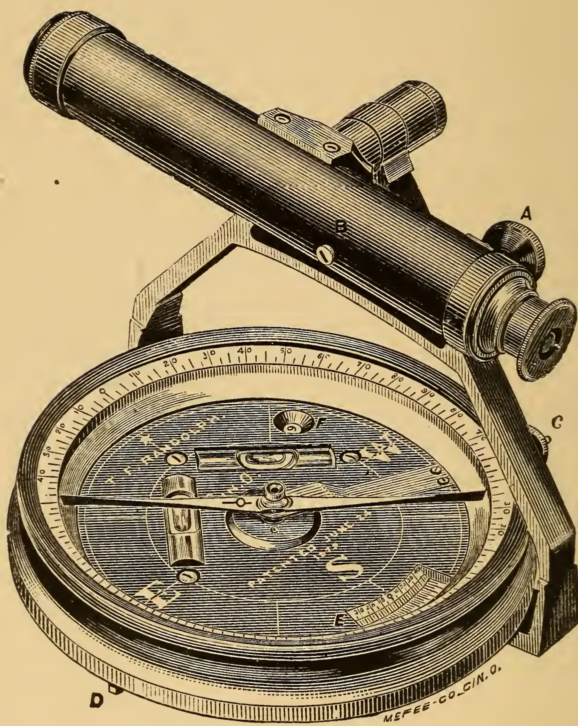
PLANE SURVEYING

BAGOT.



TELESCOPE COMPASS.

A MANUAL

OF

PLANE SURVEYING;

CONFINED TO

WORK WITH THE COMPASS.

WITH AN APPENDIX.

AMPLY ILLUSTRATED.

BY

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PREFACE.

EVERY person who studies Surveying from the text-books in general use, and afterward is called upon to discharge the duties of a surveyor, must, in the course of time, become aware of two things: (1) that he has spent time in learning much that he has never had, and probably never will have, occasion to use, and (2) that a great deal he needs to know, and must know, is not to be found in the books.

This is the case particularly under the Rectangular System, and the author's experience has led him to believe that a necessity exists for a book dealing directly with the problems continually coming up before surveyors throughout the country, and that such a book will be cordially received by every person who wishes to understand the subject as it is comprehended in general practice.

And this, dear reader, accounts for the existence of this little book. You will find it simply a brief treatise on Compass-Surveying, shorn of everything superfluous, and yet embracing all that is necessary to a good understanding of the subject. Very few geometrical or trigonometrical terms are employed, and all the problems may be mastered by any person having a moderately good knowledge of arithmetic.

The author does not claim that the book is above criticism, but, on the contrary, he is well aware of the fact that a person disposed to criticise may find in it an ample field in which to exercise his talents. He trusts, however, that a search for its faults will result in disclosing enough merit, even among so much demerit, to excuse him for writing it, and so trusting, he submits it to the public.

The plates of surveying instruments used in the book represent the excellent instruments manufactured by T. F. Randolph, Cincinnati, Ohio.

NEW MARION, INDIANA,
May, 1883.

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MANUAL

OF

PLANE SURVEYING.

CHAPTER I.

INTRODUCTION.

ART. (1). SURVEYING is that branch of applied mathematics which embraces operations for finding, (1) the relative positions of points on the earth's surace, (2) the area of any portion of its surface, and (3) the contour or shape of any part of its surface, so that it may be represented in maps and plots.

(2). It is divided into three branches:

1. *Topographical Surveying*, or *Topography*, includes operations for determining the contour of portions of the earth's surface and representing it on paper.

2. *Geodetic Surveying*, or *Geodesy*, takes into consideration the curvature of the earth's surface and is employed in extensive surveys.

3. *Plane Surveying* does not regard the curvature of the earth's surface and all lines are measured as on a plane. It is used in local work.

(3). All measurements in *surveying* are made as nearly horizontally as possible, and the area of a tract of land is not its actual surface measure, unless the tract be perfectly level, but the amount of land enclosed by its boundaries measured horizontally, instead of with the inclinations of the surface over which they run.

1. The actual area, therefore, is nearly always greater than its computed area, and increases in proportion to the inequality of the surface.

2. We may conceive a smooth surface at the level of the ocean underlying the surface of the land; then the area of a tract of land is equal to the contents of a figure formed by projecting the boundaries of the tract on the horizontal surface below.

3. Were the real surface considered, it would be impossible in many instances either to compute its area or represent its contour on paper.

(4). The extremities of lines in *surveying* are called corners, and each corner marks the vertex of an angle formed by the meeting of two lines. The corner is a mathematical point, and may or may not be marked with a monument.

(5). Lines are surveyed either with a solar or a magnetic instrument. With the former, where more precision than expedition is required; and with the latter, where expedition is of more importance than precision.

(6). The principal magnetic instruments in use are the transit and compass.

1. The transit is provided with a telescope, and at the present time is so constructed as to be adapted to the measurement of both horizontal and vertical angles. It serves many important purposes independent of the assistance of the magnetic needle, and is not strictly a magnetic instrument.

2. The compass is either supplied with sights or a telescope, and is strictly a magnetic instrument. It is not usually adapted to measuring vertical angles. The lightness, simplicity, and convenience of the compass have brought it into almost general use in common surveying, and the following chapter is devoted to a description of it. The transit may be found described in almost any comprehensive work on Surveying.

(7). Measurements of lines in surveying are made with an iron or steel chain* usually 33 feet or 2 rods long and divided into 50 links. It has a handle at each end by which it is carried during the survey, and the successive chain-lengths are maked

*For convenience in counting the links, this chain is divided into five parts of ten links each by four brass or copper tags. The real chain is 100 links or 66 feet in length, and called a Gunter's chain, from its inventor. The half-chain of 50 links is used for convenience. A link is 7.92 inches in length.

with pins of iron or steel wire, generally 10 or 12 inches in length, sharpened at one end and bent into the form of a ring at the other. In the ring is sometimes tied a bright ribbon or piece of cloth to render the pin more conspicuous, in order that it may be easily found by the person who carries the rear end of the chain, and such colors should always be chosen as will contrast most with the surface to be surveyed.

(8). In chaining up and down hill, the chain must be kept taut and horizontal, as on a level surface. In order to do this, it is sometimes necessary to drop the pin from the front end of the chain, or elevate the rear end of the chain to a point exactly over the pin that sticks in the ground.

(9). A straight staff, about $1\frac{1}{4}$ inches in diameter and 8 or 10 feet high, surmounted by a small flag of brilliant color, is used in alignment; and a good set of drawing instruments, drawing board, t-square, triangle, protractor, ruler, etc., are necessary in drawing and plotting.

(**10**). In field-work the surveyor generally needs four assistants; two chain-men, one flag-man, and a marker. The first two measure the line with the chain, the third carries the flag, and the fourth assists the chain-men by marking the line with stakes at the proper distances. To this force it is sometimes necessary to add one or more ax-men, as bushes may have to be cut out of the way and trees marked.

Questions on Chapter I.

1. Define Surveying.
2. Into what three branches is it divided?
3. What does each branch embrace?
4. How are measurements made in surveying?
5. Why is the actual area of a tract of land nearly always greater than its computed area?
6. What is a corner?
7. Name the two kinds of instruments used in surveying.
8. Name the principal magnetic instruments.
9. Describe the transit. The compass.
10. How are lines measured in surveying?
11. How should the chain be held in chaining over hills?
12. How many assistants does the surveyor usually need?

CHAPTER II.

DESCRIPTION OF THE COMPASS.

(**11**). The compass consists essentially of the compass circle, the magnetic needle, and the sights. The circle has its circumference raised and divided into 360 equal parts or degrees, and these are usually subdivided to half or quarter degrees.

(**12**). At the center of the compass circle is placed a perpendicular pin, called the center pin. Upon this pin the magnetic needle is balanced in such a way as to mark opposite points of the divided circumference of the circle.

(**13**). The degrees on the circumference are marked from two opposite points, called **0** points, up to 90° to the right and left. The 90° points are, therefore, opposite one another also. One of the **0** points is called the north point, and the other the south point, and one of the 90° points is called the east point, and the other the west point.

(**14**). The compass circle is enclosed in what is known as the compass box, and this rests upon the compass plate. The box is usually between 5 and 7 inches in diameter, and the plate about 12 or 14 inches long. At the ends of the plate perpendicular sights are placed. These sights have slits in them, and are so placed that the line of sight from one of them to the other will strike opposite points of the graduated circumference of the compass circle. Between the compass box and the sights are usually placed two spirit levels, one at right angles to the other. These are used in leveling the compass. The compass rests upon a tripod or Jacob's staff, at the head of which is a ball and socket joint, enabling a person to move the compass as he may wish.

(**15**). The needle is a magnetized steel bar, very delicately balanced upon the point of the center pin, and it is a little shorter than the diameter of the compass circle. The delicacy of the needle is determined by the number of horizontal vibrations it will make before coming to rest after being disturbed. Needles 5 or $5\frac{1}{2}$ inches in length are generally preferred by surveyors to longer or shorter ones.

(**16**). Horizontal angles are measured with the compass by turning the sights from one line of the angle to the other and noting the number of degrees passed over by the end of the needle. The sights are sometimes arranged for the measurement of vertical angles also.

(**17**). The letters "E" and "W" are reversed on the compass face, but it will be plainly seen that the arrangement enables the surveyer to take the direction (bearing) of a line more readily from the compass face, and reduces the liability to err in the reading.

(**18**). This may be illustrated by Fig. 1.

Fig. 1.

Suppose the sights set in the direction of the **0** points of the circle, and that the compass be turned until the north end of the needle marks the point midway between the north point (generally marked with a *fleur de lis*, instead of the letter N) and E, or to 45°. The sights have moved in the direction of the second hand of a watch, and the bearing of the line marked by them is N 45° E, which is read directly from the north end of the needle.

(**19**). The following is the general rule for all readings:

Note the letters between which the end of the needle comes, and to what number. Then name the letter N or S (as the case may be) that is the nearer to the end of the needle from which you are reading, next the number of degrees to which the needle points, and lastly the letter E or W that is the nearer to the same end of the needle.

(**20**). If the preceding reading had been taken from the south end of the needle, the bearing would have been S 45° W, the reverse of N 45° E, but equivalent to it, and indicating the bearing taken from the opposite end of the line.

(**21**). In taking the northerly bearing of a line, sight from the south end of the compass plate, and in taking the southerly bearing sight from the north end. N 45° E is a northerly bearing, and S 45° W is its corresponding southerly bearing.

(**22**). In running lines east, the E point of the compass circle should be turned toward the north, and in running west, the W point should be turned north. Northerly bearings should be read from the north end of the needle and southerly bearings from the south end.

(**23**). In measuring angles observe the following rules:

1. When both readings are in the same quadrant, as between N and E, N and W, S and E, or S and W, the angle is equal to the difference between the two readings. Thus, the angle between N 56° E and N. 43° E is equal to 13°.

2. When one reading is in each of either the two north quadrants or the two south quadrants, the included angle is equal to the sum of the two readings. Thus, the included angle of N 35° E and N 23° W is equal to $35° + 23° = 58°$.

3. When one reading is in each of either the two east quadrants or the two west quadrants, the included angle is equal to the sum of the readings subtracted from 180°. Thus, the angle included between N 50° E and S 37° E is equal to $180° — (50° + 37°) = 93°$.

4. When one reading is in each of two opposite quadrants, the angle is equal to the difference of the readings subtracted from

180°. Thus, the included angle of N 16° E and S 12° W is equal to 180° — (16° — 12°) = 176°.

(**24**). The reasons for these rules will be seen in Fig. 2.

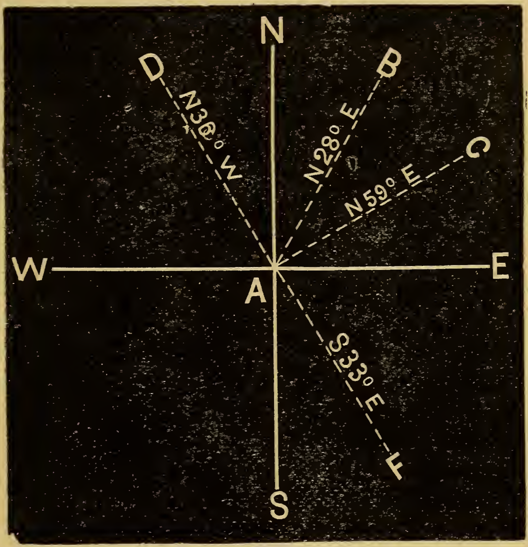

FIG. 2.

The angle included between the courses A B and A C is equal to 59° — 28° = 31°, as both readings are in the same quadrant.

The angle included between the courses A B and A D is equal to 28° + 36° = 64°, because one reading is in each of the two north quadrants.

The angle included between the courses A C and A F is equal to 180° — (59° + 33°) = 88°, because one is in each of the two east quadrants.

The angle included between the courses A D and A F is equal to 180° — (36° — 33°) = 177°, because the courses are in opposite quadrants.

(**25**). The compass box is protected by a glass covering over which fits a brass lid. Care should be taken while the brass lid is off that no electricity be excited in the glass by the friction of the hand or a cloth upon its surface, as it interferes with the working of the needle and may cause a serious error. However, when the fluid does exist, it may be removed by breathing on the glass or touching it in various places with the moistened finger.

(26). The action of the needle is also affected by pieces of iron or steel brought or kept near it. This materially interferes with its use at sea, particularly on iron ships. While surveying, nothing having a tendency to affect the action of the needle should be carried upon the person or allowed near the compass.

(27). Two kinds of compasses are in use—the sight compass and the telescope compass. Each of these may be either a plain compass or a vernier compass. The plain compass is not very extensively used, as all readings are made from its face alone, and can not be depended on for precision. In the plain compass, the line of sight lies in the direction of the **0** points of the compass circle. The vernier compass differs from the plain compass in having its compass circle, to which a "vernier" is attached, movable, generally through a short arc, about its center, thus enabling the surveyor to set the zeros or **0** points of the circle at an angle with the line of sight. This angle is read from the vernier. The movement of the circle is effected by means of a thumb-screw that gives it a slow motion. When the required angle is set off, the vernier is clamped to the plate of the compass and the readings taken. The vernier enables a surveyor to take a certain class of readings "closer" or with greater precision than is possible without it, as it gives him the advantage of a double index; yet readings down to a very small angle, say $1'$ or $30''$, are hardly ever reliable, owing to the difficulty a surveyor meets in setting the needle to exactity. The fault, however, is not in the vernier.

(28). The VERNIER* consists of an arc divided into a certain number of equal parts, and moving within another arc whose divisions are somewhat larger or smaller than its own. The first arc (vernier), as stated before, is attached to the compass circle and moves with it around a common center; the second arc is called the "limb," and is generally on the brass plate of the compass upon which the circle moves, so that the outer edge of the vernier coincides with the inner edge of the limb.

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*The vernier is so called from its inventor, and on this account the word is sometimes written with an initial capital. It was first applied to the compass by David Rittenhouse of Philadelphia.

(**29**). Let us now suppose the divisions on the limb to equal half-degrees, and that the vernier-arc, corresponding to twenty-nine divisions of the limb, is divided into thirty equal parts.

It is plain, since 30 divisions of the vernier equal 29 divisions of the limb, that one division of the limb equals $\frac{30}{29}$ divisions of the vernier, and that one division of the vernier equals $\frac{29}{30}$ division of the limb.

But each division of the limb equals 30′ or one-half of one degree; therefore, one division of the vernier will equal $\frac{30' \times 29}{30} = 29'$; which is one minute less than a division of the limb.

Now, suppose the zero of the vernier to correspond with the zero of the limb; then the **0** points of the compass circle lie in the line of sight. If now we turn the vernier until its first division from zero coincides with the first division from zero on the limb, and on the same side of zero as the division of the vernier, the **0** points of the compass will make an angle of 1′ with the line of sight; if the second division of each coincide, the angle will be 2′; if the third, it will be 3′, and so on by the same increase, so that if we make the twenty-ninth division of each correspond, the angle will be 29′; and if we turn still further until the first division of the limb coincides with zero of the vernier, the angle will be 30′. In the same manner, 30′ acting as a base, the angle may be increased to 1°, and so on.

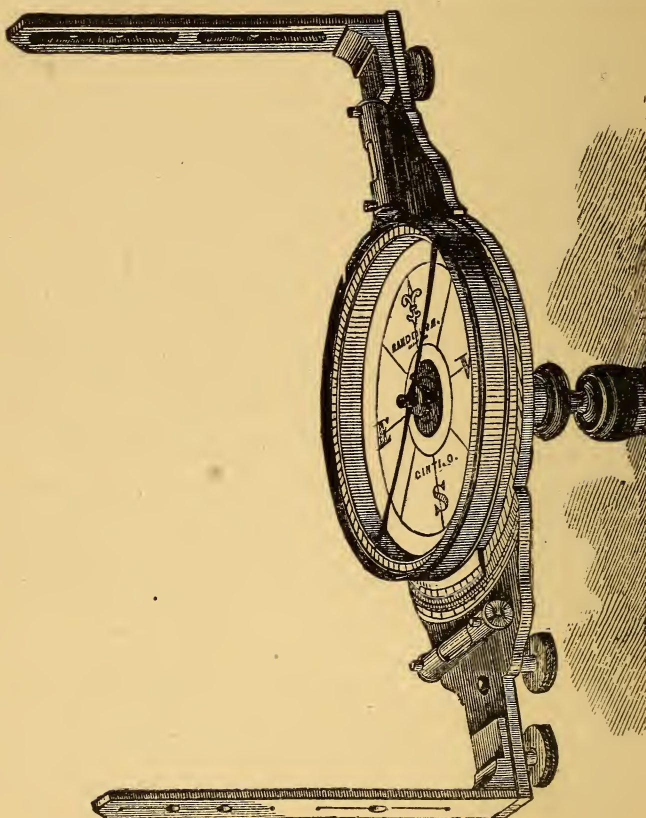
(**30**). Sometimes verniers read lower than 1′; but they are not of much practical use on magnetic instruments. They may also differ in construction from the kind described above, but they all work on the same principle. The plate on page 26 represents a vernier compass; the vernier may be observed on the compass plate in front of the box.

(**31**). As a general thing, when a compass needs repairing, either from wear or on account of some mishap, it is best to forward it to some maker of mathematical instruments. This, however, may not always be convenient or practicable, and it may be well enough to give some directions, which may be of service in case of an emergency:

1. *To Re-magnetize the Needle.*—When the needle works lazily, on account of losing a portion of its magnetism, it may be re-magnetized with a common bar or horse-shoe magnet by passing the south pole of the magnet along the north end of the needle

from the center to the extremity and bringing the magnet back to the starting point in a circle of five or six inches radius. The south end of the needle should be treated in the same manner, except that the north pole of the magnet should be used on this end. From twenty to thirty passes will give it an ample charge.

2. *To Sharpen the Center-pin.*—Sometimes the needle moves sluggishly when the center-pin upon which it turns becomes dull.



VERNIER COMPASS.

When this is the case, take out the plate in which the center-pin is set and then unscrew the pin. It may then be sharpened on a very fine stone and finished on a piece of smooth leather. Care must be taken to grind equally from every side of it.

3. *To Replace a Spirit-level.*—Remove the brass tube from the plate and take off the caps at the ends of it. Then with some pointed instrument, as an awl or a penknife, scrape out the plaster or other substance that holds the vial in place, and next force out the old vial by pressing on one end of it. Now slide the new vial into place, keeping the proper side up, and if it is too small for the tube, wedge it up with pieces of wood or paper. Notice carefully its position with regard to the opening in the tube, and when it is set in its proper place press some beeswax, boiled plaster, or putty of the proper consistency, around the ends of it, so as to fasten it firmly to the sides of the tube; then put on the brass caps and replace the tube on the compass plate. To re-adjust the level, press on the compass plate until the bubble stands in the center of the opening in the tube; then turn the compass one-half round, and if it remains there, the level is properly placed, but if it runs toward the end of the vial, and it probably will, the end toward which it settles is too high, and should be lowered or the other end raised, whichever is necessary in order to keep the tube parallel with the compass plate. After this, give the compass another half-turn and repeat the process given above until the bubble will remain in the middle of the opening in the tube in every horizontal position of the plate.

4. *To Adjust a New Sight.*—Fit it to its place on the plate, and notice how the slit lines with that of the old one on the opposite end. If it inclines to one side, remove it and file off its base on the opposite side where it rests on the plate. Then try it again, and keep up the operation until the two slits coincide throughout their whole length. If both sights need adjusting, hang a plumb, using a fine thread or hair, and regulate both sights by it. The compass should be perfectly level whenever an observation of the thread is taken, and the sights will be properly adjusted whenever they correspond with the plumb-line.

5. *To Straighten the Center-pin.*—Remove it with its base from the rest of the compass and bend it with a pair of pincers or wrench made for the purpose, always grasping it about an eighth of an inch below its point.

6. *To Straighten the Needle.*—It sometimes happens that the needle of the compass does not "cut" opposite degrees on the circle, as, for instance, when its north point is placed at **0** its south point inclines either to the right or left of the opposite **0**; when this is

the case a portion of the error may be corrected by bending the needle with the fingers and the rest by bending the center-pin.

7. *To put in a new Glass.*—First take off the brass ring that contains it (bezzle ring) and remove the putty. Then take out the old glass and put in the new by reversing the process. If the new glass is so large that it will not go in readily, hold the edge on a grindstone and grind it down. The manner in which it should be ground may generally be seen by noticing the glass just taken out.

8. The motion of the ball at the head of the Jacob's staff may be regulated by a screw-cap that fits down upon it. It should be kept reasonably tight in order that the compass may not be too easily jarred out of level. If it works loosely, screw the cap down tighter. After long usage the ball may not fit the cavity well, and in this case it may be taken out and a small piece of sheet brass placed under it, or even a piece of paper will answer for a short time.

(**32**). In carrying the compass it need only be lifted from the staff and put under the left arm so that one of the sights may project up behind the shoulder, and the staff makes a good walking stick; but in transportation over the country the sights should be taken off and all packed snugly in a box or something else that will answer the purpose. A suitable box is usually furnished with the compass by the manufacturer.

(**33**). All compasses are provided with a lever or spring with which to raise the needle from the center-pin when the compass is not in use, and this should not be neglected.

(**34**). The compass when not in use should be placed in a horizontal position and the needle allowed to assume its natural direction. If this precaution is taken, the needle will better retain its polarity.

(**35**). The telescope compass is gradually growing into favor with surveyors and seems to be taking the place of the sight compass in many localities. The telescope enables the surveyor to set a flag at longer ranges, at greater elevations and depressions, and discern it more easily among trees and bushes. It is not quite so convenient to handle as the sight compass, however, but this is no great disadvantage. Either may be used on a tripod, instead of a Jacob's staff. For plate of a telescope compass see frontispiece. This engraving represents the very fine instrument manufactured by T. F. Randolph, Cincinnati.

Questions on Chapter II.

1. Describe the compass.
2. How are the degrees numbered on the circumference of the compass circle?
3. How are the sights arranged?
4. Describe the magnetic needle. How is the delicacy of a needle determined?
5. Explain the method of measuring horizontal angles.
6. How are vertical angles sometimes approximately measured?
7. Why are the letters E and W reversed on the compass face?
8. State the rule for taking the readings from the compass circle.
9. The north end of the needle points 20° to the right of N. What is the bearing of the line of sight? What is its reverse bearing?
10. What is a northerly bearing? A southerly bearing?
11. In running lines east what letter on the compass face should be turned north? In running west, what one? Why?
12. Give the four rules for measuring angles.
13. What is the included angle in each of the following cases: N 40° E and N 62° E? S 15° W and S 39° E? N 29° W and S 43° W?
14. How is electricity excited in the glass of the compass? How may it be removed?
15. How do iron and steel affect the action of the needle?
16. What two kinds of compasses are in use?
17. How many kinds of sight compasses are there?
18. What is the difference between a plain compass and a vernier compass?
19. Describe the "vernier." Of what advantage is it?
20. How is the needle re-magnetized?
21. Explain the manner in which the center-pin is sharpened.
22. How is a spirit level replaced?
23. In what way is a new sight adjusted?
24. How do you determine, when the two extremities of the needle do not "cut" opposite degrees, whether it is the needle or the center-pin that is bent? Answer—Turn the compass and notice the amount of the error in several

positions. If it decreases in certain places and increases in others, it is the center-pin. If it remains about the same in every position, it is probable that the needle alone is bent.

25. Why should the needle be raised against the glass when the compass is not in use?
26. What is the proper position for the compass during the interval between surveys?
27. What are the advantages of the telescope compass over the sight compass?

CHAPTER III.

THE VARIATION OF THE MAGNETIC NEEDLE.

(36). The meridian of any point on the earth's surface is a due north and south line connecting the point with the poles of the earth.

(37). This is called the *true meridian* of the place, in order to distinguish it from the *magnetic meridian*, which will be considered further on.

(38). Various methods are employed in determining the true meridian, but only two of the most simple and satisfactory will be described:

1. *By a Shadow Cast by a Perpendicular Object.*—Erect a perpendicular staff on a level surface, so that its shadow will remain on

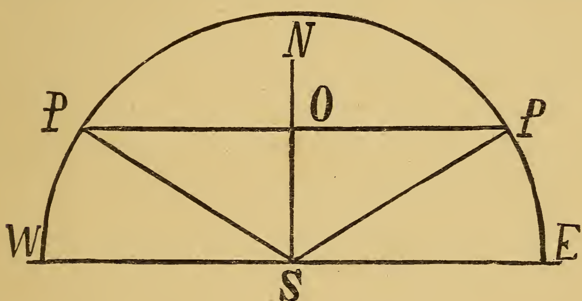


Fig. 3.

the surface from about 8 o'clock, A. M., till 4 o'clock, P. M., as indicated in the horizontal projection, Fig. 3, in which S N represents the staff.

Three or four hours before noon, with a radius, P S, shorter than the length of the shadow, and from the point S as a center, describe an arc through the point P and produce it beyond, opposite P. Then mark the point P, where the shadow last touches the arc, and in the afternoon the other point P where it first touches it again, and connect these two points with a line PP. Bisect this line at O, and the line SO, produced in either or both directions, will represent the true meridian of the place. It is not exactly correct, except at certain times during the year (at the solstices), but it is always sufficiently accurate for ordinary purposes.

2. *By the Polar Star.*—The Polar star (Polaris) is situated about $1\frac{1}{2}$ degrees from the north pole of the heavens, and appears to revolve around the pole once in 23 hr. 56 min. If, now, we suppose a vertical plane to pass through the north pole and the eye of the observer, then twice during the time of revolution Polaris will be in this plane, and consequently in the meridian of the observer (once when above the pole and again when below it). These are called its upper and lower culminations, respectively.

(*a*). On the opposite side of the pole from Polaris is a star known as *Alioth*, or more commonly as *Epsilon*, of the constellation of the *Great Bear* or "*Dipper.*" This is the first star in the handle of the Dipper, and is situated next to the four that form the quadrilateral, the outside two of which, *Dubhe* and *Merak*, are called "the pointers," because they indicate the position of Polaris.

(*b*). Since these stars, Polaris and Alioth, are almost exactly on opposite sides of the pole, it is evident that they will both be on the meridian of the observer when one of them is above the other, and it is more convenient to make the observation during the upper culmination of Polaris.

