate with the diagonal scale, and much more readily made.

SECTION III.

TABLES OF TRIGONOMETRICAL FUNCTIONS.

129. Table of Natural Sines and Cosines. This table (page 87 of the Tables) contains the sines and cosines to five decimal places for every minute of the quadrant. The table is calculated to the radius 1. As the sine and cosine are always less than radius, the figures are all decimals. In the table the decimal point is omitted. If the sine and cosine is wanted to any other radius, the number taken from the table must be multiplied by that radius.

To take out the sine or cosine of an arc from this table, look for the degrees, if less than 45, at the top of the table, and for the minutes at the left; then, in the column headed properly, and opposite the minutes, will be the function required. If the degrees are 45 or upwards they will be found at the bottom, and the minutes at the right. The name of the column is at the bottom.

Thus, the sine of 32° 17′, found under 32° and opposite 17′, is .53411.

The cosine of 53° 24′, found over 53° and opposite 24′ in the right-hand column, is .59622.

130. The table of natural sines and cosines is of but little use in trigonometrical calculations, these being generally performed by logarithms. It is principally employed in determining the latitudes and departures of lines.

131. Table of Logarithmic Sines, Cosines, &c. This table contains the logarithms of the sines, cosines, tangents, and cotangents, to every minute of the semicircle, the radius being 10 000 000 000 and its logarithm 10. The logarithmic sine of 90°, cosine of 0°, tangent of 45°, and cotangent of 45°, is each 10.

The *sine*, *cosine*, *tangent*, and *cotangent*, of every arc being equal to the *sine*, *cosine*, *tangent*, and *cotangent*, of its *supplement*, and also to the *cosine*, *sine*, *cotangent*, and *tangent*, of its *complement*, the table is only extended to forty five pages, the degrees from 0 to 44 inclusive being found at the top, those from 45 to 135 at the bottom, and from 136 to 180 at the top. The minutes are contained in the two outer columns, and agree with the degrees at the top and bottom on the same side of the page.

The columns headed Diff. 1″ contain the difference of the function for a change of 1″ in the arc. These differences are calculated by dividing the differences of the successive numbers in the columns of the functions by 60. By an inspection of these columns of difference it will be seen that, except in the first few pages, they change very slowly. In these, in consequence of the rapid change of the function, the differences vary very much. The difference set down will not, therefore, be accurate, except for about the middle of the minute. The calculations for seconds, therefore, are not in these cases to be depended on. To obviate this inconvenience, and give to the first few pages a degree of accuracy commensurate with that of the rest of the table, the sines and tangents are calculated to every 10 seconds, and these are the same as the cosines and cotangents of arcs within two degrees of 90.

132. Use of Table. To take out any function from the table, seek the degrees, if less than 45° or more than 135°, at the top of the page, and the minutes in the column on the same side of the page as the degrees. Then, in the proper column, (the title being at the top,) and opposite the minutes, will be found the value required.

If the degrees are between 45° and 135°, seek them at the bottom of the page, the minutes being found, as before, at the same side of the page as the degrees. The titles of the columns are also at the bottom.

EXAMPLES.

Ex. 1. Required the sine of 37° 17′. Ans. 9.782298.

Ex. 2. Required the cosine of 127° 43′. Ans. 9.786579.

Ex. 3. Required the cotangent of 163° 29′.
Ans. 10.527932.

Ex. 4. Required the tangent of 69° 11′.
Ans. 10.419991.

133. If there are seconds in the arc, take out the function for the degrees and minutes as before. Multiply the number in the difference column by the number of seconds, and add the product to the number first taken out, if the function is increasing, but subtract, if it is decreasing: the result will be the value required.

If the arc is less than 90° the *sine* and *tangent* are increasing, and the *cosine* and *cotangent* are decreasing; but if the arc is greater than 90° the reverse holds true.

Ex. 1. What is the tangent of 37° 42′ 25″?

The tangent of 37° 42′ is		9.888116
Diff. 1″	4.35	
	25	
	21 75	
	87 0	
Diff. 25″	108.75	+ 109
Tangent 37° 42′ 25″		9.888225

Ex. 2. What is the cosine of 129° 17′ 53″?

The cosine of 129° 17′ is		9.801511
Diff. 1″	2.57	
	53	
	7 71	
	128 5	
Diff. 53″	136.21	+ 136
Cosine 129° 17′ 53″		9.801647

Ex. 3. What is the sine of 63° 19′ 23″?

Ans. 9.951120.

Ex. 4. What is the cosine of 57° 28′ 37″?

Ans. 9.730491.

Ex. 5. What is the tangent of 143° 52′ 16″?

Ans. 9.863314.

Ex. 6. What is the sine of 172° 19′ 48″?

Ans. 9.125375.

If the sine or tangent of an arc less than 2° or more than 178°, or the cosine or cotangent of an arc between 88° and 92°, is required, it should be taken from the first pages of the table. Take out the function to the ten seconds next less than the given arc, multiply one tenth of the difference between the two numbers in the table by the odd seconds, and add or subtract as before.

The cotangent of an arc less than 2° may be found by taking out the tangent, and subtracting it from 20.000000; so likewise the tangent of an arc between 178° and 180° is found by taking the complement to 20.000000 of its cotangent.

Ex. 1. Required the sine of 1° 27′ 36″.

Sine of 1° 27′ 30″ is		8.405687
$\frac{1}{10}$ of difference	82.6	
	6	
Difference 6″	495.6	496
Sine of 1° 27′ 36″		8.406183

Ex. 2. What is the cosine of 88° 18′ 48″?

Ans. 8.468844.

Ex. 3. What is the sine of 179° 19′ 13″?

Ans. 8.074198.

134. To find the Arc corresponding to any Trigonometric Function.

If degrees and minutes only be required, seek, in the proper column, the number nearest that given; and if the title is at the top the degrees are found at the top, and the minutes under the degrees; but if the title is at the bottom the degrees are at the bottom, and the minutes on the same side as the degrees.

If seconds are desired, seek for the number corresponding to the minute next less than the true arc, and take the difference between that number and the given one: divide said difference by the number in the difference column, for the seconds.

Ex. 1. What is the arc whose sine is 9.427586?

	9.427586
Sine of 15° 31′ is	9.427354
	7.58) 232.00 (31″
	227 4
	4.60

The arc is, therefore, 15° 31′ 31″.

Ex. 2. What is the arc whose cotangent is 10.219684?

$$
\begin{array}{lr}
 & 10.219684 \\
\text{Cotangent of } 31^\circ\ 5' \text{ is} & \underline{10.219797} \\
 & 4.76)\,113.00\,(23.7'' \\
 & \underline{95\ 2} \\
 & 17\ 80 \\
 & \underline{14\ 28} \\
 & 3.52
\end{array}
$$

The arc is, therefore, 31° 5′ 24″.

Ex. 3. Required the arc the cosine of which is 9.764227.
Ans. 54° 28′ 27″.

Ex. 4. Required the arc the tangent of which is 10.876429.
Ans. 82° 25′ 44″.

Ex. 5. What is the arc the cotangent of which is 11.562147?

As this corresponds to an arc less than 2°, take it from 20.000000: the remainder, 8.437853, is the tangent. The arc is found as follows:—

$$
\begin{array}{lr}
 & 8.437853 \\
1^\circ\ 34'\ 10'' \text{ tang.} & \underline{8.437732} \\
\text{Diff. to } 1'' & 76.8\ )\ 121.0\,(1.6'' \\
 & \underline{76\ 8} \\
 & 44.20
\end{array}
$$

The angle is, therefore, 1° 34′ 11.6″.

Ex. 6. What arc corresponds to the cotangent 8.164375?
Ans. 89° 9′ 48.6″.

135. Table of Chords. This table contains the chords of arcs to 90° for every 5 minutes. Its principal use is in laying off angles, as explained in Art. 120.

SECTION IV.

ON THE NUMERICAL SOLUTION OF TRIANGLES.

136. Definition. THE solution of a triangle is the determination of the numerical value of certain parts when others are given. To determine a triangle, three independent parts must be known,—viz.: either the three sides, or two sides and an angle, or the angles and one side. The three angles are not of themselves sufficient, since they are not independent,—any one of them being equal to the difference between the sum of the others and 180°.

In the solution of triangles several cases may be distinguished; these will be treated of separately. These cases are applicable to all triangles. But as there are special rules for right-angled triangles, which are simpler than the more general ones, they will first be given.

A.—THE NUMERICAL SOLUTION OF RIGHT-ANGLED TRIANGLES.

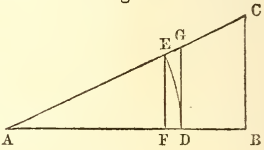
137. The following rules contain all that is necessary for solving the different cases of right-angled triangles.

1. *The hypothenuse is to either leg as radius is to the sine of the opposite angle.*

2. *The hypothenuse is to one leg as radius is to the cosine of the adjacent angle.*

3. *One leg is to the other as radius is to the tangent of the angle adjacent to the former.*

Fig. 43.



DEMONSTRATION.—Let ABC (Fig. 43) be a triangle right-angled at B. Take AD any radius, and describe the arc DE; draw EF and DG perpendicular to AB. Then EF will be the sine, AF the cosine, and DG the tangent, of the angle A. Now, from similar triangles we have—

1. AC : CB : : AE : EF : : r : sin. A. Rule 1;
2. AC : AB : : AE : AF : : r : cos. A. Rule 2;
3. AB : BC : : AD : DG : : r : tan. A. Rule 3.

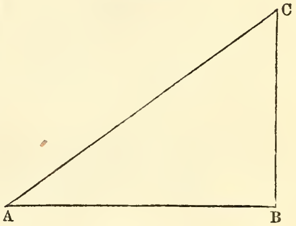
EXAMPLES.

Ex. 1. In the triangle ABC, right-angled at B, there are given the base AB = 57.23 chains, and the angle A 35° 27′ 25″, to find the other sides.

Construction.

Make AB (Fig. 44) = 57.23, taken from a scale of equal parts. At the point A make the angle BAC = 35° 27′. Erect the perpendicular BC, meeting AC in C, and ABC is the triangle required.

Fig. 44.



Calculation.

Rule 3. r : tan. A : : AB : BC.
Rule 2. cos. A : r : : AB : AC.

For facility of calculation, the proportions are generally written vertically, as below.

As rad.		log. 10.000000
: tan. A	35° 27′ 25″	9.852577
: : AB	57.23 ch.	1.757624
: BC	40.76	1.610201

As cos. A	35° 27′ 25″ Ar. Co.	0.089081
: rad.		10.000000
: : AB	57.23	1.757624
: AC	70.26	1.846705

Ex. 2. Given AB = 47.50 chains, and AC = 63.90 chains, to find the angles and side BC.

RULE 2.

As AC	63.90	Ar. Co.	8.194499
: AB	47.50		1.676694
: : rad.			10.000000
: cos. A	41° 58′ 57″		9.871193
	90		
C	48° 1′ 3″		

RULE 1.

As rad.		10.000000
: sin. A	41° 58′ 57″	9.825363
:: AC	63.90	1.805501
: CB	42.74	1.630864

Ex. 3. Given the two legs AB = 59.47 yards, and BC = 48.52 yards, to find the hypothenuse and the angles.

Ans. A 39° 12′ 36″, C 50° 47′ 24″, and AC 76.75 yds.

Ex. 4. Given the hypothenuse AC = 97.23 chains, the perpendicular BC = 75.87 chains, to find the rest.

Ans. A 51° 17′ 22″, C 38° 42′ 38″, AB 60.81 ch.

Ex. 5. Given the angle A = 42° 19′ 24″, and the perpendicular BC = 25.54 chains, to find the other sides.

Ans. AC 37.932 ch., AB 28.045 ch.

Ex. 6. Given the angle C = 72° 42′ 9″, and the hypothenuse AC = 495 chains, to find the other sides.

Ans. AB 472.612 ch., BC 147.18 ch.

Ex. 7. In the right-angled triangle ABC we have the base AB = 63.2 perches, and the angle A 42° 8′ 45″, to find the hypothenuse and the perpendicular.

Ans. BC 57.20 p., AC 85.24 p.

138. When two sides are given, the third may be found by (47.1); thus,

1. *Given the hypothenuse and one leg, to find the other.*

Rule. *From the square of the hypothenuse subtract the square of the given leg: the square root of the remainder will be the other leg;* or,

Multiply the sum of the hypothenuse and given leg by their difference: the square root of this product will be the other leg.

This is evident from (47.1) and (cor. 5.2.)

2. *Given the two legs, to find the hypothenuse.*

Rule. *Add the squares of the two legs, and extract the square root of the sum: the result will be the hypothenuse.*

EXAMPLES.

Ex. 1. Given the hypothenuse AC = 45 perches, and the leg BC = 29 perches, to find the other leg.

Rule 1. $AB = \sqrt{AC^2 - BC^2} = \sqrt{2025 - 841} = \sqrt{1184} = 34.41.$

or, $AB = \sqrt{(AC + BC).(AC - BC)} = \sqrt{74 \times 16} = \sqrt{1184} = 34.41.$

Ex. 2. The two legs AB and AC are 6 and 8 respectively: what is the hypothenuse? Ans. 10.

Ex. 3. The hypothenuse AC is 47.92 perches, and the leg AB is 29.45 perches: required the length of BC.
Ans. 37.8 perches.

Ex. 4. The hypothenuse of a right-angled triangle is 49.27 yards, and the base 37.42 yards: required the perpendicular. Ans. 32.05.

B.—THE NUMERICAL SOLUTION OF OBLIQUE-ANGLED TRIANGLES.

CASE 1.

139. *The angles and one side, or two sides and an angle opposite to one of them, being given, to find the rest.*

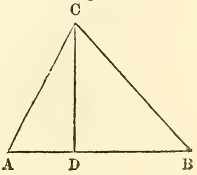
RULE.

1. *As the sine of the angle opposite the given side is to the sine of the angle opposite the required side, so is the given side to the required side.*

2. *As the side opposite the given angle is to the other given side, so is the sine of the angle opposite to the former to the sine of the angle opposite the latter.*

DEMONSTRATION.—Both the above rules are combined in the general proposition. The sides are to one another as the sines of their opposite angles.

Fig. 45.



Let ABC (Fig. 45) be any triangle. From C let fall CD perpendicular to AB. Then (Art. 137) AC : CD : : r : sin. A, and CD : CB : : sin. B : r. Whence (23.5) AC : CB : : sin. B : sin. A.

EXAMPLES.

Ex. 1. In the triangle ABC are given AB = 123.5, the angle B = 39° 47′ 20″, and C = 74° 52′ 10″: required the rest.

Construction.

The angle A = 180 − (B + C) = 180° − 114° 39′ 30″ = 65° 20′ 30″.

Draw AB (Fig. 45) = 123.5. At the points A and B draw AC, BC, making the angles BAC and ABC equal, respectively, to 65° 20′ 30″ and 39° 47′ 20″; then will ABC be the triangle required.

Calculation.

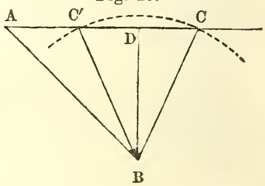
As sin. C		74° 52′ 10″	A. C. 0.015322
:	sin. B	39° 47′ 20″	9.806154
: :	AB	123.5	2.091667
:	AC	81.87	1.913143
As sin. C			A. C. 0.015322
:	sin. A	65° 20′ 30″	9.958474
: :	AB		2.091667
:	BC	116.27	2.065463

Ex. 2. Given the side AB = 327, the side BC = 238, and the angle A = 32° 27′, to determine the rest.

Construction.

Make AB (Fig. 46) = 327; and at the point A draw AC making the angle A = 32° 47′. With the centre B and radius = 238 describe an arc cutting AC in C; then will ABC be the triangle required.

Fig. 46.



Calculation. RULE 2.

As BC		238	A. C. 7.623423
:	AB	327	2.514548
: :	sin. A	32° 47′	9.733569
:	sin. C	48° 4′ 6″	9.871540
	or	131° 55′ 54″	

C acute.

As sin. C	48° 4′ 6″	A. C. 0.128460
: sin. B	99° 8′ 54″	9.994477
:: AB	327	2.514548
: AC	433.99	2.637485

C obtuse.

As sin. C	131° 55′ 54″	A. C. 0.128460
: sin. B	15° 17′ 6″	9.420979
:: AB		2.514548
: AC	115.87	2.063987

NOTE.—It will be seen that in the above example the result is uncertain. The sine of an angle being equal to the sine of its supplement, it is impossible, from the sine alone, to determine whether the angle should be taken acute or obtuse. By reference to the construction, (Fig. 46,) we see that whenever the side opposite the given angle is less than the other given side, and greater than the perpendicular BD, the triangle will admit of two forms: ABC, in which the angle opposite to the side AB is acute, and ABC′, in which it is obtuse. If BC were greater than BA, the point C′ would fall on the other side of A, and be excluded by the conditions. If it were less than BD, the circle would not meet AC, and the question would be impossible.

Ex. 3. Given the side AB 37.25 chains, the side AC = 42.59 chains, and the angle C 57° 29′ 15″, to determine the rest.

Ans. BC 32.774 chains, A = 47° 53′ 52″, and B = 74° 36′ 53″.

Ex. 4. Given the angle A 29° 47′ 29″, the angle B = 24° 15′ 17″, and the side AB 325 yards, to find the other sides.

Ans. AC = 164.93, BC = 199.48.

Ex. 5. The side AB of an obtuse-angled triangle is 127.54 yards, the side AC 106.49 yards, and the angle B 52° 27′ 18″, to determine the remaining angles and the side BC.

Ans. C = 108° 16′ 3″, A = 19° 16′ 39″, BC = 44.34.

Ex. 6. Given AB = 527.63 yards, AC = 398.47 yards, and the angle B 43° 29′ 11″, to determine the rest.

Ans. C = 65° 40′ 44″, A = 70° 50′ 5″, BC = 546.93;
or, C = 114° 19′ 16″, A = 22° 11′ 33″, BC = 218.71.

CASE 2.

140. *Two sides and the included angle being given, to determine the rest.*

RULE 1.

Subtract the given angle from 180° : *the remainder will be the sum of the remaining angles.* Then,

As the sum of the given sides is to their difference, so is the tangent of half the sum of the remaining angles to the tangent of half their difference.

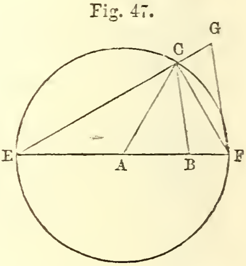
This half difference added to the half sum will give the angle opposite the greater side, and subtracted from the half sum will give the angle opposite the less side.

Then having the angles, the remaining side may be found by Case 1.

DEMONSTRATION.—The second paragraph of this rule may be enunciated in general terms; thus,

As the sum of two sides of a plane triangle is to their difference, so is the tangent of half the sum of the angles opposite those sides to the tangent of half the difference of those angles.

Fig. 47.



Let ABC (Fig. 47) be the triangle of which the side AC is greater than AB. With the centre A and radius AC describe a circle cutting AB produced in E and F. Join EC and CF, and draw FG parallel to BC. Then, because ABC and AFC have the common angle A, AFC + ACF = ABC + ACB. Whence AFC = ½ (ABC + ACB); and, since the half sum of two quantities taken from the greater leaves their half difference, CFG = EFG — EFC = ABC — EFC = ½ (ABC — ACB).

Now, since the angle ECF is an angle in a semicircle, it is a right angle. Therefore, if with the centre F and radius FC an arc be described, EC and CG will be the tangents of EFC and CFG, or of the half sum and half difference of ABC and ACB. But (2.6) EB : BF : : EC : CG.

Whence AC + AB : AC — AB : : tan. ½ (ABC + ACB) : tan. ½ (ABC — ACB).

EXAMPLES.

Ex. 1. Given AB = 527 yards, AC = 493 yards, and the angle A = 37° 49′.

Here $\quad C + B = 180° - 37° 49' = 142° 11'$, and

$AB + AC$	1020	A.C. 6.991400
$: AB - AC$	34	1.531479
$:: \tan.\dfrac{C+B}{2}$	$71°\ 5'\ 30''$	10.465290
$: \tan.\dfrac{C-B}{2}$	$5°\ 33'\ 29''$	8.988169
C	$76°\ 38'\ 59''$	
B	$65°\ 32'\ 1''$	
$: \sin. C$	$76°\ 38'\ 59''$	A.C. 0.011897
$: \sin. A$	$37°\ 49'$	9.787557
$:: AB$	527	2.721811
$: BC$	332.10	2.521265

Ex. 2. In the triangle ABC are given $AB = 1025.57$ yards, $BC = 849.53$ yards, and the angle $B = 65°\ 43'\ 20''$, to find the rest.

Ans. $A = 48°\ 52'\ 10''$, $C = 65°\ 24'\ 30''$, $AC = 1028.13$.

Ex. 3. Two sides of a triangle are 155.96 feet and 217.43 feet, and their included angle $49°\ 19'$, to find the rest.

Ans. Angles, $85°\ 4'\ 12''$, $45°\ 36'\ 48''$, side, 165.49.

RULE 2.

141. *As the less of the two given sides is to the greater, so is radius to the tangent of an angle; and as radius is to the tangent of the excess of this angle above* 45°, *so is the tangent of the half sum of the opposite angles to the tangent of their half difference.*

Having found the half difference, proceed as in Rule 1.

NOTE.—This rule is rather shorter than the last, where the two sides have been found in a preceding calculation, and thus their logarithms are known.

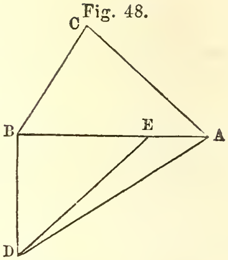


Fig. 48.

DEMONSTRATION.—Let ABC (Fig. 48) be any plane triangle. Draw BD perpendicular to AB, the greater, and equal to BC, the less side. Make BE = BD, and join ED. Then, since BE = BD, the angle BED = BDE; and since EBD is a right angle, BDE = 45°. But BED + BDE = 2 BDE = BAD + BDA, and BDE = $\frac{1}{2}$ (BDA + BAD). But the half sum of any two quantities being taken from the greater will leave the half difference: therefore ADE is the half difference of BDA and BAD.

Now, (Rule 3, Art. 137,) BD or BC : BA : : *rad.* : tan. ADB;

and (demonstration to last rule) AB + BD : AB — BD : : tan. $\frac{1}{2}$ (BDA + BAD) : tan. $\frac{1}{2}$ (BDA — BAD) : : tan. BDE : tan. ADE; but BDE being equal to 45°, its tangent = rad.

And ADE = (ADB — 45°) ∴ AB + BD : AB — BD : : r : tan. (ADB — 45°); but AB + BC : AB — BC : : tan. $\frac{1}{2}$ (ACB + BAC) : tan. $\frac{1}{2}$ (ACB — BAC); whence r : tan. (ADB — 45°) : : tan. $\frac{1}{2}$ (ACB + BAC) : tan. $\frac{1}{2}$ (ACB — BAC).

EXAMPLES.

Ex. 1. In the course of a calculation I have found the logarithm of AB = 2.596387, that of BC = 2.846392: now, the angle B being 55° 49′, required the side AC.

Calculation.

As AB		A. C. 7.403613
: BC		2.846392
: : Rad.		10.000000
: tan. x	60° 38′ 58″	10.250005
As rad.		A. C. 0.000000
: tan. $(x - 45)$	15° 38′ 58″	9.447368
: : tan. $\frac{1}{2}$ (A + C)	62° 5′ 30″	9.276004
: tan. $\frac{1}{2}$ (A — C)	27° 52′ 28″	9.723372
	A 89° 57′ 58″	

Then,

As sin. A	89° 57′ 58″	A. C. 0.000000
: sin. B	55° 49′	9.917634
: : BC		2.846392
: AC	580.8	2.764026

Ex. 2. Given the logarithms of BC and AC 3.964217 and 3.729415 respectively, and the angle $C = 63° 17' 24''$, to find AB. Ans. 8317.

Ex. 3. Given the logarithms of AB and BC 1.963425 and 2.416347, and the angle $B = 129° 42'$, to find AC.
Ans. 327.27.

CASE 3.

142. Given the three sides, to find the angles.

Rule 1.

Call the longest side the base, and on it let fall a perpendicular from the opposite angle.

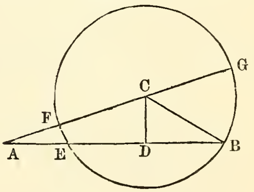
Then, as the base is to the sum of the other sides, so is the difference of those sides to the difference of the segments of the base.

Half this difference added to half the base will give the greater segment, and subtracted will give the less segment.

Having the segments of the base, and the adjacent sides, the angles may be found by Rule 2, Art. 137.

Demonstration.—Let ABC (Fig. 49) be the triangle, AB being the longest side: with the centre C and a radius CB, the less of the other sides, describe a circle, cutting AB in E and AC in F and G. Draw CD perpendicular to AB. Then (3.3) DE = DB; therefore AE is the difference of the segments of the base.

Fig. 49.



Also, AG = AC + CB; and AF = AC — CB.

Now, (36.3. cor.,) AB . AE = AG . AF;

whence (16.6) AB : AG : : AF : AE,

or AB : AC + CB : : AC — CB : AD — DB.

Examples.

Ex. 1. Given the three sides of a triangle,—viz.: AB = 467, AC = 413, and BC = 394, to find the angles.

As AB	467	Ar. Co. 7.330683
: AC + BC	807	2.906874
: : AC − BC	19	1.278754
: AD − DB	32.833	1.516311
$\frac{1}{2}$(AD − DB)	16.4165	
$\frac{1}{2}$ AB	233.5	
AD	249.9165	
BD	217.0835	

As AC	413	Ar. Co. 7.384050
: AD	249.9165	2.397794
: : r		10.000000
: cos. A	52° 45′ 44″	9.781844

As BC	394	Ar. Co. 7.404504
: BD	217.0835	2.336627
: : r		10.000000
: cos. B	56° 33′ 58″	9.741131

Whence C = 180 − (A + B) = 70° 40′ 18″.

Ex. 2. Given the three sides of a triangle, BC 167, AB 214, and AC 195 yards, respectively, to find the angles.

Ans. A = 47° 55′ 13″, B = 60° 4′ 19, C = 72° 0′ 28″.

Ex. 3. Given AB = 51.67, AC = 43.95, and BC = 27.16, to find the angles.

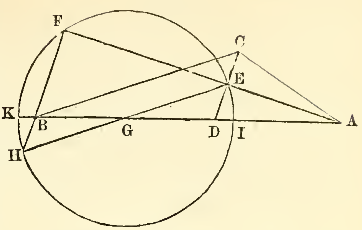
Ans. A = 31° 42′ 42″, B = 58° 16′ 34″, C = 90° 0′ 44″.

RULE 2.

143. *As the rectangle of two sides is to the rectangle of the half sum of the three sides and the excess thereof above the third side, so is the square of radius to the square of the cosine of half the angle contained by the first mentioned sides.*

DEMONSTRATION.—Let ABC (Fig. 50) be a triangle, of which AB is greater than AC. Make AD = AC. Join DC, and bisect it by AEF. Draw EH parallel and equal to CB. Join HB, and produce it to meet AEF in F. Then, since EH is equal and parallel to CB, BH is equal and parallel to CE, (33.1.) Therefore F is a right angle. Again: since BH is equal to ED, and the angle EGD = BGH and EDG = GBH, (26.1,) DG = GB and EG = GH. On EH describe a circle, and it will pass through F.

Fig. 50.



Now, 2 AK = 2 AG + 2 GK = AC + AD + 2 DG + 2 GK = AC + AB + BC;

or $\quad AK = \frac{1}{2}(AC + AB + BC)$,

and $\quad AI = AK - KI = \frac{1}{2}(AC + AB + BC) - BC.$

But, (Rule 2, Art. 137,) As $AD : AE :: r : \cos. DAE\ (\cos. \frac{1}{2} BAC)$,

and $\quad AB : AF :: r : \cos. \frac{1}{2} BAC$;

whence (23.6) $\quad AB . AD : AE . AF :: r^2 : \cos.^2 \frac{1}{2} BAC.$

But (36.3, Cor.) $AE . AF = AK . AI = \frac{1}{2}(AC + AB + BC) . \frac{1}{2}(AC + AB + BC) - BC$;

whence $AB . AC : \frac{1}{2}(AC + AB + BC) \cdot (\frac{1}{2}(AC + AB + BC) - BC) :: r^2 : \cos.^2 \frac{1}{2} BAC.$

EXAMPLES.

Ex. 1. Given AB = 467, AC = 413, and BC = 394, to find the angle C.

Here, put s = half sum of the sides: we have $s = 637$ and $s - BC = 170$; whence

As AC . BC	AC	413	A.C.	7.384050
	BC	394	A.C.	7.404504
: $s.(s-AB)$	s	637		2.804139
	$s-AB$	170		2.230449
:: R^2				20.000000
: $\cos.^2 \frac{1}{2} BCA$				2)19.823142
$\frac{1}{2}$ BCA =	35° 20′ 9″			9.911571
BCA =	70° 40′ 18″.			

In the above calculation the R^2 and its logarithm might have been omitted, since we have to deduct 20 in consequence of having taken two arithmetical complements. The sum of the logarithms is divided by 2, to extract the square root, (Art. 16.)

The rule may be expressed thus:—

Add together the arithmetical complements of the logarithms of the two sides containing the required angle, the logarithm of the half sum of the three sides, and the logarithm of the excess of the half sum above the side opposite to the required angle: the half sum of these four logarithms will be the logarithmic cosine of half that angle.

Ex. 2. Given $AB = 167$, $AC = 214$, and $BC = 195$, to find the angles.

Ans. $A = 60° 4' 22''$, $B = 72° 0' 32''$, $C = 47° 55' 14''$.

Ex. 3. Given $AB = 51.67$, $AC = 43.95$, and $BC = 27.16$, to find the angles.

Ans. $A = 31° 42' 40''$, $B = 58° 16' 28''$, $C = 90° 0' 52''$.

SECTION V.

INSTRUMENTS AND FIELD OPERATIONS.

144. The Chain. GUNTER'S CHAIN is the instrument most commonly employed for measuring distances on the ground. For surveying purposes, it is made 66 feet or 4 perches long, and is formed of one hundred links, each of which is therefore .66 feet or 7.92 inches long. The links are generally connected by two or three elliptic rings, to make the chain more flexible. A swivel link should be inserted in the middle, that the chain may turn without twisting. In order to facilitate the counting of the links, every tenth link is marked by a piece of brass, having one, two, three, or four points, according to the number of tens, reckoned from the nearest end of the chain. Sometimes the number of links is stamped on the brass. The middle link is also indicated by a round piece of brass.

The advantage of having a chain of this particular length is, that ten square chains make an acre. The calculations are therefore readily reduced to acres by simply shifting the decimal point. There being one hundred links to the chain, all measures are expressed decimally, which renders the calculations much more convenient. Eighty chains make one mile.

In railroad surveying, a chain of one hundred feet long is preferred, the dimensions being thus at once given in feet.

When the measurements are required to be made with great accuracy, rods of wood or metal, which have been made of precisely the length intended, are used. In the surveys of the American Coast Survey, the unit of length employed is the French metre, equal to the 10000000th part of the quadrant of the meridian. The metre is 39.37079 inches = 3.280899 feet = 1.093633 yards long.

It were much to be desired that the metre, or some other unit founded on the magnitude of the earth, or on some other natural length, such as that of a pendulum beating seconds at a given latitude, were universally adopted as the unit. The metre will probably gradually come into general use.

To reduce chains and links to feet, express the links decimally and multiply by 66. Thus, 7 chains 57 links = 7.57 chains are equal to $7.57 \times 66 = 499.62$ feet = 499 feet 7.4 inches.

To reduce feet and inches to chains, divide by 66, or by 6 and 11. The inches must first be reduced to a decimal of a foot. Thus, 563 feet 8 inches = 563.67 feet $= \frac{563.67}{66}$ ch. = 8.54 chains.

Instead of a chain of 66 feet, one of 33 feet, divided into fifty links, is sometimes used. This is really a half chain, and should be so recorded in the notes. The half chain is more convenient when the ground to be measured is uneven.

145. The chain is liable to become incorrect by use; its connecting rings may be pulled open, and thus the chain become too long, or its links may be bent, which will shorten the chain. Every surveyor should, therefore, have a carefully measured standard with which to compare his chain frequently. According to the laws of Pennsylvania, such a standard is directed to be marked in every county town, and all surveyors are required to compare their chain therewith every year.

If the chain is too long, it may be shortened by tightening the rings; if it is too short, which it can only become by some of the links having been bent or some rings tightened too much, these should be rectified.

It has been found that a distance measured by a perfectly accurate chain is very generally recorded too long; if then the chain is found slightly too long, say from one fourth to one third of an inch, it need not be altered, a distance measured with such a chain being more accurately recorded than if the chain were correct.

In using the chain, care should be taken to stretch it always with the same force, or the different parts of the line will not be correctly recorded. Like all other instruments, it should be carefully handled, as it is liable to injury.

146. The Pins. In using the chain, ten pins are necessary to set in the ground to mark the end of each chain measured. These are usually made of iron, and are about a foot or fifteen inches long, the upper end being formed into a ring, and the lower sharpened that they may be readily thrust into the ground. Pieces of red and white cloth should be tied to the ring, to distinguish them when measuring through grass or among dead leaves.

147. Chaining. This operation requires two persons. The leader starts with all the pins in his left hand and the end of the chain in his right; the follower, remaining at the starting point and looking at the staff set up to mark the other end of the line, directs the leader to extend the chain precisely in the proper direction. The leader then sticks one pin perpendicularly into the ground at the end of the chain. They then go on until the follower comes to this pin, when he again puts the leader in line, who places a second pin. The follower then takes up the first pin, and the same operation is repeated until the leader has expended all his pins. When he has stuck his last pin, he calls to the follower, who comes forward, bringing the pins with him. The distance measured—viz.: ten chains—is then noted. The leader, taking all the pins, again starts, and the operation is repeated as before. When the leader has arrived at the end of the line, the number of pins in possession of the follower shows the number of chains since the last "out," and the number of links from the last pin to the end of the line, the number of odd links. Thus, supposing there were two "outs," and the follower has six pins, the end of the line being 27 links from the last pin, the length would be 26.27 chains.

Some surveyors prefer eleven pins. One pin is then stuck at the beginning of the line, and at every "out" a pin is left in the ground by the leader.

If the chain-men are both equally careful, they may change duties from time to time. If otherwise, the more intelligent and careful man should act as follower, that being much the more responsible position.

148. Recording the "Outs." As every "out" indicates ten chains,—or five chains, if a two-pole chain is used,—it is of great importance to have them carefully kept. Various contrivances have been suggested for that purpose. Some chain-men carry a string, in which they tie a knot for every out; others place in one pocket a number of pebbles, and shift one to another pocket at each out. Either of these methods is sufficient if faithfully followed out. One rule, however, should be faithfully adhered to,—viz.: that the memory should never be trusted. The distractions to which the mind is subject in all such operations, necessarily call off the attention, so that a mere number, which has no associations to call it up, will be very likely to be forgotten.

Perhaps the best method of preserving the "outs" is to have nine iron pins and five or six brass ones. The leader takes all the pins and goes on until he has exhausted his iron pins; he then goes on one chain, and, sticking a brass pin, calls, "Out." The follower then advances, bringing the pins. He delivers to the leader the iron pins but retains the brass ones. On arriving at the end of the line, the brass pins in the follower's possession will show the number of "outs" and the iron pins the number of chains since the last "out." Thus, supposing he have six brass and eight iron pins, and that the end of the line is 63 links from the last pin, the distance is 68.63 chains.

149. Horizontal Measurement. In all cases where the object is to determine the area or the position of points on a survey, the measurements must either be made horizontally, or, if made up or down a slope, the distance must be reduced according to the inclination.

In chaining down a slope, the follower should hold his end of the chain firmly at the pin. The leader should then elevate his end until the chain is horizontal, and then mark the point directly under the end of the chain. This may be done by means of a staff four or five feet long, which should be held vertical, or by dropping a pin held in the hand with the ring downwards, or by a plumb-line. If the ground slopes much, the whole chain cannot be used at once. In such cases the leader should take the end of the half or the quarter, and, elevating it as before, drop his pin or make a mark. The follower then comes forward, and, holding the 50th or 25th link, as the case may be, the leader goes forward to the end of another short portion of the chain, which he holds up, as before. A pin is left only at the end of every whole chain.

Chaining up a slope is less accurate than chaining down, from the difficulty of holding the end still, under the strain to which the chain is subjected. The follower should always, in such cases, be provided with a staff four or five feet long, and a plumb-line to keep it vertical. If the slope is so steep that the whole chain cannot be used at once, the leader should take (as before) the end of a short portion, say one fourth, and proceed up hill. The follower then elevates his end, holding it firmly against the staff, which is kept vertical by the plumb-line. The leader, having made his mark, notifies the follower, who comes forward and holds up the same link that the leader used. He then goes forward as before.

150. When great accuracy is required, the chaining should be made according to the slope of the ground, leaving stakes where there is any change of the slope, and recording the distances to these stakes in the note book. The inclination of the different parts being then taken, the horizontal distance can be calculated. If a transit with a vertical arc is employed, the slope can be obtained at once, and the proper correction may be made at the time. The best way is to have a table prepared for all slopes likely to be met with, and apply the correction on the ground. Instead of deducting from the distance measured, it is best to increase the length on the slope, calling each length so increased a chain: the horizontal distance will then be correctly recorded. Thus, supposing the slope to be 5°, in order that the base may be 1 chain the hypothenuse must be 1.0038: the follower should therefore advance his end of the chain rather less than half a link.

If a compass is used, it may be furnished with a tangent scale, to be described hereafter.

The following table contains the ratio of the perpendicular to the base, the correction of the base for each chain on the slope, and the correction of the slope for each horizontal chain. If the corrections are made as the work proceeds, the last column should be used; if in the field-notes after the work is done, the third column furnishes the data.

Angle.	Slope, perp.: base.	Correction of base, in links.	Correction of hypoth. in links.	Angle.	Slope.	Correction of base, in links.	Correction of hypoth. in links.
3°	1 : 19.1	—0.14	+0.14	17°	1 : 3.3	—4.37	+4.57
4°	1 : 14.3	0.24	0.24	18°	1 : 3.1	4.89	5.15
5°	1 : 11.4	0.38	0.38	19°	1 : 2.9	5.45	5.76
6°	1 : 9.5	0.55	0.55	20°	1 : 2.7	6.03	6.42
7°	1 : 8.1	0.75	0.75	21°	1 : 2.6	6.64	7.11
8°	1 : 7.1	0.97	0.98	22°	1 : 2.5	7.28	7.85
9°	1 : 6.3	1.23	1.25	23°	1 : 2.4	7.95	8.64
10°	1 : 5.7	1.52	1.54	24°	1 : 2.2	8.65	9.46
11°	1 : 5.1	1.84	1.87	25°	1 : 2.1	9.37	10.34
12°	1 : 4.7	2.19	2.23	26°	1 : 2.1	10.12	11.26
13°	1 : 4.3	2.56	2.63	27°	1 : 2	10.90	12.23
14°	1 : 4.0	2.97	3.06	28°	1 : 1.9	11.71	13.26
15°	1 : 3.7	3.41	3.53	29°	1 : 1.8	12.54	14.34
16°	1 : 3.5	3.87	4.03	30°	1 : 1.7	13.40	15.47

151. Tape-Lines. A tape-line is sometimes used instead of a chain in measuring short distances. It is, however, very little to be depended on. If used at all, the kind that is made with a wire chain should be employed. It is much less liable to be stretched than those made wholly of linen.

152. Chaining being one of the fundamental operations of surveying, whether for trigonometrical purposes or for the calculation of the contents, it has been described minutely. If correct measurements are needful, accurate notes are no less so. The chief points to be attended to in recording the measurements are precision and conciseness. Some of the most approved methods are given in Chapter IV.

153. Angles. For surveying purposes horizontal angles alone are needed, since all the parts of the survey are reduced to a horizontal plane; but to fix the direction of a point in space not only the horizontal but vertical angles are required. With the aid of these, and the proper linear measures, its position may be fully determined.

154. Horizontal angles are measured by having a plane, properly divided, and capable of being so adjusted as to be perfectly horizontal. Movable about the centre of this plane is another plane, or a movable arm, carrying a pair of sights or a telescope, which can be placed so that the line of sight may pass through the object. If then this line be directed to one object, and the position of the two plates or of the arm on the plate be noted by an index properly situated, and then be turned so as to point to another object, the angle through which the plate or the arm has turned will be the horizontal angle contained by two planes drawn from the centre of the instrument to the two objects.

155. Vertical angles are measured by having a pair of sights or a telescope so adjusted as to move on a horizontal axis, the horizontal position of the sights or the telescope being indicated either by a plumb-line or a level.

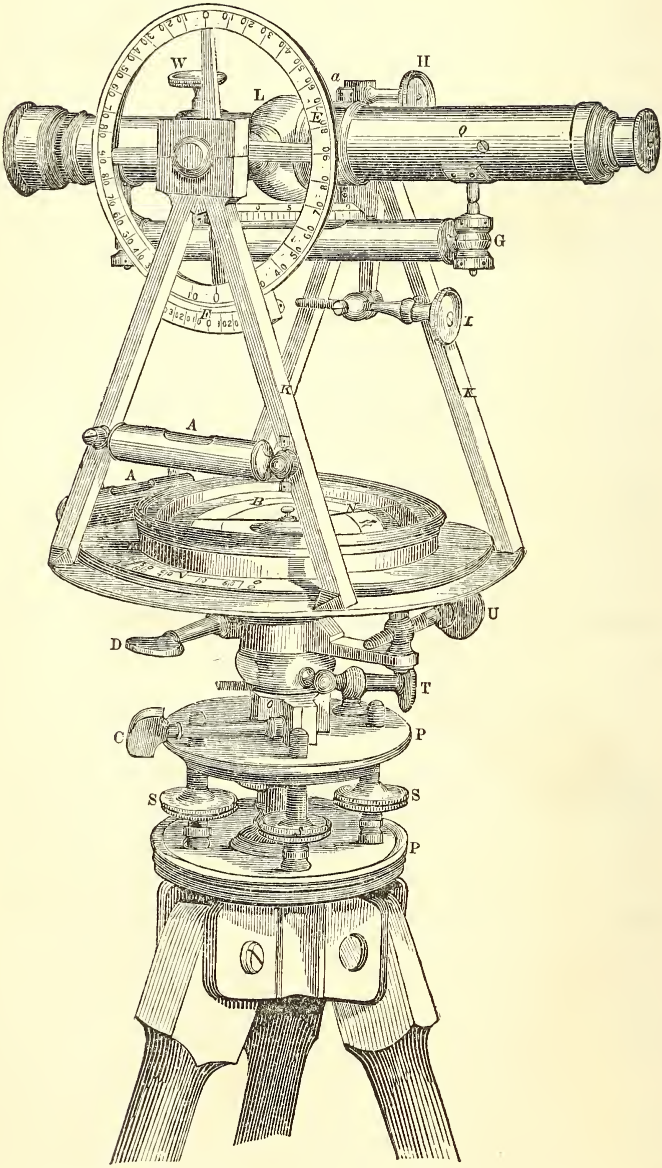
156. The transit with a vertical arc, or the theodolite, are so arranged as to perform both these offices. As a full understanding of the use of the different parts of these instruments is necessary to their proper management, we shall enter, considerably in detail, into a description of them.

THE TRANSIT AND THE THEODOLITE.

157. General Description. The Transit or the Theodolite (Figs. 51 and 52) consists of a circular plate, divided at its circumference into degrees and parts, and so supported that it can be placed in a perfectly horizontal position. This divided circle is called the limb. An axis exactly perpendicular to this plate, bearing another circular plate, passes through its centre. This plate is so adjusted as to move very nearly in contact with the former without touching it. By this arrangement the upper plate can be turned freely about their common centre. This plate carries a telescope Q, resting on two upright supports KK, upon which it is movable in a vertical plane. The telescope, having thus a horizontal and a vertical motion,

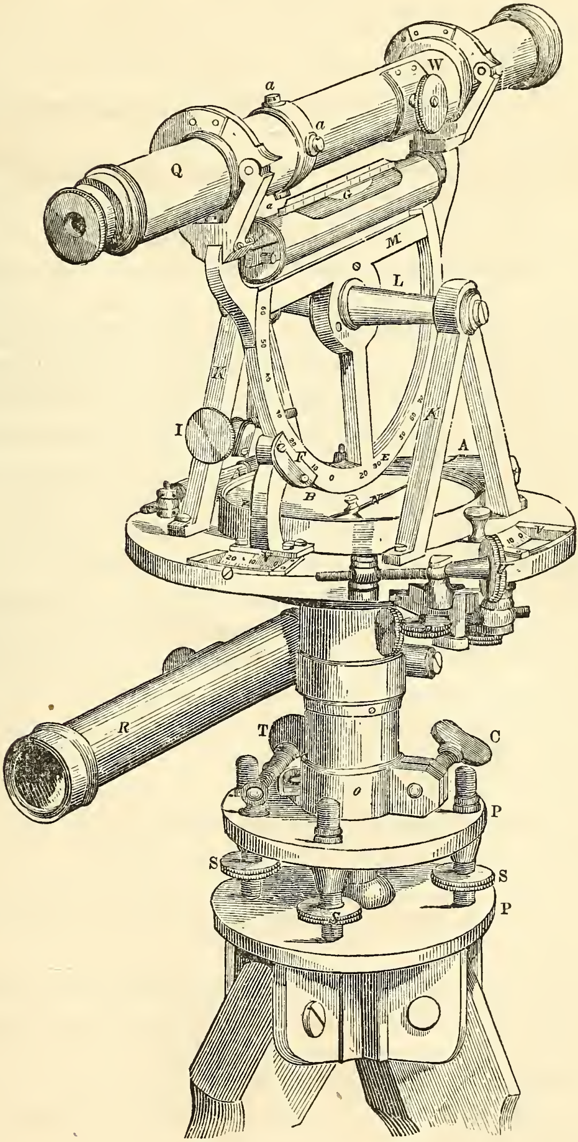
THE TRANSIT.

Fig. 51.



THE THEODOLITE.

Fig. 52.



can readily be pointed to any object. The second described plate has an index of some kind, moving in close proximity to the divided arc, so that the relative position of the plates may be determined. If then the telescope be directed to one object, and afterwards be turned to another, the index will travel over the arc which measures the horizontal angle between the objects.

In order to place the plates in a perfectly horizontal position, levelling screws and levels are required: these, as well as the other parts of the instrument, will be fully described in their proper place.

158. The above description applies to both instruments. The transit, however, is so arranged that the telescope can turn completely over; it can, therefore, be directed backwards and forwards in the same line. If the same thing is to be done by the theodolite, the telescope must be taken from its supports and have its position reversed. This operation is troublesome, and is, besides, very apt to derange the position of the instrument.

For surveying purposes, therefore, the transit is much to be preferred; and when the axis on which the telescope moves is provided with a vertical arc it serves all the purposes of a theodolite.

The theodolite has a level attached to the telescope. This is not generally found in the transit.

159. The accuracy of these instruments depends on several particulars :—

1. By means of the telescope the object can be distinctly seen at distances at which it would be invisible by the unassisted eye.

2. The circle, with its *vernier* index, enables the observer to record the position of the telescope with the same degree of precision with which it can be pointed.

3. There are arrangements for giving slow and regular motion to the parts, so as to place the telescope precisely in the position required.

4. There are other arrangements for making the plates of the instrument truly horizontal.

5. Imperfections in the relative position of the different parts of the instruments may be corrected by screws, the heads of some of which are shown in the drawings.

However complicated the arrangements for performing these various operations may make the instruments appear, that complication disappears when they are viewed in detail and properly understood.

160. In the figures of these instruments, V is the vernier, covered with a glass plate. In some theodolites the whole divided limb is seen. In others (and in the transit) but a small portion is exposed,—it being completely covered by the other plate, except the small portions near the vernier. Transits have generally but one vernier, though in some instruments there are two. The theodolite has generally two, and sometimes three or four. B is the compass box, containing the magnetic needle N. A, A, are the levels. C and D are screws; the former of which is designed to clamp the lower plate, and the latter to clamp the plates together. T and U are tangent screws, to give slow and regular motion when the plates are clamped: by the former the whole instrument is turned on its axis, and by the latter the upper plate is moved over the other. P, P are the levelling plates; and S, S, S, are three of the four levelling screws. E is the vertical circle, with its vernier F. G is a level attached to the telescope. H is a screw to clamp the horizontal axis, (not visible in the figure of the theodolite,) and I a tangent screw, to give it regular motion.

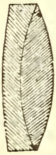
161. The Telescope. A telescope is a combination of lenses so adjusted in a tube as to give a distinct view of a distant object. It consists, essentially, of an object-glass, placed at the far end of the tube, and an eye-piece at the near end.

By the principles of optics, the rays of light proceeding from the different points of the object are brought to a focus within the tube, (Fig. 53,) there forming an inverted image. Crossing at this focus, they proceed on to the eye-piece, by the lenses of which they are again refracted, and made to issue in parallel pencils, thus giving a distinct magnified image of the object.

Fig. 53.

162. The Object-glass. Whenever a beam of light passes through a lens, it is not merely refracted, but it is likewise separated into the different colored rays of the solar spectrum. This separation of the colored rays, or the *chromatic aberration*, causes the edges of all bodies viewed with such a glass to be fringed with prismatic colors, instead of being sharply defined. It has been found, however, that the chromatic aberration may be nearly removed, by making a compound lens of flint and crown glass, as represented in Fig. 54, in which A is a concavo-convex lens of flint glass, and B a double convex lens of crown glass,—the convexity of one surface being made to agree with the concavity of the other lens. The two are pressed together by a screw in the rim of the brass box which contains them, thus forming a single compound lens. When the surfaces are properly curved, this arrangement is nearly achromatic.

Fig. 54.



The object-glass is placed in a short tube, movable by a pinion attached to the milled head W. (Figs. 51, 52.) By this means it may be moved backwards and forwards, so as to adjust it to distinct vision.

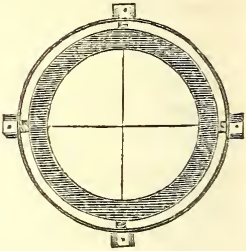
163. The Eye-piece. The eye-piece used in the telescopes employed for surveying purposes consists of two plano-convex lenses, fixed in a short tube, the convex surfaces of the lenses being towards each other. This arrangement is known as "Ramsden's Eye-piece."

164. A telescope with an object-glass and an eye-piece as above described, inverts objects. By the addition of two more lenses the rays may be made to cross each other again, and thus to give a direct image of the object. As these additional lenses absorb a portion of the light passing through them, they diminish the brightness of the image. They may therefore be considered a defect in telescopes intended for the transit or theodolite. A little practice obviates the inconvenience arising from the inversion of the image. The surveyor soon learns to direct his assistant to the right when the image appears to the left of its proper position, and *vice versâ.*

165. The Spider-Lines. The advantage gained by the telescope in producing distinct vision, would add nothing to the precision of the observations, without some means of directing the attention to the precise point which should be observed in the field of view. The whole field forms a circle, in the centre of which the object should appear at the time its position is to be noted. This centre is determined by stretching across the field precisely in the focus of the eye-piece a couple of spider-lines or fine wires, at right angles to each other. The former are generally employed. When they are properly adjusted in the focus they can be distinctly seen, and the point to be observed can be brought exactly to coincide with their intersection. The magnifying power of the eye-piece enables this to be done with the greatest precision. When it has been effected, a line through the centre of the eye-piece and the centre of the object-glass will pass directly through the object. This line is called the *line of collimation* of the telescope.

The spider-lines are attached by gum to the rim of a circular ring of brass placed in the tube of the telescope at the point indicated by the screw-heads *a*, *a*, (Figs. 51, 52,) some of which are invisible in the figure. These screws

Fig. 55.



serve to hold the ring in position, as represented in Fig 55, and to adjust it to its proper position. The eye-piece is made to slip in and out of the tube of the telescope, so that the focus may be brought to coincide exactly with the intersection of cross-wires. The perfect adjustment of the focus may be determined by moving the eye sideways. If this motion causes the wires to change their position on the object, the adjustment is not perfect: it must be made so before taking the observation.

166. Spider-lines are generally used for making the "cross-wires," though platinum wires drawn out very fine are preferable. The wire is drawn to the requisite degree of fineness by stretching a platinum wire in the axis of a cylindrical mould and casting silver around it. The compound wire thus formed is then drawn out as fine as possible and the silver removed by nitric acid. By this means Dr. Wollaston succeeded in obtaining wire not more than one thirty thousandth ($\frac{1}{30000}$) of an inch in diameter. As such wire is very difficult to procure, the spider-threads are generally substituted. The operation of placing them in their proper position is thus performed. A piece of stout wire is bent into the form of the letter U, the distance between the legs being greater than the external diameter of the ring. A cobweb is selected having a spider hanging at the end. It is gradually wound round the wire, his weight keeping it stretched: a number of strands are thus obtained extending from leg to leg of the wire: these are fixed by a little gum.

To fix them in their position, the wire is placed so that one of the lines is over notches previously made in the ring. The thread is then fixed in the position with gum or some other tenacious substance. The wire being removed, the line is left stretched across the opening in the proper position.

167. The Supports. Attached to one of the horizontal plates, usually the index-plate of the instrument, are two supports, K, K, (Figs. 51, 52,) bearing the horizontal axis L. These supports should be made of precisely the same height, so that when the plate is level the axis may be horizontal. In some instruments there is an arrangement for raising or depressing one end of the axis so as to perfect the adjustment. In most cases, however, the adjustment is made perfect by the maker, and, if found not to be so, it must be remedied by removing the support which is too high and filing some off from the bottom. This should always be done by the manufacturer.

In the transit the telescope is attached immediately to the axis; but in the theodolite the axis bears a bar M at right angles to it. This bar carries at its ends two supports, which from their shape are called Y's, in the crotch of which the telescope rests, being confined there by an arch of metal passing over the top. This arch is movable by a joint at one side, and is fastened by a pin at the other. By removing the pin and lifting the arch the telescope is released and may be taken from the support. It rotates freely on its axis when confined by the arch. The telescope, being attached thus to the horizontal axis, admits of being elevated or depressed in a vertical plane so that it may be directed to any object.

168. The Vertical Limb. In the theodolite, the vertical limb E consists of a semicircle of brass graduated on its face and attached to the bar M. This limb moves with the telescope upon the horizontal axis, and thus by means of the index F, serves to determine the angle of elevation of the object. In the transit with a vertical circle, the circle is attached to the end of the axis, as seen at E, the index then being attached to the support K. In some instruments, instead of the axis bearing a circle, an arc of from 60° to 90° is attached to the support, and the index is fixed to the axis by an arm which is either permanently fastened to it or is capable of being clamped in any position.

169. The Levels. Attached to the horizontal plate are two levels A and A set at right angles to each other, so as to determine when that plate is horizontal. They consist of glass tubes very slightly curved, the convexity being upward. They are nearly filled with alcohol, leaving a small bubble of air, which by the principles of hydrostatics will always take the highest point. If they are properly adjusted, the plate to which they are attached will, when these bubbles have been brought to the middle of their run, be level, however it may be turned about its vertical axis. To the telescope of the theodolite and also to that of some transits another level G is fixed. This should be so adjusted that when the line of collimation of the telescope is horizontal the bubble may be in the centre of its run.

170. The Levelling Plates. The four screws S, S, S, and S, called levelling screws, are arranged at intervals of 90° between the two plates P, P, which are called levelling plates or parallel plates. They screw into one plate and press on the other. By tightening one screw and loosening the opposite one at the same time, the upper plate, with the instrument above, may be tilted. To allow this motion, the column connecting them terminates in a ball, which works in a socket in the centre of the lower plate. A joint of this kind, called a ball-and-socket joint, allows movement in all directions.

To level the instrument by means of these levelling screws, loosen the clamp, and turn the plates until the telescope is directly over one pair of the screws. Then, taking hold of two opposite screws, move them in contrary directions with an equal and uniform motion, until the bubble in the tube parallel to the line joining these screws is in the middle. Then turn the other screws in like manner until the other bubble comes to the middle of its tube. When they are both brought to this position the plates are level if the instrument is in adjustment. In levelling, care should be taken to move both screws equally. If one is moved faster than the other, the instrument will not be firm, or will be cramped,

171. The Clamp and Tangent Screws. The former of these are used for binding parts of the instrument firmly together, the latter for giving a slow motion when they are so bound. The clamp C tightens the collar O clasping the vertical axis, and thus holds it and the plate attached to it firmly in their places. The other plate, moving on an axis within the former, may, notwithstanding, move freely. When this clamp is tightened, the collar may be moved slowly round by means of the tangent screw T. In its motion it carries with it the axis and attached plate. The clamp D fastens the two plates together. They may, however, when so clamped, be made to move slightly on each other by means of the tangent screw U. If both clamps are tight, the instrument is firm, and the telescope can only be turned horizontally by one of the tangent screws. If the clamp C is tight and the other loose, the telescope and upper plate will move while the lower remains fixed. If D is tight and C is loose, the two plates are firmly attached to each other; but the whole instrument can be moved horizontally.

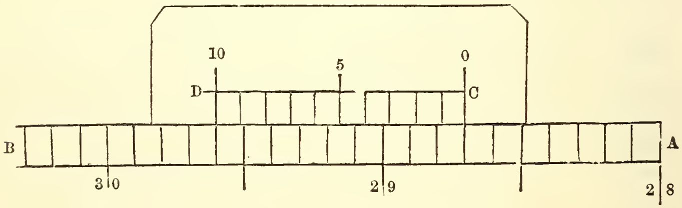
Attached to the horizontal axis there is likewise a clamp H and tangent screw I, the purposes of which are similar to those described,—the clamp fixing the axis, and the screw moving it slowly and steadily.

172. The Watch-Telescope. Connected with the lower part of theodolites of the larger class there is a second telescope R, the object of which is to determine whether the instrument has changed position during an observation. It is directed to some well defined object, and after all the observations at the station have been made, or more frequently if thought necessary, it should be examined to see whether or not it has changed its position. If it has, the divided arc has changed also. The instrument, therefore, requires readjustment.

173. Verniers. As it would be very difficult to divide a circle to the degree of minuteness to which it is desirable to read the angles, or, if it were so divided, since it would be impossible for the eye to detect the divisions, some contrivance is necessary to avoid both difficulties. These difficulties will, perhaps, be made more striking by a simple calculation. The circumference of a circle 6 inches in diameter is 18.849 inches. If the circle is divided into degrees there will be $\frac{360}{18.849} = 19.1$ divisions in the space of an inch. If the divisions are quarter degrees there will be 76.4 to the inch; and if minutes, there would be 1150 divisions to every inch. The first and second could be read; but the third, though it might by proper mechanical contrivances be made, yet it would be almost, if not entirely, impossible to distinguish the cuts so as to read the proper arc. And yet that division is not so minute as is sometimes desirable on a circle of that diameter. The *vernier* is a simple contrivance to effect this subdivision of space, in a way to be perfectly distinct and easily read.

174. The principle of the vernier will be best understood by a simple example. In the adjoining figure, (Fig. 56,) AB represents a scale with the inch divided into tenths, the figure being on a scale of 3 to 2 or $1\frac{1}{2}$ times the natural size.

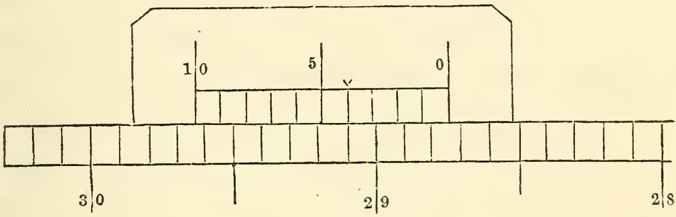
Fig. 56.



CD is another scale having a space equal nine of the divisions on AB divided into ten equal parts. This second scale is the *vernier*. Now, since ten spaces of the vernier are equal to nine of the scale, each of the former is equal to nine tenths of one of the latter. If then the 0 on the vernier corresponds to one of the divisions of the scale, the first division of the vernier will fall $\frac{1}{10}$ of a space or $\frac{1}{100}$ of an inch below the next mark on the scale, the next division will fall $\frac{2}{100}$ of an inch below, the next $\frac{3}{100}$, and so on. The 0 in the figure stands at 28.7 inches.

If now the vernier be slid up so that the first division shall correspond to a division on the scale, the 0 will have been raised $\frac{1}{100}$ inch. If the second be made to coincide, the vernier will have been raised $\frac{2}{100}$ of an inch. If it be placed as in Fig. 57, the reading will be 28.74 inches.

Fig. 57.



The student should make for himself paper scales, divided variously, with verniers on other pieces of paper, so that he may become familiar with the manner of reading them. If his scale is to represent degrees, the portion representing the arc might be drawn as a straight line, for the sake of facility in the drawing. It will illustrate the subject as well as if an arc of a circle were used. He should become particularly familiar with the one represented by Fig. 60, as it is the division most commonly used in theodolites and transits.

175. The Reading of the Vernier. To determine the *reading of the vernier*,—that is, the denomination of the parts into which it divides the spaces on the scale,—observe how many of the spaces on the scale are equal to a number on the vernier which is greater or less by one. The number of spaces on the vernier, so determined, divided into the value of one of the spaces on the scale, will give the denomination required. Thus, in Figs. 56 and 57, ten spaces of the vernier correspond with nine on the scale: the reading is therefore to $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$ of an inch.

If an arc were divided into half-degrees, and thirty spaces on the vernier were equal to twenty nine or to thirty one spaces on the arc, the reading would be to $\frac{1}{30}$ of $\frac{1}{2}^{\circ} = \frac{1}{60}^{\circ} = 1$ minute; or, as it is usually expressed, to minutes. Fig. 60 is an example of this division.

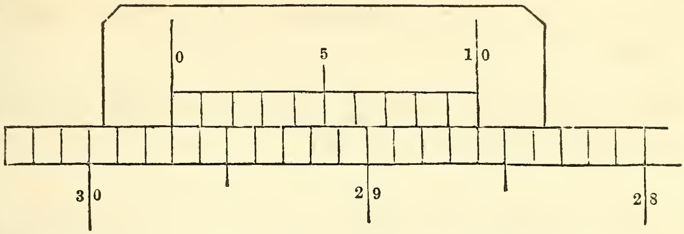
176. To read any Vernier. First, determine as above the *reading*. Then examine the zero point of the vernier. If it coincides with any division of the scale as in Fig. 56, that division gives the true reading,—28.7 inches. But if, as will generally be the case, it does not so coincide, note the division of the scale next preceding the place of the zero, and then look along the vernier until a division thereof is found which is in the same straight line as some division on the scale. This division of the vernier gives the number of parts to be added to the quantity first taken out. Thus, in Fig. 57, the 0 of the vernier is between 8.7 and 8.8, and the fourth division on the vernier is in a line with a division on the scale: the true reading is therefore 28.74 inches.

To assist the eye in determining the coincidence of the lines, a magnifying glass, or sometimes a compound microscope, is employed.

When no line is found exactly to coincide, then there will be some which will appear equally distant on opposite sides. In such cases, take the middle one.

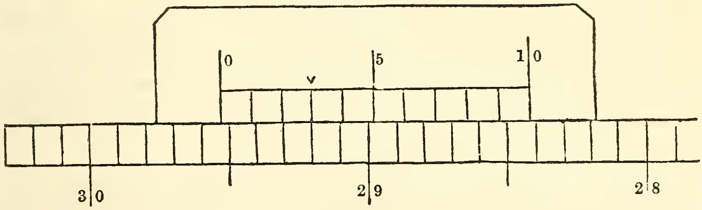
177. Retrograde Verniers. Most verniers to modern instruments are made as above described. In some instances, the vernier is made to correspond to a number of spaces on the arc one greater than that into which it is divided. Such verniers require to be read backwards, and are hence called retrograde verniers. Fig. 58 is an example of one of this kind. It is the form that is generally used in barometers. It is drawn to one and a half times the natural size: the inches are divided into tenths, and eleven spaces on the scale correspond with ten on the vernier.

Fig. 58.



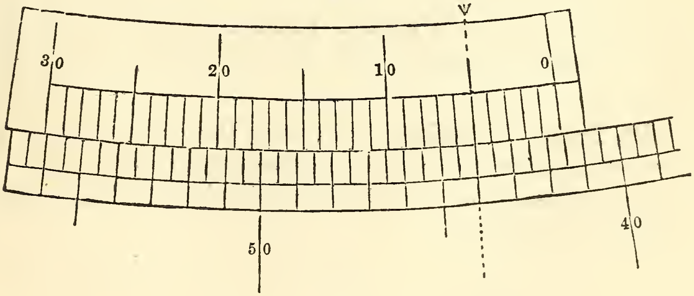
The value of one division of the vernier is $\frac{11}{10}$ inch. If therefore 0 on the vernier corresponds to a division on the scale, 1 on the vernier will be $\frac{1}{100}$ of an inch below the next on the scale, 2 will be $\frac{2}{100}$ below; and so on. If the vernier is raised so that the 1 on the vernier is in line, it is raised $\frac{1}{100}$ inch; if 2 is in line, it is raised $\frac{2}{100}$; and so on. The reading in Fig. 58 is 29.7 inches, and in Fig. 59, 29.53 inches.

Fig. 59.



178. In Fig. 60, the arc is divided by the longer lines into degrees, and by the shorter into half degrees, or 30′ spaces.

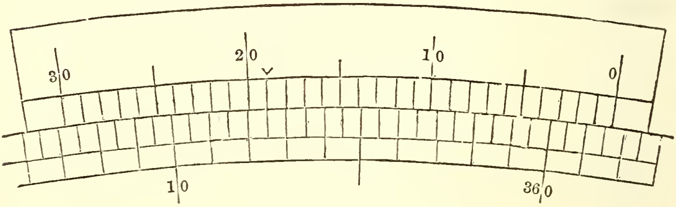
Fig. 60.



Thirty spaces on the vernier are equal to twenty nine on the arc. The reading is therefore to $\frac{1}{30}$ of 30 minutes = 1 minute. The zero of the vernier stands between 41° 30′ and 42°. On looking along the vernier, it is seen that the fifth and sixth lines coincide about equally well. The vernier therefore reads 41° 35′ 30″

179. Reading backwards. Sometimes it is required to read backwards from the zero point on the limb. When this is done, the numbers on the vernier must be read in reverse, the highest being called zero, and the zero the highest.

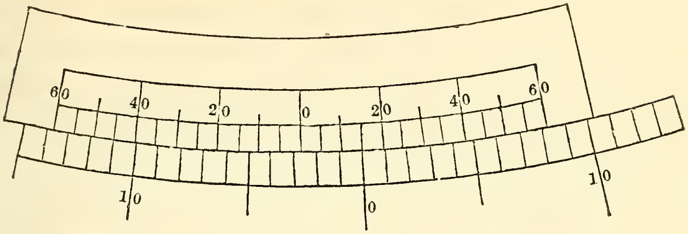
Fig. 61.



Thus, in Fig. 61, the zero of the vernier standing to the right of 360 on the limb, between 1° 30′ and 2°, and the division marked with an arrow-head being in line, the angle is 1° 41′. This mode of reading is needful when using the theodolite to take angles of *depression*, and also when using the transit to trace a line that bends backwards and forwards, the angle of deflection being then generally taken, and recorded to the right or to the left, as the case may be.

180. Double Verniers. To avoid the inconvenience of reading backwards, a double vernier is frequently made. It consists of two direct verniers having the same zero point, as shown in Fig. 62.

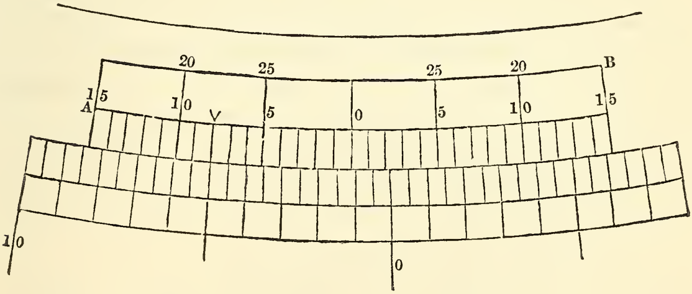
Fig. 62.



The arc in this figure is divided into degrees, and eleven spaces on the arc are equal to twelve on the vernier: the reading is therefore to 5 minutes. When the figures on the arc increase to the right, the right-hand vernier is used, and *vice versâ.* The reading on the figure is 2° 45′ to the left.

181. Another form of double vernier is shown in Fig. 63.

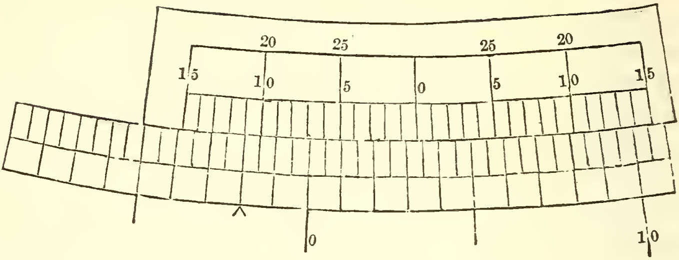
Fig. 63.



In the figure, the vernier reads to minutes. When the zero of the vernier is to the left of that on the limb, the figures begin at the zero and increase towards the left to 15′; they then pass to the right-hand extremity, and again proceed to the left; that is, they stop at A and commence again at B. The upper figures of each half are the continuation of the lower figures of the other half. The reading in Fig. 63 is 1° 8′ to the left.

In Fig. 64 the reading is 3° 19′ to the right.

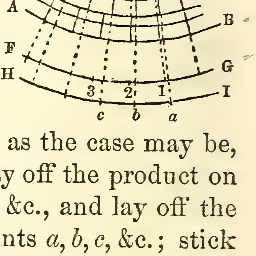
Fig. 64.



182. If the preceding descriptions have been thoroughly understood, the student will have no difficulty in reading the arc on any limb, however it may be divided. He should study the different positions until he can determine the angle with readiness, however the index may be placed. For this purpose, as before remarked, he should make for himself verniers with different scales, so that they can be placed in various positions.

The construction of such verniers is very simple. Suppose, for example, it is desired to divide the arc into degrees and subdivide it by the vernier so as to read to 5 minutes: twelve spaces on the vernier must equal eleven on the arc, or one space on the vernier will equal $\frac{11}{12}$ of a space on the arc. Let (Fig. 65) E be the centre and AB a portion of the limb, which, for the purpose intended, should not be of less radius than ten or twelve inches, and let CD be the vernier; with some other radius EG, which should be greater than EB, describe an arc GF; take EI : EG :: number of divisions on the vernier : the number that occupies the same space on the arc, —in this case, as 12 to 11. Take from the table of chords the chord of 1° or $\frac{1}{2}$°, as the case may be, and multiply it by the length of EG; lay off the product on GF, thus determining the points 1, 2, 3, &c., and lay off the same length on IH, determining the points a, b, c, &c.; stick a fine needle in the centre E; then, resting the ruler against the needle, bring it so as to coincide with I, and draw the

Fig. 65.



division on AB; then, keeping it pressed against the needle, bring it successively to the other points on GF, and draw the corresponding divisions on AB. The arc will then be divided. In the same way, resting the ruler against the needle, and bringing it successively to the points on IH, the vernier may be divided. The reason of this process is, that since $ab = 1.2$, the degrees of ab will be to the degrees of 1.2 as the radius of GF is to the radius of HI, as 11 to 12. Hence each division of the vernier is $\frac{11}{12}$ of one division of the arc.

By this means the divisions may be made with facility and accuracy.

183. Adjustments. In order that the theodolite and transit may give correct results when used, it is necessary that the different parts should bear the precise relations to each other that they are intended to have. By the term *adjustment* is meant the due relation of the parts to each other: when it is said an instrument is in *adjustment*, it is meant that every part bears to every other precisely its proper relations, so that the instrument is in perfect working order.

Before making any observations with a new instrument, it should be carefully examined to *verify* the adjustment. If the parts are not found to be properly adjusted, they must be rectified.

184. For measuring horizontal angles, the following conditions are necessary:—

1. The levels should be parallel to the plates, so that when the bubbles are in the middle of their run, the plates shall be horizontal.

2. The axes of the two horizontal plates should be perfectly parallel and perpendicular to the plane of the plates.

3. The line of collimation should be perpendicular to the horizontal axis.

4. The horizontal axis should be parallel to the plane of the plates, so that when they are horizontal it may be so likewise.

185. First Adjustment. *The levels should be parallel to the horizontal plates.*

Verification. Clamp the two plates together; loosen the clamp C, (Figs. 51, 52;) bring the telescope directly over one pair of levelling screws, and level the plates as directed in Art. 170. Turn the plates half round: if the bubbles retain their position, the plane of the levels is perpendicular to the axis on which the lower plate turns. If either of them inclines to one end of its tube, it is out of adjustment, and requires rectification.

To rectify the fault, bring the bubble half way back to the middle by means of the capstan screw attached to one end, and the other half by the levelling screws. Again reverse the position of the plate: if the bubble now remains in the middle, the rectification is complete; if not, the operation must be repeated. When both levels have been so arranged that the bubbles retain their position in the middle of their run when the plates are turned all round, the adjustment is perfect, and the axis is perpendicular to the plane of the levels.

186. Second Adjustment. *The axes of the horizontal plates should be parallel.*

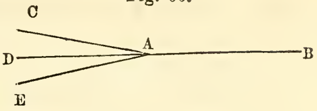
Verification. Level the plates, as directed in last article. Clamp the lower plate, and loosen the vernier-plate. Turn it half round: if both bubbles still retain their position the axes are parallel. If the plates move freely over each other without binding in any position, they are perpendicular to the axes, or, at least, the upper one is so.

If any defects be found in either of these particulars, the instrument should be returned to the maker to be rectified.

187. Third Adjustment. *The line of collimation must be perpendicular to the horizontal axis.*

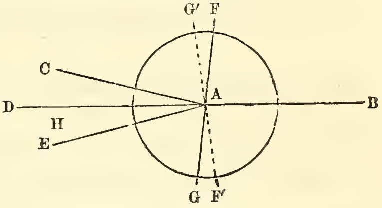
(*a.*) *Verification for the Transit.* Set the transit in a piece of level ground, as at A, (Fig. 66,) and level it carefully. At some distance—say four or five chains—set a stake B in the ground, with a nail driven in the head, and direct

Fig. 66.



the telescope so that the cross-wires may bisect exactly on the nail. Clamp the plates, turn the telescope over, and place a second stake C precisely in the line of sight. If the adjustment is perfect, the three points B, A, and C will be in a straight line. To determine whether they are so, turn the plate round until the telescope points to B; turn it over, and, if the line of sight passes again through C, the adjustment is perfect. If it does not, set up a stake at E, in the line of sight: then the prolongation of the line BA bisects EAC.

Fig. 67.



Let FG (Fig. 67) be the horizontal axis. Then, if the line of collimation makes the angle FAB acute, when the telescope is turned over it will make FAC = FAB. The angle CAD is therefore equal to twice the error. Now, if the plate is turned until the line of sight is directed to B, the axis will be in the position F′G′. Turn the telescope over, and the angle EAF′= F′AB; CAE is therefore equal to four times the error. Hence, to rectify the error, the instrument being in the second position, place a stake at H, one fourth of the distance from E to C, (Fig. 67,) and, by means of the screws *a*, *a*, (Fig. 51,) move the diaphragm horizontally till the vertical line passes through H. Verify the adjustment; and, if not precisely correct, repeat the operation.

188. (*b*.) *Verification for the Theodolite.* As the telescope of the theodolite does not turn over, the verification must be made differently. Sight to the stake B, (Fig. 66,) as directed for the transit. Take the telescope out of its Y's, and place a stake at C. Turn the plate till the telescope points to B; reverse the position of the telescope again, without moving the plate, and, if the line of sight does not pass through C, rectify as in the transit.

189. *The line of collimation of the telescope in the theodolite should be parallel to the common axis of the two cylinders on which it rests in its* Y's.

Verification. Direct the telescope so that the intersection of the wires coincides with some well defined point at a distance. Rotate the telescope so as to bring the level to the top. If the intersection is still coincident with the point, the adjustment is perfect. If the intersection has shifted horizontally, the line joining the Y's is not perpendicular to the axis. This defect must be remedied by the manufacturer. If the vertical wire is correct, but the point has shifted vertically, bring it half way back by the adjusting screws a, a. If done carefully, the wires will be in their proper position. Repeat the verification.

190. Fourth Adjustment. *The horizontal axis must be parallel to the horizontal plates.*

Verification. When this is so, the telescope will move in a vertical plane if the plates are levelled. To verify this adjustment, suspend a plumb line from some elevated point, allowing the plummet to swing in a bucket of water: then, having carefully levelled the plate, bring the intersection of the wires accurately to the line. If, on depressing the telescope, this coincidence is maintained, the adjustment is good. If it deviates to either side, the error may be corrected by filing the base of the frame on which the axis rests.

Instead of a plumb-line, the edge of a well constructed building will serve as an imperfect substitute.

Another method is, to direct the telescope to some well defined elevated object, as the spire of a steeple, and then depress it until the image of the same object is seen reflected from a vessel of mercury placed in a proper position. If the reflected image is bisected by the intersection of the wires, the adjustment is good. Instead of mercury, molasses, well boiled to free it from air-bubbles, may be substituted.

This adjustment may also be examined by directing the telescope to some well defined elevated object, and then to another on or near the ground. If none such can be found, let one be placed by an assistant; then reverse the telescope in its Y's if the instrument is a theodolite, or turn it over if the instrument is a transit, and direct it to the upper object. If the cross-wires still intersect upon the lower point when the tube is depressed, the adjustment is perfect.

191. Adjustments of the Vertical Limb. Having verified the various adjustments for horizontal motion, as described in the preceding articles, and rectified them if defective, the instrument is ready for use for horizontal work. To take angles of elevation, or to use the instrument for levelling, the following adjustments must also be examined:—

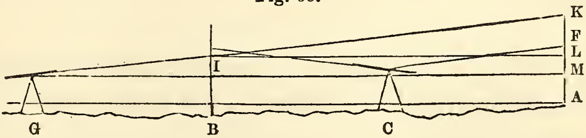
1. The level beneath the telescope must be parallel to the line of collimation.

2. The zero of the vernier must coincide with the zero of the vertical limb when the plates are level and the telescope horizontal.

192. First Adjustment. *The level must be parallel to the line of collimation.*

Verification. Select a piece of level ground, and drive two stakes, A and B, (Fig. 68,) four or five chains apart. At C, equidistant from them, set the instrument. Level the plates, and bring the bubble in the telescope level, to the middle of its run; then let an assistant hold a graduated staff on A. Note exactly the point in which the line of sight meets the staff: then let the assistant remove the staff to B, and drive the stake B until the telescope points

Fig. 68.



to the same spot on the staff. The tops of A and B are then level, whether the instrument is in adjustment or not.

Now remove the instrument to G, and level as before. Direct the telescope to the staff on B, and note the point I of intersection. Let the assistant carry the staff to A. Again note the intersection K. If the instrument is properly adjusted, these two points will coincide. If they do not, the line of collimation points too high or too low.

Take the difference between BI and AK. This difference will be LK, the difference of level as given by the instrument at G. Then say, As the distance between the stakes (BA) is to the distance from the instrument to the far stake (GA), so is the difference of apparent level of the stakes (LK) to the correction on the far staff (MK).

This correction—either taken from the height AK if too great, or added to it if too small—will give AM, the height of a point on the same level as the instrument. Direct the telescope to this point, and rectify the level, by raising or lowering one end by means of the capstan screw until the bubble is in the middle of its run. If the operation has been carefully done, the adjustment is perfect. Verify again; and, if needful, repeat the operation.

193. Second Adjustment. *The zeros of the vernier and of the vertical limb should coincide when the telescope is level.*

When the first adjustment is perfected, and the telescope is still level, examine the reading on the vertical limb carefully: if the zeros coincide, the vernier is properly adjusted; if they do not, note the error, and have it marked somewhere on the instrument under the plates, that it may not be forgotten. It must be applied to all angles of elevation taken by the instrument.

If the index-arm is movable, as is frequently the case with transits, it should be adjusted before taking vertical angles.

194. When all the preceding adjustments have been examined, and rectified if necessary, the instrument is ready for work. It would be well, however, to examine carefully the reading of the verniers, to see that they are properly divided. However placed, no *two* lines of the vernier except the first and last should coincide with divisions on the arc. If two are found to do so in any position, there is an imperfection in the graduation. If the division is very fine, a number of lines in the immediate neighborhood of the coincident lines will differ very slightly from coincidence; but, when carefully examined with a good magnifier, they should recede gradually.

Place the instrument where a good view of a fine point, some eight or ten chains distant, can be obtained. Level carefully, direct the line of sight to the point, and note the reading on the horizontal limb. Reverse the telescope in its Y's, or, if the instrument is a transit, turn it over; turn the vernier-plate till the line of sight passes again through the point, and note the reading. It should differ by 180° from that before obtained. If it does not, the divisions are not perfect, or the telescope is not over the centre of the plates. Either defect should condemn the instrument, as it can be remedied only by the maker. This verification should be tried in various positions of the divided plate. If these tests, and those formerly mentioned, are found to detect no imperfection, the instrument may be pronounced a good one.

195. Taking Angles. Set the instrument precisely over the angular point, and level it, being careful to have the levelling screws pressed tightly against the plates, that the instrument may be steady. Set the index to zero, and clamp the plates, and, if there be more than one vernier, note the minutes and seconds of the others. Loosen the lower clamp, and bring the telescope so that the wires may intersect on the left-hand object; clamp, and perfect the adjustment by the tangent screw. If there is a watch-telescope, set it upon some well-defined object,—such as a lightning-rod or the corner of a chimney,—and clamp it tightly. Loosen the vernier-plate, and turn the telescope to the other object, perfecting the adjustment by the tangent screw. Examine the watch telescope, and, if the instrument has shifted, bring it back by the tangent screw, and readjust the telescope by moving the vernier-plate.

Now read the arc by the same index as before, noting the minutes and seconds by the other verniers. Take the mean of the minutes and seconds of each position for the true reading. Then the true reading in the first position taken from that in the second will give the angle required. It is convenient to have a table prepared, with the requisite number of columns, in which to set down the readings of the different verniers. Thus, suppose there were three verniers, 120 degrees apart: rule a table, with six columns, as below:—

Occd. Sta.	Obs. Sta.	A	B	C	Mean.
A	B	0° 0′ 0″	0′ 30″	59′ 45″	0° 0′ $7\frac{1}{2}$″
A	C	75° 8′ 15″	8′ 0″	8′ 30″	75° 8′ 15″

The first column is the occupied station; the second, the observed station; the next three the readings of the verniers, and the sixth the mean.

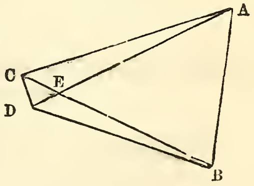
In the case above, the angle BAC would be 75° 8′ $7\frac{1}{2}$″. The instrument is supposed to read to 30″, the 15″ being taken when two lines on the vernier appear equally near coincidence.

196. Repetition of Angles. The following method of observation is sometimes employed. Suppose the angle ABC is to be measured, A being the left-hand object: direct to A, and turn to B as above directed. Clamp the vernier-plate and loosen below, and bring the telescope again to A. Clamp below, loosen the vernier, and bring the telescope again to B. The index has now traversed an arc measuring twice ABC. The operation may be repeated as often as desired, noting the number of whole revolutions the telescope has made. Then divide the whole number of degrees by the number of repetitions. The result will be the degrees of the angle required. If there is a watch-telescope, it should be set carefully before each observation. When this is done, and proper care is taken to avoid deranging the instrument, the result may be depended on as more accurate than any single reading. Any error in the final reading, being divided by the number of observations, will affect the result by but a small part of its value.

197. Verification of the Angles. When it is possible to do so, all the angles of a triangle should be measured. If their sum does not make 180°, there must be an error somewhere. Should the error be considerable, the work ought to be reviewed. But if it does not exceed two or three minutes, providing the instrument only reads to minutes, it may be distributed equally among the three angles, should there be no reason to suppose one is more accurate than another. But if more observations have been taken for some angles than for others, their determination should be most depended on, and a proportionally less part of the correction assigned to them. Suppose, for example, the angle A is the mean of five observations, B of three, while at C but one was taken, the error being 1′ 45″: we would proceed thus:—As $\frac{1}{5} + \frac{1}{3} + 1 : \frac{1}{5} :: 1' 45'' : 14''$, the correction for A. In the same manner the correction for B would be found to be 23″, and for C, 1′ 08″.

198. Reduction to the Centre. Where the object that has been observed is a spire or other portion of a building, it is impossible to set the instrument underneath the signal. In such cases, the observed angle must be reduced to what it would have been had the station been at the proper point. Thus, let C (Fig. 69) be the correct station, and D the occupied station, which should be taken as near as possible to C. Take the angle ADB. Then if A, C, D, and B are all in the circumference of a circle, this will be equal to ACB. The station should be assumed as near this as possible. Calculate BC and AC from the distance AB and the angles observed at A and B. Also measure DC, either directly or by trigonometrical methods to be explained hereafter, and take ADC.

Fig. 69.



Then, (Art. 139,) As CA : CD :: sin. ADC : sin. CAD.
And as CB : CD :: sin. BDC : sin. CBD.

Hence, ACB = AEB — CAD = ADB + CBD — CAD, becomes known.

Example. Let CA = 9647 ft.; CB = 8945 ft.; ADB = 68° 45′; DC = 150 ft.; and ADC = 97° 37′.

As	CA	9647 ft.	A. C.	6.015608
:	CD	150 ft.		2.176091
:: sin.	ADC	97° 37′		9.996151
: sin.	CAD	0° 52′ 59″		8.187850
As	CB	8945 ft.	A. C.	6.048420
:	CD	150 ft.		2.176091
:: sin.	CDB	166° 22′		9.372373
: sin.	CBD	0° 13′ 35″		7.596884

Whence ACB = ADB + CBD — CAD = 68° 5′ 36″.

199. Angles of Elevation. In measuring angles of elevation, the instrument must first be levelled; the telescope being then directed to the object, the reading of the vernier corrected for the index-error will be the angle of elevation.

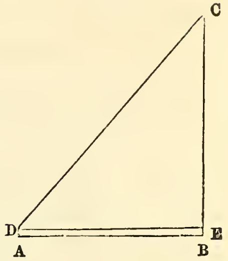
SECTION VI.

MISCELLANEOUS PROBLEMS TO ILLUSTRATE THE RULES OF PLANE TRIGONOMETRY.

Problem 1. Being desirous of determining the height of a fir-tree standing in my garden, I measured 100 feet from its base, the ground being level. I then took the angle of elevation of the top, and found it to be 47° 50′ 30″. Required the height, the theodolite being 5 feet from the ground.

Solution.

Fig. 70.



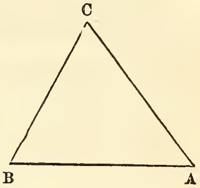
Make AB (Fig. 70) equal to 100 feet; draw AD and BC perpendicular to AB, making the former five feet from the same scale. Draw DE parallel to AB, and make EDC = 47° 50′, the given angle. Then will CB be the height of the tree.

Calculation.

As rad. : tan. EDC : : DE : EC = 110.45 feet; whence BC = 110.45 + 5 = 115.45.

Problem 2. One corner C (Fig. 71) of a tract of land being inaccessible, to determine the distances from the adjacent corners A and B, I measured AB = 9.57 chains. At A, the angle BAC was 52° 19′ 15″, and at B, the angle ABC was 63° 19′ 45″. Required the distances AC and BC.

Fig. 71.

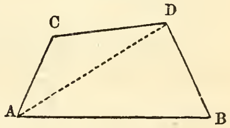


Calculation.

As sin. ACB (64°21′) : sin. A (52° 19′ 15″) : : AB (957) : BC = 840.2 links. As sin. ACB (64° 21′) : sin. B (63° 19′ 45″) : : AB : AC = 948.7 links.

Problem 3. In measuring the sides of a tract of land, one side AB (Fig. 72) was found to pass through a swamp, so that it could not be chained. I therefore selected two stations, C and D, on fast land, and took the distances and angles as follows,—viz.: AC = 37.56 chains; CD = 50.25 chains; BAC = 65° 27′ 30″; ACD = 123° 46′ 20″; CDB = 107° 29′ 15″: the corner B being inaccessible, the distance BD could not be measured. Required AB. The angle CDA could not be taken, owing to obstructions.

Fig. 72.



Solution.

Join AD. Then, from the triangle ACD, we have, (Art. 140,)

As CD + CA (87.81) : CD − CA (12.69) : : tan. $\frac{CAD + CDA}{2}$ (28° 6′ 50″) : tan. $\frac{CAD - CDA}{2} = 4° \ 24' \ 54''$;

whence CAD = 28° 6′ 50″ + 4° 24′ 54″ = 32° 31′ 44″,
and CDA = 28° 6′ 50″ − 4° 24′ 54″ = 23° 41′ 56″;
then, sin. CDA : sin. ACD : : AC : AD = 77.68.

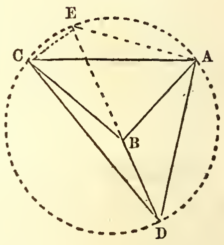
Now, in ADB we have AD = 77.68, the angle DAB = CAB − CAD = 32° 55′ 46″, and the angle ADB = BDC − ADC = 83° 47′ 19″, to find AB; thus,

As sin. B : sin. ADB : : AD : AB = 86.455 chains.

Problem 4. To determine the position of a point D on an island, I ascertained the distances of three objects on the main land as follows:—AB = 248.75 chains, BC = 213.25 chains, and AC = 325.96 chains. At D the angle ADB was found to be 29° 15′, and BDC 20° 29′ 30″. Required the distance of D from each of the objects.

Construction.

Fig. 73.



With the given distances construct the triangle ABC. At C and A make the angles ACE = 29° 15′, and CAE = 20° 29′ 30″. About AEC describe the circle ACD. Join EB, and produce it to D, which will be the point required.

For (21.3) ADB = ACE = 29° 15′, and CDB = CAE = 20° 29′ 30″.

Calculation.

1. In ABC we have the three sides to find the angle BAC = 40° 51′ 30″.

2. In CAE we have the angles and side AC to find the side AE = 208.705.

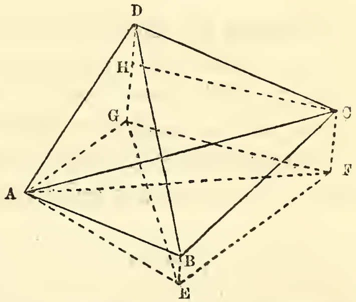
3. In BAE we have BA, AE, and the included angle BAE, to find ABE = 50° 55′ 48″, AEB = 67° 43′ 12″.

4. In ABD we have the angles and side AB, to find AD = 395.24 and BD = 188.07.

5. In ACD we have the angles and sides AC, to find CD = 379.

Problem 5.—Wishing to obtain the distance between two trees, C and D, situated on the side of a hill, and not being able to find level ground for a base, I selected a gradual slope, on which I measured the distance AB (Fig. 74) 400 yards. I then took the horizontal and vertical angles as follow:—At A, the angle BAD was 101° 47′ 15″, BAC 39° 25′ 45″. The elevation of B was 5° 32′ 45″, of C, 8° 19′ 30″, and of D, 12° 29′. At B, the angle ABD was 59° 13′ 15″, and ABC 125° 36′ 45″.

Fig. 74.



Required the distance CD, and the elevations of C and D above A.

Conceive a horizontal plane to pass through A, meeting vertical lines through B, C, and D in the points E, F, and G. Then, since the angular distances are measured horizontally, we have the following angles given,—viz.: EAG = 101° 47′ 15″, EAF = 39° 25′ 45″, AEG = 59° 13′ 15″, and AEF = 125° 36′ 45″.

Calculation.

1. To find AE, we have r : cos. BAE (5° 32′ 45″) : : AB (400) : AE = 398.13.

2. To find AG. As sin. AGE : sin. AEG : : AE : AG = 1051.07, log. 3.021631.

3. To find AF. As sin. AFE : sin. AEF : : AE : AF = 1253.96, log. 3.098284.

4. To find FG, (Art. 141.) As AG : AF : : r : tan. x = 50° 1′ 49″.

And, as rad. : tan. $(x - 45°)$: : tan. $\frac{1}{2}$ (AGF + AFG) : tan. $\frac{1}{2}$ (AGF − AFG) = 8° 16′ 34″;

then AGF = 58° 49′ 15″ + 8° 16′ 34″ = 67° 5′ 49″,

and AFG = 58° 49′ 15″ − 8° 16′ 34″ = 50° 32′ 41″.

Then, as sin. AGF : sin. FAG : : AF : GF = 1205.9.

5. To find GD and CF. As r : tan. GAD : : AG : GD = 232.69 = Elevation of D.

And as r : tan. CAF : : AF : FC = 183.49 = Elevation of C.

6. To find CD. $CD = \sqrt{CH^2 + HD^2} = 1206.9$ = Distance of CD.

Problem 6.—Being desirous to determine the height of a tower standing on the summit of a hill, I measured 75 yards from its base down the declivity, which was a regular slope. I then took the elevation of the top, 49° 37′ 45″, and of the bottom, 8° 19′, the height of the instrument being 5 feet. What was the height of the tower? Ans. 76.44 feet.