(*c*). Suspend a plumb-line from some elevated projection, as the limb of a tree or a strip nailed to the side of a building or high post, and at a point south, not so distant that Polaris will rise above the point of suspension of the plumb, arrange a short board horizontally east and west, a little below the level of the eye. On this board place some kind of a contrivance containing an opening across which a thread may be stretched so that it will be parallel to the plumb-line when the instrument is in use, and slide the instrument along the board until the thread ranges with the plumb-line and Polaris. Continue to move the instrument west as Polaris moves to its point of superior culmination, and watch also the approach of Alioth to the meridian. As soon as the plumb-line falls on both stars, fasten the instrument to the

Fig. 4.

board, and you have two points in the true meridian. The line through these may be produced at pleasure and permanently marked. Fig. 4 represents the plumb-line covering both stars.

(*d*). Still greater accuracy may be reached by following Polaris for twenty-two minutes after the plumb-line falls on it, and then marking the line.

(*e*). When the upper culmination occurs during the day, the lower culmination must be used, but Alioth is then very high.

(**39**). The following table shows the time of the upper culmination of Polaris for each tenth day. The time for intermediate days may be approximated by interpolation. The time is given to the nearest minute:

Month.	1st Day.	11th Day.	21st Day.
	h. m.	h. m.	h. m.
January . . .	6 : 30 P. M.	5 : 50 P. M.	4 : 50 P. M.
February . .	4 : 27 "	3 : 47 "	2 : 48 "
March	2 : 32 "	1 : 53 "	12 : 50 "
April	12 : 26 "	11 : 47 A.M.	10 : 49 A.M.
May	10 : 29 A.M.	9 : 49 "	8 : 50 "
June	8 : 27 "	7 : 48 "	6 : 49 "
July	6 : 29 "	5 : 50 "	4 : 51 "
August . . .	4 : 28 "	3 : 48 "	2 : 49 "
September .	2 : 26 "	1 : 47 "	12 : 48 "
October . . .	12 : 29 "	11 : 49 P. M.	10 : 50 P. M.
November . .	10 : 27 P. M.	9 : 47 "	8 : 48 "
December . .	8 : 28 "	7 : 49 "	6 : 50 "

(**40**). If a magnetic compass be placed on the true meridian thus established, the needle of the compass, pointing toward the *north magnetic pole,* instead of toward the *north pole of the earth,* marks a line called the *magnetic meridian,* which coincides with the true meridian in comparatively few places on the earth. The angle formed by the difference in direction of these two lines is called the *variation* or *declination* of the needle.

(**41**). If the north point (south pole) of the needle point to the east of the true meridian, it is called *east variation,* and if it point to the west, it is called *west variation.* The amount of variation is determined by the size of the angle.

(**42**). In the United States there is a line extending southeast through the eastern part of Michigan, western part of Lake Erie, eastern Ohio, central West Virginia, Virginia, and North Carolina, reaching the Atlantic ocean near Wilmington, that is called the *agonic line* or *line of no variation,* because at all points thereon the magnetic meridian and true meridian coincide. Places east of this line have west variation, and those west of it have east variation.

(**43**). *Isogonic lines* or *lines of equal variation* run through places having the same variation. For instance, the line of three degrees west variation passes through Chesapeake Bay, Maryland, central Pennsylvania, and western New York, and the line of three degrees east variation passes through western South Carolina, eastern Georgia, Tennessee, Kentucky and Indiana, and western North Carolina, Ohio, and Michigan. The variation increases in both directions from the agonic line, reaching about 18 degrees west variation in eastern Maine and 22 degrees east variation in the northern part of Washington Territory.

(**44**). The isogonic lines converge toward the north magnetic pole, situated at present in longitude about 96° west from Greenwich, and latitude about 70° north. This pole has been gradually moving westward for several years, and will perhaps continue to do so for several years to come. This causes the agonic line and all the isogonic lines to move in the same direction, so that west variation is constantly increasing and east variation constantly decreasing.

(**45**). The change that takes place in the variation of the needle at any place from this cause is called its secular change. This varies in different localities, and is generally greater in the northern part of the United States than in the southern part. It is determined by comparing the variation at the time of any observation with that of a preceding or succeeding one.

(**46**). But this is not the only change to which the needle is subject. Its action is modified by other influences. The north end of the needle moves westward from about 6 o'clock A. M., until about 2 o'clock P. M., and then gradually returns to the starting point. This is called its *diurnal change*, and it sometimes amounts to 10 or 12 minutes of a degree.

This change is about twice as great in summer as in winter, hence an *annual change* must be taken into consideration.

(**47**). The following table, taken from the Report of the United States Coast Survey, illustrates these changes. The *mean magnetic meridian* is the average position of the needle for the day:

Hour.	Spring.	Summer.	Autumn.	Winter.	
	′	′	′	′	
6 A. M.	3	4	2	1	Needle points E. of mean mag. mer.
7 " "	4	5	3	1	
8 " "	4	5	3	2	
9 " "	3	4	2	2	
10 " "	1	1	0	1	
11 " "	1	2	2	0	Needle points W. of mean mag. mer.
12 M.	4	4	3	2	
1 P. M.	5	6	4	3	
2 " "	5	5	3	3	
3 " "	4	4	2	2	
4 " "	3	3	1	1	
5 " "	2	2	1	1	
6 " "	1	1	0	0	

(**48**). Probably as good a time as any to get the variation of the needle is between 5 and 7 o'clock in the evening, as it then pretty nearly indicates the mean magnetic meridian. The diurnal and annual changes are usually disregarded in practical work with the compass.

(**49**). The needle is also subject to disturbances that do not appear conformable to any known law. These generally take place during a thunder storm, aurora, or other electrical phenomenon, and sometimes cause a surveyor no small amount of vexation.

(**50**). The north end of the needle is also inclined to point downward. This is called its "*dip*" or *inclination*, and increases as we go northward. It is overcome by making the north end of the needle the lighter, and in order to render the needle adjustable, a small counterpoise is arranged in a groove on its south end.

(**51**). A great many magnetic phenomena are at present very imperfectly understood, and many of the conclusions based upon partially accepted theories are liable to be overthrown at almost any time. A discussion of these belongs to Philosophy.

Questions on Chapter III.

1. What is meant by the meridian of a place?
2. What is the difference between the true meridian and the magnetic meridian?
3. Explain each of the methods given for establishing a true meridian.
4. What is meant by the culmination of Polaris?
5. To what does the north end of the needle point?
6. What do you understand by "the variation of the needle?"
7. What is east variation? West variation?
8. What determines the amount of variation?
9. Through what part of the United States does the agonic line run?
10. What are isogonic lines?
11. What is the situation of the north magnetic pole?
12. In what direction is the agonic line and all the isogonic lines moving at present?
13. What effect does this movement have on east variation? West variation?
14. What other influences act upon the needle?
15. What is the difference between the diurnal change and the annual change?
16. What is meant by "dip?"
17. How is it overcome?

CHAPTER IV.

EFFECT OF CHANGE OF VARIATION* ON OLD LINES AND METHODS OF CORRECTING BEARINGS.

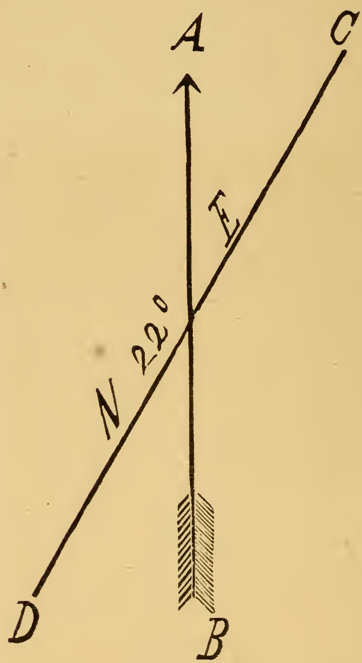


FIG. 5.

*The terms *variation* and *declination* have become synonymous in meaning, although it would be better, etymologically speaking, to use the word *variation* to denote the change of *declination*. We would then say *variation of declination*, instead of *change of variation* as at present.

(**52**). As the variation of the needle is constantly changing, so the bearing of a line surveyed at any time is also subject to constant change. This renders it necessary in the re-survey of any line to take into consideration the amount of change that has taken place since the previous survey; otherwise the location of the line will be changed.

To illustrate: Let the line A B, Fig. 5, represent the line of direction of the magnetic needle (magnetic meridian) at a certain time, and the line C D which makes an angle of 22° with the magnetic meridian, and whose course is N 22° E, represent a line surveyed while the needle marks this meridian.

Since the north end of the needle is continually moving toward the west, it is evident that the angle between A and C, formed by the crossing of the two lines, will be constantly growing larger, and that the bearing of the line C D will consequently increase

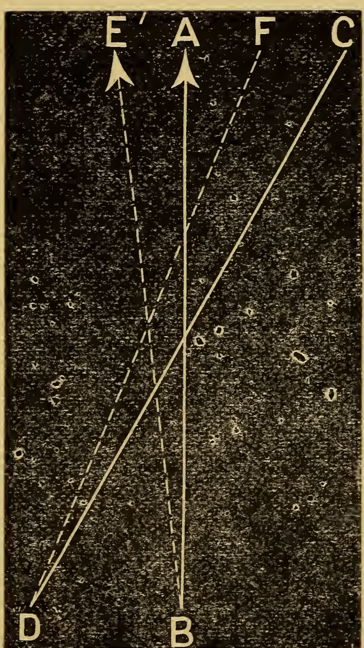


FIG. 6.

with each succeeding year, as long as the westward movement of the needle continues.

(**53**). Let us now suppose that several years after this survey is made, the needle assumes the position of the dotted line E B, Fig. 6, making an angle with A B equal to 5°. The line C D will now make an angle with the magnetic meridian equal to (22° + 5°) = 27°. It will be seen that the bearing of the line has increased 5°, the change of variation of the magnetic needle.

(**54**). If now the surveyor, through ignorance or carelessness, should neglect to take this change into consideration, and run the line at the old bearing (22°) marked in the description of the line, he would change its position to that of the dotted line D F, and it is easy to see what an error this would cause in the survey of a tract of land.

(**55**). A survey of this kind, however, does not affect the form or area of a piece of land, but simply changes its boundaries and seems partially to revolve the tract around the corner from which the surveyor starts, as a center. For instance, suppose a mistake

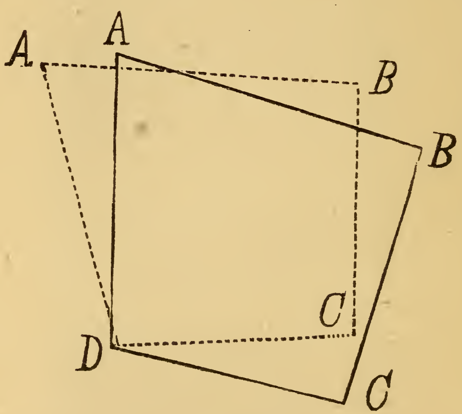


Fig. 7.

of this kind to be made in the survey of the tract A B C D, Fig. 7; its position would be changed to that of the dotted quadrilateral. In this survey the corner D was made the starting point.

(**56**). In order to determine the present bearing of a line, it is an advantage to know three things: 1. The bearing of the line at the time of a previous survey. 2. The number of years that have elapsed since that survey was made. 3. The annual amount of secular change of variation.

1 and 2. Where field-notes of the former survey have been preserved, they generally and should always, give the necessary information in regard to the bearing of the line and date of the survey; but where no record of the survey can be found, the bearing may sometimes be approximately determined from old deeds or descriptions of the land, and these frequently assist the surveyor in arriving at a conclusion with regard to the time the survey was made. Persons living in the neighborhood may also know something of the previous survey.

3. In order to determine the annual change of variation, various methods may be employed.

(1). The bearing of a line at the present time may be compared with that recorded by a competent surveyor some time previous.

(2). A true meridian may be established by the second method explained in the preceding chapter, and the variation of the needle found from this by observing the angle the magnetic meridian makes with the true meridian. This result may then be compared with the variation recorded after a previous observation.

In both of these cases the amount of change in minutes will equal the quotient obtained by dividing the difference of variation in minutes between the two observations by the number of years that have elapsed between them. For instance, if the variation of the needle at a certain time is found to be 5°, and in twenty years after an observation shows that it has decreased to 4°, the annual change will equal $(5° - 4°) = 1° = 60'$, which divided by $20 = 3'$. From this the variation for any subsequent time may be found by multiplying the annual change, $3'$, by the number of years, and adding or subtracting the product, as will be explained further on.

(3). When the annual change at several important points is known, the change at intermediate points throughout the country may be determined by interpolation, but this method is not reliable in all cases.

(57). The following table is taken, somewhat modified, from the report of the U. S. Coast Survey of a few years ago, and contains the *variation of the needle* at various important stations throughout the United States for the year 1881:

Bangor, Me.,	16° 30′ W.
Wolfsboro, N. H.,	12° 15′ W.
Middlebury, Vt.,	11° 35′ W.
Marblehead, Mass.,	12° 54′ W.
Schenectady, N. Y.,	9° 58′ W.
Chambersburg, Pa.,	4° 33′ W.
New Haven, Conn.,	9° 00′ W.
Providence, R. I.,	11° 27′ W.
Wilmington, N. C.,	1° 04′ W.
Jackson, Miss.,	6° 33′ E.
Washington, D. C.,	3° 17′ W.
Camden, N. J.,	6° 50′ W.
Columbus, O.,	0° 36′ E.
Savannah, Ga.,	1° 50′ E.
New Orleans, La.,	6° 33′ E.
Quincy, Wis.,	6° 33′ E.
Decatur, Ind.,	1° 50′ E.
Grand Rapids, Mich.,	1° 50′ E.
Havana, Cuba,	4° 46′ E.
Carlinville, Ill.,	6° 33′ E.
Aiken, S. C.,	1° 50′ E.
Monterey, Cal.,	16° 06′ E.
Sitka, Alaska,	28° 27′ E.

This list is not full enough to be of much practical use, but I think best not to extend it any further, as the surveyor can easily find the variation at any point himself.

(58). The next table, also taken from the Report of the Superintendent of the Coast Survey, gives the *annual* amount of secular *change* in certain localities in the United States.

Maine, Long Island, Delaware, Maryland, Virginia, South Florida, South Alabama and Mississippi, and New Jersey	3′
East Maine, West Tennessee, Missouri, and West Louisiana	2′

Ohio, East Tennessee, East Louisiana, and East Massachusetts	2′ 15″ to 2′ 45″
New Hampshire, West Massachusetts, Rhode Island, Connecticut, West Virginia, North Carolina, South Carolina; Georgia, and North Florida	3′ 15″ to 3′ 45″
North and West New York	4′ 30″
Vermont	5′ 30″
West Louisiana	1′

In all these places the change marked to the right indicates an increase of westerly or decrease of easterly variation.

(**59**). These tables will enable us to approximate to the variation of the needle at points included in both tables, for perhaps several years. For example, the variation of the needle at Camden, N. J., this year is 6° 50′ W; in 1885 it will be about $6° 50' + (4 \times 3') = 7° 02'$. If the variation is easterly, as in the western part of the United States; the amount of change is subtractive, instead of additive. Take this for example: What will be the variation of the needle at New Orleans in 1884?

1884—1881=3 years.

Variation in 1881 = 6° 33′.

Annual change about 2′ 30″.

Amount of change $= (3 \times 2' 30'') = 7' 30''$.

Variation in 1884 = 6° 33′—7′ 30° = 6° 25′ 30″.

(**60**). In order to find what the variation of the needle was a few years ago, we have only to reverse the rule given; that is, add where the variation is easterly, and subtract where it is westerly.

The following examples may be used for practice:

(1). The variation of the needle at a certain place in 1880 was N 5° 32′ E, and the annual change 2′ 20″; what will be the variation 1889?

(2). In 1878 the variation at a certain place was 4° 15′ W, annual change 3′ 30″; what will be the variation in 1886?

(3). In 1880 the variation at A was 2° 36′ W; what was the variation in 1874, supposing the annual change to be 4′?

(4). The annual change at B is 3′ 45″ and the variation of the needle in 1879, 3° 47′ E; what was the variation at this place in 1870?

(**61**). It would frequently be better if the bearings of lines were taken from the true meridian as a basis, as no change would then take place in the bearings.

This may be easily explained. Suppose the line A B to repre-

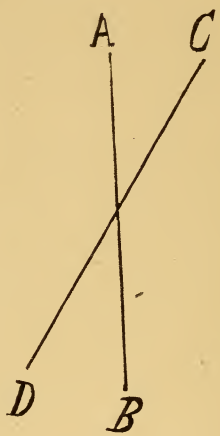


FIG. 8.

sent a section of a true meridian of the earth, and the line C D to represent one of the boundaries of a piece of land. Both of these lines are fixed, and consequently the angle between them can never change.

(**62**). If, however, the line A B represents a section of the magnetic meridian, it is constantly changing, and this of course affects the angle.

(**63**). In order to determine the true bearing of a line from its magnetic bearing, two cases must be considered :

1. *When the variation is west.* RULE.—Add the variation to the bearing for northwest and southeast courses, and take their difference for northeast and southwest courses.

In Fig. 9 let A B represent a due north and south line and C B represent the direction of the needle. D B and E B are two lines whose bearings are to be changed from the magnetic bearing to the true bearing. Suppose the variation of the needle to be 10° W; then the true bearing of the line E B will be N (25° + 10°)

W = N 35° W, and the true bearing of the line B D will be N (35° — 10°) E = N 25° E.

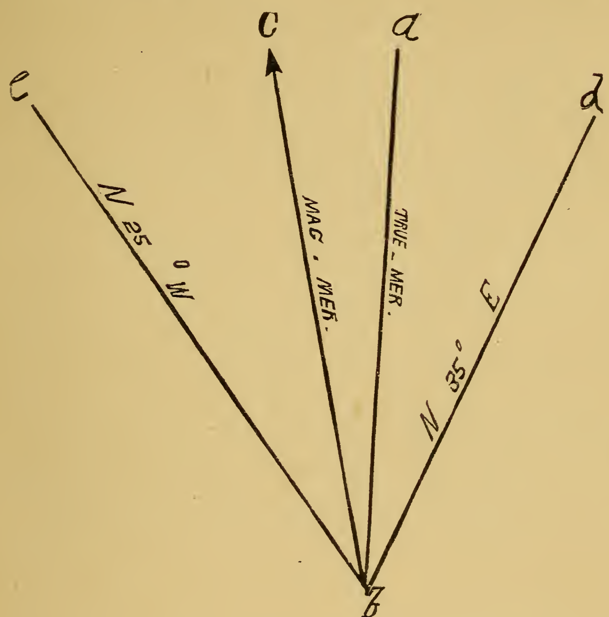


FIG. 9.

2. *When the variation is east.* RULE.—Reverse the preceding operation; that is, take the difference between the bearing and variation for northwest and southeast courses, and their sum for northeast and southwest courses.

The line A B, Fig. 10, represents a section of the true meridian, and the line B C a section of the magnetic meridian. As before, the bearings of the lines D B and E B are to be changed to the true bearings from the magnetic bearings. If the variation of the needle be 10° E, the true bearing of the line B D will be N (25° + 10°) E = N 35° E, and the true bearing of the line B E will be N (35° — 10°) W = N 25° W.

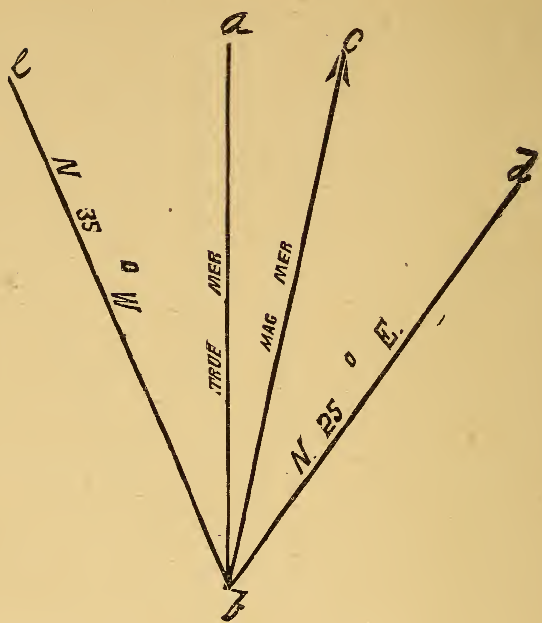


FIG. 10.

(64). The variation in the following examples is 5°; what is the true bearing of each course?

1. N 45° W.
2. N 32° W.
3. S 16° W.
4. N 17° E.
5. S 25° E.
6. N 25° E.

(65). If in any case the sum of the variation and bearing amounts to more than 90°, the supplement* of the sum must be taken, and the first letter changed to its opposite.

Take, for instance, the bearing N 88° W when the variation is 6° W. This gives us the following result: 88° + 6° = 94°, and taking 94° from 180° to get the supplement, we have (180° — 94°)

*The supplement of an arc is what remains after subtracting the arc from 180°.

= 86°. After changing the first letter of the magnetic bearing, we see that the true bearing of the line is S 86° W.

(**66**). In changing from the true to the magnetic bearing, it is necessary only to reverse the foregoing rules.

Questions on Chapter IV.

1. What effect does the change of variation of the magnetic needle have upon the bearings of lines?
2. Why must this change be taken into consideration in a re-survey?
3. Why would it not affect the figure or contents of a tract of land if this change were not regarded?
4. What three things are necessary in order to determine the present bearing of a line?
5. How is its bearing at the time of a former survey found? The annual change?
6. Two observations thirty years apart show a change of 1° 15′ in the variation of the needle. What is the annual change?
7. What is meant by *east* variation? *West* variation.
8. How do you find the true bearing of a line from the magnetic bearing? The magnetic from the true?

CHAPTER V.

METHOD OF RUNNING LINES, ETC.

(**67**). The first thing a surveyor does in making a survey is to find a corner from which to start, and this is frequently a matter of no little trouble, but its discussion belongs further on in our work. Let us suppose for the present that the monument which marks the corner (generally a stone or a stake) is still standing, or that the witnesses to it are yet to be found. These witnesses are generally trees, although any other immovable object will answer the purpose equally well. When the corner is put down the course and distance from it to one or more of these objects is carefully noted in the record (field notes) of the survey, so that, no matter if the monument that marks the corner has disappeared, its position can easily be determined as long as any remains of the witnesses exist. The witnesses, if trees, are called witness-trees, and are marked about eighteen inches from the ground on the side facing the corner. The mark consists of a blaze about six or eight inches long, which generally has a cross notch cut in it.

(**68**). If the monument at the corner remains, its center should be taken as the point from which to run the line; but if it can not be found, its position must be determined from the witnesses. It is probable that the approximate location of the corner is known, so that the surveyor or some of his attendants know where to look for it. Suppose the surveyor examines his record and finds that when the corner was set, a beech tree 18 inches in diameter was taken as a witness to it. This tree stood in a direction N 43° E from the corner, and was distant 78 links. He looks in that direction and sees a tree which he believes to be the one, and after an examination he finds the surveyor's mark and is convinced of its identity. All he has to do now is to set the compass in front of the mark on the tree and turn it until the needle indicates the reverse bearing, S 43° W; a staff is then set in the line of sight somewhat further from the tree than the distance given for the corner, and a measurement of 78 links made from the notch on the tree in the direction of the staff; the termination of this measurement is the corner.

(69). On prairies, where no trees are available for witnesses, mounds of earth thrown up around a stake are usually made to answer the purpose, and in clearings it is sometimes necessary to make the same kind of witnesses, or supply their places with stones set at suitable distances from the corner. A good monument should always be put down at the corner, and one not easily moved or destroyed does not particularly need any witnesses to it.

(70). The method of describing a witness-tree employed by surveyors in making up their field-notes, is much abbreviated from the method used above. The above description would read as follows: Be 18 N 43 E 78. The first number signifies the diameter, the second the bearing, and the third the distance in links. The bearing and distance of the tree from the corner are called its "course and distance." Likewise the course and distance of any line are its bearing and length.

(71). After the corner has been found, the next thing is to run a line from this corner to the next. In surveying, a line always connects two corners, and in plane surveying it represents the shortest distance between them.

(72). In order to run this line, its course and distance must be at least approximately known, and should be exactly known.

(73). Let us suppose that the true bearing of the line is N 3° E, and its length 6 chains and 50 links. Since the true bearing is given it has remained unchanged (Art. 61), and the surveyor has only to turn the vernier of the compass until he sets off the variation of the needle, and then set the north end of the needle at N 3° E. The sights of the compass are then set on the line, and a measurement of 6 chains and 50 links on the line of sight will bring him to the next corner.

(74). In setting off the variation of the needle on the vernier, two cases arise: (1) when the variation is east, and (2) when the variation is west.

1. *To Set Off East Variation.* RULE.—Turn the sights to the *left*

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until the angle between the line of sight and the **0** points of the compass circle equals the variation of the needle.

2. *To Set off West Variation.* RULE.—Turn the sights of the compass to the *right* until the angle between the line of sight and the **0** points of the circle equals the variation.

These rules obtain whether the movement of the needle is toward the west or east.

We are now prepared to run lines whose true bearings are known.

The course and distance of a line are N 43° E 16 chains and 50 links.

(**75**). After finding the first corner on this line, the surveyor sets off the variation of the needle on the vernier, and then sets the compass at the corner, levels it, and turns the sights until the needle indicates the bearing of the line. He then starts the flagman in the direction of the line of sight to a suitable distance from the compass, and then moves him either to the right or left until the sights strike the flag-staff. The staff is left in this position until the surveyor comes up and sets the compass where it stood, and then the flagman seeks another position further along the line.

(**76**). The chain-men commence at the center of the monument that marks the corner as soon as the surveyor has set the flag on the line. The one that takes the lead is called the "leader," and the one at the rear end of the chain is called the "follower." At setting out, the leader takes a certain number of iron or steel pins (usually ten) and puts one down at the end of the chain. The chain is then carried forward another length and its rear end held against the pin just put down while the leader sticks another into the ground. Great care should be taken to keep the chain free of "kinks," taut, and as nearly horizontal as possible, no matter how uneven the ground may be, and the follower should line each pin with the flag-staff or compass, whichever may be set at the point to which they are running.

(**77**) When ten pins are used the "marker" drives a stake* where the last pin stood; the leader takes the ten pins from the follower, and the measurement proceeds just the same as from the corner. If eleven are used the follower retains one in starting out from the corner, and as soon as the leader puts down the last of

*The points on the line marked by the stakes are called "outs," because the leader runs out of marking pins at these places.

his ten he receives ten from the follower, and the new measurement begins at the last pin put down, instead of at the stake, as before. As we said before, the chain usually employed in surveying is only two rods, fifty links, or thirty-three feet in length, and since a stake is put down at every ten lengths, it leaves the stakes twenty rods, or five chains apart. The first stake from the corner has one notch cut in it, the second two, the third three, and so on. The surveyor should sight back while the marker is setting the stake and see that it is put exactly on the line.

(**78**). The relative places on the line of the persons engaged in the survey are as follows: (1) flagman, (2) surveyor, (3) chain carriers and marker.

(**79**). The measurement is continued in the direction N 43° E until the length of the line, 16 chains and 50 links, has been measured, and if the line thus run strikes the corner, it coincides with the true line, but if it does not, the stakes must be moved, either to one side or the other, until it is made to coincide.

(**80**). Let us suppose that in the case before us the line has been found to terminate at a point $8\frac{1}{4}$ links to the left of the corner. This line is called a "random line," and it is evident that it has been diverging from the true line gradually since leaving the starting point, and that this divergence is always in proportion to the distance from this point.

Dividing now the distance between the terminations of the two lines by their length in chains, we have $8\frac{1}{4} \div 16\frac{1}{2} = \frac{1}{2}$ link for the increase of divergence for each chain from the point from which the lines start, and since the first stake is 5 chains from this point, it is $5 \times \frac{1}{2}$ link $= 2\frac{1}{2}$ links off the true line. In like manner the second stake is $10 \times \frac{1}{2}$ link $= 5$ links off the true line, and the third one $15 \times \frac{1}{2}$ link $= 7\frac{1}{2}$ links off the true line. The stakes are put on the true line by moving them the given distances in the proper direction which is always the opposite of the direction of the termination of the random line from the corner.

(**81**). This may be illustrated by the figure.

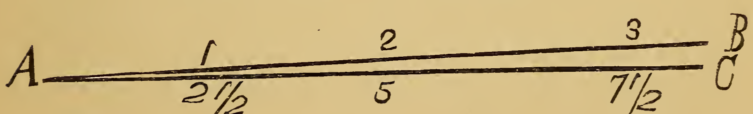

Fig. 11.

Let A C represent the true line, and A B the random line. Since the termination of the random line lies to the *left* of the true corner, the stakes must be moved to the *right*. The numbers on the random line indicate the order in which the stakes are numbered, and those on the true line represent the distance that each stake must be moved to the right.

(**82**). This enables us to deduce the following *rule for correcting the stakes:* Divide the distance between the terminations of the two lines in links by their common length in chains, and multiply the quotient by the number of chains the stake is distant from the starting point. The product will equal the number of links the stake is to be moved. Then move the stake in a direction opposite to that of the termination of the random line from the corner.

Where the length of the line is a multiple of five chains (the distance between stakes), the following rule will be found more simple: Divide the distance between the terminations of the two lines in links by the number of stakes on the line, and multiply the quotient by the number of the stake from the starting point. The product indicates the number of links the stake is to be moved as before.

(**83**). In both of these rules the terminations of the two lines and the starting point are considered as the vertices of an isosceles triangle, and the distance between the terminations of the two lines should be measured on a line that will make the same angle with one as with the other.

(**84**). In the following examples, give the distance and direction each stake should be moved:

(1). Length of line, 20 chains. Ran to the left of true corner 40 links.

(2). Line 40 chains long. Ran to the left 20 links.

(3). Line 20 chains. Ran to the right 30 links.

(4). Line 18 chains. Ran to the left 12 links.

(5). Line 16 chains. Ran to the right 32 links.

(6). Line 26 chains. Ran to the right $45\frac{1}{2}$ links.

(**85**). In making up his field-notes, the surveyor usually writes links as hundredths of a chain. For instance, ten chains and forty links would be written 10.40. He also sometimes uses the word "missed" to denote that the termination of the random line lies either to the right or left of the true corner, and accompanies it with the word "right" or "left," as the case may be. These terms are frequently abbreviated in writing by using their initial letters instead of the words themselves. Thus, "missed left" may be cut down to M. L.

(86). Owing to the imperfection of magnetic instruments it is common, particularly on long lines, for a surveyor to run either to one side or the other of the corner to which he is running, even when he knows the bearing of the line. In this case, however, he seldom misses the corner any considerable distance—perhaps only two or three links—and the bearing he has taken may be assumed to be correct, the error being due to the manipulation of the instrument.

(87). But where the bearing is only approximately known, the random line may vary greatly from the true line, and when this happens the assumed bearing must be corrected in order that the true bearing may be subsequently determined without a resurvey of the line.

(88). Suppose a line 40 chains long is to be surveyed, and from the best evidence the surveyor has, he assumes its bearing to be N 2° E. The line is accordingly surveyed, and upon reaching the other end the surveyor finds that he has missed the corner 70 links. It is plain that he assumed a bearing considerably in error, and he wishes to correct it. To do this, he may employ either of the following rules:

1. Multiply the length of the line by .01745 and divide the product by the product of the distance between the extremities of the two lines by 60. The quotient will be the correction in minutes of a degree. In this case we have

$$(.01745 \times 40) \div (70 \times 60) = 60' + = 1^\circ +.$$

All distances should be in chains and hundredths of a chain.

This rule is derived as follows: In Fig. 12 the lines A B and A C represent the true line and random line, respectively, of the survey. C B represents the distance between their extremities.

(1). The line C D is called the *sine* of the angle B A C, and where the angle is small, it does not vary much in length from the chord of C B which connects the extremities of the two lines. The line A D is called the *cosine* of the angle B A C, and where the angle is small it does not differ materially in length from the line A B.

(2). The *sine* of an angle, therefore, is a perpendicular raised

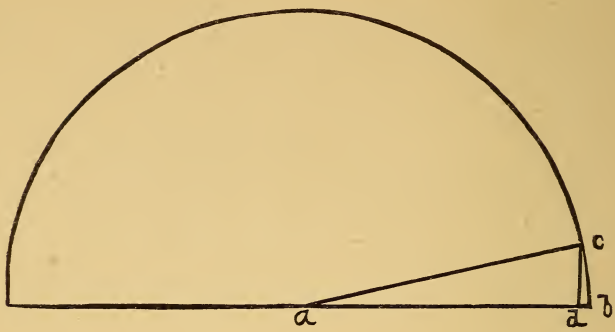

FIG. 12.

from one of its sides to the other, and the *cosine* of an angle is the portion of one of its sides intercepted between the foot of the sine and the vertex of the angle itself.

(3). Particular attention should be paid to the relation of the sine and cosine to one another, and they should also be studied in their application to the lines of the survey, as explained above. The surveyor needs to employ sines and cosines frequently.

(4). If now we call the length of the radius 1, the length of the sine of an angle of one degree will be .01745 which is the number we employed in the rule. The sine will increase in the same ratio that the radius increases. Therefore, if the radius is 40, the sine will be $(40 \times .01745) = .69800$; and as this is to the distance between the extremities of two lines, so is 60, the number of minutes in a degree, to the number of minutes of correction; hence the rule.

(5). Where the angle is large, the sine may differ considerably in length from the chord, but this is a case that will seldom come up in practice, except where the surveyor chances to make a mistake, and in this case he would better resurvey the line.

2. In rectangular surveying, nearly all the lines to be surveyed in usual practice in dividing and sub-dividing the section, are $\frac{1}{4}$ mile, $\frac{1}{2}$ mile, or one mile in length. Now, the sine of an angle of one degree for a radius of 20 chains, or $\frac{1}{4}$ mile, is very nearly 35 links; therefore, for 40 chains, or $\frac{1}{2}$ mile, it is $(2 \times 35) = 70$ links; and for 80 chains, or 1 mile it is $(4 \times 35) = 140$ links.

(1). This enables us to modify the rule already given, as follows:

Multiply the distance between the extremities of the two lines by 60, and divide the product by the sine of one degree for a radius equal to the length of the line surveyed. The quotient will be the number of minutes of correction to be made.

(2). If, for example, in a line $\frac{1}{2}$ mile long, the surveyor miss the corner 70 links, the correction is made as follows:

$$70 : 70 : : 60 : x = \frac{70 \times 60}{70} = 60' = 1°.$$

3. If, however, he has its bearing at a previous time, he may then find its present bearing by the following rule: Run a line from one extremity of it with the old bearing and distance. Measure the distance between the other extremity of this random line and the true line. Multiply this distance by 57.3 and divide the product by the length of the line surveyed. The quotient will be the change of variation expressed in degrees.

Suppose the length of the line to be 18.25 and the distance between the extremities 35 links. Then, $57.3 \times .35 = 20.055$, and $20.055 \div 18.25 = 1.09° = 1° \ 5' \ 24''$.*

(**89**). In determining the correction to be made in the following examples, use whichever one of the preceding rules appears most expeditious:

(1). Length of line, 16 chains. Distance between terminations 18 links.

(2). Length of line, 36 chains. Distance between extremities, 32 links.

(3). Length of line, 20 chains. Bearing 7° 32′. Distance between extremities, 30 links.

(4). Length of line, 40 chains. Distance between extremities, 48 links. Bearing 16°.

(**90**). Let us now see whether this correction to be made is to be added to the assumed bearing of the line, or subtracted from it.

This involves two cases:

---

*Suppose the random line and true line to represent two radii of a circle, and the line connecting their extremities to be a portion of the circumference of the same circle. We shall then have this proportion:

Whole circumference : arc : : 360° : angle of lines.

Whence, in case under consideration, we have

$(36.50 \times 3.1416) : 35 : : 360° : 1.09°$.

To generalize, let A represent the starting point of the lines and B and C their extremities. The angle formed by them will be expressed by B A C.

Then $2 \text{ A B} \times 3.1416 : \text{B C} : : 360° : \text{B A C}$.

Whence, $\text{B A C} = \frac{\text{B C}}{\text{A B}} \times 57.3$, very nearly

1. *In Northeast and Southwest Courses.* RULE.—Add the minutes of correction to the assumed bearing of the line (bearing of random line) when the random line lies to the left of the true line, and subtract when it lies to the right. The sum or difference will be the true bearing of the line.

To illustrate, let the line A B, Fig. 13, represent a section of the true meridian, A C the line to be surveyed, A D a random line lying to the left, and A E a random line lying to the right of the true line.

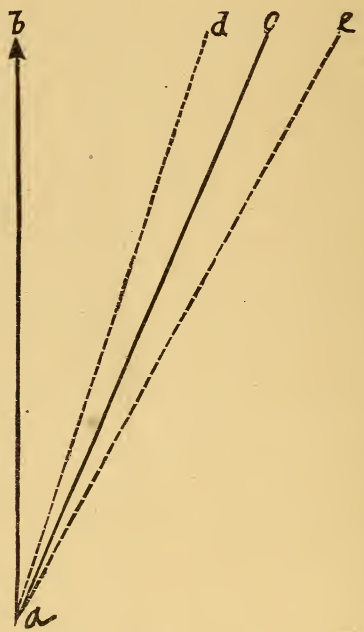

FIG. 13.

The fact is apparent that the bearing of the line A C is equal to the bearing of the line A D plus the angle D A C; and it is equally apparent that the bearing of the same line, A C, equals the bearing of the line A E minus the angle C A E.

2. *In Northwest and Southeast Courses.* RULE.—Reverse the preceding rule; i. e., subtract the minutes of correction from the assumed bearing (bearing of random line) when the random line lies to the left of the true line, and add when it lies to the right.

(**91**). In the following examples the bearings of the random lines are given, and the words "right" and "left" signify their position with regard to the true line. What is the bearing of the true line in each case?

(1).	N 40°	E	40.25,	left	.40.
(2).	S 32°	W	16.50,	"	.25.
(3).	N 71°	W	14.00,	right	.12.
(4).	N 15° 30′	E	8.40,	"	.06.
(5).	S 5° 15′	W	20.00,	"	.13.
(6).	S 6° 17′	E	20.03,	left	.13.
(7).	N 4° 06′	W	15.50,	"	.23.
(8).	N 3° 21′	E	40.00,	right	.10.

The distances are all given in chains and hundredths of a chain.

(**92**). So far, we have based all our bearings on the true meridian, but in many cases lines are surveyed and their magnetic bearings, instead of their true bearings, given.

(**93**). When this is the case, and a resurvey is to be made of these lines, it is necessary to know what change has taken place in the variation of the needle since their bearings were determined, because the bearings change with the variation. Having found this, their present bearings may be determined by the following rule:

1. *To Correct Magnetic Bearings.*—In northeast and southwest courses, add the change to the given bearing, and the *sum* will be the present bearing. In northwest and southeast courses, subtract the change from the bearing, and the *difference* will be the present bearing.

2. In Fig. 14 let A B and A C represent two courses, one bearing northeast and southwest, and the other northwest and southeast. Also, let A D represent the direction of the needle when the lines were surveyed, and A E its direction at the present time. The change of variation equals the angle E A D, and it will be seen that the present bearing of the line A C is equal to the sum of D A C and D A E. Likewise, that the present bearing of the line A B is equal to the difference between D A B and D A E

(**94**). This rule holds good while the needle moves toward the west. If its movement change, the rule will have to be reversed.

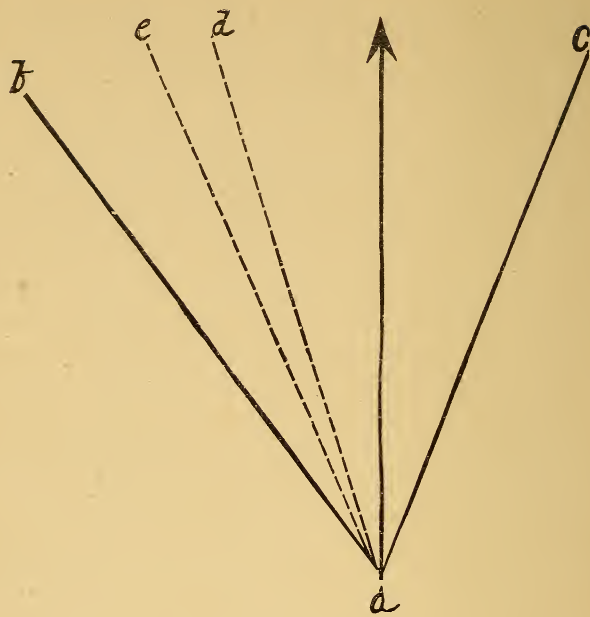

FIG. 14.

(**95**). Correct the bearing of each of the following courses for the present year:

(1). N 14° E, annual change 2′ 30″. Survey made in 1856.

(2). N 7° 34′ W, annual change 3′. Survey made in 1862.

(3). S 1° 13′ E, annual change 2′. Survey made in 1874.

(4). S 3° 02′ W, annual change 2′ 15″. Survey made in 1858.

(**96**). Sometimes the letters T. M. are placed after a bearing to denote that it is based on a true meridian, and M. M. sometimes follows a magnetic bearing to show that it is based on the magnetic meridian. These precautions are necessary, and their omission is frequently a cause of much perplexity to surveyors.

After the bearing of the line has been determined, the survey of it is entirely similar to the survey of lines whose true bearings are given.

(**97**). When the surveyor finishes a random line, he generally walks back along it and moves all the stakes to the true line, and perhaps marks trees that stand on or near the true line, so that it may be the more easily found in he future.

He then enters its course and distance in his field-notes, or surveyor's record, and this completes the survey, although he may sometimes draw a plot of the survey, particularly if it is of a tract of land.

(**98**). Flag-men, chain-men, and markers are usually sworn before the survey begins.

(**99**). The surveyor should be careful about keeping his chain of the proper length by testing it frequently with a standard measure, and should watch his assistants closely until they understand what is required of them.

It would also be well for every county to have a true meridian established, so that the variation of the needle might be found at any time, and with but little trouble.

(**100**). Back sights should always be taken at intermediate stations along the line in order to avoid possibility of deflection.

Questions on Chapter V.

1. What is a "witness?"
2. How are witnesses marked?
3. How are corners found from witnesses?
4. What is meant by the "course and distance" of a line?
5. What must we know before we can run a line?
6. How do you set off east variation on the compass vernier? West?
7. Of what length is the chain usually employed in surveying?
8. If the random line lies to the right of the true line, in what direction do you move the stakes?
9. Give the rule for correcting the stakes.
10. How should the distance between the extremities of the true line and the random line be measured?
11. How is 5 chains and 12 links usually written by a surveyor? 16 chains and 4 links?
12. Give the first rule for finding the amount of error in assumed bearing. The second.
13. What is meant by *sine?* *Cosine?*
14. Give the two rules for correcting assumed bearings.
15. Give the rule for correcting old magnetic bearings.
16. What is the meaning of N 42° E, T M? S 14° 30′ W, M M?

CHAPTER VI.

UNITED STATES RECTANGULAR SURVEYING.

(**101**). The statutes of the United States provide that, "The public lands shall be divided by north and south lines run according to the true meridian, and by others crossing them at right angles, so as to form townships of six miles square, unless where the line of an Indian reservation, or of tracts of land heretofore surveyed or patented, or the course of navigable rivers may render this impracticable; and in that case this rule must be departed from no further than such particular circumstances require."

(**102**). Again, that, "The township shall be divided into sections, containing, as nearly as may be, six hundred and forty acres each, by running through the same, each way, parallel lines at the end of every two miles, and by marking a corner on each of such lines at the end of every mile."

(**103**). There are many other important provisions relating to the subject of Public Land Surveying, but these will suffice. We shall now see how they are carried out.

(**104**). The fundamental lines upon which a survey is based are called the *principal meridian* and *base line*. The first of these is a meridian of the earth, and the second is a parallel of latitude. Their point of intersection is called the *initial point*. Upon these lines every piece of land included in the survey has a direct bearing, and the whole survey itself is located by the number or name of its meridian. For instance, the position of a small tract of land in Indiana is determined by its distance north or south of the base line and east or west of the principal meridian, but the survey of nearly the whole State, as well as of other contiguous territory, is governed by the second principal meridian which runs north and south a short distance west of the center of the State. In like manner, the survey of Ohio is based upon the first principal meridian which serves as the western boundary of the State, the survey of Michigan is regulated by the Michigan meridian, and the surveys of Minnesota are referred to the fourth and fifth principal meridians.

(**105**). The selection of an initial point is the first step in the survey of any new territory, and this is always chosen at some natural and imperishable land-mark found in or near the lands to be surveyed. From this point the principal meridian is surveyed north or south, or north and south, and the base line east or west, or east and west. Upon these lines, which are surveyed with a fine instrument and with the greatest possible precision, six-miles distances are marked for township corners, one-mile distances for section corners, and half-mile distances for quarter section corners. Each of these corners is marked with a suitable monument, and appropriate witnesses are also chosen.

(**106**). From each six-miles point on the base-line, east and west of the initial point, other meridians are surveyed, and the territory is thus divided into strips, each six miles wide, lying north and south. These strips, when divided into townships, are called ranges. The first one east of the principal meridian is called range 1 east, the second is called range 2 east, the third range 3 east, and so on. In the same way the first one west of the meridian is called range 1 west, the second range 2 west, etc.

(**107**). Similarly, lines running east and west from the six-miles points on the principal meridian, divide the territory into strips, each six miles wide, lying east and west. The meridians running north and south and the parallels running east and west thus divide the territory into townships, each of which is about six miles square, and consequently contains thirty-six square miles or sections. The first township north of the base-line in each range is called township 1 north, the second township 2 north, and so on; and those south are named 1 south, 2 south, etc., to the limit of the survey. The first township north of the base-line and east of the principal meridian is described as township 1 north, range 1 east, and, in a similar manner, every township is named with regard to its distance from the base-line and from the principal meridian. This is conveniently shown in Fig. 15.

(**108**). Since meridians converge as they approach the pole, it is evident that townships can not be quite square, and that every township must be somewhat smaller than the township south of it and larger than the one north of it, except in certain cases on the base-line. In the northern part of the United States this convergence is greater than in the southern part, and the north line of a

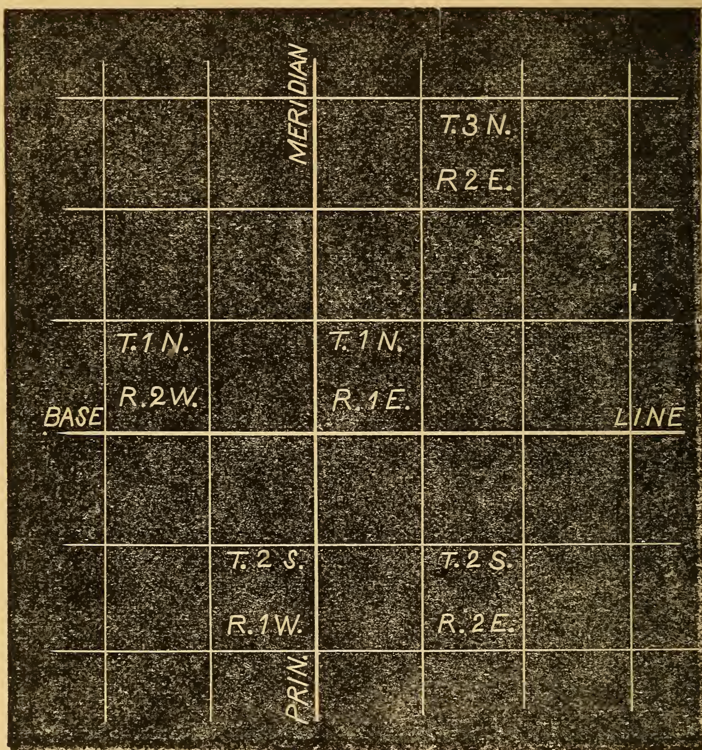

Fig. 15.

township in some places is more than one hundred feet shorter than the south line. To keep the error arising from this convergence within reasonable bounds, lines called "correction lines" are now surveyed every twenty-four miles or four townships on the north side of the base-line, and every thirty miles or five townships south of the base lne, and always parallel to it. Upon these correction parallels, the distances are measured off anew, same as on the base-line, and they become secondary bases in the survey, although townships are all referred to the base-line, just as if they did not exist.

(**109**). For convenience, auxiliary meridians are also established every eight ranges or forty-eight miles east and west of the principal meridian, and the territory is, in consequence, divided into rectangles each 48 miles long by 24 miles wide north of the base-line, and 48 miles long by 30 miles wide south of the base-line.

(**110**). The manner in which the townships are surveyed may easily be explained by Fig. 16. Those north of the base-line and east of the principal meridian are surveyed by commencing at the southeast corner of township 1 north, range 1 east, and running north, establishing section and quarter-section corners at proper distances, four hundred and eighty chains, or six miles, to the northeast corner of the township. From this point the surveyor runs west six miles, or four hundred and eighty chains, to the principal meridian and finishes the survey of the first township. He then surveys the next township north in exactly the same way, and continues, as indicated by the numbers, until he closes the first tier of townships on the correction parallel. In the same way he begins at the base-line and surveys the next tier east. The townships south of the base-line are surveyed in the same way, except that the surveyor works *toward* the base-line instead of *from* it, as when north, as shown by the numbers.

(**111**). West of the principal meridian and north of the base-line the survey begins at the southwest corner of township 1 north, range 1 west, and proceeds in the order of the numbers. South of the base-line the process is entirely similar. It will be observed that townships east of the principal meridian are surveyed by running first north and then *west*, and those west by running first north and then *east*.

(**112**). Excesses and deficiencies in the length of township lines are thrown on the north and west sides of the townships so as to fall ultimately into the north and west tiers of sections. These are called fractional sections, and will be considered in due time.

(**113**). The township (congressional township) which we have had under consideration must not be confounded with the civil township. The former is always, when not fractional, six miles square, but the latter may be any reasonable size or shape whatever.

(**114**). Let us now observe how the townships are divided into sections.

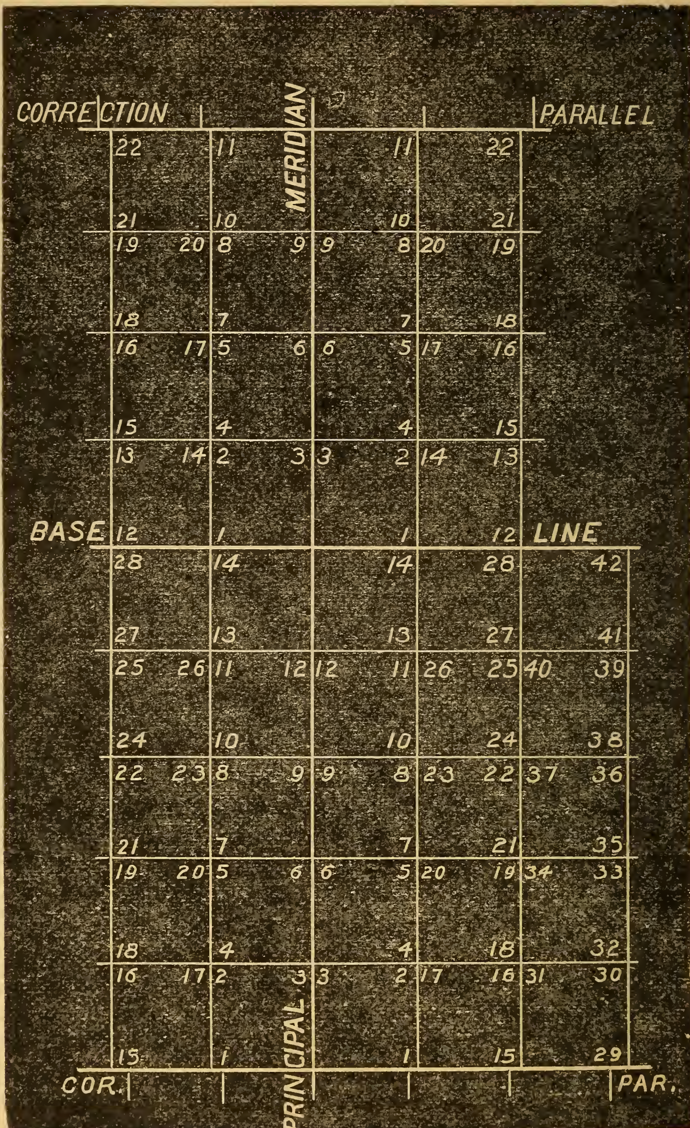

Fig. 16.

1. Before the surveyor attempts to make this division, he ascertains the bearings of the boundary lines of the township, so as to be able to run the interior lines as nearly parallel to them as possible.

2. These bearings may generally be easily obtained from the notes of the previous survey, according to the directions we have already given, but he may also obtain them by retracing a section of one or more of the exterior lines of the township and determining the bearing from the line itself. He should also measure sections of the township line to see how his chain agrees in length with the chain used in the previous survey. Having attended to these preliminaries, he is ready to begin work.

3. The townships, as we have before stated, contain thirty-six sections each, and these sections are numbered, as shown in Fig. 17.

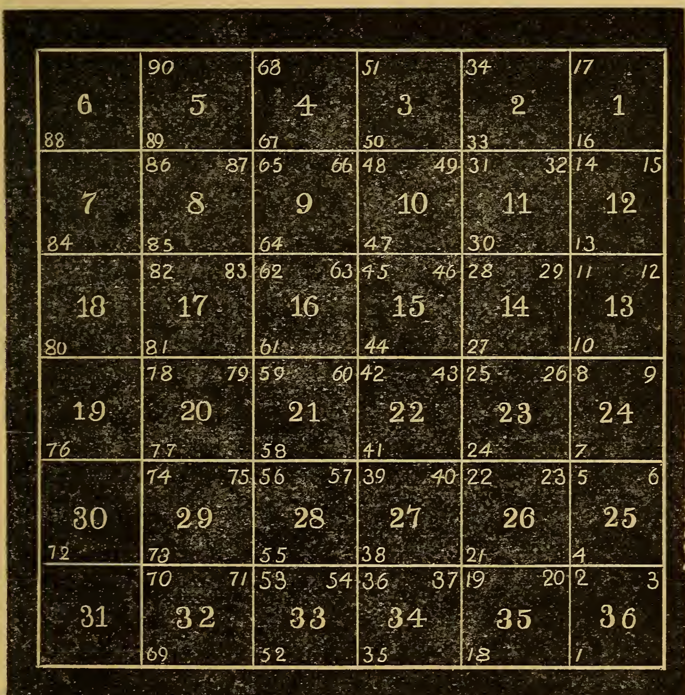

Fig. 17.

The manner in which the sections are surveyed will now be explained.

4. The surveyor goes to the south-west corner of section thirty-six to begin the survey. At this point he sets his compass at the bearing of the east line of the township, which should be, but seldom is, the true meridian, and runs north forty chains. Here he establishes a quarter-section corner between sections thirty-five and thirty-six, and then proceeds forty chains or one-half mile further to the corner of the section, or rather to the corner of sections twenty-five, twenty-six, thirty-five and thirty-six, since these four sections have a common corner here. Distances from the starting point at which brooks, creeks and other objects of importance are met on the line, are carefully noted. From this corner he runs a random line to the east line of the township. If this line intersects the township line at the first mile corner, it is marked back as the true north line of section thirty-six, but if it does not, the distance which it misses the corner, either right or left, is noted, and the line changed accordingly.