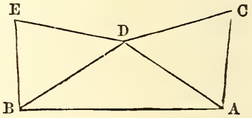
Problem 7.—To determine the height of a tree in an inaccessible situation, I took a station, and found the elevation of the top to be 38° 45′ 15″; then, measuring back 100 feet, the elevation was found to be 24° 18′. Required the altitude of the tree and its distance from the first station, the instrument being 4 feet 9 inches high.

Ans. Height, 107.95 feet; distance, 128.57 feet.

Problem 8.—To determine the distance of two objects A and B, I took two stations C and D, distant 35.75 chains, from which both could be seen. At C, the angle ACD was found to be 103° 47′, and BCD 45° 29′ 30″; at D, the angle BDC was 110° 23′ 30″, and ADC 60° 21′ 15″. Required the distance AB. Ans. 99.236 ch.

Problem 9.—The side AB (Fig. 75) of a tract of land being inaccessible, and not being able to find two stations from which both ends were visible, I measured two lines, CD, 7.75 ch., and DE, 7.92 ch., and took the angles as follow: At C, the angle ACD was 68° 15′. At D, CDA was 50° 27′, ADB 112° 46′, and BDE 43° 30′. At E, DEB was 75° 10′. What was the length of AB?

Fig. 75.



Ans. 14.10 ch.

Problem 10.—To determine the position of a point D, situated on an island, I took the angles to three objects, A, B, and C, situated on the shore, and found them to be ADB, 19° 14′ 30″, CDB, 24° 19′. I subsequently determined the distances AB = 4596 yards, AC = 5916 yards, and BC = 4153 yards. Required the distance of D from each of the objects, it being nearest to B.

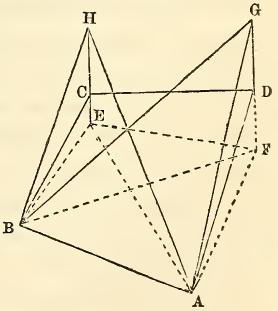
Ans. AD = 8287.2 yards; BD = 4127.7 yards; CD = 7550.8 yards.

Problem 11.—To determine the height of a mountain rising abruptly from the water of a lake, I selected a station C on the slope of the hill rising from the opposite shore, and took the angle of elevation of the summit, 47° 22′ 15″, and depression of the water's edge at the base of the mountain in the vertical plane through the summit, 12° 30′. Then measuring up the slope, directly from the rock, a distance of 800 yards, to a station D, the elevation of the summit was 25° 33′ 30″, the depression of the water's edge, 18° 15′, and of the top of a staff left at C to mark the height of the instrument, 24° 15′. Required the height of the mountain.

Ans. Height, 1390.7 yds.

Problem 12.—To determine the heights and distance of two trees C and D, standing on a hill side, I measured on level ground a base line AB 252.28 feet long, and took the following angles: At A, the angle of position of C from B was = 82° 54′ 30″, and of D from B = 89° 24′; the elevation of the base of C = 3° 45′; of top of do. = 9° 25′; of the base of D = 3° 54′; of top of do. = 10° 29′ 30″. At B, the angle of position of D from C was = 6° 14′ 30″; and of A from C = 80° 51′ 30″, and for verification the elevations at B were of base of C = 3° 44′, of top of do. = 9° 22′ 15″; of base of D = 3° 46′, and of top of do. =

Fig. 76.

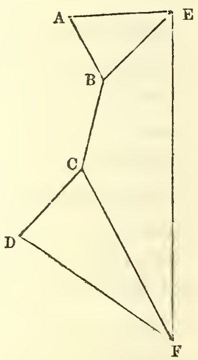


10° 7′ 30″. Required the heights of the trees, and the distance between their bases.

Ans. Height of C = 89.37 ft.; of D = 103.37 ft.; distance, 100.7 ft. With the angles of verification; height of C = 103.29 ft.; of D = 89.36 ft.

Problem 13.—One side EF (Fig. 77) of a tract of land being inaccessible, and there being no station from which the two ends could be seen, I selected four stations, A, B, C, D; A and D being in the adjoining sides, and B and C between them. The following measurements were then taken,—viz.: AB = 7.37 ch.; BC = 8.95 ch., and CD = 9.33 ch.; at A, the angle EAB was 64° 37; at B, ABE was 72° 43′, and EBC 149° 32′; at C, BCF was 139° 47′, and FCD 69° 38′; and at D, CDF was 82° 35′. Required AE, EF, FD, and the angles AEF and EFD.

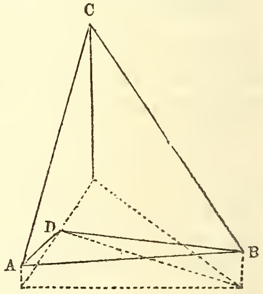
Fig. 77.



Ans. EF = 33.50; AE = 10.38; DF = 18.77;
AEF = 86° 39′; EFD = 54° 29′.

Problem 14.—Being desirous of finding the elevation and distance of an elevated peak C (Fig. 78) of a mountain rising abruptly from the shore of a river, and not being able to find a level place for a base line, or a regular slope ascending in a line from the point to be measured, I selected two stations, the one A nearly opposite the base D of a rock jutting into the water, and which was so situated that A, C, and D were in the same vertical plane, and the other station B farther up the stream, the slope between them being regular. I then took the following

Fig 78.



measurements,—viz.: AB, 850 yards. At A, the angle of position of B and C was 87° 49′; elevation of C, 35° 27′; depression of D, 3° 25′ 45″; elevation of top of a staff at B of same height as the instrument, 3° 14′ 30″. At B, the angle of position of A and D was 47° 39′, and of A and C, 70° 43′ 30″. Depression of A, 3° 14′ 30″; of D, 4° 48′ 30″; elevation of C, 33° 6′. Required the horizontal distance of C and D from A and B, and the elevation of A, B, and C above the water.

Ans. Horizontal distance of C from A, 2189.8 yds.; from B, 2318.1 yds.; of D from A, 894.3 yds.; from B, 1209.2 yds. Elevation of C, 1612.7 yds.; of A, 53.6 yds.; and of B 101.7 yds.

CHAPTER IV.

CHAIN SURVEYING.

SECTION I.

DEFINITIONS.

200. Definition. LAND SURVEYING is the art of measuring the dimensions of a tract of land, so as to furnish data for calculating the content and determining the area.

201. The position of the angular points of a tract may be determined either by measuring the lines of the survey, the diagonals, offsets, &c., or by linear measures in connection with angular distances. These different methods of fixing the points give rise to different modes of surveying,—the first of which, as it is performed principally by the chain, may be called *chain surveying.*

202. Advantages. As the chain, or some substitute, such as a tape-line or a cord, is readily procured by every one, surveying by this method may be performed where the more expensive instruments cannot readily be procured. To every farmer it may be important to know the content of a particular field, or of several fields, that he may divide them properly, or that he may know the value of crops which he is about to buy or to sell; or for various other purposes that need not be mentioned. He should, therefore, not be under the necessity of calling in a professional man to do for him what he himself, with a pair of carriage lines, can do, if not as well, yet fully well enough for all practical purposes.

In order that this very simple method may be fully understood, we shall treat of it somewhat at length. It must not be inferred from this that it is recommended in preference to the other methods to be explained hereafter, but only as a substitute to be used, when, from the circumstances of the case, these are inapplicable or inconvenient.

203. Area Horizontal. It must be remembered that, in land surveying, it is the horizontal area that is required, and not the actual surface of the ground. Every measurement must, therefore, be made horizontally, as explained in Art. 149, *et seq.*, and, where angles are taken, they must be horizontal angles.

As the method of chaining has been fully explained in the articles above referred to, it will be unnecessary to repeat the directions here. There are, however, certain preliminary operations to be performed, which will form the subject of the next section.

SECTION II.

FIELD OPERATIONS.

A.—TO RANGE OUT LINES, AND TO INTERPOLATE POINTS.

204. Ranging out Lines. THIS requires three persons, each of whom should be provided with a rod some ten or twelve feet long, one end being pointed with iron, that it may be thrust in the ground. He should also have a plumb-line, that he may set his rod upright. The first, whom we shall call A, takes his station at the point of beginning. Looking in the direction of the line, he places B in the proper direction, signalling him to the right or left as may be required. When the position is determined, B sets his rod firmly in the ground. C then goes forward, and looking back, by ranging with the rods of B and A, he puts his rod in line. A then comes forward, and, going ahead of C, puts himself in line, by ranging with C and B. They thus continue, the hindmost always coming forward, until the other end of the line is reached. At the point at which each rod was erected a stake should be driven for future reference.

Lines may be prolonged in the same manner to any extent that may be desired.

If the operation is carefully done, the rods being set plumb, the line will vary very slightly, if at all, from a straight line, even when extended several miles.

205. To interpolate points in a line. The men in chaining should keep themselves exactly in line. This may readily be done by a careful follower, when the end of the line can be seen. If, however, one end is not visible from the other, and from every point in the line, there will be nothing by which the follower can range his leader, unless there are staves set up for that purpose, at points along the line. The fixing of such points is called interpolation.

206. On level ground. If, for any purpose, such points were needed in a line on level open ground, a person, stationing himself at one end, can signal another into the proper position. As many points as are wanted can thus be determined.

207. Over a hill. If a hill intervenes, from the top of which both points may be seen, let two persons, provided with rods, put themselves as near in line as possible. Then, by alternately signalling to each other, their proper places can be found. Thus, let XY (Fig. 79) be the line to be interpolated. A will take his station in the supposed position of the line, and signal B until he ranges with X. B then places A in line with Y at C; A again signals B to D, in line with X; and so they proceed till they are both in the line XY.

Fig. 79.

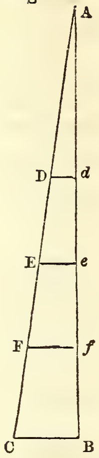
208. If an assistant is not at hand, or if but one point can be found from which both ends of the line can be seen, one person can put himself in line by having a rule with a sight at each end; wires, set upright, will do very well: lay this on some support, and then go to each end in turn, sighting to the end of the line; he can thus determine whether it is the proper position, and alter it until he finds himself rightly placed.

209. By a Random Line. When the ends cannot be seen from each other, nor from any intermediate point, it is necessary to run a random line. This is done as directed in Art. 204, following a course as near that of the line to be interpolated as possible.

When the foremost person has come opposite the end of the line, the distance to it should be measured, as well as that of the whole line run, and to each stake set up along that line. For convenience' sake, the stakes should be set at equal distances.

Then say, As the whole distance is to the distance to any one of the stakes, so is the whole deviation to the deviation of that stake.

Fig. 80.



Thus, let AB (Fig. 80) be the line to be interpolated. Run the random line AC, setting stakes at D, E, F, &c. Measure CB and the distance from A to D, E, F, and C.

Suppose AC measures 27.56 chains, AD 10 chains, AE 15 chains, AF 20 chains, and BC = 1.57 chains.

Then, 27.56 : 10 : : 1.57 : .57, the correction for D. Similarly, E*e* = .85, and F*f* = 1.14 chains.

Set off D*d*, E*e*, and F*f*, the calculated distances; set stakes at *d*, *e*, and *f*, and range out the line anew.

Instead of working out each proportion, it is more concise to divide the deviation by the number of chains in the measured length: this will give the correction for one chain. This correction, being multiplied by the distance to each stake, will give the correction for that stake.

Thus, in the above example,

$$\frac{1.57}{27.56} = .057, \text{ the correction for 1 chain.}$$

$$10 \times .057 = .57, \text{ the correction for D;}$$
$$15 \times .057 = .85, \text{ the correction for E;}$$
$$20 \times .057 = 1.14, \text{ the correction for F.}$$

210. Across a valley. When the line runs across a valley, let two points A and B be determined on opposite sides of the valley, from which the intervening ground can be seen. Then let one person take his station at A, and, holding a plumb-line over the stake, let him sight to B: he can then direct his assistant into the proper position, and thus fix as many points as are desirable.

NOTE.—These operations are all done more accurately and rapidly by means of the transit or theodolite.

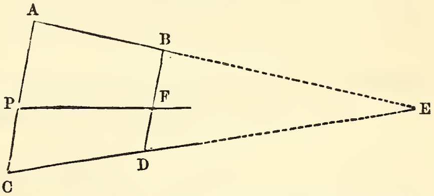
211. *To determine the point of intersection of two visual lines.*

This is most readily done by three persons, two of whom take their stations in the lines, at some distance from the point of intersection, and, looking along their lines respectively, signal the third until he ranges in both lines. A stake may then be driven at the point of intersection.

This operation may readily be performed by two persons. First, let them run out one of the lines, and stretch a cord or the chain across the course of the other. One of them then taking his station in the second line can signal the other to his proper position.

212. *To run a line towards an invisible intersection.*

Fig. 81.



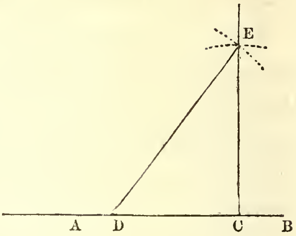
Through P (Fig. 81) run the line AC, intersecting the given lines in A and C. Then through any point B in AB set out BD parallel to AC by one of the modes to be pointed out. (See Arts. 227–229.) Divide BD in F, so that BF : FD : : AP : PC; that is, make $BF = \frac{BD \cdot AP}{AC}$. Then PF will be the required line.

B.—PERPENDICULARS.

Problem 1.—*To draw a perpendicular to a given line from a given point in it.*

213. (*a.*) **When the Point is accessible.** This may be done on the ground by the methods described in Arts. 88, 89, and 90, using the chain for a pair of compasses to sweep the circles, or by the following methods:—

Fig. 82.

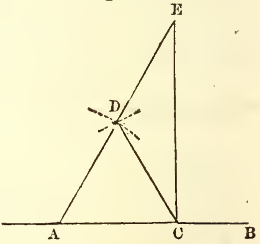


214. *First Method.* Let AB (Fig. 82) be the line and C the point at which the perpendicular is to be erected. First, lay off CD, 60 links; then, fixing one end of the chain at D, sweep an arc of a circle at E, using the whole chain (100 links) for a radius. Next, fix one end at C, and, with 80 links for a radius, sweep an arc cutting the former in E. CE will be perpendicular to AB.

Any other distances, in the same ratio as the above, will answer. Thus, DC might be 30, CE 40, and DE 50. With these numbers no circles need be struck. Lay off DC = 30 links; fix the end of the chain at D, and the end of the ninetieth link at C: then, taking the end of the fiftieth link, stretch both parts of the chain equally tight, and set a stake at the point of intersection.

These numbers are very convenient when short perpendiculars are required; but when the line is run to some distance the greater lengths are preferable.

Fig. 83.

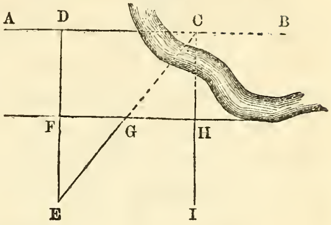


215. *Second Method.* Make AC (Fig. 83) a chain. With the whole length of the chain sweep two arcs cutting in D; range out AD, making DE = AD: then CE will be the perpendicular required.

For, ADC being equilateral, A = 60°, and A and ACD = 120°; whence DCE and DEC = 60°. But DE = DC: therefore DCE = 30°, and ACE = 90°.

216. (*b.*) **When the Point is inaccessible.**

Fig. 84.



Erect a perpendicular at some other point D (Fig. 84) of the line. Through F, a point in this perpendicular, draw FH parallel to AB, (Art. 227.) Take FE = FD : range out EC, intersecting FH in G. Make GH equal FG: then CHI will be the perpendicular required.

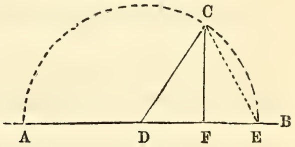
FE need not be taken equal to DF. If unequal, G will be determined by the proportion EF : FD : : FG : GH.

(*c.*) If the line is inaccessible, trigonometrical methods must be employed.

Problem 2. *To let fall a perpendicular to a line from a point without it.*

(*a.*) *When the point and line are both accessible.*

Fig. 85.



217. The methods in Arts. 91, 92, 93, may be adopted in this case; or in AB (Fig. 85) take any point D, and measure CD. Make DE = DC, and measure CE.

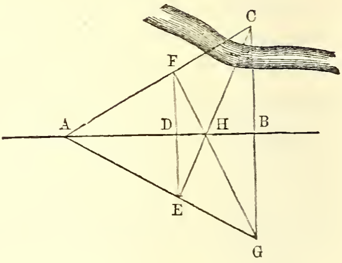
Then take $EF = \dfrac{EC^2}{2.ED}$, and F will be the foot of the perpendicular.

Describe the semicircle ECA. Then, if CF is perpendicular to AB, EC is a mean proportional between AE and EF, whence $EF = \dfrac{EC^2}{AE} = \dfrac{EC^2}{2\,DE}$.

(*b.*) *If the point is remote or inaccessible.*

218. *First Method.*—In AB (Fig. 86) take any convenient points A and D; erect the perpendicular FDE, making FD = DE; range out AE, and EC cutting AB in H, and FH intersecting AE in G: then GBC will be perpendicular to AB.

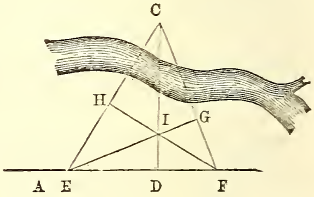
Fig. 86.



For, by construction, the triangles ADE and ADF, as also FDH and EDH, are equal in all respects. Hence, AFG and AEG, having two sides and the included angle of one equal to two sides and the included angle of the other, are equal in all respects; therefore AG = AC. Finally, ABC and ABG have two sides and their included angles respectively equal, whence B is a right angle.

219. *Second Method.*—Select any two convenient stations E and F (Fig. 87) from which C may be seen, and range out FC and EC. To these draw the perpendiculars EG and FH cutting in I: then CID will be the perpendicular required.

Fig. 87.

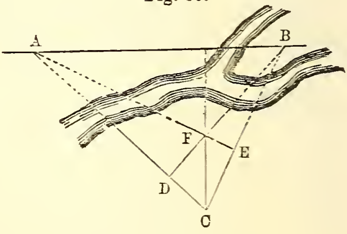


For the perpendiculars to the three sides of a triangle from the opposite angles intersect in the same point.

(*c.*) *If the line be inaccessible.*

220. From the given point C towards two visible points A and B (Fig. 88) of the given line range out CA and CB, and by one of the preceding methods draw the perpendicular EA and BD intersecting in F: CF will be the perpendicular required.

Fig. 88.



221. The preceding methods will apply in all the cases enumerated. They are, however, only to be considered as substitutes for the neater and more accurate methods by the use of the theodolite or transit. Measurements such as those directed above, when they are intended to determine the direction of an important line, require to be made with scrupulous accuracy; for every deviation will be magnified as we proceed. An error of two or three inches, which would be a matter of but little importance in a line of a chain long, would cause a deviation of from twelve to twenty feet if the line were prolonged to a mile.

In the absence of a transit or theodolite, the following simple instruments, either of which can be constructed by any one having a moderate degree of facility in the use of tools, will enable the surveyor to lay out perpendiculars with readiness and considerable accuracy.

222. The Surveyor's Cross. This consists of a block of wood four or five inches in diameter, with two saw-cuts across its centre precisely at right angles. An auger hole should be made at the bottom of each saw curf, to afford a larger field of view. The block is fastened to the top of a staff about eight or ten inches long. It should turn freely but firmly on the head of the staff.

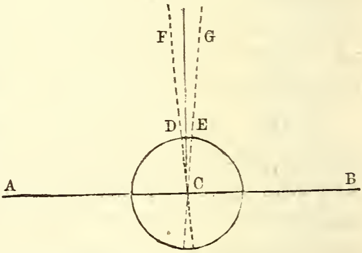
Instead of saw-cuts, four wires may be set upright at the extremities of perpendicular diameters; but, as these are likely to be deranged, the other form is better.

223. To erect a perpendicular with the cross, set it up at the point at which the perpendicular is to be drawn, and turn it round till one of the cuts ranges with the given line; then, looking through the other cut, the surveyor can direct his assistant to set a stake in the required perpendicular.

If the point is out of the line, take a station as near as the eye can judge to the position of the foot of the perpendicular, and, having set the cross so that one cut may range with the given line, look through the other, and see how far the line of sight misses the given point. Move the cross that distance and test it again. A few trials will determine the proper position.

224. To verify the Accuracy of the Cross. Place it at a given station: range with one of the cuts to a well-defined object, and place a stake in the perpendicular; then turn the cross one-quarter round, and if the stake is in the perpendicular, the cross is correct, but if not, the instrument is in error by half the observed deviation.

Fig. 89.



This will be apparent by reference to Fig. 89. If the angle ACD is acute, the stake will be placed to the left of the true position, as at F. By turning the block one-fourth round, the acute angle will be found at BCE, and the stake will be posited at G, as far to the right as it was before to the left.

225. The Optical Square. The optical square is a much more convenient instrument for drawing perpendiculars than the cross. It consists of a circular box, having a fine vertical slit cut in one side, and directly opposite a circular or oval opening with a vertical line, such as a horsehair stretched across it. The box contains a piece of looking-glass set across it, so as to make an angle of 45° with the line of sight. From the upper half of this glass the silvering must be removed. Half-way between the two openings mentioned is another, to allow the rays coming from an object in the perpendicular to fall on the mirror and be reflected to the eye.

Fig. 90.

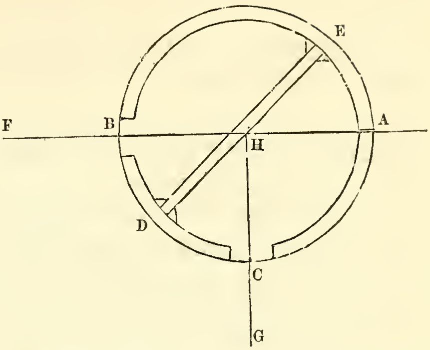


Fig. 90 represents a plan of this instrument. ABC is a section of the box, A the slit at which the eye is placed, B the opening in the line of sight, C the opening for the perpendicular, and DE the looking-glass.

The surveyor holds the box in his hand, and, looking at the other end of the line, through the openings A and B, directs his assistant, who is seen by reflection through C, to place his rod in such a position that its image shall coincide with the hair across the opening B. HG is then perpendicular to AF.

To find the point in which the perpendicular from a distant point will intersect AF, walk along the line, keeping the line of sight AB directed to the end of the line. When the image of a pole standing at the point from which the perpendicular is to be drawn appears at H, the proper position has been attained.

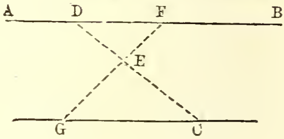
226. To test the Accuracy of the Square. Erect a perpendicular with it, as above directed. Then sight along the perpendicular, and if the original line appears perpendicular, the instrument is correct; if it does not, the deviation will equal twice the error of the instrument. Set a pole in the true perpendicular, which will be found as in Art. 224, and alter the position of the glass until the reflected image appears in the proper position. One end of the glass should be movable by screws or by little wedges, so as to allow of its position being rectified.

C.—PARALLELS.

Problem 1.—*Through a given point to run a parallel to a given accessible line.*

227. This may be done by Arts. 97, 98, or 99, or thus:—

Fig. 91.

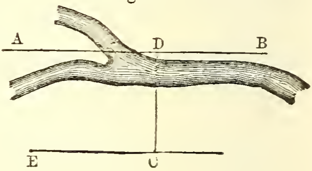


Let AB (Fig. 91) be the line, and C the point. From C to any point D in AB, run out the line CD. From E, any point in CD, run a line cutting AB in F. Then make EG a fourth proportional to DE, EF, and EC, or $EG = \frac{EF \cdot EC}{ED}$, and GC will be parallel to AB.

Problem 2.—*To draw a parallel to an inaccessible line, two points of which are visible.*

228. Let AB (Fig. 92) be the straight line, and C the given point. Run the line CD perpendicular to AB, by Art. 220; and from C set out CE perpendicular to CD. It will be the parallel required.

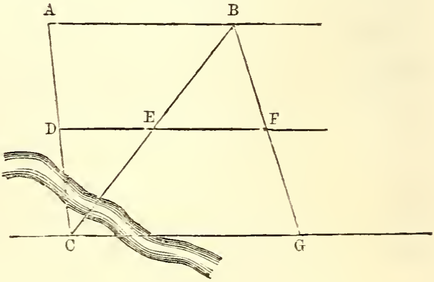
Fig. 92.



Problem 3.—*To draw a parallel to a given line through an inaccessible point.*

229. Let AB (Fig. 93) be the given line, and C the given point. From A, towards C, run AC; and in CA, or CA produced, take any point D. Run DE parallel to AB. Set off BC towards C, in-

Fig. 93.



tersecting DE in E. Measure AB and DE. Run through any point in AB the line BFG, intersecting DE in F. Make $FG = \frac{DE \cdot BF}{AB - DE}$, and CG will be parallel to AB.

For, since $FG = \frac{DE \cdot BF}{AB - DE}$, we have $AB - DE : DE :: BF : FG$.

Whence $AB : DE :: BG : FG$;

but $AB : DE :: BC : EC$;

$\therefore$ $BG : FG :: BC : EC$, and CG is parallel to EF, or to AB.

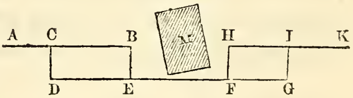
SECTION III.

OBSTACLES IN RUNNING AND MEASURING LINES.

A.—OBSTACLES IN RUNNING LINES.

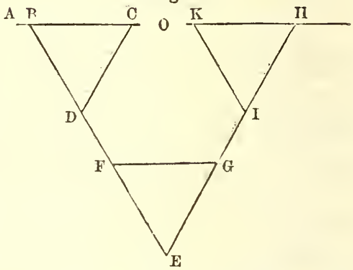
230. IN ranging out lines by the method described in Art. 204, obstacles are frequently met with which prevent the operation being directly carried on. In such cases some contrivance is necessary in order that the line may be prolonged beyond such obstacle. Various methods have been devised for this purpose. The following are among the most simple:—

Fig. 94.



231. *First Method.—By perpendiculars.* Let AB (Fig. 94) be the line, and M the obstacle. At two points C and B in AB, set off two equal perpendiculars CD and BE long enough to pass the obstacle. Through D and E run the line DG; and at two points F and G beyond the obstacle, set off perpendiculars FH and GI equal to CD. Then HIK will be the prolongation of AB.

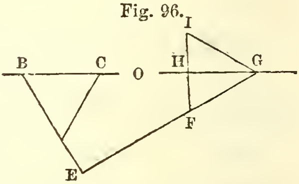
Fig. 95.



232. *Second Method.—By equilateral triangles.* Let AB (Fig. 95) be the line, the obstacle being at O. By sweeping with the chain, describe the equilateral triangle BCD. Prolong BD to E sufficiently far to pass the obstacle. Describe the equilateral triangle FEG, and prolong EG till EH = EB. Describe the equilateral triangle HKI, and KH will be the prolongation of AB.

233. Instead of making BEH an equilateral triangle, which would sometimes require the point E to be inconveniently remote, run BE (Fig. 96) as before. Set out the perpendicular EG = 1.414 × BE. Describe the equilateral triangle GFI. Bisect FI in H. Then HG will be the prolongation of BC.

Fig. 96.



B.—OBSTACLES IN MEASURING LINES.

234. When, owing to any obstructions, the distance of a line cannot be directly measured, resort should be had to trigonometrical methods. In the absence, however, of the proper instruments, it may be necessary to determine such distances. The following are a few of the many methods that may be employed in such cases:—

1. *To measure a line when both ends are accessible.*

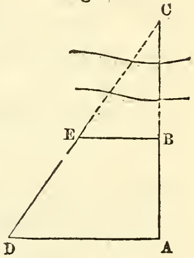
235. Arts. 231, 232, 233, furnish means of determining the distance in this case. By the method Art. 231, BH =

EF; and in that of 232, $BH = BE$. If the method Art. 233 is employed, $BG = 2\,BE$.

2. *When one end is inaccessible.*

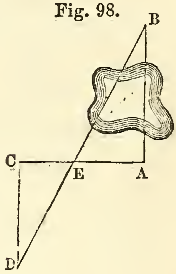
236. *First Method.*—Run BE (Fig. 97) in any direction, and AD parallel to it. Through any point D in AD, run DE towards C. Measure AD, AB, and BE: then $BC = \dfrac{AB \, . \, BE}{AD - BE}$.

Fig. 97.



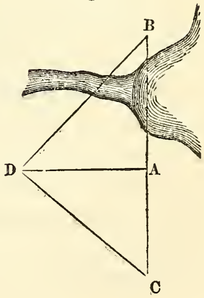
237. *Second Method.*—Set off AC (Fig. 98) in any direction, and CD parallel to AB. Run DE towards B. Measure AE, EC, and CD: then $AB = \dfrac{AE \, . \, CD}{CE}$.

Fig. 98.



238. *Third Method.*—Set off AD (Fig. 99) perpendicular to AB, and of any distance. Run DC perpendicular to DB. Measure DC and CA: then $CB = \dfrac{CD^2}{CA}$, or $AB = \dfrac{AD^2}{AC}$.

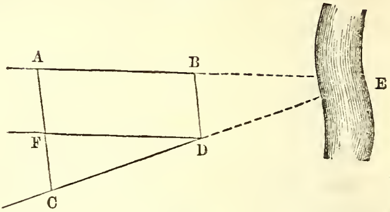
Fig. 99.



3. *When the point is the intersection of the line with another, and is inaccessible.*

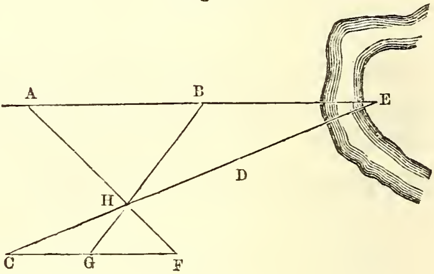
239. *First Method.*—Let AB and CD (Fig. 100) be the lines, the distances of which to their intersection are required. Set off DF parallel to BA, and run CFA. Measure CD, CF, CA, and FD. Then $BE = \dfrac{BD \cdot DF}{FC}$, and $DE = \dfrac{BD \cdot DC}{CF}$.

Fig. 100.



240. *Second Method.*—Through H, (Fig. 101,) any point in CD, run two lines AF and BG. Make FH in any ratio to HA, and GH in the same ratio to HB. Draw FGC, cutting CD in C. Measure FC and HC. Then $AE = \dfrac{AH \cdot FC}{FH}$, and $HE = \dfrac{AH \cdot HC}{FH}$.

Fig. 101.



4. *When both ends are inaccessible.*

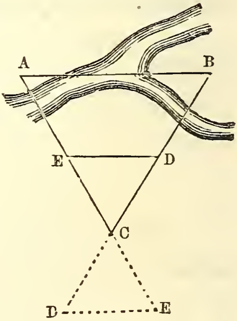
241. Let AB (Fig. 102) be the inaccessible line. From any convenient point C, run the lines CA and CB towards A and B, and, by one of the preceding methods, find CA and CB. In CA and CB, or CA and CB produced, take E and D.

So, CE : CA :: CD : CB.

Measure DE.

Then CE : CA :: ED : AB.

Fig. 102.



SECTION IV.

KEEPING FIELD-NOTES.

242. The operation next in importance to that of performing the measurements accurately is that of recording them neatly, concisely, and luminously. The first is a requisite that cannot be too much insisted on, not only in the first notes, but in all the calculations and records connected with surveying. A rough, careless mode of recording observations of any kind generally indicates an equal carelessness in making them. Carelessness in a surveyor, on whose accuracy so much depends, is intolerable. Conciseness is also necessary, but it should never be allowed to detract from the luminousness of the notes. By this last quality is meant the recording of all the observations in such a mode as to indicate, in the most clear manner, the whole configuration of the plat surveyed, and all the circumstances connected with it which it is intended to preserve. The notes should be, in fact, a full record of all the work, so as to indicate fully not only what was done, but what was left undone.

243. *First Method.—By a sketch.* The simplest mode of recording the notes is to draw a sketch of the tract to be surveyed, on which other lines can be inserted as they are measured. On this sketch may be set down the distances to the various points determined.

When the tract is large, however, or contains many base-lines, this sketch becomes so complicated as scarcely to be capable of being deciphered after the mind has been withdrawn from that particular work and the configuration of the plat has been in some measure forgotten.

244. Field-Book. Perhaps the best kind of a field-book is one that is long and comparatively narrow, faint-lined at moderate distances. The right-hand page should be ruled from top to bottom with two lines, about an inch apart, near the middle of the page. The left-hand page may be ruled in the same manner; but it is better left for remarks, sketches, and subsidiary calculations.

In the space between the vertical lines all the distances are to be inserted: offsets, and other measurements connected with the main line, may be recorded in the spaces on each side of the column.

In recording the measurements the book should be held in the direction in which the work is proceeding. The right-hand side of the column will then coincide with the right-hand side of the line, and *vice versâ*. The notes should commence at the bottom, and all offsets and other lateral distances must be recorded on the side of the columns corresponding to the side of the line to which they belong.

When marks are left for starting points for other measurements, the distance to them should be recorded in the column, and some sign should be made to indicate the purpose for which such distance was recorded. Stations of this kind are called *False Stations*, and may be designated by the letters F. S.; by a triangle, △; or circle, ○; or by surrounding the number by a circle, thus, 567. Whatever plan is adopted should be scrupulously adhered to,—changes in the notation being always liable to lead to confusion.

A regular station may be designated either by letters, A, B, or by numbers, 1, 2, 3, prefixed by the letter S or by Sta. In the field-notes in the following pages examples of most of these methods will be found.

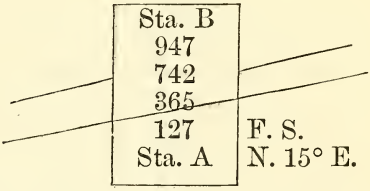
Lines are referred to, either by having them numbered on the notes as Line 1, Line 2, or by the letters or figures which designate the stations at their ends. Thus, a line from Sta. 1 to Sta. 3 would be referred to as the line 1, 3; one from Sta. B to Sta. D, as the line BD. This is perhaps the best mode. Some surveyors, however, refer to them by their lengths. Thus, a line 563 links long would be called the line 563.

False stations on a line are named by the line and distance.

Thus, a station on a line AB at 597 links would be called F. S. 597 AB, or 597 AB, or △, or ○ 597 AB. It hardly needs remark, yet it is of importance, that unity of system should be adopted. Whatever method of designating a line or station has been employed in recording it, should be used in referring to it.

The spaces on the right and left of the column will serve, in addition to the purposes already mentioned, to contain sketches of adjoining lines and short remarks to elucidate the work.

A fence, road, brook, &c. crossing the line measured, should not be sketched as crossing it in a continuous line, as at 365, marginal plan, but should consist of two lines starting at opposite points, as at 742, so that if we were to suppose the lines forming the vertical column to collapse, those representing the fence would be continuous.



When the chainmen, after closing the work on one line, begin the next at the closing station, a single horizontal line should be drawn; but if they pass to some other part of the tract, two lines should indicate the end of the line.

To indicate the direction in which a line turns, the marks ┐ or ┌ may be used, the former indicating that the new line bears to the left, and the latter to the right. Instead of these, the words *right* and *left* may be used, or the simple initials R. and L. Whichever of the means is used, the sign should be on the left hand of the column if the turn is to the left, and *vice versâ*.

The following notes will illustrate all these directions. They belong to the tract Fig. 103.

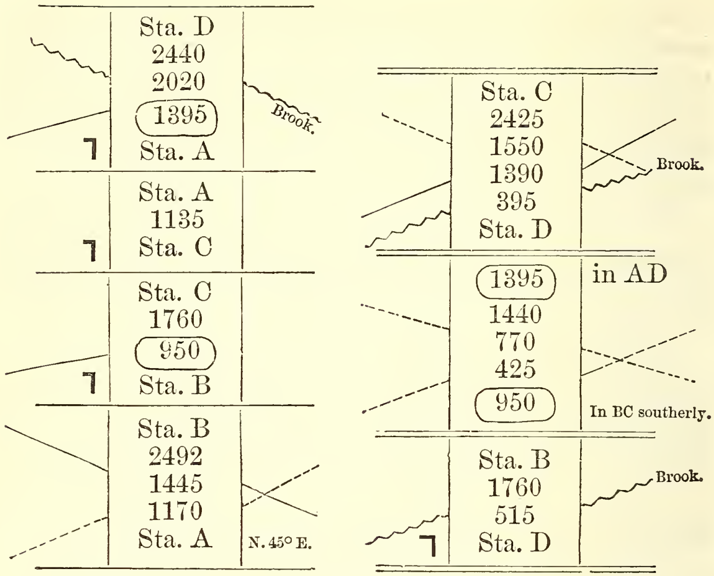
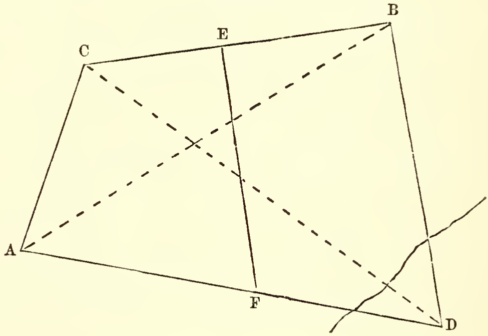


Fig. 103.



Beginning at A, the first line measured is the diagonal AB; the course N. 45° E. is set down at the right. The first point requiring notice is the intersection of the diagonals at 1170 links from A. The diagonal is represented by the dotted line crossing the columns, a continuous line being employed to designate a fence or side, and a dotted line a sight-line. At 1445 the fence EF is crossed. The whole length to B is 2492 links.

Turning to the left along BC at 950, we come to the fence bearing to the left: 950 is surrounded by a line, thus, (950) because it is to be used as a starting-point for another measurement. Having arrived at C, 1760 links from B, again turn to the left towards A: the distance CA is 1135 links. AD is next measured. At 1395 the fence EF is found: the point is marked (1395): at 2020 the brook is crossed, and at 2440 links we find the corner D. Turning to the left along DB, at 515 the brook is again crossed. This line is 1760 links long.

Passing now to E, (950) in BC, along the cross fence, the diagonal AB is passed at 425; at 770 CD is passed; 1440 links brings us to 1395 in AD. Passing to D: along DC, at 395 the brook is crossed; at 1390 the fence is found; at 1550 we cross the diagonal AB: 2425 brings us to C, which finishes the work.

245. Test-lines. In the above survey more lines have been measured than are absolutely necessary. It is always better to measure too many than too few. If the redundant lines are not needed in the calculation, they serve as tests by which to prove the work. For the mere purpose of calculation, one of the diagonals and the line EF might have been omitted: the other lines afford sufficient data for making a plat and calculating the area. An error in one of the others will not prevent the notes from being platted, and hence they do not in any way afford a criterion by which we can judge of the accuracy of the measurements; but when to these are added the length of the other diagonal we have a series of values, all of which must be correct or the map cannot be made.

246. General Directions. When about to survey a tract by this method, the surveyor should first examine the tract carefully and erect poles at the prominent points, corners, and false stations, along the boundary lines. He should stake out all diagonals and subsidiary lines which he may wish to measure, setting a stake at the points in which such lines intersect each other or cross the former lines,—in fact, at every point the position of which it may be desirable to fix on the plat.

Having made these preparations, he may, if the tract is at all complicated, make an eye-sketch. This will serve to guide him in regard to the best course to take in his measurements.

Commencing then at some convenient point of the tract, he should measure carefully the diagonals and sides in succession, passing from one line to such other as will make the least unnecessary walking, and setting down in his note-book the distance to every stake, fence, brook, or other important object met with.

When the tract is large, the work may last through several days. In such cases, each day's work should, if possible, be made complete in itself,—that it may be platted in the evening. This will prevent the accumulation of errors which might occur from a mismeasurement of one of the earlier lines.

247. Platting the Survey. To plat a survey from the notes, select three sides of a triangle and construct it. Then, on the sides of this construct other triangles, until the whole of the lines are laid down. Measure test-lines to see whether the work is correct.

In all cases commence with large triangles, and fill up the details as the work proceeds.

SECTION V.

ON THE METHOD OF SURVEYING FIELDS OF PARTICULAR FORMS.

248. Rectangles. MEASURE two adjacent sides: their product will give the area.

EXAMPLES.

Ex. 1. Let the adjacent sides of a rectangular field be 756 and 1082 links respectively, to plat the field and calculate the content.

Calculation.

Content = 1082 × 756 = 817992 square links = 8 A., 0R., 28.7 P.

Ex. 2. The adjacent sides of a rectangular tract are 578 and 924 links: required the area.

Ans. 5 A., 1 R., 14.51 P.

Ex. 3. Required the area of a tract the sides of which are 9.75 and 11.47 chains respectively.

Ans. 11 A., 0 R., 29 P.

249. Parallelograms. Measure one side and the perpendicular distance to the opposite side. Their product will be the area.

If a plat is required, a diagonal or the distance from one angle to the foot of the perpendicular let fall from the adjacent angle may be measured.

EXAMPLES.

Ex. 1. Given one side of a parallelogram 10.37 chains, and the perpendicular distance from the opposite side 7.63 chains, the distance from one end of the first side to the perpendicular thereon from the adjacent angle being 2.75 chains. Required the area and plat.

Ans. 7 A., 3 R., 25.96 P.

Ex. 2. Desiring to find the area of a field in the form of a parallelogram, I measured one side 763 links, and the perpendicular from the other end of the adjacent side 647 links, said perpendicular intersecting the first side 137 links from the beginning. Required the content and plat.

Ans. 4 A., 3 R., 29.86 P.

250. Triangles. *First Method.*—Measure one side, and the perpendicular thereon from the opposite angle; noting, if the plat is required, the distance of the foot of the perpendicular from one end of the base.

Multiply the base by the perpendicular, and half the product will be the area.

EXAMPLES.

Ex. 1. Required the area and plat of a triangular tract, the base being 7.85 chains and the perpendicular 5.47 chains, the foot of the perpendicular being 3.25 chains from one end of the base.

Calculation.

$$\text{Area} = \frac{7.85 \times 5.47}{2} = \frac{42.9395}{2} = 21.46975 \text{ chains} = 2 \text{ A.},$$
0 R., 23.5 P.

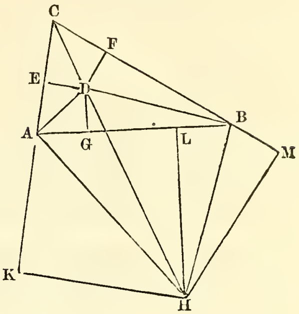
Ex. 2. Required the area and plat of a triangle, the base being 10.47 chains, and the perpendicular to a point 4.57 chains from the end, being 7.93 chains.

Ex. 3. Required the area of a triangle, the base being 1575 links, and the perpendicular 894 links.

251. *Second Method.*—Measure the three sides, and calculate by the following rule:—

From half the sum of the sides take each side severally; multiply the half-sum and the three remainders continually together, and the square root of the product will be the area.

Fig. 104.



DEMONSTRATION.—Let ABC (Fig. 104) be a triangle. Bisect the angles C and A by the lines CDH and AD, cutting each other in D. Then D is the centre of the inscribed circle. Join DB, and draw DE, DF, and DG perpendicular to the three sides. Then will DE = DF = DG, and (47.1) FB = BG, CE = CF, and AE = AG.

Bisect the exterior angle KAB by the line AH, cutting CDH in H. Draw HK, HL, and HM perpendicular to CA, AB, and CB. Join HB. Then (26.1) KH = HM, CK = CM, HL = HK, and AL = AK; also (47.1) BL = BM. Because AK = AL and BM = BL, CK + CM will be equal to the sum of the sides AB, AC, and BC; therefore CK or CM $= \frac{1}{2}$ (AB + AC + BC) $= \frac{1}{2}$ S, if S stand for the sum of the three sides. But CE + AE + BG $= \frac{1}{2}$ S; therefore CK = CM = CA + BG, and AK = AL = BG; whence AG = AE = BL = BM, and EK = AB. Now, since CK = CM $= \frac{1}{2}$ S, we have AK $= \frac{1}{2}$ S — AC, EC $= \frac{1}{2}$ S — AB, and AE = BM $= \frac{1}{2}$ S — BC.

Because the triangles CDE and CKH, as also ADE and HKA, are similar,

we have (4.6) $\quad CE : ED :: CK : KH,$

and $\quad AE : ED :: HK : KA,$

$\therefore$ (23.6) $\quad AE \cdot EC : ED^2 :: CK : KA :: CK^2 : CK \cdot KA.$

Whence, $\quad \sqrt{AE \cdot EC} : ED :: CK : \sqrt{CK \cdot KA},$

and $\quad CK \cdot ED = \sqrt{CK \cdot KA \cdot AE \cdot EC}.$

Now, $ABC = ACD + BCD + ABD = \frac{1}{2} AC \cdot ED + \frac{1}{2} BC \cdot ED + \frac{1}{2} AB \cdot ED = \frac{1}{2} S \cdot ED = CK \cdot ED.$

Wherefore, $\quad ABC = \sqrt{CK \cdot KA \cdot AE \cdot EC}.$

COR.—From the above demonstration, it is apparent that the area of a triangle is equal to the rectangle of the half-sum of the sides and the radius of the inscribed circle.

For another demonstration of this rule, see Appendix.

EXAMPLES.

Ex. 1. Required the area of a triangle, the three sides being 672, 875, and 763 links respectively.

NOTE.—In cases of this kind the operation will be much facilitated by using logarithms.

$$\frac{672 + 875 + 763}{2} = \frac{2310}{2} = 1155 = \text{half-sum of sides.}$$

$\frac{1}{2}$ sum $= 1155$	log.	3.062582
$\frac{1}{2}$ sum $- 672 = 483$	log.	2.683947
$\frac{1}{2}$ sum $- 875 = 280$	log.	2.447158
$\frac{1}{2}$ sum $- 763 = 392$	log.	2.593286
	2)	10.786973
Area, 247449 square links,		5.393486
$= 2$ A., 1 R., 35.9 P.		

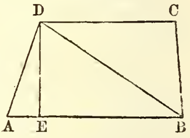
Ex. 2. Required the area of a triangular tract, the sides of which are 17.25 chains, 16.43 chains, and 14.65 chains respectively. Ans. 11 A., 0 R., 14.4 P.

Ex. 3. Given the three sides, 19.58 chains, 16.92 chains, and 12.76 chains, of a triangular field: required the area.
Ans. 10 A., 2 R., 27 P.

252. Trapezoids. Measure the parallel sides and the perpendicular distance between them.

If a plat is desired, a diagonal, or the distance AE, (Fig. 105,) may be measured.

Fig. 105.



Multiply the sum of the parallel sides by half the perpendicular: the product is the area.

DEMONSTRATION. — $ABCD = ABD + BCD = \frac{1}{2} AB \cdot DE + \frac{1}{2} DC \cdot DE = (AB + DC) \cdot \frac{1}{2} DE$.

EXAMPLES.

Ex. 1. Given AB = 7.75 chains, DC = 5.47 chains, and DE = 4.43 chains, to calculate the content and plat the map, AC being 7.00 chains.

Ans. Area, 2 A., 3 R., 28.5 P.

Ex. 2. Given the parallel sides of a trapezoid, 16.25 chains and 14.23 chains, respectively: the perpendicular from the end of the shorter side being 12.76 chains, and the distance from the foot of said perpendicular to the adjacent end of the longer side 1.37 chains. Required the area and plat.

Ans. 19 A., 1 R., 31.4 P.

253. Trapeziums. *First Method.*—Measure a diagonal, and the perpendiculars thereon, from the opposite angle.

The area of a trapezium is equal to the rectangle of the diagonal and half the sum of the perpendiculars from the opposite angles.

This is evident from the triangles of which the trapezium is composed.

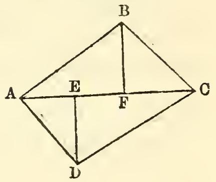
EXAMPLES.

Ex. 1. To plat and calculate the area of a trapezium, the diagonal being 15.63 chains, and the perpendiculars thereto from the opposite angles being 8.97 and 6.43 chains, and meeting the diagonal at the distances of 4.65 and 13.23 chains. Ans. Area, 12 A., 0 R., 5.6 P.

Ex. 2. Given (Fig. 106) AC = 19.68 chains, AE = 7.84 chains, AF = 16.23 chains, ED = 10.42 chains, and FB = 8.73 chains, to plat the figure and find the area.

Ans. 18 A., 3 R., 14.98 P.

Fig. 106.



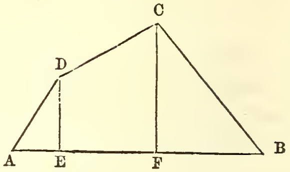
Ex. 3. Required the area of a trapezium, the diagonal being 17.63 chains, and the perpendiculars 6.47 and 12.51 chains respectively.

Ans. 16 A., 2 R., 36.94 P.

254. *Second Method.*—Measure one side, and the perpendiculars thereon from the extremities of the opposite side, with the distances of the feet of these perpendiculars from one end of the base.

EXAMPLES.

Fig. 107.



Ex. 1. Let ABCD (Fig. 107) be a trapezium, of which the following dimensions are given,—viz.: AE = 3.27 chains, AF = 10.17 chains, AB = 17.62 chains, ED = 7.29 chains, and FC = 13.19 chains. Required to plat it, and calculate the area.

Lay off the distances AE, AF, and AB; then erect the perpendiculars ED and FC, and draw AD, DC, and CB.

The trapezium is divided into two triangles and the trapezoid, the areas of which may be found by the preceding rules.

$$\begin{aligned} \text{Thus,}\quad 2\,\text{AED} &= \text{AE}.\text{ED} = 23.8383 \\ 2\,\text{EFCD} &= \text{EF}.(\text{ED}+\text{FC}) = 141.3120 \\ 2\,\text{CFB} &= \text{CF}.\text{FB} = 98.2655 \\ \text{whence}\quad \text{ABCD} &= \tfrac{1}{2}\text{ of } 263.4158 = 131.7079 \end{aligned}$$

chains = 13 A., 0 R., 27.3 P.

If either of the angles A or B were obtuse, the perpendicular would fall outside the base, and the area of the corresponding triangle should be subtracted.

Ex. 2. Plat and calculate the area of a trapezium from the following field-notes:—

	⊙ B	
	1143	
perp. 936	917	
perp. 825	415	
	⊙ A	

Ans. 7 A., 0 R., 30.3 P.

Ex. 3. Calculate the area from the following field-notes:—

perp. 892	1365	
	967	Stat. B.
perp. 568	376	
	⊙ A	

Ans. 6 A., 2 R., 2 P.

Fields of more than four sides, bounded by straight lines.

255. *First Method.*—Divide the tract into triangles and trapeziums, and calculate the areas by some of the preceding rules. In applying this method, as many of the measurements as practicable should be made on the ground; the field then being platted with care, the other distances may be measured on the map. When it is intended to depend on the map for the distances, every part of the plat should be laid down with scrupulous accuracy, on a scale of not less than three chains to the inch.

Ex. 1. To draw the map and calculate from the following field-notes the area of the pentagonal field ABCDE :—

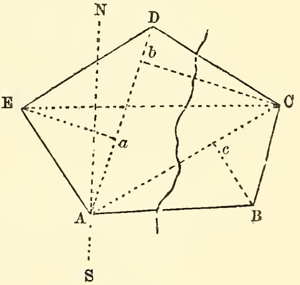
Diagonal.		⊙ D	
		690	
		570	510 C
	E. 350	280	
		⊙ A	N. 15° E.

Diagonal.	⊙ C	
	770	
	510	250 B
	360	Brook.
	⊙ A	E. of AD

Test-line.	⊙ C	
	915	
	585	
	365	Brook. AD
	⊙ E	

The construction is plain.

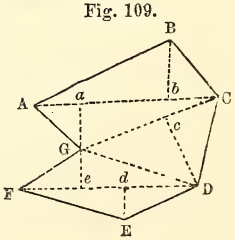
Fig. 108.



Calculation.

Twice trapezium ACDE = AD × (Ea + bC) = 6.90 × 8.60 = 59.34; twice triangle ABC = AC × Bc = 7.70 × 2.50 = 19.25; whence ABCDE = $\dfrac{59.34 + 19.25}{2}$ = 39.295 ch. = 3 A., 3 R., 28.72 P.

Fig. 109.



Ex. 2. Map the plat, and calculate the content from the following field-notes :—

	⊙ D	
	520	
	288	80 E
G 120	206	
	⊙ F	
	⊙ G	
	440	
D 230	150	
	⊙ C	L of CA
	⊙ C	
	550	
B 180	410	
	135	130 G
	⊙ A	East.

Construction.

Commencing at A, (Fig. 109,) draw the line AC east 5.50 chains, marking the points a and b at 1.35 and 4.10 chains respectively: at a and b erect the perpendiculars aG 1.30 and bB 1.80 chains. From C to G draw CG, which should be 4.40 chains long. At c, 1.50 chains from C, draw cD perpendicular to CG and equal to 2.30 chains. With the centre G and radius 1.20 chains, describe a circle, and from D draw the line DF 5.20 chains long, touching the circle at e, which should be 2.06 chains from F. At d, 2.88 chains from F, draw the perpendicular dE = .80 chains: then will A B C D E F G be the corners of the tract.

Calculation.

$$2\ \mathrm{ABCG} = \mathrm{AC}\,(\mathrm{G}a + \mathrm{B}b) = 5.50 \times 3.10 = 17.05;$$
$$2\ \mathrm{GCD} = \mathrm{GC}\,.\,c\mathrm{D} = 4.40 \times 2.30 = 10.12;$$
$$2\ \mathrm{GDEF} = \mathrm{FD}\,(\mathrm{G}e + d\mathrm{E}) = 5.20 \times 2.00 = 10.40.$$

Therefore area $= \dfrac{37.57}{2}$ chains = 18.785 chains = 1 A., 3 R., 20.56 P.

Ex. 3. Required the plans and areas of the adjoining fields, of which the following are the field-notes, the two fields to be platted on one map.

	⊙ (4)	
	970	
(3) 772	830	
	395	284 (5)
	⊙ (6)	N.E.
	⊙ (3)	
	990	
(2) 395	320	
	100	715 (6)
	⊙ (1)	N. 10° E.

Area 10 A., 2 R., 18.576 P.

	⊙ 7	
	1150	
	675	432 (11)
	⊙ (8)	
	⊙ (9)	
	1285	
(8) 565	1000	
	960	155 (10)
	⊙ (7)	L. of (7,5)
	⊙ (7)	
	1315	
	390	313 (10)
(4) 562	282	
	⊙ (5)	R. of (4)

Area 12 A., 3 R., 18.1 P.

Ex. 4. Required the plan and areas of the adjoining fields from the following field-notes, tracing thereon the course of the brooks.

	⊙ (7)	
	1051	
Brook + (6.7)—	680	
	648	540 (1)
	510	Brook.
	365	—Brook + (1.5)
(6) 380	130	
	⊙ (5)	
	⊙ (5)	
	1255	
	853	765 (1)
(4) 500	440	
	⊙ (3)	
	⊙ (3)	
	1150	
Brook + (2.3)—	850	
	490	Brook.
(2) 482	200	
	⊙ (1)	

Area 15 A., 2 R., 7 P.

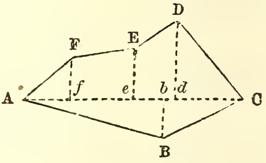
	⊙ (10)	
(11) 620	1080	
	730	465 (9)
	⊙ (8)	N.E.
	⊙ (11)	
	1395	
	1095	—Brook + (8.11)
(6) 665	748	
	325	635 (8)
	270	—Junction of Brooks.
	55	—Brook + (7.8)
	⊙ (7)	R. of (7.5)

Area 14 A., 3R., 28.24 P.

Note.—In the above field-notes the marginal references, such as "Brook + 6.7," means to the point in which the brook crosses the line (6.7.)

256. *Second Method.*—Instead of running diagonals, it may sometimes be more convenient to run one or more lines through the tract and take the perpendiculars to the several angles, as in the following example.

Fig. 110.



Let the field be of the form ABCDEF, (Fig. 110.) Run the line AC, and take the perpendiculars *f*F, *e*E, *b*B, and *d*D. The field will thus be divided into triangles and trapezoids, the area of which may be calculated by the preceding rules.

Thus, let the field-notes of the preceding tract be as follows:—

	⊙ C	
	1185	
D 420	840	
	760	200 B
E 280	590	
F 220	250	
	⊙ A	East.

Dist.	Perp.	Int. Dist.	Sum of Perp.	Double Areas.	
0	0				
250	220	250	220	55000	2 AF*f*
590	280	340	500	170000	2 *f*FE*e*
840	420	250	700	175000	2 *e*ED*d*
1185	0	345	420	144900	2 D*d*C
				544900	Left-hand areas.
1185 × 200				237000	Right " "
				2) 781900	
				39.0950 ch. = 3 A., 3 R., 25.5 P.	

The calculation being performed thus:—In the first column are placed the distances to the feet of the left-hand perpendiculars. In the second the perpendiculars themselves. The numbers in the third column are found by subtracting each number in column 1 from the succeeding number in the same column. The numbers in column 3 therefore represent the distances A*f*, *fe*, *ed*, and *d*C. The numbers in the fourth column are found by adding each number in column 2 to the succeeding number in the same column; they therefore are the sums of the adjacent perpendiculars. Those in the fifth column are found by multiplying the corresponding numbers in columns 3 and 4. They therefore are the double areas of the several trapezoids and triangles.

Ex. 2. Required to calculate the content and make plats from the following field-notes:—

	⊙ G	
	3127	
	2590	476 F
H 375	2145	
	2070	642 E
I 400	1920	
	1485	523 D
	840	516 C
K 600	790	
	200	465 B
	⊙ A	E.

Area 25 A., 1 R., 5 P.

	⊙ F	
	4025	
	3617	792 G
	3254	826 H
E 594	2846	
D 435	2137	
	1548	319 I
C 729	1026	
	429	623 K
B 237	175	
	⊙ A	N. 15° E.

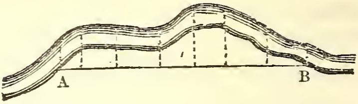
Area 38 A., 3 R., 17.5 P.

257. Offsets. In what precedes, the sides have been supposed to be right lines. This is ordinarily the case except when the tract bounds on a stream. It then, if the stream is not navigable, generally takes in half the bed. Lands bounding on tide-water go to low-water mark. In all such cases the area and configuration of the boundary are most readily determined by offsets.

A base is run near the crooked boundary, and perpendicular offsets are taken to the line, whether it be the middle of the stream or low-water mark. If the positions of these offsets are judiciously chosen, so that the part of the boundary intercepted between any two consecutive ones is nearly straight, the correct area may be calculated precisely as in last article. In the field-notes the distances are written in the column and the offsets on the right or left hand, according as they are to the right or left of the line run.

Thus, supposing it were required to find the area contained between the line AB and the stream, (Fig. 111,) the following being the field-notes.

Fig. 111.



	⊙ B	
25	865	
70	725	
165	580	
165	475	
100	355	
115	195	
90	75	
40	0	
	⊙ A	N. 10° E.

The calculation would be as below, the same formula being used as in last article.

Dist.	Offs.	Int. Dist.	Sum of Offs.	Double Areas.
0	40			
75	90	75	130	9750
195	115	120	205	24600
355	100	160	215	34400
475	165	120	265	31800
580	165	105	330	34650
725	70	145	235	34075
865	25	140	95	13300
			2)	182575
Area = 3 R., 26 P.				9.12875 ch.

Ex. 1. Required the area and plan from the following notes:—

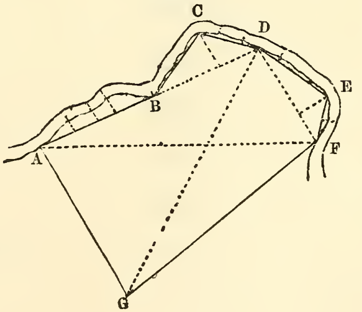
	A	
	4830	
	2040	
	F	⌜
	F	
	2175	
E 355	1428	
	D	⌜
	D	on creek-bank.
	4175	
C 665	3335	
55	(2160)	B
270	1929	
396	1408	
310	1015	
340	610	
50	0	
	A	N. 56¼° E. ⌝

	E	
60	1471	
95	930	
140	485	
60	0	
	D	⌜
	D	
60	1072	
130	750	
85	390	
55	0	
	C	⌜
	C	
55	1350	
55	0	
	(2160)	B on A.D.

	D	
	5000	
	3585	
	G	
	A	
	3000	
	G	⌜
	G	
	4241	
	F	⌜
	F	
75	826	
100	420	
60	0	
	E	⌜

Fig. 112 is a plat of this tract.

Fig. 112.



Calculation.

To find AGF, Art. 251.

AG	3000	
FG	4241	
FA	4830	
	2) 12071	
$\frac{1}{2}$ sum	6035.5	3.780713
$\frac{1}{2}s -$ AG	3035.5	3.482230
$\frac{1}{2}s -$ FG	1794.5	3.253943
$\frac{1}{2}s -$ AF	1205.5	3.081167
		2) 13.598053
AGF =	6295435	6.799026

To find AFD.

AF	4830	
AD	4175	
FD	2175	
	2) 11180	
$\frac{1}{2}$ sum	5590	3.747412
$\frac{1}{2}s -$ AF	760	2.880814
$\frac{1}{2}s -$ AD	1415	3.150756
$\frac{1}{2}s -$ FD	3415	3.533391
		2) 13.312373
AFD =	4530917	6.656186

To find BCD.

BC	1350	
BD	2015	
CD	1072	
	2) 4437	
$\frac{1}{2}$ sum	2218.5	3.346059
$\frac{1}{2}\,s$ — BC	868.5	2.938770
$\frac{1}{2}\,s$ — BD	203.5	2.308564
$\frac{1}{2}\,s$ — CD	1146.5	3.059374
		2) 11.652767
BCD =	670475	5.826383

To find DEF.

DE	1471	
EF	826	
DF	2175	
	2) 4472	
$\frac{1}{2}$ sum	2236	3.349472
$\frac{1}{2}\,s$ — DE	765	2.836661
$\frac{1}{2}\,s$ — EF	1410	3.149219
$\frac{1}{2}\,s$ — DF	61	1.785330
		2) 11.120682
DEF =	336363	5.560341

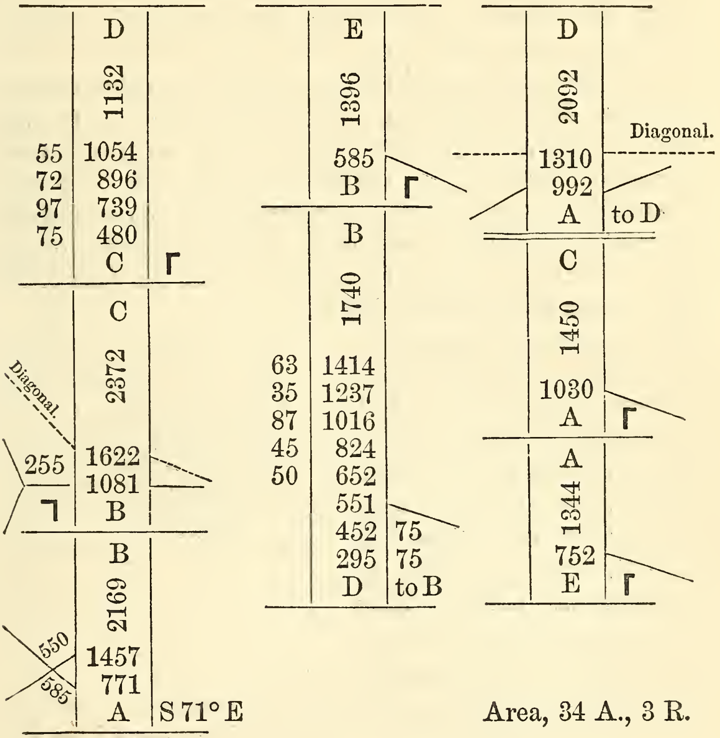
Base.	Dist.	Offsets.	Inter. Dist.	Sum of Offsets.	Double Areas.
AB	0	50			
	610	340	610	390	237900
	1015	310	405	650	263250
	1408	396	393	706	277458
	1929	270	521	666	346986
	2160	55	231	325	75075
BC			1350	110	148500
CD	0	55			
	390	85	390	140	54600
	750	130	360	215	77400
	1072	60	322	190	61180
DE	0	60			
	485	140	485	200	97000
	930	95	445	235	104575
	1471	60	541	155	83855
EF	0	60			
	420	100	420	160	67200
	826	75	406	175	71050

2) 1966029

Area of part cut off by bases,	983014.5
AGF	6295435
AFD	4530917
BCD	670475
DEF	336363
	12816204 links.

= 128 A., 0 R., 25.9 P.

The field-notes of a meadow, bounding on a river and divided into four fields, are as follows,—the measurements being to low-water mark. Required the map and the content of the whole:—



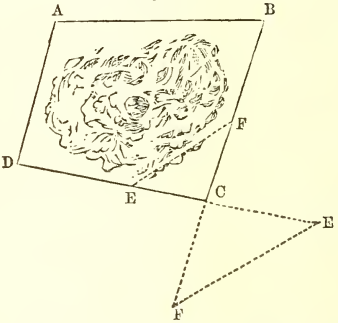
To find the contents of the several enclosures, other lines would be required: these might be measured on the plat, if it were drawn with neatness and accuracy.

SECTION VI.

TIE-LINES.

258. Tie-Lines. The external boundaries of a tract of land having more than three sides are not sufficient either for making a plat or calculating the area. In the methods heretofore laid down, diagonals were also used. In some cases, however, owing to obstructions, such as ponds, close woods, or buildings, it is difficult to run the diagonals. When this is the case, a line measured across one of the angles of a quadrilateral will determine the direction of two sides, and thus fix the relative position of all the lines of the tract. Such lines are called *tie-lines*.

Fig. 113.



For example, suppose it were required to survey the tract represented in Fig. 113, the interior of which is filled with such thick woods that the diagonals cannot be measured: the external lines AB, BC, CD, and DA might be measured as before. Then on the lines adjacent to one angle, as C, measure carefully CE and CF; also measure EF. These measures should be made with the greatest accuracy, as a slight error here will very materially affect the result. On the same account, the distances CE and CF should be taken as large as circumstances will allow.

If the tie-line cannot be run within the tract, the points may be taken at E and F in the sides produced.

To plat such a tract, commence with the triangle. This being formed, the direction of CB and CD is known.

259. To calculate the Area. First find in ECF the angle ECF, whence by trigonometry BD is found, and then the area of the triangles.

If CE = CF, EF will be the chord of the arc to the radius CE, whence the chord to radius $1 = \frac{EF}{EC}$. This quotient being found in the table of chords the corresponding arc will give the degrees and minutes of the angle ECF: or CE : $\frac{1}{2}$ EF :: rad. : sin. $\frac{1}{2}$ ECF.

260. Inaccessible Areas. By a combination of tie-lines and offsets, tracts that cannot be entered, such as a pond or a swamp, may be measured. For this purpose, surround the tract by a system of lines bound at the angles by tie-lines, and take offsets to the prominent points in the boundary of the tract.

261. Defects of this Method. Every system of measurement or drafting should commence with the longer lines and end with the shorter. By this means the errors that are unavoidable are diminished as we proceed. If, for example, a diagonal of thirty chains were measured, this would fix the distance of the ends to a degree of certainty precisely equal to that of the measurement; and if from this measurement the length of an inferior line joining two points in the sides were to be determined, the errors in the length of the diagonal would affect this length to a degree exactly proportional to its length, the error in a line of five chains long being one-sixth of that of the diagonal. Precisely the reverse is the case when the shorter line is measured: the error is magnified as we proceed. On this account, the method explained above should never be employed when it can be avoided. By the use of the compass, transit, or theodolite, this can always be done. The mode of using them for surveying purposes forms the subject of the next chapter.

CHAPTER V.

COMPASS SURVEYING.

SECTION I.

DEFINITIONS AND INSTRUMENTS.

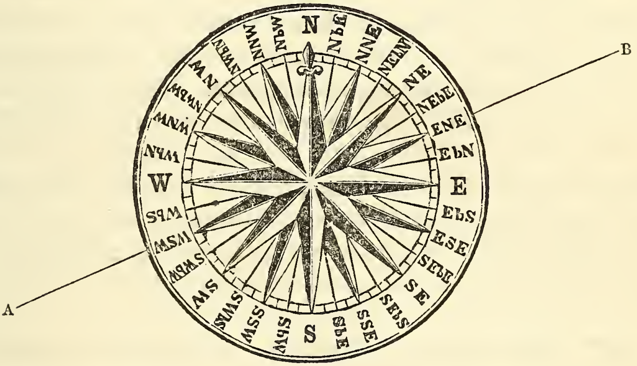
262. In chain surveying, the position of any point is determined either by directly measuring to it from other known points, or by determining its distance from such points by the indirect methods explained in last chapter. In the method about to be explained, its position is ascertained by angular measurements taken from known stations, or by its distance from a known point and the angle which it makes with the meridian.

All those methods, which have a direct reference to the meridian as the base of angular distance, are known under the head of compass surveying; whether the instrument used to determine the angle is a *theodolite*, a *transit*, or a *compass*.

263. The Meridian. If the heavens are examined during a clear night, the stars to the north will be perceived to revolve around a star elevated about 40°. This is called the pole-star, and is very nearly in the point in which the axis of the earth if produced would meet the heavens. This point is called the north pole of the heavens. The north star is not exactly at the pole, but revolves around it in a small circle. If a transit or theodolite be levelled, and the telescope directed to the centre of this circle (see chap. ix.) it will point exactly north. Depress it, and run out a line in the direction of the line of collimation. This will be a *meridian line.*

264. The Points of the Compass. If through any station a line be drawn perpendicular to the meridian it will run east and west. If we face the south, the *west* will be to the *right* hand and the *east* to the *left.* These four points—north, east, south, west—are called the cardinal points of the compass, and are used as reference for all angular distances from the meridian.

Fig. 114.



For nautical purposes, each of the quadrants into which the horizon is divided is further divided into eight parts called points, and named as in Fig. 114, commencing at the north and going to the east.

North, N.; North by East, (N.*b*E.;) North Northeast, (N.N.E.;) Northeast by North, (N.E.*b*N.;) Northeast, (N.E.;) Northeast by East, (N.E.*b*E.;) East Northeast, (E.N.E.;) East by North, (E.*b*N.;) East, (E.) and so on, E.*b*S.; E.S.E.; S.E.*b*E.; S.E.; S.E.*b*S.; S.S.E.; S.*b*E.; S.

For land surveying only the cardinal points are mentioned, the direction being determined by the angular distance from the meridian.

265. Bearing. The *bearing* of a line is the angle which it makes with a meridian through one end. It is expressed either by naming the points, as N.*b*E., S.S.E. ½ E., as is done in navigation, or by mentioning the number of degrees in the angle accompanied by the cardinal points between which it runs. Thus, if a line runs between north and west and makes an angle of 37° 25′ with the meridian, its bearing is N. 37° 25′ W. It deflects 37° 25′ from the north towards the west, and is therefore sometimes said to run from north towards the west. This expression, though convenient, is not strictly correct.

266. The Reverse Bearing. If the bearing of a line of moderate length is determined at one end, and then again at the other end, the latter is called the *reverse bearing.* It will be found to be of the same number of degrees as the bearing, but with the opposite points. Thus, if the bearing of a line be N. $27\frac{1}{4}$° E, its reverse bearing is S. $27\frac{1}{4}$° W.

If the line be long, there will be a continual variation from the initial course. Thus, if a line run N. 45° E. through its whole course, it will be found to deviate to the left from a straight line. A true east and west line in latitude 40° is a curve with a radius of about 4800 miles.

267. The Magnetic Needle. A magnetic needle is a light bar of magnetized steel suspended on a pivot, so that it may turn freely in a horizontal direction. Such a needle will always place itself in nearly the same direction, one end of it being northward and the other southward. The needle should move very freely on its pivot, so that it may always assume its proper position. The pivot should therefore be of very hard steel ground to a fine point. In the centre of the needle there should likewise be a cup of agate or some other hard material inserted for it to rest upon.

As the needle is generally balanced before being magnetized, the north end in northern latitudes will always "dip" after the magnetic force has been communicated to it. To restore the balance, a coil of fine brass wire is wrapped around the south end. This may be slipped along the bar so as perfectly to restore the balance. It serves also to distinguish the two ends of the needle.

A good needle will vibrate for a considerable time after having been disturbed. If it settles soon, it is defective in magnetic power, or the pivot is imperfect. To preserve the pivot in good order, the needle should always be lifted from it when not in use.

268. The Magnetic Meridian. The line upon the surface of the earth in the direction of the needle, when uninfluenced by disturbing causes, is called the magnetic meridian. If the needle pointed steadily to the north pole, the magnetic meridian would coincide with the true. This is, however, far from being the case. Throughout the eastern part of the United States and Canada it points west of north, the amount of the deviation (called the variation of the compass) being different in different places. This amount is subject to a gradual secular change. (See chap. x.)

269. The Magnetic Bearing. The bearing of a line from the magnetic meridian is called the magnetic bearing. This has generally been used in land surveying. Its convenience is such as to have heretofore counterbalanced its defects in the opinion of a large number of surveyors. The attention of scientific surveyors and legislators has of late been called to the difficulties arising from the use of such a false and varying standard. In Pennsylvania, by a late law, the bearings of all lines inserted in the title-deeds of real estate are required to be from the true meridian line. The surveys of United States public lands have always been made on this principle.

270. There are two modes in which the needle may be employed to enable us to determine the bearing of a line.

First. Attached to the needle may be fixed a card divided as in Fig. 114, or subdivided into degrees,—the north point of the needle being directly under the north point of the card. Such a card would always place itself in the same position with respect to the cardinal points.

To determine the bearing of a line, it would only be necessary to have a pair of sights in the line of a diameter of the card, with an index between them to show at what point of the card the line crossed. The degrees between this point and the north or south point of the card would be the bearing required. Thus, the bearing of AB would be about N. 67° E. The cardinal points on the card show the points between which the line runs.

The great defect in this plan is that, in consequence of the weight of the card, the needle settles slowly, and the pivot is very liable to wear. The card, too, must be made of some light material, which cannot be divided so accurately as metal. This form is therefore never used except for the mariner's compass.

Second. The sights may be connected with a circular box in the centre of which is the pivot,—the circumference of the box being appropriately divided. This is the plan employed in the surveyor's *compass* or *circumferentor.*

271. The Compass. The compass consists of a stiff brass plate A, (Figs. 115, 116,) carrying the circular box B, and furnished at the ends with two brass sights C, perpendicular to its plane. In the centre of the box is the pivot to support the magnetic needle.

The circumference of the box is divided into 360°, and these in the larger instruments are subdivided into halves.

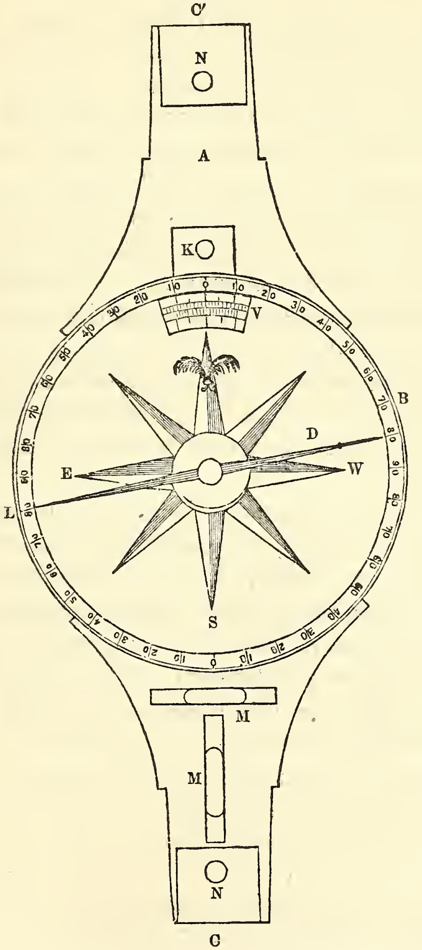
The zero-points are in the line joining the sights, one being marked for the north, and the other for the south. The degrees are counted from zero to 90° each way.

If we stand opposite the south point looking towards the north, the 90° on the left hand is marked E. and that on the right W. The cardinal points thus follow each other in an inverted order.

The reason why this should be so will appear from considering the difference between the mariner's compass and the circumferentor. In the former, the card is stationary, while the index moves; in the latter, the index, which is the needle, is stationary, while the divided circle moves: while, then, the north point of the box is moving towards the *east,* the north point of the needle will traverse it towards the *west.* In order, then, that the index should not only point to the number of degrees, but also show the cardinal points between which the line runs, those points must be engraved in a reverse order.

Thus, supposing the instrument to be in the position, (Fig. 115,) the north point of the needle at L shows the magnetic

Fig. 115.



north, and the south point the magnetic south; the point midway between these to the right is east. The line from C to C′ is therefore south of east. If then the north point of the needle is to be used as the index, it should be found between the letters S. and E. The bearing in the figure is S. 80° E.

272. The Sights. These consist of two plates of brass about an inch wide set at right angles to the plate. Each plate has a vertical slit cut in it, with larger openings at intervals, as seen in Fig. 116 at H. The faces of the sights are seen at G. The slits should be perfectly straight, and as narrow as is consistent with distinct vision. The larger openings enable the surveyor to see the object more readily than he could through the fine slits.

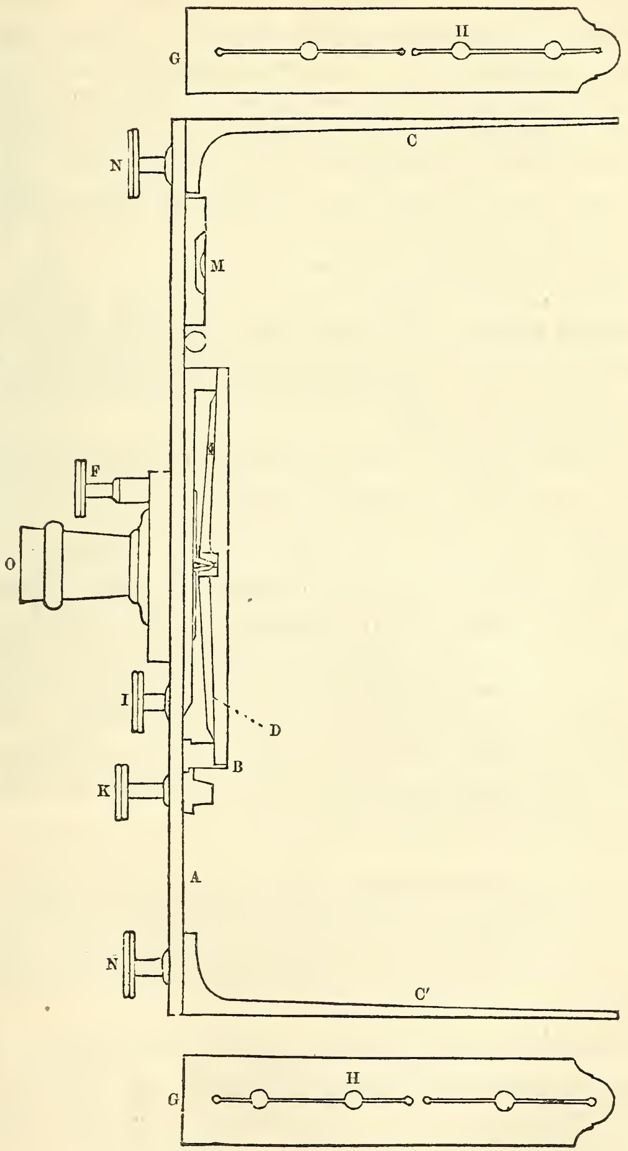
Instead of the sights, a telescope that can be elevated or depressed in a plane perpendicular to that of the plate A is sometimes employed. It has the advantage of giving more distinct vision at great distances, and, when connected with a vertical arc, of determining the angle of elevation of a hill up or down which the line may run. This object may be obtained with the sights, by having at the lower end of one of them a projection pierced with a small hole, and upon the face of the other the angles of elevation engraved. By looking through the hole at an object on the summit of the hill, the angle of elevation may be read on the face of the engraved sight.

If such a scale is not on the instrument, it may be put on by the surveyor himself; a mark being made on one sight near the bottom, or a small plate with a hole being screwed to it; on the other, at the same distance from the plate, the zero mark should be made. The distance from zero to the other marks will be the tangent of the angle of elevation to a radius equal to the distance between the sights. Measure therefore accurately the distance between the sights, and say, As rad. : tangent of the number of degrees : : the distance between the sights : the distance from the zero point to the mark for that number of degrees.

273. Attached to the plate there are generally two levels at right angles to each other, as in the transit and theodolite.

274. The Verniers. In some instruments, the compass-box is movable about its centre for a few degrees, the amount of deflection being determined by the vernier V. The purpose of this arrangement will appear hereafter.

Fig. 116.



275. In the figures 115, 116, the different parts described above are lettered as below. Different makers, however, arrange the parts differently. A is the principal plate, which bears all the other parts. B is the compass-box, sometimes movable about its centre by means of a pinion connected with the milled head I, and capable of being clamped in any position by the screw K. D is the needle, resting on a pivot in the middle of the compass-box. The needle can be raised from its pivot by the screw F. C and C′ are the sights, which are fastened to the plate by the screws N. M, M are the levels.

276. The Pivot. This should, as remarked above, be extremely hard and very sharp. It should likewise be placed exactly in the centre of the box and in the line joining the slits in the sights.

To discover whether it is properly centred, and likewise whether the needle is straight, turn the compass until the north point of the needle coincides with any given number of degrees. The south point must be 180° distant. If it is so in all positions, or, in four, distant 90°, as for instance the 0's and 90's, the needle is straight and well centred.

Draw a hair or fine silk string through the slits in the sights. If this passes over the zero-points, the centre is in line.

Or, sight to a very near object, and note the reading. Turn the instrument half round, and again note the reading: if these do not agree, the pivot is not on the line of sight. Half the difference is the actual error.

277. The Divided Circle. The accuracy of the division may be tested by turning the plate into different positions. If in all cases the opposite ends of the needle point to the same number of degrees, the probability is that the circle is correctly divided.

If the compass has a vernier, set the instrument in any direction. Then move the box through any number of degrees, and see whether the needle traverses the same number of degrees as the vernier. If it does in all positions, the arc is properly divided.

278. Adjustments. The levels may be adjusted as directed for the transit and theodolite.

The sights should be perpendicular to the plane of the instrument. To verify this, suspend a long plumb-line: level the plate, and sight to this line. If it appears equally distinct through all parts of the slit, the sight is perpendicular. Turn the instrument half round and test the other sight in the same manner. If either is found incorrect, the maker should rectify it.

279. The compass, as already remarked, is very generally used for surveying purposes, though it is fast giving place to the transit. The latter is furnished with a compass-box, which was not described with the instrument, as it was not needed at that stage of the work. It is in all respects similar to the box attached to the compass itself. The theodolite likewise has a compass. It is, however, so small as to be of very little use in accurate work.

280. The compass is generally supported on an axis inserted in the socket O. This axis terminates in a ball, which works freely but firmly in a socket. This arrangement admits of the axis being placed in any direction. The compass-plate may thus be made level.

Instead of a tripod, many surveyors prefer a single staff pointed with iron. This is called a "Jacob's Staff." Its chief defects are the difficulty of setting in hard ground or among stones, and the want of steadiness in windy weather.

281. Defects of the Compass. Though a very convenient and useful instrument, the compass is deficient in two very important particulars:—its indications are neither correct nor precise.

It is not correct, because, as already remarked, the needle (which is the standard) does not do what it professes: it does not point to the north. This would be of comparatively little importance if its direction were fixed or parallel; but neither of these is the fact. It not only varies from year to year, but from season to season, and even during the same day. These variations will be the subject of a future chapter.

The presence of ferruginous matter in the earth, or the too great proximity of the chain, or of any other piece of iron, may deflect it very seriously from its normal position.

It is not precise. The divisions on the arc are rarely smaller than half-degrees; and if they were finer it would be difficult to read to less than a quarter of a degree. A little calculation will convince one that this is a serious defect where accuracy is desired. An error of 5′ in the bearing would cause a deviation of nearly one foot in ten chains, or about seven feet eight inches in a mile.

SECTION II.

FIELD OPERATIONS.

282. Bearings. To take the bearing of a line, set the compass directly over one end; level it, and turn the plate till the other end of the line—or a rod set up in the direction of the line at a distance as great as is consistent with distinct vision—can be seen through the slits. Then, when the needle has settled, notice the number of degrees to which the end of the needle points, and the cardinal points between which it is situated: the result will be the bearing of the line.

If the north end of the compass is ahead, the north end of the needle should be used, and *vice versâ.*

If you are running with the north end of the compass ahead, and the north point of the needle is between S. and E. and points to $45\frac{1}{2}°$, the bearing is S. $45\frac{1}{2}°$ E.

In reading, the eye should be placed opposite to the other end of the needle; otherwise, owing to the parallax of the point, it will appear to stand at a different point of the arc from what it really does. Any iron about the person will be less likely to affect the needle than when in another position.

283. Use of the Vernier. When the needle does not point to one of the divisions of the arc, it is usual to estimate the fraction. Some surveyors, however, after the needle has come to rest, notice between which divisions the needle points, and then move the compass-box, by turning the milled head I, until the point of the needle is opposite one of the divisions. The amount by which the box is turned, as indicated by the vernier, will give the fraction.

This plan, though theoretically correct, adds really nothing to the correctness of the work. The liability to derangement, from handling the instrument, is so great as to neutralize any advantage it might otherwise possess.

284. Reverse Bearing. The reverse bearing of every line should be taken. To do this, set the compass at the position of the rod, and sight back to the former station. The bearing found should be the reverse of the former. If it is not, the work at the former station should be reviewed; if found correct, the difference between the two must arise from some local cause.

285. Local Attraction. When the back sight does not agree with the forward sight, some cause of derangement exists about one of the stations. This is called local attraction. It is generally caused by ferruginous matter in the earth. It is said that any high object, such as a building or even a tree, will slightly deflect the needle. In situations in which trap rocks abound, the local attraction is often very great. The author has known a variation of more than 10° in a line of two and a half chains long, produced by this cause alone. In such regions, running by the needle is very troublesome, and may cause very serious errors unless great care is taken to allow for the effect produced.

To discover where the attraction exists, select a number of positions in the neighborhood of the suspected points, and note their bearings from these stations, and also from each other. The agreement of several of these will prove their probable correctness. The points thus found to be void of local attraction may be taken as the starting points.

In surveying a farm, a very good way is to note the forward and back sights of every line. If these are found to agree on any line, they may be presumed to be right, and the others corrected accordingly.

286. To correct for back sights.

When the back sight is greater than the fore sight, subtract the difference from the next bearing, if the two lie between the same points of the compass or between points directly opposite, but add it in all other cases. If the back sight is the less, add the difference in the former case, and subtract it in the latter.

Where the local attraction is great, or the line runs nearly in the direction of one of the cardinal points, a difficulty may occur in the application of the preceding rule. A little reflection will enable the surveyor to modify it to suit the case.

287. By the Vernier. It is more convenient in practice to turn the box by the vernier until the reading for the back sight corresponds with the fore sight. The needle will then give the true bearing of the new line as though no attraction existed.

288. To survey a Farm. Commence by going round it, and verifying, so far as can be done, the landmarks, fixing stakes at the corners, so that the assistant may readily find them if he is not already familiar with their position. Then, placing the compass at one corner, send the flag-man ahead to the next corner; note the bearing of his pole; and so proceed with the sides, in succession, taking a back sight at each station.

If the end of the line cannot be seen from the beginning, let the flag-man erect his pole, in the line, at a point as distant from the beginning as possible. Sight to the pole, as before; then, going forward, set the compass by sighting to the last station. The flag-man should now be placed, exactly in line, at another station. So proceed until the end of the line has been reached.

289. Random Line. If the first position of the flag-staff were not exactly in line, the course run will deviate to the right or left of the corner. Where such is the case, measure the perpendicular distance to the corner, and determine the correction by the following rule:—

As the length of the line is to the deviation found as above, so is 57.3 degrees, or 3438 minutes, to the correction in the bearing.*

In running through woods, it is very frequently necessary to correct the bearing in this manner. In all cases, however, where back sights are taken, the compass should be allowed to stand at the last station on the random line, since the local attraction often varies very considerably in a short distance. If it is desired to run the next line precisely on its location, the corner should be sighted to from the end of the random line, and a back sight taken.

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* This rule is founded on the ordinary rule for the solution of right-angled triangles,—the length being the hypothenuse, and the deviation the perpendicular, an arc of 57.3 degrees being equal in length to the radius.

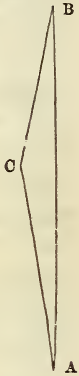
Thus, supposing, in running a line N. 35° 30′ E. 27.53 chains, the corner is found 35 links to the right hand: the calculation would be

$$27.53 : 35 :: 57.3^\circ : 0^\circ\ 43'.$$

The proper bearing would therefore be N. 36° 13′ E.

290. When the far end of the line cannot be seen, it will sometimes be found convenient to run to a station as near the middle of the line as possible, if one can be found from which both ends can be seen. Then, instead of continuing on in the same course, sight to the corner. The chain-men should note the distance to the assumed station. A very obtuse-angled triangle will thus be formed, and the correction in bearing may be readily calculated.

Fig. 117.



Thus, supposing the line were AB, (Fig. 117,) passing over an elevation at C. At A the bearing of AC was found to be N. $43\frac{3}{4}°$ W., distance 10.50 chains. At C, CB was N. 43° W., distance 7.36 chains.

We have AC : BC :: sin. B : sin. A;

or, as the angles are small, AC : BC :: B : A;

whence AC + BC : BC :: B + A : A.

That is, 17.86 : 7.36 :: 45′ : A = 19′, the required correction. The true bearing of AB is therefore N. $43\frac{1}{2}°$ W.

Where the deviation from the correct line is not much greater than in the example given, AB is sensibly equal to AC + CB. Where the deviation is considerable, the angles and side should be calculated by Trigonometry.

The above rule may be expressed thus:—

As the sum of the distances is to the last distance, so is the whole deviation to the correction to be applied at the first station.

291. Proof Bearings. In the course of the survey, bearings or angles should be taken to prominent objects. These form a test of the accuracy of the work. Three bearings are necessary to each object: two of these, being required to fix its position, will afford no check on the intermediate measurements; but their coincidence with a third will determine the probable correctness of all, and of the connecting measurements. Diagonal bearings and distances may likewise be taken as proof lines.

292. Angles of Deflection. In surveying with the transit or theodolite, it is most convenient to record the angles of deflection; that is, the angle by which the new course deviates to the right or to the left from that of the last line. This is always done in surveying roads, rivers, &c. From the angles of deflection the bearings are very readily deduced, by rules to be given hereafter. As checks to the work, the bearings of some of the lines may likewise be taken.

In a closed survey the whole deflection must equal 360°. To determine whether it is so, arrange the deflections to the left in one column, and those to the right in another. Sum the numbers in each column: the difference of these sums should equal 360°.

In practice this will rarely occur; though in open ground, where the angles can readily be taken, the error should not exceed four or five minutes in a tract of ten or twelve sides, provided a good transit or theodolite is employed.

EXAMPLE.

The following are the notes of a survey taken by the author:—1. S. 53° 10′ W.; 2. Deflect 97° 3′ to the right; 3. 97° 45′ to the right; 4. 81° 14′ to the right; 5. 30° 12′ to the left; 6. 12° 14′ to the left; 7. 27° 48′ to the right. Whence the first line deflects 98° 34′ to the right.

Right hand.	Left hand.
97° 3′	30° 12′
97° 45′	12° 14′
81° 14′	42° 26′
27° 48′	
98° 34′	
402° 24′	
42° 26′	
359° 58′,	

differing but two minutes from 360°.

Where the difference amounts to several minutes, it is best to distribute it among the angles.

The rule which is sometimes given: to determine the angles from the bearings, and ascertain whether the sum of the internal angles is equal to twice as many right angles as the figure has sides, less four right angles—proves nothing in regard to the correctness of the field work. Any set of bearings will prove in this way.

SECTION III.

OBSTACLES IN COMPASS SURVEYING.

A.—PROBLEMS IN RUNNING LINES.

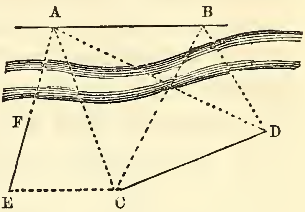
293. MANY of the obstacles that occur in angular surveying have already been alluded to. These, and all others which the operator will meet with, may be overcome by the principles of Trigonometry. As, however, there is frequently a choice in the means to be used, the following methods are given, as being perhaps the most simple:—

294. Problem 1.—*To run a line making a given angle with a given line from a given point within it.*

Place the instrument at the point, and sight along the line. Turn the plate the required number of degrees, and the sights or telescope will be in the required line.

295. Problem 2.—*To run a line making a given angle with a given inaccessible line at a given point in that line.*

Fig. 118.



Let AB (Fig. 118) be the given line, and A the given point. Take two points C and D from which A and some other point B in AB may be seen, and measure CD. Then take the angles ACD, BCD, ADC, and BDC. The distance AC and the angle CAB may be calculated.

Run CE, making ACE = CAB: CE will then be parallel to AB. Now, if we suppose AE to be drawn, we shall have in the triangle ACE all the angles and side AC to find CE. Lay off this distance from C to E, and run the line EF towards A.

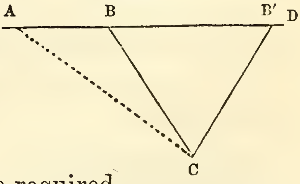
If A cannot be seen from E, calculate CEF, and run the line from E, making the proper angle with CE.

Problem 3.—*From a given point out of a line, to run a line making a given angle with that line.*

296. *Where the line is accessible.*

If the compass is used. Take the bearing of the given line. Then place the compass at the given point, and set it to same bearing. Deflect the compass the number of degrees required, and run the line.

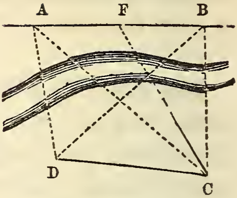
Fig. 119.



If a transit or theodolite is used. Set the instrument at some point A (Fig. 119) in the line, and take the angle BAC. Move the instrument to C, and make the angle ACB = B − A, or = 180° − (B + A), and CB or CB′ will be the line required.

In all cases, unless the line is to be a perpendicular, there will be two lines that will answer the conditions.

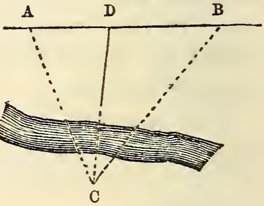
Fig. 120.



297. *If the line is inaccessible.* Let AB (Fig. 120) be the given line, and C the given point. Run any convenient base CD, and take the angles of position of two visible points A and B in the given line. Then, in the triangle ADC, we shall have DC and the angles, to find CA. Similarly, in CBD, find CB. Then, in ACB, we shall have AC, CB, and ACB to find ABC.

Run CF, making BCF = B − F, or 180° − (B + F), and it will make the required angle with AB.

Fig. 121.



298. *If the point be inaccessible.* From any convenient stations A and B (Fig. 121) in the line AB, take the angles of position of the point C, and measure AB. Then, in the triangle ABC, we shall have the angles and the side AB to find BC.

In BCD we then have the angles and side BC to find BD.

BD may be found by a single proportion, thus:—

$$\text{Sin. } ACB \,.\, \text{sin. } BDC : \text{sin. } BAC \,.\, \text{sin. } BCD :: AB : BD.$$

For we have $\quad \text{sin. } ACB : \text{sin. } BAC :: AB : BC,$

and $\quad \text{sin. } BDC : \text{sin. } BCD :: BC : BD.$

Whence (23.6)

$$\text{sin. } ACB \,.\, \text{sin. } BDC : \text{sin. } BAC \,.\, \text{sin. } BCD :: AB : BD.$$

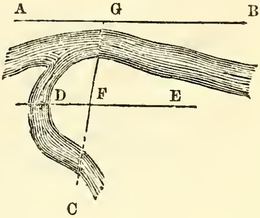
Having found BD, DC may be run towards C; or by the angle, if C be invisible from D.

If C is visible from the point D, the latter may be found by trial, thus:—

Set the instrument at a station as near the proper position as possible, and deflect the given angle. Notice whether the line passes to the right or left of the point, and move the instrument accordingly. A few trials will put it in its proper place.

299. *If the point and the line both be inaccessible.* Take any convenient station D, (Fig. 122,) and run DE parallel to AB, by Art. 302. Then run CFG, making the required angle with ED, by Art. 298; or the distance on the base DC (Fig. 125) may be calculated.

Fig. 122.



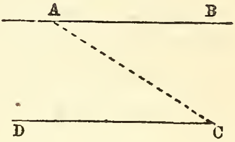
Problem 4.—*To run a line parallel to a given line through a given point.*

300. *If the line be accessible.*

With the compass. Take the bearing of the given line, and through the given point run a line with the same bearing.

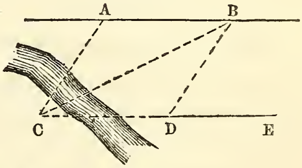
With the transit or theodolite. At any point A (Fig. 123) in the given line take the angle BAC. Remove the instrument to C, and make ACD = BAC. CD will be parallel to AB.

Fig. 123.



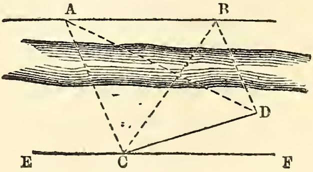
301. *If the point be inaccessible.* At A and B, (Fig. 124,) any two points in the given line, take the angles BAC and ABC. Measure AB, and calculate AC. Make CBD = ACB and BD = AC. Through D run DE in the line CD: it will be the parallel required.

Fig. 124.



302. *If the line be inaccessible.* From C (Fig. 125) run any base-line CD; and at C and D take the angles of position of two visible points A and B in the given line. Calculate the angle

Fig. 125.

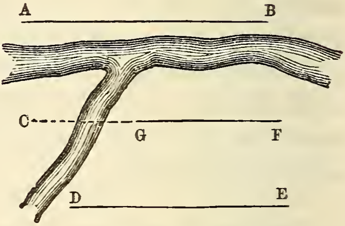


CAB. Run ECF, making ACE = CAB, and EF is the parallel required.

If the line and the point both be inaccessible.

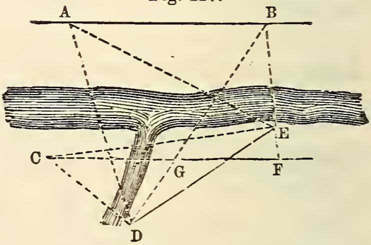
303. *First Method.*—Assume any station D, (Fig 126,) and run a line DE parallel to AB, by Art. 302, and towards C run FG parallel to DE, by Art. 301.

Fig. 126.



304. *Second Method.*—Take any convenient base DE, (Fig. 127,) and take the angles of position of C, A, and B at D and E. Calculate BE, CE, and EBA. Then CFB = 180° − EBA. In CEF, we then have the angles and CE to find EF. Lay off EF the calculated distance, and run the line from F to C.

Fig. 127.



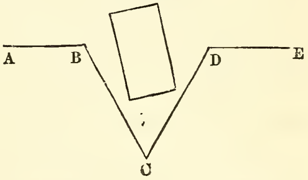
B.—PROBLEMS FOR THE PROLONGATION AND INTERPOLATION OF LINES.

305. In running a line, obstacles are often met with which it requires some ingenuity to overcome, and which will perplex the surveyor unless he has prepared himself by previous study of all cases which are likely to occur. If the total length of a line were all that it was necessary to determine, the two points at its extremity might be connected by a series of triangles, and that length calculated by Trigonometry; but it is generally desirable to have the line marked out so that the exact position of the dividing fence, if one is placed, or of the division if there be no fence, may be indicated by stakes or by marked trees. To do this, the line itself must be traced, or another run in its neighborhood, so related to that in question that the surveyor can at any time pass from the one to the other to set his landmarks. We shall treat of the different kinds of obstructions likely to occur; and, as the prolongation and interpolation of the lines are generally closely connected with the determination of their lengths, the two will be considered together.

Problem 1.—*To prolong a line beyond a building or other obstruction.*

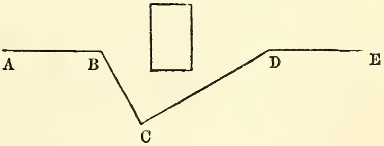
306. *First Method.*—At a point of the line erect a perpendicular of such length as to pass beyond the obstacle. Through the extremity of this run a parallel to the given line: after passing the obstacle, pass back to the required line by an equal perpendicular. The distance will be equal to that of the parallel.

Fig. 128.



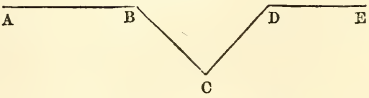
307. *Second Method.*—At B (Fig. 128) deflect 60°, and measure BC. At C deflect 120°, and measure CD = BC. Deflect 60°, and run DE, which will be in line with AB. BD = BC; for BDC is an equilateral triangle.

Fig. 129.



308. *Third Method.*—At B (Fig. 129) deflect 60°, and measure BC. At C deflect 90°, and measure CD = 1.732 times BC. At D deflect 30°, and DE will be in line with AB. BD = 2 BC.

Fig. 130.



309. *Fourth Method.*—At B (Fig. 130) deflect 45°. Measure BC. At C turn 90°, and make CD = BC. At D turn 45°, and DE will be in line. BD = 1.414 BC.

Problem 2.—*To interpolate points in a line.*

310. *If one end be visible from the other.* Set the instrument at one end and sight to the other: an assistant can then be signalled to place stakes directly in line. In crossing a valley, determine a station, as above, on the borders, from which the valley can be seen; and, placing the instrument at this point, sight to a similarly determined station on the other side. Stations may thus be determined down a very considerable declivity. With the transit almost any slope may be sighted down. In this operation, the instrument must be very carefully levelled sideways; otherwise, the points determined in the valley will be out of line.

311. By a Random line. If a wood, or other obstruction, prevents one end of the line, as B, (Fig. 131,) from being seen, run a line AC as nearly in the given course as possible, and drive a stake every five or ten chains, or oftener if desirable. When you have arrived opposite the end of the line, note the distance. Also measure the distance CB to the end. The correction of the bearing may be found as in Art. 289, and the points be interpolated as in Art. 205.

Fig. 131.

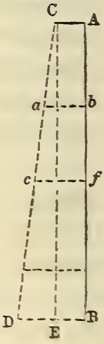
312. *If the line cannot be run from the first station.*

Lay off AC (Fig. 132) as nearly perpendicular to the line as possible, and run the random line CD. On arriving opposite the end, measure DB. Then say,—

As CD is to the difference between BD and AC, so is 57.3°, or 3438′, to the correction of bearing.

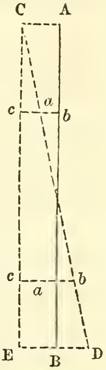
To interpolate points—Say, as CD is to the distance C*a* to any station on the random line, so is the difference between BD and AC to a fourth

Fig. 132.



term. This fourth term added to AC if BD is greater than AC., but subtracted if it be less, will give the correction for the point *a*.

Fig. 133.



If the random line crosses the other, as in Fig. 133, say, As CD is to the sum of AC and BD, so is 57.3°, or 3438′, to the correction of the bearing.

Points may be interpolated by the following rule:—

Say, As CD is to the sum of AC and BD, so is the distance C*a* to any point in the random line to a fourth term. Take the difference between this fourth term and AC.

Then if AC is the greater of the two, lay off the difference on the same side of the random line that A is; but if AC be the less, lay off the remainder on the opposite side.

Where a point in the line at a given distance from the beginning is required, measure that distance on the random line, and determine the offset as above.

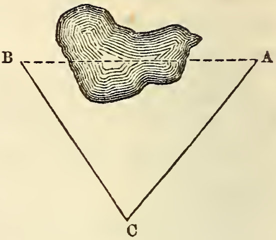
If the random line comes out very distant from the far station, it is better to run another than to depend on that as a basis for interpolation.

C.—PROBLEMS FOR THE MEASUREMENT OF INACCESSIBLE DISTANCES.

313. The various methods of determining the lengths of inaccessible points are merely applications of the rules of Trigonometry, and might, therefore, be applied by the student without further instruction. There is, however, always a choice in the method to be employed: the following are therefore given, that all that is needful in the case may be brought together.

Problem 1.—*To determine the distance between two points which are accessible and visible from each other.*

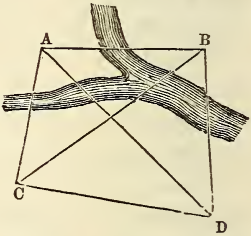
Fig. 134.



314. *First Method.*—Select any station C, (Fig. 134.) Measure BC, and take the angles BAC and ABC. Thence we can calculate AB.

315. *Second Method.*—Measure CA and CB (Fig. 134) and the angle ACB; whence, having two sides and the included angle, AB may be determined.

Fig. 135.



316. *Third Method.*—Where the angles can be taken to the extremities of an inaccessible but known base CD, (Fig. 135,) the distance AB may be calculated thus:—

In ABD we have AD : AB : : sin. ABD : sin. ADB,
and in ABC we have AB : AC : : sin. ACB : sin. ABC.

Whence (23.6) AD : AC : : sin. ABD . sin . ACB : sin. ADB . sin. ABC.

Then, in CAD having the ratio of AC to AD and the angle CAD, we may find the other angles by Art. 141, thus:—

As AD : AC, or sin. ABD . sin. ACB : sin. ADB . sin. ABC : : r : tan. x, and as rad. : tan. ($x \backsim 45°$) : : tan. $\frac{1}{2}$ (ACD + ADC) : tan. $\frac{1}{2}$ (ACD $\backsim$ ADC.)

Having now the angles and one side of ACD, AD is found; whence, in ADB, AB may be determined.

Thus, sin. CAD : sin. ACD : : CD : AD,
and sin. ABD : sin. ADB : : AD : AB.

Whence (23.6) sin. CAD . sin. ABD : sin. ACD . sin. ADB : : CD : AB.

Examples.

To determine the distance AB, accessible at its extremities, I took the angles to the ends of a line CD 10.75 chains long, as follows:—BAC = 100° 35′; BAD, 48° 19′; ABC, 46° 15′; and ABD, 85° 23′. Required the distance AB.

$$\text{ACB} = 180° - (\text{BAC} + \text{ABC}) = 33°\ 10'.$$
$$\text{ADB} = 180° - (\text{BAD} + \text{ABD}) = 46°\ 18'.$$

As AD or	sin. ABD	85° 23′	A. C.	0.001411
	sin. ACB	33° 10′	″ ″	0.261952
: AC or	sin. ADB	46° 18′		9.859119
	sin. ABC	46° 15′		9.858756
:: rad.				10.000000
: tan. x		43° 45′ 46″		9.981238
		45		
	tan. $45° - x$	1° 14′ 14″		8.334392
	tan. $\frac{\text{ACD} + \text{ADC}}{2}$	63° 52′		10.309258
	tan. $\frac{\text{ACD} - \text{ADC}}{2}$	2° 31′ 14″		8.643650
	ACD	66° 23′ 14″		
Then, As	sin. CAD	52° 16′	A. C.	0.101896
	sin. ABD	85° 23′	″ ″	0.001411
:	sin. ACD	66° 23′ 14″		9.962025
	sin. ADB	46° 18′		9.859119
:: CD		10.75 ch.		1.031408
: AB		9.034 ch.		0.955859

Problem 2.—*To determine the distance on a line to the inaccessible but visible extremity.*

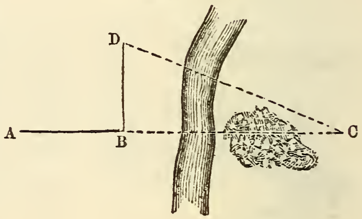
317. This may be done by the methods explained in Arts. 236, 237, and 238, using the transit or theodolite in running the lines, or by the following method:—

318. Run a base line from a point in the line making any angle therewith, and at its extremity take the angle of position of the point. A triangle is thus formed of which the angles and one side are known.

In this operation the triangle should be made as nearly equilateral as possible.

Problem 3.—*To determine the distance when the end is invisible and inaccessible.*

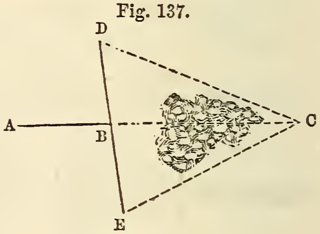
Fig. 136.



319. *First Method.*—Deflect at B (Fig. 136) by any angle, and measure BD to a point from which C is visible. Take BDC. Then calculate BC. The angle C should be made as large as possible.

If AB will not certainly pass through C, operate by the second method.

Fig. 137.

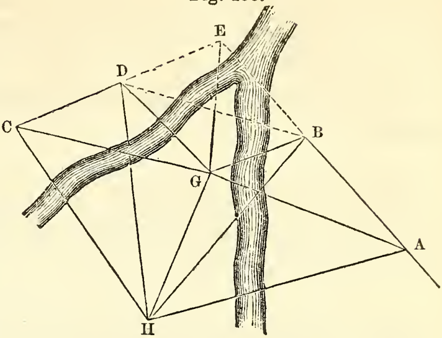


320. *Second Method.*—Run EBD making any angle with AB, (Fig. 137.) Take the angles D and E. In DEC find DC. Then in DCB we have two sides DC and DB and the included angle to find BC and DBC. If DBC is equal to ABE, C is in AB produced.

Problem 4.—*To determine the distance to the intersection of two inaccessible lines.*

321. Let AB and CD (Fig. 138) be the lines, their intersection E being both invisible and inaccessible. It is required to run a line from a given point G, that shall pass through E, and to determine GE.

Fig. 138.

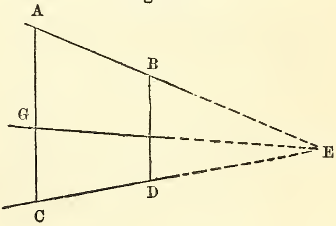


Run any base line GH, and take the angles of position of the points A, B, C, and D on the given lines.

Find GC, CD, and GDC; also GA, GB, and GBA. Then, in GBD, we have GB, GD, and BGD, to find GBD, GDB, and BD. In BDE we then have BD and the angles to find BE. Finally, in GBE we have GB, BE, and the included angle, to find BGE and GE.

If the lines AB and CD were accessible, the line GE might be run by Art. 212, and the distance determined by taking the angles C and G, (Fig. 139.)

Fig. 139.

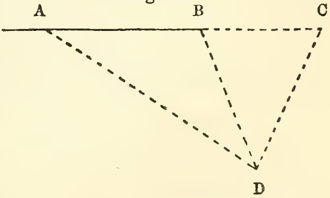


$$\text{Then GE} = \frac{\sin.\ \text{GCE}}{\sin.\ \text{GEC}} \cdot \text{GC}.$$

Problem 5.—*To determine the distance between two inaccessible points.*

322. *First Method.*—Select if possible a point C, in the direction of the line AB, (Fig. 140.) From a station D, take ADB and BDC, and measure DC. Then in CDB we have CD and the angles to find CB, and in CDA we have CD and the angles to find CA.

Fig. 140.



$$\text{AB} = \text{CA} - \text{CB}.$$

323. *Second Method.*—Take a base line CD, (Fig. 135,) which, if possible, should be chosen nearly parallel to AB, and not much shorter than it. From C and D take the angles of position of A and B, whence AB may be calculated.

324. *Third Method.*—If no two points can be found whence A and B can both be seen, the distance can be found as in Prob. 9, p. 114.

325. *Fourth Method.*—If A and B can both be seen from no one station, the distance may be found by Prob. 13, p. 116.

326. *Examples illustrative of the preceding rules.*

Ex. 1. It being necessary to run a parallel to a given inaccessible line AB, so as to pass through a given point C, also inaccessible and probably invisible from any point in the proposed line, I took a base line DE (Fig. 127) of 18 chains, and at D and E determined the following angles of position,—viz.: EDC = 106° 35′; EDA = 72° 5′; EDB = 21° 20′; DEC = 26° 50′; DEA = 61° 20′; and DEB = 120° 45′. Required the distance CG and the angle DGF; also the distance GC to the given station.

Ans. DG 8.48 ch., GC 13.47 ch., and DGF = 124° 8′ 17″.

Ex. 2. One side AB of a tract of land being inaccessible, and it being required to run from a given station C a line which shall make an angle of 67° 35′ with that side, I measured a base line CD of 7 chains, and took the angles CDA = 100° 25′; CDB = 47° 29′; DCA = 32° 17′; and DCB = 90° 3′. Required the angle DCF which the required line makes with DC; also the distance on CF to the line AB, and the distance of the point of intersection from A.

Ans. DCF = 49° 10′ 20″, CF = 7.84, AF = 2.94.

Ex. 3. The line AB not being accessible except at its extremities, which were, however, visible from each other, I took the angles as follow to the points C and D, whose distance I had previously found to be 10.78 chains, and found them to be BAD = 46° 30′; BAC = 81° 43′; ABC = 37° 23′; and ABD = 80° 47′. Required AB.

Ans. AB = 13.76 ch.

Ex. 4. To a given inaccessible line AB it being required to run a perpendicular which shall pass through a point P also inaccessible, I took a base CD of 15 chains, and measured the angles as follow,—viz.: DCP = 105° 30′; DCA = 256° 50′; DCB = 326° 42′; PDC = 38° 50′; PDA = 79° 38′; PDB = 131° 7′. Required the distance on DC from D to the proposed line.

Ans. DF = 14.36.

Ex. 5. One side AB of a tract of land being inaccessible, and it being required to locate the adjoining side AE, which makes with the former an angle BAE of 98° 17′, a base CD of 10 chains was measured. At C, the angle DCA was 95° and DCB = 37° 20′. At D, CDA was 43° 45′, and CDB = 87° 39′. Required the angle between CD and a parallel to AB; also the distance on that parallel to the point E in AE, and the distance AE.

Ans. The parallel makes with CD the angle DCE = 163° 57′, CE = 5.19 ch., and AE = 9.89 ch.

Ex. 6. In running a random line AB N. 87° E. towards a point C, after proceeding 7.50 chains I came to an impassable swamp. I therefore measured on a perpendicular N. 3° W. 4.25 chains, and S. 3° E. 5 chains to the points D and E from which C could be seen. At D, the angle CDE was 66° 39′, and at E, DEC was 67° 25′. Required the distance BC, the true course and distance of AC.

Ans. BC = 10.93 ch.; AC = 18.42 ch.; True course N. 88° 26′ E.

SECTION IV.

FIELD-NOTES.

327. THE field-notes, when the bearings are taken, are recorded in various modes.

First Method.—The simplest method is to write them after each other, as ordinary writing, thus:—

Beginning at a limestone corner of James Brown's land, N. 27¼° E. 7.75 chains, to a marked white-oak. Thence, S. 60½° E. 10.80 chains, to a limestone, &c.

In recording the boundaries, it is well to name the proprietors of the adjoining properties. These are always inserted in deeds of conveyance.

328. *Second Method.*—Rule three columns, as in the adjoining plan: in the first, insert the station; in the second, the bearing; and, in the third, the distance: the margin to the right will serve for the landmarks, adjoining proprietors, &c. The left-hand page of the book may be reserved—as directed in Chain Surveying—for remarks, subsidiary calculations, &c.

Sta.	Bearing.	Distance.	Landmarks, &c.
1	N. 27¼° E.	7.75	to a marked white-oak.
2	S. 62½° E.	10.80	" limestone.
3	S. 80° E.	9.50	" do.
4	S. 47½° E.	9.37	" forked white-oak.
5	S. 54½° W.	8.42	" limestone.
6	N. 37½° W.	23.69	" do. the place of beginning.

329. *Third Method.*—Where there are subsidiary measurements,—such as offsets, intermediate distances, &c.,—the above method is not convenient, as it requires a new table for each line along which such measurements are made. In such cases, the method by columns, with marginal sketches of fences, streams, &c., is perhaps the best. The notation for "False Stations," the crossing of lines, streams, &c., (adopted in Art. 244,) may be employed here. The bearing should be inserted diagonally in the columns, and the bearings of cross fences, proof bearings, with the offsets, should be recorded in the right or left-hand margin, according as the lines or points to which they refer are to the right or left of the line being run.

Sketches of the adjoining fences may likewise be inserted in the margin, with the distances to the intersections. By this combination of the columns and sketches, all the field-work may be recorded concisely, luminously, and accurately.

The following notes of a survey will illustrate the above:—

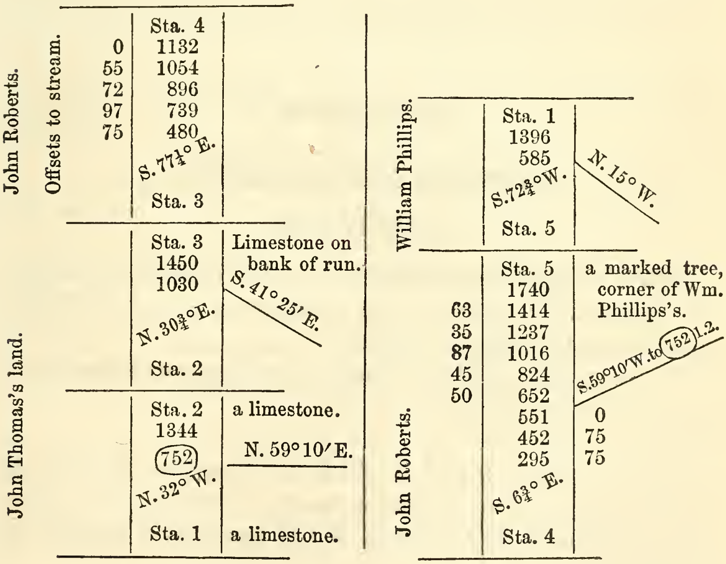
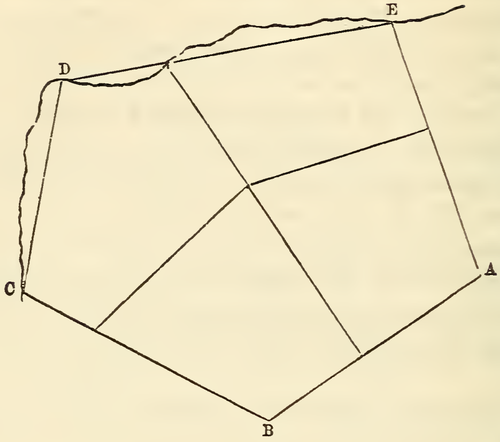


Fig. 141 is a plat of this tract.

Fig. 141.



SECTION V.

LATITUDES AND DEPARTURES.

DEFINITIONS.

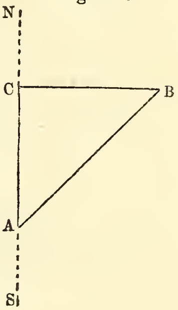
330. THE *difference* of *latitude*—or, as it is concisely called, the *latitude* of a line—is the distance one end is farther north or south than the other.

It is reckoned north or south according as the bearing is northerly or southerly.

331. The *difference* of *longitude* or *departure* of a line is the distance one end is farther east or west than the other, and is reckoned east or west as the bearing is easterly or westerly.

332. Where the course is directly north or south, the *latitude* is equal to the distance, and the departure is zero; but where the bearing is east or west, the latitude is zero, and the *departure* is equal to the distance. In all other cases the latitude and departure will each be less than the distance, the latter being the hypothenuse of a right-angled triangle, of which the others are the legs, and the angle adjacent to the latitude the bearing. Thus, AB (Fig. 142) being the line, AC is the latitude north, and CB the departure east.

Fig. 142.



Strictly speaking, the triangle is a right-angled spherical triangle; but the deviation from a plane is so small as to be absolutely unappreciable except in lines of great length. No notice is, therefore, taken of the rotundity of the earth in "Land Surveying."

333. *The latitude, departure, and distance being the sides of a right-angled triangle, of which the bearing is one of the acute angles, any two of these may be found if the others are known.*

1. Given the bearing and distance, to find latitude and departure.

As radius : cosine of bearing :: distance : latitude;
and as radius : sine of bearing :: distance : departure.

2. Given the latitude and departure, to find the bearing and distance.

As latitude : departure :: radius : tangent of bearing.
As cosine of bearing : radius :: latitude : distance.

3. Given the bearing and departure, to find the distance and latitude.

As sine of bearing : radius :: departure : distance.
As radius : cotangent of bearing :: departure : latitude.

4. Given the bearing and latitude, to find the distance and departure.

As cosine of bearing : radius :: latitude : distance.
As radius : tangent of bearing :: latitude : departure.

5. Given the distance and latitude, to find the bearing and departure.

As distance : latitude :: radius : cosine of bearing.
As radius : sine of bearing :: distance : departure.

6. Given the distance and departure, to find the bearing and latitude.

As distance : departure :: radius : sine of bearing.
As radius : cosine of bearing :: distance : latitude.

EXAMPLES.

Ex. 1. Giving the bearing and distance of a line N. 56¼° W. 37.56 chains, to find the latitude and departure.

Ans. Lat. 20.87 N.; Dep. 31.23 W.

Ex. 2. Given the difference of latitude 36.17 N., and the distance 52.95, to find the bearing and departure, east.

Ans. Bearing = N. 46° 55′ E.; Dep. = 38.67.

Ex. 3. Given the difference of latitude 19.25 N., and the departure 26.45 W., to find the bearing and distance.

Ans. Bearing = N. 53° 57′ W.; dist. = 32.71.

Ex. 4. Given the bearing S. 33½° W., and the departure 18.33 chains, to find the distance and difference of latitude.

Ans. Dist. = 33.21 ch.; Lat. = 27.69 S.

334. Traverse Table. The traverse table contains the latitudes and departures for every quarter degree of the quadrant to all distances up to ten. From these, the latitude and departure, corresponding to any bearing and distance, may readily be found by the following rule:—

If the distance be not greater than ten.—Seek the degrees at the *top* or *bottom* of the table according as their number is *less* or *greater* than 45°, and in the columns marked Latitude and Departure, opposite to the distance, will be found the latitude and departure. If the degrees are found at the bottom of the table, the name of the column is there likewise. For all degrees less than forty five, the left-hand column is the latitude, but the departure, for those greater than 45°.

If the distance be more than ten, and consist of whole tens.—Take out the number from the table as before, and remove the decimal point as many places to the right as there are ciphers at the right of the distance in the table.

If the distance is not composed simply of tens.—Take from the table the latitude and departure corresponding to every figure, removing the decimal point as many places to the right or to the left as the digit is removed to the left or the right of the unit's place, and take the sum of the results.

EXAMPLES.

Ex. 1. Required the latitude and departure of a line bearing N. $37\frac{1}{4}$° E. 8 chains.

Opposite to 8 chains, under the degrees $37\frac{1}{4}$, are found,—

Lat. 6.3680, Dep. 4.8424.

The latitude and departure required are, therefore,

6.37 N., 4.84 E.

If the distance had been 80 chains, the latitude and departure would have been

63.68 N., 48.42 E.

Ex. 2. Required the latitude and departure of a line running S. $63\frac{1}{2}$° E. 75 chains.

70 ch.	Lat. 31.234	Dep. 62.465
5 "	2.231	4.475
	33.465	66.940
Hence the result is	Lat. 33.46 S.;	Dep. 66.94 E.

Ex. 3. Required the latitude and departure of a line running N. $35\frac{3}{4}$° W. 58.65 chains.

50 ch.	Lat. 40.579	Dep. 29.212
8 "	6.493	4.674
.6	487	351
.05	41	29
	Lat. 47.600 N.	Dep. 34.266 W.

Ex. 4. What are the latitude and departure of a line bearing S. $63\frac{1}{2}°$ W. 27.49 chains?

Ans. Lat. 12.27 S.; Dep. 24.60 W.

Ex. 5. What are the latitude and departure of a line N. $55\frac{3}{4}°$ E. 27 chains? Ans. Lat. 15.20 N.; Dep. 22.32 E.

Ex. 6. What are the latitude and departure of a line bearing N. $84\frac{3}{4}°$ E. 123.56 chains?

Ans. Lat. 11.31 N.; Dep. 123.04 E.

Ex. 7. What are the latitude and departure, the bearing and distance being S. $24\frac{3}{4}°$ W. 97.56 chains?

Ans. Lat. 88.60 S.; Dep. 40.84 W.

335. *When the bearing is given to minutes.* Take out the numbers in the table for the quarter degrees between which the minutes fall. Then say,—

As 15 minutes is to the excess of the given number of minutes above the less of the two quarters, so is the difference of the numbers in the table to a fourth term, which must be subtracted from the number corresponding to the less of the two quarters if the quantity is a latitude, but added if it is a departure.

Thus, supposing the line were N. 41° 18′ E. 43.27 chains. Take the difference between the latitude for $41\frac{1}{4}°$ and that for $41\frac{1}{2}°$, and say,—

As 15′ is to the difference between $41\frac{1}{4}°$ and 41° 18′, or 3′, so is the difference between the latitudes to the correction for 3′. This correction subtracted from the latitude for $41\frac{1}{4}°$ will give the latitude required.

Do the same with the departure, except that the correction found as above must be added to the departure for $41\frac{1}{4}°$.

In the example, we have for the distance 40 in the column for

$41\frac{1}{4}°$	the Lat.	30.074	Dep. 26.374
$41\frac{1}{2}°$		29.958	26.505
	Differences	.116	.131

Then, As 15′ : 3′ : : .116 : .023, correction of latitude;
and, As 15′ : 3′ : : .131 : .026, correction of departure.

The corrected latitude and departure for 41° 18′, distance 40 chains, are Lat. 30.051., Dep. 26.400.

In like manner, the latitudes and departures for each of the remaining figures may be calculated, being as below:—

For	40 ch.	Lat. 30.051	Dep. 26.400
	3 "	2.254	1.980
	.2	150	132
	.07	53	46
		32.508 N.	28.558 E.

There will rarely be any calculation necessary for the decimal figures of the distance, as the variation caused by a quarter of a degree will seldom change more than a unit any of the figures that need be retained.

Ex. 1. The bearing and distance being N. 76° 42′ E. 39.76 chains, to find the difference of latitude and departure.

Ans. Lat. 9.147 N.; Dep. 38.694 E.

Ex. 2. Given the bearing and distance S. 37° 9′ E. 63.45 chains, to find the difference of latitude and departure.

Ans. Lat. 50.572 S.; Dep. 38.317 E.

Ex. 3. Required the difference of latitude and departure of a line running S. 29° 17′ E. 123.75 chains.

Ans. Lat. 107.937 S.; Dep. 60.529 E.

336. *By Table of Natural Sines and Cosines.* The difference of latitude and departure, when the bearing is given to minutes, is more readily found from the table of natural sines and cosines than from the traverse table. The difference of latitude and departure are the cosine and the sine of the bearing to a radius equal to the distance. Therefore, to find the difference of latitude and departure of a line, take out the natural cosine and sine of the bearing, and multiply them by the distance.

Ex. 1. Required the difference of latitude and departure of a line bearing N. 41° 18′ E. 43.27 chains.

41° 18′	Cosine .75126	Sine 66000
Dist.	Diff. Lat.	Dep.
40 ch.	30.0504	26.4000
3 "	2.2538	1.9800
.2	1503	1320
.07	526	462
	Lat. 32.5071 N.	Dep. 28.5582 E.

The result by this method may be depended on to the third decimal figure, unless the distance is several hundred chains, and then it will rarely affect the second decimal figure.

Ex. 2. Required the latitude and departure of a line N. 29° 38′ E. 26.47 chains.

29° 38′	Cosine .86921	Sine .49445
20 ch.	17.3842	9.8890
6 "	5.2153	2.9667
.4	.3477	1978
.07	608	346
	Lat. 23.0080 N.	Dep. 13.0881 E.

The calculation need not, in general, be carried beyond the third decimal place. In the above example the work would then stand thus:

29° 38′	Cosine .86921	Sine .49445
20 ch.	17.384	9.889
6 "	5.215	2.967
.4	348	198
.07	61	34
	Lat. 23.008 N.	Dep. 13.088 E.

Ex. 3. Required the latitude and departure of a line bearing S. 56° 7′ E. 63.48 chains.

Ans. Lat. 35.39 S.; Dep. 52.70 E.

Ex. 4. Required the latitude and departure of a line bearing N. 52° 49′ W. 136.75 chains.

Ans. Lat. 82.65 N.; Dep. 108.95 W.

Ex. 5. Given the bearing and distance S. 23° 47′ W. 13.62 chains, to find the latitude and departure.

Ans. Lat. 12.46 S.; Dep. 5.49 W.

337. Test of the Accuracy of the Survey. When the surveyor has gone round a tract, and has come back to the point from which he started, it is self-evident that he has travelled as far in a southerly direction as he has in a northerly, and as far easterly as westerly.

His whole northing must equal his whole southing, and his whole easting equal his whole westing. If then the north latitudes are placed in one column and the south latitudes in another, the sum of the numbers in these columns will be equal, provided the bearings and distances are correct. So also the columns of departures will balance each other.

Owing to the unavoidable errors in taking the measurements, and also to the fact that the bearings are generally taken to quarter degrees, this exact balancing rarely occurs in practice. When the sums are nearly equal, we may attribute the error to the want of precision in the instruments; but, if the error is considerable, a new survey should be made.

It not unfrequently happens that the mistake has been made on a single side. This can often be detected by taking the errors of latitude and departure, and calculating or estimating the bearing of a line which should produce such an error by a mismeasurement of its length or a mistake in its bearing. A little ingenuity will then frequently enable the surveyor to judge of the probable position of the error, and thus obviate the necessity of a complete resurvey of the tract.

It is laid down as a rule by some good surveyors that an error of one link for every five chains in the whole distance is the most that is allowable. When the transit or theodolite is used, a much closer limit should be drawn. One link for ten or fifteen chains is quite enough, unless the ground is very difficult. Every surveyor will, however, form a rule for himself, dependent on his experience of the precision to which he usually obtains. A young surveyor should set a high standard of excellence, as he will find this to be a very good method of making himself accurate. If he begins by being satisfied with poor results, the chances are that he will never attain to a high rank in his profession.

338. Correction of Latitudes and Departures.

When the northings and southings, or the eastings and westings, do not balance, the error should be distributed among the sides before making any calculations dependent upon them.

The usual mode of distributing the error is to apply to each line a portion proportioned to its length.

Rule a table, and head the columns as in the adjoining example. Take the latitudes and departures of the several sides, and place them in their proper columns.

Take the difference between the sum of the northings and that of the southings. The result is the error in latitude, and should be marked with the name of the less sum.

Do the same with the eastings and westings: the result is the error in departure, of the same name as the less sum.

Divide the error of latitude by the sum of the distances: the quotient is the correction for 1 chain.

Multiply the correction for 1 chain by the number of chains in the several sides: the products will be the corrections for those sides, which may be set down in a column prepared for the purpose, or at once applied to the latitude.

Operate the same way with the error in departure, to obtain the corrections of departure of the several sides.

The corrections are of the same name as the errors.

The corrections above found are to be applied by adding them when of the same name, but subtracting if of different names.

If one side of a tract is hilly, or otherwise difficult to measure, a larger share of the error should be attributed to that side.

When a change of bearing of a long side will lessen the error, this change should be made, especially if the survey was made with a compass.

The corrections may be made in the original columns by using red ink. New columns are, however, to be preferred.

Ex. 1. Given the bearing and distances as follows, to find the corrected latitudes and departures.

1	N. $43\frac{1}{2}°$ W.	28.43
2	N. $29\frac{3}{4}°$ E.	30.55
3	S. 80° E.	28.74
4	East.	40.00
5	S. $10\frac{1}{4}°$ E.	23.70
6	S. 64° W.	25.18
7	N. $63\frac{3}{4}°$ W.	20.82
8	S. $57\frac{1}{2}°$ W.	31.65

	Bearings.	Dist.	N.	S.	E.	W.	Cor. N.	Cor. W.	N.	S.	E.	W.
1	N.$43\frac{1}{2}°$W.	28.43	20.62			19.57		1	20.62			19.58
2	N. $29\frac{3}{4}°$E.	30.55	26.52		15.16			2	26.52		15.14	
3	S. 80° E.	28.74		4.99	28.30			2		4.99	28.28	
4	East.	40.00			40.00		1	2	.01		39.98	
5	S.$10\frac{1}{4}°$E.	23.70		23.32	4.22			1		23.32	4.21	
6	S. 64° W.	25.18		11.04		22.63		1		11.04		22.64
7	N.$63\frac{3}{4}°$W.	20.82	9.21			18.67		1	9.21			18.68
8	S.$57\frac{1}{2}°$W.	31.65		17.01		26.69		2		17.01		26.71
		229.07	56.35	56.36	87.68	87.56						
				56.35	87.56							
			Er. N.	.1	.12	Er. W.						

Ex. 2. Correct the latitudes and departures from the following notes:—1. S. 49° W. 12.93 ch.; 2. S. 88° W. 13.68 ch.; 3. N. $25\frac{1}{4}°$ W. 14.09 ch.; 4. N. $43\frac{1}{4}°$ E. 14.70 ch.; 5. N. $12\frac{1}{2}°$ W. 17.95 ch.; 6. N. $88\frac{3}{4}°$ E. 17.68 ch.; 7. S. $36\frac{1}{2}°$ E. 35.80 ch.; 8. S. $77\frac{1}{4}°$ W. 16.15 ch.

Ans. 1. S. 8.48, W. 9.76; 2. S. .48, W. 13.67; 3. N. 12.73, W. 6.01; 4. N. 10.70, E. 10.07; 5. N. 17.51, W. 3.88; 6. N. 38, E. 17.69; 7. S. 28.79, E. 21.30; 8. S. 3.57, W. 15.74.

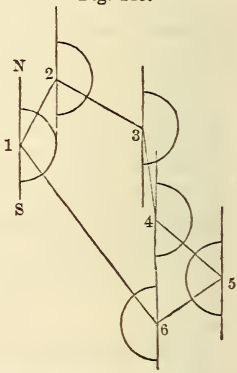
SECTION VI.

PLATTING THE SURVEY.

339. With the Protractor. *First Method.*—DRAW a line NS, on any convenient part of the paper, to represent the meridian.

Place the protractor with its straight edge to this line, and its arc turned to the right if the bearing be easterly, but to the left if it be westerly, and with a fine point mark off the number of degrees. Draw a straight line from the centre to this point, and on it lay off the distance. The point 2 (Fig. 143) will thus be determined. Through 2 draw a line parallel to N S. Place the protractor with its centre at 2 and its straight side coincident with the meridian, and prick off the degrees in the bearing of the second side. Join this point to 2, and on the line thus determined lay off 2.3 equal to the second side. Through 3 draw another meridian; and so proceed until all the bearings and distances have been laid down.

Fig. 143.



When the last line has been platted, it should end at the starting point: if it does not, either the notes are incorrect or an error has been made in the platting

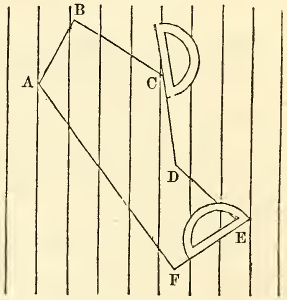
The proper position of the protractor after the first may be determined without drawing meridians, by placing the centre at the point and turning the protractor until the number of degrees in the bearing of the last line coincides with that line. Its position is then parallel to the former one, and the bearing of the next line may be pricked off.

This method is the one commonly employed. It has, however, the disadvantage of accumulating errors, since any mistake in laying down the bearing of one line will alter both the direction and position of every subsequent line on the plat.

The figure is the plat from the following field-notes:— 1. N. $27\frac{1}{2}°$ E. 7.75; 2. S. $60\frac{1}{2}°$ E. 10.80; 3. S. 8° E. 9.50; 4. S. $47\frac{1}{2}°$ E. 9.37; 5. S. $54\frac{1}{2}°$ W. 8.42; 6. N. $37\frac{1}{2}°$ W. 23.69.

340. *Second Method.*—Draw a number of parallel lines to represent meridians. They may be equidistant or not. The faint lines on ruled paper will answer very well.

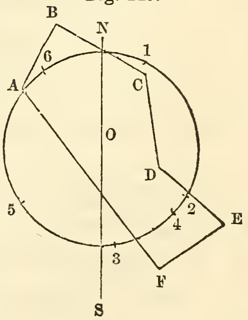
Fig. 144.



Select any convenient point for a place of beginning, and draw the line AB (Fig. 144) for the first side. Place the protractor so that its centre shall be on one of the meridians, and turn it until the number of degrees in the next side coincides with the same meridian, as at C: slip it down the line, maintaining the coincidence of the centre and degree mark with the meridian, until the straight side passes through the point Draw a line along this side. It will be the direction of the required line, on which lay off the given distance. So continue until all the sides have been platted. The figure will close, if the work is properly done.

This method is quite as accurate as the last, and admits of very rapid execution.

Fig. 145.

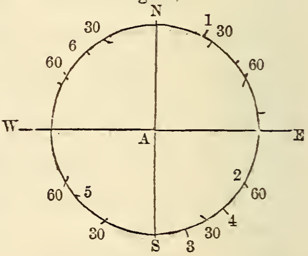


341. By a Scale of Chords. With a radius equal to the chord of 60° describe a circle near the middle of the paper. Through its centre O (Fig. 145) draw a line NS to represent the meridian. Lay off from the north and south points the different bearings, marking them 1, 2, &c. Through A, any convenient point, draw AB parallel to 0.1, and on it lay off AB equal to the length of the first side taken from any convenient scale. Through B draw BC parallel to 0.2: on it lay off BC equal to the second side. Through C draw CD parallel to 0.3; and so proceed till all the lines have been platted.

With an accurate scale of chords of a good size, this method is probably preferable to either of the others. The scale on the rule sold with cases of instruments, however, is so small that no great precision can be obtained by its use. It is still, however, preferable to the other methods if the protractor in similar cases of instruments is employed.

342. By a Table of Natural Sines. The sine of any arc is equal to half the chord of twice that arc, or to the chord of twice the number of degrees on a circle of half the radius. We may therefore use a table of natural sines to lay off angles. Its use in protracting a survey is explained below.

Fig. 146.



Describe a circle (Fig. 146) about the centre of the paper with a radius equal to 5 on a scale of equal parts. This scale should be taken as large as convenient. Through its centre A draw NS to represent the meridian, and cross the circle at the points marked 60°, with the centres N and S, and radius equal to that of the circle: also draw EW perpendicular to NS. The points marked 30° may be obtained by crossing the circle with the compasses opened to the radius and one leg at E and W.

A skeleton protractor is thus formed, having the North, South, East, and West points, as well as the 30° and 60° points, accurately laid down.

Commencing with the first bearing, which in the figure is N. $27\frac{1}{2}$ E., divide it by 2, and from the table of natural sines take out the sine of the quotient 13° 45′. It is found to be 2.3769, the decimal point being removed 1 place to the right. Take this distance 2.38 from the scale of equal parts, and lay it off from N to 1.

The second bearing is S. $60\frac{1}{2}°$ E. The half of $\frac{1}{2}°$ is 15′: the sine of this is 0.0436. Lay off .04 from 60° to 2.

The third bearing is S. 8° E.: the sine of 4° is 0.6976. Lay off .70 from S. towards E.: the point 3 is thus determined.

The fourth is S. $47\frac{1}{2}°$ E., which exceeds 30° by $17\frac{1}{2}°$: the half of $17\frac{1}{2}°$ is 8° 45′, of which the sine is 1.5212. 1.52 laid off from 30 towards E. determines the point 4.

An accurate protractor is thus formed on the paper, containing all the bearings in the field-notes. The subsequent work will be as in last article.

343. By a Table of Chords. Instead of a table of natural sines, a table of chords, when it can be procured, is more convenient.

Prepare a circle, as in last article, with the N., S., E., W., and the 30° and 60° points, the radius being 10, taken from a scale of equal parts.

Take from the table the chord of the number of degrees, or of its excess above 30° or 60°, and lay it off from the proper point, as directed in last article: an accurate protractor is thus formed on the paper, and the work proceeds as before.

The object in determining the 30° and 60° points is to avoid the necessity of laying off long distances. When the compasses are much stretched, the points strike the paper very obliquely, and are apt to sink in so as to make the distance laid off slightly too short.

This method is preferable to any of those which precede it: it is only to be excelled by the one next given.

344. By Latitudes and Departures.

Where the latitudes have been calculated and balanced, they afford the most convenient and accurate means of platting the survey.

Rule five columns, heading them Sta., N., S., E., W. Commencing at any convenient station, place the latitude and departure of the side beginning at this station opposite the next station in the table, and in their appropriate columns. When the latitude set down is of the same name as that of the next side, add them together, and place the result in the proper column of latitudes opposite the next side. But if they be of different names, take their difference, and place it in the column of the same name as the greater. Proceed in the same way with this result and the next latitude, and so continue till all the latitudes have been used. The results will be the latitude of the stations opposite which they are placed, all counted from the point at which we commenced.

Proceed in the same manner with the departures. Thus, if it were required to plat the survey of which the field-notes are given Ex. 1, Art. 338, we have the latitudes and departures, as in the following table. (See the example referred to):—

Sta.	N.	S.	E.	W.
1	20.62			19.58
2	26.52		15.14	
3		4.99	28.28	
4	.01		39.98	
5		23.32	4.21	
6		11.04		22.64
7	9.21			18.68
8		17.01		26.71

Preparing a table as above directed, and beginning at the fourth station, the total latitudes and departures will be as below:—

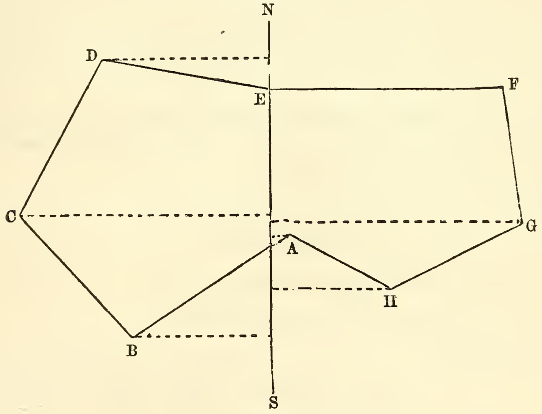
Sta.	N.	S.	E.	W.
1		42.15		23.84
2		21.53		43.42
3	4.99			28.28
4	00			0.00
5	.01		39.98	
6		23.31	44.19	
7		34.35	21.55	
8		25.14	2.87	

The latitude of the fourth side is .01 N. This is put in the column headed north, opposite the fifth station. The next latitude being south, take the difference 23.31; place it in the south: add 23.31 and 11.04, both being south, and we have 34.35 S. Subtract from this 9.21 N. leaves 25.14 S. This, added to 17.01 S., gives 42.15 S. Subtract 20.62 N. leaves 21.53 S.; 21.53 S. from 26.52 N., the next latitude, leaves 4.99 N. Finally, 4.99 N. and 4.99 S. cancel, leaving 0 for the latitude of the fourth station. In the same manner we find the total departures.

As the latitude and departure of the station with which we begin are zero, the work proves itself. It is usual to begin with the first side.

The table having been prepared as above, draw on any convenient part of the paper a meridian line, NS, (Fig. 147,) and take any point E for the starting point. From this

Fig. 147.



point, lay off the several total latitudes contained in the table *above* or *below* the point as the latitude is north or south, and number them according to the station to which they are opposite in the table.

Through these points draw perpendiculars to the meridian, and make them equal to the several total departures,—laying the distance to the right hand if the departure be east, but to the left if it be west. The corners will thus be determined. When these are joined, the plat will be completed.

SECTION VII.

PROBLEMS IN COMPASS SURVEYING.

345. Problem 1.—GIVEN the bearing of one side, and the deflection of the next, to determine its bearing.

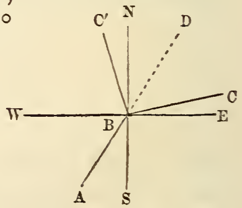
If the given bearing is *northeasterly* or *southwesterly*, *add* the deflection if it is to the *right hand*. If the sum exceeds 90°, take its supplement, and change north to south, or south to north.

If the deflection is to the *left hand*, *subtract* it from the bearing; but if it is greater than the bearing from which it is to be subtracted, take the difference, and change east to west, or west to east.

When the given bearing is *northwesterly* or *southeasterly*, *add* the *left-hand* and *subtract* the *right-hand* deflections, applying the same rules as above.

EXAMPLES.

Fig. 148.



Ex. 1. Given AB (Fig. 148) N. 37° E., and the deflection of the next side 43° 15′ to the right.

	BD =	N. 37° E.
	DBC =	43° 15′
Whence		BC is N. 80° 15′ E.

Ex. 2. Given AB N. 37° E., and the deflection of BC′ 43° 15′ to the left.

	BD =	N. 37° E.
	DBC′ =	43° 15′
Whence		BC′ is N. 6° 15′ W.

Ex. 3. Given the bearing of AB, N. 39° W., and BC deflects to the left 75° 26′: required the bearing of BC.

Ans. S. 65° 34′ W.

Ex. 4. Given the bearing of a line S. 63° 29′ E., and the deflection of the next 29° 17′ to the right: required its bearing.

Ans. S. 34° 12′ E.

Ex. 5. The bearing of one line being S. 34° 12′ E., and the deflection of the next 75° 32′ to the right: required its bearing.

Ans. S. 41° 20′ W.

346. Problem 2. — *To determine the angle of deflection between two courses.*

1. If the lines run between the same points of the compass, take the difference of their bearings.

2. If they run between points directly opposite, subtract the difference of the bearings from 180°.

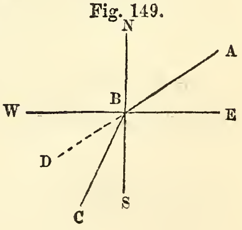
3. If they run from the same point towards different points, add the bearings.

4. If they run from different points towards the same point, take the sum of the bearings from 180°.

EXAMPLES.

Ex. 1. AB (Fig. 149) runs S. 56° W., and BC S. 25° W.: required the deflection.

Fig. 149.

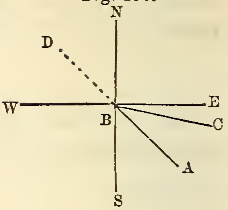


56°

25°

Deflection 31° to the left.

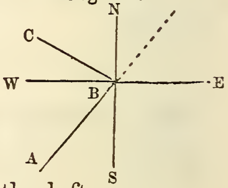
Fig. 150.



Ex. 2. Given AB (Fig. 150) N. 46 W., and BC S. 79° E.: required the deflection.

AB	N. 46° W.
BC	S. 79° E.
ABC	33°
	180°
DBC	147° = deflection to the right.

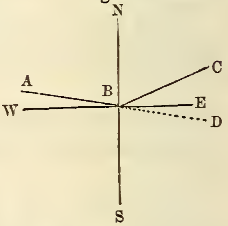
Fig. 151.



Ex. 3. Given AB (Fig. 151) N. 39° E., and BC N. 63° W., to find the deflection.

AB	N. 39° E.
BC	N. 63° W.
DBC	102° = deflection to the left.

Fig. 152.



Ex. 4. Given AB (Fig. 152) S. 82° E., and BC N. 67° E., to find the deflection.

AB	S. 82° E.
BC	N. 67° E.
	149°
	180°
DBC	31° = deflection to the left.

Ex. 5. The bearing of a line is N. 46° 30′ E., and that of the next S. 63° 29′ W.: required the deflection.

Ans. 163° 1′ to the left.

Ex. 6. What is the deflection in passing from a course S. 63° W. to one N. 29° W.?

Ans. 88° to the right.

Ex. 7. What is the deflection in passing from a course N. $82\frac{1}{2}$° W. to one N. $29\frac{1}{4}$° W.?

Ans. $53\frac{1}{4}$° to the right.

347. Angle between lines. If the angle between two lines is required, reverse the first bearing, and apply the above rules.

EXAMPLES.

Ex. 1. Given AB N. 87° E., and BC S. 25° W., to find the angle ABC. Ans. ABC = 62°.

Ex. 2. Given AB S. 63° E., and BC N. 56° E.: required the angle ABC. Ans. ABC = 119°.

Ex. 3. Given CD N. 15° W., and DE N. 56° W.: required the angle CDE. Ans. CDE = 139°.

Problem 3.—*To change the bearings of the sides of a survey.*

348. It is frequently useful to change the bearings of a survey so as to determine what they would be if one side were made a meridian. This change is made on the supposition that the whole plat is turned around without altering the relative positions of the sides. Every bearing will thus be altered by the same angle. The following rules take in all the possible cases.

The reason of these rules will be made apparent by drawing a figure to represent any particular case.

1. Deduct the bearing of the side that is to be made a meridian from all those bearings that are between the same points as it is, and also from those that are between points directly opposite to them. If it is greater than any of those bearings, take the difference, and change west to east, or east to west.

2. Add the bearing of the side that is to be made a meridian to those bearings that are neither between the same points as it is, nor between points directly opposite. If either of the sums exceeds 90°, take the supplement, and change south to north, or north to south.

EXAMPLES.

Ex. 1. The bearings of a tract of land are,—1. N. 57° E.;

2. N. 89° E.; 3. S. 49½° E.; 4. South; 5. S. 27¾° W.; 6. S. 53½° W.; 7. N. 89° W.; 8. N. 37° W.; 9. N. 43° E. to the place of beginning. Required to change the bearings, so that the ninth side may be a meridian.

1. N. 57° E.	2. N. 89° E.	3. S. 49½° E.
N. 43° E.	N. 43° E.	N. 43° E.
N. 14° E.	N. 46° E.	92½°
		180°
		N. 87½° E.
4. S. 0° W.	5. S. 27¾° W.	6. S. 53½° W.
N. 43° E.	N. 43° E.	N. 43 ° E.
S. 43° E.	S. 15¼° E.	S. 10½° W.
7. N. 89° W.	8. N. 37° W.	9. North.
N. 43° E.	N. 43° E.	
132°	N. 80° W.	
180°		
S. 48° W.		

Ex. 2. Change the bearings in the following notes, so that the second side may be a meridian:—1. N. 43° 25′ W.; 2. N. 29° 48′ E.; 3. S. 80° E.; 4. N. 89° 55′ E.; 5. S. 10° 13′ E.; 6. N. 63° 55′ W.; 7. S. 63° 45′ W.; 8. N. 57° 35′ W.

Ans. 1. N. 73° 13′ W.; 2. North; 3. N. 70° 12′ E.; 4. N. 60° 7′ E.; 5. S. 40° 1′ E.; 6. S. 86° 17′ W.; 7. S. 33° 57′ W.; 8. N. 87° 23′ W.

Ex. 3. Change the bearings in the following notes, so that the fourth side may be a meridian:—1. S. 63° E.; 2. S. 47° E.; 3. S. 59¼° W.; 4. N. 84½° W.; 5. N. 12° W.; 6. N. 17½° E., and 7. S. 29¾° W.

Ans. S. 21½° W.; 2. S. 37½° W.; 3. N. 36¼° W.; 4. North; 5. N. 72½° E.; 6. S. 78° E.; 7. N. 65¾° W.

SECTION VIII.

SUPPPLYING OMISSIONS.

349. When any two of the dimensions have been omitted to be taken, or have become obliterated from the field-notes, these may be supplied. This should never lead the surveyor to neglect to take every bearing and every distance. It is far better to use almost any means, however indirect, to obtain all the bearings and distances independently of one another than to determine any one from the rest. If one side is determined from the others, all the errors committed in the measurements are accumulated on that side, and thus the means of proving the work by the balancing of the latitudes and departures is lost. The various problems in Section 3 will enable the young surveyor to solve almost every case of difficulty that will be likely to occur in making his measurements. Should any difficulty arise to which none of the methods there developed are applicable, a knowledge of the principles of Trigonometry will afford him the means of overcoming it.

CASE 1.

350. The bearings and distances of all the sides except one, being given, to determine these.

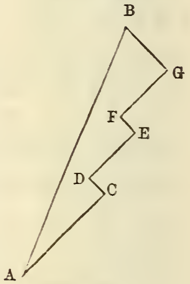
Determine the latitudes and departures of those sides of which the bearings and distances are given. Take the difference between the sums of the northings and southings, and also between the sums of the eastings and westings: the remainders will be the latitude and departure of the side the bearing and distance of which are unknown. With this latitude and departure calculate the bearing and distance by Art. 333.

This principle will enable us to determine a side when it cannot be directly measured. Thus, run a series of courses and distances, so as to join the two points to be connected.

These, with the unknown side, form a closed tract, the sides of which are all known except one.

It will likewise enable us to determine the course and distance of a straight road between two points already connected by a crooked one. In both these cases it is best, where the nature of the ground will admit of it, to run the courses at right angles to each other, as in Fig. 153, in which AB is the distance to be determined. Run AC any direction, CD perpendicular to AB, DE to CD, EF to DE, FG to EF, and, finally, GB perpendicular to FG through B.

Fig. 153.



Then, assuming AC as a meridian, AC + DE + FG will be the latitude of AB and CD + CF + GB the departure. From these calculate the distance AB and the bearing BAC. This angle applied to the true bearing of AC will give that of AB.

EXAMPLES.

Ex. 1. The bearings and distances of the sides of a tract of land being as follows, it is desired to find the bearing and distance of the third side,—viz.: 1. N. $56\frac{1}{4}°$ W. 15.35 chains; 2. N. 9° W. 19.51 ch.; 3. Unknown; 4. S. $39\frac{3}{4}°$ E. 13.35 ch.; 5. N. $82\frac{1}{2}°$ E. 12.65 ch.; 6. S. $6\frac{3}{4}°$ W. 12.18 ch.; 7. S. $52\frac{1}{2}°$ W. 20.95 ch.

Sta.	Bearing.	Distance.	N.	S.	E.	W.
1	N. $56\frac{1}{4}°$ W.	15.35	8.53			12.76
2	N. 9° W.	19.51	19.27			3.05
3						
4	S. $39\frac{3}{4}°$ E.	13.35		10.26	8.54	
5	N. $82\frac{1}{2}°$ E.	12.65	1.65		12.54	
6	S. $6\frac{3}{4}°$ W.	12.18		12.10		1.43
7	S. $52\frac{1}{2}°$ W.	20.95		12.75		16.62
			29.45	35.11	21.08	33.86
				29.45		21.08
				5.66 N.		12.78 E.

Diff. Lat.	5.66	log.	0.752816
Departure,	12.78	log.	1.106531
Bearing,	N. 66° 7′ E.	tang.	10.353715
Bearing,	66° 7′	cos.	9.607322
Diff. Lat.		log.	0.752816
Distance,	13.98		1.145494

Ex. 2. One side AB of a tract of land running through a swamp, it was impossible to take the bearing and distance directly. I therefore took the following bearings and distances on the fast land,—viz.: AC, N. 47° W. 16.55 chains; CD, N. 19° 5′ E. 11.48 ch.; DE, N. 11° 5′ W. 15.53 ch.; EF, N. 23° E. 9.72 ch., and FB, N. 75° 12′ E. 14.00 chains. Required the bearing and distance of AB.

Sta.	Bearing.	Distance.	N.	S.	E.	W.
A	N. 47° W.	16.55	11.29			12.10
C	N. 19° 5′ E.	11.48	10.85		3.75	
D	N. 11° 5′ W.	15.53	15.24			2.99
E	N. 23° E.	9.72	8.95		3.80	
F	N. 75° 12′ E.	14.00	3.58		13.54	
B				(49.91)		(6.00)
			49.91		21.09	15.09
					15.09	
					6.00	

Diff. Lat.	49.91	log. 1.698188
Departure,	6.00	log. 0.778151
Bearing AB,	N. 6° 51′ E.	tang. 9.077963
Bearing,	6° 51′	cos. 9.996889
Diff. Lat.		1.698188
Distance,	50.27	1.701299

NOTE.—In calculations of this kind, it is sufficiently accurate to confine the operations to two decimal places, unless the number of sides is large. In Ex. 2, had the work been extended to the third decimal place, it would not have made more than 15″ difference in the bearing and 1 link in the distance.

Ex. 3. Given the bearings and distances as follows,—viz.: 1. S. 29¾° E. 3.19; 2. S. 37¼° W. 5.86; 3. S. 39¼° E. 11.29; 4. N. 53° E. 19.32; 5. Unknown; 6. S. 60¾° W. 7.12; 7. S. 29¼° E. 2.18; 8. S. 60½° W. 8.12; to find the bearing and distance of the fifth side. Ans. N. 31° 5′ W. 16.26 ch.

Ex. 4. Required the bearing and distance of the third side from the following notes:—1. N. 46° 40′ W. 18.41 chains; 2. N. 54½° E. 13.45 chains; 3. Unknown; 4. S. 74° 55′ E. 17.58 chains; 5. S. 47° 50′ E. 15.86 chains; 6. S. 47° 25′ W. 16.36 chains; 7. S. 62° 35′ W. 14.67 chains.
Ans. 3d side, N. 5° 26′ W. 12.67 ch.

Ex. 5. It being impossible to take the bearing and distance of one side AB of a tract of land directly, in consequence of a marsh grown up with thick bushes, I took bearings and distances on the fast land as below,—viz.: AC S. $49\frac{1}{4}$° W. 9.30 chains; CD S. $32\frac{1}{2}$° E. 10.25 chains; DE S. $5\frac{1}{4}$° W. 6.75 chains; and EB N. $79\frac{3}{4}$° E. 8.10 chains. Required the bearing and distance of the side AB.

Ans. S. 16° 12′ E. 20.82 ch.

Ex. 6. The bearings and distances taken along the middle of a road which it is desired to straighten are as below,— 1. S. 27° 30′ E. 12.65 chains; 2. S. $10\frac{1}{4}$° E. 23.45 chains; 3. S. 14° W. 124.33 chains; 4. S. 67° E. 82.43 chains; 5. S. 17° E. 96.35 chains. Required the bearing and distance of a new road that shall connect the extremities.

Ans. S. 16° 44′ E. 291.63 ch.

CASE 2.

351. The bearings and distances of the sides of a tract of land being given, except two,—one of which has the bearing given, and the other the distance and the points between which it runs,—to determine the unknown bearing and distance.

RULE.

Change the bearings so that the side whose bearing only is given, may be a meridian.

Take out the latitudes and departures according to these changed bearings. Take the difference of the eastings and westings: this difference will be the departure of the side not made a meridian.

With this departure and the given distance, calculate by Art. 333 the changed bearing and difference of latitude, and place the latter in the column of latitude. From the changed bearing the true bearing may readily be found.

Take the difference between the northings and southings. This difference is the difference of latitude of the side made a meridian, and is equal to the distance.

NOTE.—In general, there will be no difficulty in determining whether the changed bearing found should be north or south. In some cases, however, either will render the true bearing conformable to the points given. In this case the question is ambiguous, and can only be determined from the other data, except when the true bearing is nearly known.

EXAMPLES.

Ex. 1. Given the courses and distances as below, to find the unknown bearing and distance.

Sta.	Bearing.	Changed Bearing.	Dist.	N.	S.	E.	W.
1	N. $56\frac{1}{4}$ W.	S. $57\frac{3}{4}$ W.	15.35		8.19		12.98
2	N. 9 W.	N. 75 W.	19.51	5.05			18.85
3	N. 66 E.	North.		(14.00)			
4	S. $39\frac{3}{4}$ E.	N. $74\frac{1}{4}$ E.	13.35	3.62		12.85	
5	N. E.		12.65	(12.12)		(3.62)	
6	S. $6\frac{3}{4}$ W.	S. $59\frac{1}{4}$ E.	12.18		6.23	10.47	
7	S. $52\frac{1}{2}$ W.	S. $13\frac{1}{2}$ E.	20.95		20.37	4.89	
				34.79	34.79	31.83	31.83

Dist., fifth side,	12.65	A. C. 8.897909
Dep. "	3.62	0.558709
Ch. bear. "	N. 16° 38′ E.	sin. 9.456618
	66°	
	N. 82° 38′ E., bearing of fifth side.	
Ch. bear., fifth side,	16° 38′	cos. 9.981436
Dist. "		1.102091
Diff. Lat. "	12.12	1.083527
Dist., third side,	14.00 ch.	

Ex. 2. Given—1. N. 47° W. 16.55 chains; 2. N. 19° 5′ W. 11.48 chains; 3. N. —— W. 15.53 chains; 4. N. 23° E. 9.72 chains; 5. N. $75\frac{1}{4}$° E. 14 chains; 6. S. 7° E., unknown; to determine the bearing of the third and the distance of the sixth side.

Ans. 3d side, N. $28\frac{1}{2}$° W.; 6th, 48.67 ch.

CASE 3.

352. The bearings and distances of the sides of a tract of land being given, except the distances of two sides, to determine these.

RULE.

Change the bearings so that one of the sides the distance of which is unknown may be a meridian. Take out the latitudes and departures with these changed bearings. The difference of the eastings and westings will be the departure of the side not made a meridian. With this departure and the changed bearing, find the distance and difference of latitude. Place the latter in its proper place in the table. Take the difference between the northings and southings: this difference will be the difference of latitude of the side made a meridian, and will be equal to the distance.

EXAMPLES.

Given as follow,—1. N. $56\frac{1}{4}$° W. 15.35 chains; 2. N. 9° W., unknown; 3. N. 66° E. 14.00 chains; 4. S. $39\frac{3}{4}$° E. 13.35 chains; 5. N. $82\frac{3}{4}$° E., unknown; 6. S. $6\frac{3}{4}$° W. 12.18 chains; 7. S. $52\frac{1}{2}$° W. 20.95 chains; to find the distances of the second and fifth sides.

Sta.	Bearing.	Changed Bearing.	Dist.	N.	S.	E.	W.
1	N. $56\frac{1}{4}$ W.	N. $47\frac{1}{4}$ W.	15.35	10.42			11.27
2	N. 9 W.	North.	(19.54)	(19.54)			
3	N. 66 E.	N. 75 E.	14.00	3.62		13.52	
4	S. $39\frac{3}{4}$ E.	S. $30\frac{3}{4}$ E.	13.35		11.47	6.83	
5	N. $82\frac{3}{4}$ E.	S. $88\frac{1}{4}$ E.			.39	(12.64)	
6	S. $6\frac{3}{4}$ W.	S. $15\frac{3}{4}$ W.	12.18		11.72		3.31
7	S. $52\frac{1}{2}$ W.	S. $61\frac{1}{2}$ W.	20.95		10.00		18.41
				33.58	33.58	32.99	32.99

Ch. bear., fifth side,	88° 15′	A. C. sin.	0.000203
Dep. "	12.64		1.101747
Dist. "	12.65		1.101950
Ch. bear.		cos.	8.484848
Dist.			1.101950
Diff. Lat.	0.39 S.	—	1.596798

Ex. 2. Given—1. S. $29\frac{3}{4}$° E. 3.19 chains; 2. S. $37\frac{1}{4}$° W. 5.86 chains; 3. S. $39\frac{1}{4}$° E., unknown; 4. N. 53° E. 19.32 chains; 5. N. 31° 5′ W., unknown; 6. S. $60\frac{3}{4}$° W. 7.12 chains; 7. S. $29\frac{1}{4}$° E. 2.18 chains; 8. S. $60\frac{1}{2}$° W. 8.12 chains; to find the distances of the third and fifth sides.

Ans. 3d side, 11.28 chains; 5th, 16.26 chains.

CASE 4.

353. The bearings and distances of all the sides of a tract of land being known except the bearings of two sides, to determine these.

RULE.

Take out the differences of latitude and the departures of the sides whose bearings and distances are known. The differences of the northings and southings will be the difference of latitude, and that of the eastings and westings the departure, of a line which, with the known sides of the survey, will form a closed figure, and may therefore be called the closing line.

With this closing line and the distances of the two other sides form a triangle.

Calculate two angles of this triangle. These angles applied to the bearing of the closing line will give the bearings required.

EXAMPLES.

Ex. 1. Given AB (Fig. 154) N. $56\frac{1}{4}°$ W. 15.35 chains; BC N. 9° W. 19.51 chains; CD N. — E. 14 chains; DE S. $39\frac{3}{4}°$ E. 13.35; EF N. $82\frac{1}{2}°$ E. 12.65 chains; FG S. — W. 12.18 chains; GA S. $52\frac{1}{2}°$ W. 20.95 chains; to find the bearings of the third and sixth sides.

	Bearing.	Dist.	N.	S.	E.	W.
AB	N. $56\frac{1}{4}$ W.	15.35	8.53			12.76
BC	N. 9 W.	19.51	19.27			3.05
Ce	S. $39\frac{3}{4}$ E.	13.35		10.26	8.54	
ef	N. $82\frac{1}{2}$ E.	12.65	1.65		12.54	
GA	S. $52\frac{1}{2}$ W.	20.95		12.75		16.62
			29.45	23.01	21.08	32.43
			23.01			21.08
			6.44			11.35

Diff. Lat.	6.44	A. C. 9.191114
Dep.	11.35	1.054996
Tang. closing line,	S. 60° 26′ E.	10.246110
Cos. bear.	60° 26′	A. C. 0.306769
Diff. Lat.		0.808886
Dist. closing line,	13.05	1.115655
FG	12.18	
*f*G	13.05	A. C. 8.884388
*f*F	14.00	″ ″ 8.853872
	2) 39.23	
	19.615	1.292588
	7.435	0.871281
		2) 19.902129
$\frac{1}{2}$ F*f*G	26° 41′	cos. 9.951064
F*f*G	53° 22′	

FG	12.18	A. C. 8.914353
fF	14.00	1.146128
sin. FfG	53° 22′	9.904429
sin. fGF	67° 17′	9.964910
	60° 26′ Bear. of fG	
	S. 6° 51′ W. " GF	

$$180° - (53° 22' + 60° 26') = 66° 12';$$

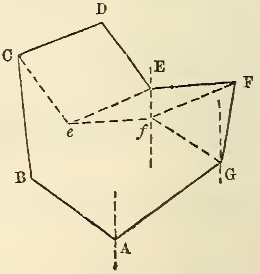
therefore, N. 66° 12′ E. is the bearing of CD.

Ex. 2. Given—1. S. $29\frac{3}{4}$° E. 3.19 chains; 2. S. $37\frac{1}{4}$° W. 5.86 chains; 3. S. — E. 11.29 chains; 4. N. 53° E. 19.32 chains; 5. N. — W. 16.26 chains; 6. S. $60\frac{3}{4}$° W. 7.12 chains; 7. S. $29\frac{1}{4}$° E. 2.18 chains; 8. S. $60\frac{1}{2}$° W. 8.12 chains; to find the bearing of the third and fifth sides.

Ans. 3d side, S. 39° 8′ E.; 5th, N. 31° W.

354. The first three of the preceding rules are so simple as hardly to need any explanation. The principle of the last will be seen from the following illustration. The figure being protracted from the field-notes in Ex 1, Case 4, these are, as will be seen, the same as Ex. 1 in the other cases.

Fig. 154.



Let ABCDEFG (Fig. 154) be the plat of the tract, the bearings of CD and FG being supposed unknown. If Ce and ef be drawn parallel to the sides DE and EF, and fG be joined, then will ABCefG form a closed figure, the bearings and distances of all the sides except fG being known. The course and distance of this side, which is the *closing line*, are found as directed in the rule. Join fF and eE. Then fF is equal and parallel to eE and therefore to CD. The sides of the triangle fFG are therefore the closing line, the side FG, and the line fF equal and parallel to the side CD. In fFG find the angles f and G: these applied to the bearing of fG will give the bearings of fF or CD and of FG.

This method might have been employed in Cases 2 and 3. Those given in the rules are, however, more concise, and are therefore to be preferred.

355. Though the methods illustrated above will serve to supply omissions in all cases where not more than two of the dimensions are unknown, yet it will not be amiss again to impress on the young practitioner the necessity, in all cases in which it is practicable, of determining each side independently of every other. The rules for supplying omissions should only be used in cases where one or more of the data have been accidentally omitted, or have become defaced on the notes. However accurate the field-work may be, there is always a liability to error, and if one side is determined by the rest no means are left of detecting any error. When a side cannot be measured directly, the best way is to determine it by some of the trigonometrical methods, taking the angles and base-lines with great care. In this way a degree of accuracy may be obtained equal to that of the sides measured directly. The latitudes and departures may then be balanced as usual.

SECTION IX.

CONTENT OF LAND.

356. From the bearings and distances of the sides of a tract of land, or from the angles and the lengths of the sides, the area may be found, however numerous the sides may be. This may be done by Problem 4, which is entirely general, it being applicable whatever the number of sides may be, provided they are straight lines. As, however, there are other more concise methods applicable to triangles and quadrilaterals, those are first given.

If one or more of the boundaries is irregular, instead of multiplying the number of sides by taking the bearings of all the sinuosities of the boundary, it is better to run one or more base lines and take offsets, as directed in chain surveying. The content within the base lines is then to be calculated, and the area cut off by the base lines, being found by the method Art. 256, is to be added to or subtracted from the former area, according as the boundary is without or within the base.

As has been already remarked, (Art. 257,) when the tract bounds on a brook or rivulet, the middle of the stream is the boundary, unless otherwise declared in the deed. Lands bordering on tide water go to low-water mark. When the stream, though not tide water, is large, the area is generally limited by the low-water mark, or by the regular banks of the stream.

If the farm bounds on a public road, the boundary is, except in special cases, the middle of the road, and the measures are to be taken accordingly.

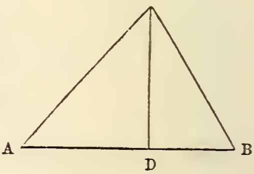
357. Problem 1.—*Given two sides and the included angle of a triangle or parallelogram, to determine the area.*

Say, As radius is to the sine of the included angle, so is the rectangle of the given sides to double the area of the triangle, or to the area of the parallelogram.

DEMONSTRATION.—We have, (Fig. 155,) by Art. 137,—

As rad. : sin. A : : AC : CD : : AB . AC : AB . CD, (Cor. 1.6); but AD . CD = 2 ABC.

Fig. 155.



EXAMPLES.

Ex. 1. Given AB = 12.36 chains, BC = 14.36 chains, and ABC = 47° 35′, to determine the area of the triangle.

As rad.		A.C. 0.000000
: sin. B	47° 35′	9.868209
: : { AB	12.36 ch.	1.092018
{ BC	14.36	1.157154
: 2 ABC	2)131.033	2.117381
	65.5165 ch. = 6 A., 2 R., 8.26 P.	

Ex. 2. Given AB N. 37° 14′ W. 17.25 chains, and BC N. 74° 29′ W. 10.87 chains, to determine the area of the triangle ABC. Ans. 5 A., 0 R., 28 P.

Ex. 3. Given AB = 23.56 chains, AC = 16.42 chains, and the angle A 126° 47′. Required the area of the triangle. Ans. 15 A., 1 R., 38.7 P.

358. Problem 2.—*The angles and one side of a triangle being given, to determine the area.*

Say, As the rectangle of radius and sine of the angle opposite the given side is to the rectangle of the sines of the other angles, so is the square of the given side to double the area.

DEMONSTRATION.—We have (Fig. 155)

$$r : \sin. A :: AC : CD \text{ (Art. 137)},$$

and

$$\sin. B : \sin. C :: AC : AB \text{ (Art. 139)}.$$

$$\therefore \quad (23.6) \quad r \,.\, \sin. B : \sin. A \,.\, \sin. C :: AC^2 : AB \,.\, CD, \text{ or } 2\,ABC.$$

EXAMPLES.

Ex. 1. Given AB = 21.62 chains, and the angle A = 47° 56′ and B = 76° 15′, to find the area.

As { rad.		A.C.	0.000000
{ sin. C	55° 49′	"	0.082366
: { sin. A	47° 56′		9.870618
{ sin. B	76° 15′		9.987372
: : { AB	21.62 ch.		1.334856
{ AB	21.62		1.334856
: 2 ABC	2) 407.444		2.610068

Area = 203.722 ch. = 20 A., 1 R., 19.5 P.

Ex. 2. Given AB 17.63 chains, and the angle A = 63° 52′ and B 73° 47′, to find the area. Ans. 19 A., 3 R., 22 P.

Ex. 3. Given one side 15.65 chains, and the adjacent angles 63° 17′ and 59° 12′, to determine the area of the triangle. Ans. 11 A., 0 R., 22 P.

359. Problem 3.—*To determine the area of a trapezium, three sides and the two included angles being given.*

RULE.

1. Consider two adjacent sides and their contained angle as the sides and included angle of a triangle, and find its double area by Prob. 1.

2. In like manner, find the double area of a triangle of which the two other adjacent sides and their contained angle are two sides and the included angle.

3. Take the difference between the sum of the given angles and 180°, and consider the two opposite given sides and this difference as two sides and the included angle of a triangle, and find its double area.

4. If the sum of the given angles is greater than 180°, add this third area to the sum of the others; but if the sum of the given angles is less than 180°, subtract the third area from the sum of the others: the result will be double the area of the trapezium.

DEMONSTRATION.—Let ABCD (Figs. 156, 157) be the trapezium, of which AB, BC, and CD, and the angles B and C, are given.

Join BD, and draw DE and CG perpendicular to AB, and CF perpendicular to ED. Then will DCF $= 180° \backsim (B + C.)$ Also, draw AH parallel to CB, and join DH.

Then will $2\,ABD = AB\,.\,DE = AB\,(EF \pm DF)$ $= AB\,.\,EF \pm AB\,.\,DF = 2\,ABC \pm 2\,CDH$.

Fig. 156.

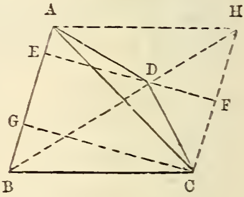
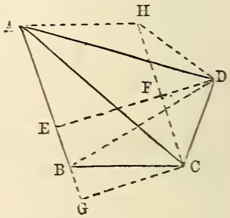


Fig. 157.



Whence $2\,ABCD = 2\,BDC + 2\,ADB = 2\,BCD + 2\,ABC \pm 2\,CDH$: the plus sign being used (Fig. 157) when the sum of the angles is greater than 180°.

EXAMPLES.

Ex. 1. Given AB = 6.95 chains, BC = 8.37 chains, CD = 5.43 chains, ABC = 85° 17′, and BCD = 54° 12′, to find the area of the trapezium.

As r		0.000000
: sin. B	85° 17′	9.998527
: : { AB	6.95	0.841985
{ BC	8.37	0.922725
: 2 ABC	57.975	1.763237

As r		0.000000
: sin. 180° − (B + C)	40° 31′	9.812692
: : { AB	6.95	0.841985
{ CD	5.43	0.743800
: 2 CDH	25.031	1.398477

As r		0.000000
: sin. C	54° 12′	9.909055
: : { BC	8.37	0.922725
{ CD	5.43	0.734800
: 2 BCD	36.862	1.566580
	57.975	
	94.837	
	25.031	
	2)69.806	
	34.903 ch. = 3 A., 1 R., 38.45 P.	

Ex. 2. Given AB S. 27° E. 12.47 chains, BC N. 66° E. 11.43, and CD N. 8° W. 9.16 chains, to find the area of the trapezium. Ans. 14 A., 0 R., 1.56 P.

Ex. 3. Given AB S. 45° W. 8.63 chains, BC S. 86° 30′ E. 9.27 chains, and CD N. 34° E. 11.23 chains, to find the area of the trapezium.

Ans. 6 A., 2 R., 9 P.

360. The above rule is a particular example of a more general problem, which may be enunciated thus:—

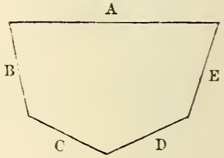
Let A, B, C, D, &c. be the sides of any polygon, and let the angle contained between the directions of any two sides, as B and D, be designated [BD]. Then, leaving out any side, we shall have the double area equal to the sum of the products of all the other pairs into the sine of their included angle. Thus, if the figure were a pentagon, we should have 2 the area = BC sin. [BC] + BD sin. [BD] + BE sin. [BE] + CD sin. [CD] + CE sin. [CE] + DE sin. [DE].

Observing that any product must be taken negative, if the angle is turned in a contrary direction from the general convexity of the figure with reference to the side A.

Thus, in Fig. 156, we have 2 ABCD = AB . BC sin. [AB . BC] + BC . CD sin. [BC . CD] − AB . CD sin. [AB . CD], the lines BA and CD meeting so as to make the angle [AB . CD] present its convexity in the opposite direction from that of the figure.

But, in Fig. 157, we have 2 ABCD = AB . BC sin. [AB . BC] + BC . CD sin. [BC . CD] + AB . CD sin. [AB . CD].

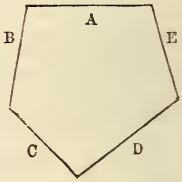
Fig. 158.



In the pentagon (Fig. 158) we shall have

2 Area = B . C . sin. [B . C.] + B . D . sin. [B . D.] + B . E . sin. [B . E.] + C . D . sin. [C . D.] + C . E . sin. [C . E.] + D . E . sin. [D . E].

Fig. 159.

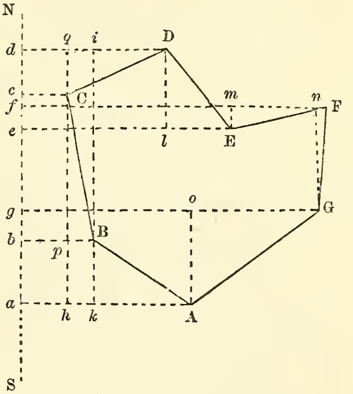


In Fig. 159 we have

2 Area = B . C . sin. [B . C.] + B . D . sin. [B . D.] − B . E . sin. [B . E.] + C . D . sin. [C . D.] + C . E . sin. [C . E.] + D . E . sin. [D . E].

361. Problem 4.—*The bearings and distances of the boundaries of a tract of land being given, to determine its area by means of the latitudes and departures of the sides.*

Fig. 160.



Let ABCDEFG (Fig. 160) be the plat of a tract, and let NS be a meridian anywhere on the map. Through the corners draw the perpendiculars Aa, Bb, &c. Then, it is evident that ABCDEFG = AagG + GgfF + DdeE − AabB − BbcC − CcdD − EefF.

Now, these various figures being trapezoids, their areas will be found by multiplying their perpendiculars by the half-sums of their parallel sides. The perpendiculars are the differences of latitude of the sides of the tract. The sums of their parallel sides may be found as follows:—

The position of the line NS being arbitrary, the sum Aa + Bb, corresponding to the first side AB, may be taken at pleasure. Now, if from Aa + Bb we take Ah, the whole departure of the two sides AB and BC, we have Bb + Cc, the sum of the parallel sides of BbcC. Similarly, if to Bb + Cc we add iD, the departure of the two sides BC and CD, we have Cc + Dd; and so on. The whole may be arranged in a tabular form, as below,—

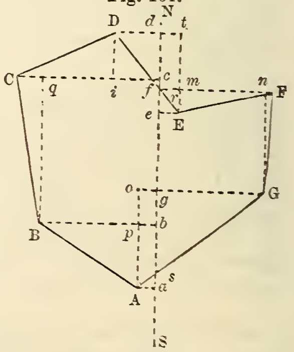
Sides.	N.	S.	E.	W.	E. D. D.	W. D. D.	Multipliers.	N. Areas.	S. Areas.
AB	Bk			Ak		Ak + Go	Aa + Bb, E.	2 AabB	
BC	pC			Bp		Ak + Bp	Bb + Cc, E.	2 BbcC	
CD	Cq		qD		qD − Bp		Cc + Dd, E.	2 CcdD	
DE		Dl	lE		qD + lE		Dd + Ee, E.		2 DdeE
EF	Em		mF		lE + mF		Ee + Ff, E.	2 EefF	
FG		nG		Fn	mF − Fn		Ff + Gg, E.		2 FfgG
GA		oA		Go		Fn + Go	Gg + Aa, E.		2 GgaA

in which the first column contains the sides, and the next four the differences of latitude and the departures; the fifth and sixth columns contain the whole departures of two consecutive sides. These may be called the double departures, and the columns headed, accordingly, E.D.D. and W.D.D. These double departures are found thus: The first, Ak + Go, is the sum of the departures of GA and AB, and is placed in the column of west double departures, because both departures are westerly; the second, Ak + Bp, is the sum of those of AB and BC, and is west; the third is Dq − Bp, and is east, because D is east of B; the fourth, Dq + El, is east; and so on. The eighth column contains the sums of the parallel sides. These may be called the multipliers. They are found by the following process. Assuming the first, Aa + Bb, at pleasure, designate it either east or west. In the figure, the line NS being to the west of AB, the multiplier is east. The double departure Ak + Bp = Ah being west, subtract it from Aa + Bb, and we have Bb + Cc. To Bb + Cc add the next double departure, qD − pB = iD, and we have Cc + Dd; qD + lE added to Cc + Dd gives Dd + Ee; lE + mF added to Dd + Ee gives Ee + Ff; mF − Fn added to Ee + Ff gives Ff + Gg; and, lastly, Fn + Go taken from Ff + Gg leaves Gg + Aa.

The areas are arranged in the last two columns, which are headed north areas and south areas for distinction. These areas are placed in the above table in the columns of the same name as the difference of latitudes of the sides to which they belong.

Had the line NS been drawn so as to intersect the plat, some of the areas would have been to the west of it, and some of the multipliers might have been west. Fig. 161 is an example of this.

Fig. 161.



In this case, we have

2 ABCDEFG = 2 AabB + 2 BbcC + 2 CcdD − 2 Ddr + 2 reE − 2 EefF + 2 FfgG + 2 Ggs − 2 saA = 2 AabB + 2 BbcC + 2 CcdD − 2 (Ddr − reE) − 2 EefF + 2 FfgG + 2 (Ggs − saA.)

But $2(\text{D}dr - re\text{E}) = \text{D}d \,.\, dr - \text{E}e \,.\, er = \text{D}d \,.\, de - \text{D}d \,.\, er - \text{E}e \,.\, de + \text{E}e \,.\, dr$;

and since $\text{D}d : dr :: \text{E}e : er$, $\text{D}d \,.\, er = \text{E}e \; dr$.

$\therefore \quad 2(\text{D}dr - re\text{E}) = \text{D}d \,.\, de - \text{E}e \,.\, de = (\text{D}d - \text{E}e)\, de.$

Whence $2\,\text{ABCDEFG} = (\text{A}a + \text{B}b)\, ab + (\text{B}b + \text{C}c)\, bc + (\text{C}c + \text{D}d)\, cd - (\text{D}d - \text{E}e)\, de - (\text{E}e + f\text{F})\, ef + (f\text{F} + \text{G}g)\, fg + (\text{G}g - \text{A}a)\, ag$.

The following table exhibits the whole.

Sides.	N.	S.	E.	W.	E. D. D.	W. D. D.	Multipliers.	N. Areas.	S. Areas.
AB	Ap			pB		pB + Go	Bb + Aa, W.		2 AabB
BC	Bq			qC		pB + qC	Bb + Cc, W.		2 BbCc
CD	Di		Ci		Ci − qC		Cc + Dd, W.		2 CcdD
DE		Et	Dt		Ci + Dt		Dd − Ee, W.	2 (Ddr − Eer)	
EF	Em		mF		Dt + Fm		Ee + Ff, E.	2 (EefF)	
FG		Gn		Fn	Fm − Fn		Ff + Gg, E.		2 FfgG
GA		Ao		Go		Fn + Go	Gg − Aa, E.		2 (Ggs − Aas)

Here the first multiplier is west, the meridian being to the east of the line AB. The subsequent multipliers are found as follow:—$(\text{B}b + \text{A}a) + (p\text{B} + q\text{C}) = \text{B}b + \text{C}c$; $(\text{B}b + \text{C}c) - (\text{C}i - q\text{C}) = \text{C}c + \text{D}d$; $(\text{C}c + \text{D}d) - (\text{C}i + \text{D}t) = \text{D}d - \text{E}e$; $(\text{D}t + \text{F}m) - (\text{D}d - \text{E}e) = (\text{E}e + \text{F}f)$, which must be marked east, not only from its position on the figure, but also from the fact that the east double departure is greater than the west multiplier, which is taken from it;—$(\text{E}e + \text{F}f) + (\text{F}m - \text{F}n) = \text{F}f + \text{G}g$; and $(\text{F}f + \text{G}g) - (\text{F}n + \text{G}o) = \text{G}g - \text{A}a$.

The areas are arranged so that the additive quantities may be in the column of south areas and the subtractive in that of north areas.

From the above investigation the following rule is derived:—

RULE.

Rule a table as in the adjoining examples. Find the corrected latitudes and departures by Art. 338. Then, if the departures of the first and last sides are of the same name, add them together, and place their sum opposite the first side in the column of double departures of that name; but if they are of different names, take their difference and place it in the column of the same name as the greater. Proceed in the same way with the departures of the first and second sides, placing the result opposite the second side; and so on.

Assume any number for a multiplier for the first side, marking it E. for east or W. for west, as may be preferred. Then, if this multiplier and the double departure corresponding to the second side are of the same name, add them together, and place the sum with that name in the column of multipliers, for a multiplier for that side; but, if the multiplier and double departure be of different names, take their difference and mark it with the name of the greater, for the next multiplier. Proceed in the same manner with the multiplier thus determined and the third double departure, to find the multiplier for the third side. So continue until all the multipliers have been found.

Multiply the difference of latitude of each side by the corresponding multiplier, for the area corresponding to that side. If the multiplier be east, place the product in the column of areas which is of the same name as the difference of latitude; but, if the multiplier be west, place the product in the column of the opposite name.

Sum the north and the south areas. Half the difference of the sums will be the area of the tract.

NOTE.—In working any area, the columns of double departures should balance.

The first multiplier is generally assumed zero. One multiplication is thus avoided. When this is done, the last multiplier will be equal to the first double departure, but of a different name.

EXAMPLES.