5. Having returned to the north-west corner of section thirty-six, he next proceeds to survey section twenty-five in the same manner, and he follows the route indicated by the figures until he completes the survey of the eastern tier of sections. It will be observed that when he completes the survey of section twelve, he then finishes up the survey of section one by running north on a random line and correcting back to the south-west quarter.

6. He next surveys the second tier of sections by beginning at the south-west corner of section thirty-five, and proceeding north in the same manner as in the survey of the first tier, and thus he continues until he reaches the fifth tier. Here, after surveying section thirty-two, he runs west from its north-west corner to the range line or meridian and completes the survey of section thirty-one, and continues to work north, surveying the fifth and sixth tiers together, until he reaches the north line of the township.

(**115**). The township is now divided into sections, each of which, except those in the north and west tiers, called fractional sections, is sold as containing six hundred and forty acres of land, more or less, and each quarter-section as containing one hundred and sixty acres, more or less. The fractional sections generally contain a greater or less quantity of land than the other sections, because all excesses and deficiencies fall to them. In surveying them, however, the quarter-section corners between them are so placed that the excesses and deficiencies fall to the exterior quarters, and the interior quarters, or those touching the other sections of the township, are sold as containing the proper amount of land—one hundred and sixty acres each. The exterior quarters are sold as containing whatever the measurements of the survey indicate that they contain. Section six is sometimes called the "double fractional," and usually contains only one exact quarter.

(116). Whenever, in the course of a survey, an impassable barrier, such as a lake or navigable river, is met, the surveyor establishes what is called a meander corner on its margin, and then runs a meander line from this corner along the edge of the obstacle. The rivers and lakes thus meandered are reserved in the sale of the public lands.

(117). The surveyor, from the time the survey of the principal meridian is begun until the township is divided into sections, marks every half-mile of true line that he surveys with a corner, and keeps an account in his field-notes of every important object he meets in the survey, as well as a topographical description of the country. He also makes two sets of corners on the correction parallels, one for townships north, and the other for townships south of the line. Aside from this it is not unusual to find two sets of corners on interior parallels and meridians, owing to discrepancies between contiguous surveys.

(118). The monuments used in marking corners are always adapted to the country in which the survey is made, and their position is generally witnessed by one or more bearing trees, or mounds of earth thrown up around a stake or a stone, whose courses and distances from the corner are carefully noted in the field-books of the survey. These witnesses, if trees, are always marked facing the corner, which enables them to be more easily found at any subsequent time.

(119). This completes the work of the Government Surveyor, or deputy, as he is usually called. He returns his notes to the Surveyor General of his district, and these notes become the basis of all subsequent surveys. The work of dividing and sub-dividing the sections, which belongs to the county and private surveyors, we shall consider in the next chapter.

(120). This beautiful system of land surveying, not unlike the old Roman system, was devised about the year 1785, for the purpose of preparing the North West Territory for settlement, and has answered the purpose in an admirable manner. It is the result of mature deliberation, and exhibits no mean knowledge of engineering skill, and, like many other great inventions, its beauty and utility consist in its extreme simplicity. It has long since outgrown the limits for which it was intended, and soon nearly the whole territory between the western boundary of Pennsylvania and the Pacific ocean will be united in one complete net-work of sections.

The readiness with which it enables a surveyor to re-trace old lines and determine the location of lost corners prevents an endless amount of litigation common to States not surveyed according to this system.

Questions on Chapter VI.

1. What are the fundamental lines of a survey?
2. What is their point of intersection called?
3. Upon what principal meridian is the survey of Indiana based?
4. What is a range? A township?
5. Draw a diagram representing town 4 south, range 3 east.
6. How is the error caused by the convergence of the meridians arrested?
7. How are townships north of the base-line and east of the principal meridian surveyed? South of the base-line and west of the principal meridian?
8. What is the difference between a congressional township and a civil township?
9. How may a surveyor ascertain the bearing of the lines that bound a township?
10. How many sections in a township? How are they numbered?
11. Where does a surveyor begin work when he divides a township into sections?
12. Describe the method of surveying the eastern tier of sections. The western.
13. Name the fractional sections. The double-fractional section.
14. What causes fractional sections?
15. How many acres in a section?

16. What quarters of fractional sections are generally full?
17. What is a meander line?
18. Why are two sets of corners needed on correction lines?
19. How far apart are the corners on lines surveyed by the government surveyor?
20. For what purpose was the rectangular system devised? When?

CHAPTER VII.

THE DIVISION AND SUB-DIVISION OF THE SECTION.

(**121**). The county or other surveyor makes all his surveys in accordance with the field-notes of the original survey, a copy of which form a part of the public records of each county.

Subsequent surveys may prove that great irregularities exist in the original survey, but none of the lines or corners can be changed.

(**122**). The *ideal* section of the young surveyor frequently

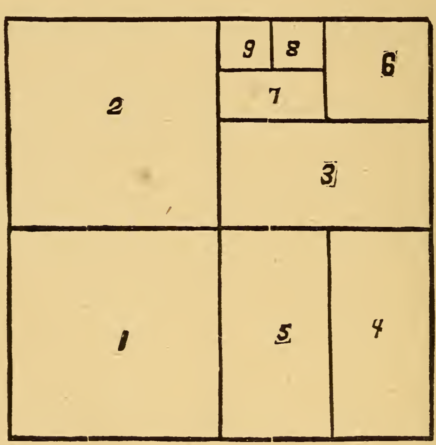

FIG. 18.

differs greatly from the *real* section he meets in practice. The ideal section is very nearly a perfect square—varying a little on account of the convergence of the meridians; it is bounded by four straight lines, and contains almost exactly 640 acres of land.

The real section may sometimes be far from square; its boundary lines may deflect every half-mile on its perimeter, and its area may exceed by several acres the area of another section adjoining. The surveyor, however, must adhere as closely as possible to the original survey, and let the section and divisions and sub-divisions of the section contain whatever the government deputy saw proper to put into them.

(**123**). Let us now see what the principal divisions and sub-divisions of the section are, and then consider the method of surveying them, locating the corners, etc.

(**124**). Fig. 18 represents a section, with the main divisions and sub-divisions laid off on its face. They are described as follows:

1. S. W. qr.
2. N. W. qr.
3. S. $\frac{1}{2}$ N. E. qr.
4. E. $\frac{1}{2}$ S. E. qr.
5. W. $\frac{1}{2}$ S. E. qr.
6. N. E. $\frac{1}{4}$ N. E. qr.
7. S. $\frac{1}{2}$ N. W. $\frac{1}{4}$ N. E. qr.
8. N. E. $\frac{1}{4}$ N. W. $\frac{1}{4}$ N. E. qr.
9. N. W. $\frac{1}{4}$ N. W. $\frac{1}{4}$ N. E. qr.

Of these tracts, Nos. 1 and 2 each contain 160 acres; 3, 4 and 5 each 80 acres; 6, 40 acres; 7, 20 acres; and 8 and 9 each 10 acres

(**125**). Whenever an instrument of writing, such as a deed or mortgage, implying responsibility to the amount of the description, bears on a tract of land, the area is usually qualified by the compound term "more or less." For instance, No. 1, above, would be described as containing 160 acres, more or less; No. 3, as containing 80 acres, more or less, and so on.

(**126**). The corners of the section, or of the different parts of it, are named from their position. The principal ones are the section corners, $\frac{1}{4}$ corners, $\frac{1}{2}$ $\frac{1}{4}$ corners, and $\frac{1}{4}$ $\frac{1}{4}$ corners. Each corner, no matter to what class it belongs, is named from its office and situation. The numbers on the diagram correspond with the names after similar numbers below.

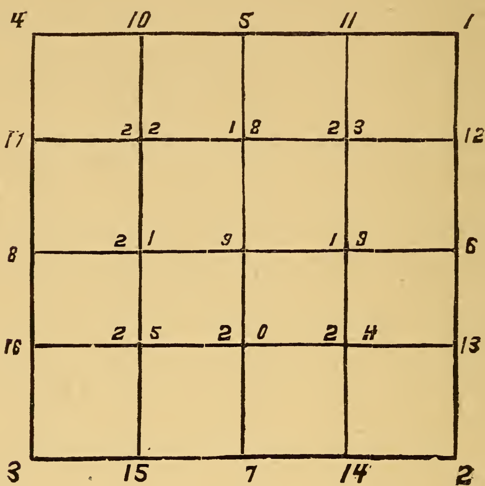

FIG. 19.

1. Section Corners:

   (1). N. E. Corner.

   (2). S. E. "

   (3). S. W. "

   (4). N. W. "

2. Quarter-Section Corners:

   (5). N. $\frac{1}{4}$ Corner.

   (6). E. $\frac{1}{4}$ "

   (7). S. $\frac{1}{4}$ "

   (8). W. $\frac{1}{4}$ "

   (9). Center "

3. Half-Quarter Corners:

(10). N. $\frac{1}{2}$ $\frac{1}{4}$ W. Corner.

(11). N. $\frac{1}{2}$ $\frac{1}{4}$ E. "

(12). E. $\frac{1}{2}$ $\frac{1}{4}$ N. "

(13). E. $\frac{1}{2}$ $\frac{1}{4}$ S. "

(14). S. $\frac{1}{2}$ $\frac{1}{4}$ E. "

(15). S. $\frac{1}{2}$ $\frac{1}{4}$ W. "

(16). W. $\frac{1}{2}$ $\frac{1}{4}$ S. "

(17). W. $\frac{1}{2}$ $\frac{1}{4}$ N. "

(18). N. $\frac{1}{2}$ $\frac{1}{4}$ "

(19). E. $\frac{1}{2}$ $\frac{1}{4}$ "

(20). S. $\frac{1}{2}$ $\frac{1}{4}$ "

(21). W. $\frac{1}{2}$ $\frac{1}{4}$ "

4. Fourth-Quarter Corners:

(22). N. W. Center Corner.

(23). N. E. " "

(24). S. E. " "

(25). S. W. " "

(**127**). The exterior lines of the section are called *section lines,* and the two lines that cross at the center of the section are called *center lines.*

(**128**). We are now ready to examine the method by which the position of each of the various classes of corners is determined.

1. The section corners, and the exterior quarter-section corners, except in an occasional case on a town or range line, are set by the Government deputy, and the surveyor who follows him sets the remaining quarter-section corner (center) and all the minor corners.

2. The section is divided and corners set according to instructions from the proper authority, and with regard to setting the center corner of the section, two methods seem to be in use:

(1). A line is run connecting the N. $\frac{1}{4}$ corner with the S. $\frac{1}{4}$ corner, and another connecting the E. $\frac{1}{4}$ corner with the W. $\frac{1}{4}$ corner. The point of intersection of these two lines is taken for the center of the section.

This is the method in general use, and is perhaps the most equitable one that could be devised:

(2). A line is surveyed connecting the E. $\frac{1}{4}$ corner with the W. $\frac{1}{4}$ corner, and the middle point of this line is taken for the center of the section.

If the section lines were straight from one corner of the section to the other, the corner determined by this method would coincide in position with the one determined by the other; but as this is not always the case, they may differ considerably in position.

3. *To Set a Half-Quarter Corner.*—Run a line along the quarter-section, on the side upon which the corner is to be set, and from one corner to the other. Bisect this line for the corner.

For example: To set the E. $\frac{1}{2}$ $\frac{1}{4}$ corner, we connect the center corner and E. $\frac{1}{4}$ corner with a line, and the middle point of this line is the required corner.

4. *To Set a Fourth-Quarter Corner.* –First set a $\frac{1}{2}$ $\frac{1}{4}$ corner on each of two opposite sides of the quarter-section. Then connect these two corners with a line, and bisect this line for the $\frac{1}{4}$ $\frac{1}{4}$ corner.

To illustrate: In order to set the N. E. center corner, it is necessary first to set the N. $\frac{1}{2}$ $\frac{1}{4}$ E. corner and the E. $\frac{1}{2}$ $\frac{1}{4}$ corner, or the E. $\frac{1}{2}$ $\frac{1}{4}$ N. corner and the N. $\frac{1}{2}$ $\frac{1}{4}$ corner. The middle point of a line connecting one corner with the other, in either set, for it is immaterial which is taken, will be the required corner.

5. The same methods are employed for setting corners to the divisions of the fourth of quarter-sections. For instance, to set the north-east corner of the S. $\frac{1}{2}$ N. W. $\frac{1}{4}$ N. E. qr., it is necessary only to bisect the line between the N. E. center corner and N. $\frac{1}{2}$ $\frac{1}{4}$ E. corner.

EXAMPLES.

(**129**). (1). How would you set the S. $\frac{1}{2}$ $\frac{1}{4}$ corner?

(3). How is the S. $\frac{1}{2}$ $\frac{1}{4}$ W. corner set?

(4). Describe the method of setting the S. W. center corner.

(**130**). We are now prepared to understand how the divisions and sub-divisions of the section are surveyed.

1. A quarter-section is surveyed by running the two exterior

half-mile lines, and the two center lines of the section, in order to locate its interior corner and fix its interior boundaries.

2. A half-quarter is surveyed by first surveying as many of the lines of the quarter as enter wholly or in part into its boundaries, then whatever other lines are necessary to determine its remaining corners, and finally the lines connecting these corners with one another, or with others.

For instance, to survey the S. $\frac{1}{2}$ S. E. qr., it would be necessary to run the south line of the quarter, the east line, the two center lines of the section, and the E. and W. center line of the south-east quarter.

3. A fourth-quarter is surveyed on the same principle as the half-quarter.

For example, in surveying the S. E. $\frac{1}{4}$ S. E. qr., it is necessary to survey the south and east lines of the quarter, the E. and W. and N. and S. center lines of the section, either the E. and W. or the N. and S. center line of the S. E. qr., and finally the north line of the fourth-quarter, if the N. and S. center line has been surveyed, or the west line, if the E. and W. center line has been surveyed.

(**131**). As a general rule for the survey of a tract of land bearing a relation to the section, the following is submitted: Run lines to connect the known corners of the tract when no unknown corner intervenes between them, then such lines as are necessary to determine the location of unknown corners, and finally lines to connect these newly located corners with one another, or with others.

(**132**). In many cases some, if not quite all, the boundary lines of a tract of land are known. There is seldom any need of re-surveying these, except where it must be done in order to establish other lines or corners.

EXAMPLES.

(**133**). What lines would a surveyor have to run in order to establish all the boundaries of each of the following described tracts?

(1). S. W. qr.
(2). N. $\frac{1}{2}$ N. E. qr.
(3). S. $\frac{1}{2}$ N. W. qr.
(4). N. E. $\frac{1}{4}$ S. W. qr.
(5). N. W. $\frac{1}{4}$ N. W. qr.

(**134**). So far we have dealt exclusively with corners, lines, and tracts which may be called independent:

The corners, because their position is determined by the division of certain lines, and is not definitely fixed, so far as distance is concerned, by any other point in the section.

The lines, because they connect the corners, and may, therefore, vary in a limited degree either in course or distance, or both.

The tracts, because they are limited by the lines and are not definitely fixed as to area or figure.

(**135**). To illustrate the preceding still further, suppose the north line of a quarter-section to be 36 chains long, instead of 40 chains long, and the contents of the quarter-section to be 148 acres, instead of 160 acres. The $\frac{1}{2}$ $\frac{1}{4}$ corner on the north line will then be 18 chains from the section corner, and the same distance from the $\frac{1}{4}$ corner, and each of the lines will be but 18 chains in length, and a fourth-quarter out of the quarter-section may fall short three or four acres. If, now, the north line of the quarter-section had been longer, the N. $\frac{1}{2}$ $\frac{1}{4}$ corner would have been further from each of its north corners, and the area of the fourth-quarter would have been greater.

(**136**). There is a class of corners, lines, and tracts, however, that may be called dependent:

The corners, because their distance is fixed from some given point, and is not obtained by bisecting a line.

The lines, because they connect the corners, and are consequently of a definite course and distance.

The tracts, because they are bounded by the lines, and their area, therefore, is not affected by the excess or deficiency of land in the section.

Dependent corners, then, are those whose position is definitely fixed; dependent lines connect dependent corners, and dependent tracts are bounded by dependent lines.

(**137**). Dependent lines and tracts are surveyed without any reference whatever to the division or sub-division of the section, although they may depend on some corner in the section as a base.

A tract may be partly dependent and partly independent.

(**138**). The following are examples of descriptions of dependent tracts:

(1). N. 45° E. 16.00; thence N. 45° W. 10.00; thence S. 45° W. 16.00; thence S. 45° E. 10.00, to the place of beginning.

(2). Commencing at the S. E. corner of section 22, town 3 N., range 4 E. and running thence N. 15° E. 12.00; thence S. 45° E. 12.00; thence S. 75° W. 12.00, to the place of beginning.

(**139**). In a dependent tract the course and distance of each of its boundary lines are usually given in the description of it, and it is then said to be described by "metes and bounds."

(**140**). The rules given for setting corners, running lines, and surveying tracts, in full sections, also apply to fractional sections, except where their fractional sides do not contain half, or much more than half, the usual amount of land found in these parts of sections, or where the amount they contain considerably exceeds the usual amount. In the first case, the outside tier of fourths-of-quarter is omitted, and in the second the excess is usually thrown to them, and the inside tier of fourths-of-quarter in the outside quarters of the sections, are left of their usual size. If the deficiency is very great, perhaps the entire outside tier of quarters is wanting.

(**141**). When a tract of land lies partly in one quarter of a section and partly in another, each part should generally be described and surveyed separately; and the same may be said of tracts extending into two or more sections, and sometimes also of those extending into different fourths of the same quarter. The following are examples:

(1). N. $\frac{1}{2}$ N. E. qr., and N. E. $\frac{1}{4}$ N. W. qr.

(2). S. W. $\frac{1}{4}$ N. E. qr., and N. $\frac{1}{2}$ N. W. $\frac{1}{4}$ S. E. qr

(3). N. E. qr. Sec. 4, and N. W. qr. Sec. 3.

(4). S. E. $\frac{1}{4}$ S. E. qr. Sec. 35, and W. $\frac{1}{2}$ S. W. qr. Sec. 36.

(5). S. E. $\frac{1}{4}$ N. W. qr., and E. $\frac{1}{2}$ N. E. $\frac{1}{4}$ N. W. qr.

In some of the cases an occasional line may be a boundary to each part of the tract, as the east line of No. 5, described above.

Questions on Chapter VII.

1. Does the section always contain exactly 640 acres?
2. Draw a diagram of a section and represent the following tracts on it: N. E. qr.; N. W. $\frac{1}{4}$ N. W. qr.; S $\frac{1}{2}$ S. W. qr.; N. $\frac{1}{2}$ N. E. $\frac{1}{4}$ N. W. qr.
3. Why is the phrase "more or less" used in descriptions of land in deeds, etc.?

4. Name the quarter-section corners. The $\frac{1}{2}$ $\frac{1}{4}$ corners. The $\frac{1}{4}$ $\frac{1}{4}$ corners.
5. What are the "center lines"?
6. What eight corners does the Government deputy establish to nearly every section?
7. Does he ever set an interior corner of a section?
8. Give the first method of finding the center of a section?
9. Prove that the first and second methods would agree, if the section lines were straight from one section corner to another.
10. How do you set a $\frac{1}{2}$ $\frac{1}{4}$ corner? A $\frac{1}{4}$ $\frac{1}{4}$ corner?
11. How would you survey a quarter section? A half-quarter? A fourth-quarter?
12. Give a general rule for the survey of a tract of land bearing a relation to the section.
13. What is the difference between independent and dependent corners? Lines? Tracts?
14. Describe an independent tract. A dependent tract.
15. When do the general methods for dividing and sub-dividing sections not hold good in fractional sections?
16. When should different parts of a tract be surveyed separately?

CHAPTER VIII.

FIELD-NOTES.

(**142**). The field-notes of sectional surveys by the Government deputy show:

(1). The witnesses taken at section and quarter-section corners.

(2). The length of the fractional lines in fractional sections.

(3). The number of acres in each of the fractional quarters of fractional sections.

(4). The offsets between section corners in one township and the corresponding ones in the adjoining township. These offsets are sometimes found on town and range lines as well as on correction parallels.

(5). The distances from the starting point of a line at which brooks and creeks are crossed, and trees and other objects met with on the line.

(6). A description of the timber, surface, soil, etc.

(7). The courses and distances of meander lines surveyed along rivers, lakes, etc.

(**143**). Each full section is supposed to contain 640 acres of land, and it is always taken for granted that the distance between a section corner and a quarter-section corner is 40 chains. The lines are also supposed to run due north and south and east and west. These suppositions, however, are strictly correct only in comparatively few cases.

Quarter-section corners, like section corners, on town and range lines answer for sections on each side of the line, but where an offset occurs aud the closing section corner is set either at one side or the other of the corner already on the line, the quarter-section corner for the closing section is omitted. In this case the closing section has only seven corners instead of eight.

(**144**). For convenience of reference a plot of the township is drawn after the survey is completed, and whatever is essential to subsequent surveys is represented on it.

(**145**). Fig. 20 will give an idea of the manner in which a plot of this kind is drawn, although space will not permit its being made as complete as it should be.

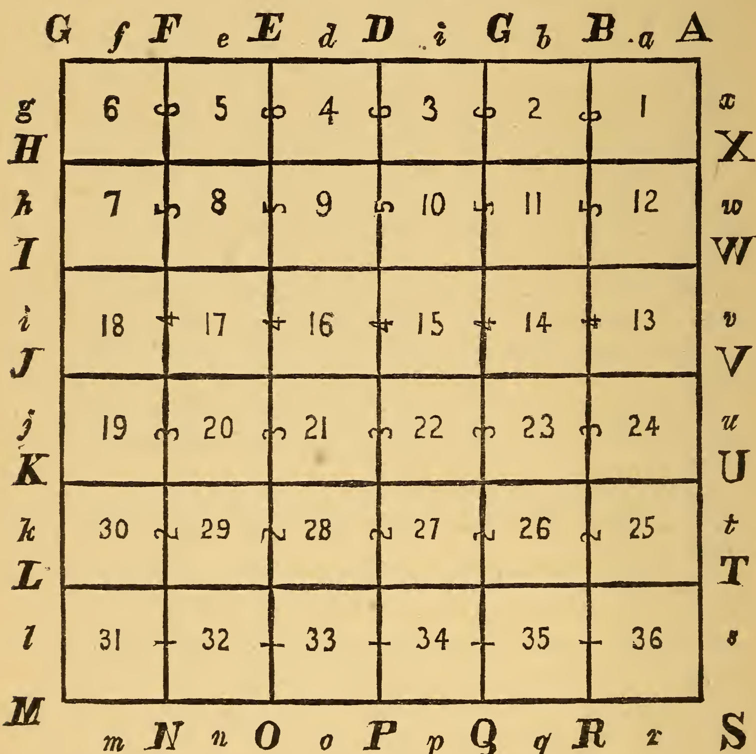

Fig. 20.

The capital letters on the margin designate the section corners on the town and range lines, and the small letters the quarter-section corners.

The interior section corners are designated by the numbers of sections. Thus, the southwest corner of section one is numbered

1, 2, 11, 12, because it serves as a corner to each of these sections.

The interior quarter-section corners are designated as corners 1 to 6, respectively, on the line B R, I W, or whatever line it may be.

(**146**). Attached to the plot of the township is a list of the witnesses at each of the corners, generally on the principle of the following:

1. Exterior corners.

Sections.

A. Be 6 N 14 W 32, Ash 16 S 12 W 17.

B. Oak 10 S 19 E 11, Hickory 18 N 5 W 6.

C. Maple 14 N 12 E 41, Poplar 28 S 72 W 19.

Quarter-sections.

a. Oak 14 N 61 W 14, Sugar 15 S 16 W 13.

b. Elm 26 S 12 E 25, Ash 9 S 63 W 14.

2. Interior corners.

Sections.

1. Cor. of 1, 2, 11, 12, Maple 15 N 10 E 19, Elm 16 S 71 E 12.

2. Cor. of 2, 3, 10, 11, Oak 36 S 15 W 12, Ash 20 82 W 41.

On the line B R (Quarter-section corners).

(a). At 1. Be 12 N 16 E 42, Gum 14 S 15 W 22.

(b). At 2. Pop. 28 S. 46 E 27, Ash 20 N 29 W 31.

In a similar manner the quarter-section corners on the lines H X, I W, J V, etc., are also numbered and the witnesses given to each.

Each of these lists is extended so as to include all the corners of that particular class.

(**147**). The following particulars are usually shown on the face of the plot:

Length of fractional lines:

B to 6, 39.07.

C to 6, 38.49.

D to 6, 38.03, and so on with all the other fractional lines.

1. Area of fractional quarters.

   N. E. qr. sec. 1, 159.17 acres.

   N. W. qr. sec. 1, 157.51 acres.

   N. E. qr. sec. 2, 155.70 acres.

2. Creeks, etc.

   N from R 31.42, creek running S. W., 43 links wide.

   E from 25, 26, 35, 36, 22.16, creek running S.W., 41 links wide.

3. Offsets.

   B, 41 links E. of corner in town north.

   L, 59 links S. of corner in range west.

Only a few instances are cited in each case to show the general plan.

SUBSEQUENT NOTES.

(**148**). Every surveyor should be provided with a copy of the original field-notes of his county, together with notes of all the surveys made by his predecessors. These notes, with the additions he himself makes from time to time (provided he and his predecessors are authorized surveyors), constitute the surveyor's records of the county.

(**149**). These records should contain a plot of each piece of land surveyed, showing its area and the course and distance of each of its boundary lines. The manner of drawing these plots is explained in the chapter on "PLOTTING."

(**150**). From the regular county record is usually drawn a pocket record for field use (1) of the original survey, and (2) of the subsequent surveys. The notes of the original survey generally fill but a small book, and may be arranged according to the method given above; but, unless the county is unusually small, it is more convenient to have a separate book for each range in which to enter the notes of the subsequent surveys.

(**151**). Each of these books should contain at least twice as many pages as there are sections in the range. The left-hand page in each one should contain a plot of a section about 4 inches square divided into quarters, and the right-hand or opposite page should be left blank, so that notes of the successive surveys in the section may be entered upon it. These refer to the plot by numbers or letters in the manner shown in the figure.

1. (Left-hand page).

SECTION ....,

TOWN ...., RANGE ....

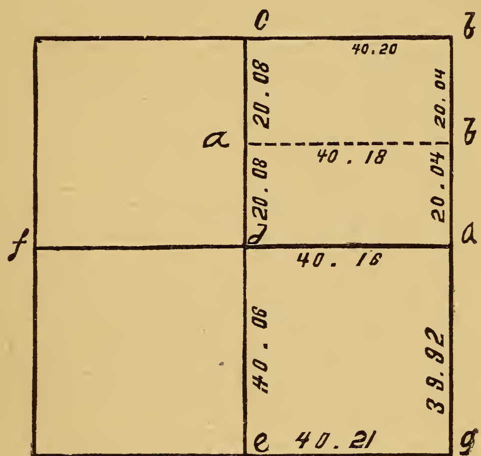

FIG. 21.

(This plat is only one-fourth the size suggested above, but will serve to illustrate).

2. (Right-hand page).

A. Be 16 N 12 E 19, Ash 14 S 6 E 12.
B. Pop 28 S 60 W 19, Walnut 30 N 16 E 41.
C. Be 13 S 16 E 17, Elm 18 N 22 E 31.
D. Pop 16 N 19 E 27, Ash 18 N 23 W 21.
a. Be 23 S 17 W 40, Elm 16 N 21 W 14.

Bearings of principal lines:
A F. N 89° 35′ E.
C E. N 1° 16′ W.

(152). In these cases the bearings are all given on the basis of the true meridian. When the magnetic bearings are given, they should be accompanied with the date at which they were taken.

(**153**). All interior surveys in the quarter-sections should be represented by proper lines on the plot. The dotted line through the north-east quarter in the figure indicates that this quarter has been divided into north and south halves. The courses of roads and creeks may also be shown on the plot.

(**154**). When the bearing of any independent line is not known, it may generally be approximated by comparing it with other lines in the section or adjoining sections. For instance, there is but little difference between the bearing of the line A B in the figure and that of the line C D, since the distance between their northern extremities differs but 4 links from the distance between their southern extremities, and both lines are about of the same length.

(**155**). The bearing of the line A B may be determined by the following method, which has been explained in a preceding chapter:

$$70 : 4 :: 60' : x = 3'+ = \text{amount of correction.}$$
$$1° 16' - 3' = 1° 13' = \text{variation of line A B.}$$

(**156**). This method of determining the bearing of one line from that of another depends on the following principles: (1) Two parallel lines have the same bearing, and (2) the difference in bearing of two lines not parallel is in proportion to their inclination to one another.

By reversing this rule we may determine the distance between two lines at successive points when their distance apart at one point and their difference of bearing are known.

(**157**). The surveyor's field-book is the memorandum he keeps of his field-work. It contains only the rough entries, which are changed in form and transmitted to the records.

(**158**). In surveys of independent tracts, perhaps the following method of keeping it is as good as any:

Let us suppose a survey of the south-east quarter of section 9, town 8, range 10, commencing at the south-east corner of the section, and made March 29, 1881. If all the corners to the quarter-section have been established previously, and the surveyor runs the east line first, the entries may be somewhat as follows:

Mar. 9, 1881.
9—8—10.

Commenced S E cor. and ran N 2° 15′ W, 40.20 to E ¼ cor., Be

20 N 15 E 36, Ash 10 N 41 W 12, M R 16 links. At 16.30 from S E cor. crossed brook 8 links wide flowing S E.

Com. E $\frac{1}{4}$ ran S 88° 40′ W, 40.12 to center of section. 19.00 crossed brook 6 links flowing S E.

Com. cen. ran S 2° 30′ E, 40.20 to S $\frac{1}{4}$.

Com. S $\frac{1}{4}$ ran N 88° 40′ E, 40.15, M L 5 links.

The first line terminated 16 links east of the corner, showing that the assumed bearing was about $\frac{1}{4}$ degree too small. The second and third lines struck the corners, but the fourth ran 5 links to the north, perhaps on account of some slight error in setting the compass, as the assumed variation was correct, if we compare with the second line run.

(**159**). Wherever the witnesses are found in bad condition new ones are taken, as was done at the E $\frac{1}{4}$ corner in this survey.

(**160**). In dependent surveys it is generally best to have the page of the field-book ruled in five vertical columns: The first giving the relative name or number of the station or corner at which the line begins; the second, its course; the third, its distance; the fourth, the number of links missed to the right; the fifth, the number of links missed to the left, as follows:

Sec. 5, Town 6, Range 4.

Mar. 18, 1881.

Sta.	Course.	Dis.	R.	L.
A	N 10° W	16.00	4	
B	N 4° W	4.00		
C	N 16° 30′ E	18.00		6
D	S 81° 15′ W	9.00		
E	N 14° W	11.24		
F	S 29° W	7.26	5	

Fig. 22.

(**161**). The station at which the surveyor begins is called "A," and the succeeding ones are named in the order of the letters that follow.

Witnesses taken at any of the corners may be described on the opposite page and referred to the corner by the proper letter, as follows:

At B. Be 24 N 23 W 16, S E corner house N 81 W 46.
" E. Elm 22 S 16 E 32, Oak 23 M 12 W 29.
" F. Large stone at corner.

The names of all the assistants in the survey are generally recorded, so that they may be known, if evidence should be needed, in case of future disputes over the survey.

(**162**). The record made by county and other authorized surveyers is taken as *prima facie* evidence in favor of the surveys made by them, and particular care should be taken in making this record, as well as in field-work, to see that no mistakes are committed.

(**163**). Other methods of keeping field-notes are also employed by surveyors, but the ones described above are perhaps the most simple, and they will answer every purpose.

Questions on Chapter VIII.

1. What particulars are enumerated in the original field-notes?
2. By referring to the original notes, how would you find the witness to the south-west corner of section 21? The W $\frac{1}{4}$ corner of section 9? The S $\frac{1}{4}$ of section 22?
3. What sections touch section 15? 29? 26?
4. What do you understand by "Subsequent Notes?"
5. What constitute the surveyor's records of a county?
6. How is the bearing of an independent line sometimes approximated?
7. The north line of a quarter-section is 40.32 long, and the south line 39.97 long. If the bearing of the east line is N 2° 19 W, what is the bearing of the west line?
8. Describe the field-book for independent tracts. For dependent tracts.
9. When are new witnesses taken to a corner?
10. What is meant by *prima facie* evidence? Answer—Evidence that establishes a fact, unless set aside by stronger evidence.

CHAPTER IX.

RE-LOCATION OF CORNERS.

(**164**). Perhaps, as a general thing, the most perplexing part of a surveyor's work consists in re-locating the corners of the original survey. As long as these or the witnesses to them remain, he seldom has any serious trouble in the survey of any independent line of the section; but if one of them chance to be lost, it must be re-located before any line bearing on it can be surveyed, and its re-location, except in certain cases, is frequently a matter of difficulty, if not, in some instances, of impossibility.

In the latter case there is no alternative, except to establish a new corner.

(**165**). This difficulty arises from the fact before stated, that section lines are usually broken at every original corner, and these parts, into which the lines are divided, differ from one another in length, so that the course and distance of one line may not be the same as that of any other line in the vicinity, and it would not do, therefore, to re-locate a corner by a line having the same course and distance of a similar line, either in the same section or any other section. The surveyor, consequently, must resort to other means.

1. In the first place a diligent search should be made for remains of the monument or witnesses at the missing corner. If the witness trees have disappeared, it may be possible to find the roots, especially if the ground has not been plowed, as traces of them remain for many years. It is probable that some person in the vicinity can give him some information relative to the corner that will enable him to judge approximately as to where he should look for the witnesses.

2. If, however, all search prove futile, and the course and dis-

tance of a line connecting this corner with some other corner that can be found, be known, the missing corner may sometimes be found by running this line from the known corner.

3. Or if the line run through the woods, it may be possible to retrace it by the blazes on the trees, and thus determine the missing corner, providing the length of the line be known.

4. And, again, where subsequent surveys have been made in one of the sections touching the corner, some of the subsequent corners, taken in connection with the original corners, may enable the surveyor to re-locate the missing corner by "projection," as follows:

(1). Suppose the S $\frac{1}{4}$ corner of the section to be missing, and that the S E corner and S $\frac{1}{2}$ $\frac{1}{4}$ E corner can both be found. Now, the S $\frac{1}{2}$ $\frac{1}{4}$ E corner was evidently set while traces of the S $\frac{1}{4}$ corner remained, and is midway between it and the S E corner. If now we begin at the S E corner and survey a line westward, measuring to the S $\frac{1}{2}$ $\frac{1}{4}$ E corner and producing the line an equal distance beyond it, the extremity of this line must mark the missing corner. If the line run either to one side or the other of the S $\frac{1}{2}$ $\frac{1}{4}$ E corner, the distance to the right or left must be noted. The S $\frac{1}{4}$ corner will be twice this distance on the same side from the extremity of the line, as may be seen by noticing Fig. 23.

The numbers on the horizontal (true) line indicate the length of each section of it, and those on the dotted vertical lines show the distance between the random line and the true line at each of the corners.

(2). This is simply the reverse of the method used in setting the S $\frac{1}{2}$ $\frac{1}{4}$ E corner.

(3). If, instead of the S E corner and S $\frac{1}{2}$ $\frac{1}{4}$ E corner, we have, for instance, the S E, E $\frac{1}{2}$ $\frac{1}{4}$, and S E center corners, it will be necessary first to project the S $\frac{1}{2}$ $\frac{1}{4}$ E from the E $\frac{1}{2}$ $\frac{1}{4}$ and S E center, and then we may project the S $\frac{1}{4}$, same as before.

EXAMPLES:

(4). 1. If we have the center and S $\frac{1}{2}$ $\frac{1}{4}$ corners, how may we find the S $\frac{1}{4}$?

2. The W $\frac{1}{4}$, S $\frac{1}{2}$ $\frac{1}{4}$, and S W center corners are known, how may the S W corner be determined?

3. The E $\frac{1}{4}$, E $\frac{1}{2}$ $\frac{1}{4}$, and S $\frac{1}{2}$ $\frac{1}{4}$, corners can be found, how may the S $\frac{1}{4}$ be re-located?

(5). It must be borne in mind that two corners must be found on the same side of a quarter-section before the third can be re-located by this method.

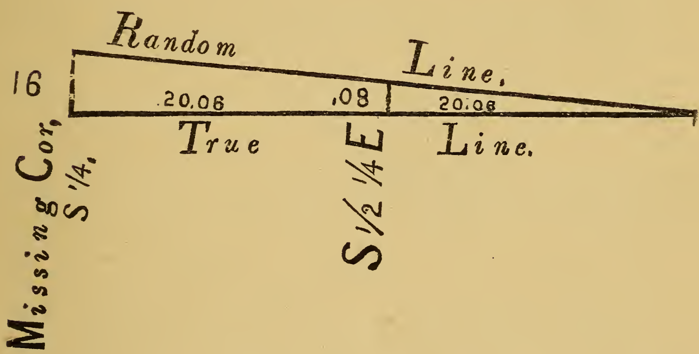

FIG. 23.

5. When the surveyor finds it impossible to re-locate a missing corner by any of the foregoing methods, or any other method, he proceeds to establish a new corner, and in doing this he presumes that the *quarter-section lines do no bend at the corner to be established*, and, if it be a quarter-section corner, that it is *midway between the corners of the section.*

(1). Suppose the missing corner to be the S $\frac{1}{4}$, the new corner would be set by bisecting the line connecting the S E and S W corners of the section in the same way that a $\frac{1}{2}$ $\frac{1}{4}$ corner is set by bisecting the line between the two corners of the quarter-section.

(2). The corner thus established may be identical with the lost corner, or may be some distance from it, but it is the best that can be done.

(3). In the figure a case is illustrated in which the new corner

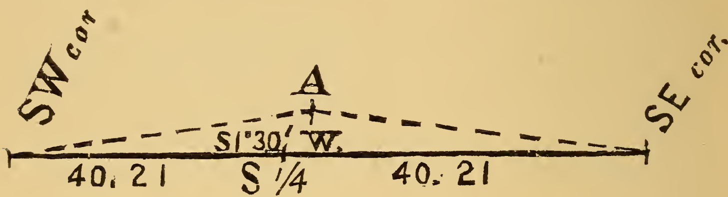

FIG. 24.

is a considerable distance from A, the supposed approximate location of the old corner. The dotted lines represent the old lines, and the numbers below the new line show the length of each section of it. Its bearing is marked above it, and in this case indicates its angle with a line running due east and west.

(4). If the corner of a section be lost, a new one is set by surveying the exterior lines of the adjacent quarter-sections as if they did not bend at the section corner. The new corner will be at the point of intersection of the two lines thus surveyed.

For instance, to set a new S E corner to section 2, connect the S $\frac{1}{4}$ corner of section 1 with the S $\frac{1}{4}$ corner of section 2, and the E $\frac{1}{4}$ corner of section 2 with the E $\frac{1}{4}$ corner of section 11. The point at which the lines cross will be the new corner.

(5). As in the former case, this corner may not be identical with the old corner. Fig. 25 represents a possible case in which they are some distance apart. In actual work, however, such extreme cases as we have noticed will seldom, if ever, come up.

(6). In any case, in setting a new section corner, if any $\frac{1}{4}$ corner can not be found, the line must be produced to the next corner that can be found. This may cause one of the lines to be $1\frac{1}{2}$ or even 2 miles long, but the corner is set at the point of intersection, same as before.

(**166**). In re-locating the original corners to the variable quarters of fractional sections any of the first four methods given above may be employed, but when a new corner must be established the 5th or last does not always hold good, except for the ¼ corner on the town or range line, which is set midway between the section corners, as in full sections.

(**167**). To set the ¼ corner between two fractional sections, run

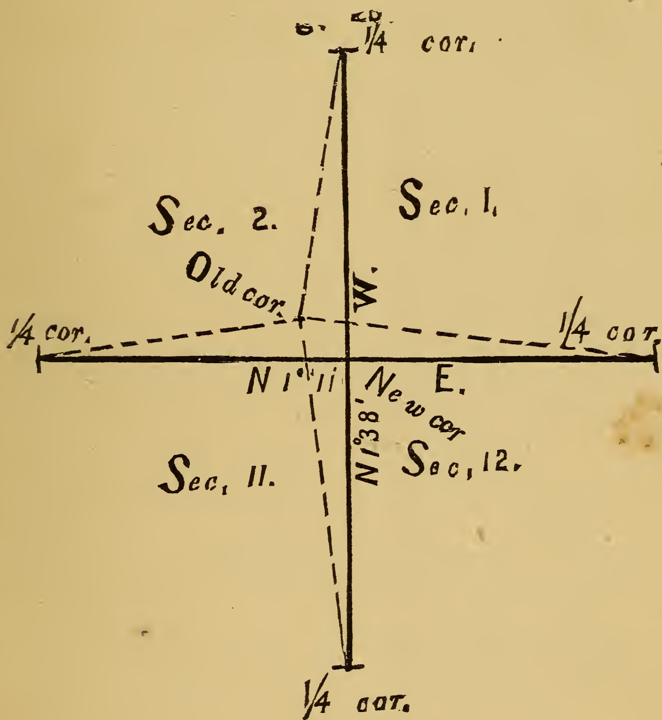

FIG. 25.

a line from the interior section corner westward or northward, as the case may be, to the exterior section corner on the town or range line, and locate the corner 40 chains from the starting point.

(**168**). To set an exterior corner to a fractional section, or to any exterior section.

1. If there be an offset between the corner of one section and that of the corresponding section in the other town or range, the

corner may be re-located by measuring this offset along the town or range line, or correction parallel, in the proper direction.

2. When there is no offset, the corner must be set by crossing the lines, according to the method used in interior sections.

EXAMPLES.

(**169**). 1. How may a new E $\frac{1}{4}$ corner be set to section 11, providing the section corners on that side can both be found?

2. What must be done, if one or both of the section corners are lost, before the quarter-section corner between them can be set?

3. How do you establish a new interior section corner?

4. If one or more of the exterior corners to the adjacent quarter-sections be lost, what must be done in order to establish the section corner?

5. How do you establish the $\frac{1}{4}$ corner section between two fractional sections?

6. How do you re-locate an exterior section corner when there is an offset on the town or range line?

(**170**). Subsequent corners, whether independent or dependent, are re-located according to the rules by which they were located at first, and the same holds good for original corners on meander lines.

QUESTIONS ON CHAPTER IX.

1. When it is found impossible to re-locate a corner, what must be done?
2. Why is not the new corner always identical with the original corner?
3. Explain each of the different methods of re-locating original corners.
4. In re-locating an original $\frac{1}{4}$ corner by "projection," the random line was found to be 16 links to the left of the $\frac{1}{2}$ $\frac{1}{4}$ corner. In what direction and how far will the re-located corner be from the extremity of the random line?
5. How many corners must be found on the side of a quarter-section before the remaining one can be re-located? Why?
6. What section in the township north corresponds with section 2? With section 5? What one in the township west corresponds with 7? With 30?
7. What townships touch T 2 N, R 3 E? T 4 S, R 5 W? T 6 N, R 1 W?
8. How are subsequent corners re-located?

CHAPTER X.

DESCRIPTIONS OF LAND.

(171). No piece of land can be sold or surveyed, unless its description is known, and this description should be just as concise and simple as possible.

(172). It would be better, if the length of lines and area of tracts were always given in surveyor's measure, instead of in ordinary linear and square measure; yet when this is not done, they may be reduced to their equivalents in surveyor's measure by the following tables:

(173). LINEAR MEASURE.

100 links	=	1 chain		=	4 rods.
1,000 "	=	10	"	=	1 furlong.
8,000 "	=	80	"	=	1 mile.

SQUARE MEASURE.

1 sq. chain		=	16 sq. rods.
10 "	"	=	1 acre.
6,400 "	"	=	1 sq. mile.

It is plain that rods may be reduced to chains by dividing by 4; furlongs to chains by multiplying by 10; and miles to chains by multiplying by 80.

EXAMPLES.

(174). 1. Reduce 15 rods to chains.

2. Reduce 1 fur. 3 rods to chains.

3. Reduce 1 mi. 3 fur. 24 rods to chains.

(175). Fractional parts of a chain should be expressed in links: Thus, $10\frac{3}{4}$ chains should be written 10 chains and seventy-five links, or simply 10. 75.

(**176**). As a general rule for reducing from ordinary long or linear measure to surveyor's measure, perhaps it would be well to use the following:

Reduce the denominations expressing the length of the line to rods, and multiply by .25. The product will be the length of the line expressed in chains and links.

To illustrate, suppose the length of a line to be 7 fur. $16\frac{1}{2}$ rods = $296\frac{1}{2}$ rods. = 296.5 rods. This multiplied by .25 equals 74.125, or 74 chains $12\frac{1}{2}$ links.

(**177**). The area of tracts in surveyor's measure is always given in acres and hundredths, instead of in acres, roods, rods, etc., as ordinarily. This will be explained in the chapter on Computation of Area.

(**178**). Whenever an independent tract is to be described, nothing whatever should be said of the course and distance of any of its boundary lines, and it should be described simply as such a division or sub-division of the section; as, for instance, the south-west quarter, or the north half of the north-east quarter, or the north-west fourth of the south-east quarter, or the north half of the south-east fourth of the north-west quarter, etc., etc.

(**179**). Errors like the following are frequently made in descriptions: Forty acres in the form of a square in the south-east corner of the section; eighty acres off the south side of the north-east quarter; one hundred and sixty acres in the north-east corner of the section; a strip twenty chains wide off the north side of the south-west quarter; and so on.

Each of these descriptions is faulty, because the independent division intended to be described may overrun or fall short in the amount of land named in the description. If the tracts were not independent, the descriptions would be good.

Correct the following descriptions:

(**180**). 1. 160 acres off west side of section.

2. Forty acres in the south-west corner of the north-east quarter.

3. Commencing at the N E cor. of the section; thence running south 20 chains; thence west 40 chains; thence north 20 chains to the N $\frac{1}{4}$ cor.; thence east to the place of beginning. Containing 80 acres.

4. 60 acres off the south side of the south-east quarter.

(**181**). Sometimes descriptions contain errors that render them worthless. The following are a few examples; tell where the error lies in each one:

1. N E $\frac{1}{2}$ N W qr.
2. S W $\frac{1}{4}$ N E qr., containing 80 acres.
3. S $\frac{1}{2}$ N $\frac{1}{4}$ N E qr.
4. 60 acres in N E qr.
5. N W qr sec. 28, containing 80 acres.
6. Running north; thence east 50 chains.
7. Running S 43° E, 11.21; thence N 32° W, 5.26; thence S 8° 30′ E, 16.32, to the place of beginning.

Mistakes like the preceding are frequently made by persons who are careless, or do not understand how lands should be described; and sometimes give rise to vexatious litigation.

(**182**). It is best in nearly all cases to qualify the area of the tract described by the phrase "more or less," as, perhaps, no two surveys of the same tract, particularly if it be large, will exactly coincide throughout, and of course the area will vary with the length of the lines.

(**183**). In describing dependent tracts the course and distance of each of their boundaries should be given, except, perhaps, in occasional cases where they have a natural or artificial boundary, as for instance, a creek or road, whose course and distance may be determined at any time; but it is always best to be definite in regard to boundaries when possible.

(**184**). The description should also state whether the bearings are based on the true meridian or on the magnetic meridian. If based on the magnetic meridian, the date at which they were taken should be given.

(**185**). Where a line is described as running north, a due north and south line is meant, and the same is true of south. Similarly, an east line means one running due east, and a west line one running due west.

(**186**). The survey of a tract of land is always made in accordance with the description, except where an obvious mistake occurs, in which case the surveyor will have to exercise his judgment in regard to the course to be pursued, as no rule can be given that will apply to all cases. However, the decisions in the "Appendix" may assist him somewhat in arriving at a conclusion.

Sometimes the mistake is made in writing the original description of the tract, and at others in copying from preceding titles and deeds. In the latter case, a comparison of the deeds will show in what it consists. As soon as a mistake is discovered in a deed or mortgage, or in any other instrument in which a great deal may depend upon the description, steps should be taken by those interested to have it corrected.

None but competent persons should be chosen to write descriptions of land.

Questions on Chapter X.

1. Why can not a tract of land be sold or surveyed without a description?
2. How many links in a rod? Chains in a mile?
3. Write 17 chains and $46\frac{1}{4}$ links decimally.
4. Give the general rule for reducing from ordinary long or linear measure to surveyor's measure.
5. How is the area of a tract of land expressed in surveyor's measure?
6. Why should not the metes and bounds of an independent tract be given in a description?
7. Why should the phrase "more or less" be inserted in a description?
8. Why is it necessary to state whether the bearings are based on the true meridian or on the magnetic meridian?
9. If on the magnetic meridian, why should the date at which they were taken be given?

CHAPTER XI.

OBSTACLES TO ALIGNMENT AND MEASUREMENT.

(187). It frequently happens in the course of a survey that the line strikes an obstacle of some kind—as, for instance, a building, or a large pond, or creek—that obstructs the measurement, if not both line of sight and measurement.

(188). These obstacles may be divided into two classes: (1) Obstacles that may be spanned by measurements along their sides or margins, as a building, a pond, etc.; (2) obstacles that can not be spanned in this way, as rivers and lakes.

(189). Various methods are employed for spanning obstacles, but only a few will be given, in order to prevent confusion.

First Class of Obstacles.

1. *By Perpendiculars.*—Fig. 26 represents an obstacle on the line

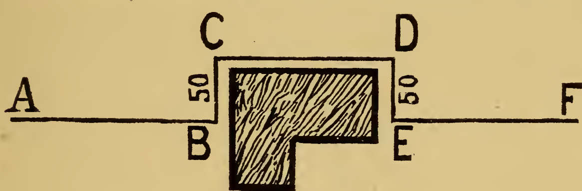

Fig. 26.

A B which runs nearly to the side of it. At the extremity B, a perpendicular, B C, is measured long enough to permit the line C D to pass the obstacle. In this case the perpendicular is fifty links long. The line C D is then run at the bearing of the line A B, and is, consequently, parallel to it. From the extremity, D, of this line another perpendicular, D E, of the same length as the first, is measured, which, of course, terminates on the original line produced through the obstacle. The survey of the line may then be continued from E in the direction E F at pleasure, and the length of C D added to the regular sections, A B and E F.

2. *By an Equilateral Triangle.*—The line A B terminates somewhat further from the side of the obstacle than before, and the

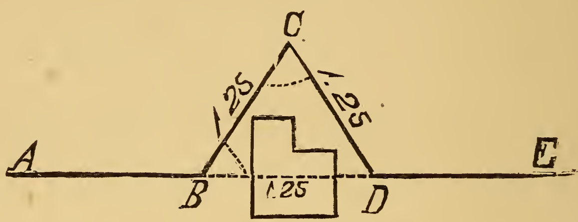

FIG. 27.

line B C is then laid off at an angle of 60° with the line A B produced and measured to a suitable distance. In the case before us it is 1 chain and 25 links in length. From the extremity, C, of this line, the line C D, of equal length with it, is surveyed at an angle of 60° with C B. We then have an equilateral triangle, and the side B D is also 1 chain and 25 links in length. The line may then be continued, and the distance through the obstacle, 1 chain and 25 links, added to the other sections, as before.

3. (*a*) *By a Right-angled Triangle.*—This method is similar to the preceding one, and differs from it only in having a right-angle

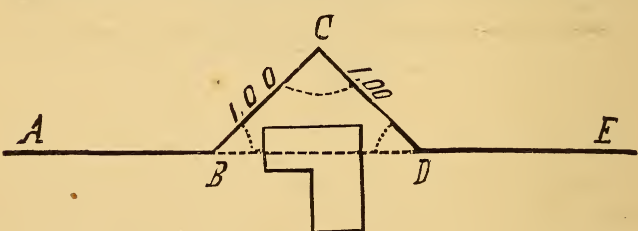

FIG. 28.

at C, and angles of 45° at B and D in the triangle used. The side B C is first surveyed, and then C D at right-angles to it and of equal length. The distance from B to D is found by extracting the square root of the sum of the squares of B C and C D, as the side B D is the hypothenuse of the triangle. In this case the distance from B to D is equal to $\sqrt{(100)^2 + (100)^2} = 1.414$.

(*b*) When the obstacle is a pond, or something that does not obstruct the line of sight, the following method will be found most convenient:

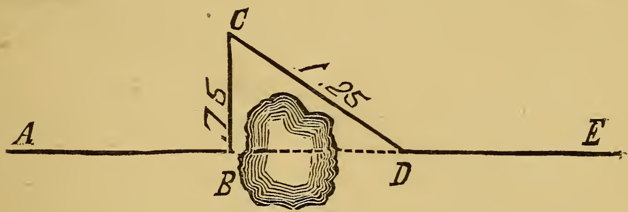

FIG. 29.

The line is measured to B, near the margin of the pond, and the flag set at D on its continuation on the opposite side. C B is then measured perpendicular to A B, and lastly the line C D is measured. We have now a right-angled triangle whose base is required and may be found by extracting the square root of the difference between the squares of C D and B C. In this example the base B D equals $\sqrt{(125)^2 - (75)^2} = 1.00$.

In every case the line is to be continued from D at the bearing of the first section, A B, and the distance through the obstacle must be added.

4. *By Symmetrical Triangles.*—When, as in the last case, the line of sight is not obstructed, the following method may sometimes be used:

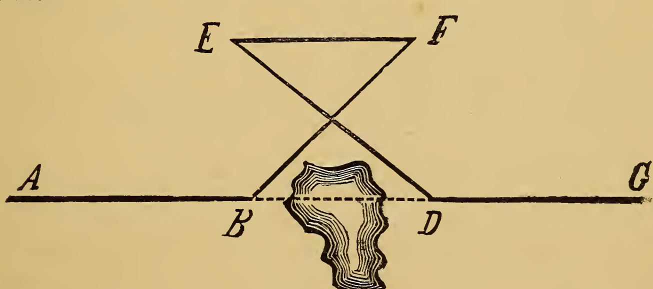

FIG. 30.

From the extremity, B, of the line A B measure a line to C and produce it to F, an equal distance beyond, and then from D measure the line D E so that C will be in the center. The line E F will then be equal to the line B D.

5. When a fence is built on a line to be surveyed, it is best to take an offset either to one side or the other, and allow for it when the stakes are set on the true line, or the stakes may be moved back a distance equal to the offset as they are set. They will thus be on the random line, and may be corrected the same as if no offset had been taken.

It is customary, after an offset has been taken, to measure back to the random line as soon as the obstruction is cleared, but if the corner be reached before this is done, the offset must not be forgotten in measuring the distance the line runs to the right or left of it.

In doing this observe the following rules:

(1). When the offset is taken either to the right or left and the offset line terminates on the opposite side of the corner, the distance missed by the random line will be equal to the distance missed by the offset line, plus the offset, and it will terminate on the same side of the corner as the offset line.