Ex. 1. Given the bearings and distances as follow, to find the area:—1. N. $56\frac{1}{4}$° W. 15.35 ch.; 2. N. 9° W. 19.51 ch.; 3. N. 66° E. 14.01 ch.; 4. S. $39\frac{3}{4}$° E. 13.35 ch.; 5. N. $82\frac{1}{2}$° E. 12.65 ch.; 6. S. $6\frac{3}{4}$° W. 12.18 ch.; 7. S. $52\frac{1}{2}$° W. 20.95 ch.; to find the area.

	Bearings.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. W.	N.	S.	E.	W.	E. D. D.	W. D. D.	Multipliers.	N. Areas.	S. Areas.
1	N. $56\frac{1}{4}$ W.	15.35	8.53			12.76	1	0	8.52			12.76		29.39	00.00 E.		
2	N. 9 W.	19.51	19.27			3.05	1	1	19.26			3.06		15.82	15.82 W.		304.6932
3	N. 66 E.	14.01	5.70		12.80		1	0	5.69		12.80		9.74		6.08 W.		34.5952
4	S. $39\frac{3}{4}$ E.	13.35		10.26	8.54		0	0		10.26	8.54		21.34		15.26 E.		156.5676
5	N. $82\frac{1}{2}$ E.	12.65	1.65		12.54		0	0	1.65		12.54		21.08		36.34 E.	59.9610	
6	S. $6\frac{3}{4}$ W.	12.18		12.10		1.43	0	0		12.10		1.43	11.11		47.45 E.		574.1450
7	S. $52\frac{1}{2}$ W.	20.95		12.75		16.62	1	1		12.76		16.63		18.06	29.39 E.		375.0164
			35.15	35.11	33.88	33.86			35.12	35.12	33.88	33.88	63.27	63.27		59.9610	1445.0174
			35.11		33.86												59.9610
		Error South	.04		.02 Error West.												2)1385.0564

$$
\begin{array}{r}
69,2.5282 \\
4 \\
\hline
1,01128 \\
40 \\
\hline
0.45120
\end{array}
$$

Area, 69 A., 1 R., 0.45 P.

Ex. 2. Given the bearings and distances as in the adjoining table, to calculate the area.

Sta.	Bearings.	Dist.	N.	S.	E.	W.	Cor. N.	Cor. W.	N.	S.	E.	W.	E. D. D.	W.D.D.	Multipliers.	N. Areas.	S. Areas.
1	N. 66° 30′ E.	7.62	3.04		6.99		0	1	3.04		6.98		6.13		00.00 E.		
2	S. 25° 9′ E.	16.18		14.65	6.88		1	1		14.64	6.87		13.85		13.85 E.		202.7640
3	S. 59° 2′ W.	10.00		5.15		8.57	1	1		5.14		8.58		1.71	12.14 E.		62.3996
4	S. 57° 50′ W.	9.29		4.95		7.86	1	1		4.94		7.87		16.45	4.31 W.	21.2914	
5	N. 35° 45′ W.	11.50	9.33			6.72	1	1	9.34			6.73		14.60	18.91 W.		176.6194
6	N. 43° 19′ E.	5.89	4.29		4.04		0	0	4.29		4.04			2.69	21.60 W.		92.6640
7	N. 25° 27′ W.	1.83	1.65			.79	0	0	1.65			.79	3.25		18.35 W.		30.2775
8	N. 66° 25′ E.	5.50	2.20		5.04		0	0	2.20		5.04		4.25		14.10 W.		31.0200
9	N. 23° 35′ W.	1.25	1.15			.50	0	0	1.15			.50	4.54		9.56 W.		10.9940
10	N. 65° 22′ E.	2.63	1.10		2.39		0	0	1.10		2.39		1.89		7.67 W.		8.4370
11	N. 23° 28′ W.	2.13	1.95			.85	0	0	1.95			.85	1.54		6.13 W.		11.9535
		73.82	24.71	24.75	25.34	25.29											627.1290
				24.71	25.29												21.2914
		Error North		.04	.05	Error West.											2)605.8376
																	30,2.9188
																	4
																	1.16752
																	40
																	6.70080

Area, 30 A., 1 R., 6.7 P.

Ex. 3. Given the bearings and distances as follow, to calculate the area:—1. N. 27° 15′ E. 7.75 ch.; 2. S. 62° 25′ E. 10.80 ch.; 3. S. 7° 55′ E. 9.50 ch.; 4. S. 47° 25′ E. 9.37 ch.; 5. S. 54° 25′ W. 8.42 ch.; 6. N. 37° 35′ W. 23.69 ch.

Ans. 22 A., 1 R., 26.17 P.

Ex. 4. Calculate the area from the following notes:—1. N. 46° 40′ W. 18.41 ch.; 2. N. 54° 30′ E. 13.45 ch.; 3. N. 5° 30′ W. 12.65 ch.; 4. S. 74° 55′ E. 17.58 ch; 5. S. 47° 50′ E. 15.86 ch.; 6. S. 47° 25′ W. 16.36 ch.; 7. S. 62° 35′ W. 14.69 ch.

Area, 66 A., 2 R., 21 P.

Ex. 5. Given the bearings and distances of the sides of a tract of land, as follow,—viz.: 1. N. 43° 25′ W. 28.43 ch.; 2. N. 29° 48′ E. 30.55 ch.; 3. S. 80° E. 28.74 ch.; 4. N. 89° 55′ E. 40 ch.; 5. S. 10° 13′ E. 23.70 ch.; 6. S. 63° 55′ W. 25.18 ch.; 7. N. 63° 45′, W. 20.82 ch.; 8. S. 57° 25′ W. 31.70 ch.: to determine the area.

Area, 251 A., 0 R., 5 P.

Ex. 6. Calculate the distances of the third and fourth sides, and the area of the tract, from the following notes:—1. S. 64° 5′ W. 11.18 ch.; 2. N. 49° 45′ W. 12.91 ch.; 3. N. 35° 20′ E., distance unknown; 4. S. 82° 25′ E., distance unknown; 5. N. 87° E. 13.82 ch.; 6. N. 49° 30′ E. 4.95 ch.; 7. S. 33° 25′ E. 10.80 ch.; 8. S. 0° 55′ E. 9.22 ch.; 9. S. 79° 10′ W. 14.30 ch.; 10. N. 52° 15′ W. 8.03 ch.

Ans. 3d side, 12.13 ch.; 4th, 9.71 ch.; Area, 57 A., 1 R., 12 P.

Ex. 7. One corner of a tract of land being in a swamp, but visible from the adjacent corners, I took the bearings and distances as follow:—1. S. 45° E. 13.65 ch.; 2. N. $38\frac{3}{4}$° E. 17.28 ch.; 3. N. 19° W. 23.43 ch.; 4. S. 58° W. 14 ch.; 5. N. 87° W. 8.14 ch.; 6. N. $45\frac{1}{2}$° W. 9.23 ch.; 7. S. $28\frac{1}{4}$° W. 14.60 ch.; 8. S. $1\frac{3}{4}$° E.; 9. N. $79\frac{1}{4}$° E. Required the distances of the last two sides and the area of the tract.

Ans. 8th side, 16.44 ch.; 9th, 20.51 ch.; Area, 92 A., 1 R., 7 P.

362. Offsets. If any of the sides border on a water-course, or are very irregular, stationary lines may be run as near the boundary as possible, and offsets be taken as directed in chain surveying. The area within the stationary lines may then be calculated as above. That of the spaces included between those lines and the true boundary is to be calculated as in Art. 256. These areas added to or subtracted from the former, according as the stationary lines are within or without the tract, will give the content required.

When the tract bounds on a stream, it is usual to consider the boundary as the middle of the stream, except in tide waters or large rivers which are navigable and are thus considered public highways. In these cases the boundary is low-water mark.

In reciting the boundaries in title-deeds, the offsets are not generally given. The description usually runs thus: —Thence S. 43½° E. 10.63 chains to a stone on the bank of Ridley Creek, and thence on the same course 1.05 chains to the middle of said creek. Thence along the bed of said creek, in a southwesterly direction, 37.63 chains; thence N. 47° W., by a marked white-oak on the banks of the creek, 25.63 chains to a limestone, corner of John Brown's land, &c.

EXAMPLES.

Ex. 1. Calculate the area from the following field-notes:—

	(4)	
55	1350	
55	0	
	(3)	N.26°45'E.

	(3)	
55	2160	
270	1929	
396	1408	
310	1015	
340	610	
50	0	
	(2)	N.56°30'E.

	3050	Mid. of do.
	3000	(2)on r.bank.
	(1)	N.36° 30'W.

	(6)	
60	1471	
95	930	
140	485	
60	0	
	(5)	S.51°30'E.

	(5)	
60	1072	
130	750	
85	390	
55	0	
	(4)	S.84°45'E.

	(1)	
	4316	
	75	(7)
Middle	of river.	S.45°15'W.

	(7)	
75	826	
100	420	
60	0	
	(6)	S.11°45'E.

Sta.	Bearings.	Dist.	N.	S.	E.	W.	E. D. D.	W.D.D.	Mult.	N.Areas.	S. Areas.
1	N. 36½ W.	30.00	24.12			17.84		47.96	0		
2	N. 56½ E.	21.60	11.92		18.01		.17		.17 E.	2.0264	
3	N. 26¾ E.	13.50	12.06		6.08		24.09		24.26 E.	292.5756	
4	S. 84¾ E.	10.72		.98	10.68		16.76		41.02 E.		40.1996
5	S. 51½ E.	14.71		9.16	11.51		22.19		63.21 E.		579.0036
6	S. 11¾ E.	8.26		8.09	1.68		13.19		76.40 E.		618.0760
7	S. 45¼ W.	42.41		29.87		30.12		28.44	47.96 E.		1432.5652
			48.10	48.10	47.96	47.96				294.6020	2669.8444
											294.6020
										2)	2375.2424
											1187.6212
										=	98.30145
										=	128.592265

Area of offsets calculated as in Ex. 1, Art. 257.

128 A., 2 R., 14.76 P.

Ex. 2. Given the field-notes as below of a meadow bounding on a small brook, to calculate the area:—

Base-line.	(2)	
0	1132	
55	1054	
72	896	
97	739	
75	480	
On brook.	S. 77¼ E. (1)	

Base-line.	(3)	
	1740	
63	1414	
35	1237	
87	1016	
45	824	
50	652	
	551	0
	452	75
	295	75
	0	0
	S. 6¾ E. (2)	

Side.	(1)	
	1450	
	N. 30¾ W. (5)	
Side.	(5)	
	1344	
	N. 32 W. (4)	
Side.	(4)	
	1396	
	S. 72¾ W. (3)	

Ans. 34 A., 3 R., 1.7 P.

Ex. 3. Required the area of the meadow bordering on a mill-race, of which the boundaries are contained in the following field-notes, the angles given being the deflections from the last course:—

Side.	(2)	
	11.28	
	(1)	S. 53° 10′ W.

Side.	(3)	
	21.65	
	(2)	⌈ 97° 03′

Side.		
	2.40	to race-bank.
	1.96	(4)
	(3)	⌈ 97° 45′.

Base.		(6)	
	32	9.89	
	30	5.50	
	132	3.00	
	40	1.08	
	35	0	
	30° 12′ ┐	(5)	
Base.		(5)	
	35	1.05	
	44	.11	
		(4)	┌ 81° 14′

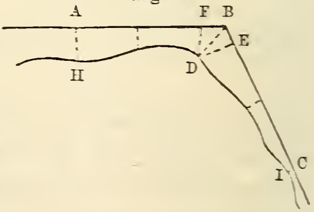
Base.		(1)	┌ 81° 26′
		9.12	
		(7)	┌ 27° 46′
Base.		(7)	
		2.40	
	corner 14	2.26	
		2.00	0
		1.75	6
		1.50	
	32	0	
	12° 14′ ┐	(6)	

In calculating the area, it will be necessary first to calculate the bearings from the observed angles.

Area, 15 A., 2 R., 11.5 P.

363. Inaccessible Areas. When it is desired to determine the area of a tract of difficult access, such as a pond, a thick copse, or a swamp, it should be surrounded by a system of lines as near the boundaries as they can be run without multiplying the number of sides unnecessarily. Offsets should then be taken to different points of the boundary, so as to determine its sinuosities. The areas of the parts determined by these offsets, taken from the area enclosed in the base lines, will leave the content required.

Fig. 162.

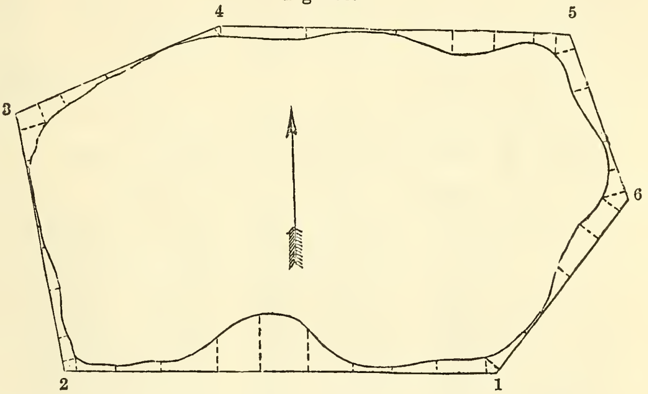


Where two base lines make an angle with each other, the first offset on each should be taken to the same point in the irregular boundary. Thus, if AB and BC (Fig. 162) are two adjacent base lines enclosing an irregular boundary HDI, the first offsets should be taken at F and E, so situated that the offsets FD and ED should meet at the same point D of the boundary. The triangular spaces BDF and BDE will then be included with the areas belonging to the lines AB and BC respectively.

The following examples of the field-notes and calculation for the area of a pond will illustrate this subject:—

Fig. 163 is a plat of Ex. 1 on a scale of 1 inch to 10 chains.

Fig. 163.



(3)	
1385	
1295	155
1125	25
975	0
775	10
430	55
155	22
47	75
(2)	┏ 78° 55′

(2)	
2280	
2215	60
2015	7
1770	33
1480	175
1240	291
1000	220
705	32
555	10
312	55
55	82
(1)	N. 88° 35′ W.

(5)	
1866	
1805	115
1675	55
1475	90
1250	105
950	22
800	0
475	42
0	42
(4)	┏ 23° 51′

(4)	
1152	
1135	42
950	0
775	0
525	11
250	70
132	120
(3)	┏ 78° 8′

(1)	┏ { 52° 52′ on (1). (2).
1140	
1100	90
875	10
750	10
500	60
250	112
75	112
(6)	┏ 56° 35′

(6)	
920	
870	122
750	32
575	17
300	73
85	97
(5)	┏ 69° 39′

Sta.	Bearings.	Dist.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipli'r.	S. Areas.
1	N. 88° 35′ W.	22.80	.55			22.78		29.88		
2	N. 9° 40′ W.	13.85	13.65			2.33		25.11	25.11 W.	342.7515
3	N. 66° 28′ E.	11.52	4.23		10.72		8.39		16.72 W.	70.7256
4	S. 87° 41′ E.	18.66		.76	18.64		29.36		12.64 E.	9.6064
5	S. 18° 2′ E.	9.20		8.75	2.85		21.49		34.13 E.	298.6375
6	S. 38° 33′ W.	11.40		8.92		7.10		4.25	29.88 E.	266.5296

2) 988.2506

Content within the base-lines, 494.1253 ch.

Base.	Dist.	Offsets.	Inter. Dist.	Sum of Offsets.	Areas.
	0.00				
	0.55	.82	.55	.82	.4510
	3.12	.55	2.57	1.37	3.5209
	5.55	.10	2.43	.65	1.5795
	7.05	.32	1.50	.42	.6300
	10.00	2.20	2.95	2.52	7.4340
	12.40	2.91	2.40	5.11	12.2640
(1) (2)	14.80	1.75	2.40	4.66	11.1840
	17.70	.33	2.90	2.08	6.0320
	20.15	.07	2.45	.40	.9800
	22.15	.60	2.00	.67	1.3400
	22.80	0	.65	.60	.3900
					45.8054
	0				
	.47	.75	.47	.75	.3525
	1.55	.22	1.08	.97	1.0476
	4.30	.55	2.75	.77	2.1175
(2) (3)	7.75	.10	3.45	.65	2.2425
	9.75	0	2.00	.10	.2000
	11.25	.25	1.50	.25	.3750
	12.95	1.55	1.70	1.80	3.0600
	13.85	0	.90	1.55	1.3950
					10.7901
	0				
	1.32	1.20	1.32	1.20	1.5840
	2.50	.70	1.18	1.90	2.2420
(3) (4)	5.25	11	2.75	81	2.2275
	7.75	0	2.50	11	.2750
	9.50	0	1.75	0	.0000
	11.35	42	1.85	42	.7770
	11.52	0	17	42	.0714

7.1769

Base.	Dist.	Offset.	Inter. Dist.	Sum of Offset.	Areas.
	.00	.42			
	4.75	.42	4.75	.84	3.9900
	8.00	.00	3.25	.42	1.3650
	9.50	.22	1.50	.22	.3300
(4) (5)	12.50	1.05	3.00	1.27	3.8100
	14.75	.90	2.25	1.95	4.3875
	16.75	.55	2.00	1.45	2.9000
	18.05	1.15	1.30	1.70	2.2100
	18.66	.00	.61	1.15	.7015
					19.6940
	.00				
	.85	.97	.85	.97	.8245
	3.00	.73	2.15	1.70	3.6550
(5) (6)	5.75	.17	2.75	.90	2.4750
	7.50	.32	1.75	.49	.8575
	8.70	1.22	1.20	1.54	1.8480
	9.20	.00	.50	1.22	.6100
					10.2700
	.00				
	.75	1.12	.75	1.12	.8400
	2.50	1.12	1.75	2.24	3.9200
(6) (1)	5.00	.60	2.50	1.82	4.5500
	7.50	.10	2.50	.70	1.7500
	8.75	.10	1.25	.20	.2500
	11.00	.90	2.25	1.00	2.2500
	11.40	.00	.40	.90	.3600
					13.9200

Area within base lines, A. 49.41253

Double area, cut off by

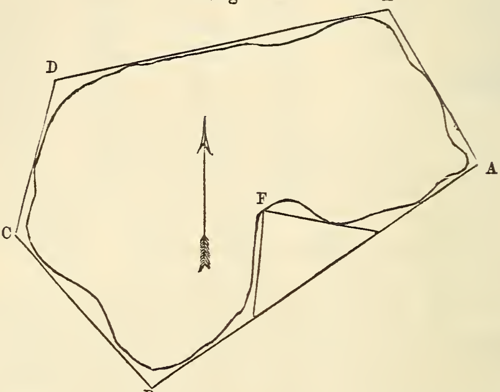
(1) (2)	4.58054
(2) (3)	1.07901
(3) (4)	.71769
(4) (5)	1.96940
(5) (6)	1.02700
(6) (1)	1.39200

$\frac{1}{2}$ of 10.76564 = 5.38282

Area of pond, 44.02971 = 44 A., 0 R., 4.75 P.

The following are the field-notes taken for the survey of a pond. The area is required. Fig. 164 is the plat, to a scale of 1 inch to 10 chains :—

Fig. 164.



Sta. C	
1090	
1050	50
937	5
675	70
475	85
275	10
150	15
45	95
Sta. B	┌ 82° 16′

Sta. B	
2100	
2035	55
1890	5
1660	25
1500	60
1460	
627	def. 44° 5′
585	82
350	10
185	25
20	15
Sta. A	S. 56° 37 W.

Sta. E	
1801	
1750	50
1600	10
1350	50
1150	50
900	10
650	20
450	5
275	25
55	92
Sta. D	┌ 63° 52′

Sta. D	
842	
785	90
650	37
400	0
225	40
15	57
Sta. C	┌ 56° 22′

	1460	AB
	580	
	560	70
	300	15
	150	20
92° 48′ ┐	Sta. F	

	Sta. F	
	627	0
	475	65
	250	0
	20	90
on AB	627	┌ 44° 5′

Sta. A	┌ { 87° 1′ on AB.
950	
900	20
750	70
500	0
400	0
300	30
150	25
27	70
Sta. E	┌ 70° 29′.

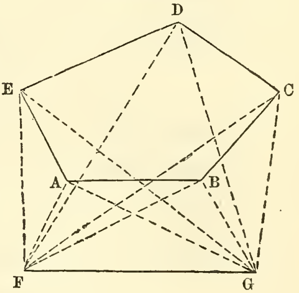
Area, 24 A., 3 R., 20 P.

364. *Compass Surveying by Triangulation.*

When the tract is bounded by straight lines, the area may be found by determining the position of each of the angular points with reference to one or more base lines properly chosen.

To do this, measure a base from the ends of which all the corners of the tract can be seen, and take their angles of position. There will thus be a system of triangles formed, giving data for calculating the content of the tract. Thus, if ABCDE (Fig. 165) represent a field, measure a base FG, and from F and G take the bearings, or the angles of position, of A, B, C, D, and E. Calculate FA, FB, FC, FD, FE, and thence the areas of the triangles FAB, FBC, FCD, FDE, and FEA.

Fig. 165.



Then, ABCDE = FBC + FCD + FDE − FEA − FAB.

EXAMPLE.

To determine the area of a field ABCDE, I measured a base line FG of 12.25 chains, and at F and G I took the angles of position, as follow:—GFA = 63° 15′, GFB = 27° 33′, GFC = 35° 35′, GFD = 58° 25′, GFE = 92° 10′, FGA = 26° 5′, FGB = 58° 30′, FGC = 97° 12′, FGD = 72° 28′, and FGE = 37° 32′. Fig. 165 is a plat of this tract, on a scale of 1 inch to 10 chains.

Calculation.

1. To find FA.

As sin. FAG	90° 40′	.000029
: sin. FGA	26° 5′	9.643135
: : FG	12.25	1.088136
: FA		0.731300

To find FB.

As sin. FBG	93° 57′	.001033
: sin. BGF	58° 30′	9.930766
: : FG		1.088136
: FB		1.019935

To find FC.

As sin. FCG	47° 13′	0.134347
: sin. FGC	97° 12′	9.996562
: : FG		1.088136
: FC		1.219045

To find FD.

As sin. FDG	49° 7′	0.121453
: sin. FGD	72° 28′	9.979340
: : FG		1.088136
: FD		1.188929

To find FE.

As sin. FEG	50° 18′	0.113848
: sin. FGE	37° 32′	9.784776
: : FG		1.088136
: FE		0.986760

To find 2 FAB.

sin. AFB	35° 42′	9.766072
FA		0.731300
FB		1.019935
2 FAB	32.9084	1.517307

To find 2 FBC.

sin BFC	8° 2′	9.145349
BF		1.019935
FC		1.219045
2 FBC	24.2286	1.384329

To find 2 FCD.

sin. CFD	22° 50′	9.588890
CF		1.219045
FD		1.188929
2 FCD	99.2805	1.996864

To find 2 FDE.

sin. DFE	33° 45′	9.744739
DF		1.188929
FE		0.986760
2 FDE	83.2585	1.920428

To find 2 FEA.

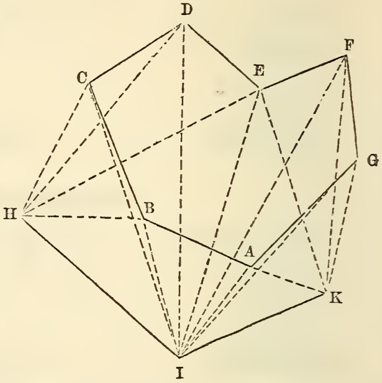
sin. AFE	28° 55′	9.684430
FE		0.986760
FA		0.731300
2 FEA	25.2633	1.402490

2 FBC		24.2286
2 FCD		99.2805
2 FDE		83.2585
		206.7676
2 FAB	32.9084	
2 FAE	25.2633	58.1717
		2) 148.5959
		74.29795 sq. ch.

= 7 A., 1 R., 28.76 P.

365. If no two points can be found from which all the corners can be seen, several points may be taken, and these all being connected by a system of triangles with a single measured base, or with several if suitable ground for measuring them can be found, the area may then be calculated.

Fig. 166.



Thus, (Fig. 166,) if ABCDEFG represent a tract, and H, I, and K, three points such that, from H, B, C, D, and E, can be seen. From I, all the corners can be seen, and from K we can see A, G, F, and E. If the angles of position of the corners relatively to the base lines HI and HK be taken, the distances IA, IB, IC, ID, &c. may be found, and thence the areas of IAB, IBC, ICD, &c.

Consequently, ABCDEFG = ICD + IDE + IEF + IFG − IGA — IAB − IBC becomes known.

366. The same principle may be applied to surveying a farm by means of one or more base lines within the tract. If such lines be run, and the corners be connected by triangles with given stations in these bases, the tract may be platted and the area calculated.

In all cases of the application of this method, care should be taken to have the triangles as nearly equilateral as possible. If any of the angles are very acute or very obtuse, the amount of error from any mistake in measuring the base or in taking the angles is much increased.

CHAPTER VI.

TRIANGULAR SURVEYING.

367. The method pursued in the last few articles of Chap. V. constitutes what is called triangular surveying. It consists in connecting prominent points with one or more base lines by means of a system of triangles,—the sides of these forming bases for other systems until the whole tract is covered.

The positions of these points having thus been accurately determined, the minuter configurations may be filled up by means of secondary triangles, or by any of the other methods of surveying of which we have already treated.

368. Base. In triangular surveying there is generally but a single base measured as a foundation for the work. This therefore requires to be measured with extreme care, since an error will vitiate the whole work. The precautions to insure extreme accuracy are such as almost to preclude the possibility of an error. Delambre, in speaking of a measurement of this kind in France, says,—

"To give some idea of the precision of the methods employed, it is sufficient to relate the following occurrence during the measurement of the base near Perpignan:—One day, a violent wind seemed every moment about to derange our rules, by slipping them on their supports. After having struggled a long time against these difficulties, we finally abandoned the work. Three days after, on a calm day, we recommenced the work of that whole day, and we only found a fourth of a line [one-twelfth of a French inch] difference between two measurements, with the one of which we were entirely satisfied, but of which the other was esteemed so doubtful that we considered it necessary to perform the whole work anew."

369. Reduction to the Level of the Sea. The base should if possible be measured on level ground. A smooth beach, if one can be found of sufficient length, affords one of the best locations. The work then requires no further reduction. If the ground is considerably elevated, the length must be reduced to what it would have been if the same arc of a great circle had been measured on the sea-level. This will be shorter than the measured arc in the ratio of the radius of the circle of which the measured arc forms part to that of the earth. Thus, suppose the arc was on ground elevated 300 feet, and a base of 5000 yards had been measured: then say, As 3912 miles + 300 feet : 3912 miles : : 5000 yards : the length required.

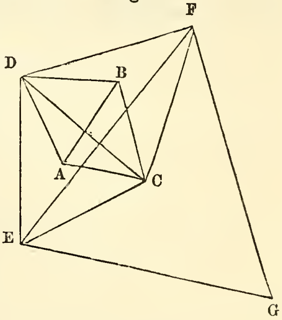
The radius used should be that belonging to the latitude in which the work was performed, it being different in different latitudes in consequence of the oblateness of the earth.

370. Signals. The base having been measured, the next object is to place signals on prominent points over the country. Any prominent object may be selected for this purpose. A tree on a hill, provided it stands so that its trunk is visible projected against the sky, the spire of a church or any other object so elevated as to be seen from a great distance, may be employed. It is in general best, however, to employ signals constructed expressly for the purpose. Perhaps one of the best is a tall mast with a flag floating from the top. The flag waving in the wind can frequently be seen when a still object would not attract the attention. The observation must, however, be taken to the centre of the mast, and not to the flag. The Drummond light, reflected in the proper direction by a parabolic mirror, is the best of all. Such a signal may be seen at the distance of sixty miles.

371. Triangulation. The signals having been placed, their relative position should then be determined by determining their angles of position with each other. In this triangulation it is very important to have all the triangles as nearly equilateral as possible. It is not always possible to obtain triangles so "well conditioned" as would be desirable. The rule should, however, be strictly observed never to employ a triangle with a very acute angle opposite to the known side, as a very small error in one of the adjacent angles will then produce a very sensible error in the calculated distance. For example, suppose the base AB were 500 yards long and the adjacent angles were A = 88° 39′ 15″ and B = 88° 17′ 45″; the third angle C would then be 3° 3′.

The calculated distance of AC would be 9394.6 yards: an error of 5″ in one of the observed angles would cause an error in this result of half a yard,—a quantity utterly inadmissible in operations of this nature.

Fig. 167.



The base generally being short, compared to the sides of the triangles which it is desirable to employ, these should be gradually enlarged, without allowing any of them to become "ill conditioned." The mode in which this is done may be seen from Fig. 167, in which AB is the base, on which two triangles ABC and ABD, both well conditioned, are founded. The line CD joining their vertices, becomes the base for two other triangles DCE and DCF, by means of which the line EF may be found.

The angles at all the points of the triangle should be measured. The sum of these should be 180°. If it differs but little, a few seconds, from this, the error should be distributed among the angles, giving one-third to each. If, however, a greater number of observations have been made at some stations than at others, they should have a proportionally less share of the error. Thus, suppose the recorded angle A is the mean of 5 observations, B the mean of 3, and C of 2: $\frac{2}{10} = \frac{1}{5}$ of the error should be applied to A, $\frac{3}{10}$ to B, and $\frac{5}{10}$ to C.

372. Base of Verification. In order to prove the correctness of the observations and calculations, some part of the work as distant as possible from the base should be connected with another carefully measured base,—the coincidence of the measured and calculated distance of which will prove the whole work. In a system of triangulation carried over the whole of France, a distance of more than 600 miles, the base of verification was found to be

	by calculation	38406.54 feet long,
and	by measurement	38407.5
The difference being only		.96 feet,

which was the total error arising from observations on more than sixty triangles. In the United States Coast Survey, carried on under the supervision of Prof. A. D. Bache, still greater accuracy has been obtained.

CHAPTER VII.

LAYING OUT AND DIVIDING LAND.

SECTION I.

LAYING OUT LAND.

Problem 1.—*To lay out a given area in the form of a square.*

373. Reduce the given area to square perches or square chains, and extract the square root. This root will be the length of one side. Erect perpendiculars at the ends equal to the base, and the thing is done.

The side of a square acre is 316.23 links = 12.65 poles = 69.57 yards.

Problem 2.—*To lay out a given area in the form of a rectangle, one side being given.*

374. Reduce the area to a denomination of the same name as the side. Divide the former by the latter, and the quotient will be the length of the other side.

Examples.

Ex. 1. Lay out 10 acres in a rectangular form, one side being 12 chains long. Required the other side.

Ans. 8.33 chains.

Ex. 2. What must be the length of one side of a rectangle, the area being 15 acres and one side 37.95 perches?

Ans. 63.24 perches.

Problem 3.—*To lay out a given area in a rectangular form, the adjacent sides to have a given ratio.*

375. Divide the given area expressed in square chains or square perches by the product of the numbers expressing the ratio. The square root of the quotient multiplied by these numbers respectively will give the length of the sides.

DEMONSTRATION.—If mx and nx represent the sides, and A the area, then will $mnx^2 = A$. Whence $x = \sqrt{\frac{A}{mn}}$.

EXAMPLES.

Ex. 1. Required to lay out an acre in a rectangular form, so that the length shall be to the breadth as 3 to 2. What must be the length of the sides?

Ans. 3.873 chains and 2.582 chains.

Ex. 2. It is desired to lay out a field of 10 acres in a rectangular form, so that the sides may be in the ratio of 4 to 5. What are these sides?

Ans. 8.944 chains and 11.18 chains.

Problem 4.—*To lay out a given area in a rectangular form, one side to exceed the other by a given difference.*

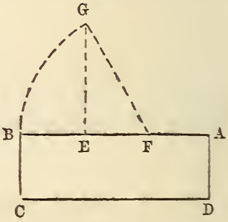
376. To the given area add the square of half the given difference of the sides. To the square root of the result add said half difference for the greater side, and subtract it for the less.

CONSTRUCTION.—Make AE (Fig. 168) equal to the given difference of the sides. Erect the perpendicular EG equal to the square root of the given area. Bisect AE in F, and make FB = FG: then will AB be the greater side, and BE the less.

For (29.6) $AB \cdot BE = EG^2$.

The rule may be proved thus: $FB^2 = FG^2 = GE^2 + EF^2$ = area + square of half the difference of the sides. Then, $AB = AF + FB$, $BC = FB - FE$.

Fig. 168.



EXAMPLES.

Ex. 1. It is desired to lay out 10 acres in the form of a rectangle, the length to exceed the breadth by 2 chains.

Ans. Length, 11.05 chains; breadth, 9.05 chains.

Ex. 2. Required to lay out 17 A., 3 R., 16 P. in a rectangular form, so that one side may exceed the other by 50 perches. Ans. Length 84, and breadth 34 perches.

Problem 5.—*To lay out a given area in the form of a triangle or parallelogram, the base being given.*

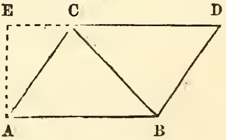
377. Divide the area of the parallelogram, or twice the area of the triangle, by the base. At any point of the base erect a perpendicular equal to the quotient. The summit will be the vertex of the triangle, or the end of a side of the parallelogram.

If through the summit of the perpendicular a parallel to the base be drawn, any point in that parallel may be taken for the vertex of the triangle.

Problem 6.—*To lay out a given area in the form of a triangle or parallelogram, one side and the adjacent angle being given.*

378. As the rectangle of a given side and sine of the given angle is to twice the area of the triangle or the area of the parallelogram, so is radius to the other side adjacent to that angle.

Fig. 169.



DEMONSTRATION.—By Art. 357 we have (Fig. 169) r : sin. A : : AB . AC : 2 ABC, or (1.6) r . AB : sin. A . AB : : AB . AC : 2 ABC; whence sin. A . AB : 2 ABC : : r . AB : AB . AC : : r : AC.

EXAMPLES.

Ex. 1. Required to lay out 43 A., 2 R. in the form of a parallelogram, one side AB being 54 chains, and the adjacent angle BAC 63°.

As AB . sin. A { AB		54	A. C.	8.267606
{ sin. A		63°	"	0.050119
: ABCD	435 ch.			2.638489
:: r				10.000000
: AC	9.04 ch.			1.956214

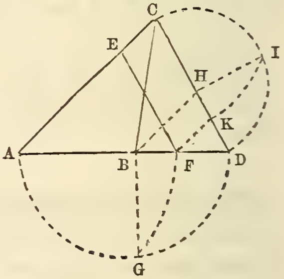
Ex. 2. Required to lay out 3.5 acres in the form of a triangle, one side being 11.25 chains, and the adjacent angle 73° 25′. Ans. AC 6.49 chains.

Ex. 3. Given AB N. 85° W. 16.37 chains, BD S. 32¼° W., to determine its length so that the parallelogram ABCD may contain 16 acres. Ans. BD = 10.99 chains.

Ex. 4. The bearings of two adjacent sides of a tract of land being N. 85° E. and S. 23° E., required to lay off 10 acres by a line running from a point in one side 14.37 chains from the angle and falling on the other side.

Ans. Distance, 14.63 chains.

Fig. 170.



379. *Lemma.*—If ABC (Fig. 170) be any triangle, and CD a line in any direction from the vertex cutting AB in D, and if AF be taken a mean proportional between AB and AD, and FE be drawn parallel to DC, the triangle AFE = ABC.

DEMONSTRATION.—Since AD : AF :: AF : AB, we have
(Cor. 2, 20.6) AD : AB :: ADC : AFE;
but (1.6) AD : AB :: ADC : ABC,
therefore ABC = AFE.

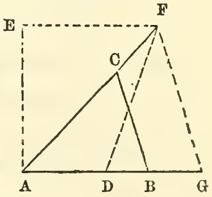
The above lemma will be found very useful in the constructions required in dividing land.

The mean proportional AF may be found by describing a semicircle on AD, erecting a perpendicular BG, and making AF = AG; or, if the point A is remote, we may draw BH parallel to AC, meeting CD in H; draw HI perpendicular to CD, cutting the semicircle on CD in I; make

CK = CI, and draw KF parallel to CA. Then, since BH and FK are parallel to AC, the line AD is divided similarly to CD (10.6); but CK is a mean proportional between CH and CD, therefore AF is a mean proportional between AB and AD.

380. Problem 7.—*Two adjacent sides of a tract of land being given in direction, to lay off a given area by a line running a given course.*

Fig. 171.



CONSTRUCTION.—Take AD (Fig. 171) any convenient length. Erect the perpendicular $AE = \frac{2\ \text{Area}}{AD}$. Draw the parallel EF cutting AF in F. Run FG the given course. Take AB a mean proportional between AD and AG or $= \sqrt{AD \cdot AG}$. Then BC parallel to GF will be the division line.

For, by construction, ADF = the given area, and, by lemma, ABC = ADF.

AB may be calculated by the following rule:—

As the rectangle of the sines of the angles adjacent to the required side is to the rectangle of radius and the sine of the angle opposite to that side, so is twice the area to be cut off to the square of that side.

The truth of this rule is evident from Art. 358.

EXAMPLES.

Ex. 1. Given AB S. 63° E. and AC N. 47° 15′ E., to lay off 7 acres by a line BC running due north. Required the distance on the first side.

Here the angles are A = 69° 45′, B = 63°, and C = 47° 15′. Whence

As { sin. A		69° 45′	Ar. Co.	0.027709
sin. B		63°	" "	0.050119
: { rad.				10.000000
sin. C		47° 15′		9.865887
::	2 ABC	140 chains		2.146128
:	AB^2			2)2.089843
	AB	11.09		1.044921.

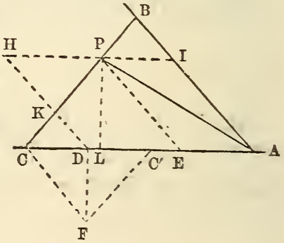
Ex. 2. Given the bearings of two adjacent sides, taken at the same station, N. 57° 15′ W. and N. 45° 30′ E., to determine the distance from the angular point of a station on the first side from which a line running N. 77° E. will cut off 5 acres. Ans. 8.648 chains.

Ex. 3. Given AB S. 57° E. and AC S. 5° 16′ W., to lay off 12 acres by a line running N. 75° E. Required the distance on the first side. Ans. 18.50 chains.

381. Problem 8.—*The directions of two adjacent sides of a tract of land being given, to lay off a given area by a line running through a given point.*

CONSTRUCTION.—Divide the given area by the perpendicular distance from P to AB, (Fig. 172.) Lay off AD equal to the quotient. Draw PE parallel to AB. Make DF perpendicular to AD and equal to AE. Lay off FC = DE. Then CPB will be the division line.

Fig. 172.



DEMONSTRATION.—Complete the parallelogram ADHI.

By construction, APD is half the required area; and, therefore, AIHD contains the required area.

Now, because the triangles PIB, HPK, and CDK are similar, and their homologous sides IP, DC, and HP are equal to the three sides DF, DC, and CF of the right-angled triangle DCF, we shall have (31.6) HPK = PBI + CDK. To these equals add AIPKD, and we have AIHD = ABC; whence ABC contains the required area.

If the directions of AB and AC and the position of the point P be given by bearings, AC may be calculated as follows:—In API find PI; also find the perpendicular PL. Then AD = area ÷ PL. Then in DFC we have DF = PI and FC = DE to find DC, which added to AD will give AC.

If FC be laid off on both sides, another point C′ will be determined, through which the line may run.

EXAMPLES.

Ex. 1. Given the bearings of AB N. 34° W., and of AC West, to lay off 18 acres by a line running through a point P bearing from A N. 41° W. 18.85 chains.

To find PI.

As sin. I	56°	A. C. 0.081426
: sin. PAI	7°	9.085894
: : AP	18.85	1.275311
: PI	2.77	0.442631

To find PL and AD.

As rad.		A. C. 0.000000
: sin. PAL	49°	9.877780
: : PA	18.85	1.275311
: PL		1.153091
Given area,	180 ch.	2.255273
AD	12.65	1.102182;

whence ED = AD − PI = 12.65 − 2.77 = 9.88.

To find DC.

FC + FD =	12.65	1.102182
FC − CD =	7.11	0.851870
		2) 1.954052
DC =	9.485	.977026;

therefore AC = AD + DC = 12.65 + 9.485 = 22.135 ch.

Ex. 2. Given the angle BAC = 85°, to lay off 6 acres by a line through a spring the perpendicular distances of which from AB and AC are 3.25 chains and 7.92 chains respectively. Required AC.

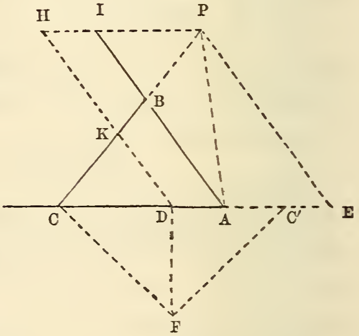
Ans. AC = 10.43 chains.

Ex. 3. A has sold B $3\frac{1}{2}$ acres, to be laid off in a corner of a field, by a line through a tree bearing North 5.64 chains from the angular point. Now, the bearings of the sides being N. 46° 15′ E. and N. 42° W., it is required to find the distance to the division line, measured on the first side.

Ans. 10.58 ch.

382. If the point P were exterior to the angle, the construction and calculation would be perfectly analogous to the preceding. The following is an example:—

Fig. 173.



Given the angle A = 60°, (Fig. 173,) EAP = 90°, and AP = 23.42 chains, to cut off 14 A. by a line running through P.

Make $AD = \frac{140}{23.42} = 5.98.$

Draw PE parallel to AB. Erect the perpendicular DF = AE, and make FC = ED. Then CB will be the division-line.

For, as before, AIHD = the given area; but PKH = PBI + CKD; ∴ HIBK = CKD, and AIHD = ABC.

$$r : \tan. 30 :: AP\ (23.42) : AE = DF = 13.52;$$

whence $CF = DE = AE + AD = 19.50,$

and $DC = \sqrt{CF^2 - FD^2} = \sqrt{33.02 \times 5.98} = 14.05;$

∴ $AC = 5.98 + 14.05 = 20.03$ chains.

Problem 9.—*Three adjacent sides of a tract of land being given in position, to lay off a given area in a quadrilateral form by a line running from the first side to the third.*

CASE 1.

383. *The division line to be parallel to the second side.*

Fig. 174.

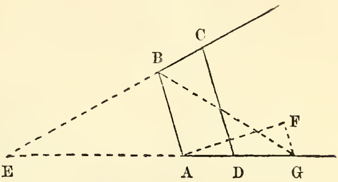
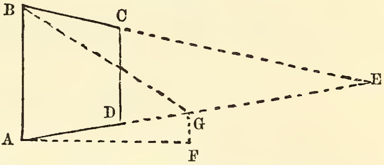


Fig. 175.



Conceive the lines CB and DA (Figs. 174, 175) to be produced till they meet, and calculate the area of ABE. Add this to the given area if the sum of the angles A and B is greater than 180°, as in Fig. 174; but if the sum be less, as in Fig. 175, subtract ABCD from ABE: the remainder will be the area of ECD.

Then say, As EAB is to ECD, so is AB^2 to CD^2. And, as sin. E is to sine of B, so is AB $\backsim$ CD to AD.

The following is a neat construction:—

Produce HB and GA to meet in E. Erect AF perpendicular to AB, and equal to double the area divided by AB. Draw FG parallel to AB, meeting AE in G. Then the triangle ABG will contain the required area. Take ED a mean proportional between EA and EG, or let ED = $\sqrt{EA \, . \, EG}$. Through D draw the division line CD: ABCD will contain the required area. For (lemma) ECD = EBG; whence ABCD = ABG.

The calculation is more concisely made by the following rule:—

As the rectangle of the sines of the angles A and B is to the rectangle of radius and the sine of E, so is twice the given area to the difference between AB^2 and CD^2.

This difference, added to AB^2 when the sum of the angles A and B is greater than 180°, but subtracted when the sum is less, will give CD^2.

Then, As sine of E is to the sine of B, so is the difference between CD and AB to the distance AD.

DEMONSTRATION. — $ECD : EBA :: CD^2 : AB^2$;

Whence, by division, $ABCD : EBA :: CD^2 \backsim AB^2 : AB^2$;

consequently, $2\,ABCD : 2\,EBA :: CD^2 \backsim AB^2 : AB^2$,

and $2\,EBA : AB^2 :: 2\,ABCD : CD^2 \backsim AB^2$.

But (Art. 380) sin. A . sin. B : rad. sin. E :: $2\,EBA : AB^2$;

whence sin. A . sin. B : rad. sin. E :: $2\,ABCD : CD^2 \backsim AB^2$.

EXAMPLES.

Ex. 1. Given—1. N. 62° 15′ E.; 2. N. 19° 12′ W. 7.92 chains; 3. S. 87° W., to cut off 5 acres by a line parallel to the second side. Required the length of the division line, and the distance on the first side.

First Method.—To find ABE, (Art. 358.)

As	rad.		A. C. 0.000000
	sin. E	24° 45′	0.378139
:	sin. A	98° 33′	9.995146
	sin. B	106° 12′	9.982404
::	AB	7.92	0.898725
	AB		0.898725
:	2 ABE	142.278	2.153139
	2 ABCD	100	
	2 ECD	242.278	

As	2 ABE	142.278	A. C. 7.846861
:	2 ECD	242.278	2.384314
::	AB^2	7.92	0.898725
		7.92	0.898725
:	CD^2		2)2.028625
	CD	10.335	1.014312

As	sin. E	24° 45′	A. C. 0.378139
:	sin. B	106° 12′	9.982404
::	CD − AB	2.415	0.382917
:	AD	5.539	0.743460

Second Method.

As	sin. A	98° 33′	A. C. 0.004854
	sin. B	106° 12′	0.017596
:	rad.		10.000000
	sin. E		9.621861
: :	2 ABCD	100 ch.	2.000000
:	$CD^2 - AB^2$	44.087	1.644311
	AB^2	62.7264	

Whence CD $= \sqrt{106.8134} = 10.335$, as before.

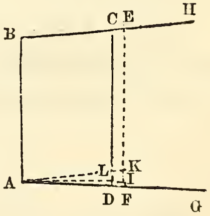
Ex. 2. Given—1. N. 26° 47′ W.; 2. N. 63° 13′ E. 12.72 chains; 3. S. 8° 17′ E., to cut off 7 acres by a line parallel to the second side. The distance on the first side and the length of the division line are required.

Ans. Division line, 10.72 chains; distance, 5.98 ch.

Ex. 3. Given the bearing of three sides of a tract of land, and the length of the middle one, as follow,—viz.: 1. N. 15° 30′ W.; 2. N. 74° 30′ E. 11.60 chains; 3. S. 45° E.: to cut off 12 acres by a line parallel to the second side. The division line and distance on the first side are required.

Ans. Division line, 16.44 chains; distance, 8.555 ch.

Fig. 176.



384. If AD and BC (Fig. 176) are nearly parallel, the following method may be employed with advantage:—

Divide the area by AB: the quotient will give the approximate length of the perpendicular AI. Draw FE parallel to AB, and AK parallel to BH. In AIK and AIF find IK and IF.

$$FK = FI \pm IK, \text{ and } FE = AB \pm FK.$$

If the sum of the angles is greater than 180°, the area cut off by EF will be too great by the small triangle AFK $= \frac{FK \cdot AI}{2}$. Make $IL = \frac{AFK}{FE} = \frac{FK \cdot AI}{2\,FE}$. Then will AL be the corrected perpendicular: AD may thence be found.

Examples.

Ex. 1. Given GA N. 87° W., AB N. 5° W. 14.25 chains, and BH S. 89° E., to lay off 10 acres by a line parallel to AB.

Here the angles are A = 98° and B = 84°: AK will therefore lie between I and F.

$$AI = \frac{100}{14.25} = 7.02 \text{ chains, nearly.}$$

In IAF we have IAF = 8° and IA = 7.02; whence IF = .987.

In IAK we have IAK = 6° and IA = 7.02; whence IK = .738.

Whence $\quad KF = .25$ and $EF = 14.50.$

Hence $\quad IL = \dfrac{KF \,.\, AI}{2\, EF} = .06$ chains,

and $\quad AL = 7.02 - .06 = 6.96$ chains;

whence $\quad AD = 7.03$ chains.

The above method is very convenient for field operations. EF may be measured *directly* on the ground; whence FK is known, and $IL = \dfrac{FK \,.\, AI}{2\, FE}$, as before.

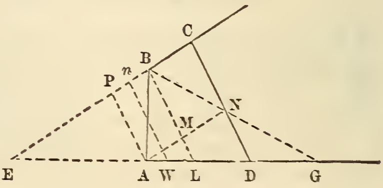
Ex. 2. Given GA North, AB N. 89° E. 7.86 chains, and BC S. 1° 30′ W., to cut off 10 acres by a line parallel to AB. Required the distance of the division line from A.

Ans. 13.00 ch.

CASE 2.

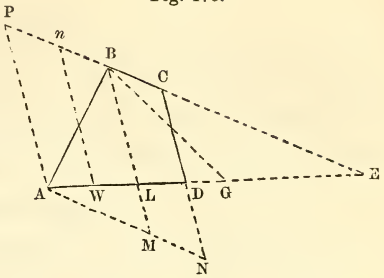
385. *By a line running a given course.*

Fig. 177.



Construct, as in last case, ABG to contain the given area. Draw BL according to the given course. Take ED a mean proportional between EL and EG: CD parallel to BL will be the division line. For, by the lemma, ECD = EBG; whence ABCD = ABG, the required area.

Fig. 178.



The calculation may be performed by the finding AE and the area of ABE; whence ECD becomes known. The distance ED may then be found by Art. 380; or,

Conceive Wn to be drawn parallel to CD, making EWn = EAB. Then say, As the rectangle of the sines of the angles C and D is to the rectangle of the sines of A and B, so is the square of AB to the square of Wn.

And, As the rectangle of the sines of C and D is to the rectangle of radius and sine of E, so is twice the given area to a fourth term.

If the sum of the angles A and B is greater than 180°, add these fourth terms together; but, if the sum of A and B is less than 180°, subtract the second fourth term from the first: the result will be the square of the division line CD.

Then, As sine of C is to sine of B, so is AB to a fourth term; take the difference between this fourth term and CD, and say, As sine of E is to the sine of C, so is this difference to AD.

DEMONSTRATION.—Since EnW = EAB, EW is a mean proportional between EA and EL. Whence nW is a mean proportional between AP and BL; therefore AP . BL = $n\text{W}^2$.

Now, by similar triangles, we have

$$\text{sin. L (sin. D) : sin. A :: AB : BL,}$$

and

$$\text{sin. P (sin. C) : sin. B :: AB : AP.}$$

Whence (23.6) sin. C . sin. D : sin. A . sin. B : : AB^2 : AP . BL = $n\text{W}^2$;

and, by demonstration to last case,

$$\text{sin. C . sin. D : rad. sin. E :: 2}\,n\text{WDC : CD}^2 \backsim n\text{W}^2.$$

Draw AMN parallel to BC. Then, in the triangle ABM, we have

$$\text{sin. M (sin. C) : sin. BAM (sin. B) :: AB : BM;}$$

and, in AND, we have

$$\text{sin. NAD (sin. E) : sin. N (sin. C) :: DN (CD} \backsim \text{BM) : AD.}$$

EXAMPLES.

Ex. 1. Given—1. N. 62° 15′ E.; 2. N. 19° 12′ W. 7.92 chains; 3. S. 87° W., to cut off 5 acres by a line perpendicular to the first side. Required the length of the division line, and its distance from the end of the first side.

First Method.

As sin. E	24° 45′	Ar. Co. 0.378139
sin. B	106° 12′	9.982404
AB	7.92	0.898725
EA	18.166	1.259268
AB		0.898725
sin. A	98° 33′	9.995146
2 ABE	142.278	2.153139
2 ABCD	100	
2 ECD	242.278	

Then, (Art. 380,)

As { sin. E	24° 45′	Ar. Co.	0.378139
sin. D	90°	" "	0.000000
: { rad.			10.000000
sin. C	65° 15′		9.958154
:: 2 ECD	242.278		2.384314
: ED^2			2)2.720607
ED	22.93		1.360303
AE	18.17		
AD	4.76		

As sin. C	65° 15′	Ar. Co. 0.041846
: sin. E	24° 45′	9.621861
:: ED		1.360303
: CD	10.57	1.024010

Second Method.

As $\begin{cases}\text{sin. C}\\ \text{sin. D}\end{cases}$	65° 15′ 90°	Ar. Co. 0.041846 " " 0.000000	
: $\begin{cases}\text{sin. A}\\ \text{sin. B}\end{cases}$	98° 33′ 106° 12′	9.995146 9.982404	
:: $\begin{cases}\text{AB}\\ \text{AB}\end{cases}$	7.92 chains "	0.898725 0.898725	
: nW^2	65.5913	1.816846	

As $\begin{cases}\text{sin. C}\\ \text{sin. D}\end{cases}$		Ar. Co. 0.041846 " " 0.000000
: $\begin{cases}\text{rad.}\\ \text{sin. E}\end{cases}$	 24° 45′	10.000000 9.621861
:: 2 ABCD	100 chains	2.000000
: $CD^2 - nW^2$	46.1006	1.663707
nW^2	65.5913	
CD =	$\sqrt{111.6919} = 10.57.$	

As sin. C	65° 15′	Ar. Co. 0.041846
: sin. B	106° 12′	9.982404
:: AB	7.92	0.898725
: BM	8.375	0.922975
CD	10.57	
DN	2.195	

As sin. E	24° 45′	Ar. Co. 0.378139
: sin. C	65° 15′	9.958154
:: DN	2.195	0.341435
: AD	4.76	0.677728

It will be seen from the above that the first method is in this case the shorter. It has the advantage, also, of first giving the value of AD, which of itself is sufficient to determine the position of the division line.

In the second method, if AG and BH are nearly parallel, the calculation for CD and DN should be carried to the third decimal figure.

The construction given for this and the preceding case admits of easy application on the ground.

Run the lines CB and GA to their point of intersection; lay out the perpendicular AF; run FG parallel to AB and BL parallel to the division line. Measure EL and EG, and make $ED = \sqrt{EL \, . \, EG}$.

Ex. 2. The bearings of three adjacent sides of a tract of land are—1. N. 26° 47′ W.; 2. N. 63° 13′ E. 12.72 chains; 3. S. 8° 17′ E., to cut off 7 acres by a line running due east. The distance on the first side and the length of the division line are required.

Ans. Distance, 3.37; division line, 11.11.

Ex. 3. The bearings of three adjacent sides of a tract of land being—1. N. 78° 17′ E; 2. N. 5° 13′ E. 15.62 chains; and 3. N. 63° 43′ W., it is desired to cut off 10 acres by a line making equal angles with the first and third sides. What is the bearing of the division line, and its distance from the end of the first side?

Ans. Bearing, N. 30° 43′ E.; distance on first side, 6.316.

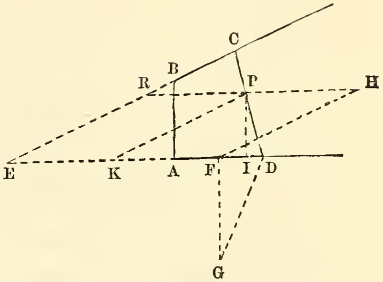
If the first and third sides are nearly parallel, the area of ABL may be calculated. This taken from ABCD, or added to it, according as BL falls within or without the tract, will give the area of BLDC, which may be parted off as directed in Art. 384.

CASE 3.

386. *By a line through a given point.*

Produce CB and DA (Fig. 179) to meet in E, and calculate the area EAB. Thence ECD is found. Proceed as in Art. 381. Thus, calculate or measure the perpendicular PI. Lay off $EF = \frac{ECD}{PI}$. Draw PK parallel to BE, meeting AE in K. Erect the perpendicular FG = EK or RP, and make GD = FK. Then will the division line pass through D.

Fig. 179.



Calculation.

Determine AE. Then $ED = EF + \sqrt{FK^2 - EK^2}$, and $AD = ED - EA$.

EXAMPLES.

Ex. 1. Given DA West, AB N. 16° 15′ W. 6.30 chains, BC N. 57° E., to cut off 3 acres by a line through a spring P, situated N. 25° 30′ E. 6.09 chains from the corner A.

To find EA, EAB, and ECD.

As sin. E	33°	Ar. Co. 0.263891
: sin. B	73° 15′	9.981171
:: AB	6.30	0.799341
: EA	11.077	1.044403
AB	6.30	0.799341
sin. A	73° 45′	9.982294
2 EAB	66.994	1.826038
2 ABCD	60.	
2 ECD =	126.994.	

To find PI and EF.

As rad.		Ar. Co. 0.000000
: sin. PAI	64° 30′	9.955488
: : AP	6.09	0.784617
: PI	5.497	0.740105
ECD	63.497	1.802753
EF	11.552	1.062648

To find AK, EK, and KF.

As sin. K	33°	Ar. Co. 0.263891
: sin. APK	31° 30′	9.718085
: : AP	6.09	0.784617
: AK	5.842	0.766593
AE	11.077	
EK = FG =	5.235	

Whence KF = GD = EF − EK = 6.317.

To find FD.

GD + GF	11.552	1.062648
GD − GF	1.082	0.034227
		2)1.096875
FD =	3.535	.548437

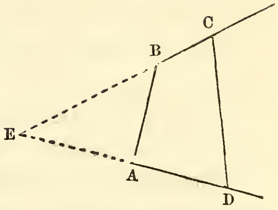
Whence AD = EF + FD — EA = 4.01.

Ex. 2. The bearings of three adjacent sides of a tract of land are as follow,—viz.: DA N. 47° E., AB N. 35° 16′ W. 15.23 chains, and BC S. 36° W., to cut off 15 acres by a line running through a spring P 9.22 chains distant from the first, and 10.55 chains from the second, side. The distance of the division line from the end of the first side is required. Ans. 10.82 chains from A.

CASE 4.

387. *By the shortest line.*

Fig. 180.



Produce the lines CB and DA (Fig. 180) to meet in E, and calculate ABE and AE, whence ECD is known. Now, the shortest line cutting off a given area will make equal angles with the sides. Therefore EC $= \text{ED}$. But $2\,\text{ECD} = \dfrac{\text{EC}\,.\,\text{ED}\,.\,\sin.\text{E}}{\text{R}}$ $= \dfrac{\text{ED}^2\,.\,\sin \text{E}}{\text{R}}$.

whence we must have $\text{AD} = \text{EA} \backsim \sqrt{\dfrac{\text{R}\,.\,2\,\text{ECD}}{\sin.\,\text{E}}}$.

Or, this case may be constructed and calculated as Case 2 by drawing BL so as to make the angles EBL and ELB equal.

Ex. 1. Given DA N. 86° W., AB N. 19° 20′ E. 16.75 ch., and BC N. 63° 30′ E., to cut off 15 acres by the shortest line. The distance on AD and the bearing of the division line are required.

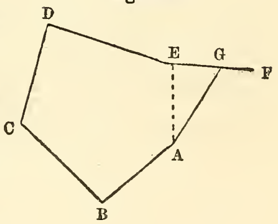
AD = 13.38; bearing of DC, N. $11\frac{1}{4}$° W.

Problem 10.—*To cut off a plat containing a given area from a larger tract of any number of sides.*

CASE 1.

388. *When the division line is to be drawn from one of the angles.*

Fig. 181.



Find by trial the side EF (Fig. 181) on which the division line will fall, and calculate the area ABCDE: subtract this area from that required; the remainder will be the area of AEG, which may be laid off as in Prob 6, Art. 378. Or,

The course and distance may be calculated directly as follows:—

Change the bearings so that the side on which the division line will fall may be a meridian.

Take out the latitudes and departures. The difference between the sums of the eastings and westings will be the departure of the division line.

Find the multipliers, assuming that corresponding to the division line to be O.

Multiply the known latitudes by the multipliers, and place the products in the columns of areas.

Subtract the difference of the sums of the north and south areas from double the required area: the remainder will be the area corresponding to the side on which the division line will fall. Divide this area by the multiplier: the quotient will be the latitude of that side. Place it in its proper column.

Take the difference between the sums of the northings and southings: this difference will be the latitude of the division line. With this latitude and the departure before determined calculate the distance and changed bearing, from which the real bearing is readily determined.

EXAMPLE.

Ex. 1. Let the bearings and distances be as follows:—1. S. 47½° W. 12.21 ch.; 2. N. 49° W. 15.28 ch.; 3. N. 13° E. 13.18 ch.; 4. S. 76½° E. 17.95 ch.; 5. S. 89¾° E., to cut off 35 acres by a line from the first angle and falling on the last side. Required the distance on the last side.

First Method.

	Bearings.	Dist.	N.	S.	E.	W.	E. D. D.	W.D.D.	Mult.	N.Areas	S. Areas.
AB	S.47½° W.	12.21		8.25		9.00		8.88	0000		
BC	N. 49° W.	15.28	10.02			11.53		20.53	20.53 W.		205.7106
CD	N. 13° E.	13.18	12.84		2.96			8.57	29.10 W.		373.6440
DE	S.76½° E.	17.95		4.19	17.45		20.41		8.69 W.	36.4111	
EA				(10.42)	(.12)		17.57		8.88 E.		92.5296
			22.86	22.86	20.53	20.53					671.8842
											36.4111
									2 ABCDE		635.4731
									2 ABCDEG		700
									2 AEG		64.5269

As diff. lat. EA	10.42	A. C. 8.982132
: dep.	.12	1.079181
:: rad.		10.000000
: tan. bear. EA	S. 0° 40′ E.	8.061313
Bear. EF	S. 89° 45′ E.	
AEF =	89° 5′	

As cos. bearing	0° 40′	A. C. 0.000029
: rad.		10.000000
:: diff. lat.		1.017868
: dist.	10.42	1.017897

Then, (Art. 378,)

As { AE	10.42	A. C. 8.982103
sin. AEG	89° 5′	" " 0.000056
: 2 AEG	64.5269	1.809741
:: r		10.000000
: EG	6.19	0.791900

Second Method.

	Bearings.	Changed Bearings.	Dist.	N.	S.	E.	W.	E. D. D.	W. D. D.	Multipliers.	N. Areas.	S. Areas.
AB	S. $47\frac{1}{2}°$ W.	N. $42\frac{3}{4}°$ W.	12.21	8.97			8.29		18.71	18.71 W.		167.8287
BC	N. 49° W.	N. $40\frac{3}{4}°$ E.	15.28	11.58		9.97		1.68		17.03 W.		197.2074
CD	N. 13° E.	S. $77\frac{1}{4}°$ E.	13.18		2.91	12.85		22.82		5.79 E.		16.8489
DE	S. $76\frac{1}{2}°$ E.	S. $13\frac{1}{4}°$ W.	17.95		17.47		4.11	8.74		14.53 E.		253.8391
EF	S. $89\frac{3}{4}°$ E.	South.	(6.17)		(6.17)		0.00		4.11	10.42 E.		
							(10.42)		10.42	000		00.0000

```
            635.7241
            700
10.42 ) 64.2759(6.17
         6252
          1755
          1042
           7139
           7294
```

Ex. 2. Given as follows:—1. N. $27\frac{1}{4}°$ W. 5 ch.; 2. N. 58° W. 9.53 ch.; 3. N. $42\frac{1}{2}°$ E. 9.60 ch.; 4. S. $81\frac{1}{4}°$ E. 14 ch.; 5. S. $28\frac{1}{2}°$ E.: to lay off 25 acres by a line from the first station. The distance on the fifth side is required.

Ans. 10.76 ch.

CASE 2.

389. *The division line to run a given course.*

Proceed as in Case 1 to find the area of the tract to a line through the ends of the sides on which the division line will fall, and the bearing and distance of the closing line. The difference between this area and the area to be laid off will be the area of a quadrilateral which may be parted off as in Art. 385.

EXAMPLES.

Ex. 1. The boundaries of a tract of land are as follows,—viz.: 1. N. 75° E. 13.70 ch.; 2. N. $20\frac{1}{2}°$ E. 10.30 ch.; 3. East 16.20 ch.; 4. S. $33\frac{1}{2}°$ W. 35.20 ch.; 5. S. 76° W. 16.00 ch.; 6. North 9.00 ch.; 7. S. 84° W. 11.60 ch.; 8. N. $53\frac{1}{4}°$ W. 11.60 ch.; 9. N. $36\frac{3}{4}°$ E. 19.60 ch.; 10. N. $22\frac{1}{2}°$ E. 14.00 ch.; 11. S. $76\frac{3}{4}°$ E. 12.00 ch.; 12. S. 15° W. 10.85 ch.; 13. S. 18° W. 10.62 ch. It is required to lay off 35 acres from the eastern end of the farm by a line perpendicular to the first side. The distance of the division line from the second corner is required.

Fig. 182.

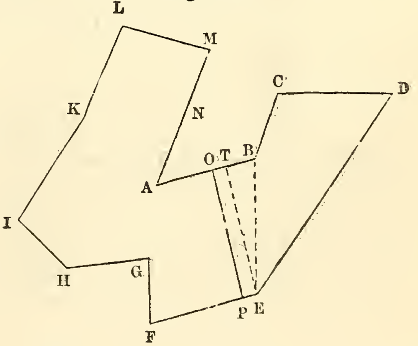


Fig. 182 is a plat of this tract.

To find BCDE and the bearing and distance of EB.

Sta.	Bearings.	Dist.	N.	S.	E.	W.	E.D.D.	W.D.D	Multipl'r.	Areas.
BC	N. 20½° E.	10.30	9.65		3.61		3.23			
CD	East.	16.20			16.20		19.81		19.81 E.	
DE	S. 33½° W.	35.20		29.35		19.43		3.23	16.58 E.	486.6230
EB			19.70			.38		19.81	3.23 W.	63.6310.

2) 550.2540

275.1270

Latitude of EB	19.70	A. C. 8.705534
Departure of EB	.38	− 1.579784
Tangent of bearing N. 1° 6′ W.		8.285318
Cosine of bearing		A. C. 0.000080
Latitude		1.294466
Distance EB	19.70	1.294546

Now, AB differing in course from FE by only 1°, the following is the best method of determining the position of the division line OP, which, by the conditions, is to be perpendicular to AB.

Draw ET perpendicular to AB, and find ET and BT: then will $BO = \frac{1}{2} BT + \frac{OBEP}{ET}$, very nearly.

To find BT and EF.

cos. EBT	76° 6′	9.380624
EB		1.294546
BT	4.733	0.675170
sin. EBT		9.987092
EB		1.294546
ET	19.127	1.281638
OBEP = 350 − 275.1270 =	74.873	1.874325
	3.915	0.592687
½ BT	2.366	
OB	6.281	

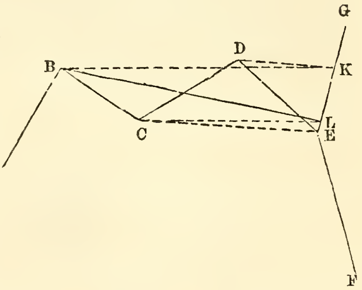
Ex. 2. The boundaries of a tract of land being as follow,—viz.: 1. N. 39° E. 12.17 chains; 2. S. 88$\frac{3}{4}$° E. 14.83 chains; 3. N. 67$\frac{1}{2}$° E. 13.32 chains; 4. S. 27$\frac{1}{4}$° E. 16.67 chains; 5. S. 57$\frac{1}{2}$° W. 21.92 chains; 6. S. 73° W. 18.23 chains; 7. S. 52$\frac{1}{4}$° W. 12.00 chains; 8. N. 37° W. 22.72 chains; 9. N. 67$\frac{1}{2}$° E. 18.00 chains,—to cut off 55 acres from the east end by a line bearing S. 37° E. Required the position of the point at which the line must commence.

Ans. On the first side, at 9.21 chains from the beginning.

Problem 11.—*To straighten boundary lines.*

390. It often becomes necessary to straighten crooked boundaries between farms, so as to leave the same quantity of land in each farm.

Fig. 183.



First Method.—If the tracts are platted, this may be done approximately by parallels. Thus, suppose BCDE (Fig. 183) was the common boundary of two farms, and it is agreed by the owners to run a straight fence from B to fall somewhere on EG. Join CE, and draw DK parallel to it; then join BK, and draw CL parallel thereto: BL will be the line required. In open ground, this work may be performed in the field by the transit.

391. *Second Method.*—Where the lines are straight, the method of latitudes and departures will enable us to run the line with accuracy. For it is evident that, if we calculate the area contained by the boundaries BCDELB, it should be 0, since the new line is intended to add to the contents of neither farm. The calculation would therefore be precisely the same in principle as in Art. 388.

Examples.

Ex. 1. Given BC S. 61° E. 16.50 chains; CD N. $53\frac{1}{4}$° E. 20.05 chains; DE S. 51° E. 18.42 chains; EG N. $10\frac{1}{2}$° E.

Rule a table as below. Then change the bearing so that the side on which the new line will fall shall be a meridian. Take out the latitudes and departures: the difference between the sums of the eastings and westings will be the departure of the new line. Find the double departures and the multipliers, assuming that corresponding to the first side equal to its double departure: that corresponding to the division line will thus be 0. Find the areas: the difference between the north and the south areas will be the area corresponding to the side on which the line will fall. Divide this area by the multiplier of that side: the quotient will be the difference of latitude of that side, which, as the changed bearing was north, will also be equal to its distance. By balancing the latitudes we may obtain the difference of latitude of the new line, and thence calculate its distance if desired.

Sta.	Bearing.	Ch. Bearing.	Dist.	N.	S.	E.	W.	E. D. D.	W. D. D.	Multipliers.	N. Areas.	S. Areas.
B	S. 61° E.	S. 71½° E.	16.50		5.24	15.64			29.80	29.80 W.	156.1520	
C	N. 53¼° E.	N. 42¾° E.	20.05	14.72		13.61		29.25		.55 W.		8.0960
D	S. 51° E.	S. 61½° E.	18.42		8.79	16.19		29.80		29.25 E.		257.1075
E	N. 10½° E.	North.		(2.40)				16.19		45.44 E.		
L					(2.09)		(45.44)		45.44	0.00 E.		

265.2035
156.1520
45.44) 109.0515 (2.40
90.88
18.171
18.176

Diff. Lat. 2.09 0.320146
Depart. 45.44 1.657438
tan. Ch. Bear. 87° 21′ 11.337292
10° 30′
97° 51′
180°
FB N. 82° 09′ W.
Dist. BF $= \sqrt{45.44^2 + 2.09^2} = 45.49.$

Ex. 2. Required to straighten the north boundary of the tract the field-notes of which are given Ex. 1, Art. 389, the new line to run from a point five chains from the beginning of the tenth side. The bearing and distance of the new line, and the position of the point where it strikes the fourth side, are desired.

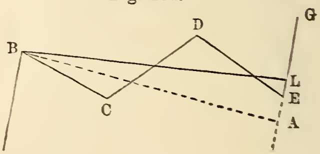
Ans. Division line, S. 83° 14′ E. 40.41 chains to a point 3.51 chains from the beginning of the fourth side.

392. *Third Method.*—When the old lines do not vary much from the position of the new, and are crooked, it will frequently be found most convenient to run a "guess-line," and take offsets from this to different points of the boundary. Then calculate the contents of the parts cut off on each side of this line. These, if the assumed line were correct, must be equal; if they are not so, divide the difference of the areas by half of the length of the "guess-line," and set the quotient off perpendicular to that line. Through the extremity of that perpendicular run a parallel to the "guess-line," meeting the side of the tract. The division line will run through this point, very nearly, if the "guess-line" did not differ much from the true one. If greater accuracy is required, the operation may be repeated, using the line determined by the first approximation as the basis of operations.

393. *Fourth Method.*—Run a random line from the starting point to the side on which the new line will fall, and calculate the area contained between this line and the original boundaries. Then, by Art. 378, run a new line to cut off the same area: this will be the line required.

Thus, (Ex. 1, Art. 390,) the bearing of EG (Fig. 184) being N. $10\frac{1}{2}$° E: run BA S. $79\frac{1}{2}$° E. 45.45 chains, falling on GE at A, distant .69 chains from E. in GE produced.

Fig. 184.



To find the area to BA.

	Bearing.	Dist.	N.	S.	E.	W.	E. D. D.	W. D. D.	Multipliers.	N. Areas.	S. Areas.
BC	S. 61° E.	16.50		8.00	14.43			30.25			
CD	N. $53\frac{1}{4}$° E.	20.05	12.00		16.07		30.50		30.50 E.	366.0000	
DE	S. 51° E.	18.42		11.59	14.31		30.38		60.88 E.		705.5992
EA	S. $10\frac{1}{2}$° W.	.69		.68		.13	14.18		75.06 E.		51.0408
AB	N. $79\frac{1}{2}$° W.	45.45	8.27			44.68		44.81	30.25 E.	250.1675	
										616.1675	756.6400
											616.1675
								Double Area			140.4725

Then, since A is a right angle,

$$AL = \frac{140.4725}{45.45} = 3.09: \text{ whence } EL = 3.09 - .69 = 2.40, \text{ as before.}$$

Problem 12.—*To run a new line between two tracts of different prices, so that the quantities cut off from each may be of equal value.*

394. This problem is in general a very complicated one, and can be best solved by approximation. Thus, run a "guess-line," and calculate the area cut off from each tract. If these areas are in the inverse ratio of the prices, the line is a correct one; if not, run a new line near this, and repeat the calculation: a few judicious trials will locate the line correctly.

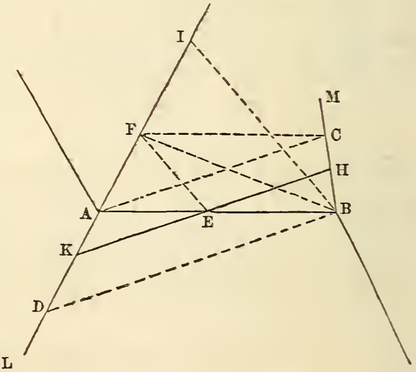
395. The following cases admit of simple solutions:—

CASE 1.

When the old line is straight, and the new line is to run a given course.

The method of solution will best be shown by an example.

Fig. 185.



Let the bearings of the lines be LA (Fig. 185) N. 46° 45′ E., AB S. 71° 20′ E., 24.10 chains, and BM N. 10° 35′ E., the land to the north of AB being estimated at $80 per acre, and that to the south at $100 per acre. It is required to run a new division line, running due east, so as not to alter the value of the two tracts.

Through B and A draw BD and AC parallel to the division line, and CF parallel to AB, meeting LA produced in F. Take AL $= \frac{10}{8}$ AD $= \frac{5}{4}$ AD, and FI a mean proportional between AL and AF. Join IB, and draw FE parallel to it, meeting AB in E. Then the division line will run through E.

DEMONSTRATION.—AL : FI :: FI : AF; ∴ AK : AF :: $FI^2 : AF^2$; but AD $= \frac{4}{5}$ AL; ∴ AD : AF :: $\frac{4}{5} FI^2 : AF^2$ :: $\frac{4}{5} BE^2 : AE^2$ :: $BE^2 : \frac{5}{4} AE^2$.

But AD : AF :: ADB : AFB (1.6) :: ADB : ABC :: $BE^2 : \frac{5}{4} AE^2$, (A)

and ABC : BEH :: $AB^2 : BE^2$;

∴ (23.5) ADB : BEH :: $AB^2 : \frac{5}{4} AE^2$;

but ADB : $\frac{5}{4}$ AEK :: $AB^2 : \frac{5}{4} AE^2$, (Cor. 2, 19.6.)

∴ BEH $= \frac{5}{4}$ AEK.

The operations in the above construction may readily be done on the ground. Thus:

Run BD, AC, and CF. Measure AF and AD. Calculate $\sqrt{\frac{5}{4} AD . AF}$, which call M. Then say, As AF + M : AF :: AB : AE. Through E run the division line.

Calculation.

To find BD. Say, As sin. ADB (43° 15′) : sin. ABD (18° 40′) :: AB (24.10) : AD = 11.26.

To find AF. Say, As sin. ACB . sin. BAF : sin. BAC . sin. ABC :: AB : AF;

that is, As sin. 79° 25′ . sin. 61° 55′ : sin. 18° 40′ . sin. 81° 55′ :: 24.10 : AF = 8.81; FI $= \sqrt{\frac{5}{4} AD . AF} = 11.13$.

Then, As AF + FI (19.94) : AF (8.81) :: AB (24.10) : AE = 10.64;

Or, As AF + FI (19.94) : AF (8.81) :: AD (11.26) : AK = 4.97.

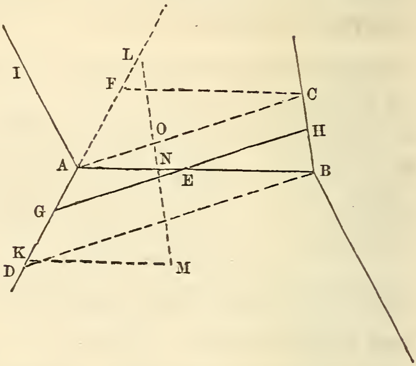
CASE 2.

396. *The division line to run through a given point* E *in* AB.

Let the bearings be as in last case. To run the division line through a point E in AB 10.64 chains distant from A.

Construction.—Take AI (Fig. 186) a third proportional to BE and AE. Let AK $= \frac{5}{4}$ AI and AL = BE. Draw LM parallel to BC, cutting AB in N; and KM parallel to AB. Make LO = MN. Join AO, and draw GEH parallel to it. Then the thing is done.

Fig. 186.



DEMONSTRATION.—Conceive BC and AL to meet in P. Then we have BE : EA : : EA : AI. ∴ (Cor. 2, 20.6) BE : AI : : BE^2 : EA^2, and LA : AK : : BE^2 : $\frac{5}{4}$ EA^2.

Again: PB : PC : : PD : PA : : PA : PF : : AD : AF;

but PB : PC : : LN : LO : : LN : NM : : LA : AK : : BE^2 : $\frac{5}{4}$ EA^2;

whence AD : AF : : BE^2 : $\frac{5}{4}$ EA^2, which agrees with (A) in the demonstration of last case. Then, following the steps of that demonstration, we find BEH = $\frac{5}{4}$ AEG.

This, like the last case, may readily be done on the ground, thus; Calculate $AI = \frac{AE^2}{EB}$, and make AK $= \frac{5}{4}$ AI. Lay off on DA produced AL = BE: run LNM and KM. Lay off LO = NM, and run GEH parallel to AO.

Calculation.

$$AK = \frac{5\ AE^2}{4\ EB} = 10.51.$$

Then sin. M (81° 55′) : sin. AKM (61° 55′) : : AK (10.51) : NM = 9.37 = LO;

and, As LA + LO (22.83) : LA − LO (4.09) : : tan. $\frac{LOA - LAO}{2}$ (71° 55′) : tan. $\frac{LOA - LAO}{2}$ = 28° 45′;

∴ LAO = 71° 55′ − 28° 45′ = 43° 10′.

But AF bears N. 46° 45′ E.; hence GH bears N. 89° 55′ E.

CASE 3.

397. *When the starting point is in the line* AD.

Given as before to run the line from a point G in AD at 4.97 chains from A.

Produce DA and BC (Fig. 186) to meet in P. Calculate AP: let the given ratio $\frac{5}{4}$ be represented by r: then, As sin. P (36° 10′) : sin. ABC (81° 55′) :: AB (24.10) : AP = 40.432.

Put $$\frac{r \, . \, AG^2}{AP} = .7636 = A;$$

and $$M^2 = A \, . \, PG = 34.67.$$

Lay off $GD = \frac{1}{2} A \pm \sqrt{\frac{1}{4} A^2 + M^2} = .382 + 5.900 = 6.282$, (the lower sign being used when G is between A and P.)

Then GH parallel to DB will be the division line.

DEMONSTRATION.—Since $GD = \frac{1}{2} A + \sqrt{\frac{1}{4} A^2 + M^2}$,
we have $GD - \frac{1}{2} A = \sqrt{\frac{1}{4} A^2 + M^2}$, and $GD^2 - A \, . \, GD = M^2$,
or $GD \, (GD - A) = A \, . \, PG$; whence PG : DG :: DG — A : A,
and composition, PD : DG :: DG : $A \left(\frac{r \, . \, AG^2}{AP}\right)$ :: AP . DG : r . AG²;
whence $$r \, . \, PD \, . \, AG^2 = AP \, . \, DG^2,$$
and r . AG² : DG² :: AP : PD :: PC : PB :: PF : PA :: AF : AD,
or, r . AE² : EB² :: AF : AD. As this agrees with (A) in the demonstration to Case 1, the truth of the work is clear.

Having found AD, the bearing of DB, which is parallel to GH, may be found by calculating the angle ADB; thus: As (AB + AD) 35.352 : (AB — AD) 12.848 :: tan. $\frac{ADB + ABD}{2}$ 30° 57½′ : tan. $\frac{ADB - ABD}{2}$ = 12° 17′ 55″. Whence the angle ADB is 43° 15′ 25″, and the bearing of DB or GH is S. 89° 59′ 35″ E.

The whole of the preceding construction might be made geometrically, but some of the lines required would be so small that no dependence could be had on the work; the method is therefore omitted.

If the given point were not on one of the lines, the problem becomes very complicated. It may, however, be solved by running "guess-lines."

SECTION II.

DIVISION OF LAND.

Problem 1.—*To divide a triangle into two parts having a given ratio.*

CASE 1.

398. *By a line through one of the corners.*

Divide the base into two parts having the same ratio as the parts into which the triangle is to be divided, and draw a line from the point of section to the opposite angle, (1.6).

EXAMPLES.

Ex. 1. A triangular field ABC contains 10 acres, the base AB being 22.50 chains. It is required to cut off $4\frac{1}{2}$ acres towards the point A by a line CD from the angle C. What is the distance AD?

Calculation.

As $10 : 4\frac{1}{2} :: \text{AB } (22.50) : \text{AD} = 10.125$ chains.

Ex. 2. The area of a triangle ABC is 7 acres, the side AC being 15 chains. To determine the distance AD to a point in AC, so that the triangle ABD may contain 3 acres.

Ans. AD = 6.43 chains.

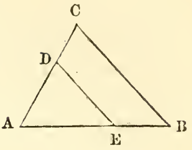
CASE 2.

399. *By a line through a given point in one of the sides.*

Say, As the whole area is to the area of the part to be cut off, so is the rectangle of the sides about the angle towards which the required part is to lie, to a fourth term.

This fourth term divided by the given distance will give the distance on the other side.

Fig. 187.



DEMONSTRATION.—Let ABC (Fig. 187) be the given triangle, and ADE the part cut off. Then we shall have (Art. 357) rad. : sin. A : : AB . AC : 2 ABC, and rad. : sin. A : : AD . AE : 2 ADE; wherefore 2 ABC : 2 ADE : : AB . AC : AD . AE, or ABC : ADE : : AB . AC : AD . AE.

EXAMPLES.

Ex. 1. Given the side AB = 25 chains, AC = 20 chains, and the distance AD = 12 chains, to find a point E in AB, such that the triangle cut off by DE may be to the whole triangle as 2 is to 5.

Calculation.

As 5 : 2 : : AB . AC (500) : AD . AE (200);

whence $$AE = \frac{200}{12} = 16.66 \text{ chains.}$$

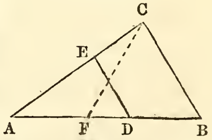
Ex. 2. Given AB = 12.25 chains, AC = 10.42 chains, and the area of ABC = 5 A. 3 R. 8 P., to cut off 3 acres towards the angle A by a line running through a point E in AB 8.50 chains from the point A. Required the distance on AC. Ans. 7.77 chains.

CASE 3.

400. *By a line parallel to one of the sides.*

Since the part cut off will be similar to the whole, say, As the whole area is to the area to be cut off, so is the square of one of the sides to the square of the corresponding side of the part.

Fig. 188.



The problem may be constructed thus: Let ABC (Fig. 188) be the given triangle. Divide AB in F, so that AF may be to FB in the ratio of the parts into which the triangle is to be divided. Take AD a mean proportional between AF and AB. Then, DE parallel to BC will divide the triangle as required.

For AFC : FCB : : AF : FB, and (lemma) ADE = AFC; therefore ADE : DECB : : AF : FB.

EXAMPLES.

Ex. 1. The three sides of a triangle are AB = 25 chains, AC = 20 chains, and BC = 17 chains, to divide it into two parts ADE and DECD, having the ratio of 4 to 3, by a line parallel to BC.

Say, As $7 : 4 :: AB^2 (625) : AD^2 = 357.1428$;
whence $AD = 18.90$ chains.

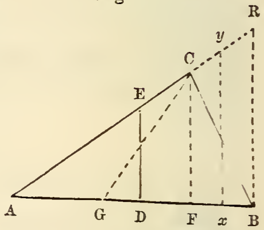
Ex. 2. The three sides of a triangle are AB = 25 chains, AC = 20 chains, and BC = 15 chains, to divide it into two parts ADE and DECB, which shall be to each other as 2 to 3, by a line parallel to BC. What is the distance on AC to the division line? Ans. 12.65 chains.

CASE 4.

401. *By a line running a given course.*

Construction.—Divide AB in G, (Fig. 189,) so that AG may be to GB in the ratio of the parts of the triangle. Run CF according to the given course. Take AD a mean proportional between AF and AG. Then DE parallel to CF is the division line.

Fig. 189.



For ACG : CGB :: AG : GB, and, by the lemma, ADE = ACG.

$\therefore$ ADE : DECB :: AG : GB.

Calculation.

In ACF find AF. Then $AD = \sqrt{AG \cdot AF}$; or say, As the rectangle of the sines of D and E is to the rectangle of the sines of B and C, so is the square of BC to a fourth term.

Then, if the ratio of the parts is to be as m to n, m corresponding to the triangular portion, multiply this fourth term by m, and divide by $m + n$: the quotient will be the square of DE. Whence AD is readily found.

DEMONSTRATION.—Draw xy parallel to CF, making $Axy = ABC$, and draw BR parallel to xy. Then, as was shown in Art. 385, sin. D . sin. E : sin. B . sin. C : : $BC^2 : xy^2$, and (Cor. 2, 20.6) Axy : ADE or $m + n : m :: xy^2 : DE^2$

EXAMPLES.

Ex. 1. The bearings and distances of the sides of a triangular plat of ground are AB N. 71° E. 17.49 chains, BC S. 15° W. 12.66 chains, and CA N. $63\frac{3}{4}$° W. 14.78 chains, to divide it into two parts ADE and DECB, in the ratio of 2 to 3, by a line running due north. The distance AD is required.

First Method.

As	sin. F	71°	A. C. 0.024330
:	sin. ACF	63° 45′	9.952731
: :	AC	14.78	1.169674
:	AF		1.146735
	AG = $\frac{2}{5}$ AB = 6.996		0.844850
			2)1.991585
	AD = 9.904 ch.		.995792

Second Method.

As	sin. D	71°	A. C. 0.024330
	sin. E	63° 45′	0.047269
:	sin. B	56°	9.918574
	sin. C	78° 45′	9.991574
: :	BC	12.66	1.102434
	BC	"	1.102434
:	xy^2	153.68	2.186615
		2	
		5)307.36	

$$DE = \sqrt{61.472} = 7.841.$$

As	sin. A	45° 15′	A. C. 0.148628
:	sin. E	63° 45′	9.952731
: :	DE	7.841	0.894371
:	AD	9.902	0.995730

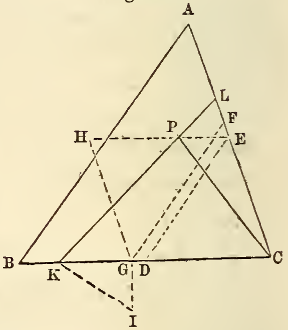
Ex. 2. Given AB N. 63° W. 12.73 ch., BC S. 10° 15′ W. 8.84 ch., and CA N. 77° 15′ E. 13.24 ch., to determine the distance AD on AB so that DE perpendicular to AB will divide the triangle into two equal parts.

Ans. AD = 8.049 ch.

CASE 5.

402. *By a line through a given point.*

Fig. 190.



Let ABC (Fig. 190) be the triangle to be divided into two parts CLK and ABKL, which shall be to each other as the numbers m and n: the division line to run through a given point P.

Construction.

Bisect BC in D; divide CA in F, so that CF : FA :: m : n. Through P draw HPE parallel to BC. Join ED; draw FG parallel to it, and complete the parallelogram CH. Make GI perpendicular to BC and equal to EP. With the centre I and the radius PH, describe an arc cutting BC in K; then KPL will be the division line.

If IG is greater than IK, the question is impossible in the terms proposed. The triangular part will then be adjacent to one of the other angular points, and a construction altogether analogous to the above will fix the position of the division line.

DEMONSTRATION.—Conceive DA, DF, and EG to be joined. Then, since CD $= \frac{1}{2}$ BC, ADC $= \frac{1}{2}$ ABC, and, because CF : FA :: $m : n$, we have by composition CA : CF :: $m + n : m$; whence CFD $= \dfrac{m}{m+n}$ CAD. But CDF = CEG, and CH = 2 CEG ∴ CH $= \dfrac{m}{m+n}$ CAB, and by demonstration (Art. 381) CKL = CH; therefore CKL $= \dfrac{m}{m+n}$ CAB.

Calculation.

Find PE, EC, and FC $= \dfrac{m}{m+n}$ AC; then CE : CF : : CD ($\frac{1}{2}$ BC) : CG, and KG $= \sqrt{KI^2 - IG^2} = \sqrt{PH^2 - PE^2}$. Finally, CK = CG ± GK.

EXAMPLES.

Ex. 1. Given the bearings and distances of the adjacent sides of a triangular tract,—viz.: CA N. 10° 17′ W. 13.25 ch., CB N. 82° 5′ W. 13.75 ch.,—to divide it into two portions ABKL and KLC in the ratio of 4 to 5, by a line through a point P N. 28 W. 7.85 chains from the corner C. The distance CK is required.

Calculation.

To find PE and EC.

As sin. PEC	108° 12′	A. C. 0.022289
: sin. PCE	17° 43′	9.483316
: : PC	7.85	0.894870
: PE	2.515	0.400475
As sin. PEC	108° 12′	A. C. 0.022289
: sin. CPE	54° 5′	9.908416
: : PC		0.894870
: CE	6.692	0.825575

To find CG.

As CE	6.692	A. C. 9.174425
: CF = $\frac{5}{9}$ CA	7.361	0.866937
: : CD = $\frac{1}{2}$ CB	6.875	0.837273
: CG = EH	7.562	0.878635
EP	2.515	
PH = IK =	5.047	

To find KG and CK.

KI + IG	7.562	0.878635
KI − IG	2.532	0.403464
		2)1.282099
KG =	4.376	.641049
CG =	7.562	
CK =	11.938	

Ex. 2. Given AB N. 46° 15′ E. 8.80 ch., AC S. 65° 15′ E. 11.87 ch., to determine the distance AK to a point K in AB so that a line from K through a spring P N. 80° E. 5.90 ch. from A may divide the triangle into two equal parts.

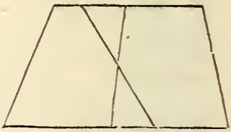
Ans. AK = 8.58 ch., or 6.244 ch.

Problem 2. *To divide a trapezoid into two parts having a given ratio.*

CASE 1.

403. *By a line cutting the parallel sides.*

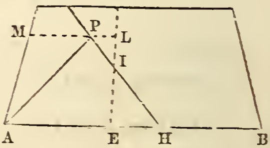
Fig. 191.



a. Divide DC and AB (Fig. 191) in F and E so that the parts may have the same ratio as the parts into which the trapezoid is to be divided: join EF and the thing is done.

b. If the division line is to pass through a given point G in one of the parallel sides. Determine F and E as before; then lay off EH = FG, and GH will be the division line.

Fig. 192.



c. If the division line is to pass through a point P (Fig. 192) not in AD or CD. Determine EF as before. Bisect it in I. Through P and I draw the division line GH.

Should GH cut either of the non-parallel sides before it does both of these, one of the portions will be a triangle. It will then be necessary to calculate the area of the whole tract, whence that of each portion is found. Then, by Art. 381, lay off a triangle by a line through P so as to contain the required area.

Calculation.

Through P draw MPL parallel to AB, and from the data given find AM and MP.

Then DA : AM : : AE — DF : AE — LM; whence LM and PL are known.

But AM — $\frac{1}{2}$ AD : $\frac{1}{2}$ AD : : PL : GF = EH; and DG = DF — FG.

EXAMPLES.

Ex. 1. Given AB E. 9.10 ch., BC N. 14° 20′ W. 4.40 ch., CD W. 6.95 ch., and DA S. 14° W. 4.39 ch., to divide the tract into two parts having a ratio of 3 to 4 by a line HG through a spring N. 47° E. 4.40 ch. from the corner A; the smaller division to be next to AD. Required the distances of the division line from A and D.

Calculation.

To find AM and MP.

As sin. M	76°	A. C. 0.013096
: sin. APM	43°	9.833783
: : AP	4.40	0.643453
: AM	3.093	0.490332
And As sin. M		A. C. 0.013096
: sin. PAM	33°	9.736109
: : AP		0.643453
: PM	2.470	0.392658

To find EH, AH, and DG.

DF = $\frac{3}{7}$ DC = 2.979, and AE = $\frac{3}{7}$ AB = 3.90.

Then, As AD (4.39) : AM (3.093) : : AE — DF (.921) : AE — ML = .649;

whence ML = 3.251, and PL = 3.251 — 2.470 = .781.