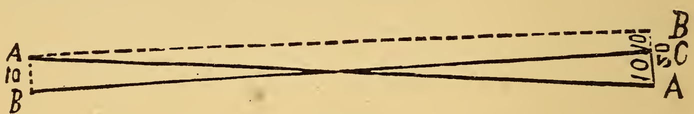

Fig. 31.

In the figure an offset, A B, of 10 links was taken to the right, and the offset line, B C, ran 10 links to the left of the corner A. The random line, A B, will therefore terminate 20 links to the left of the corner.

(2). When the offset line terminates on the same side as that on which the offset is taken.

This involves two cases: (*a*) When the distance missed is greater than the offset, and (*b*) when the offset is greater than the distance missed.

(*a*). Subtract the offset from the distance that the offset line misses the corner; the remainder will be the distance missed by the random line. The termination of the random line will lie on the same side of the corner as the termination of the offset line.

(*b*). Subtract the distance missed by the offset line from the offset; the difference will equal the distance missed by the random line. The termination of the random line will be on the opposite side of the corner from the termination of the offset line.

The offset line is always parallel to the random line.

In correcting the stakes on the offset line, it is best to correct as if they were on the random line, and then move them a distance equal to the size of the offset, and in a direction opposite to that in which the offset is taken. This will put them on the true line.

6. Sometimes, when surveys are made over hills, it is impossible for the chain-men to see the compass or flag to which they are running. In this case a stake should be put up at some prominent point on the line by the surveyor, to which they may meas-

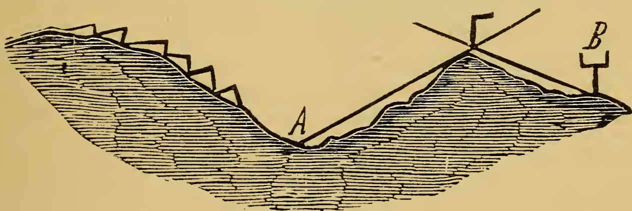

Fig. 32.

ure until they come in sight of the compass or flag. For instance, if the chain-men are down in the valley A, of the figure, a stake or flag should be set on the ridge, as it is impossible for them to see the compass at B.

In chaining up and down hill it is frequently necessary to double the chain or divide it into two sections, so that it may be held in a horizontal position. A light steel chain is always preferable to a clumsy iron one, as it will not sag so much.

Second Class of Obstacles.

(**190**). 1. Suppose the obstacle to be a large creek. The line is surveyed up somewhere near the edge and the flag set on the line on the opposite shore. In Fig. 33, let A represent the point to which the line is measured, and B the flag set on the opposite shore. From the point A, a line of indefinite length is sighted at right angles to A B. The compass is then set at any point not too near A, as C, on this line, and turned so the sights will strike B. The size of the angle A C B is then noted, and the point D on the line A E sighted at an equal angle on the other side of A C. The distance from A to B will then equal the distance from A to D.

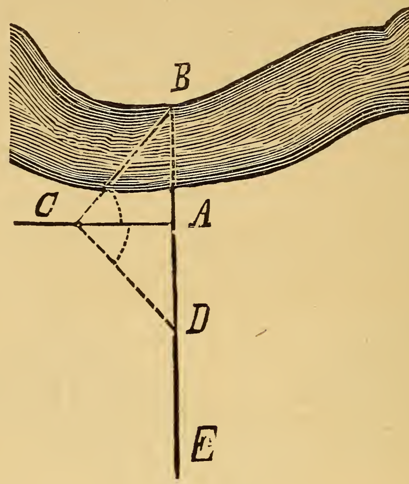

Fig. 33.

2. From the point A, a perpendicular, A C, may be sighted and another, C D, set off from its extremity. The point E, on the line

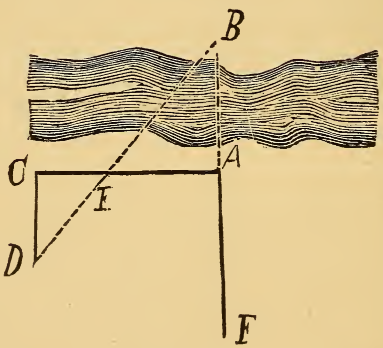

Fig. 34.

A C, is then found, and each of the sections, E C and E A, measured. The distance from A to B may then be found by the following proportion:

$$CE : EA :: CD : (x = AB);$$

$$\text{whence } AB = \frac{EA \times CD}{CE}$$

3. A perpendicular is set off from the line B F at F, and another at A, extended to the line B D. The distances, A F, A C, and

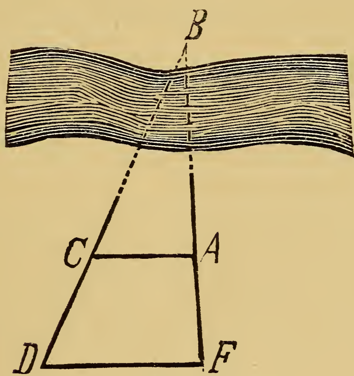

FIG. 35.

D F, are measured and the distance A B, found as follows:

$$(DF - AC) : AC :: AF : (x = AB);$$

$$\text{whence } AB = \frac{AC \times AF}{DF - AC}$$

(**191**). A great many other methods might easily be given, but these will suffice.

These methods will, of course, answer equally well where the point B is inaccessible and at the termination of a line.

In field-work the method that seems best adapted to the peculiarities of the case should be adopted.

QUESTIONS ON CHAPTER XI.

1. What is meant by an obstacle to measurement? To alignment?
2. How many classes of obstacles are there? Name one of each class.

3. Describe the method of spanning an obstacle by perpendiculars. By an equilateral triangle.
4. In the survey of a certain line an obstacle is met. A line is then surveyed 1 chain and 40 links, bearing from the termination of the main line so as to pass the obstacle. From the end of this line a perpendicular 90 links long is measured back to the original line produced through the obstacle. What is the distance through the obstacle?
5. Describe the method by symmetrical triangles.
6. What is an offset?
7. What is the difference between the offset line and the random line?
8. An offset of 12 links is taken to the right, and the offset line misses 13 links to the left. How far will the random line miss the corner?
9. What two cases arise when the offset and termination of the offset line both lie on the same side of the corner?
10. Give the rule for correcting the stakes on an offset line.
11. How do you set an intermediate stake for the flag-men to run to when an elevation of land prevents them from seeing the compass or flagstaff?
12. When is it necessary to double the chain or divide it into two sections?
13. Explain each of the methods used in the second class of obstacles.
14. In Fig. 34, A E = 1.40, C E = 90, and C D = 1.10. What is the length of A B?
15. In Fig. 35, A C = 1.22, A F = 98, and D F = 1.54. What is the length of A B?

CHAPTER XII.

COMPUTATION OF AREA.

(**192**). In computing areas the length of all lines should be expressed in chains, chains and links, or links, as the case may be, and the areas may then be reduced to acres and decimals of an acre by the rules for multiplication and division of decimals. Thus:

(1). $11.25 \times 2.50 = 28.1250$ sq ch.;
$28.1250 \div 10 = 2.81250$ acres.

(2). $21.32 \times 8 = 170.56$ sq. ch.;
$170.56 \div 10 = 17.056$ acres.

(3). $22 \times 15 = 330$ sq. ch.;
$330 \div 10 = 33.0$ acres.

(**193**). The following special rules are deduced from the preceding processes:

(1). When each dimension contains hundredths of a chain (links), the product may be reduced to acres by pointing off *five* decimal places.

(2). When one contains hundredths and the other is expressed in full chains, the product is reduced to acres by pointing off *three* decimal places.

(3). When each dimension is in full chains, reduce to acres by pointing off *one* decimal place.

If either dimension contain a fraction of a link, point off *as many additional places in the prodnct as are necessary to express the fraction.* Thus:

(1). $5.125 \times 3.20 = 1.640000$ acres.

(2). $2.3725 \times 5 = 1.18625$ acres.

EXAMPLES.

(194).	(1).	21.52	$\times$ 12.40	= how many acres?
	(2).	40.24	$\times$ 20.31	= " " "
	(3).	16.42	$\times$ 18	= " " "
	(4).	12	$\times$ 13	= " " "
	(5).	12.165	$\times$ 15.30	= " " "
	(6).	15.2425	$\times$ 12.125	= " " "

(**195**). Every tract of land is in figure a polygon, and its area is computed according to the rule for finding the area of the particular polygon representing its contour.

(**196**). RECTANGLES.

1. Multiply the length by the breadth and the product will be the area.

2. If the rectangle be a square, the area may be found by squaring one side.

(**197**). PARALLELOGRAMS.

Multiply the length of one side by the perpendicular distance between this side and the opposite. The product will equal the area.

In Fig. 36 the line A B represents the perpendicular, and the

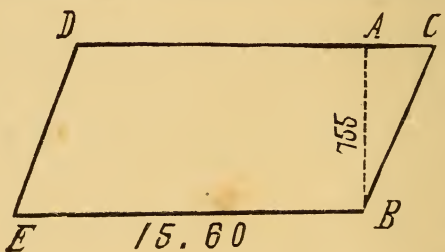

FIG. 36.

area of the parallelogram is therefore equal to the product of 15.60 by 7.55 = 11.778 acres.

If either of the sides B C or D E be given instead of B E or C D, the perpendicular must be measured in the direction of the length of the figure.

(198). TRAPEZOIDS.

Multiply the sum of the parallel sides by the perpendicular distance between them, and half the product will be the area.

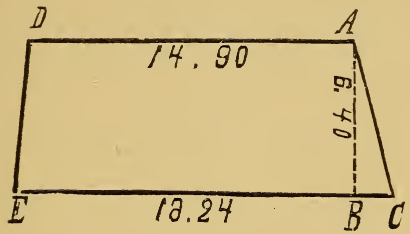

FIG. 37.

In Fig. 37 the line A B represents the distance between the parallel sides, and the area is equal to $((16.24 + 14.90) \times 6.40) \div 2 =$ 9.4648 acres.

When the trapezoid contains two right-angles, the line between them represents the perpendicular distance between the parallel sides.

(199). TRIANGLES.

In computing the area of triangles, two general classes will be considered:

1. Triangles whose base and altitude are given.

The area of triangles of this class is found by multiplying the base by the altitude and taking half the product.

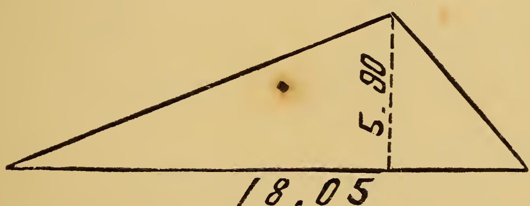

FIG. 38.

Fig. 38 represents a triangle whose base is 18.05 and altitude 5.90. Its area, therefore, is $(18.05 \times 5.90) \div 2 = 5.32475$ acres.

Right-angle triangles belong to this class.

When the three sides are given, isosceles triangles may be brought under this class by the following rule:

Square one of the equal sides and subtract the square of half of the odd side. The square root of the difference will equal the altitude.

The altitude of an equilateral triangle is found by extracting the square root of the square of one side minus the square of half of one side.

EXAMPLES.

(1). The base is 14.75 and the altitude 2.90. What is the area of the triangle?

(2). The base and perpendicular of a right-angle triangle are 5.60 and 7.42, respectively. What is its area?

(3). In an isosceles triangle the even sides are each 9.16 in length and the odd side 7.45. What is the area?

(4). What is the area of an equilateral triangle each of whose sides is 7.25 in length?

2. Triangles whose altitude is not given.

The general rule for triangles of this class is the following:

(*a*). Take half the sum of the three sides.

(*b*). Subtract from the half sum each side severally.

(*c*). Multiply the half sum and three remainders together.

(*d*). Extract the square root of the product for the area.

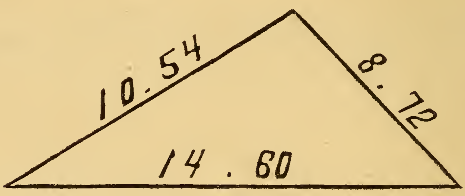

FIG. 39.

Let Fig. 39 represents a triangle whose area is to be computed.

$$\text{Then, } 10.54 + 14.60 + 8.72 = 33.86$$
$$33.86 \div 2 = 16.93$$
$$16.93 - 10.54 = 6.93$$
$$16.93 - 14.60 = 2.33$$
$$16.93 - 8.72 = 8.21$$
$$\sqrt{16.93 \times 6.39 \times 2.33 \times 8.21} = \text{area.}$$

EXAMPLES.

1. What is the area of a triangle, the sides of which are 4.20, 2.65, 3.71, respectively?

2. The sides of a triangle are 2.91, 6.90, and 5.42, respectively. What is its area?

It is sometimes more convenient to measure the altitude, and thus place the triangle under the first class.

(**200**). TRAPEZIUMS.

Divide the trapezium into two triangles. The sum of their areas will be the area of the trapezium.

To do this, measure a diagonal of the trapezium.

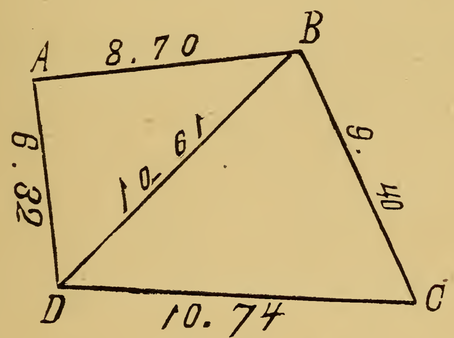

FIG. 40.

Fig. 40 represents a trapezium, one of whose diagonals has been

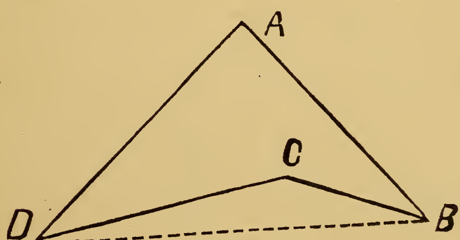

FIG. 41.

measured. It will be seen that its area will equal the sum of the areas of the triangles A B D and B C D.

A serious mistake is sometimes made by incompetent persons by multiplying together the half-sums of the opposite sides for the area.

When one angle of the trapezium is re-entrant, as in Fig. 41, the area may be found by subtracting the area of the triangle B C D from that of the triangle A B D; or it may be computed the same as when the angles are all salient by omitting the triangle B C D and measuring a diagonal from A to C.

(201). Any Figure.

Divide the figure into triangles and compute their areas separately. The sum of the areas of the triangles will be the area of the figure.

The area of the tract of land represented in Fig. 42 is equal to

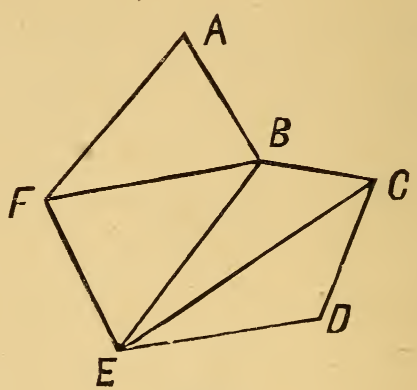

Fig. 42.

the sum of the areas of the triangles A B F, B E F, B C E, and C D E.

Sometimes, when a tract of land is narrow and has one irregular boundary, its area may be approximated by dividing it into trapezoids. Fig. 43 represents a tract of this kind bounded on one side by a creek. In cases of this kind the area of the tract is equal to the sum of the areas of the trapezoids that compose it.

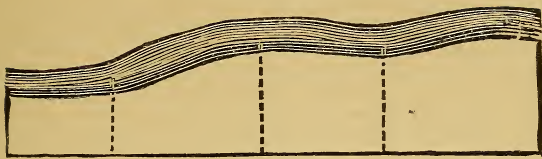

FIG. 43.

(202).

COMPUTATION OF AREA BY LATITUDES AND DEPARTURES.

The method of latitudes and departures now to be developed is simple, precise, expeditious, and universal in application, if the course and distance of each of the boundary lines of the tract whose area is to be computed is given.

(203). In plane surveying, meridians, like parallels of latitude, are supposed to be parallel to one another, and the *latitude* of a course is the distance between two parallels running through its extremities, while the *departure* of a course is the distance between two meridians drawn through its extremities. In Fig. 44 the latitude of the course A B is represented by B C, and its de-

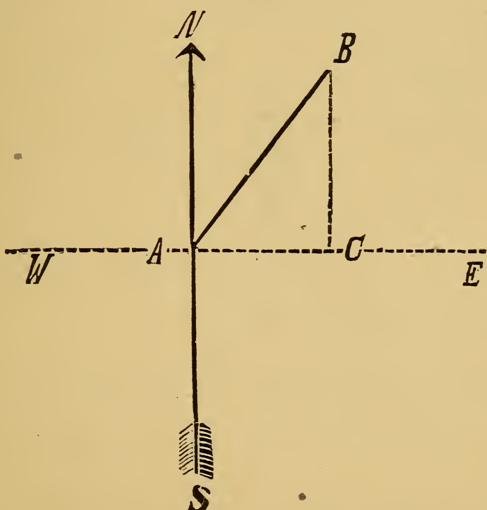

FIG. 44.

parture by A C. It is evident that the latitude of a course is equal to the difference of latitude of its extremities, and that its departure is equal to the difference of longitude of its extremities.

(**204**). The latitudes of courses bearing north are called *north latitudes* or *northings*, and of those bearing south, *south latitudes* or *southings*. Likewise the departures of courses bearing east are called *east departures* or *eastings*, and of those bearing west, *west departures* or *westings*.

In Fig. 45 the latitudes of A B and A F are northings, and of

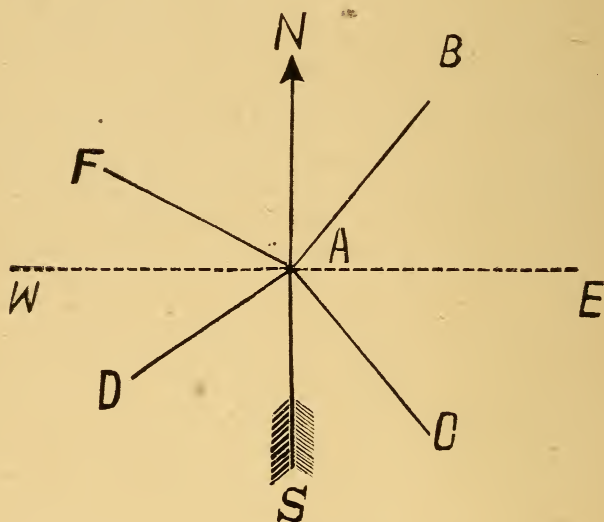

Fig. 45.

A C and A D southings; while the departures of A B and A C are eastings, and of A D and A F westings.

(**205**). North latitudes are *additive* and are marked with the sign +, *plus*, while south latitudes are *subtractive* and marked with the sign —, *minus*. In the same manner, east departures are *additive* and marked +, and west departures are *subtractive* and marked —.

(**206**). If we now refer to Fig. 12, we shall see that the radius A F, may represent the course A C, Fig. 46, whose latitude and departure we wish to find. Then will C E, the *departure*, equal the *sine* of the angle B A C, and E A, the *latitude*, equal the *cosine* of the angle B A C.

(**207**). A Table of Natural* Sines and Cosines is given in the

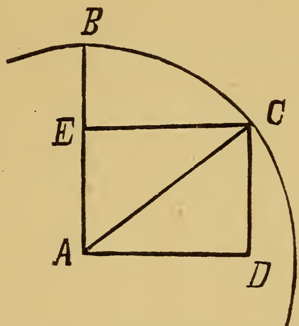

FIG. 46.

APPENDIX, by which the latitude and departure of any course may be easily found. In this table the length of the sine and cosine is given for a radius equal to unity, for each degree and minute of arc between 0 and 90°; and, hence, to find the latitude of any course, it is necessary only to *multiply the cosine of its bearing by the length of the course*, and to find the departure of any course, to *multiply the sine of its bearing by the length of the course.*†

For instance, suppose it is required to find the latitude and departure of a course bearing N 42° 33′ E, and 20.22 in length.

By referring to the Table, we find the cosine of the bearing to to be .73669, and the sine of the bearing to be .67623.

Therefore, the latitude of the course will equal

$$.73669 \times 20.22 = 14.8958,$$

and the departure of the course will equal

$$.67623 \times 20.22 = 13.6733.$$

In using the Table, when the bearing is 45° or less, take the degrees from the top of the page and the minutes from the left-hand

* Called *natural sines and cosines* to distinguish them from *logarithmic sines and cosines.*

† In this rule observe that the angles are measured to the right and left of the vertical radii. If, as in Fig. 12, they were measured from horizontal radii, the word "sine" would be used for "cosine," and "cosine" for "sine" in the rule.

column, and when the bearing is greater than 45°, use the degrees at the bottom of the page and the minutes in the right-hand column.

(**208**). EXAMPLES.

The course and distance is given in each of the following cases. Find the latitudes and departures :

(1). N 52° 16′ W, 10.12.

This bearing is greater than 45°; so the degrees must be taken from the bottom of the page. Having found the double column marked 52°, ascend it to the line marked 16′ on the right. We now find the cosine to be .61199, and the sine to be .79087; therefore the Latitude $= .61199 \times 10.12$, and the Departure $= .79087 \times 10.12$.

(2). S 15° 40′ E, 11.41.

(3). N 21° 32′ W, 19.71.

(4). S 88° 56′ E, 73.98.

(5). N 66° 25′ E, 46.12.

(**209**). It will be seen that the columns in the table marked "sine" at the top are marked "cosine" at the bottom, and that those marked "cosine" at the head are marked "sine" below. Care must be taken to use the heading for bearings read from the top, *i. e.*, for bearings not greater than 45°; and the bottom markings for bearings read from below, *i. e.*, for bearings greater than 45°.

(**210**). TRAVERSE TABLES are sometimes used instead of the Table of Natural Sines and Cosines in determining the latitudes and departures of courses, and somewhat facilitate calculations in many cases; but they are usually computed only to quarter-degrees, and it has been thought best to use in the present work only the more accurate method of natural sines and cosines.

(**211**). In the survey of every tract of land, the sum of the north latitudes should equal the sum of the south latitudes, and the sum of the east departures should equal the sum of the west departures; and, hence, in plotting a survey, or making preparations to compute the area of the tract, we have an almost infallible means of testing the accuracy of the survey by which the course and distance of each of its boundaries were determined.

(**212**). Let us now make an application to the survey of the following described tract of land: Running N 10° E, 5.60; thence S 35° 30′ E, 4.00; thence S 55° 30′ W, 4.00, to the place of beginning.

Taking each course separately, we find the respective latitudes and departures.

(1). Latitude of first course equals $.98481 \times 5.60 = 5.51$.
Departure equals $.17366 \times 5.60 = .97$.

(2). Latitude of second course equals $.81412 \times 4.00 = 3.25$.
Departure equals $.58070 \times 4.00 = 2.32$.

(3). Latitude of third course equals $.56641 \times 4.00 = 2.26$.
Departure equals $.82413 \times 4.00 = 3.29$.

The latitude of the first course is a north latitude, and must be marked +, and the latitudes of the second and third courses are south latitudes, and take the sign —. Likewise, the departures of the first and second courses are east departures and should be marked +, while the departure of the third course is a west departure and should be marked —.

The separate courses, with the latitude and departure for each one, may be entered in a diagram similar to the one used in keeping field-notes (Art. 160), and a space left at the bottom for the footings, as follows:

Sta.	Course.	Dis.	Lat.		Dep.	
			+	—	+	—
A	N 10° 00′ E.	5.60	5.51		.97	
B	S 35° 30′ E.	4.00		3.25	2.32	
C	C 55° 30′ W	4.00		2.26		3 29
			5.51	5.51	3.29	3.29

FIG. 47.

(**213**). The reason why the east departures should balance the west departures, and the north balance the south latitudes will be seen by noticing Fig. 48, which represents the above tract of land. The north and south lines represent the latitudes, and the east and west lines the departures.

(**214**). When the + latitudes balance the — latitudes, and the + departures balance the — departures, as in the case just considered, the survey is said to "close." Usually, however, owing to slight inaccuracies in sighting the flag, reading the bearing of the line, measuring the line, or, perhaps, all combined, neither the latitudes nor departures balance. If the disagreement is considerable, a re-survey should be made, as there is probably an error

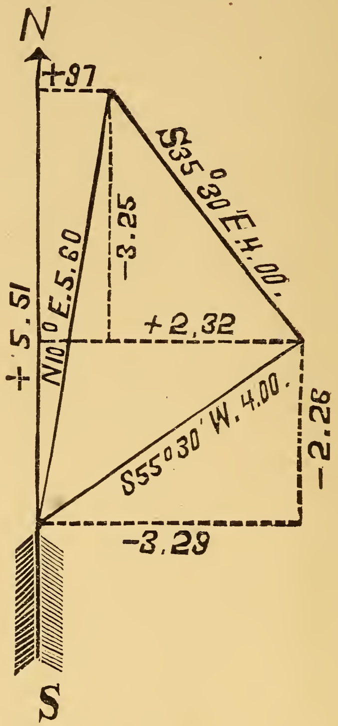

Fig. 48.

somewhere in the work; but if it is only slight, as, for instance, 1 or 2 links in 7 or 8 chains, it is probably due to some unavoidable inaccuracy in the survey, and may be corrected by the following rule:

Find the amount of the error for each chain, and distribute it among the latitudes or departures, as the case may be, in proportion to their respective lengths. Adding to those that are too small, and subtracting from those that are too large.

This will cause them to balance and answer all ordinary purposes.

(**215**). The *longitude* or *meridian distance* of a line is its mean distance from an initial line or meridian. Preparatory to finding the area of a tract of land, this meridian is conceived to be drawn through its extreme western or eastern corner—usually the western—and the longitude of each of the courses of the tract is computed from this meridian as a base.

In Fig. 49 this meridian is drawn through the western corner

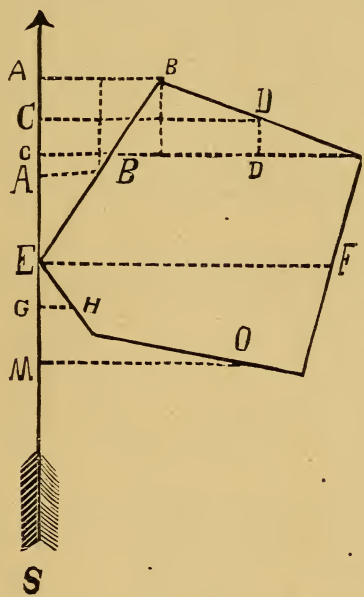

FIG. 49.

of the tract, and the lines, A B, C D, E F, G H, and M O, represent the longitudes of the various courses.

(**216**). It will be observed that there is a difference between longitudes and departures: The former show the mean distance of the line from the meridian, while the latter indicate the difference in longitude of the two ends of the line.

(**217**). By referring to the figure, it will be seen that the longitude, A B, of the first course is equal to half of its departure, *a b;* and also that the longitude, C D, of the second course is equal to *c d*, which equals the longitude of the first course, plus half the departure of the first course, plus half the departure of the second course, and it may easily be shown that the *longitude of any course is equal to the longitude of the preceding course, plus half the departure of the preceding course, plus half the departure of the course itsef.*

(**218**). It must be borne in mind that the *algebraic sum* is meant, and that west departures, having the minus sign, are really subtractive.

(**219**). In order to simplify the rule, and at the same time avoid fractions, it will be preferable to double each of the preceding expressions and use *double longitudes.* The following will then be the general rule for finding the *double longitudes* of courses.

The double longitude of the first course is equal to its departure.

The double longitude of the second course is equal to the double longitude of the first course + the departure of the first course + the departure of the second course.

The double longitude of any course is equal to the double longitude of the preceding course + the departure of the preceding course + the departure of the course itself.

Computation of Area.

(**220**). We are now prepared to compute areas by means of longitudes. Take for example the tract of land described in Art. 212.

The area of the triangle A B C, Fig. 50, is equal to the area of the trapezoid E A B D, plus the area of the triangle B C D, minus the area of the triangle A C E.

Finding the area of each of these figures, respectively, we have:

Area of trapezoid E A B D = D E × a b = the product of the latitude of the course A B by its longitude = (3.25 × 2.13) = .692 acre. (See Fig. 48.)

Area of triangle B C D = C D × e f = the product of the latitude of the course B C by its longitude = (2.26 × 1.645) = .371 acre.

Area of triangle A C E = C E × c d = the product of the latitude of A C by its longitude = (5.51 × .485) = .267 acre.

Therefore, the area of the triangle A B C = .692 + .371 — .267 = .796 acre.

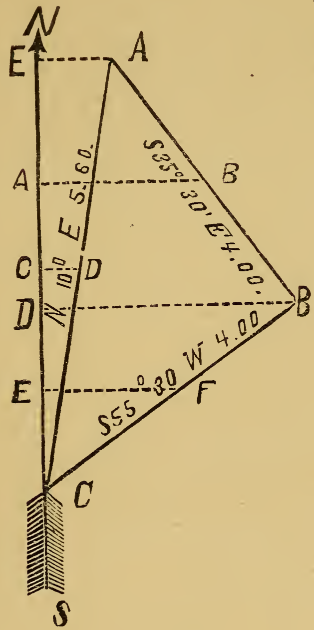

FIG. 50.

(**221**). In computations of this kind the *product of a longitude by a north latitude* is called a *north product, and by a south latitude,* is called a *south product,* and *the difference between the north products and south products is the area of the tract.*

(**222**). Hereafter *double longitudes* will be used, and the difference between the north products and the south products will then be *double the area of the tract.*

(**223**). The different steps in the process of computation may be shown very nicely, and the work kept in compact form, by ruling a sheet of paper in fourteen columns, adding seven to the right of the seven shown in Fig. 47. In the first four of the added seven write the corrected latitudes and departures, in the fifth the double longitudes, and in the sixth and seventh the north and south products or areas marked + and —, same as latitudes.

The following will serve as an illustration, and at the same time indicate the process used in computation.

Sta.	Course.	Dis.	Lat. +	Lat. −	Dep. +	Dep. −	Cor. Lat. +	Cor. Lat. −	Cor. Dep. +	Cor. Dep. −	Dou. Lon.	+ Areas.	− Areas.
A	N 52° 00′ E	10.64	6.55		8.38		6.56	.	8.39		8.39	5.495	
B	S 29° 45′ E	4.09		3.55	2.03			3.55	2.03		18.81		6.677
C	S 31° 45′ W	7.68		6.53		4.04		6.52		4.04	16.80		10.970
D	N 61° 00′ W	7.24	3.51			6.39	3.51	10.07		6.38	6.38	2.239	
			10.06	10.08	10.41	10.43	10.07	10.42	10.42	10.42		7.734	17.647 7.734 2) 9.913 4.956 Acres.

FIG. 51.

In this example the error in latitudes and departures amounts to very little in each case, and might have been disregarded in the calculation.

(**224**). The following is the general rule for computing areas by double longitudes:

Multiply the double longitude of each course by its latitude.

If the latitude is north or plus, write the product in the column of plus areas. If south or minus, write the product in the column of minus areas.

Half the difference between the sums of the areas of these two columns will be the area of the tract.

This rule holds good for any tract of land bounded by straight lines.

(**225**). When the most westerly corner of the tract can not be determined readily it is best to draw a plot of the tract according to the directions given in the chapter on PLOTTING.

(**226**). Compute the areas of the following tracts:

(1). N 34° 15′ E, 2.73.
N 85° 00′ E, 1.28.
S 56° 45′ E, 2.20.
S 34° 15′ W, 3.53.
N 56° 30′ W, 3.20.

(2). S 73° 15′ E, 19.08.
S 19° 30′ W, 13.68.
N 69° 15′ W, 10.34.
S 20° 15′ W, 11.36.
N 68° 00′ W, 9.06.
N 20° 15′ E, 23.56.

(3). South, 3.75; S 35° 00′ E, 1.04; S 86° 30′ E, 5.02; N 82° 00′ E, 1.72; S 34° 30′ E 2.46; S 77° 30′ E, 4. 25; N 45° 30′ E, 9.78; N 2° 40′ W, 233; West, 2.18; N 4° 00′ E, 1.30; N 83° 45′ W, 5.35; S 76° 00′ W, 1.94; S 60° 15′ W, 2.27; S 76° 00′ W, 3.47; N 73° 30′ W, 2.90; N 57° 30′ W, 1.65; S 21° 00′ W. 2.67.

(**227**). It is, of course, plain that a due north or south course has no departure, and that its latitude is equal to its length. Likewise, that a due east or west course has no latitude, and its departure is equal to its length.

(**228**). It is not absolutely necessary that the meridian should be drawn through the most westerly station in calculating the contents, but it is generally more convenient to compute from a meridian so drawn. Sometimes the surveyor imagines the meridian to pass through the most easterly station.

If necessary, the areas may be expressed in the ordinary denominations of land measure (acres, roods, and rods), instead of in acres and decimals of an acre, by reducing the decimals to integers. Thus, .82 of an acre = $(.82 \times 4) = 3.28$ R. $.28 \times 40 = 11.20$ sq. rods. Hence, .82 acre = 3 R. 11.2 rods.

Questions on Chapter XII.

1. Why should the length of lines be written in chains and links?
2. When links are multiplied by links, how many decimals are pointed off in reducing the product to acres? Links by chains? Chains by chains?
3. Give the rule for finding the area of a rectangle. A parallelogram. A trapezoid.
4. How is the area of a triangle found when the base and altitude are given? When the three sides are given?
5. How may the altitude of an isosceles triangle be found? Of an equilateral triangle?
6. State the rule for finding the area of a trapezium.
7. When may the area of a figure be computed by dividing it into trapezoids?
8. What is meant by the latitude of a course? Departure?
9. What are north latitudes? South latitudes? East departures? West departures?
10. Describe the Table of Natural Sines and Cosines.
11. How do you find the latitude of a course from the Table? The departure?
12. Give the rule for correcting latitudes and departures.
13. What is meant by the longitude of a line? What is the difference between the longitude of a line and its departure?
14. State the rule for finding the double longitudes of courses.
15. What are north products or areas? South products or areas?
16. Give the general rule for computing areas by double longitudes.

CHAPTER XIII.

LAYING OUT AND DIVIDING UP LAND.

(**229**). No general rule can be given either for laying out or dividing up land, and in the present chapter only the most common cases that arise in practice will be considered. This is necessary in order to keep our work within its intended limits as well as to avoid the confusion that a multiplication of details would cause.

As a general thing, a little ingenuity on the part of the surveyor will enable him to devise a method to meet the exigencies of the case he may have on hand when it can not be reached by any of the rules given in this chapter.

(**230**). In the problems now to be taken up, the area and one or more of the boundaries are in nearly all cases supposed to be known, and it is required to find from these the length of certain other boundary lines necessary to a survey of the tract. The processes used in work of this kind are generally the reverse of those employed in the last chapter, so that a careful study of operations in computing areas will materially assist in the work now before us.

Laying Out Land.

(**231**). *To Lay Out a Square.*—The square root of the area expressed in square chains and decimals will represent the length of one of its sides.

Thus, each side of a square tract of land containing 5 acres equals $\sqrt{50} = 7.07$.

EXAMPLES.

1. What is the length of each side of a square tract containing 1 acre?

2. A piece of land in the form of a square contains 11 A. 3 R. 26 P. What is the length of its sides?

(**232**). *To Lay Out a Rectangle.*—Divide the area by the length of the given side. The quotient will be the length of the required side.

Thus, if a rectangle contain 4 acres, and the given length be 5 chains, the length of the required side will equal $(40 \div 5) = 8$ chains.

EXAMPLES.

1. The area of a rectangle is 14 acres, and the length 15.00, what is the breadth?

2. The area of a rectangular tract of land equals 10 A. 2 R. 20 P., and it is 63 rods long. What is the breadth?

Process—10 A. 2 R. 20 P. = 10.625 acres.
63 rods = 15.75.
$(10.625 \times 10) \div 15.75 = 6.746.$

(**233**). *To Lay Out a Parallelogram.*—(*a*). Divide the area by the given length. The quotient will be the perpendicular distance between the given sides. (Art. 197).

(*b*). Find one of the angles of the parallelogram according to the methods explained in Articles (22) and (23).

(*c*). Divide the length of the perpendicular by the sine of the angle thus found, and the quotient will equal the required side.

If the angle of the parallelogram be greater than 90°, its supplement* must be used in its stead.

Let Fig. 52 represent the parallelogram to be laid out; its area

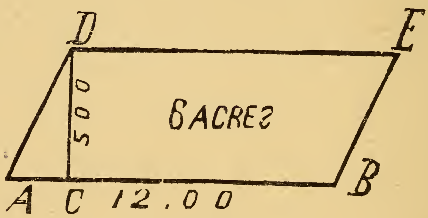

FIG. 52.

being 6 acres, and the length of the side A B, 12.00. Dividing $(6 \times 10) = 60$ sq. chains by 12, we find the perpendicular C D, to be 5.00 long.

* The sine of an angle is always equal to the sine of the supplement of the angle.

Suppose the angle A D E to equal 108°; then its supplement will be (180° — 108°) = 72°, the sine of which is .95106.

Dividing 5.00 by .95106, we find the length of the required side, A D, to be 5.257.

Had the angle B A D been used, instead of A D E, the supplement need not have been taken, as it is less than 90°.

EXAMPLES.

1. In a parallelogram, the area is 12 acres, the length of one side 14.00, and the measured angle equal to 61°. What is the length of the required side?

2. A field in the form of a parallelogram is 22.00 long and contains 25 acres. The size of one of the angles is 96° 45′; what is the length of the required side?

(**234**). *To Lay Out a Right-Angled Triangle.*—Let it be required

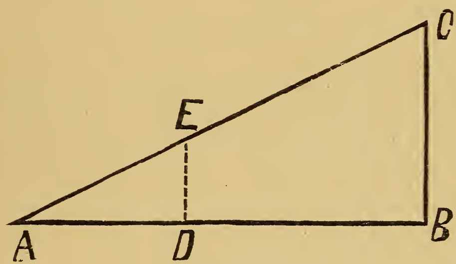

FIG. 53.

to lay out a right-angled triangle containing .3 acre by a line perpendicular to A B, Fig. 53.

(*a*). Measure the angle B A C and find its sine.

(*b*). Multiply the sine by any length of base, as A D, less than the required base. The product will represent the altitude D E of the triangle A D E.

(*c*). Compute the area of this triangle, and the length of the side A B may be found by the following proportion:

$$\text{Area } ADE : \text{Area } ABC :: (AD)^2 : (AB)^2.$$

If A D = 1.20, and D E .80, the area of the triangle A D E will be .048 acre. Then,

$$.048 : .3 :: (1.20)^2 : (AB)^2; \text{ whence } (AB)^2 = 9, \text{ and } AB = 3 \text{ chains.}$$

The length of the side A C may be found by a similar proportion:

$$\text{Area A D E} : \text{Area A B C} :: (\text{A E})^2 : (\text{A C})^2.$$

(**235**). *To Lay Out a Trapezoid.*—Approximate the distance between the parallel sides by treating it as a parallelogram. The distance thus found will be too short if the sides not parallel converge, and too long if they diverge. Let Fig. 54 represent a tract to be laid out or parted off.

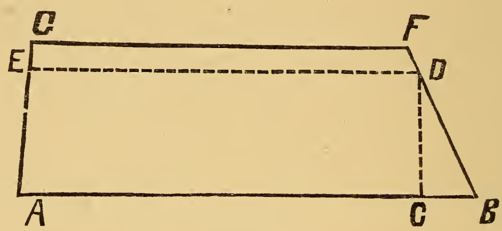

FIG. 54.

By dividing the area by the length of the line A B, the perpendicular is found to equal C D. The guess line E D is then measured, and the area of the trapezoid A B D E computed. The deficiency of area is then added outside of D E, and the trapezoid A B F G will then contain the required amount of land. If it still vary a little from the exact amount, the line F G may be moved further out or in, as the case may be.

In case of divergence of the sides, the overplus of area must be subtracted from the computed area, and the guess line moved back instead of further out.

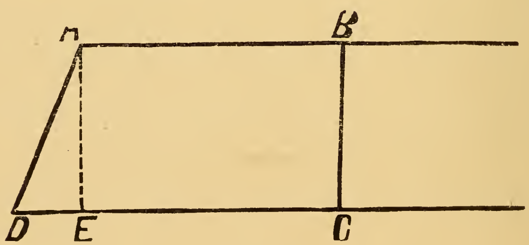

FIG. 55.

When the difference in length of the parallel sides can be determined without a measurement, no guess line need be surveyed, providing the distance between the parallel sides be known. Let A B C D, Fig. 55, represent a tract of land to be laid out or parted off the main tract by a line perpendicular to its parallel sides.

Divide its area by the perpendicular distance between its parallel sides. The quotient will be the mean length of the trapezoid. From this subtract half the distance D E for the shorter side, and add for the longer side.

(236). *To Lay Off any Figure.*—When the underlying principles of the particular problem differ from those obtaining in any of the cases considered, it will probably be best to depend on corrections made from guess lines, as in Art. 235, and thus reach the result by approximations. Yet, in many instances, easy and beautiful solutions may be reached by close observation and study.

Dividing up Land.

(237). Problems in dividing up land are such as grow out of division of estates, generally among heirs. This division is made with reference to the value of the respective shares (considering location, improvements, quality of soil, etc.), and not with regard to the quantity of land each share contains. If, however, taking all these things into consideration, the value of the land is uniform throughout the tract to be divided, and the shares of the persons among whom it is to be partitioned are equal to each other, each should receive the same quantity.

In making a partition of land no share should be taken out in such a way that it will injure any other share, when it possibly can be avoided.

(238). The problems in dividing land introduced into this chapter are of the nature of those that usually come up in practice where the land has been surveyed according to the Rectangular System. Only simple ones have been chosen.

(239). *To Divide a Rectangle into Equal Parts by Lines Parallel to a Side.*—Divide each of the lines upon which all of these parts are to rest into as many equal sections as there are shares. Connect the extremities of these sections by perpendiculars, and these perpendiculars will be the division lines of the shares.

Let Fig. 56 represent the south half of a quarter-section con-

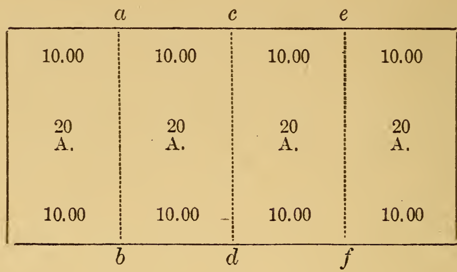

FIG. 56.

taining exactly 80 acres. The perpendiculars, *a b*, *c d*, and *e f*, divide it into four parts, each containing 20 acres.

(**240**). *To divide a rectangle into any number of unequal parts bearing a given relation to one another, by lines running parallel to a side.*—Suppose that, on account of the varying value of the land in the tract to be divided, the shares are to be to one another as the numbers 1, 2, and 5. In this case, divide the base lines into parts bearing the same relation to one another as the shares, and con-

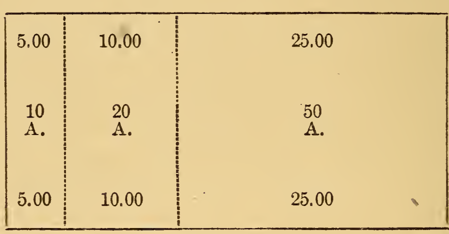

FIG. 57.

nect the points of division by perpendiculars, as shown in the division of the 80 acre tract in the figure.

Sometimes it is possible to divide land of varying quality so that each share shall contain its portion both of the best and worst. If we consider the unequal division in Fig. 57 to have been caused by the difference in quality of the land in various parts of the tract, it might have been possible to make the shares all equal by dividing the tract in the direction of its length.

The figure of the shares may be almost as variable as the quantity of land they contain. The rectangular form is preferred, but of course can not always be preserved, even in the territory surveyed according to the Rectangular System.

(**241**). *Problems.*—1. Divide a quarter-section of land into five shares in the series 1, 2, 3, 4, 5, by lines running parallel to a side.

2. The commissioners, in a certain partition of a quarter-section, set off the widow's dower of 30 acres in the form of a square in the south-east corner, and divided the remainder, by lines running north and south, equally among five children. What was the width of each share?

3. In a sale of a quarter-section for taxes, the lowest bid was for fifteen acres, and this amount was set off in the form of a square in the north-west corner. A few years afterward the remainder of the quarter was offered again, and the lowest bid was for twelve acres, which area was set off next to that first sold. What were the dimensions of each piece?

4. Divide the following described tract into four equal shares by north and south lines: South half north-east quarter, and east half north-east fourth north-east quarter, and south half north-west fourth north-east quarter.

5. Divide a quarter-section into 7 equal shares by lines running north and south, and write a description of each share, giving metes and bounds.

6. Divide the following tract into 5 equal shares by north and south lines, and write a description of each share: North half south-west quarter, and south-west fourth north-west quarter, and west half north-west fourth south-east quarter, giving metes and bounds.

7. Divide the north-west quarter, and north half north-east quarter, and north half north-east fourth south-west quarter, by east and west lines, into 3 shares that will be to each other as 1, 2, and 3, and write a description of each share, giving metes and bounds.

In the above examples each tract is supposed to contain exactly the prescribed amount of land, and the boundaries to run due east and west, or north and south, as the case may be.

9

Questions on Chapter XIII.

1. Why will a study of methods used in computation of area assist in laying out and dividing up land?
2. How do you determine a side of a square from the area? Of a rectangle?
3. Give the rule for finding the required side of a parallelogram.
4. Explain the method given for laying out a right-angle triangle.
5. How do you lay out a trapezoid from the area and length of one of the parallel sides?
6. In partitioning land, which is considered, quantity of land or value?
7. Explain the method of dividing a rectangle into equal parts by lines running parallel to a side.
8. In dividing lands, what figure for the shares is preferred?

CHAPTER XIV.

SURVEYING TOWN LOTS.

(**242**). The dimensions of town lots are usually given in feet, instead of in chains and links, and, as a general thing, the lots are all of the same size and numbered in regular order from 1 up, as shown in Fig. 58. The larger figures indicate the numbers of the

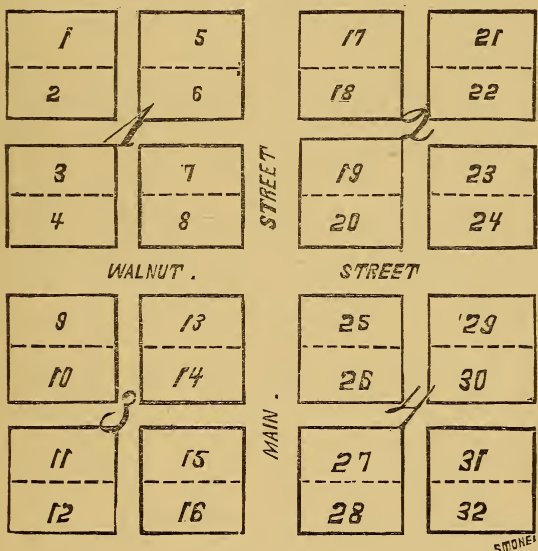

Fig. 58.

blocks, and sometimes the lots in each block are numbered separately; as lot 5, block 2; lot 2, block 9, and so on. As the town grows from the original plot, the lots in each addition are frequently numbered and referred to the particular addition to which they belong; as lot 8, Brown's addition; lot 7, Johnson's addition, etc.

(**243**). The survey of the town is generally based on some independent corner of the section in which it is situated, and important corners in various parts of the town should be marked with durable monuments. Fig. 59 shows a few lots in a town located on a section line, and the distance is given from the section corner to the south-east corner of lot number 1. It will be seen

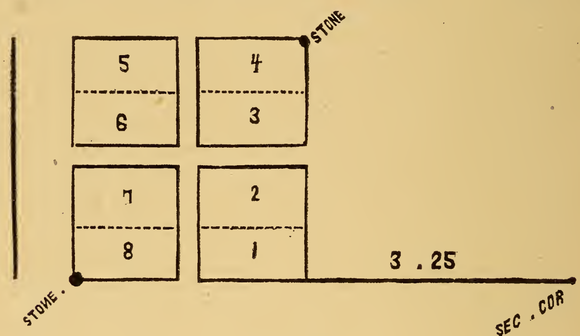

FIG. 59.

that stones are placed at the north-east corner of lot number 4 and the south-west corner of lot number 8. These will enable the surveyor at any subsequent time to make a survey in the town without going to the section corner to find a starting point, thus saving him time and trouble.

(**244**). Lots are usually rectangular in shape and about twice as long as they are wide, but this is not always the case. They may be any reasonable shape or size that adapts them to the plan of the town. Likewise, streets and alleys generally cross one another at right angles, though by no means always.

(**245**). The plot of a town should always be accompanied by full explanations showing,

(1). The size of each of the lots.

(2). The width of each of the streets and alleys.

(3). The name of each of the additions.

(4). Any other explanations necessary to determine the bearings of any of the lines which would have to be run in a survey of the town.

(**246**). Suppose that in Fig. 58 the lots are each 100 feet long and 50 feet wide, the streets 50 feet wide, and the alleys 16 feet wide. Since the lots are rectangular, the north and south lines are at right angles to the east and west lines. It is evident that after finding a starting point, the surveyor need experience no difficulty in the survey of any of the lots.

If, for instance, he wishes to survey lot number 13, and can find no corner except the one marked with a stone at the south-east corner of the town, he may start at the center of this stone, run west 266 feet to the west side of Main street, and thence north 166 feet to the south-east corner of the lot to be surveyed. He can then survey the lot without any trouble. Instead of running first west and then north, he may run first north to the south-east corner of lot number 29, and thence west to the corner of the lot to be surveyed; or he may take other routes.

(**247**). Fig. 60 represents a portion of a town in which the lots

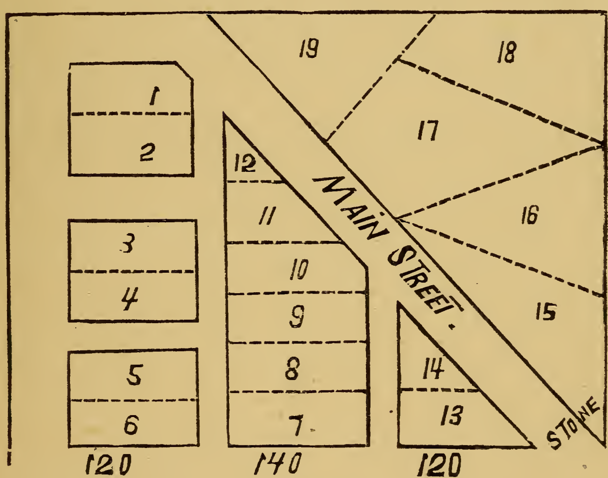

FIG. 60.

are of different sizes and shapes. All the lots west of Main street are 50 ft. wide, except number 12, which is 60 ft wide, and number 14, which is 75 feet wide.

Main street bears N 40° W.

Lots number 15, 16, 17, 18 and 19 do not belong to the regular plot of the town, and are called out-lots.

The two alleys running north into Main street, and the one between lots 4 and 5 are each 20 ft. wide, and Main street and the short street between lots 2 and 3 are each 50 ft. wide.

A stone monument marks the south-east corner of lot number 13.

The width of each tier of lots is marked in feet at the foot of the tier.

(**248**). It is now a very easy matter to survey any of the lots in the regular plot of the town. Take, for example, number 11. To survey this lot, measure first west 280 ft., and thence north 200 ft. to its south-west corner. From this point set off a perpendicular and extend it to the street; then measure north 50 ft. further and set off another perpendicular as before.

(**249**). EXAMPLES.

1. How would lot number 1 be surveyed ?
2. Explain a method of surveying lot number 14.
3. If the out-lots east of Main street were separated by lines perpendicular to the street, what would be the bearing of the lines?

(**250**). Town lots are measured either with a chain or tape.

The chain used for this purpose is usually 50 or 100 feet long and divided into links, each 1 foot in length. It is made light, and as greater accuracy is generally required in surveying town lots than in ordinary surveying, its length should be frequently tested by comparison with a standard measure. The length of the chain is affected by wear, temperature, and accidents.

All measures used in surveying should be subjected to frequent tests. Even in surveys where tolerable accuracy is sufficient, there is no excuse for neglecting anything that would be conducive to greater accuracy.

Tapes used in measuring town lots usually consist of a jointed steel ribbon, but sometimes a linen tape through which a fine brass wire is interwoven with the thread, is used. Common linen tapes contract when wet and are not trustworthy. The steel tape is the best.

Questions on Chapter XIV.

1. How are town lots numbered?
2. Upon what is the survey of a town usually based?
3. What advantage is there in having important corners marked by monuments?
4. What is the usual shape of town lots?
5. State the explanations that should accompany the plot of a town.
6. A street bears N 29° 32′ E. What is the bearing (obverse and reverse) of a line perpendicular to it?
7. What kind of measures are employed in the survey of town lots?
8. How is the length of the chain affected?

CHAPTER XV.

PLOTTING.

(**251**). Plotting is the operation of drawing to a scale upon paper the lines of a survey, so that the plot will be a correct representation of the actual lines surveyed.

(**252**). The instruments used in plotting are a drawing board, t-square, ruler, drawing pen, dividers, protractor, and a diagonal scale.

1. The drawing board should be made of pine, and its surface should be perfectly smooth and level. The paper is fastened to the drawing board while the plot is drawing. Perhaps the most suitable size for a drawing board is about 30 inches square, but 24 by 28 inches makes a very nice board. The paper should be stretched evenly, and the edges pulled down over the edges of the board and glued or tacked.

2. The t-square, so called on account of its resemblance to the letter T, consists of a thin blade with parallel edges, to which is attached a cross-head somewhat thicker than the blade, so as to form a shoulder. The blade is usually about 24 inches in length, and the cross-head about 10 inches. By laying the blade on the paper and pressing the shoulder against the edge of the drawing board, as shown in Fig. 61, perpendiculars may be drawn to any edge of the paper.

3. A good box-wood ruler, about 12 inches long, divided to 16ths of an inch, will answer every purpose in plotting. This ruler should have one beveled edge, upon which the divisions are marked, and one projecting edge, along which the pen should be pressed in drawing lines.

4. The drawing pen consists of two steel blades, whose distance apart is regulated by a thumb-screw. A little practice will en-

able any person to draw nice smooth lines of any desirable width with the drawing pen. India ink should be used, as it flows more smoothly from the pen than common ink.

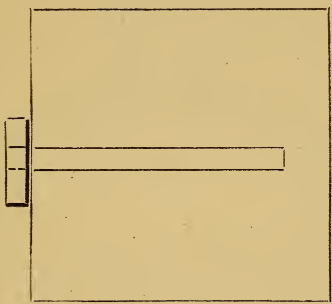

FIG. 61.

5. The dividers or compasses is an instrument used in drawing arcs, sub-tending angles, etc., and consists of two arms which open and shut by a hinge joint at the end. Each of these arms terminates in a sharp point, and one, if not both, is usually jointed so as to permit the point to be taken out and a drawing pen put in its stead. In drawing large arcs a lengthening bar is inserted in

FIG. 62.

the jointed arm. Fig. 62 represents a pair of plain dividers. It will be seen that the arms are not jointed in this pair.