As AM — $\frac{1}{2}$ AD (.898) : $\frac{1}{2}$ AD (2.195) : : PL (.781) : FG = EH = 1.909. Finally, AH = AE + EH = 5.81, and DG = DF — FG = 1.07.

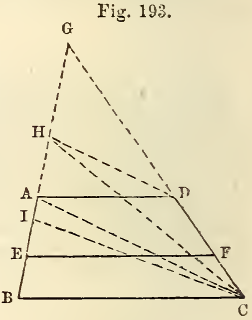
Ex. 2. Given AB S. 62° 50′ E. 14.93 ch., BC N. 7° 30′ W. 6.29 ch., CD N. 62° 50′ W. 11.88 ch., DA S. 21 W. 5.18 ch., to determine DG and AH so that a line joining G and H will pass through P N. 75° 50′ E. 6.20 ch. from A, and cut off one-third of the area of the tract towards AD.

Ans. AH = 3.40 ch.; DG = 5.53 ch.

CASE 2.

404. *The division line to be parallel to the parallel sides.*

Fig. 193.



Let ABCD (Fig. 193) be the trapezoid to be divided into two parts AEFD and FEBC having the ratio of two numbers m and n by a line EF parallel to AD or BC.

Construction.

Join CA, and draw DH parallel to it. Join CH. Divide HB in I so that HI : IB : : m : n. Produce CD and BA to meet in G, and take GE a mean proportional between GI and GB. Join CI, and draw EF parallel to AD: then will EF be the division line required.

DEMONSTRATION.—Because DH is parallel to CA, AHC = ADC (37.1); ∴ ABCD = BCH, and, since HB is divided in I so that HI : IB : : m : n, we have CHI : CIB : : m : n (1.6.) These triangles are therefore equal to the parts into which the trapezoid is to be divided. But (lemma) GEF = GIC: therefore EBCF = ICB, and EF is the division line.

Calculation.

EF may be found by the formula $EF^2 = \dfrac{m\ BC^2 + n\ AD^2}{m + n}$;

then BC ∾ AD : EF ∾ AD : : AB : AE.

DEMONSTRATION.—GBC : GAD : : $BC^2 : AD^2$; ∴ (17.5) ABCD : GAD : : $BC^2 - AD^2 : AD^2$.

Similarly, GEF : GAD : : $EF^2 : AD^2$ ∴ (17.5) AEFD : GAD : : $FE^2 - AD^2 : AD^2$;

whence ABCD : AEFD : : $BC^2 - AD^2 : FE^2 - AD^2$;

or, $m + n : m : : BC^2 - AD^2 : FE^2 - AD^2$:

consequently $(m + n)\ FE^2 - m\ AD^2 - n\ AD^2 = m\ BC^2 - m\ AD^2$;

or, $(m + n)\ FE^2 = m\ BC^2 + n\ AD^2$, and $FE^2 = \dfrac{m\ BC^2 + n\ AD^2}{m + n}$.

Again: Draw AKL parallel to DC. Then BL : EK : : AB : AE;

or, BC — AD : FE — AD : : AB : AE.

Second Method.

The distance AE may be calculated thus :—

Find GA and GD; thence GC and GB are known: then GC : GD : : GA : GH; whence HB and HI are known, and therefore $GE = \sqrt{GI.GB}$ is known.

EXAMPLES.

Ex. 1. Given AB S. 14° W. 4.39 ch., BC E. 9.10 ch., CD N. 14° 20′ W. 4.40 chains, and DA W. 6.95 chains, to divide the trapezoid into two parts AEFD and BEFC in the ratio of 2 to 3, by a line EF parallel to the sides BC and DA. Required the distance AE on the first side.

$$EF^2 = \frac{m.BC^2 + n.AD^2}{m+n} = \frac{165.62 + 144.9075}{5}$$

$$= \frac{310.5275}{5} \doteq 62.1055;$$

whence $$EF = \sqrt{62.1055} = 7.88.$$

And BC − AD (2.15) : EF − AD (.93) : : AB (4.39) : AE = 1.90.

Ex. 2. Given AB S. 87° 15′ E. 6.47 chains, BC N. 23° 30′ E. 10.32 chains, CD S. 64° 45′ W. 9.30 chains, and DA S. 23° 30′ W. 5.55 chains, to determine the distance AE of a point E, situated in AB, such that EF parallel to AD may divide the trapezoid into two parts AEFD and EBCF having the ratio of 4 to 5.

Ans. AE = 3.36 chains.

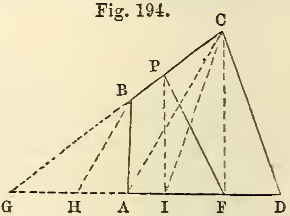
Problem 3.—*To divide a trapezium into two parts having a given ratio.*

CASE 1.

405. *The division line to run through a given point in one of the sides.*

Let ABCD (Fig. 194) represent the trapezium and P the given point; and let $m : n$ represent the given ratio.

Fig. 194.



CONSTRUCTION.—Determine I, as in Art. 404. Join PI, and draw CF parallel to it: then will PF be the division line.

For if CH and CI be joined, CHD = ABCD; and, since HCI : ICD :: $m : n$, HCI and ICD will be equal to the two parts into which the quadrilateral is to be divided. But, since PI is parallel to CF, we have

GC : GP :: GF : GI; ∴ (15.6) GPF = GCI, and PFDC = CID.

Calculation.

In GAB find GA and GB.

Then GC : GB :: GA : GH;

whence HD and HI become known;

and GP : GC :: GI : GF.

Finally, AF = GF − GA.

EXAMPLES.

Ex. 1. Given AB N. 25¾° E. 4.65 chains, BC N. 77° E. 6.30 chains, CD South 7.30 chains, and DA N. 78¼° W. 8.35 chains, to divide the trapezium into two equal parts by a line EF running through a point P in BC distant 2.50 chains from B. AF is required.

Calculation.

To find GA and GB.

As sin. G	24° 45′	A. C. 0.378139
: sin. GBA	51° 15′	9.892030
: : AB	4.65	0.667453
: AG	8.662	0.937622
AD	8.35	
GD	17.012	

As sin. G	24° 45′	A. C. 0.378139
: sin. GAB	104°	9.986904
: : AB		0.667453
: BG	10.777	1.032496
BC	6.30	
GC	17.077	

To find GH.

As GC	17.077	A. C. 8.767588
: GB	10.777	1.032496
: : GA	8.662	0.937622
: GH	5.466	0.737706

$HI = \frac{1}{2}(GD - GH) = 5.773$ and $GI = GH + HI = 11.239$.

To find GF and AF.

As GP	13.277	A. C. 8.876900
: GC	17.077	1.232412
: : GI	11.239	1.050727
: GF	14.456	1.160039
AG	8.662	
AF	5.794.	

Ex. 2. Given AB N. 27½° W. 19.55 chains, BC East 18.92 chains, CD S. 1½° E. 10.49 chains, and DA S. 56° W. 12.25 chains, to find BF, so that a line run from a point

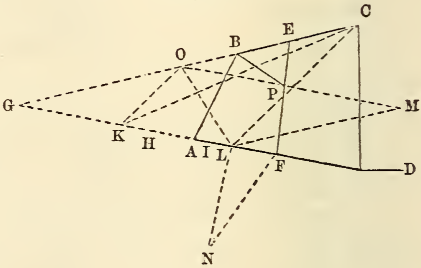
P in AD 6 chains from A may divide the trapezium into two parts ABFP and PFCD having the ratio of 5 to 4.

Ans. BF = 9.00 ch.

CASE 2.

406. *The division line to run through any point.*

Fig. 195.



Let ABCD (Fig. 195) be the given trapezium and P the given point. Determine I, as in the last two articles, and bisect GI in K. Through P draw OPM parallel to GD, meeting GB in O. Join KO, and draw CL parallel to it. Through L draw LM parallel to GB. Make LN perpendicular to AD and equal to OP. With the centre N and radius equal to PM, describe an arc cutting AD in F. Then FPE will be the division line.

DEMONSTRATION.—As was proven, Art. 381, GFE = GOML = 2 GOL = 2 GCK = GCI: whence ABEF = ABCI. But CI divides the trapezium into two parts having the given ratio; therefore, EF does so likewise.

Calculation.

Find GB, GA, GH, and GI. Then in OBP find OB and OP: thus GO is known. And because GO : GC : : GK : GL, GL is known; but PM = GL − OP. Hence, in LNF we have LN and NF to find LF.

EXAMPLES.

Ex. 1. Given AB N. $25\frac{3}{4}°$ E. 4.65 chains, BC N. 77° E. 6.30 chains, CD South 7.30 chains, and DA N. $78\frac{1}{4}°$ W. 8.35 chains, to part off two-fifths of the tract next to AB by a line through a spring S. $54\frac{3}{4}°$ E. 2.95 chains from the second corner. The distance AF is required.

Calculation.

As in Ex. 1, last case: GB = 10.777, GA = 8.662, GC = 17.077, GD = 17.012, GH = 5.466, GI = (GH + $\frac{2}{5}$ HD) = 10.084, and GK = 5.042.

To find OB and OP.

As sin. BOP	24° 45′	A. C. 0.378139
: sin. BPO	23° 30′	9.600700
: : BP	2.95	0.469822
: OB	2.81	0.448661
GB	10.777	
GO	7.967	

As sin. BOP	24° 45′	A. C. 0.378139
: sin. OBP	131° 45′	9.872772
: : BP		0.469822
: OP	5.257	0.720733

To find GL.

As GO	7.967	9.098705
: GC	17.077	1.232412
: : GK	5.042	0.702603
: GL	10.807	1.033720

$$NF = GL - OP = 5.55.$$

Whence $LF = \sqrt{NF^2 - LN^2} = 1.779;$

whence $AF = GL + LF - GA = 3.924.$

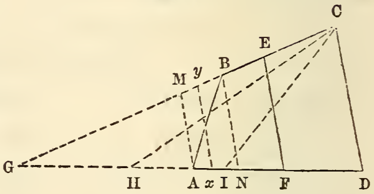
Ex. 2. Given AB N. 27½° W. 19.55 chains, BC East 18.92 chains, CD S. 1½° E. 10.49 chains, and DA S. 56° W. 12.25 chains, to divide the quadrilateral into two parts ABEF and FECD in the ratio of 5 to 4, by a line EF through a spring P, which bears from B S. 70¼° E. 11.52 chains. The distance AF is required.

Ans. AF = 5.01 ch.

CASE 3.

407. *The division line to be parallel to one side.*

Fig. 196.



Let ABCD (Fig. 196) represent the trapezium which is to be divided into two parts having the ratio of m to n by a line parallel to CD.

CONSTRUCTION. — Determine H and I, as in the preceding articles. Take GF a mean proportional between GI and GD: then EF, parallel to CD, will be the division line.

For, as was demonstrated, (Art. 404,)

$$ABCD = HCD,$$

and $\qquad CHI : CID :: m : n.$

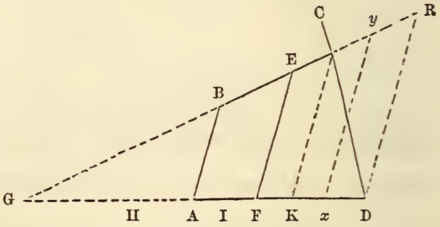
But (lemma) $\qquad GCI = GEF;$

$\therefore \qquad ICD = EFDC,$

and $\qquad HCI = ABEF:$

whence $\qquad ABEF : FECD :: m : n.$

Fig. 197.



If the division line is to be parallel to the shorter side AB (Fig. 197.) Draw CK parallel to AB, and take GF a mean proportional between GI and GK; or, join BD, and draw CH′ parallel to it. Divide AH′ in I′, so that

$$AI' : I'H' :: m : n,$$

and take GF a mean proportional between GA and GI′. Then will EF, parallel to AB, be the division line.

Calculation.

First Method.—Find, as in the preceding articles, GH and GI. Then GF $= \sqrt{\mathrm{GI}\,.\,\mathrm{GD}}$, or$= \sqrt{\mathrm{GI}\,.\,\mathrm{GK}}$.

Second Method.—Draw xy (Fig. 196) parallel to EF, so as to make Gxy = GAB, or Gxy = GCD, (Fig. 197.) Then we shall have

$$\sin. \mathrm{E}\,.\sin. \mathrm{F} : \sin. \mathrm{A}\,.\sin. \mathrm{B} :: \mathrm{AB}^2 : xy^2, \text{ (Fig. 196,)}$$

or $$\sin. \mathrm{E}\,.\sin. \mathrm{F} : \sin. \mathrm{C}\,.\sin. \mathrm{D} :: \mathrm{CD}^2 : xy^2; \text{ (Fig. 197;)}$$

and (Art. 404) $$\mathrm{EF}^2 = \frac{m\,.\,\mathrm{CD}^2 + n\,.\,xy^2}{m+n}, \text{ (Fig. 196;)}$$

or $$\mathrm{EF}^2 = \frac{m\,.\,xy^2 + n\,.\,\mathrm{AB}^2}{m+n}, \text{ (Fig. 197.)}$$

DEMONSTRATION.—Draw AM and BN (Fig. 196) parallel to EF.

Then $\sin. \mathrm{M}\,.\,(\sin. \mathrm{E}) : \sin. \mathrm{B} :: \mathrm{AB} : \mathrm{AM}$,

and $\sin. \mathrm{N}\,.\,(\sin. \mathrm{F}) : \sin. \mathrm{A} :: \mathrm{AB} : \mathrm{BN}$;

$\therefore$ (23.6) $\sin. \mathrm{E}\,.\sin. \mathrm{F} : \sin. \mathrm{A}\,.\sin. \mathrm{B} :: \mathrm{AB}^2 : \mathrm{AM}\,.\,\mathrm{BN}$.

Now, since Gxy = GAB, Gx is a mean proportional between GA and GN. Wherefore xy is a mean proportional between AM and BN. Hence, AM . BN $= xy^2$;

consequently, $\sin. \mathrm{E}\,.\sin. \mathrm{F} : \sin. \mathrm{A}\,.\sin. \mathrm{B} :: \mathrm{AB}^2 : xy^2$.

If EF is parallel to AB, (Fig. 197,) the demonstration will be precisely similar to the above.

EXAMPLES.

Ex. 1. Given the bearings and distances as follow,—viz.: AB N. $25\frac{3}{4}°$ E. 4.65 chains, BC N. 77° E. 6.30 chains, CD South 7.30 chains, and DA N. $78\frac{1}{4}°$ W. 8.35 chains,—to divide the trapezium into two parts ABEF and FECD, having the ratio of 2 to 3, by a line EF parallel to AB. AF and EF are required.

Calculation.

First Method.—As in Ex. 1 of Art. 405, we find GA = 8.662, GB = 10.777, GC = 17.077, GD = 17.012, GH = 5.466, and GI = GH + $\frac{2}{5}$ HD = 10.084.

To find GK and GF.

As GB	10.777	A. C. 8.967504
: GA	8.662	0.937622
: : GC	17.077	1.232412
: GK		1.137538
GI	10.084	1.003633
		2) 2.141171
$GF = \sqrt{GI \cdot GK} =$	11.765	1.070585
GA =	8.662	
AF =	3.103	

To find EF.

As GA	8.662	A. C. 9.062378
: AB	4.65	0.667453
: : GF	11.765	1.070585
: EF	6.316	1.800416

Second Method.

As { sin. E	128° 45′	A. C. 0.107970
sin. F	76°	" " 0.013096
: { sin. C	77°	9.988724
sin. D	78° 15′	9.990803
: : { CD	7.30	0.863323
CD		0.863323
: xy^2	67.18	1.827239
	2	
	134.36	
3 AB^2	64.8675	
	5) 199.2275	

$EF = \sqrt{39.8455} = 6.312.$

To find AF.

As sin. G	24° 45′	A. C. 0.378139
: sin. E	128° 45′	9.892030
: : FE − AB	1.662	0.220631
: AF	3.096	0.490800

Ex. 2. Given the bearings and distances as in Ex. 1, to divide the trapezium into two parts AFED and FECB, having the ratio of 3 to 2, by a line EF parallel to BC. AF and EF are required.

Ans. AF = 1.60 chains; EF = 7.66 chains.

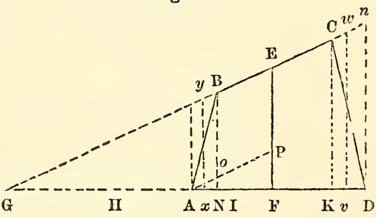
Ex. 3. Given as in Ex. 1, to divide the trapezium into two parts ABEF and FECD, in the ratio of 2 to 3, by a line EF parallel to CD. AF and EF are required.

Ans. AF = 3.91 chains; EF = 5.62 chains.

CASE 4.

408. *The division line to run any direction.*

Fig. 198.



Let ABCD (Fig. 198) be the trapezium to be divided into two parts ABEF and FECD, in the ratio of m to n, by a line EF running any course.

The construction of this case is the same as that of the last,—CK being drawn so as to be of the same course as EF.

Calculation.

Conceive xy and vw to be drawn so as to make Gxy = GAB, and Gvw = GCD: then will $vwyx$ be equal to ABCD. It will also be divided by EF into two parts having the ratio of m to n.

Find xy^2 and vw^2 by the proportions

$$\sin. E . \sin. F : \sin. A . \sin. B :: AB^2 : xy^2,$$

and $$\sin. E . \sin. F : \sin. C . \sin. D :: CD^2 : vw^2,$$

the truth of which has been proven in the demonstration to rule for Art. 407.

Then (Art. 404) $EF^2 = \dfrac{m . vw^2 + n . xy^2}{m + n}$.

Draw AOP parallel to BC, meeting BN and EF in O and P.

Then sin. BOA (sin. E) : sin. BAO (sin. B) : : AB : BO, and sin. PAF (sin. G) : sin. P (sin. E) : : PF (EF — BO) : AF.

The calculation may otherwise be made by finding GH and GI, as in Arts. 406, 407, and also GK. Then GF = $\sqrt{GI \cdot GK}$.

EXAMPLE.

Ex. 1. The bearings and distances being as in the examples in last case, it is required to divide the trapezium into two parts ABEF and FECD, having the ratio of 2 to 3, by a line perpendicular to AD. To find AF and EF.

Ans. AF = 3.84; EF = 5.76.

CHAPTER VIII.

MISCELLANEOUS EXAMPLES.

Ex. 1. Two sides of a triangle are 32 and 50 perches respectively. Required the third side, so that the area may be 3 acres. Ans. 31.05 P. or 78 P.

Ex. 2. A gentleman has a garden in the form of a rectangle, the adjacent sides being 120 and 100 yards respectively. There is a walk half round the garden, which takes up one-eighth of the ground. What is its width?

Ans. 7.05 yards.

Ex. 3. The three sides of a triangle are in the ratio of the numbers 3, 4, and 5. What are their lengths, the area being 2 A., 1 R., 24 P.?

Ans. 6 chains, 8 chains, and 10 chains.

Ex. 4. The diameter of a circular grass-plat is 150 feet, and the area of the walk that surrounds it is one-fourth of that of the plat. Required the width.

Ans. 8.85 feet.

Ex. 5. To determine the height of a liberty-pole which had been inclined by a blast of wind, I measured 75 feet from its base, the ground being level, and took the angle of elevation of its top $67° 43' 30''$, the angle of position of the base and top being $5° 37'$. Then, measuring 100 feet farther, I found the angle of position of the bottom and top to be $2° 29'$. Required the length of the pole.

Ans. 194 feet.

Ex. 6. The distances from the three corners of a field in the form of an equilateral triangle to a well situated within it are 5.62 chains, 6.23 chains, and 4.95 chains respectively. What is the area? Ans. 4 A., 0 R., 6 P.

Ex. 7. At a station on the side of a pond, elevated 30 feet above the water, the elevation of the summit of a cliff on the opposite shore was found to be 37° 43′ and the depression of the image 45° 26′. Required the elevation of the cliff. Ans. 221.8 ft.

Ex. 8. To find the altitude of a tower on the brow of a hill, I measured, on slightly-inclined ground, a base-line AB 157 yards, A being on a level with the base of the hill. At A the angle of position of B and C was 87° 45′; elevation of B, 2° 17′; of base of tower, 39° 43′, and of top, 52° 13′. At B the depression of A was 2° 17′; the angle of position of A and C, 54° 23′; elevation of base of tower, 33° 4′, and of top, 45° 42′. Required the height of the hill and also of the tower.

Ans. Height of hill, 172.5 ft.; of tower, 95.5 ft.

Ex. 9. To determine the height of a tree C standing on the opposite shore of a river, I measured a base-line AB of 100 feet. At A the angle BAC was 90°, and the angle of depression of the image of the top of the tree was 39° 48′. At B the angle of depression was 32°. Required the height, the instrument having been 10 feet above the water at each station. Ans. 84.47 feet.

Ex. 10. Not being able to measure directly the three sides of a triangle, the corners of which were visible from each other, I took the angles as follow,—viz.: A = 57° 29′, B = 72° 41′, and C = 49° 50′. I also measured the distances from the corners to a point within the triangle, and found them to be AD = 7.56 chains, BD = 9.43 chains, and CD = 8.42 chains. Required the lengths of the sides.

Ans. AB = 12.63 chains, AC = 15.78 chains, and BC = 13.94 chains.

Ex. 11. The base of a triangle being 50 perches, and the area 5 acres, what are the other sides, their sum being 85 perches? Ans. 33.3785 P. and 51.6215 P.

Ex. 12. It is required to lay out 7 acres in a triangular form, one side being 20 chains, and the others in the ratio of 2 to 3.

Ans. The other sides are 9.86 and 14.79 chains, or 39.58 and 59.37 chains.

Ex. 13. The bearings of the dividing lines of two farms being as follow,—viz.: 1. N. $83\frac{1}{2}°$ E. 2.37 chains; 2. S. 47° E. 6.25 chains; 3. N. $62\frac{3}{4}°$ E. 5.17 chains; 4. S. $56\frac{1}{2}°$ E. 3.92 chains, and 5. N. $14\frac{1}{2}°$ E.,—it is required to straighten the boundary, the new line to start from the beginning of the first side and fall on the last. The bearing of the new line is required, and also the distance on the last side.

Ans. Bearing, S. 74° 40′ E. to a point .25 chains back from the commencement of the last side.

Ex. 14. One side of a tract running through a thick copse, I took a station S. $26\frac{1}{2}°$ E. 1.53 chains from the corner, and ran a "guess-line" bearing N. $60\frac{1}{2}°$ E. 19.37 chains, when the other end bore N. $28\frac{1}{2}°$ W. 3.27 chains. What is the course and distance of the line, and what must be the course and distance of an offset from a point 8.53 chains on the random line, that it may strike a stone in the side 8.53 chains from the point of beginning?

Ans. Side, N. 55° 22′ E. 19.42 chains;
Offset, N. 28° 8′ W. 2.29 chains.

Ex. 15. Three observers, A, B, and C, whose distances asunder are AB = 1000 yards, BC = 1180 yards, and AC = 1690 yards, take the altitude of a balloon at the same instant, and find it to be as follow,—viz.: At A, 53° 43′, at B, 46° 40′, and at C, 52° 46′. Required the height of the balloon. Ans. 1461.4 yards or 2411 yards.

Ex. 16. The bearings and distances of the sides of a tract of land are,—1. N. 61° 20′ W. 22.55 chains; 2. N. 10° W. 16.05 chains; 3. N. 60° 45′ E. 14.30 chains; 4. S. 66° 40′ E. 17.03 chains; 5. S. 86° E. 22.40 chains; 6. S. 31° 40′ E. 19.10 chains, and 7. S. 76° 35′ W. 39 chains,—to divide it into two equal parts by a line running due north. The position of the division line is desired.

Ans. The division line runs from a point on the 7th side 3.77 chains from the end thereof.

Ex. 17. Not being able to run a line directly, on account of a projecting cliff, I took the angles of deflection and the distances as follow,—viz.: 1. to the right, 67° 35′ 10 chains; 2. to the left, 48° 43′ 7.25 chains; 3. to the left, 11° 45′ 5.43 chains, and 4. to the left, 65° 17′. How far on the last course must I run before coming in line again? at what angle must I deflect to continue the former direction? and what is the distance on the first line?

Ans. Distance on the last course, 14.42 chains; on the first, 23.67 chains; deflection, 58° 10′ to the right.

Ex. 18. To find the length of a tree leaning to the south, I measured due north from its base 70 yards, and found the elevation of the top to be 25° 10′; then, measuring due east 60 yards, the elevation of the top was 20° 4′. What was the length and inclination of the tree?

Ans. Length, 35.1 yards; inclination, 83° 11′.

Ex. 19. The bearings and distances being as in Ex. 16, it is required to divide the tract into two equal parts by a line running from the first corner. The bearing of the division line is required.

Ans. N. 14° 59′ E. 27.66 chains to a point on the fifth side 1.60 from beginning.

Ex. 20. The boundaries of a quadrilateral are,—1. N. 35¼° E. 23 chains; 2. N. 75½° E. 30.50 chains; 3. S. 3¼° E. 46.49 chains, and 4. N. 66¼° W. 49.64 chains,—to divide the tract into four equal parts by two straight lines, one of which shall be parallel to the third side. Required the distance of the parallel line from the first corner, the bearing of the other division line and its distance from the same corner, measured on the first side.

Ans. Distance of parallel division, 32.50 chains; bearing of the other, S. 88° 22′ E.; distance from the first corner, 5.99 chains.

CHAPTER IX.

MERIDIANS, LATITUDE, AND TIME.

SECTION I.

MERIDIANS.

409. The *meridian* of a place is a true north and south line through that place; or it may be defined to be a great circle of the earth passing through the pole and the place.

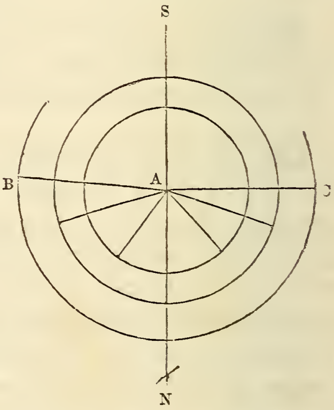
410. As it is of great importance to the surveyor to be able to trace accurately a meridian line, the following methods are given. Any one of these is sufficiently accurate for his purposes. Those which require the employment of the transit or the theodolite are to be preferred, if one of these instruments is at hand. When the observations are performed with the proper care, and the instruments are to be depended on, the line may be run within a few seconds of its proper position.

411. Although the methods to be explained in the following articles are in theory perfectly accurate, yet the results to which they lead cannot be relied on with the same certainty when the observations are made with surveyors' instruments, as if the larger and more expensive instruments to be found in fixed observatories were employed. These instruments generally rest on permanent supports: their positions and adjustments may therefore be tested, and corrected when found defective, and thus their proper position be finally obtained with almost perfect accuracy. Not so with the theodolite or the surveyors' transit. The adjustments in their position must be made at the time, and renewed for every fresh observation. The *results* alone are to be corrected by subsequent observation, and not the *position* of the instrument. Notwithstanding these difficulties, which must always prevent his attaining the precision of the astronomer, yet, with ordinary care, the surveyor may run his lines with all the accuracy which is necessary for his operations.

Problem 1.—*To run a meridian line.*

412. *First Method.—By equal altitudes of the sun.*

Fig. 199.



Select a level surface, exposed to the south, and erect an upright staff upon it. Around the foot of this staff A (Fig. 199) as a centre describe a circle. Observe carefully the point B at which the end of the shadow crosses this circle in the morning, and likewise the point C where it crosses in the evening. Bisect the angle BAC by the line NS, which will be a meridian. If a number of circles be described around A, several observations may be made on the same day, and the mean of the whole taken.

If the staff is not vertical, let fall a plumb-line from the summit, and describe the circles around the point in which this line cuts the surface. A piece of tin, with a small circular hole through it for the sun's rays to pass through, is better than the top of the staff, the image being definite.

Where much accuracy is not required, the above method is sufficient. It supposes the declination of the sun to remain unchanged during the observation. This is not true except at the solstices,—21st of June and 22d of December.

Those days—or at least a time not very remote from them —should therefore be chosen for determining the meridian by this method.

413. *Second Method.—By a meridian observation of the North Star.*

The Pole Star (*Polaris*, or *a Ursæ Minoris*) is situated very nearly at the North Pole of the heavens. If it were exactly so, all that would be necessary to determine the direction of the meridian would be to sight to the star at any time. The North Star, being, however, about $1\frac{1}{2}°$ from the pole, is only on the meridian twice in twenty-four hours.

There is another star, (*Alioth*,) in the tail of the Great Bear, (*Ursæ Majoris*,) which is on the meridian very nearly at the same time as the Pole Star.

The constellation in which *Alioth* is situated is one of the most generally known. It is often called the Plough, the Dipper, the Wagon and Horses, or Charles's Wain. The two stars in the quadrangle farthest from the handle, or tail, are called the Pointers, from the fact that the line joining them will, when produced, pass nearly through the Pole Star. The star in the handle of the dipper, nearest the quadrangle, is *Alioth*.

414. *To determine the direction of the meridian.*

Suspend a long plumb-line from some fixed elevated point. If a window can be found properly situated, a staff may be projected from it to afford a support. The plummet should be heavy, and be allowed to swing in a vessel of water, so as to lessen the effect of the currents in the air. At some distance to the south of the line set two posts, east and west from each other, making their tops level, and nail upon them a horizontal board. To another board screw a compass-sight. This may be moved steadily to the east or west upon the other board. Then, some time before *Polaris* is on the meridian, place the compass-sight so that by looking through it *Alioth* may be hidden by the plumb-line. As the star recedes from the line, move the sight, so as to keep the line and star in the same direction; at last *Polaris* will also be covered by the line. The eye and plumb-line are then very nearly in the meridian. If the time is noted, and *Polaris* sighted to seventeen minutes after the former observation, the meridian will be much more accurately determined. The compass-sight may now be firmly clamped till morning. In making the above described observation, it will generally be necessary for an assistant to illuminate the line if the night is dark.

415. *To determine the time Polaris is on the meridian.*

1. Take from the American Almanac, or other Ephemeris, the sun's right ascension, or sidereal time of mean noon, for the noon preceding the time for which the transit is wanted. The sidereal time is given in the American Almanac for mean noon at Greenwich (England) for every day in the year, and may be calculated for any other meridian by interpolation, thus:—

The difference between the sidereal times for two successive days being 3 minutes 56.555 seconds, say, As *twenty-four hours* is to *the longitude expressed in time*, so is 3 minutes 56.555 seconds to *the correction to be applied to the sidereal time at noon of the given day at Greenwich.* This correction—added to the sidereal time taken from the almanac if the longitude be west, but subtracted if it be east—will give the sidereal time at mean noon at the given place.

The above correction, having been once determined for the given place, will serve for all the calculations that may be wanted.

EXAMPLE.

Ex. 1. Let it be required to find the sidereal time at mean noon, at Philadelphia, long. 5 h. 0 m. 40 sec. W., on the 11th of August, 1855.

The sidereal time at mean noon, Greenwich, August 11, is 9 hours, 17 minutes, 32.74 seconds, as taken from the American Almanac of that year.

And, As 24 h. : 5 h. 0 m. 40. s. :: 3m. 56.555 s. : 49.391.

	h.	m.	sec.
Then, sidereal time at Greenwich, mean noon	9	17	32.74
Correction for difference of long.			49.39
Sidereal time at Philadelphia, mean noon	9	18	22.13

2. Subtract the sidereal time above determined from the right ascension of the star, taken from the same almanac, increasing the latter by 24 hours, if necessary to make the subtraction possible. The remainder is the time of the transit expressed in sidereal hours.

To convert these into solar hours. Say, *As 24 hours is to the number of hours in the above time, so is 3 minutes* 55.9 *seconds to the correction.* This correction, subtracted from the sidereal time, will give the mean solar time of the upper transit.

The time thus determined will be astronomical time. The astronomical day begins at noon, the hours being counted to twenty-four. The first twelve hours, therefore, correspond with the hours in the afternoon of the same civil day; but the last twelve agree with the hours of the morning of the next succeeding day.

Thus, August 11, 8 h. 15 m., astronomical time, corresponds
with August 11, 8 h. 15 m. P.M., civil time;
but August 11, 16 h. 15 m., astronomical time, agrees with
August 12, 4 h. 15 m. A.M., civil time.

If, therefore, the number of hours of a date expressed in astronomical time be greater than twelve, to convert it into civil time the days must be increased by one and the hours diminished by twelve.

Required the time of the upper transit of Polaris, September 11, 1855, for Philadelphia.

	h.	m.	sec.
Sidereal time at mean noon, Greenwich, September 10	11	15	49.38
Correction for Philadelphia			49.39
Sidereal time, mean noon, at Phila. (A)	11	16	38.77
Right ascension of Polaris, Sept. 11 (B)	1	7	2.71
(B) – (A)	13	50	23.94
Correction for 13 h. 50 m. 24 sec.		2	16.04
Astronomical time, September 10	13	48	7.90
agreeing with civil time, Sept. 11	1	48	7.90 A.M.

416. The times of the upper transit of Polaris for every tenth day of the year is given in the following table. The calculation is made for the meridian of Philadelphia, the year 1855. As a change of six hours, or 90° of longitude, will only make a change of one minute in the time of the transit, the table is sufficiently accurate for any place within the United States:—

Time of Polaris crossing the meridian, upper transit.

Months.	1st.	11th.	21st.
	h. m.	h. m.	h. m.
January	6 22 P.M.	5 43 P.M.	5 3 P.M.
February	4 20 "	3 40 "	3 1 "
March	2 29 "	1 50 "	1 11 "
April	0 27 "	11 48 A.M.	11 9 A.M.
May	10 30 A.M.	9 50 "	9 11 "
June	8 28 "	7 49 "	7 10 "
July	6 30 "	5 51 "	5 12 "
August	4 29 "	3 50 "	3 11 "
September	2 27 "	1 48 "	1 9 "
October	0 30 "	11 46 P.M.	11 7 P.M.
November	10 24 P.M.	9 44 "	9 5 "
December	8 26 "	7 46 "	7 7 "

If the time of the passage of the star for any day not given in the table be desired, take out the time of passage for the day next preceding, and deduct four minutes for each day that elapses between the date in the table and that for which the time of transit is required; or, more accurately, thus:—

Say, *As the number of days between those given in the table is to the number between the preceding date and that for which the time of transit is desired, so is the difference between the times of transit given in the table to the time to be subtracted from that corresponding to the earlier of the two days.*

Let the time of transit, August 27, be desired.

	Time.
Aug. 21,	3 h. 11 m.
Sept. 1,	2 27
Difference	44

As 11 d. : 6 d. :: 44 : 24;

therefore 3 h. 11 m. — 24 m. = 2 h. 47 m. is the time required.

417. If the time of the lower transit be desired, it may be obtained from the table by changing A.M. into P.M. and *diminishing* the minutes by 2, or changing P.M. into A.M. and increasing the minutes by 2.

418. The above table is calculated for the year 1855. It will, however, serve for the observation described in Art. 414 for many years, the time of the meridian passage being determined in that method by the time of Polaris and Alioth being in the same vertical. When the time is more accurately needed, as in *Method* 3 (Art. 419) for determining the meridian, it will be necessary to correct the numbers in the table for the years that elapse between 1855 and the current year.

The Pole Star passes the meridian about 21 seconds—more accurately, 20.6 seconds—later every year than the preceding one, so that in 1860 the time will be 1 minute, 43 seconds later than those given in the table; in 1870, 5 minutes; in 1880, 8 minutes 35 seconds; and, in 1890, 12 minutes later.

419. *Third Method.—By a meridian passage observed with a transit or theodolite.*

Having accurately levelled the instrument, sight to Polaris when on the meridian. Then, depressing the telescope, set up an object in the line of sight: a line drawn from the instrument to that object will be a meridian.

In observing with the transit or theodolite at night, it is needful that the wires should be illuminated. This may be done by an assistant reflecting the rays of a lamp into the tube by a sheet of white paper.

An error of 5 minutes in the time of the transit of Polaris will make an error of about $1\frac{1}{2}'$ in the bearing of the star, so that if the observation is not made near the proper time, it must be corrected.

This may be done thus:—Deduct the star's polar distance from the complement of the latitude. Then say, *As sine of this difference is to the sine of the polar distance of the star,* (1° 28′ at present,) *so is sine of the error in time* (expressed in degrees) *to the sine of the bearing of the star. East* if the time be too *early*, but *west* if it be too *late.*

The time is reduced to degrees by multiplying by 15: thus, 5 minutes = 1° 15′.

EXAMPLE.

Required the bearing of Polaris 5 minutes after the upper meridian passage, the latitude of the place being 40°.

$$50° - 1° 28' = 48° 32'$$

As sine of	48° 32′	Ar. Co. 0.125320
: sine of star's polar distance	1° 28′	8.408161
:: sine of time, in degrees,	1° 15′	8.338753
: sine of star's bearing	1′ 37″ W.	6.872234

420. *Fourth Method.—By an observation of Polaris at its greatest elongation.*

As a circumpolar star revolves round the pole, it gradually recedes from the meridian towards the west until it attains its most remote point: here it apparently remains stationary, or at least appears to move directly towards the horizon for a few minutes, and then gradually moves eastward towards the meridian, which it crosses below the pole. Continuing its course, in about six hours it reaches its greatest eastern deviation, when it again becomes stationary. When most remote from the meridian, it is said to have its greatest elongation.

As the star is apparently stationary at the time of its greatest eastern or western elongation, this time is a very favorable one for observing it. A variation of a few minutes in the time will then make no appreciable error in the bearing of the line.

421. The subjoined table contains the times of the greatest eastern or western elongations, according as the one or the other occurs at a time of day favorable for observation. The times of greatest elongations are calculated thus: Take from one of the almanacs mentioned in Art. 415 the polar distance of the star at the given time, and call it P. Call the latitude of the place L. Then find the semidiurnal arc by the following formula:—

$$\text{R . cosine } x = \text{tan. P . tan. L.}$$

Reduce x to time by dividing by 15, calling the degrees hours, and correct for the sidereal acceleration: the result will be the semidiurnal arc expressed in time. Call it t. Then, if T be the time of greatest elongation, and T′ be the time of the upper meridian passage of the star, $T = T' + t$ or $T' - t$, according as the time of the western or eastern elongation is desired.

The hour angle for Polaris at its greatest elongation, July 1, 1855, in lat. 40° N., was 5 hours 54 minutes; but, as the polar distance of the star is diminishing at the rate of 19.23″ per annum, the semidiurnal arc is slowly increasing. The change is so small, however,—being about one second per year,—that it may be entirely neglected. As the time of the meridian passage of the star is later by 20.6 seconds each year than the preceding one, the times of greatest eastern and greatest western elongation will be similarly affected: in 1860 they will be 1 minute 43 seconds later than the times given in the table; in 1870, 5 minutes; and, in 1880, 8 minutes 35 seconds later.

422. *Table of Times of Greatest Elongation of Polaris for 1855. Latitude,* 40° N.

Months.		1st.	11th.	21st.
		h. m.	h. m.	h. m.
January...	West	0 16 A.M.	11 37 P.M.	10 57 P.M.
February..	West	10 14 P.M.	9 35 "	8 55 "
March......	West	8 23 "	7 44 "	7 4 "
April.......	East	6 33 A.M.	5 54 A.M.	5 15 A.M.
May........	East	4 35 "	3 56 "	3 17 "
June.......	East	2 34 "	1 55 "	1 15 "
July........	East	0 36 "	11 53 P.M.	11 14 P.M.
August....	East	10 31 P.M.	9 51 "	9 12 "
September	East	8 29 "	7 50 "	7 11 "
October ...	West	6 24 A.M.	5 44 A.M.	5 5 A.M.
November	West	4 22 "	3 42 "	3 3 "
December	West	2 24 "	1 45 "	1 5 "

The above table is calculated for lat. 40°, for which latitude the hour angle is 5 h. 54 m. 6 sec.;
for latitude 50° the hour angle is 5 52 2,
and for lat. 30° " " " 5 55 38;
therefore, for lat. 50° the eastern elongation occurs two minutes later, and the western two minutes earlier, than those given in the table; for lat. 30° the times of the eastern elongation must be diminished, and those of the western increased, by 1 minute 32 seconds.

423. The observation for the meridian is made as directed Art. 414. Suspend the plumb-line, and, having placed the compass-sight on the table, as the star moves one way move the sight the other, so as to keep the star always hid by the line. At the time of greatest elongation the star will appear stationary behind the line. Clamp the board to which the compass-sight is attached. If the plumb-line is suspended from a point that is not liable to derangement, the remainder of the work may be left till daylight; otherwise, let an assistant take a short stake, with a candle attached to it, to a distance of 8 or 10 chains. He may then be placed exactly in line with the plumb. When the stake has been so adjusted, it should be driven firmly into the ground and its position again tested.

Measure accurately the distance between the compass-sight and the stake. Call it D. Take the azimuth of the star from the following table and call it A.

Calculate $$x = \frac{D \,.\, \tan.\, A}{R},$$

and set off the distance x to the east or west of the stake, according as the western or eastern elongation was observed. The point thus determined will be on the meridian passing through the compass-sight. Permanent marks may then be fixed at any convenient points in this line.

If a transit or theodolite is at hand, direct the telescope to the stake first set up. Turn it through an angle equal to the azimuth: it will then be in the meridian: or direct the telescope to the star when at its greatest elongation, and then turn the plate through an angle equal to the azimuth.

424. The azimuth of a star is its bearing, and may be determined by the following formula,—A being the azimuth, L the latitude of the place, and P the polar distance of the star:—

$$\text{Sin. } A = \frac{R \,.\, \sin.\, P}{\cos.\, L}.$$

Azimuths of the Pole Star at its Greatest Elongation.

Lat.	1855.	1860.	1865.	1870.
°	° ′ ″	° ′ ″	° ′ ″	° ′ ″
30	1 41 21	1 39 32	1 37 42	1 35 49
35	1 47 11	1 45 14	1 43 16	1 41 19
40	1 54 37	1 52 32	1 50 27	1 48 20
45	2 4 11	2 1 55	1 59 35	1 57 18

The above are calculated from the mean place of the star as given in Loomis's "Practical Astronomy."

425. *Fifth Method.—By equal altitudes of a star.*

If a theodolite or a transit with a vertical arc is at hand, the meridian may be run very accurately by observing a star when at equal altitudes before and after passing the meridian.

For this purpose select a star situated near the equator, and, having levelled the instrument with great care, take the altitude of the star about two or three hours before it passes the meridian, and notice carefully the horizontal reading. When the star is about as far to the west of the meridian, set the telescope to the same elevation, and follow the star by the horizontal motion until its altitude is the same as before, and again notice the reading.

Then if the zero is not between the two observed readings, take half their sum, and turn the telescope until the vernier is at that number of degrees and minutes: the telescope will then be in the meridian. If the vernier has passed the zero, add 360 to the less reading before taking the sum.

Thus, if the first reading were $150° \, 37' \, 30''$, and the second $280° \, 25'$, the half sum $\dfrac{431° \, 2' \, 30''}{2} = 215° \, 31' \, 15''$ would be the reading for the meridian.

Instead of taking the readings, a stake may be set up at any distance—say ten chains—in each observed course: then bisect the line joining the stakes, and run a line from the instrument to the point of bisection.

The mean of a few observations taken in this manner will determine the meridian with considerable precision.

SECTION II.

LATITUDE.

THE latitude of a place may be determined in various modes.

426. *First Method.—By a meridian altitude of the Pole Star.*

The altitude of the pole is equal to the latitude of the place. Take the altitude of Polaris when on the meridian, and from the result subtract the refraction taken from the following table. Increase or diminish the remainder by the polar distance of the star according as the lower or upper transit was observed: the result will be the latitude.

427. *Refraction to be taken from the apparent latitude.*

App. Alt.	Ref.		App. Alt.	Ref.		App. Alt.	Ref.		App. Alt.	Ref.		App. Alt.	Ref.	
°	′	″	°	′	″	°	′	″	°	′	″	°	′	″
20	2	39	30	1	40	40	1	9	50	0	49	60	0	34
21	2	30	31	1	37	41	1	7	51	0	47	61	0	32
22	2	23	32	1	33	42	1	5	52	0	45	62	0	31
23	2	16	33	1	29	43	1	2	53	0	44	63	0	30
24	2	10	34	1	26	44	1	0	54	0	42	64	0	28
25	2	4	35	1	23	45	0	58	55	0	41	65	0	27
26	1	59	36	1	20	46	0	56	56	0	39	66	0	26
27	1	54	37	1	17	47	0	54	57	0	38	67	0	25
28	1	49	38	1	14	48	0	52	58	0	36	68	0	24
29	1	45	39	1	12	49	0	50	59	0	35	69	0	22

428. *Second Method.*—Take the altitude of the star six hours before or after its meridian passage. The result, corrected for refraction, will be the latitude.

429. *Third Method.—By a meridian altitude of the sun.*

Take the meridian altitude of the upper or the lower limb of the sun, and correct for refraction. The result, increased or diminished by the semidiameter of the sun according as the lower or the upper limb was observed, will be the altitude of the sun's centre. (The apparent semidiameter of the sun is given in the American Almanac for every day of the year.)

To the altitude of the sun's centre, add his declination (taken from the same almanac) if south, but subtract it if north: the result subtracted from 90° will give the latitude.

Instead of the sun, a bright star, the declination of which is small, may be observed.

430. If the exact direction of the meridian is not known, the telescope must be fixed on the body some time before it is south. As the sun or star approaches the meridian its altitude increases, and it will therefore rise above the horizontal wire. Move the telescope in altitude and azimuth so as to follow the body until it ceases to leave the wire. The reading will then give the observed meridian altitude. The altitude alters very slowly for some minutes before and after its meridian passage, thus affording ample time to direct the telescope accurately towards the object.

431. *Fourth Method.—By an observation of a star in the prime vertical.*

Any great circle passing through the zenith is called a *vertical circle.* All such circles are perpendicular to the horizon.

That vertical circle which is perpendicular to the meridian is called the *prime vertical:* it cuts the horizon in the east and west points.

Level the plates of the transit or theodolite carefully, and direct the telescope to the east or west, so that it may move in the prime vertical or nearly so. Then, having selected some bright star which passes the meridian a little south of the zenith, (the declination of such a star is rather less than the latitude of the place,) observe the time of its crossing the vertical wire of the telescope before passing the meridian, and again, when in the west, after its meridian passage. Let these times be called T and T′. Let the interval between T and T′ be called x, which must be reduced to sidereal time by adding to the solar time 3 minutes 56.55 seconds for 24 hours, or 9.85 seconds per hour; also, let L be the latitude of the place, and D be the declination of the star.

Then $$\tan. L = \frac{R. \tan. D}{\cos. x}.$$

Thus, for example, the transit of *α Lyræ* over the prime vertical was observed July 1, 1855, at 10 h. 43 m. 4 sec., and again at 13 h. 3 m. 48 sec., mean solar time. Required the latitude,—the apparent right ascension of the star (as given in the American Almanac) being 18 h. 32 m. 4 sec., and the declination 38° 39′ 0.4″.

Here the interval is 2 h. 20 m. 44 sec., solar time.

Reduction			23
	2)2	21	7
	1 h.	10 m.	33.5 sec. = 17° 38′ 22″.

Cos. x	17° 38′ 22″	A. C. 0.020915
tan. D	38° 39′ 0.4″	9.902940
tan. L	40° 0′ 4″	9.923855

432. Half the sum of the observed times is the time of meridian passage in mean solar time. If this is reduced to sidereal time and increased by the sidereal time of mean noon at the given place, the result should be equal to the right ascension of the star.

In the example before us the times of observation are

	h.	m.	sec.
	10	43	4
and	13	3	48
Sum	2)23	46	52
Half sum	11	53	26
Reduction for sidereal time		1	57
(A)	11	55	23

Sidereal time, mean noon, at Greenwich	6 h.	35 m.	54 sec.
Add for difference of meridians			49
	6	36	33
Add (A)	18	31	56
Right ascension of star	18	32	4
Error in position of the instrument			8″

A slight error in the position of the instrument will make no appreciable error in the result. Hence, this method affords perhaps the best means of determining the latitude.

SECTION III.

TO FIND THE TIME OF DAY.

433. *First Method.*—If a good meridian line has been run, the transit or theodolite may be placed in that line, and, being well levelled, the telescope, if adjusted by being directed to the meridian mark, will, when elevated, move in the meridian.

Observe the time that the western limb of the sun comes to the vertical wire, and also when the eastern limb leaves it. The mean between these will be the time that the centre of the sun is on the meridian, or *apparent noon.* Increase or diminish the observed time of the passage of the centre by the equation of time according as the sun is too slow or too fast, and the result will be the time of mean noon as given by the watch. The difference between this and twelve hours will be the error of the watch.

434. *Second Method.*—Calculate the time that a fixed star having but little declination will pass the meridian as directed for Polaris, Art. 415. Then the difference between the observed and the calculated time will be the error of the watch.

435. *Third Method.*—If the meridian line has not been determined, the time may be obtained by an altitude of the sun or of a star when out of the meridian.

Take the altitude of the sun when three or four hours from the meridian, noting the time by the watch, and correct it for refraction and semidiameter. The altitude of the upper limb should be taken in the afternoon, and the lower in the morning, as the wire then crosses the face of the sun before the observation, and may be distinctly seen.

Call the altitude of the sun A, the polar distance D, the latitude L, and the hour angle H.

Then $$\sin.^2 \tfrac{1}{2} H = \frac{\cos. \tfrac{1}{2}(A + L + D) \sin. \tfrac{1}{2}(L + D - A)}{\sin. D \,.\, \cos. L},$$

or, if $S = \frac{1}{2}(A + L + D)$, then $S - A = \frac{1}{2}(L + D - A)$,

and $$\sin.^2 \tfrac{1}{2} H = \frac{\cos. S \,.\, \sin. (S - A)}{\sin. D \,.\, \cos. L}.$$

RULE.

Call the corrected altitude A. From the Ephemeris take the sun's declination at the time of observation, (the watch-time will be sufficiently accurate); if north, subtract it from 90°, but if south, add it to 90°: the result will be the sun's polar distance, which call D. Call the latitude of the place L. Let $S = \frac{1}{2}(A + L + D)$. Add together Ar. Co. sin. D, Ar. Co. cos. L, cos. S, and sin. $(S - A)$, divide the result by 2, and the quotient will be the sine of half the hour angle of the sun at the time of observation. If the observation is made in the afternoon, the hour angle reduced to time is the apparent time; but, if the observation is in the morning, the hour angle subtracted from 12 is the apparent time. To the apparent time apply the equation of time, and the result is the mean time of the observation. The difference between the calculated time and that shown by the watch is the error of the watch.

Several observations may be made in the course of a few minutes, and the mean of the results taken. If the observation is carefully made with a good transit or theodolite, the time obtained by this method will not differ more than a small fraction of a minute from the true time.

436. If a star is observed instead of the sun, the mode of calculation is the same. The hour angle will then be in sidereal hours, which must be converted into solar hours. The result, added to or subtracted from the time of the meridian passage of the star, according as the observation was made after or before the star had passed the meridian, will give the mean time of observation.

437. If two altitudes of the sun or a star be taken, and the times noted by a watch, the true time and the latitude may both be found. But, as other and preferable methods have already been given for finding the latitude, it is unnecessary to give the rule here.

CHAPTER X.

VARIATION OF THE COMPASS.

438. It has been mentioned (Art. 268) that the magnetic and the geographical meridian do not generally coincide; the difference between the directions of the two being called the variation of the compass. If this variation were constant, it would be of no practical importance to the surveyor. A line run by the compass at one time could be retraced on the same bearing at any other. The variation is, however, subject to continual changes,—some of them having a period of many years, perhaps several centuries, others being annual or diurnal, and some accidental or temporary.

439. Secular Change. From the time of the earliest observations made in this country on the position of the magnetic needle till about the commencement of the present century, the north point was gradually moving to the west. Since then, the direction of its motion has been reversed. This motion constitutes what is called the *secular change*. To give an idea of the extent of this deviation, the following table of observations, made at Paris, is presented:—

Year.	Variation.			Year.	Variation.		
1541	7°		East.	1816	22°	25′	West.
1580	11	30′	"	1823	22	23	"
1618	8		"	1827	22	20	"
1663	0		"	1828	22	5	"
1700	8	10	West.	1829	22	12	"
1780	19	55	"	1835	22	3	"
1805	22	5	"	1853	20	17	"
1814	22	34	"				

From this table, it appears that in 1580 the needle had attained its greatest eastern deviation. From that time to about the year 1814 it moved towards the west, the greatest deviation being 22° 34′. Since 1814 it has been moving to the east.

From observations made at various places in Europe and America, it appears that similar changes have been going on throughout all these countries.

440. The following table, mostly taken from the "Report of the Superintendent of the United States' Coast Survey" for 1855, gives the variation and secular change for some of the more important places in this country:—

Locality.	Lat.	Lon.	Date.	Variation.	Change in 1850.
Montreal, C.W.....	45° 30′	73° 35′	1850	+ 9° 28′	+4′
Toronto, "	49° 40′	79° 21′	1850	1° 36′	
Burlington, Vt.....	44° 27′	73° 10′	1855	9° 57′.1	4′.9
Portland, Me.......	43° 39′	70° 16′	1851	11° 41′	
Boston, Mass.......	44° 20′	71° 2′	1854	9° 31′	5′.2
Providence, R.I....	41° 50′	71° 24′	1855	9° 31′.5	6′.0
New Haven, Conn.	41° 18′	72° 54′	1845	6° 17′.3	4′.8
New York City.....	40° 43′	74° 0′	1845	6° 25′.3	5′.2
Albany, N.Y........	42° 39′	73° 44′	1836	6° 47′	7′.2
Philadelphia, Pa...	39° 58′	75° 10′	1855	4° 31′.7	6′.8
Pittsburg, Pa	40° 26′	79° 58′	1845	33′	3′.5
Wilmington, Del...	39° 45′	75° 34′	1846	2° 30′.7	
Baltimore, Md.	39° 16′	76° 34′	1847	2° 18′.6	
Washington, D.C...	38° 53′	77° 1′	1855	2° 25′	5′.0
Petersburg, Va....	37° 14′	77° 24′	1852	0° 26′.5	
Columbia, S.C......	34°	81° 2′	1854	— 3° 1′.7	
Savannah, Ga......	32° 5′	81° 5′	1852	— 3° 40′.3	
Cincinnati, O.......	39° 6′	84° 22′	1845	— 4° 4′	4′
Richmond, Ind.....	39° 49′	84° 47′	1845	— 4° 52′	4′
Detroit, Mich.......	42° 24′	82° 58′	1840	— 2° 0′	1′
San Francisco, Cal.	37° 48′	122° 27′	1852	—15° 27′	

The above are derived from the best data that could be procured; but many of the observations are doubtless very imperfect.

441. Line of no Variation. From a map published by Professor Loomis, it appears that in 1840 the lines of equal variation crossed the United States in a direction to the east of south, tending more to the east in the New England States. At that date, the line of no variation passed a little to the west of Pittsburg and to the east of Raleigh, N.C.,—all those portions of the country to the east of that line having western variation. From a similar map, published in the Report above referred to, it appears that the line of no variation had shifted to the west a few miles since that time. It also results from the calculations in the same report, that the rate of *change* in variation has now attained its maximum, and is beginning to diminish.

442. As it is frequently of importance to know the former variation, the following information is added:—

The variation in

Burlington, Vt., in	1792	7° 38′ W.;	1818, 7° 30′ W.
Salem, Mass.,	1781	7° 2′ W.;	1805, 5° 57′ W.
New Haven, Ct.,	1761	5° 47′ W.;	1775, 5° 25′ W.
" "	1819	4° 35′ W.	
New York,	1686	8° 45′ W.;	1750, 6° 22′ W.
" "	1789	4° 20′ W.;	1824, 4° 40′ W.
Philadelphia,	1710	8° 30′ W.;	1750, 5° 45′ W.
"	1793	1° 30′ W.;	1837, 3° 52′ W.

443. From the table, (Art. 440,) the variation for any time not far remote from those given may readily be found. This will also apply for places not very far distant from the line of equal variation passing through that place. As, however, the *rate* of change varies, calculations based on such a table can only be considered correct when the interval of time is comparatively small. In all cases, when it can be done, the variation should be found by direct observation by the methods explained in the next article.

444. *To determine the change in variation by old lines.*

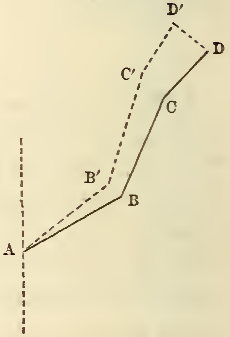
As the rate of change varies, the above rule can only be considered as true when the interval of time has not been great. If a number of years have elapsed since the prior survey, and no observations can be found relating to the immediate neighborhood, the change of variation can be found, nearly, by comparison with other places where such observations have been made.

When any well-established marks can be found, the change may be determined by taking the bearings of these and comparing them with the records. The difference will give the change that has taken place between the dates of the two surveys.

If the two marks are not on the same line, they may still be used for this purpose. Thus, according to an old deed, the bearings of three adjacent sides of a tract were as follows,—viz.: 1. Beginning at a marked locust, N. 60½° E. 200 perches to a chestnut; 2. N. 25¼° E. 183 perches to a post; 3. N. 45° E. 105.3 perches to a white-oak. The locust is gone, but the stump remains, and the white-oak is still standing. The intermediate corners are entirely lost.

Setting the instrument over the stump, run N. 60½° E. 200 perches; thence N. 25¼° E. 183 perches; and thence N. 45° E. 105.3 perches.

Fig. 200.



If no change had taken place in the variation, and both surveys had been accurately made, the last distance would have been terminated at the white-oak. Instead of this, however, the tree bears S. 54° 25′ E. 2.93 perches. Fig. 200 is a draft of the above.

From the bearings of AB, BC, and CD, calculate that of AD, which (Art. 350) will be found to be N. 43° 59′ E. 470.38 perches. This, therefore, was the bearing and distance of AD at the time of the former survey. It is now the bearing and distance of AD′.

With the latitude and departure of AD′ and that of DD′, calculate the present bearing and distance of AD (Art. 350.) It will be found to be N. 47° 54′ E. 476.25 perches. The change of variation has therefore been 3° 55′ W. There is likewise a variation of 5.87 perches in the measurement, from which it is inferred that the chain used in the former survey was 101.25 links in length, or 1¼ links too long.

In order, therefore, correctly to trace the lines of the tract, the vernier of the compass must be set 3° 55′ W., and all the distances be increased $1\frac{1}{4}$ links per chain, or $1\frac{1}{4}$ perches per hundred. The magnetic bearings and the distances of the three sides are now,—1. N. 64° 25′ E. 202.5 perches; 2. N. 29° 10′ E. 185.3 perches; 3. N. 48° 55′ E. 106.6 perches.

445. Diurnal Change. If the position of the needle be accurately noted at sunrise on a clear summer day, and the observation be repeated at intervals, it will be found that the north pole will gradually be deflected to the west, attaining its maximum deviation about 2 or 3 o'clock. During the afternoon it will gradually return towards its former position, which it will regain about 8 or 9 o'clock in the evening. This deviation from the normal position is known as the diurnal change. It amounts sometimes to as much as a quarter of a degree, being greater in a clear day than when the sky is overcast, and not being perceptible if the day is entirely cloudy. It is likewise greater in summer than in winter.

In consequence of this diurnal change, it is evident that a line run in the morning cannot be retraced with the same bearings at noon. The surveyor should therefore record not merely the date at which a survey is made, but also the time of day at which any important line was run, and also the state of the weather, whether clear or otherwise.

446. Irregular Changes. Besides the secular and diurnal changes, the needle is subject to disturbance from the passage of thunder storms, or from the occurrence of aurora boreali. It is likewise sometimes violently agitated when no apparent cause exists. Such disturbances probably result from the occurrence of a distant magnetic storm, which would otherwise be unperceived, or from the passage of electric currents through the atmosphere.

447. From the preceding articles it will be apparent that the needle, though an invaluable instrument for many purposes, is little to be depended on where precision is required. It would be very desirable that prominent marks, the bearings of which were fully known, were established over the country, and that all important lines should be determined, by triangulation, from these. The true bearings should always be recorded. There would then be no difficulty in retracing old lines. In the State of Pennsylvania, and perhaps in some others, this is now required by law, though it is very doubtful whether the law is yet carried out in a way to be of much practical benefit, owing to the want of scientific knowledge on the part of much the larger number of those who undertake the business of surveying.

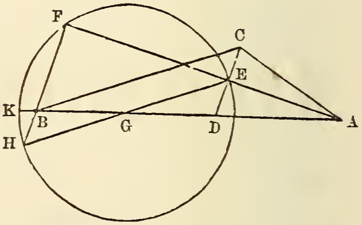
Until there is a more general diffusion of theoretical as well as practical science among those whose business it is to settle the boundaries of estates, cases will continually occur in which confusing lines will be found to exist. This could never occur if all the bearings were made to the true meridian, the surveyor being careful to determine the local attraction and to allow for it in making his record. In no instance should a station be left before the back-sight had been taken, since, even in those regions where but little such influence exists, it will sometimes be found at particular points. It sometimes likewise extends, without any change, over a considerable space, and thus may deflect the needle similarly at a number of stations. An instance of this kind was related to the author, a short time since, by a surveyor of great practical experience.

A line was in dispute. One of the parties called in a surveyor, whom we shall call A., who ran the line, coming out at a stone. The other party, not being satisfied, called upon B., who traced a line agreeing exactly with the one run by A. until he came to a certain point: he then deviated from the former line some 4° to the west. He likewise arrived at a *stone*. Both parties were now dissatisfied. The first called on A. again, who retraced his line, following exactly his former course. B. was again employed. His course deviated at the same point as before from A.'s. It was then concluded to have them together. B., being the older hand, went ahead. When they arrived at the point at which their lines separated, B. called on A. to look through the sights, saying, "Is not this right, Mr. A.?" "It looks very well," he replied: "but look back, Mr. B." On doing so, he found he was really running 4° to the west of his former course. The attraction was first manifest at that point, and continued, without change, at all the subsequent stations along the line he had traversed.

APPENDIX.

THE following demonstration of the rule for finding the area of a triangle when three sides are given is more concise than that given in Art. 251. As the former, however, develops some important properties respecting the centre of the inscribed circle, it was thought best to retain it :—

Fig. 201.



Let ABC (Fig. 201) be the triangle, the construction being the same as in Fig. 50, p. 75.

Then, as was proved in the demonstration of the Rule in Art. 143,

$$AK = \tfrac{1}{2}(AB + BC + AC) = \tfrac{1}{2}\,s.$$
$$AI = \tfrac{1}{2}\,s - BC.$$

We have also

$$KD = BI = \tfrac{1}{2}\,s - AC, \text{ and } KB = \tfrac{1}{2}\,s - AB.$$

Now, from similar triangles, ADE and AFB, we have

$$AE : ED :: AF : FB.$$

But $$AF : ED :: AF : ED;$$

whence (23.6) $$AE \,.\, AF : ED^2 :: AF^2 : ED \,.\, FB.$$

But $$AE \,.\, AF = AK \,.\, AI \text{ (Cor. 36.3)},$$

and $$ED \,.\, FB = HB \,.\, FB = IB \,.\, BK \text{ (35.3)};$$

$\therefore$ $$AI \,.\, AK : ED^2 :: AF^2 : IB \,.\, BK,$$

and $$\sqrt{AI \,.\, AK \,.\, IB \,.\, BK} = ED \,.\, AF = ED \,.\, (AE + EF)$$
$$= ADC + BDC = ABC.$$

MATHEMATICAL TABLES.

MATHEMATICAL TABLES.

TRAVERSE TABLE;

OR,

DIFFERENCE OF LATITUDE

AND

DEPARTURE.

LATITUDES AND DEPARTURES.

D.	¼ Deg.		½ Deg.		¾ Deg.		1 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	1.0000	.0044	1.0000	.0087	.9999	.0131	.9998	.0175	1
2	2.0000	.0087	1.9999	.0175	1.9998	.0262	1.9997	.0349	2
3	3.0000	.0131	2.9999	.0262	2.9997	.0393	2.9995	.0524	3
4	4.0000	.0175	3.9998	.0349	3.9997	.0524	3.9994	.0698	4
5	5.0000	.0218	4.9998	.0436	4.9996	.0654	4.9992	.0873	5
6	5.9999	.0262	5.9998	.0524	5.9995	.0785	5.9991	.1047	6
7	6.9999	.0305	6.9997	.0611	6.9994	.0916	6.9989	.1222	7
8	7.9999	.0349	7.9997	.0698	7.9993	.1047	7.9988	.1396	8
9	8.9999	.0393	8.9997	.0785	8.9992	.1178	8.9986	.1571	9
10	9.9999	.0436	9.9996	.0873	9.9991	.1309	9.9985	.1745	10
	89¾ Deg.		89½ Deg.		89¼ Deg.		89 Deg.		
	1¼ Deg.		1½ Deg.		1¾ Deg.		2 Deg.		
1	.9998	.0218	.9997	.0262	.9995	.0305	.9994	.0349	1
2	1.9995	.0436	1.9993	.0524	1.9991	.0611	1.9988	.0698	2
3	2.9993	.0654	2.9990	.0785	2.9986	.0916	2.9982	.1047	3
4	3.9990	.0873	3.9986	.1047	3.9981	.1222	3.9976	.1396	4
5	4.9988	.1091	4.9983	.1309	4.9977	.1527	4.9970	.1745	5
6	5.9986	.1309	5.9979	.1571	5.9972	.1832	5.9963	.2094	6
7	6.9983	.1527	6.9976	.1832	6.9967	.2138	6.9957	.2443	7
8	7.9981	.1745	7.9973	.2094	7.9963	.2443	7.9951	.2792	8
9	8.9979	.1963	8.9969	.2356	8.9958	.2748	8.9945	.3141	9
10	9.9976	.2181	9.9966	.2618	9.9953	.3054	9.9939	.3490	10
	88¾ Deg.		88½ Deg.		88¼ Deg.		88 Deg.		
	2¼ Deg.		2½ Deg.		2¾ Deg.		3 Deg.		
1	.9992	.0393	.9990	.0436	.9988	.0480	.9986	.0523	1
2	1.9985	.0785	1.9981	.0872	1.9977	.0960	1.9973	.1047	2
3	2.9977	.1178	2.9971	.1308	2.9965	.1439	2.9959	.1570	3
4	3.9969	.1570	3.9962	.1745	3.9954	.1919	3.9945	.2093	4
5	4.9961	.1963	4.9952	.2181	4.9942	.2399	4.9931	.2617	5
6	5.9954	.2356	5.9943	.2617	5.9931	.2879	5.9918	.3140	6
7	6.9946	.2748	6.9933	.3053	6.9919	.3358	6.9904	.3664	7
8	7.9938	.3140	7.9924	.3490	7.9908	.3838	7.9890	.4187	8
9	8.9931	.3533	8.9914	.3926	8.9896	.4318	8.9877	.4710	9
10	9.9913	.3926	9.9905	.4362	9.9885	.4798	9.9863	.5234	10
	87¾ Deg.		87½ Deg.		87¼ Deg.		87 Deg.		
	3¼ Deg.		3½ Deg.		3¾ Deg.		4 Deg.		
1	.9984	.0567	.9981	.0610	.9979	.0654	.9976	.0698	1
2	1.9968	.1134	1.9963	.1221	1.9957	.1308	1.9951	.1395	2
3	2.9952	.1701	2.9944	.1831	2.9936	.1962	2.9927	.2093	3
4	3.9936	.2268	3.9925	.2442	3.9914	.2616	3.9903	.2790	4
5	4.9920	.2835	4.9907	.3052	4.9893	.3270	4.9878	.3488	5
6	5.9904	.3402	5.9888	.3663	5.9872	.3924	5.9854	.4185	6
7	6.9887	.3968	6.9869	.4273	6.9850	.4578	6.9829	.4883	7
8	7.9871	.4535	7.9851	.4884	7.9829	.5232	7.9805	.5581	8
9	8.9855	.5102	8.9832	.5494	8.9807	.5886	8.9781	.6278	9
10	9.9839	.5669	9.9813	.6105	9.9786	.6540	9.9756	.6976	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	86¾ Deg.		86½ Deg.		86¼ Deg.		86 Deg.		

LATITUDES AND DEPARTURES.

D.	4¼ Deg.		4½ Deg.		4¾ Deg.		5 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.9973	.0741	.9969	.0785	.9966	.0828	.9962	.0872	1
2	1.9945	.1482	1.9938	.1569	1.9931	.1656	1.9924	.1743	2
3	2.9918	.2223	2.9908	.2354	2.9897	.2484	2.9886	.2615	3
4	3.9890	.2964	3.9877	.3138	3.9863	.3312	3.9848	.3486	4
5	4.9863	.3705	4.9846	.3923	4.9828	.4140	4.9810	.4358	5
6	5.9835	.4447	5.9815	.4708	5.9794	.4968	5.9772	.5229	6
7	6.9808	.5188	6.9784	.5492	6.9760	.5797	6.9734	.6101	7
8	7.9780	.5929	7.9753	.6277	7.9725	.6625	7.9696	.6972	8
9	8.9753	.6670	8.9723	.7061	8.9691	.7453	8.9658	.7844	9
10	9.9725	.7411	9.9692	.7846	9.9657	.8281	9.9619	.8716	10
	85¾ Deg.		85½ Deg.		85¼ Deg.		85 Deg.		
	5¼ Deg.		5½ Deg.		5¾ Deg.		6 Deg.		
1	.9958	.0915	.9954	.0958	.9950	.1002	.9945	.1045	1
2	1.9916	.1830	1.9908	.1917	1.9899	.2004	1.9890	.2091	2
3	2.9874	.2745	2.9862	.2875	2.9849	.3006	2.9836	.3136	3
4	3.9832	.3660	3.9816	.3834	3.9799	.4008	3.9781	.4181	4
5	4.9790	.4575	4.9770	.4792	4.9748	.5009	4.9726	.5226	5
6	5.9748	.5490	5.9724	.5751	5.9698	.6011	5.9671	.6272	6
7	6.9706	.6405	6.9678	.6709	6.9648	.7013	6.9617	.7317	7
8	7.9664	.7320	7.9632	.7668	7.9597	.8015	7.9562	.8362	8
9	8.9622	.8235	8.9586	.8626	8.9547	.9017	8.9507	.9408	9
10	9.9580	.9150	9.9540	.9585	9.9497	1.0019	9.9452	1.0453	10
	84¾ Deg.		84½ Deg.		84¼ Deg.		84 Deg.		
	6¼ Deg.		6½ Deg.		6¾ Deg.		7 Deg.		
1	.9941	.1089	.9936	.1132	.9931	.1175	.9925	.1219	1
2	1.9881	.2177	1.9871	.2264	1.9861	.2351	1.9851	.2437	2
3	2.9822	.3266	2.9807	.3396	2.9792	.3526	2.9776	.3656	3
4	3.9762	.4355	3.9743	.4528	3.9723	.4701	3.9702	.4875	4
5	4.9703	.5443	4.9679	.5660	4.9653	.5877	4.9627	.6093	5
6	5.9643	.6532	5.9614	.6792	5.9584	.7052	5.9553	.7312	6
7	6.9584	.7621	6.9550	.7924	6.9515	.8228	6.9478	.8531	7
8	7.9524	.8709	7.9486	.9056	7.9445	.9403	7.9404	.9750	8
9	8.9465	.9798	8.9421	1.0188	8.9376	1.0578	8.9329	1.0968	9
10	9.9406	1.0887	9.9357	1.1320	9.9307	1.1754	9.9255	1.2187	10
	83¾ Deg.		83½ Deg.		83¼ Deg.		83 Deg.		
	7¼ Deg.		7½ Deg.		7¾ Deg.		8 Deg.		
1	.9920	.1262	.9914	.1305	.9909	.1349	.9903	.1392	1
2	1.9840	.2524	1.9829	.2611	1.9817	.2697	1.9805	.2783	2
3	2.9760	.3786	2.9743	.3916	2.9726	.4046	2.9708	.4175	3
4	3.9680	.5048	3.9658	.5221	3.9635	.5394	3.9611	.5567	4
5	4.9600	.6310	4.9572	.6526	4.9543	.6743	4.9513	.6959	5
6	5.9520	.7572	5.9487	.7832	5.9452	.8091	5.9416	.8350	6
7	6.9440	.8834	6.9401	.9137	6.9361	.9440	6.9319	.9742	7
8	7.9360	1.0096	7.9316	1.0442	7.9269	1.0788	7.9221	1.1134	8
9	8.9280	1.1358	8.9230	1.1747	8.9178	1.2137	8.9124	1.2526	9
10	9.9200	1.2620	9.9144	1.3053	9.9087	1.3485	9.9027	1.3917	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	82¾ Deg.		82½ Deg.		82¼ Deg.		82 Deg.		

LATITUDES AND DEPARTURES.

D.	8¼ Deg.		8½ Deg.		8¾ Deg.		9 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.9897	.1435	.9890	.1478	.9884	.1521	.9877	.1564	1
2	1.9793	.2870	1.9780	.2956	1.9767	.3042	1.9754	.3129	2
3	2.9690	.4305	2.9670	.4434	2.9651	.4564	2.9631	.4693	3
4	3.9586	.5740	3.9561	.5912	3.9534	.6085	3.9508	.6257	4
5	4.9483	.7175	4.9451	.7390	4.9418	.7606	4.9384	.7822	5
6	5.9379	.8610	5.9341	.8869	5.9302	.9127	5.9261	.9386	6
7	6.9276	1.0044	6.9231	1.0347	6.9185	1.0649	6.9138	1.0950	7
8	7.9172	1.1479	7.9121	1.1825	7.9069	1.2170	7.9015	1.2515	8
9	8.9069	1.2914	8.9011	1.3303	8.8953	1.3691	8.8892	1.4079	9
10	9.8965	1.4349	9.8902	1.4781	9.8836	1.5212	9.8769	1.5643	10
	81¾ Deg.		81½ Deg.		81¼ Deg.		81 Deg.		
	9¼ Deg.		9½ Deg.		9¾ Deg.		10 Deg.		
1	.9870	.1607	.9863	.1650	.9856	.1693	.9848	.1736	1
2	1.9740	.3215	1.9726	.3301	1.9711	.3387	1.9696	.3473	2
3	2.9610	.4822	2.9589	.4951	2.9567	.5080	2.9544	.5209	3
4	3.9480	.6430	3.9451	.6602	3.9422	.6774	3.9392	.6946	4
5	4.9350	.8037	4.9314	.8252	4.9278	.8467	4.9240	.8682	5
6	5.9220	.9645	5.9177	.9903	5.9133	1.0161	5.9088	1.0419	6
7	6.9090	1.1252	6.9040	1.1553	6.8989	1.1854	6.8937	1.2155	7
8	7.8960	1.2859	7.8903	1.3204	7.8844	1.3548	7.8785	1.3892	8
9	8.8830	1.4467	8.8766	1.4854	8.8700	1.5241	8.8633	1.5628	9
10	9.8700	1.6074	9.8629	1.6505	9.8556	1.6935	9.8481	1.7365	10
	80¾ Deg.		80½ Deg.		80¼ Deg.		80 Deg.		
	10¼ Deg.		10½ Deg.		10¾ Deg.		11 Deg.		
1	.9840	.1779	.9833	.1822	.9825	.1865	.9816	.1908	1
2	1.9681	.3559	1.9665	.3645	1.9649	.3730	1.9633	.3816	2
3	2.9521	.5338	2.9498	.5467	2.9474	.5596	2.9449	.5724	3
4	3.9362	.7118	3.9330	.7289	3.9298	.7461	3.9265	.7632	4
5	4.9202	.8897	4.9163	.9112	4.9123	.9326	4.9081	.9540	5
6	5.9042	1.0677	5.8995	1.0934	5.8947	1.1191	5.8898	1.1449	6
7	6.8883	1.2456	6.8828	1.2756	6.8772	1.3057	6.8714	1.3357	7
8	7.8723	1.4235	7.8660	1.4579	7.8596	1.4922	7.8530	1.5265	8
9	8.8564	1.6015	8.8493	1.6401	8.8421	1.6787	8.8346	1.7173	9
10	9.8404	1.7794	9.8325	1.8224	9.8245	1.8652	9.8163	1.9081	10
	79¾ Deg.		79½ Deg.		79¼ Deg.		79 Deg.		
	11¼ Deg.		11½ Deg.		11¾ Deg.		12 Deg.		
1	.9808	.1951	.9799	.1994	.9790	.2036	.9781	.2079	1
2	1.9616	.3902	1.9598	.3987	1.9581	.4073	1.9563	.4158	2
3	2.9424	.5853	2.9398	.5981	2.9371	.6109	2.9344	.6237	3
4	3.9231	.7804	3.9197	.7975	3.9162	.8146	3.9126	.8316	4
5	4.9039	.9755	4.8996	.9968	4.8952	1.0182	4.8907	1.0396	5
6	5.8847	1.1705	5.8795	1.1962	5.8743	1.2219	5.8689	1.2475	6
7	6.8655	1.3656	6.8595	1.3956	6.8533	1.4255	6.8470	1.4554	7
8	7.8463	1.5607	7.8394	1.5949	7.8324	1.6291	7.8252	1.6633	8
9	8.8271	1.7558	8.8193	1.7943	8.8114	1.8328	8.8033	1.8712	9
10	9.8079	1.9509	9.7992	1.9937	9.7905	2.0364	9.7815	2.0791	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	78¾ Deg.		78½ Deg.		78¼ Deg.		78 Deg.		

LATITUDES AND DEPARTURES.

D.	12¼ Deg.		12½ Deg.		12¾ Deg.		13 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.9772	.2122	.9763	.2164	.9753	.2207	.9744	.2250	1
2	1.9545	.4244	1.9526	.4329	1.9507	.4414	1.9487	.4499	2
3	2.9317	.6365	2.9289	.6493	2.9260	.6621	2.9231	.6749	3
4	3.9089	.8487	3.9052	.8658	3.9014	.8828	3.8975	.8998	4
5	4.8862	1.0609	4.8815	1.0822	4.8767	1.1035	4.8719	1.1248	5
6	5.8634	1.2731	5.8578	1.2986	5.8521	1.3242	5.8462	1.3497	6
7	6.8406	1.4852	6.8341	1.5151	6.8274	1.5449	6.8206	1.5747	7
8	7.8178	1.6974	7.8104	1.7315	7.8027	1.7656	7.7950	1.7996	8
9	8.7951	1.9096	8.7867	1.9480	8.7781	1.9863	8.7693	2.0246	9
10	9.7723	2.1218	9.7630	2.1644	9.7534	2.2070	9.7437	2.2495	10
	77¾ Deg.		77½ Deg.		77¼ Deg.		77 Deg.		

D.	13¼ Deg.		13½ Deg.		13¾ Deg.		14 Deg.		D.
1	.9734	.2292	.9724	.2334	.9713	.2377	.9703	.2419	1
2	1.9468	.4584	1.9447	.4669	1.9427	.4754	1.9406	.4838	2
3	2.9201	.6876	2.9171	.7003	2.9140	.7131	2.9109	.7258	3
4	3.8935	.9168	3.8895	.9338	3.8854	.9507	3.8812	.9677	4
5	4.8669	1.1460	4.8618	1.1672	4.8567	1.1884	4.8515	1.2096	5
6	5.8403	1.3752	5.8342	1.4007	5.8281	1.4261	5.8218	1.4515	6
7	6.8137	1.6044	6.8066	1.6341	6.7994	1.6638	6.7921	1.6935	7
8	7.7870	1.8336	7.7790	1.8676	7.7707	1.9015	7.7624	1.9354	8
9	8.7604	2.0628	8.7513	2.1010	8.7421	2.1392	8.7327	2.1773	9
10	9.7338	2.2920	9.7237	2.3345	9.7134	2.3769	9.7030	2.4192	10
	76¾ Deg.		76½ Deg.		76¼ Deg.		76 Deg.		

D.	14¼ Deg.		14½ Deg.		14¾ Deg.		15 Deg.		D.
1	.9692	.2462	.9681	.2504	.9670	.2546	.9659	.2588	1
2	1.9385	.4923	1.9363	.5008	1.9341	.5092	1.9319	.5176	2
3	2.9077	.7385	2.9044	.7511	2.9011	.7638	2.8978	.7765	3
4	3.8769	.9846	3.8726	1.0015	3.8682	1.0184	3.8637	1.0353	4
5	4.8462	1.2308	4.8407	1.2519	4.8352	1.2730	4.8296	1.2941	5
6	5.8154	1.4769	5.8089	1.5023	5.8023	1.5276	5.7956	1.5529	6
7	6.7846	1.7231	6.7770	1.7527	6.7693	1.7822	6.7615	1.8117	7
8	7.7538	1.9692	7.7452	2.0030	7.7364	2.0368	7.7274	2.0706	8
9	8.7231	2.2154	8.7133	2.2534	8.7034	2.2914	8.6933	2.3294	9
10	9.6923	2.4615	9.6815	2.5038	9.6705	2.5460	9.6593	2.5882	10
	75¾ Deg.		75½ Deg.		75¼ Deg.		75 Deg.		

D.	15¼ Deg.		15½ Deg.		15¾ Deg.		16 Deg.		D.
1	.9648	.2630	.9636	.2672	.9625	.2714	.9613	.2756	1
2	1.9296	.5261	1.9273	.5345	1.9249	.5429	1.9225	.5513	2
3	2.8944	.7891	2.8909	.8017	2.8874	.8143	2.8838	.8269	3
4	3.8591	1.0521	3.8545	1.0690	3.8498	1.0858	3.8450	1.1025	4
5	4.8239	1.3152	4.8182	1.3362	4.8123	1.3572	4.8063	1.3782	5
6	5.7887	1.5782	5.7818	1.6034	5.7747	1.6286	5.7676	1.6538	6
7	6.7535	1.8412	6.7454	1.8707	6.7372	1.9001	6.7288	1.9295	7
8	7.7183	2.1042	7.7090	2.1379	7.6996	2.1715	7.6901	2.2051	8
9	8.6831	2.3673	8.6727	2.4051	8.6621	2.4430	8.6514	2.4807	9
10	9.6479	2.6303	9.6363	2.6724	9.6246	2.7144	9.6126	2.7564	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	74¾ Deg.		74½ Deg.		74¼ Deg.		74 Deg.		

LATITUDES AND DEPARTURES.

D.	16¼ Deg.		16½ Deg.		16¾ Deg.		17 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.9600	.2798	.9588	.2840	.9576	.2882	.9563	.2924	1
2	1.9201	.5597	1.9176	.5680	1.9151	.5764	1.9126	.5847	2
3	2.8801	.8395	2.8765	.8520	2.8727	.8646	2.8689	.8771	3
4	3.8402	1.1193	3.8353	1.1361	3.8303	1.1528	3.8252	1.1695	4
5	4.8002	1.3991	4.7941	1.4201	4.7879	1.4410	4.7815	1.4619	5
6	5.7603	1.6790	5.7529	1.7041	5.7454	1.7292	5.7378	1.7542	6
7	6.7203	1.9588	6.7117	1.9881	6.7030	2.0174	6.6941	2.0466	7
8	7.6804	2.2386	7.6706	2.2721	7.6606	2.3056	7.6504	2.3390	8
9	8.6404	2.5185	8.6294	2.5561	8.6181	2.5938	8.6067	2.6313	9
10	9.6005	2.7983	9.5882	2.8402	9.5757	2.8820	9.5630	2.9237	10
	73¾ Deg.		73½ Deg.		73¼ Deg.		73 Deg.		

D.	17¼ Deg.		17½ Deg.		17¾ Deg.		18 Deg.		D.
1	.9550	.2965	.9537	.3007	.9524	.3049	.9511	.3090	1
2	1.9100	.5931	1.9074	.6014	1.9048	.6097	1.9021	.6180	2
3	2.8651	.8896	2.8612	.9021	2.8572	.9146	2.8532	.9271	3
4	3.8201	1.1862	3.8149	1.2028	3.8096	1.2195	3.8042	1.2361	4
5	4.7751	1.4827	4.7686	1.5035	4.7620	1.5243	4.7553	1.5451	5
6	5.7301	1.7792	5.7223	1.8042	5.7144	1.8292	5.7063	1.8541	6
7	6.6851	2.0758	6.6760	2.1049	6.6668	2.1341	6.6574	2.1631	7
8	7.6402	2.3723	7.6297	2.4056	7.6192	2.4389	7.6085	2.4721	8
9	8.5952	2.6689	8.5835	2.7064	8.5716	2.7438	8.5595	2.7812	9
10	9.5502	2.9654	9.5372	3.0071	9.5240	3.0486	9.5106	3.0902	10
	72¾ Deg.		72½ Deg.		72¼ Deg.		72 Deg.		

D.	18¼ Deg.		18½ Deg.		18¾ Deg.		19 Deg.		D.
1	.9497	.3132	.9483	.3173	.9469	.3214	.9455	.3256	1
2	1.8994	.6263	1.8966	.6346	1.8939	.6429	1.8910	.6511	2
3	2.8491	.9395	2.8450	.9519	2.8408	.9643	2.8366	.9767	3
4	3.7988	1.2527	3.7933	1.2692	3.7877	1.2858	3.7821	1.3023	4
5	4.7485	1.5658	4.7416	1.5865	4.7347	1.6072	4.7276	1.6278	5
6	5.6982	1.8790	5.6899	1.9038	5.6816	1.9286	5.6731	1.9534	6
7	6.6479	2.1921	6.6383	2.2211	6.6285	2.2501	6.6186	2.2790	7
8	7.5976	2.5053	7.5866	2.5384	7.5754	2.5715	7.5641	2.6045	8
9	8.5473	2.8185	8.5349	2.8557	8.5224	2.8930	8.5097	2.9301	9
10	9.4970	3.1316	9.4832	3.1730	9.4693	3.2144	9.4552	3.2557	10
	71¾ Deg.		71½ Deg.		71¼ Deg.		71 Deg.		

D.	19¼ Deg.		19½ Deg.		19¾ Deg.		20 Deg.		D.
1	.9441	.3297	.9426	.3338	.9412	.3379	.9397	.3420	1
2	1.8882	.6594	1.8853	.6676	1.8824	.6758	1.8794	.6840	2
3	2.8323	.9891	2.8279	1.0014	2.8235	1.0138	2.8191	1.0261	3
4	3.7764	1.3188	3.7706	1.3352	3.7647	1.3517	3.7588	1.3681	4
5	4.7204	1.6485	4.7132	1.6690	4.7059	1.6896	4.6985	1.7101	5
6	5.6645	1.9781	5.6558	2.0028	5.6471	2.0275	5.6382	2.0521	6
7	6.6086	2.3078	6.5985	2.3366	6.5882	2.3654	6.5778	2.3941	7
8	7.5527	2.6375	7.5411	2.6705	7.5294	2.7033	7.5175	2.7362	8
9	8.4968	2.9672	8.4838	3.0043	8.4706	3.0413	8.4572	3.0782	9
10	9.4409	3.2969	9.4264	3.3381	9.4118	3.3792	9.3969	3.4202	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	70¾ Deg.		70½ Deg.		70¼ Deg.		70 Deg.		

D.	20¼ Deg.		20½ Deg.		20¾ Deg.		21 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.9382	.3461	.9367	.3502	.9351	.3543	.9336	.3584	1
2	1.8764	.6922	1.8733	.7004	1.8703	.7086	1.8672	.7167	2
3	2.8146	1.0384	2.8100	1.0506	2.8054	1.0629	2.8007	1.0751	3
4	3.7528	1.3845	3.7467	1.4008	3.7405	1.4172	3.7343	1.4335	4
5	4.6910	1.7306	4.6834	1.7510	4.6757	1.7715	4.6679	1.7918	5
6	5.6291	2.0767	5.6200	2.1012	5.6108	2.1257	5.6015	2.1502	6
7	6.5673	2.4228	6.5567	2.4515	6.5459	2.4800	6.5351	2.5086	7
8	7.5055	2.7689	7.4934	2.8017	7.4811	2.8343	7.4686	2.8669	8
9	8.4437	3.1151	8.4300	3.1519	8.4162	3.1886	8.4022	3.2253	9
10	9.3819	3.4612	9.3667	3.5021	9.3514	3.5429	9.3358	3.5837	10
	69¾ Deg.		69½ Deg.		69¼ Deg.		69 Deg.		
	21¼ Deg.		21½ Deg.		21¾ Deg.		22 Deg.		
1	.9320	.3624	.9304	.3665	.9288	.3706	.9272	.3746	1
2	1.8640	.7249	1.8608	.7330	1.8576	.7411	1.8544	.7492	2
3	2.7960	1.0873	2.7913	1.0995	2.7864	1.1117	2.7816	1.1238	3
4	3.7280	1.4498	3.7217	1.4660	3.7152	1.4822	3.7087	1.4984	4
5	4.6600	1.8122	4.6521	1.8325	4.6440	1.8528	4.6359	1.8730	5
6	5.5920	2.1746	5.5825	2.1990	5.5729	2.2233	5.5631	2.2476	6
7	6.5241	2.5371	6.5129	2.5655	6.5017	2.5939	6.4903	2.6222	7
8	7.4561	2.8995	7.4433	2.9320	7.4305	2.9645	7.4175	2.9969	8
9	8.3881	3.2619	8.3738	3.2985	8.3593	3.3350	8.3447	3.3715	9
10	9.3201	3.6244	9.3042	3.6650	9.2881	3.7056	9.2718	3.7461	10
	68¾ Deg.		68½ Deg.		68¼ Deg.		68 Deg.		
	22¼ Deg.		22½ Deg.		22¾ Deg.		23 Deg.		
1	.9255	.3786	.9239	.3827	.9222	.3867	.9205	.3907	1
2	1.8511	.7573	1.8478	.7654	1.8444	.7734	1.8410	.7815	2
3	2.7766	1.1359	2.7716	1.1481	2.7666	1.1601	2.7615	1.1722	3
4	3.7022	1.5146	3.6955	1.5307	3.6888	1.5468	3.6820	1.5629	4
5	4.6277	1.8932	4.6194	1.9134	4.6110	1.9336	4.6025	1.9537	5
6	5.5532	2.2719	5.5433	2.2961	5.5332	2.3203	5.5230	2.3444	6
7	6.4788	2.6505	6.4672	2.6788	6.4554	2.7070	6.4435	2.7351	7
8	7.4043	3.0292	7.3910	3.0615	7.3776	3.0937	7.3640	3.1258	8
9	8.3299	3.4078	8.3149	3.4442	8.2998	3.4804	8.2845	3.5166	9
10	9.2554	3.7865	9.2388	3.8268	9.2220	3.8671	9.2050	3.9073	10
	67¾ Deg.		67½ Deg.		67¼ Deg.		67 Deg.		
	23¼ Deg.		23½ Deg.		23¾ Deg.		24 Deg.		
1	.9188	.3947	.9171	.3987	.9153	.4027	.9135	.4067	1
2	1.8376	.7895	1.8341	.7975	1.8306	.8055	1.8271	.8135	2
3	2.7564	1.1842	2.7512	1.1962	2.7459	1.2082	2.7406	1.2202	3
4	3.6752	1.5790	3.6682	1.5950	3.6612	1.6110	3.6542	1.6269	4
5	4.5940	1.9737	4.5853	1.9937	4.5766	2.0137	4.5677	2.0337	5
6	5.5127	2.3685	5.5024	2.3925	5.4919	2.4165	5.4813	2.4404	6
7	6.4315	2.7632	6.4194	2.7912	6.4072	2.8192	6.3948	2.8472	7
8	7.3503	3.1580	7.3365	3.1900	7.3225	3.2220	7.3084	3.2539	8
9	8.2691	3.5527	8.2535	3.5887	8.2378	3.6247	8.2219	3.6606	9
10	9.1879	3.9474	9.1706	3.9875	9.1531	4.0275	9.1355	4.0674	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	66¾ Deg.		66½ Deg.		66¼ Deg.		66 Deg.		

LATITUDES AND DEPARTURES.