6. The protractor is an instrument used in laying out angles. It is usually nothing more than a semicircle divided to degrees, half-degrees, or quarter-degrees. The degrees are numbered from 0° to 180° in one or both directions from opposite extremities of the arc. The best protractors are made of silver or German silver, but the more common ones are made of brass or horn, and sometimes of paper. Fig. 63 represents a small protractor.

Where great accuracy is required, protractors are supplied with an arm to which a vernier, like the compass vernier, is attached.

Sometimes rectangular protractors are used instead of semicircular.

7. The diagonal scale of equal parts is a flat scale a given num-

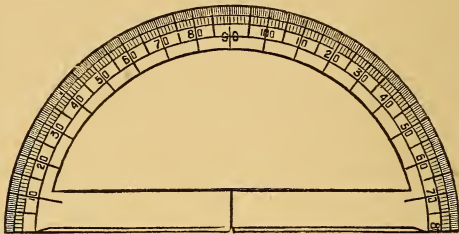

FIG. 63.

ber of units, say inches, in length, and has the space devoted to one unit at the end divided by diagonals as shown in Fig. 64. These diagonals with the assistance of the lines running parallel to the edges of the scale, enable a person to take the length of a

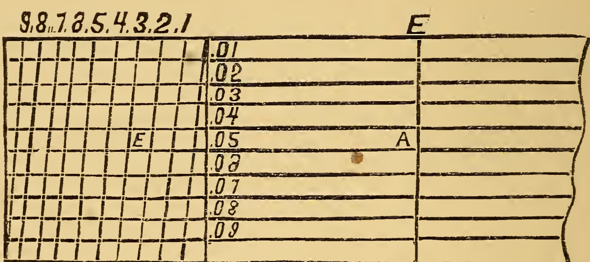

FIG. 64.

line to $\frac{1}{100}$ of the unit of the scale. If the unit of the scale be 1 inch, then the length of a line may be taken to $\frac{1}{100}$ of an inch.

(**253**). In drawing plots and maps a unit of the scale represents a certain number of units of the line to be represented on the plot. Suppose the real line is 20 chains long, and it is to be plotted to a scale of 5 chains to the inch. The line on the plot will therefore be $(20 \div 5) = 4$ inches long. Fig. 65 represents a line 2 chains in length plotted to different scales.

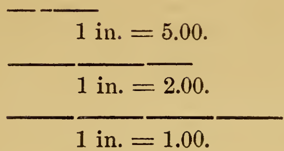

Fig. 65.

(**254**). Let us now employ the drawing instruments in plotting lines.

Suppose an east and west line 2.27 in length is to be drawn to a scale of 1 chain to an inch.

Arrange the drawing paper with its edges parallel to the edges of the board, and then place the t-square, as shown in Fig. 61, with its shoulder fitting squarely to the left-hand edge, and the edge of the blade just moved up to the point from which the line is to be drawn. Then spread the dividers so that when one arm is placed two units from the inner edge of the divided unit and on the line marked .07, the other will just reach the point where this line crosses the line marked .2 at the top of the scale. The arms then embrace the proper length of line.

Next place one arm of the dividers against the t-square with its point on the point from which the line is to be drawn and swinging the free arm round in the proper direction until it too touches the same edge of the blade. Connect these two points by a line with the drawing pen, and it will be the required line.

In like manner, a line 1.25 long to the same scale may be embraced by the dividers by placing one point at *a* on the diagonal scale, and the other at *e;* and a line 1.40 long by placing one point at *c* and the other at the point marked .4 at the top of the scale.

If the diagonal scale is not long enough to permit the required line to be taken off, it may be extended by means of a ruler.

(**255**). EXAMPLES.

Draw lines representing the following distances:

(1). 2.50; scale — 1 in. = 1.00.
(2). 3.79; scale — 1 in. = 1.00.
(3). 4.75; scale — 1 in. = 2.00.
(4). 6.42; scale — 1 in. = 5.00.
(5). 10.00; scale — 1 in. = 5.00.
(6). 12.31; scale — 1 in. = 10.00.

(**256**). Rectangular tracts of land may be plotted with the instruments used in drawing lines already described.

Take, for instance, a rectangular tract 7.15 long, 4.35 wide.

First draw a line representing the length of the tract, then another perpendicular to this at one end representing the breadth, then from the end of this another parallel to the first and of equal length, and close by connecting the extremities of the first and third with one another.

If no diagonal scale is at hand, a common ruler will answer for rough work.

(**257**). The protractor is used in nearly all cases where the courses and distances of the boundaries of the tract to be plotted are given, and its use will now be explained.

The bearing of a line represents the angle the line makes either with the magnetic meridian or the true meridian drawn through the point from which the bearing is taken; and to determine this angle and represent the line on the plot, meridians should be drawn through each station of the survey as soon as the station is located.

Begin at any important station to draw the plot by laying out a meridian on the proper part of the paper and locating the station on this meridian; then draw the first course at the proper angle with this meridian, producing it the required distance, as explained in Art. 254; then draw another meridian through the other extremity of the course, and lay out the second course in the same way; proceed in this way until the lines are all drawn, and the last line should terminate at the station taken as the starting point in the plot.

(**258**). Let us now draw a plot of the following tract of land: N 62° 45′ E, 9.25; thence S 36° E, 7.60; thence S 45° 30′ W, 10.40; thence N 31° 30′ W, 10.00.

In this case we may commence at the first station in the description. Draw a meridian as N S, Fig. 66 and locate the station at some point, as A, on the meridian. Then place the protractor so its center will fall on the station and its edge coincide with the meridian, and with the point of a pin mark the termination of an arc of 66° 45′ from the north end of the protractor. Then draw the line A B from the station through this point and determine its length by the method explained in Art. 254. Draw

B C, C D, and D A in the same manner, and the plot will be complete.

The plot, Fig. 66, is constructed to a scale of 5 chains to the inch.

In northerly courses the angle or bearing should be read from the north end of the protractor, and in southerly courses from the south end.

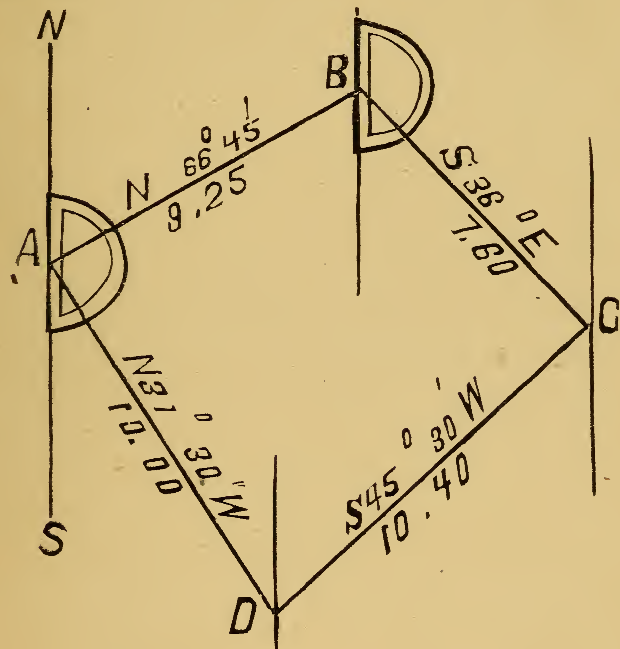

FIG. 66.

If the last course lack but a little of terminating at the first station, the discrepancy may be the result of the imperfection of the instruments employed; but if the extremities of the lines are a considerable distance apart, it is probable that a mistake has been made somewhere. If it be tested by latitudes and departures, and they balance (Art. 212), the mistake is in the plot, but if they do not balance, the error is in some of the previous work.

The descriptions given in Art. 226 may be used as examples in plotting.

Various other methods are also used in drawing plots, but the one given is, perhaps, the most speedy and simple, and will answer every purpose.

(259). The pantograph is an instrument used for copying plots, etc., either in a reduced or enlarged form. It consists of four rulers arranged somewhat in the form of a parallelogram. By fastening the instrument on the drawing-board and moving a point on one arm along the plot to be copied, another arm to which a pencil is attached sketches a precise copy on the sheet placed under it.

(260). Buildings, springs, etc., may be located on the plot if their courses from certain points on the boundaries of the tract are known.

For example, in surveying the west half of a quarter-section, a line from the north-east corner to the north-west corner of a house was found to bear S 45° W, and one from a point 12.00 west of the north-east corner was found to bear S 13° E. While constructing the plot these lines may be laid out from the proper places with the protractor, and the place where they meet will be the north-west corner of the house, as shown in Fig. 67.

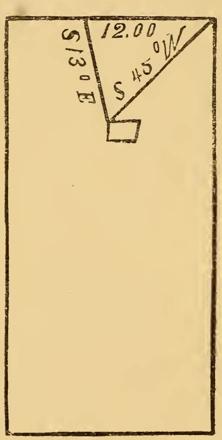

FIG. 67

(**261**). Plots and maps may be colored with crayon pencils or with water-colors; but when water-colors are used care must be taken to keep them from running into one another and injuring the shades. The paper should be dampened preparatory to applying them. For inexperienced persons, crayons will prove the most satisfactory.

Questions on Chapter XV.

1. What is plotting?
2. Name the instruments used in plotting.
3. Describe the diagonal scale.
4. Describe the method of using the diagonal scale.
5. Give the length of each of the following lines plotted to scales of 1 chain to an inch, 2 chains to an inch, and 10 chains to an inch: 12.50; 15.00; 18.375; 11.25.
6. Describe the method of using the protractor.
7. Why should the last course in a plot terminate at the first station?
8. For what is the pantograph used?
9. How may buildings and other objects be located on a plot?
10. How are plots and maps colored?

CHAPTER XVI.

SURVEYING WITHOUT A COMPASS.

(**262**). A great many surveys can be made without the compass, and a few pages will now be devoted to the consideration of the most common cases in which it may be dispensed with. It must be borne in mind, however, that the compass could be advantageously used in nearly all the cases here cited, and that the methods given are intended for use only in emergencies.

(**263**). *Setting Corners.* — Where two witness trees taken to a corner can be found, the corner may be located from them by their distances measured from the sides upon which the blazes are made. Suppose one tree is 15 links from the corner, and the other 19 links. Measure off 15 links on one end of a cord and 19 links on the other end, and tie a knot where the two measurements terminate. Then have the long end of the cord held against the

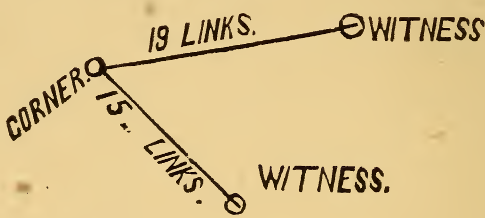

FIG. 68.

blaze on the most distant tree, and the short end against the blaze on the other. Stretch both ends of the cord tightly, and the knot will mark the corner, as shown in Fig. 68.

The corner is always in front of the blazes on the trees.

The distances of the witness trees from the corner may be found from the field-notes of the tract.

Where only one witness can be found, the corner can not be located with certainty without a compass.

This same method may be employed, slightly modified, in locating a corner by the two lines meeting there, when the length of each line and the location of each of the corners at the other end of each, are known.

(**264**). *Establishing Lines.*—When one corner is visible from the corner at the other end of the line, a stake may be put up, and intermediate points on the line may be marked at pleasure.

When one corner is not visible from the other, but its direction is approximately known, the line may be "ranged" from one to the other. To do this, put up a stake or flag at the corner from which the line is ranged, and at a certain distance, say 50 or 100 steps, in the direction of the other corner, set up another stake or flag. Then walk ahead an equal distance and set another stake in line with the first and second. Proceed in the same manner, always setting the stakes at equal distances from one another, and ranging the last one with the two previously put up, until the other corner is reached. The distance that the line misses, either to the right or left, can then be noted, and the stakes corrected in a manner entirely similar to that already explained.

For instance, if there are 12 stakes on the line, and it terminates 30 links to the right of the corner, each stake must be moved to the left. The distance it is to be moved is found by dividing the distance missed by the number of stakes and multiplying the quotient by the number of the stake from the starting point. In this case, the 11th stake must be moved to the left $\left(\frac{11 \times 30}{12}\right) = 27\frac{1}{2}$ links, the 10th $\left(\frac{10 \times 30}{12}\right) = 25$ links, and so on.

The stake put down at the corner at starting is not counted, and the next is called the first.

Of course, the location of the corners must be known before the line can be established.

(**265**). *Setting Out Perpendiculars.*—Almost any kind of a contrivance with two lines of sight at right angles to one another, will answer for this purpose. It may be a sort of cross-staff with four upright sights provided with slits or threads, two marking each line of sight. The sights need not be more than 18 inches apart, and the apparatus should be made to rest on a staff about $4\frac{1}{2}$ feet high. It may be rude in construction, but the lines of sight should be exactly at right angles to one another.

(**266**). Rectangular tracts of land may be readily surveyed in many instances with this instrument, but it should be used with care in independent divisions of the section, as they are not often exactly rectangular in form. One of the lines of sight may also be used in sighting lines, and is a good substitute for the method of "ranging" described in the preceding article.

(**267**). *Measurements.*—Lines may be measured with a cord, tape-line, or pole, and distances may be given in feet or links, as best suit the case at hand.

APPENDIX.

ABSTRACT OF DECISIONS.

ABSTRACT OF DECISIONS

OF THE

UNITED STATES AND VARIOUS STATE COURTS

RELATING TO CONTRACTS, SURVEYS, ETC.

(Nearly all of the following Decisions have been taken by permission from DUNN'S LAND DECISIONS, a valuable book for surveyors, published by George H. Frost, New York.)

BOUNDARIES.

1. Course and distance must yield to natural and artificial objects of description. Gaveny *vs.* Hinton, 2 Greene (Iowa) 344.

2. Boundaries marked on the land are to govern courses and distances. Blaisdell *vs.* Bissell, 6 Barr (Pa.) 478.

3. The lines marked on the ground constitute the actual survey and control the return of the surveyor, even where a natural or other fixed boundary is called for by the survey, though the space between the two is but twelve perches in breadth. Walker *vs.* Smith, 2 Barr (Pa.) 43; Hall *vs.* Tanner, 4 Barr (Pa.) 244.

4. A grant called for a certain number of poles "to a stake, crossing the river." Held, that the line must cross the river, though the distance terminated before entering it. Whiteside *vs.* Singleton, 1 Meigs (Tenn.) 207.

5. A survey must be closed in some way or other. If this can be done only by following the course the proper distance, then it would seem that distance should prevail; but when distance falls

short of closing, and the course will do it, the reason for observing distance fails. Doe *vs.* King, 3 How. (Miss) 125.

6. Where a deed describes lands by its admeasurements, and, at the same time, by known and visible monuments, these latter shall govern. Mayhew *vs.* Norton, 17 Peck (Mass.) 357; Massengille *vs.* Boyles, 4 Humph. (Tenn.) 205; Woods *vs.* Kennedy, 5 Monr. (Ky.) 174; Nelson *vs.* Hall, 1 McLean (U. S.) 518; Campbell *vs.* Clark, 8 Mis. 553.

7. The rule that monuments control in boundaries is, however, not inflexible; and in case where no mistake could reasonably be supposed in the courses and distances, the reasons of the rule were held to fail, and the rule itself was not applied. Davis *vs.* Rainsford, 17 Mass. 207.

8. A line is to be extended to reach a boundary in the direction called for, disregarding the distance. Witherspoon *vs.* Blanks, 1 Taylor (N. C.) 110.

9. If a vendor hold two tracts adjoining, and sell a certain quantity by metes and bounds, though the deeds call for one tract, yet if the metes and bounds run into the other, the purchaser shall hold according to the metes and bounds. Wallace *vs.* Maxwell, 1 J. J. Marsh (Ky.) 447; Mundell *vs.* Perry, 2 Gill & Johns (Md.) 206.

10. Posts set up at corners, between adjoining owners of land, control the calls for course and distance and establish the boundary where they are mentioned and recognized in the deeds. Alshire *vs.* Hulse, 5 Ham. (Ohio) 534.

11. Where land is described as running a certain distance by admeasurement, to an ascertained line, though without a visible boundary, such line will control the admeasurement and determine the extent of the grant. Flagg *vs.* Thurston, 13 Pick. (N. Y.) 145; Carroll *vs.* Norwood, 5 Har. & J. (Md.) 163.

12. Where the line or course of an adjoining tract, being sufficiently established, are called up in a patent or deed, the lines shall be extended to them without regard to distance. Cherry *vs.* Slade, 3 Murph. (N. C.) 82.

13. Where the boundaries of land are fixed, known, and unquestionable monuments, although neither courses, nor distances, nor the computed contents correspond, the monuments must govern. Pernam *vs.* Wead, 6 Mass. 131; Calhoun *vs.* Wall, 2 Har. & McHen. (Mo.) 416.

14. If a deed from the government of the U. S., or an individual, describes land as partly bounded by a river, the river boundary will be adhered to, though it does not correspond with established corners and monuments. Shelton *vs.* Mauphin, 16 Mo. 124.

15. If nothing exists to control the call for courses and distances, the land must be bounded by the course and distances of the grant, according to the magnetic meridian; but courses and distances must yield to natural objects. 16 Ga. 141.

17. The corners established by the original surveyors of public lands under the authority of the United States, are conclusive as to the boundaries of sections and divisions thereof, and no error in placing them can be corrected by any survey made by individuals or by a state surveyor. Arnier *vs.* Wallace, 28 Miss. 556.

18. Whenever natural or permanent objects are embraced in the calls of either a survey or a patent, these have absolute control, and both course and distance must yield to them. Brown *vs.* Huger, 21 Howard (U. S.) 305.

19. In determining boundaries under a grant, natural objects, as landmarks, are to be considered before courses and distances. Daggett *vs.* Wiley, 6 Fa. 482.

20. Where adjoining proprietors abut on opposite banks of a stream, their boundary line will follow the natural and imperceptible alterations in its course, but not changes caused by artificial means. Halsey *vs.* McCormick, 3 Kernan (N. Y.) 296.

21. When lands are described in a deed or grant as bounded by river not navigable, the center of the stream is to be considered the boundary. Claremont *vs.* Carlton, 2 N. Hamp. 369; Palmer *vs.* Mulligan, 3 Caines (N. Y.) 407, 319; Hayes *vs.* Bowman, 1 Rand (Va.) 417; Ingraham *vs.* Wilkinson, 4 Pick. (Mass.) 268; Gavil *vs.* Chambers, 3 Ham. (Ohio) 496; Brown *vs.* Kennedy, 5 Har. & J. (Md.) 195; Arnold *vs.* Mundy, 1 Halst. (N. J.) 1.

Quantity of Land.

22. A conveyance by metes and bounds will carry all the land contained in them. Belden *vs.* Seymour, 8 Conn. 19; Jackson *vs.* Ives, 9 Cow. (N. Y.) 661. Although it be more or less than is stated in the deed. Butler *vs.* Widger, 7 Cow. (N. Y.) 723.

23. Where a specified tract of land is sold for a gross sum, the boundaries of the tract control the description of the quantity it contains, and neither party can have a remedy against the other

for an excess or deficiency in the quantity, unless such excess or deficiency is so great as to furnish evidence of fraud or misrepresentation. Voorhees *vs.* De Meyer, 2 Bar. Sup. Ct. Rep. (N. Y.) 87.

24. Where a person purchases land by metes and bounds said to contain a certain number of acres, more or less, he is entitled to all the land within the limits, whatever the number of acres may be. Bratton *vs.* Clawson, 3 Strobh. (S. C.) 127.

24. Quantity, although the least reliable and last to be resorted to of all descriptions in a deed, in determining the boundaries of the premises conveyed, may sometimes be considered in corroboration of other proof. McClintock *vs.* Rogers, 11 Ills. 279.

Figure of Tracts of Land.

25. If the order for a survey of land do not certainly determine the form in which it should be made, the survey ought to be in a square. Kennedy *vs.* Paine, Hardin, 10.

26. "Seventy acres, being and lying in the south-west corner" of a section, is a good description, and the land will be in a square. 2 Ham. (Ohio) 327; Cockrell *vs.* McQuinn, 4 Monr. (Ky.) 63.

27. The rectangular figure will be preserved in preference to any other in fixing locations. Massie *vs.* Watts, 6 Cranch, 148; Holmes *vs.* Trout, 7 Pet. 171.

Acquiescence in Boundaries.

28. Acquiescence for a long time (*e. g.* for eighteen years), in an erroneous location, is conclusive on the party making or acquiescing in such location. Rockwell *vs.* Adams, 6 Wend. (N. Y.) 469.

29. Where a boundary is disputed between parties who own adjoining tracts, and the parties employ a surveyor, who runs out the line, and marks it on a plat in their presence, as a boundary, after twenty years corresponding possession, they are concluded by it. Boyd *vs.* Graves, 4 Wheat. 513.

30. An acquiescence for twenty years is, as a general rule, necessary to support an implied agreement in respect to a boundary different from that clearly expressed in the title deeds. Ball *vs.* Cox, 7 Ind. 453.

31. Where two persons own equal parts of a lot of land in severalty, but not divided by visible monuments, if both are in possession of their respective parts for fifteen years, acquiescing in an

imaginary line of division during that time, that line is thereby established as a divisional line. Beecher *vs.* Parmele, 9 Ver. 352; also 18 Ver. 395.

32. Maintaining a fence for many years is strong but not conclusive evidence of limitation of claim to the boundary. Potts *vs.* Everhart, 26 Pa. 493.

33. A party is precluded upon principles of public policy, from setting up or insisting upon a boundary line, in opposition to one which has been steadily adhered to upon both sides for more than forty years. Baldwin *vs.* Brown, 16 N. Y. 359.

34. A division fence of more than twenty-one years' standing, although crooked, constitutes the line between adjacent land owners, even though the deeds of both parties call for a straight line between acknowledged landmarks. McCoy *vs.* Hance, 28 Tenn. 149.

35. Where adjoining proprietors, being unable to ascertain the division line, agree verbally upon a certain line, the agreement is binding, and improvements by one up to the line is notice thereof to a purchaser from the other. Houston *vs.* Sneed, 15 Texas, 307.

36. An ancient line of division marked on the ground by adjoining owners, and afterwards acted upon by them, will become the boundary between the lots, although different from the line described in the original deeds. Hathaway *vs.* Evans, 108 Mass. 267.

37. A possession for twenty years of a part of the land in dispute, in reference to a line conflicting with another tract, of which another party may be also in actual possession, but outside of the disputed territory, may be enough to presume the execution of a deed conveying the land in dispute to the party in possession. Amick *vs.* Holman, 13 Shobh. (S. C.) 132.

38. Parties are not bound by a consent to boundaries, which have been fixed under an evident error, unless perhaps by the prescription of thirty years. Gray *vs.* Couvillon, 12 La. Ann. 730.

TABLE OF

NATURAL SINES AND COSINES.

	0°		1°		2°		3°		4°		
M.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	M.
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01341	99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	89°		88°		87°		86°		85°		

M.	5° Sine.	5° Cosin.	6° Sine.	6° Cosin.	7° Sine.	7° Cosin.	8° Sine	8° Cosin.	9° Sine.	9° Cosin.	M.
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	11147	.99277	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	84°		83°		82°		81°		80°		

M.	10° Sine.	10° Cosin.	11° Sine.	11° Cosin.	12° Sine.	12° Cosin.	13° Sine.	13° Cosin.	14° Sine.	14° Cosin.	M.
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.35291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	79°		78°		77°		76°		75°		

M.	15° Sine.	15° Cosin.	16° Sine.	16° Cosin.	17° Sine.	17° Cosin.	18° Sine.	18° Cosin.	19° Sine.	19° Cosin.	M.
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.96006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94720	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.14721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.59757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	74°		73°		72°		71°		70°		

	20°		21°		22°		23°		24°		
M.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	M.
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.96731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	69°		68°		67°		66°		65°		

M.	25° Sine.	25° Cosin.	26° Sine.	26° Cosin.	27° Sine.	27° Cosin.	28° Sine.	28° Cosin	29° Sine.	29° Cosin.	M.
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.43604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.54594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.98193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
M.	64° Cosin.	64° Sine.	63° Cosin.	63° Sine.	62° Cosin.	62° Sine.	61° Cosin.	61° Sine.	60° Cosin.	60° Sine.	M.

M.	30°		31°		32°		33°		34°		M.
	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	59°		58°		57°		56°		55°		

	35°		36°		37°		38°		39°		
M.	Sine.	Cosin.	Sine.	Cosin	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	M.
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61958	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	64279	.76604	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	54°		53°		52°		51°		50°		

M.	40° Sine.	40° Cosin.	41° Sine.	41° Cosin.	42° Sine.	42° Cosin.	43° Sine.	43° Cosin.	44° Sine.	44° Cosin.	M.
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.66548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67322	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.67173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74614	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70281	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69712	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.97944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	49°		48°		47°		46°		45°		

"QUEER QUERIES."

A BOOK FOR THE STUDENT.

A BOOK FOR THE TEACHER.

A BOOK FOR EVERYBODY.

A COLLECTION OF QUESTIONS ON DIFFERENT BRANCHES OF STUDY.

This system of teaching "things not in the books" has been in use in many or the public schools for several years, and has met with almost unlimited success in being the means of incucating facts and principles into the youthful mind which can hardly be impressed upon the memory in any other way. It will lead to investigations and researches on the part of the student which cannot prove otherwise than beneficial. Creates great interest in schools, at Institutes, wherever used.

PREFATORY AND EXPLANATORY.

Queer Queries were collected in the following manner, viz: pupils were requested to bring any query which they thought would interest others or which they could not answer themselves, to the teacher.

The teacher then placed *ten* of the first queries found in this little book upon the black-board and allowed them to remain there from Monday morning till Friday evening, when they were answered in a general exercise in which all the pupils shared equally.

The result was that the school closed with a good understanding of why the time in China and America are not the same, of why the feet of the Chinese point toward our own; of why the sun seems to rise in the east, of why Patagonia has no Capital, &c.

The time occupied in this work was not to exceed ten minutes.

The teacher tried this experiment the next week with the succeeding *ten* questions with the school thoroughly alive to this new departure: every question was intelligently discussed by the pupils, both old and young.

The third week two or three heads of families sent queries (See Nos. 23, 27 and 29), and the interest increased. The teacher kept up this system with no visible indication of lagging interest for *one hundred weeks* with the very best results.

The demand for queries has been so great that we have consented to publish our first *one thousand* "Queer Queries."

How to use QEER QUERIES: take the book on Friday evening and call the attention of the school to such queries as you may have selected by having the pupils *to mark* them by numbers; thus if you think it not best foryour school to take them in regular order and you should select Nos. 1, 2, 3, 4, 5, 7, 13, &c.;—have the pupils "check" those numbers telling them they may study the questions at odd times till the next Friday evening when they may see who can answer the greatest number out of the *ten* selected. Pupils will individually ask you during the week to answer certain questions which they fail to find satisfactory theory for. Cite them to text-books, authors or persons within your knowledge where they will probably obtain the desired information; in no case should you give the desired information direct to the individual; but should the school as a body not be able to answer a question satisfactorily, then will be the time to help it out of the dilemma by gradually and pleasantly leading the school *to see* and *know* the *why* and *wherefore* of the subject under consideration.

Object of Queer Queries:

1st. To lessen the care of the teacher and make the school more attractive for the pupils by adding spice to at least one exercise for the week. (*The last day's work should be the most pleasant*).

2nd. *To form habits of close observation in the growing pupil, and in forming these habits which will cling to him through life, give him a fund of information which will well repay for all the trouble and time which such a plan imposes.*

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