D.	24¼ Deg.		24½ Deg.		24¾ Deg.		25 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.9118	.4107	.9100	.4147	.9081	.4187	.9063	.4226	1
2	1.8235	.8214	1.8199	.8294	1.8163	.8373	1.8126	.8452	2
3	2.7353	1.2322	2.7299	1.2441	2.7244	1.2560	2.7189	1.2679	3
4	3.6470	1.6429	3.6398	1.6588	3.6326	1.6746	3.6252	1.6905	4
5	4.5588	2.0536	4.5498	2.0735	4.5407	2.0933	4.5315	2.1131	5
6	5.4706	2.4643	5.4598	2.4882	5.4489	2.5120	5.4378	2.5357	6
7	6.3823	2.8750	6.3697	2.9029	6.3570	2.9306	6.3442	2.9583	7
8	7.2941	3.2858	7.2797	3.3175	7.2651	3.3493	7.2505	3.3809	8
9	8.2059	3.6965	8.1897	3.7322	8.1733	3.7679	8.1568	3.8036	9
10	9.1176	4.1072	9.0996	4.1469	9.0814	4.1866	9.0631	4.2262	10
	65¾ Deg.		65½ Deg.		65¼ Deg.		65 Deg.		
	25¼ Deg.		25½ Deg.		25¾ Deg.		26 Deg.		
1	.9045	.4266	.9026	.4305	.9007	.4344	.8988	.4384	1
2	1.8089	.8531	1.8052	.8610	1.8014	.8689	1.7976	.8767	2
3	2.7134	1.2797	2.7078	1.2915	2.7021	1.3033	2.6964	1.3151	3
4	3.6178	1.7063	3.6103	1.7220	3.6028	1.7378	3.5952	1.7535	4
5	4.5223	2.1328	4.5129	2.1526	4.5035	2.1722	4.4940	2.1919	5
6	5.4267	2.5594	5.4155	2.5831	5.4042	2.6067	5.3928	2.6302	6
7	6.3312	2.9860	6.3181	3.0136	6.3049	3.0411	6.2916	3.0686	7
8	7.2356	3.4125	7.2207	3.4441	7.2056	3.4756	7.1904	3.5070	8
9	8.1401	3.8391	8.1233	3.8746	8.1063	3.9100	8.0891	3.9453	9
10	9.0446	4.2657	9.0259	4.3051	9.0070	4.3445	8.9879	4.3837	10
	64¾ Deg.		64½ Deg.		64¼ Deg.		64 Deg.		
	26¼ Deg.		26½ Deg.		26¾ Deg.		27 Deg.		
1	.8969	.4423	.8949	.4462	.8930	.4501	.8910	.4540	1
2	1.7937	.8846	1.7899	.8924	1.7860	.9002	1.7820	.9080	2
3	2.6906	1.3269	2.6848	1.3386	2.6789	1.3503	2.6730	1.3620	3
4	3.5875	1.7692	3.5797	1.7848	3.5719	1.8004	3.5640	1.8160	4
5	4.4844	2.2114	4.4747	2.2310	4.4649	2.2505	4.4550	2.2700	5
6	5.3812	2.6537	5.3696	2.6772	5.3579	2.7006	5.3460	2.7239	6
7	6.2781	3.0960	6.2645	3.1234	6.2509	3.1507	6.2370	3.1779	7
8	7.1750	3.5383	7.1595	3.5696	7.1438	3.6008	7.1281	3.6319	8
9	8.0719	3.9806	8.0544	4.0158	8.0368	4.0509	8.0191	4.0859	9
10	8.9687	4.4229	8.9493	4.4620	8.9298	4.5010	8.9101	4.5399	10
	63¾ Deg.		63½ Deg.		63¼ Deg.		63 Deg.		
	27¼ Deg.		27½ Deg.		27¾ Deg.		28 Deg.		
1	.8890	.4579	.8870	.4617	.8850	.4656	.8829	.4695	1
2	1.7780	.9157	1.7740	.9235	1.7700	.9312	1.7659	.9389	2
3	2.6671	1.3736	2.6610	1.3852	2.6550	1.3968	2.6488	1.4084	3
4	3.5561	1.8315	3.5480	1.8470	3.5400	1.8625	3.5318	1.8779	4
5	4.4451	2.2894	4.4351	2.3087	4.4249	2.3281	4.4147	2.3474	5
6	5.3341	2.7472	5.3221	2.7705	5.3099	2.7937	5.2977	2.8168	6
7	6.2231	3.2051	6.2091	3.2322	6.1949	3.2593	6.1806	3.2863	7
8	7.1121	3.6630	7.0961	3.6940	7.0799	3.7249	7.0636	3.7558	8
9	8.0012	4.1209	7.9831	4.1557	7.9649	4.1905	7.9465	4.2252	9
10	8.8902	4.5787	8.8701	4.6175	8.8499	4.6561	8.8295	4.6947	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	62¾ Deg.		62½ Deg.		62¼ Deg.		62 Deg.		

D.	28¼ Deg.		28½ Deg.		28¾ Deg.		29 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.8809	.4733	.8788	.4772	.8767	.4810	.8746	.4848	1
2	1.7618	.9466	1.7576	.9543	1.7535	.9620	1.7492	.9696	2
3	2.6427	1.4200	2.6365	1.4315	2.6302	1.4430	2.6239	1.4544	3
4	3.5236	1.8933	3.5153	1.9086	3.5069	1.9240	3.4985	1.9392	4
5	4.4045	2.3666	4.3941	2.3858	4.3836	2.4049	4.3731	2.4240	5
6	5.2853	2.8399	5.2729	2.8630	5.2604	2.8859	5.2477	2.9089	6
7	6.1662	3.3132	6.1517	3.3401	6.1371	3.3669	6.1223	3.3937	7
8	7.0471	3.7866	7.0305	3.8173	7.0138	3.8479	6.9970	3.8785	8
9	7.9280	4.2599	7.9094	4.2944	7.8905	4.3289	7.8716	4.3633	9
10	8.8089	4.7332	8.7882	4.7716	8.7673	4.8099	8.7462	4.8481	10
	61¾ Deg.		61½ Deg.		61¼ Deg.		61 Deg.		
	29¼ Deg.		29½ Deg.		29¾ Deg.		30 Deg.		
1	.8725	.4886	.8704	.4924	.8682	.4962	.8660	.5000	1
2	1.7450	.9772	1.7407	.9848	1.7364	.9924	1.7321	1.0000	2
3	2.6175	1.4659	2.6111	1.4773	2.6046	1.4886	2.5981	1.5000	3
4	3.4900	1.9545	3.4814	1.9697	3.4728	1.9849	3.4641	2.0000	4
5	4.3625	2.4431	4.3518	2.4621	4.3410	2.4811	4.3301	2.5000	5
6	5.2350	2.9317	5.2221	2.9545	5.2092	2.9773	5.1962	3.0000	6
7	6.1075	3.4203	6.0925	3.4470	6.0774	3.4735	6.0622	3.5000	7
8	6.9800	3.9090	6.9628	3.9394	6.9456	3.9697	6.9282	4.0000	8
9	7.8525	4.3976	7.8332	4.4318	7.8138	4.4659	7.7942	4.5000	9
10	8.7250	4.8862	8.7036	4.9242	8.6820	4.9622	8.6603	5.0000	10
	60¾ Deg.		60½ Deg.		60¼ Deg.		60 Deg.		
	30¼ Deg.		30½ Deg.		30¾ Deg.		31 Deg.		
1	.8638	.5038	.8616	.5075	.8594	.5113	.8572	.5150	1
2	1.7277	1.0075	1.7233	1.0151	1.7188	1.0226	1.7143	1.0301	2
3	2.5915	1.5113	2.5849	1.5226	2.5782	1.5339	2.5715	1.5451	3
4	3.4553	2.0151	3.4465	2.0302	3.4376	2.0452	3.4287	2.0602	4
5	4.3192	2.5189	4.3081	2.5377	4.2970	2.5565	4.2858	2.5752	5
6	5.1830	3.0226	5.1698	3.0452	5.1564	3.0678	5.1430	3.0902	6
7	6.0468	3.5264	6.0314	3.5528	6.0158	3.5791	6.0002	3.6053	7
8	6.9107	4.0302	6.8930	4.0603	6.8753	4.0903	6.8573	4.1203	8
9	7.7745	4.5340	7.7547	4.5678	7.7347	4.6016	7.7145	4.6353	9
10	8.6384	5.0377	8.6163	5.0754	8.5941	5.1129	8.5717	5.1504	10
	59¾ Deg.		59½ Deg.		59¼ Deg.		59 Deg.		
	31¼ Deg.		31½ Deg.		31¾ Deg.		32 Deg.		
1	.8549	.5188	.8526	.5225	.8504	.5262	.8480	.5299	1
2	1.7098	1.0375	1.7053	1.0450	1.7007	1.0524	1.6961	1.0598	2
3	2.5647	1.5563	2.5579	1.5675	2.5511	1.5786	2.5441	1.5898	3
4	3.4196	2.0751	3.4106	2.0900	3.4014	2.1049	3.3922	2.1197	4
5	4.2746	2.5939	4.2632	2.6125	4.2518	2.6311	4.2402	2.6496	5
6	5.1295	3.1126	5.1158	3.1350	5.1021	3.1573	5.0883	3.1795	6
7	5.9844	3.6314	5.9685	3.6575	5.9525	3.6835	5.9363	3.7094	7
8	6.8393	4.1502	6.8211	4.1800	6.8028	4.2097	6.7844	4.2394	8
9	7.6942	4.6690	7.6738	4.7025	7.6532	4.7359	7.6324	4.7693	9
10	8.5491	5.1877	8.5264	5.2250	8.5035	5.2621	8.4805	5.2992	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	58¾ Deg.		58½ Deg.		58¼ Deg.		58 Deg.		

LATITUDES AND DEPARTURES.

D.	32¼ Deg.		32½ Deg.		32¾ Deg.		33 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.8457	.5336	.8434	.5373	.8410	.5410	.8387	.5446	1
2	1.6915	1.0672	1.6868	1.0746	1.6821	1.0819	1.6773	1.0893	2
3	2.5372	1.6008	2.5302	1.6119	2.5231	1.6229	2.5160	1.6339	3
4	3.3829	2.1345	3.3736	2.1492	3.3642	2.1639	3.3547	2.1786	4
5	4.2286	2.6681	4.2170	2.6865	4.2052	2.7049	4.1934	2.7232	5
6	5.0744	3.2017	5.0603	3.2238	5.0462	3.2458	5.0320	3.2678	6
7	5.9201	3.7353	5.9037	3.7611	5.8873	3.7868	5.8707	3.8125	7
8	6.7658	4.2689	6.7471	4.2984	6.7283	4.3278	6.7094	4.3571	8
9	7.6116	4.8025	7.5905	4.8357	7.5694	4.8688	7.5480	4.9018	9
10	8.4573	5.3361	8.4339	5.3730	8.4104	5.4097	8.3867	5.4464	10
	57¾ Deg.		57½ Deg.		57¼ Deg.		57 Deg.		
	33¼ Deg.		33½ Deg.		33¾ Deg.		34 Deg.		
1	.8363	.5483	.8339	.5519	.8315	.5556	.8290	.5592	1
2	1.6726	1.0966	1.6678	1.1039	1.6629	1.1111	1.6581	1.1184	2
3	2.5089	1.6449	2.5017	1.6558	2.4944	1.6667	2.4871	1.6776	3
4	3.3451	2.1932	3.3355	2.2077	3.3259	2.2223	3.3162	2.2368	4
5	4.1814	2.7415	4.1694	2.7597	4.1573	2.7779	4.1452	2.7960	5
6	5.0177	3.2898	5.0033	3.3116	4.9888	3.3334	4.9742	3.3552	6
7	5.8540	3.8381	5.8372	3.8636	5.8203	3.8890	5.8033	3.9144	7
8	6.6903	4.3863	6.6711	4.4155	6.6518	4.4446	6.6323	4.4735	8
9	7.5266	4.9346	7.5050	4.9674	7.4832	5.0001	7.4613	5.0327	9
10	8.3629	5.4829	8.3389	5.5194	8.3147	5.5557	8.2904	5.5919	10
	56¾ Deg.		56½ Deg.		56¼ Deg.		56 Deg.		
	34¼ Deg.		34½ Deg.		34¾ Deg.		35 Deg.		
1	.8266	.5628	.8241	.5664	.8216	.5700	.8192	.5736	1
2	1.6532	1.1256	1.6483	1.1328	1.6433	1.1400	1.6383	1.1472	2
3	2.4798	1.6884	2.4724	1.6992	2.4649	1.7100	2.4575	1.7207	3
4	3.3064	2.2512	3.2965	2.2656	3.2866	2.2800	3.2766	2.2943	4
5	4.1329	2.8140	4.1206	2.8320	4.1082	2.8500	4.0958	2.8679	5
6	4.9595	3.3768	4.9448	3.3984	4.9299	3.4200	4.9149	3.4415	6
7	5.7861	3.9396	5.7689	3.9648	5.7515	3.9900	5.7341	4.0150	7
8	6.6127	4.5024	6.5930	4.5312	6.5732	4.5600	6.5532	4.5886	8
9	7.4393	5.0652	7.4171	5.0977	7.3948	5.1300	7.3724	5.1622	9
10	8.2659	5.6280	8.2413	5.6641	8.2165	5.7000	8.1915	5.7358	10
	55¾ Deg.		55½ Deg.		55¼ Deg.		55 Deg.		
	35¼ Deg.		35½ Deg.		35¾ Deg.		36 Deg.		
1	.8166	.5771	.8141	.5807	.8116	.5842	.8090	.5878	1
2	1.6333	1.1543	1.6282	1.1614	1.6231	1.1685	1.6180	1.1756	2
3	2.4499	1.7314	2.4423	1.7421	2.4347	1.7527	2.4271	1.7634	3
4	3.2666	2.3086	3.2565	2.3228	3.2463	2.3370	3.2361	2.3511	4
5	4.0832	2.8857	4.0706	2.9035	4.0579	2.9212	4.0451	2.9389	5
6	4.8998	3.4629	4.8847	3.4842	4.8694	3.5055	4.8541	3.5267	6
7	5.7165	4.0400	5.6988	4.0649	5.6810	4.0897	5.6631	4.1145	7
8	6.5331	4.6172	6.5129	4.6456	6.4926	4.6740	6.4721	4.7023	8
9	7.3498	5.1943	7.3270	5.2263	7.3042	5.2582	7.2812	5.2901	9
10	8.1664	5.7715	8.1412	5.8070	8.1157	5.8425	8.0902	5.8779	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	54¾ Deg.		54½ Deg.		54¼ Deg.		54 Deg.		

LATITUDES AND DEPARTURES.

D.	36¼ Deg.		36½ Deg.		36¾ Deg.		37 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.8064	.5913	.8039	.5948	.8013	.5983	.7986	.6018	1
2	1.6129	1.1826	1.6077	1.1896	1.6025	1.1966	1.5973	1.2036	2
3	2.4193	1.7739	2.4116	1.7845	2.4038	1.7950	2.3959	1.8054	3
4	3.2258	2.3652	3.2154	2.3793	3.2050	2.3933	3.1945	2.4073	4
5	4.0322	2.9565	4.0193	2.9741	4.0063	2.9916	3.9932	3.0091	5
6	4.8387	3.5479	4.8231	3.5689	4.8075	3.5899	4.7918	3.6109	6
7	5.6451	4.1392	5.6270	4.1638	5.6088	4.1883	5.5904	4.2127	7
8	6.4516	4.7305	6.4309	4.7586	6.4100	4.7866	6.3891	4.8145	8
9	7.2580	5.3218	7.2347	5.3534	7.2113	5.3849	7.1877	5.4163	9
10	8.0644	5.9131	8.0386	5.9482	8.0125	5.9832	7.9864	6.0181	10
	53¾ Deg.		53½ Deg.		53¼ Deg.		53 Deg.		
	37¼ Deg.		37½ Deg.		37¾ Deg.		38 Deg.		
1	.7960	.6053	.7934	.6088	.7907	.6122	.7880	.6157	1
2	1.5920	1.2106	1.5867	1.2175	1.5814	1.2244	1.5760	1.2313	2
3	2.3880	1.8159	2.3801	1.8263	2.3721	1.8367	2.3640	1.8470	3
4	3.1840	2.4212	3.1734	2.4350	3.1628	2.4489	3.1520	2.4626	4
5	3.9800	3.0265	3.9668	3.0438	3.9534	3.0611	3.9401	3.0783	5
6	4.7760	3.6318	4.7601	3.6526	4.7441	3.6733	4.7281	3.6940	6
7	5.5720	4.2371	5.5535	4.2613	5.5348	4.2855	5.5161	4.3096	7
8	6.3680	4.8424	6.3468	4.8701	6.3255	4.8977	6.3041	4.9253	8
9	7.1640	5.4476	7.1402	5.4789	7.1162	5.5100	7.0921	5.5410	9
10	7.9600	6.0529	7.9335	6.0876	7.9069	6.1222	7.8801	6.1566	10
	52¾ Deg.		52½ Deg.		52¼ Deg.		52 Deg.		
	38¼ Deg.		38½ Deg.		38¾ Deg.		39 Deg.		
1	.7853	.6191	.7826	.6225	.7799	.6259	.7771	.6293	1
2	1.5706	1.2382	1.5652	1.2450	1.5598	1.2518	1.5543	1.2586	2
3	2.3560	1.8573	2.3478	1.8675	2.3397	1.8778	2.3314	1.8880	3
4	3.1413	2.4764	3.1304	2.4901	3.1195	2.5037	3.1086	2.5173	4
5	3.9266	3.0955	3.9130	3.1126	3.8994	3.1296	3.8857	3.1466	5
6	4.7119	3.7146	4.6956	3.7351	4.6793	3.7555	4.6629	3.7759	6
7	5.4972	4.3337	5.4783	4.3576	5.4592	4.3815	5.4400	4.4052	7
8	6.2825	4.9528	6.2609	4.9801	6.2391	5.0074	6.2172	5.0346	8
9	7.0679	5.5718	7.0435	5.6026	7.0190	5.6333	6.9943	5.6639	9
10	7.8532	6.1909	7.8261	6.2251	7.7988	6.2592	7.7715	6.2932	10
	51¾ Deg.		51½ Deg.		51¼ Deg.		51 Deg.		
	39¼ Deg.		39½ Deg.		39¾ Deg.		40 Deg.		
1	.7744	.6327	.7716	.6361	.7688	.6394	.7660	.6428	1
2	1.5488	1.2654	1.5432	1.2722	1.5377	1.2789	1.5321	1.2856	2
3	2.3232	1.8981	2.3149	1.9082	2.3065	1.9183	2.2981	1.9284	3
4	3.0976	2.5308	3.0865	2.5443	3.0754	2.5578	3.0642	2.5712	4
5	3.8720	3.1635	3.8581	3.1804	3.8442	3.1972	3.8302	3.2139	5
6	4.6464	3.7962	4.6297	3.8165	4.6131	3.8366	4.5963	3.8567	6
7	5.4207	4.4289	5.4014	4.4525	5.3819	4.4761	5.3623	4.4995	7
8	6.1951	5.0616	6.1730	5.0886	6.1507	5.1155	6.1284	5.1423	8
9	6.9695	5.6943	6.9446	5.7247	6.9196	5.7550	6.8944	5.7851	9
10	7.7439	6.3271	7.7162	6.3608	7.6884	6.3944	7.6604	6.4279	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	50¾ Deg.		50½ Deg.		50¼ Deg.		50 Deg.		

LATITUDES AND DEPARTURES.

D.	40¼ Deg.		40½ Deg.		40¾ Deg.		41 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.7632	.6461	.7604	.6494	.7576	.6528	.7547	.6561	1
2	1.5265	1.2922	1.5208	1.2989	1.5151	1.3055	1.5094	1.3121	2
3	2.2897	1.9384	2.2812	1.9483	2.2727	1.9583	2.2641	1.9682	3
4	3.0529	2.5845	3.0416	2.5978	3.0303	2.6110	3.0188	2.6242	4
5	3.8162	3.2306	3.8020	3.2472	3.7878	3.2638	3.7735	3.2803	5
6	4.5794	3.8767	4.5624	3.8967	4.5454	3.9166	4.5283	3.9364	6
7	5.3426	4.5229	5.3228	4.5461	5.3030	4.5693	5.2830	4.5924	7
8	6.1059	5.1690	6.0832	5.1956	6.0605	5.2221	6.0377	5.2485	8
9	6.8691	5.8151	6.8437	5.8450	6.8181	5.8748	6.7924	5.9045	9
10	7.6323	6.4612	7.6041	6.4945	7.5756	6.5276	7.5471	6.5606	10
	49¾ Deg.		49½ Deg.		49¼ Deg.		49 Deg.		

D.	41¼ Deg.		41½ Deg.		41¾ Deg.		42 Deg.		D.
1	.7518	.6593	.7490	.6626	.7461	.6659	.7431	.6691	1
2	1.5037	1.3187	1.4979	1.3252	1.4921	1.3318	1.4863	1.3383	2
3	2.2555	1.9780	2.2469	1.9879	2.2382	1.9976	2.2294	2.0074	3
4	3.0074	2.6374	2.9958	2.6505	2.9842	2.6635	2.9726	2.6765	4
5	3.7592	3.2967	3.7448	3.3131	3.7303	3.3294	3.7157	3.3457	5
6	4.5110	3.9561	4.4937	3.9757	4.4763	3.9953	4.4589	4.0148	6
7	5.2629	4.6154	5.2427	4.6383	5.2224	4.6612	5.2020	4.6839	7
8	6.0147	5.2748	5.9916	5.3010	5.9685	5.3271	5.9452	5.3530	8
9	6.7666	5.9341	6.7406	5.9636	6.7145	5.9929	6.6883	6.0222	9
10	7.5184	6.5935	7.4896	6.6262	7.4606	6.6588	7.4314	6.6913	10
	48¾ Deg.		48½ Deg.		48¼ Deg.		48 Deg.		

D.	42¼ Deg.		42½ Deg.		42¾ Deg.		43 Deg.		D.
1	.7402	.6724	.7373	.6756	.7343	.6788	.7314	.6820	1
2	1.4804	1.3447	1.4746	1.3512	1.4686	1.3576	1.4627	1.3640	2
3	2.2207	2.0171	2.2118	2.0268	2.2030	2.0364	2.1941	2.0460	3
4	2.9609	2.6895	2.9491	2.7024	2.9373	2.7152	2.9254	2.7280	4
5	3.7011	3.3618	3.6864	3.3780	3.6716	3.3940	3.6568	3.4100	5
6	4.4413	4.0342	4.4237	4.0535	4.4059	4.0728	4.3881	4.0920	6
7	5.1815	4.7066	5.1609	4.7291	5.1403	4.7516	5.1195	4.7740	7
8	5.9217	5.3789	5.8982	5.4047	5.8746	5.4304	5.8508	5.4560	8
9	6.6620	6.0513	6.6355	6.0803	6.6089	6.1092	6.5822	6.1380	9
10	7.4022	6.7237	7.3728	6.7559	7.3432	6.7880	7.3135	6.8200	10
	47¾ Deg.		47½ Deg.		47¼ Deg.		47 Deg.		

D.	43¼ Deg.		43½ Deg.		43¾ Deg.		44 Deg.		D.
1	.7284	.6852	.7254	.6884	.7224	.6915	.7193	.6947	1
2	1.4567	1.3704	1.4507	1.3767	1.4447	1.3830	1.4387	1.3893	2
3	2.1851	2.0555	2.1761	2.0651	2.1671	2.0745	2.1580	2.0840	3
4	2.9135	2.7407	2.9015	2.7534	2.8895	2.7661	2.8774	2.7786	4
5	3.6419	3.4259	3.6269	3.4418	3.6118	3.4576	3.5967	3.4733	5
6	4.3702	4.1111	4.3522	4.1301	4.3342	4.1491	4.3160	4.1680	6
7	5.0986	4.7963	5.0776	4.8185	5.0565	4.8406	5.0354	4.8626	7
8	5.8270	5.4815	5.8030	5.5068	5.7789	5.5321	5.7547	5.5573	8
9	6.5553	6.1666	6.5284	6.1952	6.5013	6.2236	6.4741	6.2519	9
10	7.2837	6.8518	7.2537	6.8835	7.2236	6.9151	7.1934	6.9466	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	46¾ Deg.		46½ Deg.		46¼ Deg.		46 Deg.		

LATITUDES AND DEPARTURES.

D.	44¼ Deg.		44½ Deg.		44¾ Deg.		45 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.7163	.6978	.7133	.7009	.7102	.7040	.7071	.7071	1
2	1.4326	1.3956	1.4265	1.4018	1.4204	1.4080	1.4142	1.4142	2
3	2.1489	2.0934	2.1398	2.1027	2.1306	2.1120	2.1213	2.1213	3
4	2.8652	2.7912	2.8530	2.8036	2.8407	2.8161	2.8284	2.8284	4
5	3.5815	3.4890	3.5663	3.5045	3.5509	3.5201	3.5355	3.5355	5
6	4.2978	4.1867	4.2795	4.2055	4.2611	4.2241	4.2426	4.2426	6
7	5.0141	4.8845	4.9928	4.9064	4.9713	4.9281	4.9497	4.9497	7
8	5.7304	5.5823	5.7060	5.6073	5.6815	5.6321	5.6569	5.6569	8
9	6.4467	6.2801	6.4193	6.3082	6.3917	6.3361	6.3640	6.3640	9
10	7.1630	6.9779	7.1325	7.0091	7.1019	7.0401	7.0711	7.0711	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	45¾ Deg.		45½ Deg.		45¼ Deg.		45 Deg.		

TABLE OF USEFUL NUMBERS.

		Logarithms.
Ratio of circumference to diameter π =	3.1415926536	0.4971499
Area of circle to radius 1 =	"	"
Surface of sphere to diameter 1 =	"	"
Area of circle to diameter 1 =	.7853981634	—1.8950899
Base of Napierian Logarithms =	2.7182818285	.4342945
Modulus of common " =	.4342944819	—1.6377843
Equatorial radius of the earth, in feet =	20923599.98	7.3206364
Polar " " " =	20853657.16	7.3191823

Length of seconds pendulum, in London, in inches = 39.13929.
" " " Paris " = 39.1285.
" " " New York " = 39.1012.

U. S. standard gallon contains 231 c. in., or 58372.175 grains = 8.338882 lbs. avoirdupois of water at 39.8° Fahr.

U. S. standard bushel contains 2150.42 c. in., or 77.627413 lbs. av. of water at 39.8° Fahr.

British imperial gallon contains 277.274 c. in., = 1.2003 wine gallons of 231 c. in.

French metre = 39.37079 in. = 3.28089917 feet.
" toise = 6.39459252 feet.
" are = 100 sq. metres = 1076.4299 sq. ft.
" hectare = 100 ares = 2.471143 acres = 107642.9936 sq. ft.
" litre = 1 cubic decimeter = 61.02705 c. in. = .26418637 gallons of 231 c. in.
" hectolitre = 100 litres = 26.418637 gallons.

1 pound avoirdupois = 7000 grs. = 1.215277 pounds Troy.
1 " Troy = 5760 grs. = .822857 pounds avoir.
1 gramme = 15.442 grains.
1 kilogramme = 1000 grammes = 15442 grs. = 2.20607 lbs. avoir.
Tropical year = 365 d. 5 h. 45 m. 47.588 sec.

TABLE

OF THE

LOGARITHMS OF NUMBERS,

FROM

1 TO 10,000.

A TABLE

OF THE

LOGARITHMS OF NUMBERS

FROM 1 TO 10,000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	26	1.414973	51	1.707570	76	1.880814
2	0.301030	27	1.431364	52	1.716003	77	1.886491
3	0.477121	28	1.447158	53	1.724276	78	1.892095
4	0.602060	29	1.462398	54	1.732394	79	1.897627
5	0.698970	30	1.477121	55	1.740363	80	1.903090
6	0.778151	31	1.491362	56	1.748188	81	1.908485
7	0.845098	32	1.505150	57	1.755875	82	1.913814
8	0.903090	33	1.518514	58	1.763428	83	1.919078
9	0.954243	34	1.531479	59	1.770852	84	1.924279
10	1.000000	35	1.544068	60	1.778151	85	1.929419
11	1.041393	36	1.556303	61	1.785330	86	1.934498
12	1.079181	37	1.568202	62	1.792392	87	1.939519
13	1.113943	38	1.579784	63	1.799341	88	1.944483
14	1.146128	39	1.591065	64	1.806180	89	1.949390
15	1.176091	40	1.602060	65	1.812913	90	1.954243
16	1.204120	41	1.612784	66	1.819544	91	1.959041
17	1.230449	42	1.623249	67	1.826075	92	1.963788
18	1.255273	43	1.633468	68	1.832509	93	1.968483
19	1.278754	44	1.643453	69	1.838849	94	1.973128
20	1.301030	45	1.653213	70	1.845098	95	1.977724
21	1.322219	46	1.662758	71	1.851258	96	1.982271
22	1.342423	47	1.672098	72	1.857332	97	1.986772
23	1.361728	48	1.681241	73	1.863323	98	1.991226
24	1.380211	49	1.690196	74	1.869232	99	1.995635
25	1.397940	50	1.698970	75	1.875061	100	2.000000

N.	0	1	2	3	4	5	6	7	8	9
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174
102	8600	9026	9451	9876	^{0}300	^{0}724	^{1}147	^{1}570	^{1}993	^{2}415
103	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616
104	7033	7451	7868	8284	8700	9116	9532	9947	^{0}361	^{0}775
105	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978
107	9384	9789	^{0}195	^{0}600	^{1}004	^{1}408	^{1}812	^{2}216	^{2}619	^{3}021
108	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028
109	7426	7825	8223	8620	9017	9414	9811	^{0}207	^{0}602	^{0}998
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830
112	9218	9606	9993	^{0}380	^{0}766	^{1}153	^{1}538	^{1}924	^{2}309	^{2}694
113	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	^{0}320
115	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815
117	8186	8557	8928	9298	9668	^{0}038	^{0}407	^{0}776	^{1}145	^{1}514
118	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819
120	079181	9543	9904	^{0}266	^{0}626	^{0}987	^{1}347	^{1}707	^{2}067	^{2}426
121	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552
123	9905	^{0}258	^{0}611	^{0}963	^{1}315	^{1}667	^{2}018	^{2}370	^{2}721	^{3}071
124	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562
125	6910	7257	7604	7951	8298	8644	8990	9335	9681	^{0}026
126	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871
128	7210	7549	7888	8227	8565	8903	9241	9579	9916	^{0}253
129	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609
130	113943	4277	4611	4944	5278	5611	5943	6276	6608	6940
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	^{0}245
132	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781
134	7105	7429	7753	8076	8399	8722	9045	9368	9690	^{0}012
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564
138	9879	^{0}194	^{0}508	^{0}822	^{1}136	^{1}450	^{1}763	^{2}076	^{2}389	^{2}702
139	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818
140	146128	6438	6748	7058	7367	7676	7985	8294	8603	8911
141	9219	9527	9835	^{0}142	^{0}449	^{0}756	^{1}063	^{1}370	^{1}676	^{1}982
142	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061
144	8362	8664	8965	9266	9567	9868	^{0}168	^{0}469	^{0}769	^{1}068
145	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968
148	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802
150	176091	6381	6670	6959	7248	7536	7825	8113	8401	8689
151	8977	9264	9552	9839	^{0}126	^{0}413	^{0}699	^{0}986	^{1}272	558
152	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	^{0}051
155	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623
157	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382
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855	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448
860	934498	4549	4599	4650	4700	4751	4801	4852	4902	4953
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457
862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960
863	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463
864	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966
865	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468
866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969
867	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470
868	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970
869	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469
870	939519	9569	9619	9669	9719	9769	9819	9869	9918	9968
871	940018	0068	0118	0168	0218	0267	0317	0367	0417	0467
872	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964
873	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462
874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958
875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433
N.	0	1	2	3	4	5	6	7	8	9

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880	944483	4532	4581	4631	4680	4729	4779	4828	4877	4927
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894
885	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341
890	949390	9439	9488	9536	9585	9634	9683	9731	9780	9829
891	9878	9926	9975	^{0}024	^{0}073	^{0}121	^{0}170	^{0}219	^{0}267	^{0}316
892	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601
905	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994
910	959041	9089	9137	9185	9232	9280	9328	9375	9423	9471
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947
912	9995	^{0}042	^{0}090	^{0}138	^{0}185	^{0}233	^{0}280	^{0}328	^{0}376	^{0}423
913	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899
914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374
915	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741
920	963788	3835	3882	3929	3977	4024	4071	4118	4165	4212
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095
925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436
930	968483	8530	8576	8623	8670	8716	8763	8810	8856	8903
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369
932	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835
933	9882	9928	9975	^{0}021	^{0}068	^{0}114	^{0}161	^{0}207	^{0}254	^{0}300
934	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765
935	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082
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940	973128	3174	3220	3266	3313	3359	3405	3451	3497	3543
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678
950	977724	7769	7815	7861	7906	7952	7998	8043	8089	8135
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958
955	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226
960	982271	2316	2362	2407	2452	2497	2543	2588	2633	2678
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727
970	986772	6817	6861	6906	6951	6996	7040	7085	7130	7175
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850
977	9895	9939	9983	0028	0072	0117	0161	0206	0250	0294
978	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182
980	991226	1270	1315	1359	1403	1448	1492	1536	1580	1625
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957
N.	0	1	2	3	4	5	6	7	8	9

TABLE

OF

LOGARITHMIC SINES

AND

TANGENTS.

M.	Sec.	Sine.	Tang.			M.	Sec.	Sine.	Tang.		
0					60	10		7.463725	7.463727		50
	10	5.685575	5.685575	50			10	70904	70906	50	
	20	5.986605	5.986605	40			20	77966	77968	40	
	30	6.162696	6.162696	30			30	84915	84917	30	
	40	.287635	.287635	20			40	91754	7.491756	20	
	50	.384545	.384545	10			50	7.498487	7.598490	10	
1		.463726	.463726		59	11		7.505118	05120		49
	10	.530673	.530673	50			10	11649	11651	50	
	20	.588665	.588665	40			20	18083	18085	40	
	30	.639817	.639817	30			30	24423	24426	30	
	40	.685575	.685575	20			40	30672	30675	20	
	50	.726968	.726968	10			50	36832	36835	10	
2		.764756	.764756		58	12		42906	42909		48
	10	.799518	.799518	50			10	48897	48899	50	
	20	.831703	.831703	40			20	54806	54808	40	
	30	.861666	.861666	30			30	60635	60638	30	
	40	.889695	.889695	20			40	66387	66390	20	
	50	.916024	.916024	10			50	72065	72068	10	
3		.940847	.940847		57	13		77668	77671		47
	10	.964328	.964329	50			10	83201	83204	50	
	20	6.986605	6.986605	40			20	88664	88667	40	
	30	7.007794	7.077794	30			30	94059	94062	30	
	40	27998	27998	20			40	7.599388	7.599391	20	
	50	47303	47303	10			50	7.604652	7.604655	10	
4		65786	65786		56	14		09853	09857		46
	10	7.083515	7.083515	50			10	14993	14996	50	
	20	7.100548	7.100548	40			20	20072	20076	40	
	30	16938	16939	30			30	25093	25097	30	
	40	32733	32733	20			40	30056	30060	20	
	50	47973	47973	10			50	34963	34968	10	
5		62696	62696		55	15		39816	39820		45
	10	76936	76937	50			10	44615	44619	50	
	20	7.190725	7.190725	40			20	49361	49366	40	
	30	7.204089	7.204089	30			30	54056	54061	30	
	40	17054	17054	20			40	58701	58706	20	
	50	29643	29643	10			50	63297	63301	10	
6		41877	41878		54	16		67845	67849		44
	10	53776	53777	50			10	72345	72350	50	
	20	65358	65359	40			20	76799	76804	40	
	30	76639	76640	30			30	81208	81213	30	
	40	87635	87635	20			40	85573	85578	20	
	50	7.298358	7.298359	10			50	89894	89900	10	
7		7.308824	7.308825		53	17		94173	94179		43
	10	19043	19044	50			10	7.698410	7.698416	50	
	20	29027	29028	40			20	7.702606	7.702612	40	
	30	38787	38788	30			30	06762	06768	30	
	40	48332	48333	20			40	10879	10885	20	
	50	57672	57673	10			50	14957	14962	10	
8		66816	66817		52	18		18997	19003		42
	10	75770	75772	50			10	22999	23005	50	
	20	84544	84546	40			20	26966	26972	40	
	30	7.393145	7.393146	30			30	30896	30902	30	
	40	7.401578	7.401579	20			40	34791	34797	20	
	50	09850	09852	10			50	38651	38658	10	
9		17968	17970		51	19		42477	42484		41
	10	25937	25939	50			10	46270	46277	50	
	20	33762	33764	40			20	50031	50037	40	
	30	41449	41451	30			30	53758	53765	30	
	40	49002	49004	20			40	57454	57462	20	
	50	56426	56428	10			50	61119	61127	10	
10		7.463725	7.463727		50	20		7.764754	7.764761		40
		Cosine.	Cotang.	Sec.	M.			Cosine.	Cotang.	Sec.	M.

90° 89°

0° SINES AND TANGENTS. 179°

M.	Sec.	Sine.	Tang.			M.	Sec.	Sine.	Tang.		
20		7·764754	7·764761		40	30		7·940842	7·940858		30
	10	68358	68365	50			10	43248	43265	50	
	20	71932	71940	40			20	45641	45657	40	
	30	75477	75485	30			30	48020	48037	30	
	40	78994	79002	20			40	50387	50404	20	
	50	82482	82490	10			50	52741	52758	10	
21		85943	85951		39	31		55082	55100		29
	10	89376	89384	50			10	57410	57428	50	
	20	92782	92790	40			20	59727	59745	40	
	30	96162	96170	30			30	62031	62049	30	
	40	7·799515	7·799524	20			40	64322	64341	20	
	50	7·802843	7·802852	10			50	66602	66621	10	
22		06146	06155		38	32		68870	68889		28
	10	09423	09432	50			10	71126	71145	50	
	20	12677	12686	40			20	73370	73389	40	
	30	15905	15915	30			30	75603	75622	30	
	40	19111	19120	20			40	77824	77844	20	
	50	22292	22302	10			50	80034	80054	10	
23		25451	25460		37	33		82233	82253		27
	10	28586	28596	50			10	84421	84441	50	
	20	31700	31710	40			20	86598	86618	40	
	30	34791	34801	30			30	88764	88785	30	
	40	37860	37870	20			40	90919	90940	20	
	50	40907	40918	10			50	93064	93085	10	
24		43934	43944		36	34		95198	95219		26
	10	46939	46950	50			10	97322	97343	50	
	20	49924	49935	40			20	7·999435	7·999456	40	
	30	52888	52900	30			30	8.001538	8.001560	30	
	40	55833	55844	20			40	03631	03653	20	
	50	58757	58769	10			50	05714	05736	10	
25		61662	61674		35	35		07787	07809		25
	10	64548	64560	50			10	09850	09872	50	
	20	67414	67426	40			20	11903	11926	40	
	30	70262	70274	30			30	13947	13970	30	
	40	73092	73104	20			40	15981	16004	20	
	50	75902	75915	10			50	18005	18029	10	
26		78695	78708		34	36		20021	20044		24
	10	81470	81483	50			10	22027	22051	50	
	20	84228	84240	40			20	24023	24047	40	
	30	86968	86981	30			30	26011	26035	30	
	40	89690	89704	20			40	27989	28014	20	
	50	92396	92410	10			50	29959	29984	10	
27		95085	95099		33	37		31919	31945		23
	10	7·897758	7·897771	50			10	33871	33897	50	
	20	7·900414	7·900428	40			20	35814	35840	40	
	30	03054	03068	30			30	37749	37775	30	
	40	05678	05692	20			40	39675	39701	20	
	50	08287	08301	10			50	41592	41618	10	
28		10879	10894		32	38		43501	43527		22
	10	13457	13471	50			10	45401	45428	50	
	20	16019	16034	40			20	47294	47321	40	
	30	18566	18581	30			30	49178	49205	30	
	40	21098	21113	20			40	51054	51081	20	
	50	23616	23631	10			50	52922	52949	10	
29		26119	26134		31	39		54781	54809		21
	10	28608	28623	50			10	56633	56661	50	
	20	31082	31098	40			20	58477	58506	40	
	30	33543	33559	30			30	60314	60342	30	
	40	35989	36006	20			40	62142	62171	20	
	50	38422	38439	10			50	63963	63992	10	
30		7·940842	7·940858		30	40		8.065776	8.065806		20
		Cosine.	Cotang.	Sec.	M.			Cosine.	Cotang.	Sec.	M.

90° 89°

M.	Sec.	Sine.	Tang.			M.	Sec.	Sine.	Tang.		
40		8.065776	8.065806		20	50		8.162681	8.162727		10
	10	67582	67612	50			10	64126	64172	50	
	20	69380	69410	40			20	65566	65613	40	
	30	71171	71201	30			30	67002	67049	30	
	40	72955	72985	20			40	68433	68480	20	
	50	74731	74761	10			50	69859	69906	10	
41		76500	76531		19	51		71280	71328		9
	10	78261	78293	50			10	72697	72745	50	
	20	80016	80047	40			20	74109	74158	40	
	30	81764	81795	30			30	75517	75566	30	
	40	83504	83536	20			40	76920	76969	20	
	50	85238	85270	10			50	78319	78368	10	
42		86965	86997		18	52		79713	79763		8
	10	88684	88717	50			10	81102	81152	50	
	20	90398	90430	40			20	82488	82538	40	
	30	92104	92137	30			30	83868	83919	30	
	40	93804	93837	20			40	85245	85296	20	
	50	95497	95530	10			50	86617	86668	10	
43		97183	97217		17	53		87985	88036		7
	10	8.098863	8.098897	50			10	89348	89400	50	
	20	8.100537	8.100571	40			20	90707	90760	40	
	30	02204	02239	30			30	92062	92115	30	
	40	03864	03899	20			40	93413	93466	20	
	50	05519	05554	10			50	94760	94813	10	
44		07167	07202		16	54		96102	96156		6
	10	08809	08845	50			10	97440	97494	50	
	20	10444	10481	40			20	8.198774	8.198829	40	
	30	12074	12110	30			30	8.200104	8.200159	30	
	40	13697	13734	20			40	01430	01485	20	
	50	15315	15352	10			50	02752	02808	10	
45		16926	16963		15	55		04070	04126		5
	10	18532	18569	50			10	05384	05440	50	
	20	20131	20169	40			20	06694	06750	40	
	30	21725	21763	30			30	08000	08057	30	
	40	23313	23351	20			40	09302	09359	20	
	50	24895	24933	10			50	10601	10658	10	
46		8.126471	8.126510		14	56		11895	11953		4
	10	28042	28081	50			10	13185	13243	50	
	20	29606	29646	40			20	14472	14530	40	
	30	31166	31206	30			30	15755	15814	30	
	40	32720	32760	20			40	17034	17093	20	
	50	34268	34308	10			50	18309	18369	10	
47		35810	35851		13	57		8.219581	8.219641		3
	10	37348	37389	50			10	20849	20909	50	
	20	38879	38921	40			20	22113	22174	40	
	30	40406	40447	30			30	23374	23434	30	
	40	41927	41969	20			40	24631	24692	20	
	50	43443	43485	10			50	25884	25945	10	
48		44953	44996		12	58		27133	27195		2
	10	46458	46501	50			10	28380	28442	50	
	20	47959	48001	40			20	29622	29685	40	
	30	49453	49497	30			30	30861	30924	30	
	40	50943	50987	20			40	32096	32160	20	
	50	52428	52472	10			50	33328	33392	10	
49		53907	53952		11	59		34557	34621		1
	10	55382	55426	50			10	35782	35846	50	
	20	56852	56896	40			20	37003	37068	40	
	30	58316	58361	30			30	38221	38286	30	
	40	59776	59821	20			40	39436	39501	20	
	50	61231	61276	10			50	40647	40713	10	
50		8.162681	8.162727		10	60		8.241855	8.241921		0
		Cosine.	Cotang.	Sec.	M.			Cosine.	Cotang.	Sec.	M.

90° 89°

M.	Sec.	Sine.	Tang.			M.	Sec.	Sine.	Tang.		
0		8.241855	8.241921		60	10		8.308794	8.308884		50
	10	3060	3126	50			10	8.309827	8.309917	50	
	20	4261	4328	40			20	8.310857	8.310948	40	
	30	5459	5526	30			30	1885	1976	30	
	40	6654	6721	20			40	2910	3002	20	
	50	7845	7913	10			50	3933	4025	10	
1		8.249033	8.249101		59	11		4954	5046		49
	10	8.250218	8.250287	50			10	5972	6065	50	
	20	1400	1469	40			20	6987	7081	40	
	30	2578	2648	30			30	8001	8095	30	
	40	3753	3823	20			40	8.319012	8.319106	20	
	50	4925	4996	10			50	8.320021	8.320115	10	
2		6094	6165		58	12		1027	1122		48
	10	7260	7331	50			10	2031	2127	50	
	20	8423	8494	40			20	3033	3129	40	
	30	8.259582	8.259654	30			30	4032	4128	30	
	40	8.260739	8.260811	20			40	5029	5126	20	
	50	1892	1965	10			50	6024	6121	10	
3		3042	3115		57	13		7016	7114		47
	10	4190	4263	50			10	8007	8105	50	
	20	5334	5408	40			20	8995	8.329093	40	
	30	6475	6549	30			30	8.329980	8.330080	30	
	40	7613	7688	20			40	8.330964	1064	20	
	50	8749	8824	10			50	1945	2045	10	
4		8.269881	8.269956		56	14		2924	3025		46
	10	8.271010	8.271086	50			10	3901	4002	50	
	20	2137	2213	40			20	4876	4977	40	
	30	3260	3337	30			30	5848	5950	30	
	40	4381	4458	20			40	6819	6921	20	
	50	5499	5576	10			50	7787	7890	10	
5		6614	6691		55	15		8753	8856		45
	10	7726	7804	50			10	8.339717	8.339821	50	
	20	8835	8.278913	40			20	8.340679	8.340783	40	
	30	8.279941	8.280020	30			30	1638	1743	30	
	40	8.281045	1124	20			40	2596	2701	20	
	50	2145	2225	10			50	3551	3657	10	
6		3243	3323		54	16		4504	4610		44
	10	4339	4419	50			10	5456	5562	50	
	20	5431	5512	40			20	6405	6512	40	
	30	6521	6602	30			30	7352	7459	30	
	40	7608	7689	20			40	8297	8405	20	
	50	8692	8774	10			50	8.349240	8.349348	10	
7		8.289773	8.289856		53	17		8.350181	8.350289		43
	10	8.290852	8.290935	50			10	1119	1229	50	
	20	1928	2012	40			20	2056	2166	40	
	30	3002	3086	30			30	2991	3101	30	
	40	4073	4157	20			40	3924	4035	20	
	50	5141	5226	10			50	4855	4966	10	
8		6207	6292		52	18		5783	5895		42
	10	7270	7355	50			10	6710	6823	50	
	20	8330	8416	40			20	7635	7748	40	
	30	8.299388	8.299474	30			30	8558	8671	30	
	40	8.300443	8.300530	20			40	8.359479	8.359593	20	
	50	1496	1583	10			50	8.360398	8.360512	10	
9		2546	2633		51	19		1315	1430		41
	10	3594	3682	50			10	2230	2345	50	
	20	4639	4727	40			20	3143	3259	40	
	30	5681	5770	30			30	4054	4171	30	
	40	6721	6811	20			40	4964	5080	20	
	50	7759	7849	10			50	5871	5988	10	
10		8.308794	8.308884		50	20		8.366777	8.366894		40
		Cosine.	Cotang.	Sec.	M.			Cosine.	Cotang.	Sec.	M.

M.	Sec.	Sine.	Tang.			M.	Sec.	Sine.	Tang.		
20		8.366777	8.366894		40	30		8.417919	8.418068		30
	10	7681	7799	50			10	8722	8872	50	
	20	8582	8701	40			20	8.419524	8.419674	40	
	30	8.369482	8.369601	30			30	8.420324	8.420475	30	
	40	8.370380	8.370500	20			40	1123	1274	20	
	50	1277	1397	10			50	1921	2072	10	
21		2171	2291		39	31		2717	2869		29
	10	3063	3184	50			10	3511	3664	50	
	20	3954	4076	40			20	4304	4458	40	
	30	4843	4965	30			30	5096	5250	30	
	40	5730	5853	20			40	5886	6040	20	
	50	6615	6738	10			50	6675	6830	10	
22		7499	7622		38	32		7462	7618		28
	10	8380	8504	50			10	8248	8404	50	
	20	8.379260	8.379385	40			20	9032	9189	40	
	30	8.380138	8.380263	30			30	8.429815	8.429973	30	
	40	1015	1140	20			40	8.430597	8.430755	20	
	50	1889	2015	10			50	1377	1536	10	
23		2762	2889		37	33		2156	2315		27
	10	3633	3760	50			10	2933	3093	50	
	20	4502	4630	40			20	3709	3870	40	
	30	5370	5498	30			30	4484	4645	30	
	40	6236	6364	20			40	5257	5419	20	
	50	7100	7229	10			50	6029	6191	10	
24		7962	8092		36	34		6800	6962		26
	10	8823	8953	50			10	7569	7732	50	
	20	8.389682	8.389812	40			20	8337	8500	40	
	30	8.390539	8.390670	30			30	9103	8.439267	30	
	40	1395	1526	20			40	8.439868	8.440033	20	
	50	2249	2381	10			50	8.440632	0797	10	
25		3101	3234		35	35		1394	1560		25
	10	3951	4085	50			10	2155	2322	50	
	20	4800	4934	40			20	2915	3082	40	
	30	5647	5782	30			30	3674	3841	30	
	40	6493	6628	20			40	4431	4599	20	
	50	7337	7472	10			50	5186	5355	10	
26		8179	8315		34	36		5941	6110		24
	10	9020	9156	50			10	6694	6864	50	
	20	8.399859	8.399996	40			20	7446	7616	40	
	30	8.400696	8.400834	30			30	8196	8367	30	
	40	1532	1670	20			40	8946	9117	20	
	50	2366	2505	10			50	8.449694	8.449866	10	
27		3199	3338		33	37		8.450440	8.450613		23
	10	4030	4170	50			10	1186	1359	50	
	20	4859	5000	40			20	1930	2104	40	
	30	5687	5828	30			30	2672	2847	30	
	40	6513	6655	20			40	3414	3589	20	
	50	7338	7480	10			50	4154	4330	10	
28		8161	8304		32	38		4893	5070		22
	10	8983	9126	50			10	5631	5808	50	
	20	8.409803	8.409946	40			20	6368	6545	40	
	30	8.410621	8.410765	30			30	7103	7281	30	
	40	1438	1583	20			40	7837	8016	20	
	50	2254	2399	10			50	8570	8749	10	
29		3068	3213		31	39		8.459301	8.459481		21
	10	3880	4026	50			10	8.460032	8.460212	50	
	20	4691	4837	40			20	0761	0942	40	
	30	5500	5647	30			30	1489	1670	30	
	40	6308	6456	20			40	2215	2398	20	
	50	7114	7262	10			50	2941	3124	10	
30		8.417919	8.418068		30	40		8.463665	8.463849		20
		Cosine.	Cotang.	Sec.	M.			Cosine.	Cotang.	Sec.	M.

M.	Sec.	Sine.	Tang.			M.	Sec.	Sine.	Tang.		
40		8.463665	8.463849		20	50		8.505045	8.505267		10
	10	4388	4572	50			10	5702	5925	50	
	20	5110	5295	40			20	6358	6582	40	
	30	5830	6016	30			30	7014	7238	30	
	40	6550	6736	20			40	7668	7893	20	
	50	7268	7455	10			50	8321	8547	10	
41		7985	8172		19	51		8974	9200		9
	10	8701	8889	50			10	8.509625	8.509852	50	
	20	8.469416	8.469604	40			20	8.510275	8.510503	40	
	30	8.470129	8.470318	30			30	0925	1153	30	
	40	0841	1031	20			40	1573	1802	20	
	50	1553	1743	10			50	2221	2451	10	
42		2263	2454		18	52		2867	3098		8
	10	2971	3163	50			10	3513	3744	50	
	20	3679	3871	40			20	4157	4389	40	
	30	4386	4579	30			30	4801	5034	30	
	40	5091	5285	20			40	5444	5677	20	
	50	5795	5990	10			50	6086	6319	10	
43		6498	6693		17	53		6726	6961		7
	10	7200	7396	50			10	7366	7602	50	
	20	7901	8097	40			20	8005	8241	40	
	30	8601	8798	30			30	8643	8880	30	
	40	9299	8.479497	20			40	9280	8.519517	20	
	50	8.479997	8.480195	10			50	8.519916	8.520154	10	
44		8.480693	0892		16	54		8.520551	0790		6
	10	1388	1588	50			10	1186	1425	50	
	20	2082	2283	40			20	1819	2059	40	
	30	2775	2976	30			30	2451	2692	30	
	40	3467	3669	20			40	3083	3324	20	
	50	4158	4360	10			50	3713	3956	10	
45		4848	5050		15	55		4343	4586		5
	10	5536	5740	50			10	4972	5215	50	
	20	6224	6428	40			20	5599	5844	40	
	30	6910	7115	30			30	6226	6472	30	
	40	7596	7801	20			40	6852	7098	20	
	50	8280	8486	10			50	7477	7724	10	
46		8963	9170		14	56		8102	8349		4
	10	8.489645	8.489852	50			10	8725	8973	50	
	20	8.490326	8.490534	40			20	9347	8.529596	40	
	30	1006	1215	30			30	8.529969	8.530218	30	
	40	1685	1894	20			40	8.530589	0840	20	
	50	2363	2573	10			50	1209	1460	10	
47		3040	3250		13	57		1828	2080		3
	10	3715	3927	50			10	2446	2698	50	
	20	4390	4602	40			20	3063	3316	40	
	30	5064	5276	30			30	3679	3933	30	
	40	5736	5949	20			40	4295	4549	20	
	50	6408	6622	10			50	4909	5164	10	
48		7078	7293		12	58		5523	5779		2
	10	7748	7963	50			10	6136	6392	50	
	20	8416	8632	40			20	6747	7005	40	
	30	9084	9300	30			30	7358	7616	30	
	40	8.499750	8.499967	20			40	7969	8227	20	
	50	8.500415	8.500633	10			50	8578	8837	10	
49		1080	1298		11	59		9186	8.539447		1
	10	1743	1962	50			10	8.539794	8.540055	50	
	20	2405	2625	40			20	8.540401	0662	40	
	30	3067	3287	30			30	1007	1269	30	
	40	3727	3948	20			40	1612	1875	20	
	50	4386	4608	10			50	2216	2480	10	
50		8.505045	8.505267		10	60		8.542819	8.543084		0
		Cosine.	Cotang.	Sec.	M.			Cosine.	Cotang.	Sec.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	Inf. neg.		10.000000	.00	Inf. neg.		Infinite.	60
1	6.463726	5017.17	0		6.463726	5017.17	13.536274	59
2	764756	2934.85	0		764756	2934.85	235244	58
3	6.940847	2082.31	0		6.940847	2082.31	13.059153	57
4	7.065786	1615.17	0		7.065786	1615.17	12.934214	56
5	162696	1319.68	0	.00	162696	1319.69	837304	55
6	241877	1115.78	9.999999	.01	241878	1115.78	758122	54
7	308824	966.53	99		308825	966.54	691175	53
8	366816	852.54	99		366817	852.54	633183	52
9	417968	762.62	99		417970	762.63	582030	51
10	463725	689.88	98		463727	689.88	536273	50
11	7.505118	629.81	9.999998		7.505120	629.81	12.494880	49
12	542906	579.36	97		542909	579.37	457091	48
13	577668	536.41	97		577672	536.42	422328	47
14	609853	499.38	96		609857	499.39	390143	46
15	639816	467.14	96		639820	467.15	360180	45
16	667845	438.81	95		667849	438.82	332151	44
17	694173	413.72	95		694179	413.73	305821	43
18	718997	391.35	94		719003	391.36	280997	42
19	742477	371.27	93		742484	371.28	257516	41
20	764754	353.15	93		764761	353.16	235239	40
21	7.785943	336.72	9.999992		7.785951	336.73	12.214049	39
22	806146	321.75	91		806155	321.76	193845	38
23	825451	308.05	90	.01	825460	308.07	174540	37
24	843934	295.47	89	.02	843944	295.49	156056	36
25	861662	283.88	88		861674	283.90	138326	35
26	878695	273.17	88		878708	273.18	121292	34
27	895085	263.23	87		895099	263.25	104901	33
28	910879	253.99	86		910894	254.01	089106	32
29	926119	245.38	85		926134	245.40	073866	31
30	940842	237.33	83		940858	237.35	059142	30
31	7.955082	229.80	9.999982		7.955100	229.82	12.044900	29
32	968870	222.73	81		968889	222.75	031111	28
33	982233	216.08	80		982253	216.10	017747	27
34	7.995198	209.81	79		7.995219	209.83	12.004781	26
35	8.007787	203.90	77		8.007809	203.92	11.992191	25
36	020021	198.31	76		020045	198.33	979955	24
37	031919	193.02	75		031945	193.05	968055	23
38	043501	188.01	73		043527	188.03	956473	22
39	054781	183.25	72		054809	183.27	945191	21
40	065776	178.72	71		065806	178.75	934194	20
41	8.076500	174.41	9.999969		8.076531	174.44	11.923469	19
42	086965	170.31	68		086997	170.34	913003	18
43	097183	166.39	66	.02	097217	166.42	902783	17
44	107167	162.65	64	.03	107202	162.68	892798	16
45	116926	159.08	63		116963	159.11	883037	15
46	126471	155.66	61		126510	155.68	873490	14
47	135810	152.38	59		135851	152.41	864149	13
48	144953	149.24	58		144996	149.27	855004	12
49	153907	146.22	56		153952	146.25	846048	11
50	162681	143.33	54		162727	143.36	837273	10
51	8.171280	140.54	9.999952		8.171328	140.57	11.828672	9
52	179713	137.86	50		179763	137.90	820237	8
53	187985	135.29	48		188036	135.32	811964	7
54	196102	132.80	46		196156	132.84	803844	6
55	204070	130.41	44	.03	204126	130.44	795874	5
56	211895	128.10	42	.04	211953	128.14	788047	4
57	219581	125.87	40		219641	125.91	780359	3
58	227134	123.72	38		227195	123.76	772805	2
59	234557	121.64	36	.04	234621	121.68	765379	1
60	8.241855		9.999934		8.241921		11.758079	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

90° 89°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	8.241855	119.63	9.999934	.04	8.241921	119.67	11.758079	60
1	249033	117.68	932		249102	117.72	750898	59
2	256094	115.80	929		256165	115.84	743835	58
3	263042	113.98	927		263115	114.02	736885	57
4	269881	112.21	925		269956	112.25	730044	56
5	276614	110.50	922		276691	110.54	723309	55
6	283243	108.83	920		283323	108.87	716677	54
7	289773	107.22	917		289856	107.26	710144	53
8	296207	105.65	915		296292	105.70	703708	52
9	302546	104.13	912		302634	104.18	697366	51
10	308794	102.66	910		308884	102.70	691116	50
11	8.314954	101.22	9.999907		8.315046	101.26	11.684954	49
12	321027	99.82	905		321122	99.87	678878	48
13	327016	98.47	902	.04	327114	98.51	672886	47
14	332924	97.14	899	.05	333025	97.19	666975	46
15	338753	95.86	897		338856	95.90	661144	45
16	344504	94.60	894		344610	94.65	655390	44
17	350180	93.38	891		350289	93.43	649711	43
18	355783	92.19	888		355895	92.24	644105	42
19	361315	91.03	885		361430	91.08	638570	41
20	366777	89.90	882		366895	89.95	633105	40
21	8.372171	88.80	9.999879		8.372292	88.85	11.627708	39
22	377499	87.72	876		377622	87.77	622378	38
23	382762	86.67	873		382889	86.72	617111	37
24	387962	85.64	870		388092	85.70	611908	36
25	393101	84.64	867		393234	84.69	606766	35
26	398179	83.66	864		398315	83.71	601685	34
27	403199	82.71	861		403338	82.76	596662	33
28	408161	81.77	858		408304	81.82	591696	32
29	413068	80.86	854	.05	413213	80.91	586787	31
30	417919	79.96	851	.06	418068	80.02	581932	30
31	8.422717	79.09	9.999848		8.422869	79.14	11.577131	29
32	427462	78.23	844		427618	78.29	572382	28
33	432156	77.40	841		432315	77.45	567685	27
34	436800	76.57	838		436962	76.63	563038	26
35	441394	75.77	834		441560	75.83	558440	25
36	445941	74.99	831		446110	75.05	553890	24
37	450440	74.22	827		450613	74.28	549387	23
38	454893	73.46	823		455070	73.52	544930	22
39	459301	72.73	820		459481	72.79	540519	21
40	463665	72.00	816		463849	72.06	536151	20
41	8.467985	71.29	9.999813		8.468173	71.35	11.531827	19
42	472263	70.60	809		472454	70.66	527546	18
43	476498	69.91	805		476693	69.98	523307	17
44	480693	69.24	801	.06	480892	69.31	519108	16
45	484848	68.59	797	.07	485050	68.65	514950	15
46	488963	67.94	793		489170	68.01	510830	14
47	493040	67.31	790		493250	67.38	506750	13
48	497078	66.69	786		497293	66.76	502707	12
49	501080	66.08	782		501298	66.15	498702	11
50	505045	65.48	778		505267	65.55	494733	10
51	8.508974	64.89	9.999774		8.509200	64.96	11.490800	9
52	512867	64.32	769		513098	64.39	486902	8
53	516726	63.75	765		516961	63.82	483039	7
54	520551	63.19	761		520790	63.26	479210	6
55	524343	62.64	757		524586	62.72	475414	5
56	528102	62.11	753		528349	62.18	471651	4
57	531828	61.58	748		532080	61.65	467920	3
58	535523	61.06	744		535779	61.13	464221	2
59	539186	60.55	740	.07	539447	60.62	460553	1
60	8.542819		9.999735		8.543084		11.456916	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	8.542819	60.04	9.999735	.07	8.543084	60.12	11.456916	60
1	46422	59.55	731	.07	46691	59.62	53309	59
2	49995	59.06	726	.07	50268	59.14	49732	58
3	53539	58.58	722	.08	53817	58.66	46183	57
4	57054	58.11	717		57336	58.19	42664	56
5	60540	57.65	713		60828	57.73	39172	55
6	63999	57.19	708		64291	57.27	35709	54
7	67431	56.74	704		67727	56.82	32273	53
8	70836	56.30	699		71137	56.38	28863	52
9	74214	55.87	694		74520	55.95	25480	51
10	77566	55.44	689		77877	55.52	22123	50
11	8.580892	55.02	9.999685		8.581208	55.10	11.418792	49
12	84193	54.60	680		84514	54.68	15486	48
13	87469	54.19	675		87795	54.27	12205	47
14	90721	53.79	670		91051	53.87	08949	46
15	93948	53.39	665		94283	53.47	05717	45
16	8.597152	53.00	660		8.597492	53.08	11.402508	44
17	8.600332	52.61	655		8.600677	52.70	11.399323	43
18	03489	52.23	650	.08	03839	52.32	96161	42
19	06623	51.86	645	.09	06978	51.94	93022	41
20	09734	51.49	640		10094	51.58	89906	40
21	8.612823	51.12	9.999635		8.613189	51.21	11.386811	39
22	15891	50.76	629		16262	50.85	83738	38
23	18937	50.41	624		19313	50.50	80687	37
24	21962	50.06	619		22343	50.15	77657	36
25	24965	49.72	614		25352	49.81	74648	35
26	27948	49.38	608		28340	49.47	71660	34
27	30911	49.04	603		31308	49.13	68692	33
28	33854	48.71	597		34256	48.80	65744	32
29	36776	48.39	592		37184	48.48	62816	31
30	39680	48.06	586		40093	48.16	59907	30
31	8.642563	47.75	9.999581		8.642983	47.84	11.357017	29
32	45428	47.43	575		45853	47.53	54147	28
33	48274	47.12	570		48704	47.22	51296	27
34	51102	46.82	564	.09	51537	46.91	48463	26
35	53911	46.52	558	.10	54352	46.61	45648	25
36	56702	46.22	553		57149	46.31	42851	24
37	59475	45.92	547		59928	46.02	40072	23
38	62230	45.63	541		62689	45.73	37311	22
39	64968	45.35	535		65433	45.44	34567	21
40	67689	45.06	529		68160	45.16	31840	20
41	8.670393	44.79	9.999524		8.670870	44.88	11.329130	19
42	73080	44.51	518		73563	44.61	26437	18
43	75751	44.24	512		76239	44.34	23761	17
44	78405	43.97	506		78900	44.07	21100	16
45	81043	43.70	500		81544	43.80	18456	15
46	83665	43.44	493		84172	43.54	15828	14
47	86272	43.18	487		86784	43.28	13216	13
48	88863	42.92	481		89381	43.03	10619	12
49	91438	42.67	475		91963	42.77	08037	11
50	93998	42.42	469	.10	94529	42.52	05471	10
51	96543	42.17	9.999463	.11	97081	42.28	02919	9
52	8.699073	41.92	456		8.699617	42.03	11.300383	8
53	8.701589	41.68	450		8.702139	41.79	11.297861	7
54	04090	41.44	443		04646	41.55	95354	6
55	06577	41.21	437		07140	41.32	92860	5
56	09049	40.97	431		09618	41.08	90382	4
57	11507	40.74	424		12083	40.85	87917	3
58	13952	40.51	418		14535	40.62	85465	2
59	16383	40.29	411	.11	16972	40.40	83028	1
60	8.718800		9.999404		8.719396		11.280604	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

92° 87°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	8.718800	40.06	9.999404	.11	8.719396	40.17	11.280604	60
1	21204	39.84	9398		21806	39.95	78194	59
2	23595	39.62	9391		24203	39.74	75797	58
3	25972	39.41	9384		26588	39.52	73412	57
4	28337	39.19	9378		28959	39.31	71041	56
5	30688	38.98	9371	.11	31317	39.09	68683	55
6	33027	38.77	9364	.12	33663	38.89	66337	54
7	35354	38.57	9357		35996	38.68	64004	53
8	37667	38.36	9350		38317	38.48	61683	52
9	39969	38.16	9343		40626	38.27	59374	51
10	42259	37.96	9336		42922	38.07	57078	50
11	8.744536	37.76	9.999329		8.745207	37.87	11.254793	49
12	46802	37.56	9322		47479	37.68	52521	48
13	49055	37.37	9315		49740	37.49	50260	47
14	51297	37.17	9308		51989	37.29	48011	46
15	53528	36.98	9301		54227	37.10	45773	45
16	55747	36.79	9294		56453	36.92	43547	44
17	57955	36.61	9286		58668	36.73	41332	43
18	60151	36.42	9279		60872	36.55	39128	42
19	62337	36.24	9272		63065	36.36	36935	41
20	64511	36.06	9265		65246	36.18	34754	40
21	8.766675	35.88	9.999257	.12	8.767417	36.00	11.232583	39
22	68828	35.70	9250	.13	69578	35.83	30422	38
23	70970	35.53	9242		71727	35.65	28273	37
24	73101	35.35	9235		73866	35.48	26134	36
25	75223	35.18	9227		75995	35.31	24005	35
26	77333	35.01	9220		78114	35.14	21886	34
27	79434	34.84	9212		80222	34.97	19778	33
28	81524	34.67	9205		82320	34.80	17680	32
29	83605	34.51	9197		84408	34.64	15592	31
30	85675	34.34	9189		86486	34.47	13514	30
31	8.787736	34.18	9.999181		8.788554	34.31	11.211446	29
32	89787	34.02	9174		90613	34.15	09387	28
33	91828	33.86	9166		92662	33.99	07338	27
34	93859	33.70	9158		94701	33.83	05299	26
35	95881	33.54	9150		96731	33.68	03269	25
36	97894	33.39	9142		8.798752	33.52	11.201248	24
37	8.799897	33.23	9134		8.800763	33.37	11.199237	23
38	8.801892	33.08	9126		02765	33.22	97235	22
39	03876	32.93	9118		04758	33.07	95242	21
40	05852	32.78	9110		06742	32.92	93258	20
41	8.807819	32.63	9.999102	.13	8.808717	32.77	11.191283	19
42	09777	32.49	9094	.14	10683	32.62	89317	18
43	11726	32.34	9086		12641	32.48	87359	17
44	13667	32.19	9077		14589	32.33	85411	16
45	15599	32.05	9069		16529	32.19	83471	15
46	17522	31.91	9061		18461	32.05	81539	14
47	19436	31.77	9053		20384	31.91	79616	13
48	21343	31.63	9044		22298	31.77	77702	12
49	23240	31.49	9036		24205	31.63	75795	11
50	25130	31.35	9027		26103	31.50	73897	10
51	8.827011	31.22	9.999019		8.827992	31.36	11.172008	9
52	28884	31.08	9010		29874	31.23	70126	8
53	30749	30.95	9002		31748	31.09	68252	7
54	32607	30.82	8993		33613	30.96	66387	6
55	34456	30.69	8984		35471	30.83	64529	5
56	36297	30.56	8976	.14	37321	30.70	62679	4
57	38130	30.43	8967	.15	39163	30.57	60837	3
58	39956	30.30	8958	.15	40998	30.45	59002	2
59	41774	30.17	8950	.15	42825	30.32	57175	1
60	8.843585		9.998941		8.844644		11.155356	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	8.843585	30.05	9.998941	.15	8.844644	30.19	11.155356	60
1	45387	29.92	932		46455	30.07	53545	59
2	47183	29.80	923		48260	29.95	51740	58
3	48971	29.67	914		50057	29.82	49943	57
4	50751	29.55	905		51846	29.70	48154	56
5	52525	29.43	896		53628	29.58	46372	55
6	54291	29.31	887		55403	29.46	44597	54
7	56049	29.19	878		57171	29.35	42829	53
8	57801	29.08	869		58932	29.23	41068	52
9	59546	28.96	860		60686	29.11	39314	51
10	61283	28.84	851		62433	29.00	37567	50
11	8.863014	28.73	9.998841		8.864173	28.88	11.135827	49
12	64738	28.61	832	.15	65906	28.77	34094	48
13	66455	28.50	823	.16	67632	28.66	32368	47
14	68165	28.39	813		69351	28.54	30649	46
15	69868	28.28	804		71064	28.43	28936	45
16	71565	28.17	795		72770	28.32	27230	44
17	73255	28.06	785		74469	28.21	25531	43
18	74938	27.95	776		76162	28.11	23838	42
19	76615	27.84	766		77849	28.00	22151	41
20	78285	27.73	757		79529	27.89	20471	40
21	8.879949	27.63	9.998747		8.881202	27.79	11.118798	39
22	81607	27.52	738		82869	27.68	17131	38
23	83258	27.42	728		84530	27.58	15470	37
24	84903	27.31	718		86185	27.47	13815	36
25	86542	27.21	708		87833	27.37	12167	35
26	88174	27.11	699		89476	27.27	10524	34
27	89801	27.00	689		91112	27.17	08888	33
28	91421	26.90	679	.16	92742	27.07	07258	32
29	93035	26.80	669	.17	94366	26.97	05634	31
30	94643	26.70	659		95984	26.87	04016	30
31	96245	26.60	9.998649		97596	26.77	02404	29
32	97842	26.51	639		8.899203	26.67	11.100797	28
33	8.899432	26.41	629		8.900803	26.58	11.099197	27
34	8.901017	26.31	619		02398	26.48	97602	26
35	02596	26.22	609		03987	26.38	96013	25
36	04169	26.12	599		05570	26.29	94430	24
37	05736	26.03	589		07147	26.20	92853	23
38	07297	25.93	578		08719	26.10	91281	22
39	08853	25.84	568		10285	26.01	89715	21
40	10404	25.75	558		11846	25.92	88154	20
41	8.911949	25.66	9.998548		8.913401	25.83	11.086599	19
42	13488	25.56	537		14951	25.74	85049	18
43	15022	25.47	527	.17	16495	25.65	83505	17
44	16550	25.38	516	.18	18034	25.56	81966	16
45	18073	25.29	506		19568	25.47	80432	15
46	19591	25.20	495		21096	25.38	78904	14
47	21103	25.12	485		22619	25.30	77381	13
48	22610	25.03	474		24136	25.21	75864	12
49	24112	24.94	464		25649	25.12	74351	11
50	25609	24.86	453		27156	25.03	72844	10
51	8.927100	24.77	9.998442		8.928658	24.95	11.071342	9
52	28587	24.69	431		30155	24.86	69845	8
53	30068	24.60	421		31647	24.78	68353	7
54	31544	24.52	410		33134	24.70	66866	6
55	33015	24.43	399		34616	24.61	65384	5
56	34481	24.35	388		36093	24.53	63907	4
57	35942	24.27	377		37565	24.45	62435	3
58	37398	24.19	366		39032	24.37	60968	2
59	38850	24.11	355	.18	40494	24.29	59506	1
60	8.940296		9.998344		8.941952		11.058048	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	8.940296	24.03	9.998344	.19	8.941952	24.21	11.058048	60
1	41738	23.94	333		43404	24.13	56596	59
2	43174	23.87	322		44852	24.05	55148	58
3	44606	23.79	311		46295	23.97	53705	57
4	46034	23.71	300		47734	23.90	52266	56
5	47456	23.63	289		49168	23.82	50832	55
6	48874	23.55	277		50597	23.74	49403	54
7	50287	23.48	266		52021	23.66	47979	53
8	51696	23.40	255		53441	23.59	46559	52
9	53100	23.32	243		54856	23.51	45144	51
10	54499	23.25	232		56267	23.44	43733	50
11	8.955894	23.17	9.998220		8.957674	23.37	11.042326	49
12	57284	23.10	209		59075	23.29	40925	48
13	58670	23.02	197		60473	23.22	39527	47
14	60052	22.95	186		61866	23.14	38134	46
15	61429	22.88	174		63255	23.07	36745	45
16	62801	22.80	163		64639	23.00	35361	44
17	64170	22.73	151	.19	66019	22.93	33981	43
18	65534	22.66	139	.20	67394	22.86	32606	42
19	66893	22.59	128		68766	22.79	31234	41
20	68249	22.52	116		70133	22.71	29867	40
21	8.969600	22.45	9.998104		8.971496	22.65	11.028504	39
22	70947	22.38	092		72855	22.57	27145	38
23	72289	22.31	080		74209	22.51	25791	37
24	73628	22.24	068		75560	22.44	24440	36
25	74962	22.17	056		76906	22.37	23094	35
26	76293	22.10	044		78248	22.30	21752	34
27	77619	22.03	032		79586	22.23	20414	33
28	78941	21.97	020		80921	22.17	19079	32
29	80259	21.90	9.998008		82251	22.10	17749	31
30	81573	21.83	9.997996		83577	22.04	16423	30
31	8.982883	21.77	984		8.984899	21.97	11.015101	29
32	84189	21.70	972		86217	21.91	13783	28
33	85491	21.63	959		87532	21.84	12468	27
34	86789	21.57	947	.20	88842	21.78	11158	26
35	88083	21.50	935	.21	90149	21.71	09851	25
36	89374	21.44	922		91451	21.65	08549	24
37	90660	21.38	910		92750	21.58	07250	23
38	91943	21.31	897		94045	21.52	05955	22
39	93222	21.25	885		95337	21.46	04663	21
40	94497	21.19	872		96624	21.40	03376	20
41	8.995768	21.12	9.997860		97908	21.34	02092	19
42	97036	21.06	847		8.999188	21.27	11.000812	18
43	98299	21.00	835		9.000465	21.21	10.999535	17
44	8.999560	20.94	822		01738	21.15	98262	16
45	9.000816	20.88	809		03007	21.09	96993	15
46	02069	20.82	797		04272	21.03	95728	14
47	03318	20.76	784		05534	20.97	94466	13
48	04563	20.70	771		06792	20.91	93208	12
49	05805	20.64	758		08047	20.85	91953	11
50	07044	20.58	745		09298	20.80	90702	10
51	9.008278	20.52	9.997732		9.010546	20.74	10.989454	9
52	09510	20.46	719		11790	20.68	88210	8
53	10737	20.40	706	.21	13031	20.62	86969	7
54	11962	20.34	693	.22	14268	20.56	85732	6
55	13182	20.29	680		15502	20.51	84498	5
56	14400	20.23	667		16732	20.45	83268	4
57	15613	20.17	654		17959	20.40	82041	3
58	16824	20.12	641		19183	20.34	80817	2
59	18031	20.06	628	.22	20403	20.28	79597	1
60	9.019235		9.997614		9.021620		10.978380	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.019235	20.00	9.997614	.22	9.021620	20.23	10.978380	60
1	20435	19.95	601		22834	20.17	77166	59
2	21632	19.89	588		24044	20.11	75956	58
3	22825	19.84	574		25251	20.06	74749	57
4	24016	19.78	561		26455	20.01	73545	56
5	25203	19.73	547	.22	27655	19.95	72345	55
6	26386	19.67	534	.23	28852	19.90	71148	54
7	27567	19.62	520		30046	19.85	69954	53
8	28744	19.57	507		31237	19.79	68763	52
9	29918	19.51	493		32425	19.74	67575	51
10	31089	19.46	480		33609	19.69	66391	50
11	9.032257	19.41	9.997466		9.034791	19.64	10.965209	49
12	33421	19.36	452		35969	19.58	64031	48
13	34582	19.30	439		37144	19.53	62856	47
14	35741	19.25	425		38316	19.48	61684	46
15	36896	19.20	411		39485	19.43	60515	45
16	38048	19.15	397		40651	19.38	59349	44
17	39197	19.10	383		41813	19.33	58187	43
18	40342	19.05	369		42973	19.28	57027	42
19	41485	18.99	355		44130	19.23	55870	41
20	42625	18.95	341	.23	45284	19.18	54716	40
21	9.043762	18.89	9.997327	.24	9.046434	19.13	10.953566	39
22	44895	18.84	313		47582	19.08	52418	38
23	46026	18.79	299		48727	19.03	51273	37
24	47154	18.75	285		49869	18.98	50131	36
25	48279	18.70	271		51008	18.93	48992	35
26	49400	18.65	257		52144	18.89	47856	34
27	50519	18.60	242		53277	18.84	46723	33
28	51635	18.55	228		54407	18.79	45593	32
29	52749	18.50	214		55535	18.74	44465	31
30	53859	18.45	199		56659	18.70	43341	30
31	9.054966	18.41	9.997185		9.057781	18.65	10.942219	29
32	56071	18.36	170		58900	18.60	41100	28
33	57172	18.31	156		60016	18.55	39984	27
34	58271	18.27	141		61130	18.51	38870	26
35	59367	18.22	127		62240	18.46	37760	25
36	60460	18.17	112		63348	18.42	36652	24
37	61551	18.13	098	.24	64453	18.37	35547	23
38	62639	18.08	083	.25	65556	18.33	34444	22
39	63724	18.04	068		66655	18.28	33345	21
40	64806	17.99	053		67752	18.24	32248	20
41	9.065885	17.94	9.997039		9.068846	18.19	10.931154	19
42	66962	17.90	024		69938	18.15	30062	18
43	68036	17.86	9.997009		71027	18.10	28973	17
44	69107	17.81	9.996994		72113	18.06	27887	16
45	70176	17.77	979		73197	18.02	26803	15
46	71242	17.72	964		74278	17.97	25722	14
47	72305	17.68	949		75356	17.93	24644	13
48	73366	17.63	934		76432	17.89	23568	12
49	74424	17.59	919		77505	17.84	22495	11
50	75480	17.55	904		78576	17.80	21424	10
51	9.076533	17.50	9.996889		9.079644	17.76	10.920356	9
52	77583	17.46	874		80710	17.72	19290	8
53	78631	17.42	858		81773	17.67	18227	7
54	79676	17.38	843		82833	17.63	17167	6
55	80719	17.33	828	.25	83891	17.59	16109	5
56	81759	17.29	812	.26	84947	17.55	15053	4
57	82797	17.25	797		86000	17.51	14000	3
58	83832	17.21	782		87050	17.47	12950	2
59	84864	17.17	766	.26	88098	17.43	11902	1
60	9.085894		9.996751		9.089144		10.910856	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.085894	17.13	9.996751	.26	9.089144	17.38	10.910856	60
1	86922	17.09	735		90187	17.35	09813	59
2	87947	17.04	720		91228	17.30	08772	58
3	88970	17.00	704		92266	17.27	07734	57
4	89990	16.96	688		93302	17.22	06698	56
5	91008	16.92	673		94335	17.19	05664	55
6	92024	16.88	657		95367	17.15	04633	54
7	93037	16.84	641		96395	17.11	03605	53
8	94047	16.80	625		97422	17.07	02578	52
9	95056	16.76	610		98446	17.03	01554	51
10	96062	16.73	594	.26	99468	16.99	10.900532	50
11	9.097065	16.68	9.996578	.27	9.100487	16.95	10.899513	49
12	98066	16.65	562		01504	16.91	98496	48
13	9.099065	16.61	546		02519	16.87	97481	47
14	9.100062	16.57	530		03532	16.84	96468	46
15	01056	16.53	514		04542	16.80	95458	45
16	02048	16.49	498		05550	16.76	94450	44
17	03037	16.45	482		06556	16.72	93444	43
18	04025	16.42	465		07559	16.69	92441	42
19	05010	16.38	449		08560	16.65	91440	41
20	05992	16.34	433		09559	16.61	90441	40
21	9.106973	16.30	9.996417		9.110556	16.58	10.889444	39
22	07951	16.27	400		11551	16.54	88449	38
23	08927	16.23	384		12543	16.50	87457	37
24	09901	16.19	368		13533	16.47	86467	36
25	10873	16.16	351		14521	16.43	85479	35
26	11842	16.12	335		15507	16.39	84493	34
27	12809	16.08	318	.27	16491	16.36	83509	33
28	13774	16.05	302	.28	17472	16.32	82528	32
29	14737	16.01	285		18452	16.29	81548	31
30	15698	15.97	269		19429	16.25	80571	30
31	9.116656	15.94	9.996252		9.120404	16.22	10.879596	29
32	17613	15.90	235		21377	16.18	78623	28
33	18567	15.87	219		22348	16.15	77652	27
34	19519	15.83	202		23317	16.11	76683	26
35	20469	15.80	185		24284	16.08	75716	25
36	21417	15.76	168		25249	16.04	74751	24
37	22362	15.73	151		26211	16.01	73789	23
38	23306	15.69	134		27172	15.97	72828	22
39	24248	15.66	117		28130	15.94	71870	21
40	25187	15.62	100	.28	29087	15.91	70913	20
41	9.126125	15.59	9.996083	.29	9.130041	15.87	10.869959	19
42	27060	15.56	066		30994	15.84	69006	18
43	27993	15.52	049		31944	15.81	68056	17
44	28925	15.49	032		32893	15.77	67107	16
45	29854	15.45	9.996015		33839	15.74	66161	15
46	30781	15.42	9.995998		34784	15.71	65216	14
47	31706	15.39	980		35726	15.67	64274	13
48	32630	15.35	963		36667	15.64	63333	12
49	33551	15.32	946		37605	15.61	62395	11
50	34470	15.29	928		38542	15.58	61458	10
51	9.135387	15.25	9.995911		9.139476	15.55	10.860524	9
52	36303	15.22	894		40409	15.51	59591	8
53	37216	15.19	876		41340	15.48	58660	7
54	38128	15.16	859		42269	15.45	57731	6
55	39037	15.12	841		43196	15.42	56804	5
56	39944	15.09	823		44121	15.39	55879	4
57	40850	15.06	806		45044	15.35	54956	3
58	41754	15.03	788		45966	15.32	54034	2
59	42655	15.00	771	.29	46885	15.29	53115	1
60	9.143555		9.995753		9.147803		10.852197	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.143555	14.96	9.995753	.30	9.147803	15.26	10.852197	60
1	4453	14.93	735		8718	15.23	1282	59
2	5349	14.90	717		9.149632	15.20	10.850368	58
3	6243	14.87	699		9.150544	15.17	10.849456	57
4	7136	14.84	681		1454	15.14	8546	56
5	8026	14.81	664		2363	15.11	7637	55
6	8915	14.78	646		3269	15.08	6731	54
7	9.149802	14.75	628		4174	15.05	5826	53
8	9.150686	14.72	610		5077	15.02	4923	52
9	1569	14.69	591		5978	14.99	4022	51
10	2451	14.66	573		6877	14.96	3123	50
11	9.153330	14.63	9.995555		9.157775	14.93	10.842225	49
12	4208	14.60	537		8671	14.90	1329	48
13	5083	14.57	519	.30	9.159565	14.87	10.840435	47
14	5957	14.54	501	.31	9.160457	14.84	10.839543	46
15	6830	14.51	482		1347	14.81	8653	45
16	7700	14.48	464		2236	14.78	7764	44
17	8569	14.45	446		3123	14.75	6877	43
18	9.159435	14.42	427		4008	14.73	5992	42
19	9.160301	14.39	409		4892	14.70	5108	41
20	1164	14.36	390		5774	14.67	4226	40
21	9.162025	14.33	9.995372		9.166654	14.64	10.833346	39
22	2885	14.30	353		7532	14.61	2468	38
23	3743	14.27	334		8409	14.58	1591	37
24	4600	14.24	316		9.169284	14.55	10.830716	36
25	5454	14.22	297		9.170157	14.53	10.829843	35
26	6307	14.19	278		1029	14.50	8971	34
27	7159	14.16	260	.31	1899	14.47	8101	33
28	8008	14.13	241	.32	2767	14.44	7233	32
29	8856	14.10	222		3634	14.42	6366	31
30	9.169702	14.07	203		4499	14.39	5501	30
31	9.170547	14.05	9.995184		9.175362	14.36	10.824638	29
32	1389	14.02	165		6224	14.33	3776	28
33	2230	13.99	146		7084	14.31	2916	27
34	3070	13.96	127		7942	14.28	2058	26
35	3908	13.94	108		8799	14.25	1201	25
36	4744	13.91	089		9.179655	14.23	10.820345	24
37	5578	13.88	070		9.180508	14.20	10.819492	23
38	6411	13.86	051		1360	14.17	8640	22
39	7242	13.83	032		2211	14.15	7789	21
40	8072	13.80	9.995013		3059	14.12	6941	20
41	8900	13.77	9.994993		9.183907	14.09	10.816093	19
42	9.179726	13.74	974		4752	14.07	5248	18
43	9.180551	13.72	955		5597	14.04	4403	17
44	1374	13.69	935	.32	6439	14.02	3561	16
45	2196	13.67	916	.33	7280	13.99	2720	15
46	3016	13.64	896		8120	13.96	1880	14
47	3834	13.61	877		8958	13.94	1042	13
48	4651	13.59	857		9.189794	13.91	10.810206	12
49	5466	13.56	838		9.190629	13.89	10.809371	11
50	6280	13.53	818		1462	13.86	8538	10
51	9.187092	13.51	9.994798		9.192294	13.84	10.807706	9
52	7903	13.48	779		3124	13.81	6876	8
53	8712	13.46	759		3953	13.79	6047	7
54	9.189519	13.43	739		4780	13.76	5220	6
55	9.190325	13.41	719		5606	13.74	4394	5
56	1130	13.38	700		6430	13.71	3570	4
57	1933	13.36	680		7253	13.69	2747	3
58	2734	13.33	660		8074	13.66	1926	2
59	3534	13.30	640	.33	8894	13.64	1106	1
60	9.194332		9.994620		9.199713		10.800287	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

98° 81°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.194332	13.28	9.994620	.33	9.199713	13.61	10.800287	60
1	5129	13.26	600	.33	9.200529	13.59	10.799471	59
2	5925	13.23	580	.33	1345	13.56	8655	58
3	6719	13.21	560	.34	2159	13.54	7841	57
4	7511	13.18	540		2971	13.52	7029	56
5	8302	13.16	519		3782	13.49	6218	55
6	9091	13.13	499		4592	13.47	5408	54
7	9.199879	13.11	479		5400	13.45	4600	53
8	9.200666	13.08	459		6207	13.42	3793	52
9	1451	13.06	438		7013	13.40	2987	51
10	2234	13.04	418		7817	13.38	2183	50
11	9.203017	13.01	9.994397		8619	13.35	1381	49
12	3797	12.99	377		9.209420	13.33	10.790580	48
13	4577	12.96	357		9.210220	13.31	10.789780	47
14	5354	12.94	336		1018	13.28	8982	46
15	6131	12.92	316		1815	13.26	8185	45
16	6906	12.89	295	.34	2611	13.24	7389	44
17	7679	12.87	274	.35	3405	13.21	6595	43
18	8452	12.85	254		4198	13.19	5802	42
19	9222	12.82	233		4989	13.17	5011	41
20	9.209992	12.80	212		5780	13.15	4220	40
21	9.210760	12.78	9.994191		9.216568	13.12	10.783432	39
22	1526	12.75	171		7356	13.10	2644	38
23	2291	12.73	150		8142	13.08	1858	37
24	3055	12.71	129		8926	13.05	1074	36
25	3818	12.68	108		9.219710	13.03	10.780290	35
26	4579	12.66	087		9.220492	13.01	10.779508	34
27	5338	12.64	066		1272	12.99	8728	33
28	6097	12.61	045		2052	12.97	7948	32
29	6854	12.59	024		2830	12.94	7170	31
30	7609	12.57	9.994003		3606	12.92	6394	30
31	9.218363	12.55	9.993981		9.224382	12.90	10.775618	29
32	9116	12.53	960		5156	12.88	4844	28
33	9.219868	12.50	939		5929	12.86	4071	27
34	9.220618	12.48	918	.35	6700	12.84	3300	26
35	1367	12.46	896	.36	7471	12.81	2529	25
36	2115	12.44	875		8239	12.79	1761	24
37	2861	12.42	854		9007	12.77	0993	23
38	3606	12.39	832		9.229773	12.75	10.770227	22
39	4349	12.37	811		9.230539	12.73	10.769461	21
40	5092	12.35	789		1302	12.71	8698	20
41	9.225833	12.33	9.993768		9.232065	12.69	10.767935	19
42	6573	12.31	746		2826	12.67	7174	18
43	7311	12.28	725		3586	12.65	6414	17
44	8048	12.26	703		4345	12.62	5655	16
45	8784	12.24	681		5103	12.60	4897	15
46	9.229518	12.22	660		5859	12.58	4141	14
47	9.230252	12.20	638		6614	12.56	3386	13
48	0984	12.18	616	.36	7368	12.54	2632	12
49	1714	12.16	594	.37	8120	12.52	1880	11
50	2444	12.14	572		8872	12.50	1128	10
51	9.233172	12.12	9.993550		9.239622	12.48	10.760378	9
52	3899	12.09	528		9.240371	12.46	10.759629	8
53	4625	12.07	506		1118	12.44	8882	7
54	5349	12.05	484		1865	12.42	8135	6
55	6073	12.03	462		2610	12.40	7390	5
56	6795	12.01	440		3354	12.38	6646	4
57	7515	11.99	418		4097	12.36	5903	3
58	8235	11.97	396		4839	12.34	5161	2
59	8953	11.95	374	.37	5579	12.32	4421	1
60	9.239670		9.993351		9.246319		10.753681	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.239670	11.93	9.993351	·37	9.246319	12.30	10.753681	60
1	9.240386	11.91	329		7057	12.28	2943	59
2	1101	11.89	307		7794	12.26	2206	58
3	1814	11.87	285		8530	12.24	1470	57
4	2526	11.85	262		9264	12.22	0736	56
5	3237	11.83	240	·37	9.249998	12.20	10.750002	55
6	3947	11.81	217	·38	9.250730	12.18	10.749270	54
7	4656	11.79	195		1461	12.17	8539	53
8	5363	11.77	172		2191	12.15	7809	52
9	6069	11.75	149		2920	12.13	7080	51
10	6775	11.73	127		3648	12.11	6352	50
11	9.247478	11.71	9.993104		9.254374	12.09	10.745626	49
12	8181	11.69	081		5100	12.07	4900	48
13	8883	11.67	059		5824	12.05	4176	47
14	9.249583	11.65	036		6547	12.03	3453	46
15	9.250282	11.63	9.993013		7269	12.01	2731	45
16	0980	11.61	9.992990		7990	12.00	2010	44
17	1677	11.59	967		8710	11.98	1290	43
18	2373	11.58	944		9.259429	11.96	10.740571	42
19	3067	11.56	921		9.260146	11.94	10.739854	41
20	3761	11.54	898		0863	11.92	9137	40
21	9.254453	11.52	9.992875		9.261578	11.90	10.738422	39
22	5144	11.50	852	·38	2292	11.89	7708	38
23	5834	11.48	829	·39	3005	11.87	6995	37
24	6523	11.46	806		3717	11.85	6283	36
25	7211	11.44	783		4428	11.83	5572	35
26	7898	11.42	759		5138	11.81	4862	34
27	8583	11.41	736		5847	11.79	4153	33
28	9268	11.39	713		6555	11.78	3445	32
29	9.259951	11.37	690		7261	11.76	2739	31
30	9.260633	11.35	666		7967	11.74	2033	30
31	1314	11.33	9.992643		8671	11.72	1329	29
32	1994	11.31	619		9.269375	11.70	10.730625	28
33	2673	11.30	596		9.270077	11.69	10.729923	27
34	3351	11.28	572		0779	11.67	9221	26
35	4027	11.26	549		1479	11.65	8521	25
36	4703	11.24	525		2178	11.64	7822	24
37	5377	11.22	501	·39	2876	11.62	7124	23
38	6051	11.20	478	·40	3573	11.60	6427	22
39	6723	11.19	454		4269	11.58	5731	21
40	7395	11.17	430		4964	11.57	5036	20
41	9.268065	11.15	9.992406		9.275658	11.55	10.724342	19
42	8734	11.13	382		6351	11.53	3649	18
43	9.269402	11.12	359		7043	11.51	2957	17
44	9.270069	11.10	335		7734	11.50	2266	16
45	0735	11.08	311		8424	11.48	1576	15
46	1400	11.06	287		9113	11.46	0887	14
47	2064	11.05	263		9.279801	11.45	10.720199	13
48	2726	11.03	239		9.280488	11.43	10.719512	12
49	3388	11.01	214		1174	11.41	8826	11
50	4049	10.99	190		1858	11.40	8142	10
51	9.274708	10.98	9.992166		9.282542	11.38	10.717458	9
52	5367	10.96	142	·40	3225	11.36	6775	8
53	6024	10.94	117	·41	3907	11.35	6093	7
54	6681	10.92	093		4588	11.33	5412	6
55	7337	10.91	069		5268	11.31	4732	5
56	7991	10.89	044		5947	11.30	4053	4
57	8644	10.87	9.992020		6624	11.28	3376	3
58	9297	10.86	9.991996		7301	11.26	2699	2
59	9.279948	10.84	971	·41	7977	11.25	2023	1
60	9.280599		9.991947		9.288652		10.711348	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.280599	10.82	9.991947	.41	9.288652	11.23	10.711348	60
1	1248	10.81	922		9326	11.22	0674	59
2	1897	10.79	897		9.289999	11.20	10.710001	58
3	2544	10.77	873		9.290671	11.18	10.709329	57
4	3190	10.76	848		1342	11.17	8658	56
5	3836	10.74	823		2013	11.15	7987	55
6	4480	10.72	799	.41	2682	11.14	7318	54
7	5124	10.71	774	.42	3350	11.12	6650	53
8	5766	10.69	749		4017	11.11	5983	52
9	6408	10.67	724		4684	11.09	5316	51
10	7048	10.66	699		5349	11.07	4651	50
11	9.287687	10.64	9.991674		9.296013	11.06	10.703987	49
12	8326	10.63	649		6677	11.04	3323	48
13	8964	10.61	624		7339	11.03	2661	47
14	9.289600	10.59	599		8001	11.01	1999	46
15	9.290236	10.58	574		8662	11.00	1338	45
16	0870	10.56	549		9322	10.98	0678	44
17	1504	10.54	524		9.299980	10.96	10.700020	43
18	2137	10.53	498		9.300638	10.95	10.699362	42
19	2768	10.51	473		1295	10.93	8705	41
20	3399	10.50	448		1951	10.92	8049	40
21	9.294029	10.48	9.991422		9.302607	10.90	10.697393	39
22	4658	10.46	397	.42	3261	10.89	6739	38
23	5286	10.45	372	.43	3914	10.87	6086	37
24	5913	10.43	346		4567	10.86	5433	36
25	6539	10.42	321		5218	10.84	4782	35
26	7164	10.40	295		5869	10.83	4131	34
27	7788	10.39	270		6519	10.81	3481	33
28	8412	10.37	244		7168	10.80	2832	32
29	9034	10.36	218		7815	10.78	2185	31
30	9.299655	10.34	193		8463	10.77	1537	30
31	9.300276	10.32	9.991167		9109	10.75	0891	29
32	0895	10.31	141		9.309754	10.74	10.690246	28
33	1514	10.29	115		9.310398	10.73	10.689602	27
34	2132	10.28	090		1042	10.71	8958	26
35	2748	10.26	064		1685	10.70	8315	25
36	3364	10.25	038		2327	10.68	7673	24
37	3979	10.23	9.991012		2967	10.67	7033	23
38	4593	10.22	9.990986		3608	10.65	6392	22
39	5207	10.20	960	.43	4247	10.64	5753	21
40	5819	10.19	934	.44	4885	10.62	5115	20
41	9.306430	10.17	9.990908		9.315523	10.61	10.684477	19
42	7041	10.16	882		6159	10.60	3841	18
43	7650	10.14	855		6795	10.58	3205	17
44	8259	10.13	829		7430	10.57	2570	16
45	8867	10.11	803		8064	10.55	1936	15
46	9.309474	10.10	777		8697	10.54	1303	14
47	9.310080	10.08	750		9329	10.53	0671	13
48	0685	10.07	724		9.319961	10.51	10.680039	12
49	1289	10.06	697		9.320592	10.50	10.679408	11
50	1893	10.04	671		1222	10.48	8778	10
51	9.312495	10.03	9.990644		9.321851	10.47	10.678149	9
52	3097	10.01	618		2479	10.45	7521	8
53	3698	10.00	591		3106	10.44	6894	7
54	4297	9.98	565		3733	10.43	6267	6
55	4897	9.97	538	.44	4358	10.41	5642	5
56	5495	9.96	511	.45	4983	10.40	5017	4
57	6092	9.94	485		5607	10.39	4393	3
58	6689	9.93	458		6231	10.37	3769	2
59	7284	9.91	431	.45	6853	10.36	3147	1
60	9.317879		9.990404		9.327475		10.672525	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.317879	9.90	9.990404	.45	9.327474	10.35	10.672526	60
1	8473	9.88	378		8095	10.33	1905	59
2	9066	9.87	351		8715	10.32	1285	58
3	9.319658	9.86	324		9334	10.30	0666	57
4	9.320249	9.84	297		9.329953	10.29	10.670047	56
5	0840	9.83	270		9.330570	10.28	10.669430	55
6	1430	9.82	243		1187	10.26	8813	54
7	2019	9.80	215		1803	10.25	8197	53
8	2607	9.79	188		2418	10.24	7582	52
9	3194	9.77	161		3033	10.23	6967	51
10	3780	9.76	134	.45	3646	10.21	6354	50
11	9.324366	9.75	9.990107	.46	9.334259	10.20	10.665741	49
12	4950	9.73	079		4871	10.19	5129	48
13	5534	9.72	052		5482	10.17	4518	47
14	6117	9.70	9.990025		6093	10.16	3907	46
15	6700	9.69	9.989997		6702	10.15	3298	45
16	7281	9.68	970		7311	10.13	2689	44
17	7862	9.66	942		7919	10.12	2081	43
18	8442	9.65	915		8527	10.11	1473	42
19	9021	9.64	887		9133	10.10	0867	41
20	9.329599	9.62	860		9.339739	10.08	10.660261	40
21	9.330176	9.61	9.989832		9.340344	10.07	10.659656	39
22	0753	9.60	804		0948	10.06	9052	38
23	1329	9.58	777	.46	1552	10.04	8448	37
24	1903	9.57	749	.47	2155	10.03	7845	36
25	2478	9.56	721		2757	10.02	7243	35
26	3051	9.54	693		3358	10.01	6642	34
27	3624	9.53	665		3958	9.99	6042	33
28	4195	9.52	637		4558	9.98	5442	32
29	4766	9.50	609		5157	9.97	4843	31
30	5337	9.49	582		5755	9.96	4245	30
31	9.335906	9.48	9.989553		9.346353	9.94	10.653647	29
32	6475	9.46	525		6949	9.93	3051	28
33	7043	9.45	497		7545	9.92	2455	27
34	7610	9.44	469		8141	9.91	1859	26
35	8176	9.43	441		8735	9.90	1265	25
36	8742	9.41	413		9329	9.88	0671	24
37	9306	9.40	384		9.349922	9.87	10.650078	23
38	9.339871	9.39	356		9.350514	9.86	10.649486	22
39	9.340434	9.37	328		1106	9.85	8894	21
40	0996	9.36	300		1697	9.83	8303	20
41	9.341558	9.35	9.989271		9.352287	9.82	10.647713	19
42	2119	9.34	243		2876	9.81	7124	18
43	2679	9.32	214		3465	9.80	6535	17
44	3239	9.31	186		4053	9.79	5947	16
45	3797	9.30	157	.47	4640	9.77	5360	15
46	4355	9.29	128	.48	5227	9.76	4773	14
47	4912	9.27	100		5813	9.75	4187	13
48	5469	9.26	071		6398	9.74	3602	12
49	6024	9.25	042		6982	9.73	3018	11
50	6579	9.24	9.989014		7566	9.71	2434	10
51	9.347134	9.22	9.988985		9.358149	9.70	10.641851	9
52	7687	9.21	956		8731	9.69	1269	8
53	8240	9.20	927		9313	9.68	0687	7
54	8792	9.19	898		9.359893	9.67	10.640107	6
55	9343	9.17	869		9.360474	9.66	10.639526	5
56	9.349893	9.16	840	.48	1053	9.65	8947	4
57	9.350443	9.15	811	.49	1632	9.63	8368	3
58	0992	9.14	782	.49	2210	9.62	7790	2
59	1540	9.13	753	.49	2787	9.61	7213	1
60	9.352088		9.988724		9.363364		10.636636	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff.1″	Tang.	Diff. 1″	Cotang.	
0	9.352088	9.11	9.988724	.49	9.363364	9.60	10.636636	60
1	2635	9.10	8695		3940	9.59	6060	59
2	3181	9.09	8666		4515	9.58	5485	58
3	3726	9.08	8636		5090	9.57	4910	57
4	4271	9.07	8607		5664	9.55	4336	56
5	4815	9.05	8578		6237	9.54	3763	55
6	5358	9.04	8548		6810	9.53	3190	54
7	5901	9.03	8519		7382	9.52	2618	53
8	6443	9.02	8489		7953	9.51	2047	52
9	6984	9.01	8460		8524	9.50	1476	51
10	7524	8.99	8430		9094	9.49	0906	50
11	9.358064	8.98	9.988401		9.369663	9.48	10.630337	49
12	8603	8.97	8371		9.370232	9.46	10.629768	48
13	9141	8.96	8342	.49	0799	9.45	9201	47
14	9.359678	8.95	8312	.50	1367	9.44	8633	46
15	9.360215	8.93	8282		1933	9.43	8067	45
16	0752	8.92	8252		2499	9.42	7501	44
17	1287	8.91	8223		3064	9.41	6936	43
18	1822	8.90	8193		3629	9.40	6371	42
19	2356	8.89	8163		4193	9.39	5807	41
20	2889	8.88	8133		4756	9.38	5244	40
21	9.363422	8.87	9.988103		9.375319	9.37	10.624681	39
22	3954	8.85	8073		5881	9.35	4119	38
23	4485	8.84	8043		6442	9.34	3558	37
24	5016	8.83	8013		7003	9.33	2997	36
25	5546	8.82	7983		7563	9.32	2437	35
26	6075	8.81	7953		8122	9.31	1878	34
27	6604	8.80	7922		8681	9.30	1319	33
28	7131	8.79	7892		9239	9.29	0761	32
29	7659	8.78	7862	.50	9797	9.28	0203	31
30	8185	8.76	7832	.51	9.380354	9.27	10.619646	30
31	9.368711	8.75	9.987801		9.380910	9.26	10.619090	29
32	9236	8.74	7771		1466	9.25	8534	28
33	9.369761	8.73	7740		2020	9.24	7980	27
34	9.370285	8.72	7710		2575	9.23	7425	26
35	0808	8.71	7679		3129	9.22	6871	25
36	1330	8.70	7649		3682	9.21	6318	24
37	1852	8.69	7618		4234	9.20	5766	23
38	2373	8.67	7588		4786	9.19	5214	22
39	2894	8.66	7557		5337	9.18	4663	21
40	3414	8.65	7526		5888	9.17	4112	20
41	9.373933	8.64	9.987496		9.386438	9.15	10.613562	19
42	4452	8.63	7465		6987	9.14	3013	18
43	4970	8.62	7434	.51	7536	9.13	2464	17
44	5487	8.61	7403	.52	8084	9.12	1916	16
45	6003	8.60	7372		8631	9.11	1369	15
46	6519	8.59	7341		9178	9.10	0822	14
47	7035	8.58	7310		9.389724	9.09	10.610276	13
48	7549	8.57	7279		9.390270	9.08	10.609730	12
49	8063	8.56	7248		0815	9.07	9185	11
50	8577	8.54	7217		1360	9.06	8640	10
51	9089	8.53	9.987186		9.391903	9.05	10.608097	9
52	9.379601	8.52	7155		2447	9.04	7553	8
53	9.380113	8.51	7124		2989	9.03	7011	7
54	0624	8.50	7092		3531	9.02	6469	6
55	1134	8.49	7061		4073	9.01	5927	5
56	1643	8.48	7030		4614	9.00	5386	4
57	2152	8.47	6998		5154	8.99	4846	3
58	2661	8.46	6967		5694	8.98	4306	2
59	3168	8.45	6936	.52	6233	8.97	3767	1
60	9.383675		9.986904		9.396771		10.603229	0
	Cosine.	Diff. 1″	Sine.	Diff.1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.383675	8.44	9.986904	.52	9.396771	8.96	10.603229	60
1	4182	8.43	6873	.53	7309	8.96	2691	59
2	4687	8.42	6841		7846	8.95	2154	58
3	5192	8.41	6809		8383	8.94	1617	57
4	5697	8.40	6778		8919	8.93	1081	56
5	6201	8.39	6746		9455	8.92	0545	55
6	6704	8.38	6714		9.399990	8.91	10.600010	54
7	7207	8.37	6683		9.400524	8.90	10.599476	53
8	7709	8.36	6651		1058	8.89	8942	52
9	8210	8.35	6619		1591	8.88	8409	51
10	8711	8.34	6587		2124	8.87	7876	50
11	9211	8.33	9.986555		9.402656	8.86	10.597344	49
12	9.389711	8.32	6523		3187	8.85	6813	48
13	9.390210	8.31	6491		3718	8.84	6282	47
14	0708	8.30	6459		4249	8.83	5751	46
15	1206	8.28	6427		4778	8.82	5222	45
16	1703	8.27	6395	.53	5308	8.81	4692	44
17	2199	8.26	6363	.54	5836	8.80	4164	43
18	2695	8.25	6331		6364	8.79	3636	42
19	3191	8.24	6299		6892	8.78	3108	41
20	3685	8.23	6266		7419	8.77	2581	40
21	9.394179	8.22	9.986234		9.407945	8.76	10.592055	39
22	4673	8.21	6202		8471	8.75	1529	38
23	5166	8.20	6169		8997	8.74	1003	37
24	5658	8.20	6137		9.409521	8.74	10.590479	36
25	6150	8.18	6104		9.410045	8.73	10.589955	35
26	6641	8.17	6072		0569	8.72	9431	34
27	7132	8.17	6039		1092	8.71	8908	33
28	7621	8.16	6007		1615	8.70	8385	32
29	8111	8.15	5974		2137	8.69	7863	31
30	8600	8.14	5942	.54	2658	8.68	7342	30
31	9088	8.13	9.985909	.55	9.413179	8.67	10.586821	29
32	9.399575	8.12	5876		3699	8.66	6301	28
33	9.400062	8.11	5843		4219	8.65	5781	27
34	0549	8.10	5811		4738	8.64	5262	26
35	1035	8.09	5778		5257	8.64	4743	25
36	1520	8.08	5745		5775	8.63	4225	24
37	2005	8.07	5712		6293	8.62	3707	23
38	2489	8.06	5679		6810	8.61	3190	22
39	2972	8.05	5646		7326	8.60	2674	21
40	3455	8.04	5613		7842	8.59	2158	20
41	9.403938	8.03	9.985580		9.418358	8.58	10.581642	19
42	4420	8.02	5547		8873	8.57	1127	18
43	4901	8.01	5514		9387	8.56	0613	17
44	5382	8.00	5480		9.419901	8.55	10.580099	16
45	5862	7.99	5447	.55	9.420415	8.55	10.579585	15
46	6341	7.98	5414	.56	0927	8.54	9073	14
47	6820	7.97	5380		1440	8.53	8560	13
48	7299	7.96	5347		1952	8.52	8048	12
49	7777	7.95	5314		2463	8.51	7537	11
50	8254	7.94	5280		2974	8.50	7026	10
51	9.408731	7.94	9.985247		9.423484	8.49	10.576516	9
52	9207	7.93	5213		3993	8.48	6007	8
53	9.409682	7.92	5180		4503	8.48	5497	7
54	9.410157	7.91	5146		5011	8.47	4989	6
55	0632	7.90	5113		5519	8.46	4481	5
56	1106	7.89	5079		6027	8.45	3973	4
57	1579	7.88	5045		6534	8.44	3466	3
58	2052	7.87	5011		7041	8.43	2959	2
59	2524	7.86	4978	.56	7547	8.43	2453	1
60	9.412996		9.984944		9.428052		10.571948	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9·412996	7·85	9·984944	·57	9·428052	8·42	10·571948	60
1	3467	7·84	4910		8557	8·41	1443	59
2	3938	7·83	4876		9062	8·40	0938	58
3	4408	7·83	4842		9·429566	8·39	10·570434	57
4	4878	7·82	4808		9·430070	8·38	10·569930	56
5	5347	7·81	4774		0573	8·38	9427	55
6	5815	7·80	4740		1075	8·37	8925	54
7	6283	7·79	4706		1577	8·36	8423	53
8	6751	7·78	4672		2079	8·35	7921	52
9	7217	7·77	4637		2580	8·34	7420	51
10	7684	7·76	4603		3080	8·33	6920	50
11	9·418150	7·75	9·984569		9·433580	8·32	10·566420	49
12	8615	7·74	4535		4080	8·32	5920	48
13	9079	7·73	4500		4579	8·31	5421	47
14	9·419544	7·73	4466	·57	5078	8·30	4922	46
15	9·420007	7·72	4432	·58	5576	8·29	4424	45
16	0470	7·71	4397		6073	8·28	3927	44
17	0933	7·70	4363		6570	8·28	3430	43
18	1395	7·69	4328		7067	8·27	2933	42
19	1857	7·68	4294		7563	8·26	2437	41
20	2318	7·67	4259		8059	8·25	1941	40
21	9·422778	7·67	9·984224		9·438554	8·24	10·561446	39
22	3238	7·66	4190		9048	8·23	0952	38
23	3697	7·65	4155		9·439543	8·23	10·560457	37
24	4156	7·64	4120		9·440036	8·22	10·559964	36
25	4615	7·63	4085		0529	8·21	9471	35
26	5073	7·62	4050		1022	8·20	8978	34
27	5530	7·61	4015		1514	8·19	8486	33
28	5987	7·60	3981		2006	8·19	7994	32
29	6443	7·60	3946		2497	8·18	7503	31
30	6899	7·59	3911		2988	8·17	7012	30
31	9·427354	7·58	9·983875	·58	9·443479	8·16	10·556521	29
32	7809	7·57	3840	·59	3968	8·16	6032	28
33	8263	7·56	3805		4458	8·15	5542	27
34	8717	7·55	3770		4947	8·14	5053	26
35	9170	7·54	3735		5435	8·13	4565	25
36	9·429623	7·54	3700		5923	8·12	4077	24
37	9·430075	7·53	3664		6411	8·12	3589	23
38	0527	7·52	3629		6898	8·11	3102	22
39	0978	7·51	3594		7384	8·10	2616	21
40	1429	7·50	3558		7870	8·09	2130	20
41	9·431879	7·49	9·983523		9·448356	8·09	10·551644	19
42	2329	7·49	3487		8841	8·08	1159	18
43	2778	7·48	3452		9326	8·07	0674	17
44	3226	7·47	3416		9·449810	8·06	10·550190	16
45	3675	7·46	3381		9·450294	8·06	10·549706	15
46	4122	7·45	3345		0777	8·05	9223	14
47	4569	7·44	3309	·59	1260	8·04	8740	13
48	5016	7·44	3273	·60	1743	8·03	8257	12
49	5462	7·43	3238		2225	8·02	7775	11
50	5908	7·42	3202		2706	8·02	7294	10
51	9·436353	7·41	9·983166		9·453187	8·01	10·546813	9
52	6798	7·40	3130		3668	8·00	6332	8
53	7242	7·40	3094		4148	7·99	5852	7
54	7686	7·39	3058		4628	7·99	5372	6
55	8129	7·38	3022		5107	7·98	4893	5
56	8572	7·37	2986		5586	7·97	4414	4
57	9014	7·36	2950		6064	7·96	3936	3
58	9456	7·36	2914		6542	7·96	3458	2
59	9·439897	7·35	2878	·60	7019	7·95	2981	1
60	9·440338		9·982842		9·457496		10·542504	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.440338	7.34	9.982842	.60	9.457496	7.94	10.542504	60
1	0778	7.33	2805	.60	7973	7.93	2027	59
2	1218	7.32	2769	.61	8449	7.93	1551	58
3	1658	7.31	2733		8925	7.92	1075	57
4	2096	7.31	2696		9400	7.91	0600	56
5	2535	7.30	2660		9.459875	7.90	10.540125	55
6	2973	7.29	2624		9.460349	7.90	10.539651	54
7	3410	7.28	2587		0823	7.89	9177	53
8	3847	7.27	2551		1297	7.88	8703	52
9	4284	7.27	2514		1770	7.88	8230	51
10	4720	7.26	2477		2242	7.87	7758	50
11	9.445155	7.25	9.982441		9.462714	7.86	10.537286	49
12	5590	7.24	2404		3186	7.85	6814	48
13	6025	7.23	2367		3658	7.85	6342	47
14	6459	7.23	2331		4129	7.84	5871	46
15	6893	7.22	2294		4599	7.83	5401	45
16	7326	7.21	2257	.61	5069	7.83	4931	44
17	7759	7.20	2220	.62	5539	7.82	4461	43
18	8191	7.20	2183		6008	7.81	3992	42
19	8623	7.19	2146		6476	7.80	3524	41
20	9054	7.18	2109		6945	7.80	3055	40
21	9485	7.17	9.982072		9.467413	7.79	10.532587	39
22	9.449915	7.16	2035		7880	7.78	2120	38
23	9.450345	7.16	1998		8347	7.78	1653	37
24	0775	7.15	1961		8814	7.77	1186	36
25	1204	7.14	1924		9280	7.76	0720	35
26	1632	7.13	1886		9.469746	7.75	10.530254	34
27	2060	7.13	1849		9.470211	7.75	10.529789	33
28	2488	7.12	1812		0676	7.74	9324	32
29	2915	7.11	1774		1141	7.73	8859	31
30	3342	7.10	1737	.62	1605	7.73	8395	30
31	9.453768	7.10	9.981699	.63	9.472068	7.72	10.527932	29
32	4194	7.09	1662		2532	7.71	7468	28
33	4619	7.08	1625		2995	7.71	7005	27
34	5044	7.07	1587		3457	7.70	6543	26
35	5469	7.07	1549		3919	7.69	6081	25
36	5893	7.06	1512		4381	7.69	5619	24
37	6316	7.05	1474		4842	7.68	5158	23
38	6739	7.04	1436		5303	7.67	4697	22
39	7162	7.04	1399		5763	7.67	4237	21
40	7584	7.03	1361		6223	7.66	3777	20
41	9.458006	7.02	9.981323		9.476683	7.65	10.523317	19
42	8427	7.01	1285		7142	7.65	2858	18
43	8848	7.01	1247		7601	7.64	2399	17
44	9268	7.00	1209		8059	7.63	1941	16
45	9.459688	6.99	1171	.63	8517	7.63	1483	15
46	9.460108	6.98	1133	.64	8975	7.62	1025	14
47	0527	6.98	1095		9432	7.61	0568	13
48	0946	6.97	1057		9.479889	7.61	10.520111	12
49	1364	6.96	1019		9.480345	7.60	10.519655	11
50	1782	6.95	0981		0801	7.59	9199	10
51	9.462199	6.95	9.980942		9.481257	7.59	10.518743	9
52	2616	6.94	0904		1712	7.58	8288	8
53	3032	6.93	0866		2167	7.57	7833	7
54	3448	6.93	0827		2621	7.57	7379	6
55	3864	6.92	0789		3075	7.56	6925	5
56	4279	6.91	0750		3529	7.55	6471	4
57	4694	6.90	0712		3982	7.55	6018	3
58	5108	6.90	0673		4435	7.54	5565	2
59	5522	6.89	0635	.64	4887	7.53	5113	1
60	9.465935		9.980596		9.485339		10.514661	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.465935	6.88	9.980596	.64	9.485339	7.53	10.514661	60
1	6348	6.88	0558	.64	5791	7.52	4209	59
2	6761	6.87	0519	.65	6242	7.51	3758	58
3	7173	6.86	0480		6693	7.51	3307	57
4	7585	6.85	0442		7143	7.50	2857	56
5	7996	6.85	0403		7593	7.49	2407	55
6	8407	6.84	0364		8043	7.49	1957	54
7	8817	6.83	0325		8492	7.48	1508	53
8	9227	6.83	0286		8941	7.47	1059	52
9	9.469637	6.82	0247		9390	7.47	0610	51
10	9.470046	6.81	0208		9.489838	7.46	10.510162	50
11	0455	6.80	9.980169		9.490286	7.46	10.509714	49
12	0863	6.80	0130		0733	7.45	9267	48
13	1271	6.79	0091		1180	7.44	8820	47
14	1679	6.78	0052		1627	7.44	8373	46
15	2086	6.78	9.980012		2073	7.43	7927	45
16	2492	6.77	9.979973	.65	2519	7.43	7481	44
17	2898	6.76	9934	.66	2965	7.42	7035	43
18	3304	6.76	9895		3410	7.41	6590	42
19	3710	6.75	9855		3854	7.40	6146	41
20	4115	6.74	9816		4299	7.40	5701	40
21	9.474519	6.74	9.979776		9.494743	7.39	10.505257	39
22	4923	6.73	9737		5186	7.39	4814	38
23	5327	6.72	9697		5630	7.38	4370	37
24	5730	6.72	9658		6073	7.37	3927	36
25	6133	6.71	9618		6515	7.37	3485	35
26	6536	6.70	9579		6957	7.36	3043	34
27	6938	6.69	9539		7399	7.36	2601	33
28	7340	6.69	9499		7841	7.35	2159	32
29	7741	6.68	9459		8282	7.34	1718	31
30	8142	6.67	9420		8722	7.34	1278	30
31	9.478542	6.67	9.979380		9163	7.33	0837	29
32	8942	6.66	9340	.66	9.499603	7.33	10.500397	28
33	9342	6.65	9300	.67	9.500042	7.32	10.499958	27
34	9.479741	6.65	9260		0481	7.31	9519	26
35	9.480140	6.64	9220		0920	7.31	9080	25
36	0539	6.63	9180		1359	7.30	8641	24
37	0937	6.63	9140		1797	7.30	8203	23
38	1334	6.62	9100		2235	7.29	7765	22
39	1731	6.61	9059		2672	7.28	7328	21
40	2128	6.61	9019		3109	7.28	6891	20
41	9.482525	6.60	9.978979		9.503546	7.27	10.496454	19
42	2921	6.59	8939		3982	7.27	6018	18
43	3316	6.59	8898		4418	7.26	5582	17
44	3712	6.58	8858		4854	7.25	5146	16
45	4107	6.57	8817		5289	7.25	4711	15
46	4501	6.57	8777		5724	7.24	4276	14
47	4895	6.56	8736	.67	6159	7.24	3841	13
48	5289	6.55	8696	.68	6593	7.23	3407	12
49	5682	6.55	8655		7027	7.22	2973	11
50	6075	6.54	8615		7460	7.22	2540	10
51	9.486467	6.53	9.978574		9.507893	7.21	10.492107	9
52	6860	6.53	8533		8326	7.21	1674	8
53	7251	6.52	8493		8759	7.20	1241	7
54	7643	6.51	8452		9191	7.19	0809	6
55	8034	6.51	8411		9.509622	7.19	10.490378	5
56	8424	6.50	8370		9.510054	7.18	10.489946	4
57	8814	6.50	8329		0485	7.18	9515	3
58	9204	6.49	8288		0916	7.17	9084	2
59	9593	6.48	8247	.68	1346	7.17	8654	1
60	9.489982		9.978206		9.511776		10.488224	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.489982	6.48	9.978206	.68	9.511776	7.16	10.488224	60
1	9.490371	6.47	8165		2206	7.16	7794	59
2	0759	6.46	8124	.68	2635	7.15	7365	58
3	1147	6.46	8083	.69	3064	7.14	6936	57
4	1535	6.45	8042		3493	7.14	6507	56
5	1922	6.44	8001		3921	7.13	6079	55
6	2308	6.44	7959		4349	7.13	5651	54
7	2695	6.43	7918		4777	7.12	5223	53
8	3081	6.42	7877		5204	7.12	4796	52
9	3466	6.42	7835		5631	7.11	4369	51
10	3851	6.41	7794		6057	7.10	3943	50
11	9.494236	6.41	9.977752		9.516484	7.10	10.483516	49
12	4621	6.40	7711		6910	7.09	3090	48
13	5005	6.39	7669		7335	7.09	2665	47
14	5388	6.39	7628		7761	7.08	2239	46
15	5772	6.38	7586	.69	8185	7.08	1815	45
16	6154	6.37	7544	.70	8610	7.07	1390	44
17	6537	6.37	7503		9034	7.06	0966	43
18	6919	6.36	7461		9458	7.06	0542	42
19	7301	6.36	7419		9.519882	7.05	10.480118	41
20	7682	6.35	7377		9.520305	7.05	10.479695	40
21	9.498064	6.34	9.977335		0728	7.04	9272	39
22	8444	6.34	7293		1151	7.04	8849	38
23	8825	6.33	7251		1573	7.03	8427	37
24	9204	6.32	7209		1995	7.03	8005	36
25	9584	6.32	7167		2417	7.02	7583	35
26	9.499963	6.31	7125		2838	7.02	7162	34
27	9.500342	6.31	7083		3259	7.01	6741	33
28	0721	6.30	7041		3680	7.01	6320	32
29	1099	6.29	6999		4100	7.00	5900	31
30	1476	6.29	6957		4520	6.99	5480	30
31	9.501854	6.28	9.976914	.70	9.524939	6.99	10.475061	29
32	2231	6.28	6872	.71	5359	6.98	4641	28
33	2607	6.27	6830		5778	6.98	4222	27
34	2984	6.26	6787		6197	6.97	3803	26
35	3360	6.26	6745		6615	6.97	3385	25
36	3735	6.25	6702		7033	6.96	2967	24
37	4110	6.25	6660		7451	6.96	2549	23
38	4485	6.24	6617		7868	6.95	2132	22
39	4860	6.23	6574		8285	6.95	1715	21
40	5234	6.23	6532		8702	6.94	1298	20
41	9.505608	6.22	9.976489		9.529119	6.93	10.470881	19
42	5981	6.22	6446		9535	6.93	0465	18
43	6354	6.21	6404		9.529950	6.93	10.470050	17
44	6727	6.20	6361		9.530366	6.92	10.469634	16
45	7099	6.20	6318		0781	6.91	9219	15
46	7471	6.19	6275	.71	1196	6.91	8804	14
47	7843	6.19	6232	.72	1611	6.90	8389	13
48	8214	6.18	6189		2025	6.90	7975	12
49	8585	6.18	6146		2439	6.89	7561	11
50	8956	6.17	6103		2853	6.89	7147	10
51	9326	6.16	9.976060		9.533266	6.88	10.466734	9
52	9.509696	6.16	6017		3679	6.88	6321	8
53	9.510065	6.15	5974		4092	6.87	5908	7
54	0434	6.15	5930		4504	6.87	5496	6
55	0803	6.14	5887		4916	6.86	5084	5
56	1172	6.13	5844		5328	6.86	4672	4
57	1540	6.13	5800		5739	6.85	4261	3
58	1907	6.12	5757		6150	6.85	3850	2
59	2275	6.12	5714	.72	6561	6.84	3439	1
60	9.512642		9.975670		9.536972		10.463028	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

108° 71°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.512642	6.11	9.975670	·73	9.536972	6.84	10.463028	60
1	3009	6.11	5627		7382	6.83	2618	59
2	3375	6.10	5583		7792	6.83	2208	58
3	3741	6.09	5539		8202	6.82	1798	57
4	4107	6.09	5496		8611	6.82	1389	56
5	4472	6.08	5452		9020	6.81	0980	55
6	4837	6.08	5408		9429	6.81	0571	54
7	5202	6.07	5365		9.539837	6.80	10.460163	53
8	5566	6.07	5321		9.540245	6.80	10.459755	52
9	5930	6.06	5277		0653	6.79	9347	51
10	6294	6.05	5233		1061	6.79	8939	50
11	9.516657	6.05	9.975189		9.541468	6.78	10.458532	49
12	7020	6.04	5145		1875	6.78	8125	48
13	7382	6.04	5101		2281	6.77	7719	47
14	7745	6.03	5057		2688	6.77	7312	46
15	8107	6.03	5013	·73	3094	6.76	6906	45
16	8468	6.02	4969	·74	3499	6.76	6501	44
17	8829	6.01	4925		3905	6.75	6095	43
18	9190	6.01	4880		4310	6.75	5690	42
19	9551	6.00	4836		4715	6.74	5285	41
20	9.519911	6.00	4792		5119	6.74	4881	40
21	9.520271	5.99	9.974748		9.545524	6.73	10.454476	39
22	0631	5.99	4703		5928	6.73	4072	38
23	0990	5.98	4659		6331	6.72	3669	37
24	1349	5.98	4614		6735	6.72	3265	36
25	1707	5.97	4570		7138	6.71	2862	35
26	2066	5.96	4525		7540	6.71	2460	34
27	2424	5.96	4481		7943	6.70	2057	33
28	2781	5.95	4436		8345	6.70	1655	32
29	3138	5.95	4391	·74	8747	6.69	1253	31
30	3495	5.94	4347	·75	9149	6.69	0851	30
31	9.523852	5.94	9.974302		9550	6.68	0450	29
32	4208	5.93	4257		9.549951	6.68	10.450049	28
33	4564	5.93	4212		9.550352	6.67	10.449648	27
34	4920	5.92	4167		0752	6.67	9248	26
35	5275	5.91	4122		1152	6.66	8848	25
36	5630	5.91	4077		1552	6.66	8448	24
37	5984	5.90	4032		1952	6.65	8048	23
38	6339	5.90	3987		2351	6.65	7649	22
39	6693	5.89	3942		2750	6.65	7250	21
40	7046	5.89	3897		3149	6.64	6851	20
41	9.527400	5.88	9.973852		9.553548	6.64	10.446452	19
42	7753	5.88	3807		3946	6.63	6054	18
43	8105	5.87	3761	·75	4344	6.63	5656	17
44	8458	5.87	3716	·76	4741	6.62	5259	16
45	8810	5.86	3671		5139	6.62	4861	15
46	9161	5.86	3625		5536	6.61	4464	14
47	9513	5.85	3580		5933	6.61	4067	13
48	9.529864	5.85	3535		6329	6.60	3671	12
49	9.530215	5.84	3489		6725	6.60	3275	11
50	0565	5.84	3444		7121	6.59	2879	10
51	9.530915	5.83	9.973398		9.557517	6.59	10.442483	9
52	1265	5.82	3352		7913	6.59	2087	8
53	1614	5.82	3307		8308	6.58	1692	7
54	1963	5.81	3261		8702	6.58	1298	6
55	2312	5.81	3215		9097	6.57	0903	5
56	2661	5.80	3169		9491	6.57	0509	4
57	3009	5.80	3124		9.559885	6.56	10.440115	3
58	3357	5.79	3078	·76	9.560279	6.56	10.439721	2
59	3704	5.79	3032	·77	0673	6.55	9327	1
60	9.534052		9.972986		9.561066		10.438934	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9·534052	5·78	9·972986	·77	9·561066	6.55	10.438934	60
1	4399	5·78	2940		1459	6.54	8541	59
2	4745	5·77	2894		1851	6.54	8149	58
3	5092	5·77	2848		2244	6.53	7756	57
4	5438	5·76	2802		2636	6.53	7364	56
5	5783	5·76	2755		3028	6.53	6972	55
6	6129	5·75	2709		3419	6.52	6581	54
7	6474	5·74	2663		3811	6.52	6189	53
8	6818	5·74	2617		4202	6.51	5798	52
9	7163	5·73	2570		4592	6.51	5408	51
10	7507	5·73	2524		4983	6.50	5017	50
11	9·537851	5·72	9·972478	·77	9·565373	6.50	10.434627	49
12	8194	5·72	2431	·78	5763	6.49	4237	48
13	8538	5·71	2385		6153	6.49	3847	47
14	8880	5·71	2338		6542	6.49	3458	46
15	9223	5·70	2291		6932	6.48	3068	45
16	9565	5·70	2245		7320	6.48	2680	44
17	9·539907	5·69	2198		7709	6.47	2291	43
18	9·540249	5·69	2151		8098	6.47	1902	42
19	0590	5·68	2105		8486	6.46	1514	41
20	0931	5·68	2058		8873	6.46	1127	40
21	9·541272	5·67	9·972011		9261	6.45	0739	39
22	1613	5·67	1964		9·569648	6.45	10.430352	38
23	1953	5·66	1917		9·570035	6.45	10.429965	37
24	2293	5·66	1870		0422	6.44	9578	36
25	2632	5·65	1823		0809	6.44	9191	35
26	2971	5·65	1776	·78	1195	6.43	8805	34
27	3310	5·64	1729	·79	1581	6.43	8419	33
28	3649	5·64	1682		1967	6.42	8033	32
29	3987	5·63	1635		2352	6.42	7648	31
30	4325	5·63	1588		2738	6.42	7262	30
31	9·544663	5·62	9·971540		9·573123	6.41	10.426877	29
32	5000	5·62	1493		3507	6.41	6493	28
33	5338	5·61	1446		3892	6.40	6108	27
34	5674	5·61	1398		4276	6.40	5724	26
35	6011	5·60	1351		4660	6.39	5340	25
36	6347	5·60	1303		5044	6.39	4956	24
37	6683	5·59	1256		5427	6.39	4573	23
38	7019	5·59	1208		5810	6.38	4190	22
39	7354	5·58	1161		6193	6.38	3807	21
40	7689	5·58	1113	·79	6576	6.37	3424	20
41	9·548024	5·57	9·971066	·80	9·576958	6.37	10.423042	19
42	8359	5·57	1018		7341	6.36	2659	18
43	8693	5·56	0970		7723	6.36	2277	17
44	9027	5·56	0922		8104	6.36	1896	16
45	9360	5·55	0874		8486	6.35	1514	15
46	9·549693	5·55	0827		8867	6.35	1133	14
47	9·550026	5·54	0779		9248	6.34	0752	13
48	0359	5·54	0731		9·579629	6.34	10.420371	12
49	0692	5·53	0683		9·580009	6.34	10.419991	11
50	1024	5·53	0635		0389	6.33	9611	10
51	9·551356	5·52	9·970586		9·580769	6.33	10.419231	9
52	1687	5·52	0538		1149	6.32	8851	8
53	2018	5·52	0490		1528	6.32	8472	7
54	2349	5·51	0442		1907	6.32	8093	6
55	2680	5·51	0394	·80	2286	6.31	7714	5
56	3010	5·50	0345	·81	2665	6.31	7335	4
57	3341	5·50	0297		3043	6.30	6957	3
58	3670	5·49	0249		3422	6.30	6578	2
59	4000	5·49	0200	·81	3800	6.29	6200	1
60	9·554329		9·970152		9·584177		10.415823	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

110° 69°

M.	Sine.	Diff. 1″	Cosine.	Diff.1″	Tang.	Diff. 1″	Cotang.	
0	9.554329	5.48	9.970152	.81	9.584177	6.29	10.415823	60
1	4658	5.48	0103		4555	6.29	5445	59
2	4987	5.47	0055		4932	6.28	5068	58
3	5315	5.47	9.970006		5309	6.28	4691	57
4	5643	5.46	9.969957		5686	6.27	4314	56
5	5971	5.46	9909		6062	6.27	3938	55
6	6299	5.45	9860		6439	6.27	3561	54
7	6626	5.45	9811		6815	6.26	3185	53
8	6953	5.44	9762		7190	6.26	2810	52
9	7280	5.44	9714		7566	6.25	2434	51
10	7606	5.43	9665	.81	7941	6.25	2059	50
11	9.557932	5.43	9.969616	.82	9.588316	6.25	10.411684	49
12	8258	5.43	9567		8691	6.24	1309	48
13	8583	5.42	9518		9066	6.24	0934	47
14	8909	5.42	9469		9440	6.23	0560	46
15	9234	5.41	9420		9.589814	6.23	10.410186	45
16	9558	5.41	9370		9.590188	6.23	10.409812	44
17	9.559883	5.40	9321		0562	6.22	9438	43
18	9.560207	5.40	9272		0935	6.22	9065	42
19	0531	5.39	9223		1308	6.22	8692	41
20	0855	5.39	9173		1681	6.21	8319	40
21	9.561178	5.38	9.969124		9.592054	6.21	10.407946	39
22	1501	5.38	9075		2426	6.20	7574	38
23	1824	5.37	9025		2798	6.20	7202	37
24	2146	5.37	8976	.82	3171	6.20	6829	36
25	2468	5.36	8926	.83	3542	6.19	6458	35
26	2790	5.36	8877		3914	6.19	6086	34
27	3112	5.36	8827		4285	6.18	5715	33
28	3433	5.35	8777		4656	6.18	5344	32
29	3755	5.35	8728		5027	6.18	4973	31
30	4075	5.34	8678		5398	6.17	4602	30
31	9.564396	5.34	9.968628		9.595768	6.17	10.404232	29
32	4716	5.33	8578		6138	6.16	3862	28
33	5036	5.33	8528		6508	6.16	3492	27
34	5356	5.32	8479		6878	6.16	3122	26
35	5676	5.32	8429		7247	6.15	2753	25
36	5995	5.31	8379		7616	6.15	2384	24
37	6314	5.31	8329		7985	6.15	2015	23
38	6632	5.31	8278	.83	8354	6.14	1646	22
39	6951	5.30	8228	.84	8722	6.14	1278	21
40	7269	5.30	8178		9091	6.13	0909	20
41	9.567587	5.29	9.968128		9459	6.13	0541	19
42	7904	5.29	8078		9.599827	6.13	10.400173	18
43	8222	5.28	8027		9.600194	6.12	10.399806	17
44	8539	5.28	7977		0562	6.12	9438	16
45	8856	5.28	7927		0929	6.11	9071	15
46	9172	5.27	7876		1296	6.11	8704	14
47	9488	5.27	7826		1662	6.11	8338	13
48	9.569804	5.26	7775		2029	6.10	7971	12
49	9.570120	5.26	7725		2395	6.10	7605	11
50	0435	5.25	7674		2761	6.10	7239	10
51	9.570751	5.25	9.967624		9.603127	6.09	10.396873	9
52	1066	5.24	7573	.84	3493	6.09	6507	8
53	1380	5.24	7522	.85	3858	6.09	6142	7
54	1695	5.23	7471		4223	6.08	5777	6
55	2009	5.23	7421		4588	6.08	5412	5
56	2323	5.23	7370		4953	6.07	5047	4
57	2636	5.22	7319		5317	6.07	4683	3
58	2950	5.22	7268		5682	6.07	4318	2
59	3263	5.21	7217	.85	6046	6.06	3954	1
60	9.573575		9.967166		9.606410		10.393590	0
	Cosine.	Diff. 1″	Sine.	Diff.1″	Cotang.	Diff. 1″	Tang.	M.

111° 68°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.573575	5.21	9.967166	.85	9.606410	6.06	10.393590	60
1	3888	5.20	7115		6773	6.06	3227	59
2	4200	5.20	7064		7137	6.05	2863	58
3	4512	5.19	7013		7500	6.05	2500	57
4	4824	5.19	6961		7863	6.04	2137	56
5	5136	5.19	6910		8225	6.04	1775	55
6	5447	5.18	6859		8588	6.04	1412	54
7	5758	5.18	6808	.85	8950	6.03	1050	53
8	6069	5.17	6756	.86	9312	6.03	0688	52
9	6379	5.17	6705		9.609674	6.03	10.390326	51
10	6689	5.16	6653		9.610036	6.02	10.389964	50
11	9.576999	5.16	9.966602		0397	6.02	10.389603	49
12	7309	5.16	6550		0759	6.02	9241	48
13	7618	5.15	6499		1120	6.01	8880	47
14	7927	5.15	6447		1480	6.01	8520	46
15	8236	5.14	6395		1841	6.01	8159	45
16	8545	5.14	6344		2201	6.00	7799	44
17	8853	5.13	6292		2561	6.00	7439	43
18	9162	5.13	6240		2921	6.00	7079	42
19	9470	5.13	6188		3281	5.99	6719	41
20	9.579777	5.12	6136	.86	3641	5.99	6359	40
21	9.580085	5.12	9.966085	.87	9.614000	5.98	10.386000	39
22	0392	5.11	6033		4359	5.98	5641	38
23	0699	5.11	5981		4718	5.98	5282	37
24	1005	5.11	5928		5077	5.97	4923	36
25	1312	5.10	5876		5435	5.97	4565	35
26	1618	5.10	5824		5793	5.97	4207	34
27	1924	5.09	5772		6151	5.96	3849	33
28	2229	5.09	5720		6509	5.96	3491	32
29	2535	5.09	5668		6867	5.96	3133	31
30	2840	5.08	5615		7224	5.95	2776	30
31	9.583145	5.08	9.965563		9.617582	5.95	10.382418	29
32	3449	5.07	5511		7939	5.95	2061	28
33	3754	5.07	5458		8295	5.94	1705	27
34	4058	5.06	5406	.87	8652	5.94	1348	26
35	4361	5.06	5353	.88	9008	5.94	0992	25
36	4665	5.06	5301		9364	5.93	0636	24
37	4968	5.05	5248		9.619721	5.93	10.380279	23
38	5272	5.05	5195		9.620076	5.93	10.379924	22
39	5574	5.04	5143		0432	5.92	9568	21
40	5877	5.04	5090		0787	5.92	9213	20
41	9.586179	5.03	9.965037		9.621142	5.92	10.378858	19
42	6482	5.03	4984		1497	5.91	8503	18
43	6783	5.03	4931		1852	5.91	8148	17
44	7085	5.02	4879		2207	5.90	7793	16
45	7386	5.02	4826		2561	5.90	7439	15
46	7688	5.01	4773		2915	5.90	7085	14
47	7989	5.01	4719	.88	3269	5.89	6731	13
48	8289	5.01	4666	.89	3623	5.89	6377	12
49	8590	5.00	4613		3976	5.89	6024	11
50	8890	5.00	4560		4330	5.88	5670	10
51	9.589190	4.99	9.964507		9.624683	5.88	10.375317	9
52	9489	4.99	4454		5036	5.88	4964	8
53	9.589789	4.99	4400		5388	5.87	4612	7
54	9.590088	4.98	4347		5741	5.87	4259	6
55	0387	4.98	4294		6093	5.87	3907	5
56	0686	4.97	4240		6445	5.86	3555	4
57	0984	4.97	4187		6797	5.86	3203	3
58	1282	4.97	4133		7149	5.86	2851	2
59	1580	4.96	4080	.89	7501	5.85	2499	1
60	9.591878		9.964026		9.627852		10.372148	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff.1″	Tang.	Diff. 1″	Cotang.	
0	9.591878	4.96	9.964026	.89	9.627852	5.85	10.372148	60
1	2176	4.95	3972	.89	8203	5.85	1797	59
2	2473	4.95	3919	.89	8554	5.85	1446	58
3	2770	4.95	3865	.90	8905	5.84	1095	57
4	3067	4.94	3811		9255	5.84	0745	56
5	3363	4.94	3757		9606	5.83	0394	55
6	3659	4.93	3704		9.629956	5.83	10.370044	54
7	3955	4.93	3650		9.630306	5.83	10.369694	53
8	4251	4.93	3596		0656	5.83	9344	52
9	4547	4.92	3542		1005	5.82	8995	51
10	4842	4.92	3488		1355	5.82	8645	50
11	9.595137	4.91	9.963434		9.631704	5.82	10.368296	49
12	5432	4.91	3379		2053	5.81	7947	48
13	5727	4.91	3325		2401	5.81	7599	47
14	6021	4.90	3271		2750	5.81	7250	46
15	6315	4.90	3217		3098	5.80	6902	45
16	6609	4.89	3163	.90	3447	5.80	6553	44
17	6903	4.89	3108	.91	3795	5.80	6205	43
18	7196	4.89	3054		4143	5.79	5857	42
19	7490	4.88	2999		4490	5.79	5510	41
20	7783	4.88	2945		4838	5.79	5162	40
21	9.598075	4.87	9.962890		9.635185	5.78	10.364815	39
22	8368	4.87	2836		5532	5.78	4468	38
23	8660	4.87	2781		5879	5.78	4121	37
24	8952	4.86	2727		6226	5.77	3774	36
25	9244	4.86	2672		6572	5.77	3428	35
26	9536	4.85	2617		6919	5.77	3081	34
27	9.599827	4.85	2562		7265	5.77	2735	33
28	9.600118	4.85	2508		7611	5.76	2389	32
29	0409	4.84	2453	.91	7956	5.76	2044	31
30	0700	4.84	2398	.92	8302	5.76	1698	30
31	9.600990	4.84	9.962343		9.638647	5.75	10.361353	29
32	1280	4.83	2288		8992	5.75	1008	28
33	1570	4.83	2233		9337	5.75	0663	27
34	1860	4.82	2178		9.639682	5.74	10.360318	26
35	2150	4.82	2123		9.640027	5.74	10.359973	25
36	2439	4.82	2067		0371	5.74	9629	24
37	2728	4.81	2012		0716	5.73	9284	23
38	3017	4.81	1957		1060	5.73	8940	22
39	3305	4.81	1902		1404	5.73	8596	21
40	3594	4.80	1846		1747	5.72	8253	20
41	9.603882	4.80	9.961791		9.642091	5.72	10.357909	19
42	4170	4.79	1735		2434	5.72	7566	18
43	4457	4.79	1680	.92	2777	5.72	7223	17
44	4745	4.79	1624	.93	3120	5.71	6880	16
45	5032	4.78	1569		3463	5.71	6537	15
46	5319	4.78	1513		3806	5.71	6194	14
47	5606	4.78	1458		4148	5.70	5852	13
48	5892	4.77	1402		4490	5.70	5510	12
49	6179	4.77	1346		4832	5.70	5168	11
50	6465	4.76	1290		5174	5.69	4826	10
51	9.606751	4.76	9.961235		9.645516	5.69	10.354484	9
52	7036	4.76	1179		5857	5.69	4143	8
53	7322	4.75	1123		6199	5.69	3801	7
54	7607	4.75	1067		6540	5.68	3460	6
55	7892	4.74	1011		6881	5.68	3119	5
56	8177	4.74	0955		7222	5.68	2778	4
57	8461	4.74	0899	.93	7562	5.67	2438	3
58	8745	4.73	0843	.94	7903	5.67	2097	2
59	9029	4.73	0786	.94	8243	5.67	1757	1
60	9.609313		9.960730		9.648583		10.351417	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.609313	4.73	9.960730	.94	9.648583	5.66	10.351417	60
1	9597	4.72	0674		8923	5.66	1077	59
2	9.609880	4.72	0618		9263	5.66	0737	58
3	9.610164	4.72	0561		9602	5.66	0398	57
4	0447	4.71	0505		9.649942	5.65	10.350058	56
5	0729	4.71	0448		9.650281	5.65	10.349719	55
6	1012	4.70	0392		0620	5.65	9380	54
7	1294	4.70	0335		0959	5.64	9041	53
8	1576	4.70	0279		1297	5.64	8703	52
9	1858	4.69	0222		1636	5.64	8364	51
10	2140	4.69	0165	.94	1974	5.63	8026	50
11	9.612421	4.69	0109	.95	9.652312	5.63	10.347688	49
12	2702	4.68	9.960052		2650	5.63	7350	48
13	2983	4.68	9.959995		2988	5.63	7012	47
14	3264	4.67	9938		3326	5.62	6674	46
15	3545	4.67	9882		3663	5.62	6337	45
16	3825	4.67	9825		4000	5.62	6000	44
17	4105	4.66	9768		4337	5.61	5663	43
18	4385	4.66	9711		4674	5.61	5326	42
19	4665	4.66	9654		5011	5.61	4989	41
20	4944	4.65	9596		5348	5.61	4652	40
21	9.615223	4.65	9.959539		9.655684	5.60	10.344316	39
22	5502	4.65	9482		6020	5.60	3980	38
23	5781	4.64	9425		6356	5.60	3644	37
24	6060	4.64	9368	.95	6692	5.59	3308	36
25	6338	4.64	9310	.96	7028	5.59	2972	35
26	6616	4.63	9253		7364	5.59	2636	34
27	6894	4.63	9195		7699	5.59	2301	33
28	7172	4.62	9138		8034	5.58	1966	32
29	7450	4.62	9081		8369	5.58	1631	31
30	7727	4.62	9023		8704	5.58	1296	30
31	9.618004	4.61	9.958965		9.659039	5.58	10.340961	29
32	8281	4.61	8908		9373	5.57	0627	28
33	8558	4.61	8850		9.659708	5.57	10.340292	27
34	8834	4.60	8792		9.660042	5.57	10.339958	26
35	9110	4.60	8734		0376	5.57	9624	25
36	9386	4.60	8677		0710	5.56	9290	24
37	9662	4.59	8619		1043	5.56	8957	23
38	9.619938	4.59	8561	.96	1377	5.56	8623	22
39	9.620213	4.59	8503	.97	1710	5.55	8290	21
40	0488	4.58	8445		2043	5.55	7957	20
41	0763	4.58	9.958387		9.662376	5.55	10.337624	19
42	1038	4.57	8329		2709	5.54	7291	18
43	1313	4.57	8271		3042	5.54	6958	17
44	1587	4.57	8213		3375	5.54	6625	16
45	1861	4.56	8154		3707	5.54	6293	15
46	2135	4.56	8096		4039	5.53	5961	14
47	2409	4.56	8038		4371	5.53	5629	13
48	2682	4.55	7979		4703	5.53	5297	12
49	2956	4.55	7921		5035	5.53	4965	11
50	3229	4.55	7863		5366	5.52	4634	10
51	9.623502	4.54	9.957804	.97	9.665697	5.52	10.334303	9
52	3774	4.54	7746	.98	6029	5.52	3971	8
53	4047	4.54	7687		6360	5.51	3640	7
54	4319	4.53	7628		6691	5.51	3309	6
55	4591	4.53	7570		7021	5.51	2979	5
56	4863	4.53	7511		7352	5.51	2648	4
57	5135	4.52	7452		7682	5.50	2318	3
58	5406	4.52	7393		8013	5.50	1987	2
59	5677	4.52	7335	.98	8343	5.50	1657	1
60	9.625948		9.957276		9.668672		10.331328	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.625948	4.51	9.957276	.98	9.668673	5.50	10.331327	60
1	6219	4.51	7217		9002	5.49	0998	59
2	6490	4.51	7158		9332	5.49	0668	58
3	6760	4.50	7099		9661	5.49	0339	57
4	7030	4.50	7040		9.669991	5.48	10.330009	56
5	7300	4.50	6981	.98	9.670320	5.48	10.329680	55
6	7570	4.49	6921	.99	0649	5.48	9351	54
7	7840	4.49	6862		0977	5.48	9023	53
8	8109	4.49	6803		1306	5.47	8694	52
9	8378	4.48	6744		1634	5.47	8366	51
10	8647	4.48	6684		1963	5.47	8037	50
11	9.628916	4.47	9.956625		9.672291	5.47	10.327709	49
12	9185	4.47	6566		2619	5.46	7381	48
13	9453	4.47	6506		2947	5.46	7053	47
14	9721	4.46	6447		3274	5.46	6726	46
15	9.629989	4.46	6387		3602	5.46	6398	45
16	9.630257	4.46	6327		3929	5.45	6071	44
17	0524	4.46	6268	.99	4257	5.45	5743	43
18	0792	4.45	6208	1.00	4584	5.45	5416	42
19	1059	4.45	6148		4910	5.44	5090	41
20	1326	4.45	6089		5237	5.44	4763	40
21	9.631593	4.44	9.956029		9.675564	5.44	10.324436	39
22	1859	4.44	5969		5890	5.44	4110	38
23	2125	4.44	5909		6216	5.43	3784	37
24	2392	4.43	5849		6543	5.43	3457	36
25	2658	4.43	5789		6869	5.43	3131	35
26	2923	4.43	5729		7194	5.43	2806	34
27	3189	4.42	5669		7520	5.42	2480	33
28	3454	4.42	5609		7846	5.42	2154	32
29	3719	4.42	5548		8171	5.42	1829	31
30	3984	4.41	5488	1.00	8496	5.42	1504	30
31	9.634249	4.41	9.955428	1.01	9.678821	5.41	10.321179	29
32	4514	4.40	5368		9146	5.41	0854	28
33	4778	4.40	5307		9471	5.41	0529	27
34	5042	4.40	5247		9.679795	5.41	10.320205	26
35	5306	4.39	5186		9.680120	5.40	10.319880	25
36	5570	4.39	5126		0444	5.40	9556	24
37	5834	4.39	5065		0768	5.40	9232	23
38	6097	4.39	5005		1092	5.40	8908	22
39	6360	4.38	4944		1416	5.39	8584	21
40	6623	4.38	4883		1740	5.39	8260	20
41	9.636886	4.37	9.954823		9.682063	5.39	10.317937	19
42	7148	4.37	4762		2387	5.39	7613	18
43	7411	4.37	4701		2710	5.38	7290	17
44	7673	4.37	4640		3033	5.38	6967	16
45	7935	4.36	4579	1.01	3356	5.38	6644	15
46	8197	4.36	4518	1.02	3679	5.38	6321	14
47	8458	4.36	4457		4001	5.37	5999	13
48	8720	4.35	4396		4324	5.37	5676	12
49	8981	4.35	4335		4646	5.37	5354	11
50	9242	4.35	4274		4968	5.37	5032	10
51	9503	4.34	9.954213		9.685290	5.36	10.314710	9
52	9.639764	4.34	4152		5612	5.36	4388	8
53	9.640024	4.34	4090		5934	5.36	4066	7
54	0284	4.33	4029		6255	5.36	3745	6
55	0544	4.33	3968		6577	5.35	3423	5
56	0804	4.33	3906		6898	5.35	3102	4
57	1064	4.32	3845		7219	5.35	2781	3
58	1324	4.32	3783	1.02	7540	5.35	2460	2
59	1583	4.32	3722	1.03	7861	5.34	2139	1
60	9.641842		9.953660		9.688182		10.311818	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.641842	4.31	9.953660	1.03	9.688182	5.34	10.311818	60
1	2101	4.31	3599		8502	5.34	1498	59
2	2360	4.31	3537		8823	5.34	1177	58
3	2618	4.30	3475		9143	5.33	0857	57
4	2877	4.30	3413		9463	5.33	0537	56
5	3135	4.30	3352		9.689783	5.33	10.310217	55
6	3393	4.30	3290		9.690103	5.33	10.309897	54
7	3650	4.29	3228		0423	5.33	9577	53
8	3908	4.29	3166		0742	5.32	9258	52
9	4165	4.29	3104		1062	5.32	8938	51
10	4423	4.28	3042	1.03	1381	5.32	8619	50
11	9.644680	4.28	9.952980	1.04	9.691700	5.31	10.308300	49
12	4936	4.28	2918		2019	5.31	7981	48
13	5193	4.27	2855		2338	5.31	7662	47
14	5450	4.27	2793		2656	5.31	7344	46
15	5706	4.27	2731		2975	5.31	7025	45
16	5962	4.26	2669		3293	5.30	6707	44
17	6218	4.26	2606		3612	5.30	6388	43
18	6474	4.26	2544		3930	5.30	6070	42
19	6729	4.26	2481		4248	5.30	5752	41
20	6984	4.25	2419		4566	5.29	5434	40
21	9.647240	4.25	9.952356		9.694883	5.29	10.305117	39
22	7494	4.24	2294		5201	5.29	4799	38
23	7749	4.24	2231	1.04	5518	5.29	4482	37
24	8004	4.24	2168	1.05	5836	5.29	4164	36
25	8258	4.24	2106		6153	5.28	3847	35
26	8512	4.23	2043		6470	5.28	3530	34
27	8766	4.23	1980		6787	5.28	3213	33
28	9020	4.23	1917		7103	5.28	2897	32
29	9274	4.22	1854		7420	5.27	2580	31
30	9527	4.22	1791		7736	5.27	2264	30
31	9.649781	4.22	9.951728		9.698053	5.27	10.301947	29
32	9.650034	4.22	1665		8369	5.27	1631	28
33	0287	4.21	1602		8685	5.26	1315	27
34	0539	4.21	1539		9001	5.26	0999	26
35	0792	4.21	1476		9316	5.26	0684	25
36	1044	4.20	1412	1.05	9632	5.26	0368	24
37	1297	4.20	1349	1.06	9.699947	5.26	10.300053	23
38	1549	4.20	1286		9.700263	5.25	10.299737	22
39	1800	4.19	1222		0578	5.25	9422	21
40	2052	4.19	1159		0893	5.25	9107	20
41	9.652304	4.19	9.951096		9.701208	5.24	10.298792	19
42	2555	4.18	1032		1523	5.24	8477	18
43	2806	4.18	0968		1837	5.24	8163	17
44	3057	4.18	0905		2152	5.24	7848	16
45	3308	4.18	0841		2466	5.24	7534	15
46	3558	4.17	0778		2780	5.23	7220	14
47	3808	4.17	0714		3095	5.23	6905	13
48	4059	4.17	0650		3409	5.23	6591	12
49	4309	4.16	0586	1.06	3723	5.23	6277	11
50	4558	4.16	0522	1.07	4036	5.22	5964	10
51	9.654808	4.16	9.950458		9.704350	5.22	10.295650	9
52	5058	4.16	0394		4663	5.22	5337	8
53	5307	4.15	0330		4977	5.22	5023	7
54	5556	4.15	0266		5290	5.22	4710	6
55	5805	4.15	0202		5603	5.21	4397	5
56	6054	4.14	0138		5916	5.21	4084	4
57	6302	4.14	0074		6228	5.21	3772	3
58	6551	4.14	9.950010		6541	5.21	3459	2
59	6799	4.13	9.949945	1.07	6854	5.21	3146	1
60	9.657047		9.949881		9.707166		10.292834	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

116° 63°

M.	Sine.	Diff. 1″	Cosine.	Diff.1″	Tang.	Diff. 1″	Cotang.	
0	9.657047	4.13	9.949881	1.07	9.707166	5.20	10.292834	60
1	7295	4.13	9816	1.07	7478	5.20	2522	59
2	7542	4.12	9752	1.07	7790	5.20	2210	58
3	7790	4.12	9688	1.08	8102	5.20	1898	57
4	8037	4.12	9623		8414	5.19	1586	56
5	8284	4.12	9558		8726	5.19	1274	55
6	8531	4.11	9494		9037	5.19	0963	54
7	8778	4.11	9429		9349	5.19	0651	53
8	9025	4.11	9364		9660	5.19	0340	52
9	9271	4.10	9300		9.709971	5.18	10.290029	51
10	9517	4.10	9235		9.710282	5.18	10.289718	50
11	9.659763	4.10	9.949170		0593	5.18	9407	49
12	9.660009	4.09	9105		0904	5.18	9096	48
13	0255	4.09	9040		1215	5.18	8785	47
14	0501	4.09	8975		1525	5.17	8475	46
15	0746	4.09	8910		1836	5.17	8164	45
16	0991	4.08	8845	1.08	2146	5.17	7854	44
17	1236	4.08	8780	1.09	2456	5.17	7544	43
18	1481	4.08	8715		2766	5.16	7234	42
19	1726	4.07	8650		3076	5.16	6924	41
20	1970	4.07	8584		3386	5.16	6614	40
21	9.662214	4.07	9.948519		9.713696	5.16	10.286304	39
22	2459	4.07	8454		4005	5.16	5995	38
23	2703	4.06	8388		4314	5.15	5686	37
24	2946	4.06	8323		4624	5.15	5376	36
25	3190	4.06	8257		4933	5.15	5067	35
26	3433	4.05	8192		5242	5.15	4758	34
27	3677	4.05	8126		5551	5.14	4449	33
28	3920	4.05	8060	1.09	5860	5.14	4140	32
29	4163	4.05	7995	1.10	6168	5.14	3832	31
30	4406	4.04	7929		6477	5.14	3523	30
31	9.664648	4.04	9.947863		9.716785	5.14	10.283215	29
32	4891	4.04	7797		7093	5.13	2907	28
33	5133	4.03	7731		7401	5.13	2599	27
34	5375	4.03	7665		7709	5.13	2291	26
35	5617	4.03	7600		8017	5.13	1983	25
36	5859	4.02	7533		8325	5.13	1675	24
37	6100	4.02	7467		8633	5.12	1367	23
38	6342	4.02	7401		8940	5.12	1060	22
39	6583	4.02	7335		9248	5.12	0752	21
40	6824	4.01	7269		9555	5.12	0445	20
41	9.667065	4.01	9.947203	1.10	9.719862	5.12	10.280138	19
42	7305	4.01	7136	1.11	9.720169	5.11	10.279831	18
43	7546	4.01	7070		0476	5.11	9524	17
44	7786	4.00	7004		0783	5.11	9217	16
45	8027	4.00	6937		1089	5.11	8911	15
46	8267	4.00	6871		1396	5.11	8604	14
47	8506	3.99	6804		1702	5.10	8298	13
48	8746	3.99	6738		2009	5.10	7991	12
49	8986	3.99	6671		2315	5.10	7685	11
50	9225	3.99	6604		2621	5.10	7379	10
51	9464	3.98	9.946538		9.722927	5.10	10.277073	9
52	9703	3.98	6471		3232	5.09	6768	8
53	9.669942	3.98	6404		3538	5.09	6462	7
54	9.670181	3.97	6337	1.11	3844	5.09	6156	6
55	0419	3.97	6270	1.12	4149	5.09	5851	5
56	0658	3.97	6203		4454	5.09	5546	4
57	0896	3.97	6136		4759	5.08	5241	3
58	1134	3.96	6069		5065	5.08	4935	2
59	1372	3.96	6002	1.12	5369	5.08	4631	1
60	9.671609		9.945935		9.725674		10.274326	0
	Cosine.	Diff. 1″	Sine.	Diff.1″	Cotang.	Diff. 1″	Tang.	M.

117° 62°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.671609	3.96	9.945935	1.12	9.725674	5.08	10.274326	60
1	1847	3.95	5868		5979	5.08	4021	59
2	2084	3.95	5800		6284	5.07	3716	58
3	2321	3.95	5733		6588	5.07	3412	57
4	2558	3.95	5666		6892	5.07	3108	56
5	2795	3.94	5598		7197	5.07	2803	55
6	3032	3.94	5531	1.12	7501	5.07	2499	54
7	3268	3.94	5464	1.13	7805	5.06	2195	53
8	3505	3.94	5396		8109	5.06	1891	52
9	3741	3.93	5328		8412	5.06	1588	51
10	3977	3.93	5261		8716	5.06	1284	50
11	9.674213	3.93	9.945193		9.729020	5.06	10.270980	49
12	4448	3.92	5125		9323	5.05	0677	48
13	4684	3.92	5058		9626	5.05	0374	47
14	4919	3.92	4990		9.729929	5.05	10.270071	46
15	5155	3.92	4922		9.730233	5.05	10.269767	45
16	5390	3.91	4854		0535	5.05	9465	44
17	5624	3.91	4786		0838	5.04	9162	43
18	5859	3.91	4718		1141	5.04	8859	42
19	6094	3.91	4650	1.13	1444	5.04	8556	41
20	6328	3.90	4582	1.14	1746	5.04	8254	40
21	9.676562	3.90	9.944514		9.732048	5.04	10.267952	39
22	6796	3.90	4446		2351	5.03	7649	38
23	7030	3.90	4377		2653	5.03	7347	37
24	7264	3.89	4309		2955	5.03	7045	36
25	7498	3.89	4241		3257	5.03	6743	35
26	7731	3.89	4172		3558	5.03	6442	34
27	7964	3.88	4104		3860	5.02	6140	33
28	8197	3.88	4036		4162	5.02	5838	32
29	8430	3.88	3967		4463	5.02	5537	31
30	8663	3.88	3899		4764	5.02	5236	30
31	9.678895	3.87	9.943830		9.735066	5.02	10.264934	29
32	9128	3.87	3761	1.14	5367	5.02	4633	28
33	9360	3.87	3693	1.15	5668	5.01	4332	27
34	9592	3.87	3624		5969	5.01	4031	26
35	9.679824	3.86	3555		6269	5.01	3731	25
36	9.680056	3.86	3486		6570	5.01	3430	24
37	0288	3.86	3417		6871	5.01	3129	23
38	0519	3.85	3348		7171	5.00	2829	22
39	0750	3.85	3279		7471	5.00	2529	21
40	0982	3.85	3210		7771	5.00	2229	20
41	9.681213	3.85	9.943141		9.738071	5.00	10.261929	19
42	1443	3.84	3072		8371	5.00	1629	18
43	1674	3.84	3003		8671	4.99	1329	17
44	1905	3.84	2934		8971	4.99	1029	16
45	2135	3.84	2864	1.15	9271	4.99	0729	15
46	2365	3.83	2795	1.16	9570	4.99	0430	14
47	2595	3.83	2726		9.739870	4.99	10.260130	13
48	2825	3.83	2656		9.740169	4.99	10.259831	12
49	3055	3.83	2587		0468	4.98	9532	11
50	3284	3.82	2517		0767	4.98	9233	10
51	9.683514	3.82	9.942448		9.741066	4.98	10.258934	9
52	3743	3.82	2378		1365	4.98	8635	8
53	3972	3.82	2308		1664	4.98	8336	7
54	4201	3.81	2239		1962	4.97	8038	6
55	4430	3.81	2169		2261	4.97	7739	5
56	4658	3.81	2099		2559	4.97	7441	4
57	4887	3.80	2029		2858	4.97	7142	3
58	5115	3.80	1959	1.16	3156	4.97	6844	2
59	5343	3.80	1889	1.17	3454	4.97	6546	1
60	9.685571		9.941819		9.743752		10.256248	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

118° 61°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.685571	3.80	9.941819	1.17	9.743752	4.96	10.256248	60
1	5799	3.79	1749		4050	4.96	5950	59
2	6027	3.79	1679		4348	4.96	5652	58
3	6254	3.79	1609		4645	4.96	5355	57
4	6482	3.79	1539		4943	4.96	5057	56
5	6709	3.78	1469		5240	4.95	4760	55
6	6936	3.78	1398		5538	4.95	4462	54
7	7163	3.78	1328		5835	4.95	4165	53
8	7389	3.78	1258		6132	4.95	3868	52
9	7616	3.77	1187		6429	4.95	3571	51
10	7843	3.77	1117	1.17	6726	4.95	3274	50
11	9.688069	3.77	9.941046	1.18	9.747023	4.94	10.252977	49
12	8295	3.77	0975		7319	4.94	2681	48
13	8521	3.76	0905		7616	4.94	2384	47
14	8747	3.76	0834		7913	4.94	2087	46
15	8972	3.76	0763		8209	4.94	1791	45
16	9198	3.76	0693		8505	4.93	1495	44
17	9423	3.75	0622		8801	4.93	1199	43
18	9648	3.75	0551		9097	4.93	0903	42
19	9.689873	3.75	0480		9393	4.93	0607	41
20	9.690098	3.75	0409		9689	4.93	0311	40
21	0323	3.74	9.940338		9.749985	4.93	10.250015	39
22	0548	3.74	0267		9.750281	4.93	10.249719	38
23	0772	3.74	0196	1.18	0576	4.92	9424	37
24	0996	3.74	0125	1.19	0872	4.92	9128	36
25	1220	3.73	9.940054		1167	4.92	8833	35
26	1444	3.73	9.939982		1462	4.92	8538	34
27	1668	3.73	9911		1757	4.92	8243	33
28	1892	3.73	9840		2052	4.91	7948	32
29	2115	3.72	9768		2347	4.91	7653	31
30	2339	3.72	9697		2642	4.91	7358	30
31	9.692562	3.72	9.939625		9.752937	4.91	10.247063	29
32	2785	3.71	9554		3231	4.91	6769	28
33	3008	3.71	9482		3526	4.91	6474	27
34	3231	3.71	9410		3820	4.90	6180	26
35	3453	3.71	9339	1.19	4115	4.90	5885	25
36	3676	3.70	9267	1.20	4409	4.90	5591	24
37	3898	3.70	9195		4703	4.90	5297	23
38	4120	3.70	9123		4997	4.90	5003	22
39	4342	3.70	9052		5291	4.90	4709	21
40	4564	3.69	8980		5585	4.89	4415	20
41	9.694786	3.69	9.938908		9.755878	4.89	10.244122	19
42	5007	3.69	8836		6172	4.89	3828	18
43	5229	3.69	8763		6465	4.89	3535	17
44	5450	3.68	8691		6759	4.89	3241	16
45	5671	3.68	8619		7052	4.89	2948	15
46	5892	3.68	8547		7345	4.88	2655	14
47	6113	3.68	8475	1.20	7638	4.88	2362	13
48	6334	3.67	8402	1.21	7931	4.88	2069	12
49	6554	3.67	8330		8224	4.88	1776	11
50	6775	3.67	8258		8517	4.88	1483	10
51	9.696995	3.67	9.938185		9.758810	4.88	10.241190	9
52	7215	3.66	8113		9102	4.87	0898	8
53	7435	3.66	8040		9395	4.87	0605	7
54	7654	3.66	7967		9687	4.87	0313	6
55	7874	3.66	7895		9.759979	4.87	10.240021	5
56	8094	3.65	7822		9.760272	4.87	10.239728	4
57	8313	3.65	7749		0564	4.87	9436	3
58	8532	3.65	7676		0856	4.86	9144	2
59	8751	3.65	7604	1.21	1148	4.86	8852	1
60	9.698970		9.937531		9.761439		10.238561	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

119° 60°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.698970	3.64	9.937531	1.21	9.761439	4.86	10.238561	60
1	9189	3.64	7458	1.22	1731	4.86	8269	59
2	9407	3.64	7385		2023	4.86	7977	58
3	9626	3.64	7312		2314	4.86	7686	57
4	9.699844	3.63	7238		2606	4.85	7394	56
5	9.700062	3.63	7165		2897	4.85	7103	55
6	0280	3.63	7092		3188	4.85	6812	54
7	0498	3.63	7019		3479	4.85	6521	53
8	0716	3.63	6946		3770	4.85	6230	52
9	0933	3.62	6872		4061	4.85	5939	51
10	1151	3.62	6799		4352	4.84	5648	50
11	9.701368	3.62	9.936725	1.22	9.764643	4.84	10.235357	49
12	1585	3.62	6652	1.23	4933	4.84	5067	48
13	1802	3.61	6578		5224	4.84	4776	47
14	2019	3.61	6505		5514	4.84	4486	46
15	2236	3.61	6431		5805	4.84	4195	45
16	2452	3.61	6357		6095	4.84	3905	44
17	2669	3.60	6284		6385	4.83	3615	43
18	2885	3.60	6210		6675	4.83	3325	42
19	3101	3.60	6136		6965	4.83	3035	41
20	3317	3.60	6062		7255	4.83	2745	40
21	9.703533	3.59	9.935988		9.767545	4.83	10.232455	39
22	3749	3.59	5914		7834	4.83	2166	38
23	3964	3.59	5840	1.23	8124	4.82	1876	37
24	4179	3.59	5766	1.24	8413	4.82	1587	36
25	4395	3.59	5692		8703	4.82	1297	35
26	4610	3.58	5618		8992	4.82	1008	34
27	4825	3.58	5543		9281	4.82	0719	33
28	5040	3.58	5469		9570	4.82	0430	32
29	5254	3.58	5395		9.769860	4.81	10.230140	31
30	5469	3.57	5320		9.770148	4.81	10.229852	30
31	9.705683	3.57	9.935246		0437	4.81	9563	29
32	5898	3.57	5171		0726	4.81	9274	28
33	6112	3.57	5097		1015	4.81	8985	27
34	6326	3.56	5022		1303	4.81	8697	26
35	6539	3.56	4948		1592	4.81	8408	25
36	6753	3.56	4873	1.24	1880	4.80	8120	24
37	6967	3.56	4798	1.25	2168	4.80	7832	23
38	7180	3.55	4723		2457	4.80	7543	22
39	7393	3.55	4649		2745	4.80	7255	21
40	7606	3.55	4574		3033	4.80	6967	20
41	9.707819	3.55	9.934499		9.773321	4.80	10.226679	19
42	8032	3.54	4424		3608	4.79	6392	18
43	8245	3.54	4349		3896	4.79	6104	17
44	8458	3.54	4274		4184	4.79	5816	16
45	8670	3.54	4199		4471	4.79	5529	15
46	8882	3.53	4123		4759	4.79	5241	14
47	9094	3.53	4048		5046	4.79	4954	13
48	9306	3.53	3973	1.25	5333	4.79	4667	12
49	9518	3.53	3898	1.26	5621	4.78	4379	11
50	9730	3.53	3822		5908	4.78	4092	10
51	9.709941	3.52	9.933747		9.776195	4.78	10.223805	9
52	9.710153	3.52	3671		6482	4.78	3518	8
53	0364	3.52	3596		6769	4.78	3231	7
54	0575	3.52	3520		7055	4.78	2945	6
55	0786	3.51	3445		7342	4.78	2658	5
56	0997	3.51	3369		7628	4.77	2372	4
57	1208	3.51	3293		7915	4.77	2085	3
58	1419	3.51	3217		8201	4.77	1799	2
59	1629	3.50	3141	1.26	8487	4.77	1513	1
60	9.711839		9.933066		9.778774		10.221226	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.711839	3.50	9.933066	1.26	9.778774	4.77	10.221226	60
1	2050	3.50	2990	1.27	9060	4.77	0940	59
2	2260	3.50	2914		9346	4.76	0654	58
3	2469	3.49	2838		9632	4.76	0368	57
4	2679	3.49	2762		9.779918	4.76	10.220082	56
5	2889	3.49	2685		9.780203	4.76	10.219797	55
6	3098	3.49	2609		0489	4.76	9511	54
7	3308	3.49	2533		0775	4.76	9225	53
8	3517	3.48	2457		1060	4.76	8940	52
9	3726	3.48	2380		1346	4.75	8654	51
10	3935	3.48	2304		1631	4.75	8369	50
11	9.714144	3.48	9.932228		9.781916	4.75	10.218084	49
12	4352	3.47	2151	1.27	2201	4.75	7799	48
13	4561	3.47	2075	1.28	2486	4.75	7514	47
14	4769	3.47	1998		2771	4.75	7229	46
15	4978	3.47	1921		3056	4.75	6944	45
16	5186	3.47	1845		3341	4.75	6659	44
17	5394	3.46	1768		3626	4.74	6374	43
18	5602	3.46	1691		3910	4.74	6090	42
19	5809	3.46	1614		4195	4.74	5805	41
20	6017	3.46	1537		4479	4.74	5521	40
21	9.716224	3.45	9.931460		9.784764	4.74	10.215236	39
22	6432	3.45	1383		5048	4.74	4952	38
23	6639	3.45	1306	1.28	5332	4.73	4668	37
24	6846	3.45	1229	1.29	5616	4.73	4384	36
25	7053	3.45	1152		5900	4.73	4100	35
26	7259	3.44	1075		6184	4.73	3816	34
27	7466	3.44	0998		6468	4.73	3532	33
28	7673	3.44	0921		6752	4.73	3248	32
29	7879	3.44	0843		7036	4.73	2964	31
30	8085	3.43	0766		7319	4.72	2681	30
31	9.718291	3.43	9.930688		9.787603	4.72	10.212397	29
32	8497	3.43	0611		7886	4.72	2114	28
33	8703	3.43	0533		8170	4.72	1830	27
34	8909	3.43	0456		8453	4.72	1547	26
35	9114	3.42	0378	1.29	8736	4.72	1264	25
36	9320	3.42	0300	1.30	9019	4.72	0981	24
37	9525	3.42	0223		9302	4.71	0698	23
38	9730	3.42	0145		9585	4.71	0415	22
39	9.719935	3.41	9.930067		9.789868	4.71	10.210132	21
40	9.720140	3.41	9.929989		9.790151	4.71	10.209849	20
41	0345	3.41	9911		0433	4.71	9567	19
42	0549	3.41	9833		0716	4.71	9284	18
43	0754	3.40	9755		0999	4.71	9001	17
44	0958	3.40	9677		1281	4.71	8719	16
45	1162	3.40	9599		1563	4.70	8437	15
46	1366	3.40	9521		1846	4.70	8154	14
47	1570	3.40	9442	1.30	2128	4.70	7872	13
48	1774	3.39	9364	1.31	2410	4.70	7590	12
49	1978	3.39	9286		2692	4.70	7308	11
50	2181	3.39	9207		2974	4.70	7026	10
51	9.722385	3.39	9.929129		9.793256	4.70	10.206744	9
52	2588	3.39	9050		3538	4.69	6462	8
53	2791	3.38	8972		3819	4.69	6181	7
54	2994	3.38	8893		4101	4.69	5899	6
55	3197	3.38	8815		4383	4.69	5617	5
56	3400	3.38	8736		4664	4.69	5336	4
57	3603	3.37	8657		4945	4.69	5055	3
58	3805	3.37	8578		5227	4.69	4773	2
59	4007	3.37	8499	1.31	5508	4.68	4492	1
60	9.724210		9.928420		9.795789		10.204211	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.724210	3.37	9.928420	1.32	9.795789	4.68	10.204211	60
1	4412	3.37	8342		6070	4.68	3930	59
2	4614	3.36	8263		6351	4.68	3649	58
3	4816	3.36	8183		6632	4.68	3368	57
4	5017	3.36	8104		6913	4.68	3087	56
5	5219	3.36	8025		7194	4.68	2806	55
6	5420	3.35	7946		7475	4.68	2525	54
7	5622	3.35	7867		7755	4.68	2245	53
8	5823	3.35	7787		8036	4.67	1964	52
9	6024	3.35	7708		8316	4.67	1684	51
10	6225	3.35	7629		8596	4.67	1404	50
11	9.726426	3.34	9.927549	1.32	9.798877	4.67	10.201123	49
12	6626	3.34	7470	1.33	9157	4.67	0843	48
13	6827	3.34	7390		9437	4.67	0563	47
14	7027	3.34	7310		9717	4.67	0283	46
15	7228	3.34	7231		9.799997	4.66	10.200003	45
16	7428	3.33	7151		9.800277	4.66	10.199723	44
17	7628	3.33	7071		0557	4.66	9443	43
18	7828	3.33	6991		0836	4.66	9164	42
19	8027	3.33	6911		1116	4.66	8884	41
20	8227	3.33	6831		1396	4.66	8604	40
21	9.728427	3.32	9.926751		9.801675	4.66	10.198325	39
22	8626	3.32	6671		1955	4.66	8045	38
23	8825	3.32	6591	1.33	2234	4.65	7766	37
24	9024	3.32	6511	1.34	2513	4.65	7487	36
25	9223	3.31	6431		2792	4.65	7208	35
26	9422	3.31	6351		3072	4.65	6928	34
27	9621	3.31	6270		3351	4.65	6649	33
28	9.729820	3.31	6190		3630	4.65	6370	32
29	9.730018	3.30	6110		3908	4.65	6092	31
30	0216	3.30	6029		4187	4.65	5813	30
31	0415	3.30	9.925949		9.804466	4.64	10.195534	29
32	0613	3.30	5868		4745	4.64	5255	28
33	0811	3.30	5788		5023	4.64	4977	27
34	1009	3.29	5707		5302	4.64	4698	26
35	1206	3.29	5626	1.34	5580	4.64	4420	25
36	1404	3.29	5545	1.35	5859	4.64	4141	24
37	1602	3.29	5465		6137	4.64	3863	23
38	1799	3.29	5384		6415	4.63	3585	22
39	1996	3.28	5303		6693	4.63	3307	21
40	2193	3.28	5222		6971	4.63	3029	20
41	9.732390	3.28	9.925141		9.807249	4.63	10.192751	19
42	2587	3.28	5060		7527	4.63	2473	18
43	2784	3.28	4979		7805	4.63	2195	17
44	2980	3.27	4897		8083	4.63	1917	16
45	3177	3.27	4816	1.35	8361	4.63	1639	15
46	3373	3.27	4735	1.36	8638	4.62	1362	14
47	3569	3.27	4654		8916	4.62	1084	13
48	3765	3.27	4572		9193	4.62	0807	12
49	3961	3.26	4491		9471	4.62	0529	11
50	4157	3.26	4409		9.809748	4.62	10.190252	10
51	9.734353	3.26	9.924328		9.810025	4.62	10.189975	9
52	4549	3.26	4246		0302	4.62	9698	8
53	4744	3.25	4164		0580	4.62	9420	7
54	4939	3.25	4083		0857	4.62	9143	6
55	5135	3.25	4001		1134	4.61	8866	5
56	5330	3.25	3919		1410	4.61	8590	4
57	5525	3.25	3837	1.36	1687	4.61	8313	3
58	5719	3.24	3755	1.37	1964	4.61	8036	2
59	5914	3.24	3673	1.37	2241	4.61	7759	1
60	9.736109		9.923591		9.812517		10.187483	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff.1″	Tang.	Diff. 1″	Cotang.	
0	9.736109	3.24	9.923591	1.37	9.812517	4.61	10.187483	60
1	6303	3.24	3509		2794	4.61	7206	59
2	6498	3.24	3427		3070	4.61	6930	58
3	6692	3.23	3345		3347	4.60	6653	57
4	6886	3.23	3263		3623	4.60	6377	56
5	7080	3.23	3181		3899	4.60	6101	55
6	7274	3.23	3098		4175	4.60	5825	54
7	7467	3.23	3016		4452	4.60	5548	53
8	7661	3.22	2933		4728	4.60	5272	52
9	7855	3.22	2851	1.37	5004	4.60	4996	51
10	8048	3.22	2768	1.38	5279	4.60	4721	50
11	9.738241	3.22	9.922686		9.815555	4.59	10.184445	49
12	8434	3.22	2603		5831	4.59	4169	48
13	8627	3.21	2520		6107	4.59	3893	47
14	8820	3.21	2438		6382	4.59	3618	46
15	9013	3.21	2355		6658	4.59	3342	45
16	9206	3.21	2272		6933	4.59	3067	44
17	9398	3.21	2189		7209	4.59	2791	43
18	9590	3.20	2106		7484	4.59	2516	42
19	9783	3.20	2023		7759	4.59	2241	41
20	9.739975	3.20	1940	1.38	8035	4.58	1965	40
21	9.740167	3.20	9.921857	1.39	9.818310	4.58	10.181690	39
22	0359	3.20	1774		8585	4.58	1415	38
23	0550	3.19	1691		8860	4.58	1140	37
24	0742	3.19	1607		9135	4.58	0865	36
25	0934	3.19	1524		9410	4.58	0590	35
26	1125	3.19	1441		9684	4.58	0316	34
27	1316	3.19	1357		9.819959	4.58	10.180041	33
28	1508	3.18	1274		9.820234	4.58	10.179766	32
29	1699	3.18	1190		0508	4.57	9492	31
30	1889	3.18	1107		0783	4.57	9217	30
31	9.742080	3.18	9.921023	1.39	9.821057	4.57	10.178943	29
32	2271	3.18	0939	1.40	1332	4.57	8668	28
33	2462	3.17	0856		1606	4.57	8394	27
34	2652	3.17	0772		1880	4.57	8120	26
35	2842	3.17	0688		2154	4.57	7846	25
36	3033	3.17	0604		2429	4.57	7571	24
37	3223	3.17	0520		2703	4.57	7297	23
38	3413	3.16	0436		2977	4.56	7023	22
39	3602	3.16	0352		3250	4.56	6750	21
40	3792	3.16	0268		3524	4.56	6476	20
41	9.743982	3.16	9.920184		9.823798	4.56	10.176202	19
42	4171	3.16	0099		4072	4.56	5928	18
43	4361	3.15	9.920015	1.40	4345	4.56	5655	17
44	4550	3.15	9.919931	1.41	4619	4.56	5381	16
45	4739	3.15	9846		4893	4.56	5107	15
46	4928	3.15	9762		5166	4.56	4834	14
47	5117	3.15	9677		5439	4.55	4561	13
48	5306	3.14	9593		5713	4.55	4287	12
49	5494	3.14	9508		5986	4.55	4014	11
50	5683	3.14	9424		6259	4.55	3741	10
51	9.745871	3.14	9.919339		9.826532	4.55	10.173468	9
52	6059	3.14	9254		6805	4.55	3195	8
53	6248	3.13	9169		7078	4.55	2922	7
54	6436	3.13	9085	1.41	7351	4.55	2649	6
55	6624	3.13	9000	1.42	7624	4.55	2376	5
56	6812	3.13	8915		7897	4.54	2103	4
57	6999	3.13	8830		8170	4.54	1830	3
58	7187	3.12	8745		8442	4.54	1558	2
59	7374	3.12	8659	1.42	8715	4.54	1285	1
60	9.747562		9.918574		9.828987		10.171013	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.747562	3.12	9.918574	1.42	9.828987	4.54	10.171013	60
1	7749	3.12	8489		9260	4.54	0740	59
2	7936	3.12	8404		9532	4.54	0468	58
3	8123	3.11	8318		9.829805	4.54	10.170195	57
4	8310	3.11	8233		9.830077	4.54	10.169923	56
5	8497	3.11	8147	1.42	0349	4.53	9651	55
6	8683	3.11	8062	1.43	0621	4.53	9379	54
7	8870	3.11	7976		0893	4.53	9107	53
8	9056	3.10	7891		1165	4.53	8835	52
9	9243	3.10	7805		1437	4.53	8563	51
10	9429	3.10	7719		1709	4.53	8291	50
11	9.749615	3.10	9.917634		9.831981	4.53	10.168019	49
12	9801	3.10	7548		2253	4.53	7747	48
13	9.749987	3.09	7462		2525	4.53	7475	47
14	9.750172	3.09	7376		2796	4.53	7204	46
15	0358	3.09	7290		3068	4.52	6932	45
16	0543	3.09	7204	1.43	3339	4.52	6661	44
17	0729	3.09	7118	1.44	3611	4.52	6389	43
18	0914	3.08	7032		3882	4.52	6118	42
19	1099	3.08	6946		4154	4.52	5846	41
20	1284	3.08	6859		4425	4.52	5575	40
21	9.751469	3.08	9.916773		9.834696	4.52	10.165304	39
22	1654	3.08	6687		4967	4.52	5033	38
23	1839	3.08	6600		5238	4.52	4762	37
24	2023	3.07	6514		5509	4.52	4491	36
25	2208	3.07	6427		5780	4.51	4220	35
26	2392	3.07	6341		6051	4.51	3949	34
27	2576	3.07	6254	1.44	6322	4.51	3678	33
28	2760	3.07	6167	1.45	6593	4.51	3407	32
29	2944	3.06	6081		6864	4.51	3136	31
30	3128	3.06	5994		7134	4.51	2866	30
31	9.753312	3.06	9.915907		9.837405	4.51	10.162595	29
32	3495	3.06	5820		7675	4.51	2325	28
33	3679	3.06	5733		7946	4.51	2054	27
34	3862	3.05	5646		8216	4.50	1784	26
35	4046	3.05	5559		8487	4.50	1513	25
36	4229	3.05	5472		8757	4.50	1243	24
37	4412	3.05	5385		9027	4.50	0973	23
38	4595	3.05	5297		9297	4.50	0703	22
39	4778	3.04	5210	1.45	9568	4.50	0432	21
40	4960	3.04	5123	1.46	9.839838	4.50	10.160162	20
41	9.755143	3.04	9.915035		9.840108	4.50	10.159892	19
42	5326	3.04	4948		0378	4.50	9622	18
43	5508	3.04	4860		0647	4.50	9353	17
44	5690	3.04	4773		0917	4.49	9083	16
45	5872	3.03	4685		1187	4.49	8813	15
46	6054	3.03	4598		1457	4.49	8543	14
47	6236	3.03	4510		1726	4.49	8274	13
48	6418	3.03	4422		1996	4.49	8004	12
49	6600	3.03	4334	1.46	2266	4.49	7734	11
50	6782	3.02	4246	1.47	2535	4.49	7465	10
51	9.756963	3.02	9.914158		9.842805	4.49	10.157195	9
52	7144	3.02	4070		3074	4.49	6926	8
53	7326	3.02	3982		3343	4.49	6657	7
54	7507	3.02	3894		3612	4.49	6388	6
55	7688	3.01	3806		3882	4.48	6118	5
56	7869	3.01	3718		4151	4.48	5849	4
57	8050	3.01	3630		4420	4.48	5580	3
58	8230	3.01	3541		4689	4.48	5311	2
59	8411	3.01	3453	1.47	4958	4.48	5042	1
60	9.758591		9.913365		9.845227		10.154773	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.758591	3.01	9.913365	1.47	9.845227	4.48	10.154773	60
1	8772	3.00	3276	1.47	5496	4.48	4504	59
2	8952	3.00	3187	1.48	5764	4.48	4236	58
3	9132	3.00	3099		6033	4.48	3967	57
4	9312	3.00	3010		6302	4.48	3698	56
5	9492	3.00	2922		6570	4.47	3430	55
6	9672	2.99	2833		6839	4.47	3161	54
7	9.759852	2.99	2744		7107	4.47	2893	53
8	9.760031	2.99	2655		7376	4.47	2624	52
9	0211	2.99	2566		7644	4.47	2356	51
10	0390	2.99	2477		7913	4.47	2087	50
11	9.760569	2.98	9.912388	1.48	9.848181	4.47	10.151819	49
12	0748	2.98	2299	1.49	8449	4.47	1551	48
13	0927	2.98	2210		8717	4.47	1283	47
14	1106	2.98	2121		8986	4.47	1014	46
15	1285	2.98	2031		9254	4.47	0746	45
16	1464	2.98	1942		9522	4.47	0478	44
17	1642	2.97	1853		9.849790	4.46	10.150210	43
18	1821	2.97	1763		9.850058	4.46	10.149942	42
19	1999	2.97	1674		0325	4.46	9675	41
20	2177	2.97	1584		0593	4.46	9407	40
21	9.762356	2.97	9.911495		9.850861	4.46	10.149139	39
22	2534	2.96	1405	1.49	1129	4.46	8871	38
23	2712	2.96	1315	1.50	1396	4.46	8604	37
24	2889	2.96	1226		1664	4.46	8336	36
25	3067	2.96	1136		1931	4.46	8069	35
26	3245	2.96	1046		2199	4.46	7801	34
27	3422	2.96	0956		2466	4.46	7534	33
28	3600	2.95	0866		2733	4.45	7267	32
29	3777	2.95	0776		3001	4.45	6999	31
30	3954	2.95	0686		3268	4.45	6732	30
31	9.764131	2.95	9.910596		9.853535	4.45	10.146465	29
32	4308	2.95	0506	1.50	3802	4.45	6198	28
33	4485	2.94	0415	1.51	4069	4.45	5931	27
34	4662	2.94	0325		4336	4.45	5664	26
35	4838	2.94	0235		4603	4.45	5397	25
36	5015	2.94	0144		4870	4.45	5130	24
37	5191	2.94	9.910054		5137	4.45	4863	23
38	5367	2.94	9.909963		5404	4.45	4596	22
39	5544	2.93	9873		5671	4.44	4329	21
40	5720	2.93	9782		5938	4.44	4062	20
41	9.765896	2.93	9.909691		9.856204	4.44	10.143796	19
42	6072	2.93	9601		6471	4.44	3529	18
43	6247	2.93	9510		6737	4.44	3263	17
44	6423	2.93	9419	1.51	7004	4.44	2996	16
45	6598	2.92	9328	1.52	7270	4.44	2730	15
46	6774	2.92	9237		7537	4.44	2463	14
47	6949	2.92	9146		7803	4.44	2197	13
48	7124	2.92	9055		8069	4.44	1931	12
49	7300	2.92	8964		8336	4.44	1664	11
50	7475	2.91	8873		8602	4.43	1398	10
51	9.767649	2.91	9.908781		9.858868	4.43	10.141132	9
52	7824	2.91	8690		9134	4.43	0866	8
53	7999	2.91	8599		9400	4.43	0600	7
54	8173	2.91	8507	1.52	9666	4.43	0334	6
55	8348	2.90	8416	1.53	9.859932	4.43	10.140068	5
56	8522	2.90	8324		9.860198	4.43	10.139802	4
57	8697	2.90	8233		0464	4.43	9536	3
58	8871	2.90	8141		0730	4.43	9270	2
59	9045	2.90	8049	1.53	0995	4.43	9005	1
60	9.769219		9.907958		9.861261		10.138739	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

125° 54°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.769219	2.90	9.907958	1.53	9.861261	4.43	10.138739	60
1	9393	2.89	7866		1527	4.43	8473	59
2	9566	2.89	7774		1792	4.42	8208	58
3	9740	2.89	7682		2058	4.42	7942	57
4	9.769913	2.89	7590		2323	4.42	7677	56
5	9.770087	2.89	7498		2589	4.42	7411	55
6	0260	2.88	7406	1.53	2854	4.42	7146	54
7	0433	2.88	7314	1.54	3119	4.42	6881	53
8	0606	2.88	7222		3385	4.42	6615	52
9	0779	2.88	7129		3650	4.42	6350	51
10	0952	2.88	7037		3915	4.42	6085	50
11	9.771125	2.88	9.906945		9.864180	4.42	10.135820	49
12	1298	2.88	6852		4445	4.42	5555	48
13	1470	2.87	6760		4710	4.42	5290	47
14	1643	2.87	6667		4975	4.41	5025	46
15	1815	2.87	6575		5240	4.41	4760	45
16	1987	2.87	6482	1.54	5505	4.41	4495	44
17	2159	2.87	6389	1.55	5770	4.41	4230	43
18	2331	2.86	6296		6035	4.41	3965	42
19	2503	2.86	6204		6300	4.41	3700	41
20	2675	2.86	6111		6564	4.41	3436	40
21	9.772847	2.86	9.906018		9.866829	4.41	10.133171	39
22	3018	2.86	5925		7094	4.41	2906	38
23	3190	2.86	5832		7358	4.41	2642	37
24	3361	2.85	5739		7623	4.41	2377	36
25	3533	2.85	5645		7887	4.41	2113	35
26	3704	2.85	5552		8152	4.40	1848	34
27	3875	2.85	5459	1.55	8416	4.40	1584	33
28	4046	2.85	5366	1.56	8680	4.40	1320	32
29	4217	2.85	5272		8945	4.40	1055	31
30	4388	2.84	5179		9209	4.40	0791	30
31	9.774558	2.84	9.905085		9473	4.40	0527	29
32	4729	2.84	4992		9.869737	4.40	10.130263	28
33	4899	2.84	4898		9.870001	4.40	10.129999	27
34	5070	2.84	4804		0265	4.40	9735	26
35	5240	2.84	4711		0529	4.40	9471	25
36	5410	2.83	4617		0793	4.40	9207	24
37	5580	2.83	4523	1.56	1057	4.40	8943	23
38	5750	2.83	4429	1.57	1321	4.40	8679	22
39	5920	2.83	4335		1585	4.40	8415	21
40	6090	2.83	4241		1849	4.39	8151	20
41	9.776259	2.83	9.904147		9.872112	4.39	10.127888	19
42	6429	2.82	4053		2376	4.39	7624	18
43	6598	2.82	3959		2640	4.39	7360	17
44	6768	2.82	3864		2903	4.39	7097	16
45	6937	2.82	3770		3167	4.39	6833	15
46	7106	2.82	3676		3430	4.39	6570	14
47	7275	2.81	3581		3694	4.39	6306	13
48	7444	2.81	3487	1.57	3957	4.39	6043	12
49	7613	2.81	3392	1.58	4220	4.39	5780	11
50	7781	2.81	3298		4484	4.39	5516	10
51	9.777950	2.81	9.903203		9.874747	4.39	10.125253	9
52	8119	2.81	3108		5010	4.39	4990	8
53	8287	2.80	3014		5273	4.38	4727	7
54	8455	2.80	2919		5536	4.38	4464	6
55	8624	2.80	2824		5800	4.38	4200	5
56	8792	2.80	2729		6063	4.38	3937	4
57	8960	2.80	2634		6326	4.38	3674	3
58	9128	2.80	2539	1.58	6589	4.38	3411	2
59	9295	2.79	2444	1.59	6851	4.38	3149	1
60	9.779463		9.902349		9.877114		10.122886	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.779463	2.79	9.902349	1.59	9.877114	4.38	10.122886	60
1	9631	2.79	2253		7377	4.38	2623	59
2	9798	2.79	2158		7640	4.38	2360	58
3	9.779966	2.79	2063		7903	4.38	2097	57
4	9.780133	2.79	1967		8165	4.38	1835	56
5	0300	2.78	1872		8428	4.38	1572	55
6	0467	2.78	1776		8691	4.38	1309	54
7	0634	2.78	1681		8953	4.37	1047	53
8	0801	2.78	1585		9216	4.37	0784	52
9	0968	2.78	1490	1.59	9478	4.37	0522	51
10	1134	2.78	1394	1.60	9.879741	4.37	10.120259	50
11	9.781301	2.77	9.901298		9.880003	4.37	10.119997	49
12	1468	2.77	1202		0265	4.37	9735	48
13	1634	2.77	1106		0528	4.37	9472	47
14	1800	2.77	1010		0790	4.37	9210	46
15	1966	2.77	0914		1052	4.37	8948	45
16	2132	2.77	0818		1314	4.37	8686	44
17	2298	2.76	0722		1576	4.37	8424	43
18	2464	2.76	0626		1839	4.37	8161	42
19	2630	2.76	0529	1.60	2101	4.37	7899	41
20	2796	2.76	0433	1.61	2363	4.36	7637	40
21	9.782961	2.76	9.900337		9.882625	4.36	10.117375	39
22	3127	2.76	0240		2887	4.36	7113	38
23	3292	2.75	0144		3148	4.36	6852	37
24	3458	2.75	9.900047		3410	4.36	6590	36
25	3623	2.75	9.899951		3672	4.36	6328	35
26	3788	2.75	9854		3934	4.36	6066	34
27	3953	2.75	9757		4196	4.36	5804	33
28	4118	2.74	9660		4457	4.36	5543	32
29	4282	2.74	9564	1.61	4719	4.36	5281	31
30	4447	2.74	9467	1.62	4980	4.36	5020	30
31	9.784612	2.74	9.899370		9.885242	4.36	10.114758	29
32	4776	2.74	9273		5503	4.36	4497	28
33	4941	2.74	9176		5765	4.36	4235	27
34	5105	2.74	9078		6026	4.36	3974	26
35	5269	2.73	8981		6288	4.36	3712	25
36	5433	2.73	8884		6549	4.35	3451	24
37	5597	2.73	8787		6810	4.35	3190	23
38	5761	2.73	8689		7072	4.35	2928	22
39	5925	2.73	8592	1.62	7333	4.35	2667	21
40	6089	2.73	8494	1.63	7594	4.35	2406	20
41	9.786252	2.72	9.898397		9.887855	4.35	10.112145	19
42	6416	2.72	8299		8116	4.35	1884	18
43	6579	2.72	8202		8377	4.35	1623	17
44	6742	2.72	8104		8639	4.35	1361	16
45	6906	2.72	8006		8900	4.35	1100	15
46	7069	2.72	7908		9160	4.35	0840	14
47	7232	2.71	7810		9421	4.35	0579	13
48	7395	2.71	7712		9682	4.35	0318	12
49	7557	2.71	7614		9.889943	4.35	10.110057	11
50	7720	2.71	7516	1.63	9.890204	4.34	10.109796	10
51	9.787883	2.71	9.897418	1.64	0465	4.34	9535	9
52	8045	2.71	7320		0725	4.34	9275	8
53	8208	2.71	7222		0986	4.34	9014	7
54	8370	2.70	7123		1247	4.34	8753	6
55	8532	2.70	7025		1507	4.34	8493	5
56	8694	2.70	6926		1768	4.34	8232	4
57	8856	2.70	6828		2028	4.34	7972	3
58	9018	2.70	6729		2289	4.34	7711	2
59	9180	2.70	6631	1.64	2549	4.34	7451	1
60	9.789342		9.896532		9.892810		10.107190	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.789342	2.69	9.896532	1.64	9.892810	4.34	10.107190	60
1	9504	2.69	6433	1.65	3070	4.34	6930	59
2	9665	2.69	6335		3331	4.34	6669	58
3	9827	2.69	6236		3591	4.34	6409	57
4	9.789988	2.69	6137		3851	4.34	6149	56
5	9.790149	2.69	6038		4111	4.34	5889	55
6	0310	2.68	5939		4371	4.34	5629	54
7	0471	2.68	5840		4632	4.33	5368	53
8	0632	2.68	5741		4892	4.33	5108	52
9	0793	2.68	5641		5152	4.33	4848	51
10	0954	2.68	5542	1.65	5412	4.33	4588	50
11	9.791115	2.68	9.895443	1.66	9.895672	4.33	10.104328	49
12	1275	2.67	5343		5932	4.33	4068	48
13	1436	2.67	5244		6192	4.33	3808	47
14	1596	2.67	5145		6452	4.33	3548	46
15	1757	2.67	5045		6712	4.33	3288	45
16	1917	2.67	4945		6971	4.33	3029	44
17	2077	2.67	4846		7231	4.33	2769	43
18	2237	2.66	4746		7491	4.33	2509	42
19	2397	2.66	4646		7751	4.33	2249	41
20	2557	2.66	4546	1.66	8010	4.33	1990	40
21	9.792716	2.66	9.894446	1.67	9.898270	4.33	10.101730	39
22	2876	2.66	4346		8530	4.33	1470	38
23	3035	2.66	4246		8789	4.32	1211	37
24	3195	2.66	4146		9049	4.32	0951	36
25	3354	2.65	4046		9308	4.32	0692	35
26	3514	2.65	3946		9568	4.32	0432	34
27	3673	2.65	3846		9.899827	4.32	10.100173	33
28	3832	2.65	3745		9.900086	4.32	10.099914	32
29	3991	2.65	3645		0346	4.32	9654	31
30	4150	2.64	3544	1.67	0605	4.32	9395	30
31	9.794308	2.64	9.893444	1.68	9.900864	4.32	10.099136	29
32	4467	2.64	3343		1124	4.32	8876	28
33	4626	2.64	3243		1383	4.32	8617	27
34	4784	2.64	3142		1642	4.32	8358	26
35	4942	2.64	3041		1901	4.32	8099	25
36	5101	2.64	2940		2160	4.32	7840	24
37	5259	2.63	2839		2419	4.32	7581	23
38	5417	2.63	2739		2679	4.32	7321	22
39	5575	2.63	2638		2938	4.32	7062	21
40	5733	2.63	2536	1.68	3197	4.31	6803	20
41	9.795891	2.63	9.892435	1.69	9.903455	4.31	10.096545	19
42	6049	2.63	2334		3714	4.31	6286	18
43	6206	2.63	2233		3973	4.31	6027	17
44	6364	2.62	2132		4232	4.31	5768	16
45	6521	2.62	2030		4491	4.31	5509	15
46	6679	2.62	1929		4750	4.31	5250	14
47	6836	2.62	1827		5008	4.31	4992	13
48	6993	2.62	1726		5267	4.31	4733	12
49	7150	2.62	1624	1.69	5526	4.31	4474	11
50	7307	2.61	1523	1.70	5784	4.31	4216	10
51	9.797464	2.61	9.891421		9.906043	4.31	10.093957	9
52	7621	2.61	1319		6302	4.31	3698	8
53	7777	2.61	1217		6560	4.31	3440	7
54	7934	2.61	1115		6819	4.31	3181	6
55	8091	2.61	1013		7077	4.31	2923	5
56	8247	2.61	0911		7336	4.31	2664	4
57	8403	2.60	0809		7594	4.31	2406	3
58	8560	2.60	0707		7852	4.31	2148	2
59	8716	2.60	0605	1.70	8111	4.30	1889	1
60	9.798872		9.890503		9.908369		10.091631	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.798872	2.60	9.890503	1.70	9.908369	4.30	10.091631	60
1	9028	2.60	0400	1.71	8628	4.30	1372	59
2	9184	2.60	0298		8886	4.30	1114	58
3	9339	2.59	0195		9144	4.30	0856	57
4	9495	2.59	9.890093		9402	4.30	0598	56
5	9651	2.59	9.889990		9660	4.30	0340	55
6	9806	2.59	9888		9.909918	4.30	10.090082	54
7	9.799962	2.59	9785		9.910177	4.30	10.089823	53
8	9.800117	2.59	9682		0435	4.30	9565	52
9	0272	2.58	9579		0693	4.30	9307	51
10	0427	2.58	9477	1.71	0951	4.30	9049	50
11	9.800582	2.58	9.889374	1.72	9.911209	4.30	10.088791	49
12	0737	2.58	9271		1467	4.30	8533	48
13	0892	2.58	9168		1724	4.30	8276	47
14	1047	2.58	9064		1982	4.30	8018	46
15	1201	2.58	8961		2240	4.30	7760	45
16	1356	2.57	8858		2498	4.30	7502	44
17	1511	2.57	8755		2756	4.30	7244	43
18	1665	2.57	8651		3014	4.29	6986	42
19	1819	2.57	8548	1.72	3271	4.29	6729	41
20	1973	2.57	8444	1.73	3529	4.29	6471	40
21	9.802128	2.57	9.888341		9.913787	4.29	10.086213	39
22	2282	2.56	8237		4044	4.29	5956	38
23	2436	2.56	8134		4302	4.29	5698	37
24	2589	2.56	8030		4560	4.29	5440	36
25	2743	2.56	7926		4817	4.29	5183	35
26	2897	2.56	7822		5075	4.29	4925	34
27	3050	2.56	7718		5332	4.29	4668	33
28	3204	2.56	7614		5590	4.29	4410	32
29	3357	2.55	7510	1.73	5847	4.29	4153	31
30	3511	2.55	7406	1.74	6104	4.29	3896	30
31	9.803664	2.55	9.887302		9.916362	4.29	10.083638	29
32	3817	2.55	7198		6619	4.29	3381	28
33	3970	2.55	7093		6877	4.29	3123	27
34	4123	2.55	6989		7134	4.29	2866	26
35	4276	2.54	6885		7391	4.29	2609	25
36	4428	2.54	6780		7648	4.29	2352	24
37	4581	2.54	6676		7905	4.29	2095	23
38	4734	2.54	6571		8163	4.28	1837	22
39	4886	2.54	6466	1.74	8420	4.28	1580	21
40	5039	2.54	6362	1.75	8677	4.28	1323	20
41	9.805191	2.54	9.886257		9.918934	4.28	10.081066	19
42	5343	2.53	6152		9191	4.28	0809	18
43	5495	2.53	6047		9448	4.28	0552	17
44	5647	2.53	5942		9705	4.28	0295	16
45	5799	2.53	5837		9.919962	4.28	10.080038	15
46	5951	2.53	5732		9.920219	4.28	10.079781	14
47	6103	2.53	5627		0476	4.28	9524	13
48	6254	2.53	5522		0733	4.28	9267	12
49	6406	2.52	5416	1.75	0990	4.28	9010	11
50	6557	2.52	5311	1.76	1247	4.28	8753	10
51	9.806709	2.52	9.885205		9.921503	4.28	10.078497	9
52	6860	2.52	5100		1760	4.28	8240	8
53	7011	2.52	4994		2017	4.28	7983	7
54	7163	2.52	4889		2274	4.28	7726	6
55	7314	2.52	4783		2530	4.28	7470	5
56	7465	2.51	4677		2787	4.28	7213	4
57	7615	2.51	4572	1.76	3044	4.28	6956	3
58	7766	2.51	4466	1.77	3300	4.28	6700	2
59	7917	2.51	4360	1.77	3557	4.27	6443	1
60	9.808067		9.884254		9.923813		10.076187	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

129° 50°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.808067	2.51	9.884254	1.77	9.923813	4.28	10.076187	60
1	8218	2.51	4148		4070	4.27	5930	59
2	8368	2.51	4042		4327	4.27	5673	58
3	8519	2.50	3936		4583	4.27	5417	57
4	8669	2.50	3829		4840	4.27	5160	56
5	8819	2.50	3723		5096	4.27	4904	55
6	8969	2.50	3617		5352	4.27	4648	54
7	9119	2.50	3510		5609	4.27	4391	53
8	9269	2.50	3404	1.77	5865	4.27	4135	52
9	9419	2.49	3297	1.78	6122	4.27	3878	51
10	9569	2.49	3191		6378	4.27	3622	50
11	9718	2.49	9.883084		9.926634	4.27	10.073366	49
12	9.809868	2.49	2977		6890	4.27	3110	48
13	9.810017	2.49	2871		7147	4.27	2853	47
14	0167	2.49	2764		7403	4.27	2597	46
15	0316	2.48	2657		7659	4.27	2341	45
16	0465	2.48	2550		7915	4.27	2085	44
17	0614	2.48	2443	1.78	8171	4.27	1829	43
18	0763	2.48	2336	1.79	8427	4.27	1573	42
19	0912	2.48	2229		8683	4.27	1317	41
20	1061	2.48	2121		8940	4.27	1060	40
21	9.811210	2.48	9.882014		9.929196	4.27	10.070804	39
22	1358	2.48	1907		9452	4.27	0548	38
23	1507	2.47	1799		9708	4.27	0292	37
24	1655	2.47	1692		9.929964	4.27	10.070036	36
25	1804	2.47	1584		9.930220	4.26	10.069780	35
26	1952	2.47	1477		0475	4.26	9525	34
27	2100	2.47	1369	1.79	0731	4.26	9269	33
28	2248	2.47	1261	1.80	0987	4.26	9013	32
29	2396	2.46	1153		1243	4.26	8757	31
30	2544	2.46	1046		1499	4.26	8501	30
31	9.812692	2.46	9.880938		9.931755	4.26	10.068245	29
32	2840	2.46	0830		2010	4.26	7990	28
33	2988	2.46	0722		2266	4.26	7734	27
34	3135	2.46	0613		2522	4.26	7478	26
35	3283	2.46	0505		2778	4.26	7222	25
36	3430	2.46	0397	1.80	3033	4.26	6967	24
37	3578	2.45	0289	1.81	3289	4.26	6711	23
38	3725	2.45	0180		3545	4.26	6455	22
39	3872	2.45	9.880072		3800	4.26	6200	21
40	4019	2.45	9.879963		4056	4.26	5944	20
41	9.814166	2.45	9855		9.934311	4.26	10.065689	19
42	4313	2.45	9746		4567	4.26	5433	18
43	4460	2.44	9637		4823	4.26	5177	17
44	4607	2.44	9529		5078	4.26	4922	16
45	4753	2.44	9420		5333	4.26	4667	15
46	4900	2.44	9311	1.81	5589	4.26	4411	14
47	5046	2.44	9202	1.82	5844	4.26	4156	13
48	5193	2.44	9093		6100	4.26	3900	12
49	5339	2.44	8984		6355	4.26	3645	11
50	5485	2.43	8875		6610	4.26	3390	10
51	9.815631	2.43	9.878766		9.936866	4.25	10.063134	9
52	5778	2.43	8656		7121	4.25	2879	8
53	5924	2.43	8547		7376	4.25	2624	7
54	6069	2.43	8438		7632	4.25	2368	6
55	6215	2.43	8328	1.82	7887	4.25	2113	5
56	6361	2.43	8219	1.83	8142	4.25	1858	4
57	6507	2.42	8109		8398	4.25	1602	3
58	6652	2.42	7999		8653	4.25	1347	2
59	6798	2.42	7890	1.83	8908	4.25	1092	1
60	9.816943		9.877780		9.939163		10.060837	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

130° 49°

41° SINES AND TANGENTS. 138°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.816943	2.42	9.877780	1.83	9.939163	4.25	10.060837	60
1	7088	2.42	7670		9418	4.25	0582	59
2	7233	2.42	7560		9673	4.25	0327	58
3	7379	2.42	7450		9.939928	4.25	10.060072	57
4	7524	2.42	7340	1.83	9.940183	4.25	10.059817	56
5	7668	2.41	7230	1.84	0438	4.25	9562	55
6	7813	2.41	7120		0694	4.25	9306	54
7	7958	2.41	7010		0949	4.25	9051	53
8	8103	2.41	6899		1204	4.25	8796	52
9	8247	2.41	6789		1458	4.25	8542	51
10	8392	2.41	6678		1714	4.25	8286	50
11	9.818536	2.40	9.876568		9.941968	4.25	10.058032	49
12	8681	2.40	6457		2223	4.25	7777	48
13	8825	2.40	6347	1.84	2478	4.25	7522	47
14	8969	2.40	6236	1.85	2733	4.25	7267	46
15	9113	2.40	6125		2988	4.25	7012	45
16	9257	2.40	6014		3243	4.25	6757	44
17	9401	2.40	5904		3498	4.25	6502	43
18	9545	2.40	5793		3752	4.25	6248	42
19	9689	2.39	5682		4007	4.25	5993	41
20	9832	2.39	5571		4262	4.25	5738	40
21	9.819976	2.39	9.875459		9.944517	4.25	10.055483	39
22	9.820120	2.39	5348		4771	4.24	5229	38
23	0263	2.39	5237	1.85	5026	4.24	4974	37
24	0406	2.39	5126	1.86	5281	4.24	4719	36
25	0550	2.38	5014		5535	4.24	4465	35
26	0693	2.38	4903		5790	4.24	4210	34
27	0836	2.38	4791		6045	4.24	3955	33
28	0979	2.38	4680		6299	4.24	3701	32
29	1122	2.38	4568		6554	4.24	3446	31
30	1265	2.38	4456		6808	4.24	3192	30
31	9.821407	2.38	9.874344	1.86	9.947063	4.24	10.052937	29
32	1550	2.38	4232	1.87	7318	4.24	2682	28
33	1693	2.37	4121		7572	4.24	2428	27
34	1835	2.37	4009		7826	4.24	2174	26
35	1977	2.37	3896		8081	4.24	1919	25
36	2120	2.37	3784		8336	4.24	1664	24
37	2262	2.37	3672		8590	4.24	1410	23
38	2404	2.37	3560		8844	4.24	1156	22
39	2546	2.37	3448		9099	4.24	0901	21
40	2688	2.36	3335		9353	4.24	0647	20
41	9.822830	2.36	9.873223	1.87	9607	4.24	0393	19
42	2972	2.36	3110	1.88	9.949862	4.24	10.050138	18
43	3114	2.36	2998		9.950116	4.24	10.049884	17
44	3255	2.36	2885		0370	4.24	9630	16
45	3397	2.36	2772		0625	4.24	9375	15
46	3539	2.36	2659		0879	4.24	9121	14
47	3680	2.35	2547		1133	4.24	8867	13
48	3821	2.35	2434		1388	4.24	8612	12
49	3963	2.35	2321		1642	4.24	8358	11
50	4104	2.35	2208	1.88	1896	4.24	8104	10
51	9.824245	2.35	9.872095	1.89	9.952150	4.24	10.047850	9
52	4386	2.35	1981		2405	4.24	7595	8
53	4527	2.35	1868		2659	4.24	7341	7
54	4668	2.34	1755		2913	4.24	7087	6
55	4808	2.34	1641		3167	4.23	6833	5
56	4949	2.34	1528		3421	4.23	6579	4
57	5090	2.34	1414		3675	4.23	6325	3
58	5230	2.34	1301		3929	4.23	6071	2
59	5371	2.34	1187	1.89	4183	4.23	5817	1
60	9.825511		9.871073		9.954437		10.045563	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

131° 48°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.825511	2.34	9.871073	1.90	9.954437	4.23	10.045563	60
1	5651	2.33	0960		4691	4.23	5309	59
2	5791	2.33	0846		4945	4.23	5055	58
3	5931	2.33	0732		5200	4.23	4800	57
4	6071	2.33	0618		5454	4.23	4546	56
5	6211	2.33	0504		5707	4.23	4293	55
6	6351	2.33	0390		5961	4.23	4039	54
7	6491	2.33	0276		6215	4.23	3785	53
8	6631	2.33	0161	1.90	6469	4.23	3531	52
9	6770	2.32	9.870047	1.91	6723	4.23	3277	51
10	6910	2.32	9.869933		6977	4.23	3023	50
11	9.827049	2.32	9818		9.957231	4.23	10.042769	49
12	7189	2.32	9704		7485	4.23	2515	48
13	7328	2.32	9589		7739	4.23	2261	47
14	7467	2.32	9474		7993	4.23	2007	46
15	7606	2.32	9360		8246	4.23	1754	45
16	7745	2.32	9245		8500	4.23	1500	44
17	7884	2.31	9130	1.91	8754	4.23	1246	43
18	8023	2.31	9015	1.92	9008	4.23	0992	42
19	8162	2.31	8900		9262	4.23	0738	41
20	8301	2.31	8785		9516	4.23	0484	40
21	9.828439	2.31	9.868670		9.959769	4.23	10.040231	39
22	8578	2.31	8555		9.960023	4.23	10.039977	38
23	8716	2.31	8440		0277	4.23	9723	37
24	8855	2.30	8324		0531	4.23	9469	36
25	8993	2.30	8209		0784	4.23	9216	35
26	9131	2.30	8093	1.92	1038	4.23	8962	34
27	9269	2.30	7978	1.93	1291	4.23	8709	33
28	9407	2.30	7862		1545	4.23	8455	32
29	9545	2.30	7747		1799	4.23	8201	31
30	9683	2.30	7631		2052	4.23	7948	30
31	9821	2.29	9.867515		9.962306	4.23	10.037694	29
32	9.829959	2.29	7399		2560	4.23	7440	28
33	9.830097	2.29	7283		2813	4.23	7187	27
34	0234	2.29	7167		3067	4.23	6933	26
35	0372	2.29	7051	1.93	3320	4.23	6680	25
36	0509	2.29	6935	1.94	3574	4.23	6426	24
37	0646	2.29	6819		3827	4.23	6173	23
38	0784	2.29	6703		4081	4.23	5919	22
39	0921	2.28	6586		4335	4.23	5665	21
40	1058	2.28	6470		4588	4.22	5412	20
41	9.831195	2.28	9.866353		9.964842	4.22	10.035158	19
42	1332	2.28	6237		5095	4.22	4905	18
43	1469	2.28	6120	1.94	5349	4.22	4651	17
44	1606	2.28	6004	1.95	5602	4.22	4398	16
45	1742	2.28	5887		5855	4.22	4145	15
46	1879	2.28	5770		6109	4.22	3891	14
47	2015	2.27	5653		6362	4.22	3638	13
48	2152	2.27	5536		6616	4.22	3384	12
49	2288	2.27	5419		6869	4.22	2131	11
50	2425	2.27	5302		7123	4.22	3877	10
51	9.832561	2.27	9.865185		9.967376	4.22	10.032624	9
52	2697	2.27	5068		7629	4.22	2371	8
53	2833	2.27	4950	1.95	7883	4.22	2117	7
54	2969	2.26	4833	1.96	8136	4.22	1864	6
55	3105	2.26	4716		8389	4.22	1611	5
56	3241	2.26	4598		8643	4.22	1357	4
57	3377	2.26	4481		8896	4.22	1104	3
58	3512	2.26	4363		9149	4.22	0851	2
59	3648	2.26	4245	1.96	9403	4.22	0597	1
60	9.833783		9.864127		9.969656		10.030344	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.833783	2.26	9.864127	1.96	9.969656	4.22	10.030344	60
1	3919	2.25	4010	1.96	9909	4.22	0091	59
2	4054	2.25	3892	1.97	9.970162	4.22	10.029838	58
3	4189	2.25	3774		0416	4.22	9584	57
4	4325	2.25	3656		0669	4.22	9331	56
5	4460	2.25	3538		0922	4.22	9078	55
6	4595	2.25	3419		1175	4.22	8825	54
7	4730	2.25	3301		1429	4.22	8571	53
8	4865	2.25	3183		1682	4.22	8318	52
9	4999	2.24	3064	1.97	1935	4.22	8065	51
10	5134	2.24	2946	1.98	2188	4.22	7812	50
11	9.835269	2.24	9.862827		9.972441	4.22	10.027559	49
12	5403	2.24	2709		2694	4.22	7306	48
13	5538	2.24	2590		2948	4.22	7052	47
14	5672	2.24	2471		3201	4.22	6799	46
15	5807	2.24	2353		3454	4.22	6546	45
16	5941	2.24	2234		3707	4.22	6293	44
17	6075	2.23	2115		3960	4.22	6040	43
18	6209	2.23	1996		4213	4.22	5787	42
19	6343	2.23	1877	1.98	4466	4.22	5534	41
20	6477	2.23	1758	1.99	4719	4.22	5281	40
21	9.836611	2.23	9.861638		9.974973	4.22	10.025027	39
22	6745	2.23	1519		5226	4.22	4774	38
23	6878	2.23	1400		5479	4.22	4521	37
24	7012	2.22	1280		5732	4.22	4268	36
25	7146	2.22	1161		5985	4.22	4015	35
26	7279	2.22	1041		6238	4.22	3762	34
27	7412	2.22	0922		6491	4.22	3509	33
28	7546	2.22	0802	1.99	6744	4.22	3256	32
29	7679	2.22	0682	2.00	6997	4.22	3003	31
30	7812	2.22	0562		7250	4.22	2750	30
31	9.837945	2.22	9.860442		9.977503	4.22	10.022497	29
32	8078	2.21	0322		7756	4.22	2244	28
33	8211	2.21	0202		8009	4.22	1991	27
34	8344	2.21	9.860082		8262	4.22	1738	26
35	8477	2.21	9.859962		8515	4.22	1485	25
36	8610	2.21	9842	2.00	8768	4.22	1232	24
37	8742	2.21	9721	2.01	9021	4.22	0979	23
38	8875	2.21	9601		9274	4.22	0726	22
39	9007	2.21	9480		9527	4.22	0473	21
40	9140	2.20	9360		9.979780	4.22	10.020220	20
41	9.839272	2.20	9.859239		9.980033	4.22	10.019967	19
42	9404	2.20	9119		0286	4.22	9714	18
43	9536	2.20	8998		0538	4.22	9462	17
44	9668	2.20	8877	2.01	0791	4.21	9209	16
45	9800	2.20	8756	2.02	1044	4.21	8956	15
46	9.839932	2.20	8635		1297	4.21	8703	14
47	9.840064	2.19	8514		1550	4.21	8450	13
48	0196	2.19	8393		1803	4.21	8197	12
49	0328	2.19	8272		2056	4.21	7944	11
50	0459	2.19	8151		2309	4.21	7691	10
51	9.840591	2.19	9.858029		9.982562	4.21	10.017438	9
52	0722	2.19	7908		2814	4.21	7186	8
53	0854	2.19	7786	2.02	3067	4.21	6933	7
54	0985	2.19	7665	2.03	3320	4.21	6680	6
55	1116	2.19	7543		3573	4.21	6427	5
56	1247	2.18	7422		3826	4.21	6174	4
57	1378	2.18	7300		4079	4.21	5921	3
58	1509	2.18	7178		4331	4.21	5669	2
59	1640	2.18	7056	2.03	4584	4.21	5416	1
60	9.841771		9.856934		9.984837		10.015163	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	
0	9.841771	2.18	9.856934	2.03	9.984837	4.21	10.015163	60
1	1902	2.18	6812	2.03	5090	4.21	4910	59
2	2033	2.18	6690	2.04	5343	4.21	4657	58
3	2163	2.17	6568		5596	4.21	4404	57
4	2294	2.17	6446		5848	4.21	4152	56
5	2424	2.17	6323		6101	4.21	3899	55
6	2555	2.17	6201		6354	4.21	3646	54
7	2685	2.17	6078		6607	4.21	3393	53
8	2815	2.17	5956		6860	4.21	3140	52
9	2946	2.17	5833	2.04	7112	4.21	2888	51
10	3076	2.17	5711	2.05	7365	4.21	2635	50
11	9.843206	2.16	9.855588		9.987618	4.21	10.012382	49
12	3336	2.16	5465		7871	4.21	2129	48
13	3466	2.16	5342		8123	4.21	1877	47
14	3595	2.16	5219		8376	4.21	1624	46
15	3725	2.16	5096		8629	4.21	1371	45
16	3855	2.16	4973		8882	4.21	1118	44
17	3984	2.16	4850		9134	4.21	0866	43
18	4114	2.16	4727	2.05	9387	4.21	0613	42
19	4243	2.15	4603	2.06	9640	4.21	0360	41
20	4372	2.15	4480		9.989893	4.21	10.010107	40
21	9.844502	2.15	9.854356		9.990145	4.21	10.009855	39
22	4631	2.15	4233		0398	4.21	9602	38
23	4760	2.15	4109		0651	4.21	9349	37
24	4889	2.15	3986		0903	4.21	9097	36
25	5018	2.15	3862		1156	4.21	8844	35
26	5147	2.15	3738	2.06	1409	4.21	8591	34
27	5276	2.14	3614	2.07	1662	4.21	8338	33
28	5405	2.14	3490		1914	4.21	8086	32
29	5533	2.14	3366		2167	4.21	7833	31
30	5662	2.14	3242		2420	4.21	7580	30
31	9.845790	2.14	9.853118		9.992672	4.21	10.007328	29
32	5919	2.14	2994		2925	4.21	7075	28
33	6047	2.14	2869		3178	4.21	6822	27
34	6175	2.14	2745		3430	4.21	6570	26
35	6304	2.14	2620	2.07	3683	4.21	6317	25
36	6432	2.13	2496	2.08	3936	4.21	6064	24
37	6560	2.13	2371		4189	4.21	5811	23
38	6688	2.13	2247		4441	4.21	5559	22
39	6816	2.13	2122		4694	4.21	5306	21
40	6944	2.13	1997		4947	4.21	5053	20
41	9.847071	2.13	9.851872		9.995199	4.21	10.004801	19
42	7199	2.13	1747		5452	4.21	4548	18
43	7327	2.13	1622	2.08	5705	4.21	4295	17
44	7454	2.12	1497	2.09	5957	4.21	4043	16
45	7582	2.12	1372		6210	4.21	3790	15
46	7709	2.12	1246		6463	4.21	3537	14
47	7836	2.12	1121		6715	4.21	3285	13
48	7964	2.12	0996		6968	4.21	3032	12
49	8091	2.12	0870		7221	4.21	2779	11
50	8218	2.12	0745		7473	4.21	2527	10
51	9.848345	2.12	9.850619	2.09	9.997726	4.21	10.002274	9
52	8472	2.11	0493	2.10	7979	4.21	2021	8
53	8599	2.11	0368		8231	4.21	1769	7
54	8726	2.11	0242		8484	4.21	1516	6
55	8852	2.11	9.850116		8737	4.21	1263	5
56	8979	2.11	9.849990		8989	4.21	1011	4
57	9106	2.11	9864		9242	4.21	0758	3
58	9232	2.11	9738		9495	4.21	0505	2
59	9359	2.11	9611	2.10	9.999747	4.21	0253	1
60	9.849485		9.849485		10.000000		10.000000	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	M.

TABLE

OF

NATURAL SINES

AND

COSINES.

NATURAL SINES AND COSINES.

′	0°		1°		2°		3°		4°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	00000	Unit.	01745	99985	03490	99939	05234	99863	06976	99756	60
1	00029	Unit.	01774	99984	03519	99938	05263	99861	07005	99754	59
2	00058	Unit.	01803	99984	03548	99937	05292	99860	07034	99752	58
3	00087	Unit.	01832	99983	03577	99936	05321	99858	07063	99750	57
4	00116	Unit.	01862	99983	03606	99935	05350	99857	07092	99748	56
5	00145	Unit.	01891	99982	03635	99934	05379	99855	07121	99746	55
6	00175	Unit.	01920	99982	03664	99933	05408	99854	07150	99744	54
7	00204	Unit.	01949	99981	03693	99932	05437	99852	07179	99742	53
8	00233	Unit.	01978	99980	03723	99931	05466	99851	07208	99740	52
9	00262	Unit.	02007	99980	03752	99930	05495	99849	07237	99738	51
10	00291	Unit.	02036	99979	03781	99929	05524	99847	07266	99736	50
11	00320	99999	02065	99979	03810	99927	05553	99846	07295	99734	49
12	00349	99999	02094	99978	03839	99926	05582	99844	07324	99731	48
13	00378	99999	02123	99977	03868	99925	05611	99842	07353	99729	47
14	00407	99999	02152	99977	03897	99924	05640	99841	07382	99727	46
15	00436	99999	02181	99976	03926	99923	05669	99839	07411	99725	45
16	00465	99999	02211	99976	03955	99922	05698	99838	07440	99723	44
17	00495	99999	02240	99975	03984	99921	05727	99836	07469	99721	43
18	00524	99999	02269	99974	04013	99919	05756	99834	07498	99719	42
19	00553	99998	02298	99974	04042	99918	05785	99833	07527	99716	41
20	00582	99998	02327	99973	04071	99917	05814	99831	07556	99714	40
21	00611	99998	02356	99972	04100	99916	05844	99829	07585	99712	39
22	00640	99998	02385	99972	04129	99915	05873	99827	07614	99710	38
23	00669	99998	02414	99971	04159	99913	05902	99826	07643	99708	37
24	00698	99998	02443	99970	04188	99912	05931	99824	07672	99705	36
25	00727	99997	02472	99969	04217	99911	05960	99822	07701	99703	35
26	00756	99997	02501	99969	04246	99910	05989	99821	07730	99701	34
27	00785	99997	02530	99968	04275	99909	06018	99819	07759	99699	33
28	00814	99997	02560	99967	04304	99907	06047	99817	07788	99696	32
29	00844	99996	02589	99966	04333	99906	06076	99815	07817	99694	31
30	00873	99996	02618	99966	04362	99905	06105	99813	07846	99692	30
31	00902	99996	02647	99965	04391	99904	06134	99812	07875	99689	29
32	00931	99996	02676	99964	04420	99902	06163	99810	07904	99687	28
33	00960	99995	02705	99963	04449	99901	06192	99808	07933	99685	27
34	00989	99995	02734	99963	04478	99900	06221	99806	07962	99683	26
35	01018	99995	02763	99962	04507	99898	06250	99804	07991	99680	25
36	01047	99995	02792	99961	04536	99897	06279	99803	08020	99678	24
37	01076	99994	02821	99960	04565	99896	06308	99801	08049	99676	23
38	01105	99994	02850	99959	04594	99894	06337	99799	08078	99673	22
39	01134	99994	02879	99959	04623	99893	06366	99797	08107	99671	21
40	01164	99993	02908	99958	04653	99892	06395	99795	08136	99668	20
41	01193	99993	02938	99957	04682	99890	06424	99793	08165	99666	19
42	01222	99993	02967	99956	04711	99889	06453	99792	08194	99664	18
43	01251	99992	02996	99955	04740	99888	06482	99790	08223	99661	17
44	01280	99992	03025	99954	04769	99886	06511	99788	08252	99659	16
45	01309	99991	03054	99953	04798	99885	06540	99786	08281	99657	15
46	01338	99991	03083	99952	04827	99883	06569	99784	08310	99654	14
47	01367	99991	03112	99952	04856	99882	06598	99782	08339	99652	13
48	01396	99990	03141	99951	04885	99881	06627	99780	08368	99649	12
49	01425	99990	03170	99950	04914	99879	06656	99778	08397	99647	11
50	01454	99989	03199	99949	04943	99878	06685	99776	08426	99644	10
51	01483	99989	03228	99948	04972	99876	06714	99774	08455	99642	9
52	01513	99989	03257	99947	05001	99875	06743	99772	08484	99639	8
53	01542	99988	03286	99946	05030	99873	06773	99770	08513	99637	7
54	01571	99988	03316	99945	05059	99872	06802	99768	08542	99635	6
55	01600	99987	03345	99944	05088	99870	06831	99766	08571	99632	5
56	01629	99987	03374	99943	05117	99869	06860	99764	08600	99630	4
57	01658	99986	03403	99942	05146	99867	06889	99762	08629	99627	3
58	01687	99986	03432	99941	05175	99866	06918	99760	08658	99625	2
59	01716	99985	03461	99940	05205	99864	06947	99758	08687	99622	1
60	01745	99985	03490	99939	05234	99863	06976	99756	08716	99619	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	89°		88°		87°		86°		85°		′

NATURAL SINES AND COSINES.

′	5°		6°		7°		8°		9°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	08716	99619	10453	99452	12187	99255	13917	99027	15643	98769	60
1	08745	99617	10482	99449	12216	99251	13946	99023	15672	98764	59
2	08774	99614	10511	99446	12245	99248	13975	99019	15701	98760	58
3	08803	99612	10540	99443	12274	99244	14004	99015	15730	98755	57
4	08831	99609	10569	99440	12302	99240	14033	99011	15758	98751	56
5	08860	99607	10597	99437	12331	99237	14061	99006	15787	98746	55
6	08889	99604	10626	99434	12360	99233	14090	99002	15816	98741	54
7	08918	99602	10655	99431	12389	99230	14119	98998	15845	98737	53
8	08947	99599	10684	99428	12418	99226	14148	98994	15873	98732	52
9	08976	99596	10713	99424	12447	99222	14177	98990	15902	98728	51
10	09005	99594	10742	99421	12476	99219	14205	98986	15931	98723	50
11	09034	99591	10771	99418	12504	99215	14234	98982	15959	98718	49
12	09063	99588	10800	99415	12533	99211	14263	98978	15988	98714	48
13	09092	99586	10829	99412	12562	99208	14292	98973	16017	98709	47
14	09121	99583	10858	99409	12591	99204	14320	98969	16046	98704	46
15	09150	99580	10887	99406	12620	99200	14349	98965	16074	98700	45
16	09179	99578	10916	99402	12649	99197	14378	98961	16103	98695	44
17	09208	99575	10945	99399	12678	99193	14407	98957	16132	98690	43
18	09237	99572	10973	99396	12706	99189	14436	98953	16160	98686	42
19	09266	99570	11002	99393	12735	99186	14464	98948	16189	98681	41
20	09295	99567	11031	99390	12764	99182	14493	98944	16218	98676	40
21	09324	99564	11060	99386	12793	99178	14522	98940	16246	98671	39
22	09353	99562	11089	99383	12822	99175	14551	98936	16275	98667	38
23	09382	99559	11118	99380	12851	99171	14580	98931	16304	98662	37
24	09411	99556	11147	99377	12880	99167	14608	98927	16333	98657	36
25	09440	99553	11176	99374	12908	99163	14637	98923	16361	98652	35
26	09469	99551	11205	99370	12937	99160	14666	98919	16390	98648	34
27	09498	99548	11234	99367	12966	99156	14695	98914	16419	98643	33
28	09527	99545	11263	99364	12995	99152	14723	98910	16447	98638	32
29	09556	99542	11291	99360	13024	99148	14752	98906	16476	98633	31
30	09585	99540	11320	99357	13053	99144	14781	98902	16505	98629	30
31	09614	99537	11349	99354	13081	99141	14810	98897	16533	98624	29
32	09642	99534	11378	99351	13110	99137	14838	98893	16562	98619	28
33	09671	99531	11407	99347	13139	99133	14867	98889	16591	98614	27
34	09700	99528	11436	99344	13168	99129	14896	98884	16620	98609	26
35	09729	99526	11465	99341	13197	99125	14925	98880	16648	98604	25
36	09758	99523	11494	99337	13226	99122	14954	98876	16677	98600	24
37	09787	99520	11523	99334	13254	99118	14982	98871	16706	98595	23
38	09816	99517	11552	99331	13283	99114	15011	98867	16734	98590	22
39	09845	99514	11580	99327	13312	99110	15040	98863	16763	98585	21
40	09874	99511	11609	99324	13341	99106	15069	98858	16792	98580	20
41	09903	99508	11638	99320	13370	99102	15097	98854	16820	98575	19
42	09932	99506	11667	99317	13399	99098	15126	98849	16849	98570	18
43	09961	99503	11696	99314	13427	99094	15155	98845	16878	98565	17
44	09990	99500	11725	99310	13456	99091	15184	98841	16906	98561	16
45	10019	99497	11754	99307	13485	99087	15212	98836	16935	98556	15
46	10048	99494	11783	99303	13514	99083	15241	98832	16964	98551	14
47	10077	99491	11812	99300	13543	99079	15270	98827	16992	98546	13
48	10106	99488	11840	99297	13572	99075	15299	98823	17021	98541	12
49	10135	99485	11869	99293	13600	99071	15327	98818	17050	98536	11
50	10164	99482	11898	99290	13629	99067	15356	98814	17078	98531	10
51	10192	99479	11927	99286	13658	99063	15385	98809	17107	98526	9
52	10221	99476	11956	99283	13687	99059	15414	98805	17136	98521	8
53	10250	99473	11985	99279	13716	99055	15442	98800	17164	98516	7
54	10279	99470	12014	99276	13744	99051	15471	98796	17193	98511	6
55	10308	99467	12043	99272	13773	99047	15500	98791	17222	98506	5
56	10337	99464	12071	99269	13802	99043	15529	98787	17250	98501	4
57	10366	99461	12100	99265	13831	99039	15557	98782	17279	98496	3
58	10395	99458	12129	99262	13860	99035	15586	98778	17308	98491	2
59	10424	99455	12158	99258	13889	99031	15615	98773	17336	98486	1
60	10453	99452	12187	99255	13917	99027	15643	98769	17365	98481	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	84°		83°		82°		81°		80°		′

NATURAL SINES AND COSINES.

′	10° Sine.	10° Cosine.	11° Sine.	11° Cosine.	12° Sine.	12° Cosine.	13° Sine.	13° Cosine.	14° Sine.	14° Cosine.	′
0	17365	98481	19081	98163	20791	97815	22495	97437	24192	97030	60
1	17393	98476	19109	98157	20820	97809	22523	97430	24220	97023	59
2	17422	98471	19138	98152	20848	97803	22552	97424	24249	97015	58
3	17451	98466	19167	98146	20877	97797	22580	97417	24277	97008	57
4	17479	98461	19195	98140	20905	97791	22608	97411	24305	97001	56
5	17508	98455	19224	98135	20933	97784	22637	97404	24333	96994	55
6	17537	98450	19252	98129	20962	97778	22665	97398	24362	96987	54
7	17565	98445	19281	98124	20990	97772	22693	97391	24390	96980	53
8	17594	98440	19309	98118	21019	97766	22722	97384	24418	96973	52
9	17623	98435	19338	98112	21047	97760	22750	97378	24446	96966	51
10	17651	98430	19366	98107	21076	97754	22778	97371	24474	96959	50
11	17680	98425	19395	98101	21104	97748	22807	97365	24503	96952	49
12	17708	98420	19423	98096	21132	97742	22835	97358	24531	96945	48
13	17737	98414	19452	98090	21161	97735	22863	97351	24559	96937	47
14	17766	98409	19481	98084	21189	97729	22892	97345	24587	96930	46
15	17794	98404	19509	98079	21218	97723	22920	97338	24615	96923	45
16	17823	98399	19538	98073	21246	97717	22948	97331	24644	96916	44
17	17852	98394	19566	98067	21275	97711	22977	97325	24672	96909	43
18	17880	98389	19595	98061	21303	97705	23005	97318	24700	96902	42
19	17909	98383	19623	98056	21331	97698	23033	97311	24728	96894	41
20	17937	98378	19652	98050	21360	97692	23062	97304	24756	96887	40
21	17966	98373	19680	98044	21388	97686	23090	97298	24784	96880	39
22	17995	98368	19709	98039	21417	97680	23118	97291	24813	96873	38
23	18023	98362	19737	98033	21445	97673	23146	97284	24841	96866	37
24	18052	98357	19766	98027	21474	97667	23175	97278	24869	96858	36
25	18081	98352	19794	98021	21502	97661	23203	97271	24897	96851	35
26	18109	98347	19823	98016	21530	97655	23231	97264	24925	96844	34
27	18138	98341	19851	98010	21559	97648	23260	97257	24954	96837	33
28	18166	98336	19880	98004	21587	97642	23288	97251	24982	96829	32
29	18195	98331	19908	97998	21616	97636	23316	97244	25010	96822	31
30	18224	98325	19937	97992	21644	97630	23345	97237	25038	96815	30
31	18252	98320	19965	97987	21672	97623	23373	97230	25066	96807	29
32	18281	98315	19994	97981	21701	97617	23401	97223	25094	96800	28
33	18309	98310	20022	97975	21729	97611	23429	97217	25122	96793	27
34	18338	98304	20051	97969	21758	97604	23458	97210	25151	96786	26
35	18367	98299	20079	97963	21786	97598	23486	97203	25179	96778	25
36	18395	98294	20108	97958	21814	97592	23514	97196	25207	96771	24
37	18424	98288	20136	97952	21843	97585	23542	97189	25235	96764	23
38	18452	98283	20165	97946	21871	97579	23571	97182	25263	96756	22
39	18481	98277	20193	97940	21899	97573	23599	97176	25291	96749	21
40	18509	98272	20222	97934	21928	97566	23627	97169	25320	96742	20
41	18538	98267	20250	97928	21956	97560	23656	97162	25348	96734	19
42	18567	98261	20279	97922	21985	97553	23684	97155	25376	96727	18
43	18595	98256	20307	97916	22013	97547	23712	97148	25404	96719	17
44	18624	98250	20336	97910	22041	97541	23740	97141	25432	96712	16
45	18652	98245	20364	97905	22070	97534	23769	97134	25460	96705	15
46	18681	98240	20393	97899	22098	97528	23797	97127	25488	96697	14
47	18710	98234	20421	97893	22126	97521	23825	97120	25516	96690	13
48	18738	98229	20450	97887	22155	97515	23853	97113	25545	96682	12
49	18767	98223	20478	97881	22183	97508	23882	97106	25573	96675	11
50	18795	98218	20507	97875	22212	97502	23910	97100	25601	96667	10
51	18824	98212	20535	97869	22240	97496	23938	97093	25629	96660	9
52	18852	98207	20563	97863	22268	97489	23966	97086	25657	96653	8
53	18881	98201	20592	97857	22297	97483	23995	97079	25685	96645	7
54	18910	98196	20620	97851	22325	97476	24023	97072	25713	96638	6
55	18938	98190	20649	97845	22353	97470	24051	97065	25741	96630	5
56	18967	98185	20677	97839	22382	97463	24079	97058	25769	96623	4
57	18995	98179	20706	97833	22410	97457	24108	97051	25798	96615	3
58	19024	98174	20734	97827	22438	97450	24136	97044	25826	96608	2
59	19052	98168	20763	97821	22467	97444	24164	97037	25854	96600	1
60	19081	98163	20791	97815	22495	97437	24192	97030	25882	96593	0
′	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	′
	79°		78°		77°		76°		75°		

NATURAL SINES AND COSINES.

′	15°		16°		17°		18°		19°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	25882	96593	27564	96126	29237	95630	30902	95106	32557	94552	60
1	25910	96585	27592	96118	29265	95622	30929	95097	32584	94542	59
2	25938	96578	27620	96110	29293	95613	30957	95088	32612	94533	58
3	25966	96570	27648	96102	29321	95605	30985	95079	32639	94523	57
4	25994	96562	27676	96094	29348	95596	31012	95070	32667	94514	56
5	26022	96555	27704	96086	29376	95588	31040	95061	32694	94504	55
6	26050	96547	27731	96078	29404	95579	31068	95052	32722	94495	54
7	26079	96540	27759	96070	29432	95571	31095	95043	32749	94485	53
8	26107	96532	27787	96062	29460	95562	31123	95033	32777	94476	52
9	26135	96524	27815	96054	29487	95554	31151	95024	32804	94466	51
10	26163	96517	27843	96046	29515	95545	31178	95015	32832	94457	50
11	26191	96509	27871	96037	29543	95536	31206	95006	32859	94447	49
12	26219	96502	27899	96029	29571	95528	31233	94997	32887	94438	48
13	26247	96494	27927	96021	29599	95519	31261	94988	32914	94428	47
14	26275	96486	27955	96013	29626	95511	31289	94979	32942	94418	46
15	26303	96479	27983	96005	29654	95502	31316	94970	32969	94409	45
16	26331	96471	28011	95997	29682	95493	31344	94961	32997	94399	44
17	26359	96463	28039	95989	29710	95485	31372	94952	33024	94390	43
18	26387	96456	28067	95981	29737	95476	31399	94943	33051	94380	42
19	26415	96448	28095	95972	29765	95467	31427	94933	33079	94370	41
20	26443	96440	28123	95964	29793	95459	31454	94924	33106	94361	40
21	26471	96433	28150	95956	29821	95450	31482	94915	33134	94351	39
22	26500	96425	28178	95948	29849	95441	31510	94906	33161	94342	38
23	26528	96417	28206	95940	29876	95433	31537	94897	33189	94332	37
24	26556	96410	28234	95931	29904	95424	31565	94888	33216	94322	36
25	26584	96402	28262	95923	29932	95415	31593	94878	33244	94313	35
26	26612	96394	28290	95915	29960	95407	31620	94869	33271	94303	34
27	26640	96386	28318	95907	29987	95398	31648	94860	33298	94293	33
28	26668	96379	28346	95898	30015	95389	31675	94851	33326	94284	32
29	26696	96371	28374	95890	30043	95380	31703	94842	33353	94274	31
30	26724	96363	28402	95882	30071	95372	31730	94832	33381	94264	30
31	26752	96355	28429	95874	30098	95363	31758	94823	33408	94254	29
32	26780	96347	28457	95865	30126	95354	31786	94814	33436	94245	28
33	26808	96340	28485	95857	30154	95345	31813	94805	33463	94235	27
34	26836	96332	28513	95849	30182	95337	31841	94795	33490	94225	26
35	26864	96324	28541	95841	30209	95328	31868	94786	33518	94215	25
36	26892	96316	28569	95832	30237	95319	31896	94777	33545	94206	24
37	26920	96308	28597	95824	30265	95310	31923	94768	33573	94196	23
38	26948	96301	28625	95816	30292	95301	31951	94758	33600	94186	22
39	26976	96293	28652	95807	30320	95293	31979	94749	33627	94176	21
40	27004	96285	28680	95799	30348	95284	32006	94740	33655	94167	20
41	27032	96277	28708	95791	30376	95275	32034	94730	33682	94157	19
42	27060	96269	28736	95782	30403	95266	32061	94721	33710	94147	18
43	27088	96261	28764	95774	30431	95257	32089	94712	33737	94137	17
44	27116	96253	28792	95766	30459	95248	32116	94702	33764	94127	16
45	27144	96246	28820	95757	30486	95240	32144	94693	33792	94118	15
46	27172	96238	28847	95749	30514	95231	32171	94684	33819	94108	14
47	27200	96230	28875	95740	30542	95222	32199	94674	33846	94098	13
48	27228	96222	28903	95732	30570	95213	32227	94665	33874	94088	12
49	27256	96214	28931	95724	30597	95204	32254	94656	33901	94078	11
50	27284	96206	28959	95715	30625	95195	32282	94646	33929	94068	10
51	27312	96198	28987	95707	30653	95186	32309	94637	33956	94058	9
52	27340	96190	29015	95698	30680	95177	32337	94627	33983	94049	8
53	27368	96182	29042	95690	30708	95168	32364	94618	34011	94039	7
54	27396	96174	29070	95681	30736	95159	32392	94609	34038	94029	6
55	27424	96166	29098	95673	30763	95150	32419	94599	34065	94019	5
56	27452	96158	29126	95664	30791	95142	32447	94590	34093	94009	4
57	27480	96150	29154	95656	30819	95133	32474	94580	34120	93999	3
58	27508	96142	29182	95647	30846	95124	32502	94571	34147	93989	2
59	27536	96134	29209	95639	30874	95115	32529	94561	34175	93979	1
60	27564	96126	29237	95630	30902	95106	32557	94552	34202	93969	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	74°		73°		72°		71°		70°		′

NATURAL SINES AND COSINES.

′	20° Sine.	20° Cosine.	21° Sine.	21° Cosine.	22° Sine.	22° Cosine.	23° Sine.	23° Cosine.	24° Sine.	24° Cosine.	′
0	34202	93969	35837	93358	37461	92718	39073	92050	40674	91355	60
1	34229	93959	35864	93348	37488	92707	39100	92039	40700	91343	59
2	34257	93949	35891	93337	37515	92697	39127	92028	40727	91331	58
3	34284	93939	35918	93327	37542	92686	39153	92016	40753	91319	57
4	34311	93929	35945	93316	37569	92675	39180	92005	40780	91307	56
5	34339	93919	35973	93306	37595	92664	39207	91994	40806	91295	55
6	34366	93909	36000	93295	37622	92653	39234	91982	40833	91283	54
7	34393	93899	36027	93285	37649	92642	39260	91971	40860	91272	53
8	34421	93889	36054	93274	37676	92631	39287	91959	40886	91260	52
9	34448	93879	36081	93264	37703	92620	39314	91948	40913	91248	51
10	34475	93869	36108	93253	37730	92609	39341	91936	40939	91236	50
11	34503	93859	36135	93243	37757	92598	39367	91925	40966	91224	49
12	34530	93849	36162	93232	37784	92587	39394	91914	40992	91212	48
13	34557	93839	36190	93222	37811	92576	39421	91902	41019	91200	47
14	34584	93829	36217	93211	37838	92565	39448	91891	41045	91188	46
15	34612	93819	36244	93201	37865	92554	39474	91879	41072	91176	45
16	34639	93809	36271	93190	37892	92543	39501	91868	41098	91164	44
17	34666	93799	36298	93180	37919	92532	39528	91856	41125	91152	43
18	34694	93789	36325	93169	37946	92521	39555	91845	41151	91140	42
19	34721	93779	36352	93159	37973	92510	39581	91833	41178	91128	41
20	34748	93769	36379	93148	37999	92499	39608	91822	41204	91116	40
21	34775	93759	36406	93137	38026	92488	39635	91810	41231	91104	39
22	34803	93748	36434	93127	38053	92477	39661	91799	41257	91092	38
23	34830	93738	36461	93116	38080	92466	39688	91787	41284	91080	37
24	34857	93728	36488	93106	38107	92455	39715	91775	41310	91068	36
25	34884	93718	36515	93095	38134	92444	39741	91764	41337	91056	35
26	34912	93708	36542	93084	38161	92432	39768	91752	41363	91044	34
27	34939	93698	36569	93074	38188	92421	39795	91741	41390	91032	33
28	34966	93688	36596	93063	38215	92410	39822	91729	41416	91020	32
29	34993	93677	36623	93052	38241	92399	39848	91718	41443	91008	31
30	35021	93667	36650	93042	38268	92388	39875	91706	41469	90996	30
31	35048	93657	36677	93031	38295	92377	39902	91694	41496	90984	29
32	35075	93647	36704	93020	38322	92366	39928	91683	41522	90972	28
33	35102	93637	36731	93010	38349	92355	39955	91671	41549	90960	27
34	35130	93626	36758	92999	38376	92343	39982	91660	41575	90948	26
35	35157	93616	36785	92988	38403	92332	40008	91648	41602	90936	25
36	35184	93606	36812	92978	38430	92321	40035	91636	41628	90924	24
37	35211	93596	36839	92967	38456	92310	40062	91625	41655	90911	23
38	35239	93585	36867	92956	38483	92299	40088	91613	41681	90899	22
39	35266	93575	36894	92945	38510	92287	40115	91601	41707	90887	21
40	35293	93565	36921	92935	38537	92276	40141	91590	41734	90875	20
41	35320	93555	36948	92924	38564	92265	40168	91578	41760	90863	19
42	35347	93544	36975	92913	38591	92254	40195	91566	41787	90851	18
43	35375	93534	37002	92902	38617	92243	40221	91555	41813	90839	17
44	35402	93524	37029	92892	38644	92231	40248	91543	41840	90826	16
45	35429	93514	37056	92881	38671	92220	40275	91531	41866	90814	15
46	35456	93503	37083	92870	38698	92209	40301	91519	41892	90802	14
47	35484	93493	37110	92859	38725	92198	40328	91508	41919	90790	13
48	35511	93483	37137	92849	38752	92186	40355	91496	41945	90778	12
49	35538	93472	37164	92838	38778	92175	40381	91484	41972	90766	11
50	35565	93462	37191	92827	38805	92164	40408	91472	41998	90753	10
51	35592	93452	37218	92816	38832	92152	40434	91461	42024	90741	9
52	35619	93441	37245	92805	38859	92141	40461	91449	42051	90729	8
53	35647	93431	37272	92794	38886	92130	40488	91437	42077	90717	7
54	35674	93420	37299	92784	38912	92119	40514	91425	42104	90704	6
55	35701	93410	37326	92773	38939	92107	40541	91414	42130	90692	5
56	35728	93400	37353	92762	38966	92096	40567	91402	42156	90680	4
57	35755	93389	37380	92751	38993	92085	40594	91390	42183	90668	3
58	35782	93379	37407	92740	39020	92073	40621	91378	42209	90655	2
59	35810	93368	37434	92729	39046	92062	40647	91366	42235	90643	1
60	35837	93358	37461	92718	39073	92050	40674	91355	42262	90631	0
′	69° Cosine.	69° Sine.	68° Cosine.	68° Sine.	67° Cosine.	67° Sine.	66° Cosine.	66° Sine.	65° Cosine.	65° Sine.	′

NATURAL SINES AND COSINES.

′	25°		26°		27°		28°		29°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60
1	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59
2	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58
3	42341	90594	43916	89841	45477	89061	47024	88254	48557	87420	57
4	42367	90582	43942	89828	45503	89048	47050	88240	48583	87406	56
5	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55
6	42420	90557	43994	89803	45554	89021	47101	88213	48634	87377	54
7	42446	90545	44020	89790	45580	89008	47127	88199	48659	87363	53
8	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52
9	42499	90520	44072	89764	45632	88981	47178	88172	48710	87335	51
10	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50
11	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49
12	42578	90483	44151	89726	45710	88942	47255	88130	48786	87292	48
13	42604	90470	44177	89713	45736	88928	47281	88117	48811	87278	47
14	42631	90458	44203	89700	45762	88915	47306	88103	48837	87264	46
15	42657	90446	44229	89687	45787	88902	47332	88089	48862	87250	45
16	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44
17	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43
18	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42
19	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41
20	42788	90383	44359	89623	45917	88835	47460	88020	48989	87178	40
21	42815	90371	44385	89610	45942	88822	47486	88006	49014	87164	39
22	42841	90358	44411	89597	45968	88808	47511	87993	49040	87150	38
23	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37
24	42894	90334	44464	89571	46020	88782	47562	87965	49090	87121	36
25	42920	90321	44490	89558	46046	88768	47588	87951	49116	87107	35
26	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34
27	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33
28	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32
29	43025	90271	44594	89506	46149	88715	47690	87896	49217	87050	31
30	43051	90259	44620	89493	46175	88701	47716	87882	49242	87036	30
31	43077	90246	44646	89480	46201	88688	47741	87868	49268	87021	29
32	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28
33	43130	90221	44698	89454	46252	88661	47793	87840	49318	86993	27
34	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26
35	43182	90196	44750	89428	46304	88634	47844	87812	49369	86964	25
36	43209	90183	44776	89415	46330	88620	47869	87798	49394	86949	24
37	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23
38	43261	90158	44828	89389	46381	88593	47920	87770	49445	86921	22
39	43287	90146	44854	89376	46407	88580	47946	87756	49470	86906	21
40	43313	90133	44880	89363	46433	88566	47971	87743	49495	86892	20
41	43340	90120	44906	89350	46458	88553	47997	87729	49521	86878	19
42	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18
43	43392	90095	44958	89324	46510	88526	48048	87701	49571	86849	17
44	43418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16
45	43445	90070	45010	89298	46561	88499	48099	87673	49622	86820	15
46	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14
47	43497	90045	45062	89272	46613	88472	48150	87645	49672	86791	13
48	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12
49	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11
50	43575	90007	45140	89232	46690	88431	48226	87603	49748	86748	10
51	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9
52	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8
53	43654	89968	45218	89193	46767	88390	48303	87561	49824	86704	7
54	43680	89956	45243	89180	46793	88377	48328	87546	49849	86690	6
55	43706	89943	45269	89167	46819	88363	48354	87532	49874	86675	5
56	43733	89930	45295	89153	46844	88349	48379	87518	49899	86661	4
57	43759	89918	45321	89140	46870	88336	48405	87504	49924	86646	3
58	43785	89905	45347	89127	46896	88322	48430	87490	49950	86632	2
59	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1
60	43837	89879	45399	89101	46947	88295	48481	87462	50000	86603	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	64°		63°		62°		61°		60°		′

NATURAL SINES AND COSINES.

′	30°		31°		32°		33°		34°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	50000	86603	51504	85717	52992	84805	54464	83867	55919	82904	60
1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	59
2	50050	86573	51554	85687	53041	84774	54513	83835	55968	82871	58
3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57
4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	56
5	50126	86530	51628	85642	53115	84728	54586	83788	56040	82822	55
6	50151	86515	51653	85627	53140	84712	54610	83772	56064	82806	54
7	50176	86501	51678	85612	53164	84697	54635	83756	56088	82790	53
8	50201	86486	51703	85597	53189	84681	54659	83740	56112	82773	52
9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	51
10	50252	86457	51753	85567	53238	84650	54708	83708	56160	82741	50
11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	49
12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	48
13	50327	86413	51828	85521	53312	84604	54781	83660	56232	82692	47
14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	46
15	50377	86384	51877	85491	53361	84573	54829	83629	56280	82659	45
16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	44
17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	43
18	50453	86340	51952	85446	53435	84526	54902	83581	56353	82610	42
19	50478	86325	51977	85431	53460	84511	54927	83565	56377	82593	41
20	50503	86310	52002	85416	53484	84495	54951	83549	56401	82577	40
21	50528	86295	52026	85401	53509	84480	54975	83533	56425	82561	39
22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	38
23	50578	86266	52076	85370	53558	84448	55024	83501	56473	82528	37
24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	36
25	50628	86237	52126	85340	53607	84417	55072	83469	56521	82495	35
26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	34
27	50679	86207	52175	85310	53656	84386	55121	83437	56569	82462	33
28	50704	86192	52200	85294	53681	84370	55145	83421	56593	82446	32
29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	31
30	50754	86163	52250	85264	53730	84339	55194	83389	56641	82413	30
31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	29
32	50804	86133	52299	85234	53779	84308	55242	83356	56689	82380	28
33	50829	86119	52324	85218	53804	84292	55266	83340	56713	82363	27
34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	26
35	50879	86089	52374	85188	53853	84261	55315	83308	56760	82330	25
36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	24
37	50929	86059	52423	85157	53902	84230	55363	83276	56808	82297	23
38	50954	86045	52448	85142	53926	84214	55388	83260	56832	82281	22
39	50979	86030	52473	85127	53951	84198	55412	83244	56856	82264	21
40	51004	86015	52498	85112	53975	84182	55436	83228	56880	82248	20
41	51029	86000	52522	85096	54000	84167	55460	83212	56904	82231	19
42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	18
43	51079	85970	52572	85066	54049	84135	55509	83179	56952	82198	17
44	51104	85956	52597	85051	54073	84120	55533	83163	56976	82181	16
45	51129	85941	52621	85035	54097	84104	55557	83147	57000	82165	15
46	51154	85926	52646	85020	54122	84088	55581	83131	57024	82148	14
47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	13
48	51204	85896	52696	84989	54171	84057	55630	83098	57071	82115	12
49	51229	85881	52720	84974	54195	84041	55654	83082	57095	82098	11
50	51254	85866	52745	84959	54220	84025	55678	83066	57119	82082	10
51	51279	85851	52770	84943	54244	84009	55702	83050	57143	82065	9
52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	8
53	51329	85821	52819	84913	54293	83978	55750	83017	57191	82032	7
54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	6
55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	5
56	51404	85777	52893	84866	54366	83930	55823	82969	57262	81982	4
57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	3
58	51454	85747	52943	84836	54415	83899	55871	82936	57310	81949	2
59	51479	85732	52967	84820	54440	83883	55895	82920	57334	81932	1
60	51504	85717	52992	84805	54464	83867	55919	82904	57358	81915	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	59°		58°		57°		56°		55°		′

NATURAL SINES AND COSINES.

′	35° Sine.	35° Cosine.	36° Sine.	36° Cosine.	37° Sine.	37° Cosine.	38° Sine.	38° Cosine.	39° Sine.	39° Cosine.	′
0	57358	81915	58779	80902	60182	79864	61566	78801	62932	77715	60
1	57381	81899	58802	80885	60205	79846	61589	78783	62955	77696	59
2	57405	81882	58826	80867	60228	79829	61612	78765	62977	77678	58
3	57429	81865	58849	80850	60251	79811	61635	78747	63000	77660	57
4	57453	81848	58873	80833	60274	79793	61658	78729	63022	77641	56
5	57477	81832	58896	80816	60298	79776	61681	78711	63045	77623	55
6	57501	81815	58920	80799	60321	79758	61704	78694	63068	77605	54
7	57524	81798	58943	80782	60344	79741	61726	78676	63090	77586	53
8	57548	81782	58967	80765	60367	79723	61749	78658	63113	77568	52
9	57572	81765	58990	80748	60390	79706	61772	78640	63135	77550	51
10	57596	81748	59014	80730	60414	79688	61795	78622	63158	77531	50
11	57619	81731	59037	89713	60437	79671	61818	78604	63180	77513	49
12	57643	81714	59061	80696	60460	79653	61841	78586	63203	77494	48
13	57667	81698	59084	80679	60483	79635	61864	78568	63225	77476	47
14	57691	81681	59108	80662	60506	79618	61887	78550	63248	77458	46
15	57715	81664	59131	80644	60529	79600	61909	78532	63271	77439	45
16	57738	81647	59154	80627	60553	79583	61932	78514	63293	77421	44
17	57762	81631	59178	80610	60576	79565	61955	78496	63316	77402	43
18	57786	81614	59201	80593	60599	79547	61978	78478	63338	77384	42
19	57810	81597	59225	80576	60622	79530	62001	78460	63361	77366	41
20	57833	81580	59248	80558	60645	79512	62024	78442	63383	77347	40
21	57857	81563	59272	80541	60668	79494	62046	78424	63406	77329	39
22	57881	81546	59295	80524	60691	79477	62069	78405	63428	77310	38
23	57904	81530	59318	80507	60714	79459	62092	78387	63451	77292	37
24	57928	81513	59342	80489	60738	79441	62115	78369	63473	77273	36
25	57952	81496	59365	80472	60761	79424	62138	78351	63496	77255	35
26	57976	81479	59389	80455	60784	79406	62160	78333	63518	77236	34
27	57999	81462	59412	80438	60807	79388	62183	78315	63540	77218	33
28	58023	81445	59436	80420	60830	79371	62206	78297	63563	77199	32
29	58047	81428	59459	80403	60853	79353	62229	78279	63585	77181	31
30	58070	81412	59482	80386	60876	79335	62251	78261	63608	77162	30
31	58094	81395	59506	80368	60899	79318	62274	78243	63630	77144	29
32	58118	81378	59529	80351	60922	79300	62297	78225	63653	77125	28
33	58141	81361	59552	80334	60945	79282	62320	78206	63675	77107	27
34	58165	81344	59576	80316	60968	79264	62342	78188	63698	77088	26
35	58189	81327	59599	80299	60991	79247	62365	78170	63720	77070	25
36	58212	81310	59622	80282	61015	79229	62388	78152	63742	77051	24
37	58236	81293	59646	80264	61038	79211	62411	78134	63765	77033	23
38	58260	81276	59669	80247	61061	79193	62433	78116	63787	77014	22
39	58283	81259	59693	80230	61084	79176	62456	78098	63810	76996	21
40	58307	81242	59716	80212	61107	79158	62479	78079	63832	76977	20
41	58330	81225	59739	80195	61130	79140	62502	78061	63854	76959	19
42	58354	81208	59763	80178	61153	79122	62524	78043	63877	76940	18
43	58378	81191	59786	80160	61176	79105	62547	78025	63899	76921	17
44	58401	81174	59809	80143	61199	79087	62570	78007	63922	76903	16
45	58425	81157	59832	80125	61222	79069	62592	77988	63944	76884	15
46	58449	81140	59856	80108	61245	79051	62615	77970	63966	76866	14
47	58472	81123	59879	80091	61268	79033	62638	77952	63989	76847	13
48	58496	81106	59902	80073	61291	79016	62660	77934	64011	76828	12
49	58519	81089	59926	80056	61314	78998	62683	77916	64033	76810	11
50	58543	81072	59949	80038	61337	78980	62706	77897	64056	76791	10
51	58567	81055	59972	80021	61360	78962	62728	77879	64078	76772	9
52	58590	81038	59995	80003	61383	78944	62751	77861	64100	76754	8
53	58614	81021	60019	79986	61406	78926	62774	77843	64123	76735	7
54	58637	81004	60042	79968	61429	78908	62796	77824	64145	76717	6
55	58661	80987	60065	79951	61451	78891	62819	77806	64167	76698	5
56	58684	80970	60089	79934	61474	78873	62842	77788	64190	76679	4
57	58708	80953	60112	79916	61497	78855	62864	77769	64212	76661	3
58	58731	80936	60135	79899	61520	78837	62887	77751	64234	76642	2
59	58755	80919	60158	79881	61543	78819	62909	77733	64256	76623	1
60	58779	80902	60182	79864	61566	78801	62932	77715	64279	76604	0
′	54° Cosine.	54° Sine.	53° Cosine.	53° Sine.	52° Cosine.	52° Sine.	51° Cosine.	51° Sine.	50° Cosine.	50° Sine.	′

NATURAL SINES AND COSINES.

′	40°		41°		42°		43°		44°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	64279	76604	65606	75471	66913	74314	68200	73135	69466	71934	60
1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59
2	64323	76567	65650	75433	66956	74276	68242	73096	69508	71894	58
3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57
4	64368	76530	65694	75395	66999	74237	68285	73056	69549	71853	56
5	64390	76511	65716	75375	67021	74217	68306	73036	69570	71833	55
6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54
7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53
8	64457	76455	65781	75318	67086	74159	68370	72976	69633	71772	52
9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51
10	64501	76417	65825	75280	67129	74120	68412	72937	69675	71732	50
11	64524	76398	65847	75261	67151	74100	68434	72917	69696	71711	49
12	64546	76380	65869	75241	67172	74080	68455	72897	69717	71691	48
13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47
14	64590	76342	65913	75203	67215	74041	68497	72857	69758	71650	46
15	64612	76323	65935	75184	67237	74022	68518	72837	69779	71630	45
16	64635	76304	65956	75165	67258	74002	68539	72817	69800	71610	44
17	64657	76286	65978	75146	67280	73983	68561	72797	69821	71590	43
18	64679	76267	66000	75126	67301	73963	68582	72777	69842	71569	42
19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41
20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40
21	64746	76210	66066	75069	67366	73904	68645	72717	69904	71508	39
22	64768	76192	66088	75050	67387	73885	68666	72697	69925	71488	38
23	64790	76173	66109	75030	67409	73865	68688	72677	69946	71468	37
24	64812	76154	66131	75011	67430	73846	68709	72657	69966	71447	36
25	64834	76135	66153	74992	67452	73826	68730	72637	69987	71427	35
26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34
27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33
28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32
29	64923	76059	66240	74915	67538	73747	68814	72557	70070	71345	31
30	64945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30
31	64967	76022	66284	74876	67580	73708	68857	72517	70112	71305	29
32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28
33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27
34	65033	75965	66349	74818	67645	73649	68920	72457	70174	71243	26
35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25
36	65077	75927	66393	74780	67688	73610	68962	72417	70215	71203	24
37	65100	75908	66414	74760	67709	73590	68983	72397	70236	71182	23
38	65122	75889	66436	74741	67730	73570	69004	72377	70257	71162	22
39	65144	75870	66458	74722	67752	73551	69025	72357	70277	71141	21
40	65166	75851	66480	74703	67773	73531	69046	72337	70298	71121	20
41	65188	75832	66501	74683	67795	73511	69067	72317	70319	71100	19
42	65210	75813	66523	74664	67816	73491	69088	72297	70339	71080	18
43	65232	75794	66545	74644	67837	73472	69109	72277	70360	71059	17
44	65254	75775	66566	74625	67859	73452	69130	72257	70381	71039	16
45	65276	75756	66588	74606	67880	73432	69151	72236	70401	71019	15
46	65298	75738	66610	74586	67901	73413	69172	72216	70422	70998	14
47	65320	75719	66632	74567	67923	73393	69193	72196	70443	70978	13
48	65342	75700	66653	74548	67944	73373	69214	72176	70463	70957	12
49	65364	75680	66675	74528	67965	73353	69235	72156	70484	70937	11
50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10
51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9
52	65430	75623	66740	74470	68029	73294	69298	72095	70546	70875	8
53	65452	75604	66762	74451	68051	73274	69319	72075	70567	70855	7
54	65474	75585	66783	74431	68072	73254	69340	72055	70587	70834	6
55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5
56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4
57	65540	75528	66848	74373	68136	73195	69403	71995	70649	70772	3
58	65562	75509	66870	74353	68157	73175	69424	71974	70670	70752	2
59	65584	75490	66891	74334	68179	73155	69445	71954	70690	70731	1
60	65606	75471	66913	74314	68200	73135	69466	71934	70711	70711	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	49°		48°		47°		46°		45°		′

TABLE OF CHORDS.

A TABLE OF CHORDS.

M.	0°	1°	2°	3°	4°	5°	6°	7°	8°	M.
0	.0000	.0175	.0349	.0524	.0698	.0872	.1047	.1221	.1395	0
5	.0015	.0189	.0364	.0538	.0713	.0887	.1061	.1235	.1410	5
10	.0029	.0204	.0378	.0553	.0727	.0901	.1076	.1250	.1424	10
15	.0044	.0218	.0393	.0567	.0742	.0916	.1090	.1265	.1439	15
20	.0058	.0233	.0407	.0582	.0756	.0931	.1105	.1279	.1453	20
25	.0073	.0247	.0422	.0596	.0771	.0945	.1119	.1294	.1468	25
30	.0087	.0262	.0436	.0611	.0785	.0960	.1134	.1308	.1482	30
35	.0102	.0276	.0451	.0625	.0800	.0974	.1148	.1323	.1497	35
40	.0116	.0291	.0465	.0640	.0814	.0989	.1163	.1337	.1511	40
45	.0131	.0305	.0480	.0654	.0829	.1003	.1177	.1352	.1526	45
50	.0145	.0320	.0494	.0669	.0843	.1018	.1192	.1366	.1540	50
55	.0160	.0335	.0509	.0683	.0858	.1032	.1206	.1381	.1555	55
60	.0175	.0349	.0524	.0698	.0872	.1047	.1221	.1395	.1569	60
	9°	10°	11°	12°	13°	14°	15°	16°	17°	
0	.1569	.1743	.1917	.2091	.2264	.2437	.2611	.2783	.2956	0
5	.1584	.1758	.1931	.2105	.2279	.2452	.2625	.2798	.2971	5
10	.1598	.1772	.1946	.2119	.2293	.2466	.2639	.2812	.2985	10
15	.1613	.1787	.1960	.2134	.2307	.2481	.2654	.2827	.2999	15
20	.1627	.1801	.1975	.2148	.2322	.2495	.2668	.2841	.3014	20
25	.1642	.1816	.1989	.2163	.2336	.2510	.2683	.2855	.3028	25
30	.1656	.1830	.2004	.2177	.2351	.2524	.2697	.2870	.3042	30
35	.1671	.1845	.2018	.2192	.2365	.2538	.2711	.2884	.3057	35
40	.1685	.1859	.2033	.2206	.2380	.2553	.2726	.2899	.3071	40
45	.1700	.1873	.2047	.2221	.2394	.2567	.2740	.2913	.3086	45
50	.1714	.1888	.2062	.2235	.2409	.2582	.2755	.2927	.3100	50
55	.1729	.1902	.2076	.2250	.2423	.2596	.2769	.2942	.3114	55
60	.1743	.1917	.2091	.2264	.2437	.2611	.2783	.2956	.3129	60

TABLE OF CHORDS.

M.	18°	19°	20°	21°	22°	23°	24°	25°	26°	M.
0	.3129	.3301	.3473	.3645	.3816	.3987	.4158	.4329	.4499	0
5	.3143	.3315	.3487	.3659	.3830	.4002	.4172	.4343	.4513	5
10	.3157	.3330	.3502	.3673	.3845	.4016	.4187	.4357	.4527	10
15	.3172	.3344	.3516	.3688	.3859	.4030	.4201	.4371	.4542	15
20	.3186	.3358	.3530	.3702	.3873	.4044	.4215	.4386	.4556	20
25	.3200	.3373	.3545	.3716	.3888	.4059	.4229	.4400	.4570	25
30	.3215	.3387	.3559	.3730	.3902	.4073	.4244	.4414	.4584	30
35	.3229	.3401	.3573	.3745	.3916	.4087	.4258	.4428	.4598	35
40	.3244	.3416	.3587	.3759	.3930	.4101	.4272	.4442	.4612	40
45	.3258	.3430	.3602	.3773	.3945	.4116	.4286	.4456	.4626	45
50	.3272	.3444	.3616	.3788	.3959	.4130	.4300	.4471	.4641	50
55	.3287	.3459	.3630	.3802	.3973	.4144	.4315	.4485	.4655	55
60	.3301	.3473	.3645	.3816	.3987	.4158	.4329	.4499	.4669	60

M.	27°	28°	29°	30°	31°	32°	33°	34°	35°	M.
0	.4669	.4838	.5008	.5176	.5345	.5513	.5680	.5847	.6014	0
5	.4683	.4853	.5022	.5190	.5359	.5527	.5694	.5861	.6028	5
10	.4697	.4867	.5036	.5204	.5373	.5541	.5708	.5875	.6042	10
15	.4711	.4881	.5050	.5219	.5387	.5555	.5722	.5889	.6056	15
20	.4725	.4895	.5064	.5233	.5401	.5569	.5736	.5903	.6070	20
25	.4740	.4909	.5078	.5247	.5415	.5583	.5750	.5917	.6083	25
30	.4754	.4923	.5092	.5261	.5429	.5597	.5764	.5931	.6097	30
35	.4768	.4937	.5106	.5275	.5443	.5611	.5778	.5945	.6111	35
40	.4782	.4951	.5120	.5289	.5457	.5625	.5792	.5959	.6125	40
45	.4796	.4965	.5134	.5303	.5471	.5638	.5806	.5972	.6139	45
50	.4810	.4979	.5148	.5317	.5485	.5652	.5820	.5986	.6153	50
55	.4824	.4994	.5162	.5331	.5499	.5666	.5833	.6000	.6167	55
60	.4838	.5008	.5176	.5345	.5513	.5680	.5847	.6014	.6180	60

M.	36°	37°	38°	39°	40°	41°	42°	43°	44°	M.
0	.6180	.6346	.6511	.6676	.6840	.7004	.7167	.7330	.7492	0
5	.6194	.6360	.6525	.6690	.6854	.7018	.7181	.7344	.7506	5
10	.6208	.6374	.6539	.6704	.6868	.7031	.7195	.7357	.7519	10
15	.6222	.6387	.6553	.6717	.6881	.7045	.7208	.7371	.7533	15
20	.6236	.6401	.6566	.6731	.6895	.7059	.7222	.7384	.7546	20
25	.6249	.6415	.6580	.6745	.6909	.7072	.7235	.7398	.7560	25
30	.6263	.6429	.6594	.6758	.6922	.7086	.7249	.7411	.7573	30
35	.6277	.6443	.6608	.6772	.6936	.7099	.7262	.7425	.7586	35
40	.6291	.6456	.6621	.6786	.6950	.7113	.7276	.7438	.7600	40
45	.6305	.6470	.6635	.6799	.6963	.7127	.7289	.7452	.7613	45
50	.6319	.6484	.6649	.6813	.6977	.7140	.7303	.7465	.7627	50
55	.6332	.6498	.6662	.6827	.6991	.7154	.7316	.7479	.7640	55
60	.6346	.6511	.6676	.6840	.7004	.7167	.7330	.7492	.7654	60

M.	45°	46°	47°	48°	49°	50°	51°	52°	53°	M.
0	.7654	.7815	.7975	.8135	.8294	.8452	.8610	.8767	.8924	0
5	.7667	.7828	.7988	.8148	.8307	.8466	.8623	.8780	.8937	5
10	.7681	.7841	.8002	.8161	.8320	.8479	.8636	.8794	.8950	10
15	.7694	.7855	.8015	.8175	.8334	.8492	.8650	.8807	.8963	15
20	.7707	.7868	.8028	.8188	.8347	.8505	.8663	.8820	.8976	20
25	.7721	.7882	.8042	.8201	.8360	.8518	.8676	.8833	.8989	25
30	.7734	.7895	.8055	.8214	.8373	.8531	.8689	.8846	.9002	30
35	.7748	.7908	.8068	.8228	.8386	.8545	.8702	.8859	.9015	35
40	.7761	.7922	.8082	.8241	.8400	.8558	.8715	.8872	.9028	40
45	.7774	.7935	.8095	.8254	.8413	.8571	.8728	.8885	.9041	45
50	.7788	.7948	.8108	.8267	.8426	.8584	.8741	.8898	.9054	50
55	.7801	.7962	.8121	.8281	.8439	.8597	.8754	.8911	.9067	55
60	.7815	.7975	.8135	.8294	.8452	.8610	.8767	.8924	.9080	60

TABLE OF CHORDS.

M.	54°	55°	56°	57°	58°	59°	60°	61°	62°	M.
0	.9080	.9235	.9389	.9543	.9696	.9848	1.0000	1.0151	1.0301	0
5	.9093	.9248	.9402	.9556	.9709	.9861	1.0013	1.0163	1.0313	5
10	.9106	.9261	.9415	.9569	.9722	.9874	1.0025	1.0176	1.0326	10
15	.9119	.9274	.9428	.9581	.9734	.9886	1.0038	1.0188	1.0338	15
20	.9132	.9287	.9441	.9594	.9747	.9899	1.0050	1.0201	1.0351	20
25	.9145	.9299	.9454	.9607	.9760	.9912	1.0063	1.0213	1.0363	25
30	.9157	.9312	.9466	.9620	.9772	.9924	1.0075	1.0226	1.0375	30
35	.9170	.9325	.9479	.9633	.9785	.9937	1.0088	1.0238	1.0388	35
40	.9183	.9338	.9492	.9645	.9798	.9950	1.0101	1.0251	1.0400	40
45	.9196	.9351	.9505	.9658	.9810	.9962	1.0113	1.0263	1.0413	45
50	.9209	.9364	.9518	.9671	.9823	.9975	1.0126	1.0276	1.0425	50
55	.9222	.9377	.9530	.9683	.9836	.9987	1.0138	1.0288	1.0438	55
60	.9235	.9389	.9543	.9696	.9848	1.0000	1.0151	1.0301	1.0450	60

M.	63°	64°	65°	66°	67°	68°	69°	70°	71°	M.
0	1.0450	1.0598	1.0746	1.0893	1.1039	1.1184	1.1328	1.1472	1.1614	0
5	1.0462	1.0611	1.0758	1.0905	1.1051	1.1196	1.1340	1.1483	1.1626	5
10	1.0475	1.0623	1.0771	1.0917	1.1063	1.1208	1.1352	1.1495	1.1638	10
15	1.0487	1.0635	1.0783	1.0929	1.1075	1.1220	1.1364	1.1507	1.1650	15
20	1.0500	1.0648	1.0795	1.0942	1.1087	1.1232	1.1376	1.1519	1.1661	20
25	1.0512	1.0660	1.0807	1.0954	1.1099	1.1244	1.1388	1.1531	1.1673	25
30	1.0524	1.0672	1.0820	1.0966	1.1111	1.1256	1.1400	1.1543	1.1685	30
35	1.0537	1.0685	1.0832	1.0978	1.1123	1.1268	1.1412	1.1555	1.1697	35
40	1.0549	1.0697	1.0844	1.0990	1.1136	1.1280	1.1424	1.1567	1.1709	40
45	1.0561	1.0709	1.0856	1.1002	1.1148	1.1292	1.1436	1.1579	1.1720	45
50	1.0574	1.0721	1.0868	1.1014	1.1160	1.1304	1.1448	1.1590	1.1732	50
55	1.0586	1.0734	1.0881	1.1027	1.1172	1.1316	1.1460	1.1602	1.1744	55
60	1.0598	1.0746	1.0893	1.1039	1.1184	1.1328	1.1472	1.1614	1.1756	60

M.	72°	73°	74°	75°	76°	77°	78°	79°	80°	M.
0	1.1756	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	0
5	1.1767	1.1908	1.2048	1.2187	1.2325	1.2462	1.2598	1.2733	1.2867	5
10	1.1779	1.1920	1.2060	1.2198	1.2336	1.2473	1.2609	1.2744	1.2878	10
15	1.1791	1.1931	1.2071	1.2210	1.2348	1.2484	1.2620	1.2755	1.2889	15
20	1.1803	1.1943	1.2083	1.2221	1.2359	1.2496	1.2632	1.2766	1.2900	20
25	1.1814	1.1955	1.2094	1.2233	1.2370	1.2507	1.2643	1.2778	1.2911	25
30	1.1826	1.1966	1.2106	1.2244	1.2382	1.2518	1.2654	1.2789	1.2922	30
35	1.1838	1.1978	1.2117	1.2256	1.2393	1.2530	1.2665	1.2800	1.2934	35
40	1.1850	1.1990	1.2129	1.2267	1.2405	1.2541	1.2677	1.2811	1.2945	40
45	1.1861	1.2001	1.2141	1.2279	1.2416	1.2552	1.2688	1.2822	1.2956	45
50	1.1873	1.2013	1.2152	1.2290	1.2428	1.2564	1.2699	1.2833	1.2967	50
55	1.1885	1.2025	1.2164	1.2302	1.2439	1.2575	1.2710	1.2845	1.2978	55
60	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	1.2989	60

M.	81°	82°	83°	84°	85°	86°	87°	88°	89°	M.
0	1.2989	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	0
5	1.3000	1.3132	1.3263	1.3393	1.3523	1.3651	1.3778	1.3904	1.4029	5
10	1.3011	1.3143	1.3274	1.3404	1.3533	1.3661	1.3788	1.3914	1.4039	10
15	1.3022	1.3154	1.3285	1.3415	1.3544	1.3672	1.3799	1.3925	1.4049	15
20	1.3033	1.3165	1.3296	1.3426	1.3555	1.3682	1.3809	1.3935	1.4060	20
25	1.3044	1.3176	1.3307	1.3437	1.3565	1.3693	1.3820	1.3945	1.4070	25
30	1.3055	1.3187	1.3318	1.3447	1.3576	1.3704	1.3830	1.3956	1.4080	30
35	1.3066	1.3198	1.3328	1.3458	1.3587	1.3714	1.3841	1.3966	1.4091	35
40	1.3077	1.3209	1.3339	1.3469	1.3597	1.3725	1.3851	1.3977	1.4101	40
45	1.3088	1.3220	1.3350	1.3480	1.3608	1.3735	1.3862	1.3987	1.4111	45
50	1.3099	1.3231	1.3361	1.3490	1.3619	1.3746	1.3872	1.3997	1.4122	50
55	1.3110	1.3242	1.3372	1.3501	1.3629	1.3757	1.3883	1.4008	1.4132	55
60	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	1.4142	60