A MANUAL

OF

LAND SURVEYING

COMPRISING

AN ELEMENTARY COURSE OF PRACTICE
WITH INSTRUMENTS

AND A TREATISE UPON THE

Survey of Public and Private Lands,

PREPARED

For use of Schools and Surveyors.

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"Let things that have to be done be learned by doing them."

F HODGMAN,
CLIMAX, MICHIGAN,
1897.

PREFACE.

This addition to the already numerous treatises on land surveying was caused by the demand of the surveyors of Michigan for a treatise which would deal with the practical questions which meet the surveyor in his every day work in the field. Several admirable treatises were already in existence which dealt amply with the mathematical and instrumental part of surveying. But the perplexing questions which meet the surveyor are not questions of mathematical calculation or of the use of instruments. On the contrary they are, for the most part, questions of how to apply the principles of common law and statutory enactment to the location of boundary lines. These are the controlling considerations in all resurveys; a class which comprises probably nine-tenths of all the land surveys which are made. Scarcely an allusion to these principles was to be found in any of the works on surveying extant. In 1880 the Michigan Association of Surveyors and Civil Engineers appointed a committee on manual, to prepare a work which would give authoritative answers to the many questions of practice which came up before them. The committee spent their spare time for five years in an exhaustive research of the laws and the decisions of the highest courts in the land. The chairman attended the meetings of various surveyors' associations and collected their reports. From the great mass of material thus collected, the leading points in the laws of the United States and the decisions of the courts of last resort were selected, covering, as nearly as possible, all the points relative to surveys and boundary lines which arise in the land surveyor's practice. The legal decisions quoted are a part of the Common law of the whole country and apply wherever the Common law prevails, whether in Canada, England, or the United

States. It should be remembered, however, that different courts do not always expound the law alike, and sometimes a court reverses its own decisions. Whenever there appears to be a conflict of authorities, the Surveyor should follow the latest decisions in his own State if there be any. It seemed to the committee to be important that the student in land surveying should be taught these things; that they were as necessary for the beginner to know as for the older practitioner, and hence might properly be incorporated in the text book. Having this in view, it was decided to extend the scope of the manual by including such mathematical work as would make it equally adapted to the use of the student as a text book and the practical surveyor as a book of reference. In preparing this portion of the work, the leading idea has been that, so far as possible, the student should be taught by actual practice in the field, as well as in the class room; that he should learn to survey by surveying. The solution of a problem in surveying in actual practice is always worked out upon the ground, hence suggestions are made to the student how problems may be solved, instead of giving any formal solution. It is pre-supposed that every successful teacher will have methods of his own for conveying instruction, and will use these suggestions or make different ones as may seem best to him. Doubtless things have been omitted which some would regard as important to have introduced. Such omissions will be supplied by teachers at their pleasure and convenience. We acknowledge our indebtedness to the authors of many treatises which have been consulted in the preparation of this volume, especially to the works of Davies, Gillespie, Hawes and Dunn, also to Messrs. W. & L. E. Gurley for many favors received, and to the officers and members of the Surveyors' Associations of Michigan, Ohio, Indiana, Illinois and Missouri for many valuable suggestions, sympathy and assistance.

F. HODGMAN.

Climax, Mich., 1891.

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A MANUAL

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LAND SURVEYING.

CHAPTER I.

I. Definitions. Field Work, &c.

1. **Land Surveying** is the art of measuring distances and running lines on the earth's surface to determine the boundaries or to ascertain the areas of tracts of land. The lines run are not mathematical lines, but are representations of them, traced upon the earth's surface by means of various instruments, and marked to the eye by chops and notches cut upon trees, or rocks, or by stakes or stones set in the ground, or any other means to render them visible.

2. **Original Surveys** are the surveys which are first made for the purpose of locating upon the ground the boundaries of tracts of land, and marking them by visible objects. This work is called the Field Work. A full description of what is done is kept by the surveyor and is called the field notes. The field notes furnish the data from which to make a map of the land and calculate the area. They also furnish the evidence from which to again find and identify the boundaries upon the ground.

3. **Resurveys** are those which are made for the purpose of finding the boundaries which were marked when the original survey was made.

4. The instruments most commonly used in land surveying are *the Chain and Tape* for measuring distances, and the *Picket, Compass, Solar Compass and Transit* for running lines.

II. Instruments for Measuring Distances and Their Use.

1. The Chain. The word chain is used to represent a distance of 66 feet and also an instrument used for measuring distances. The chain in most general use for land surveying is that invented by Gunter, and known as the Gunter chain. It is 66 feet long and divided into 100 equal parts, called links. The chain is made of wire, in links somewhat less than eight inches long. These are joined by two small, round or oval rings at each joint. The length of one of these longer links, with the two rings or short links taken together, make the distance known as a link.

The best surveyor's chains are made of steel wire, having the links brazed to prevent stretching by opening of the joints. Chains have every tenth link marked with a brass tag. The tags at the end of the tenth link from each end have one point; those at the twentieth links have two points; those at the thirtieth links have three points; those at the fortieth links have four points; while that in the centre or fiftieth link is rounded and has no point. Heavy chains of iron wire, with open joints, are of little value. It is very difficult to measure correctly with them, over rough ground, owing to their weight. They stretch rapidly by wear and by the opening of the joints. Chains fifty links long are used to measure over rough ground.

2. Chains Stretch by use, chiefly from wear in the joints. The best steel brazed chains, when in constant use on gritty ground, will stretch six inches or more in a year from this cause alone. They may be corrected in several ways. They may be shortened a limited amount by turning up the nuts or burrs which hold the handles in place. They may be shortened by taking out short links or rings. The better way is to distribute the correction evenly throughout the chain, by putting each link in a vise and striking lightly on the end with a hammer, shortening it in that way.

The links in the chain get bent by use. When many of them are bent, the chain becomes elastic and will elongate from one to two inches when pulled. Chains should be examined before using and the links straightened. They should be frequently compared with a standard, that their length may be known, and they should be kept near the true length.

3. Steel Tapes are made for the use of land surveyors. They are light, so that they may be readily leveled up in measuring over rough ground or on a slope. They do not stretch. There are no links to get kinked and thus cause a false measure. They are in every way more accurate and convenient than the chain. The best tapes for general use are made of the best quality of steel ribbon, polished and blued. from ⅛ to ⅜ of an inch wide, and No. 30 to 32 thick. The wider thinner tapes are nearly useless for field work.

Tapes are made of any length and graduated to suit the work for which they are designed. A tape 66 feet long, graduated to links, is best adapted to general use. Tapes 50 or 100 feet long, graduated to feet and hundredths, are better adapted for use in many cities. Tapes from 200 to 400 feet long or even longer are made for special uses. With them long lines may be rapidly measured with an accuracy fairly comparable with the best work of the coast survey.

Two precautions need to be observed with steel tapes. When in use they should be kept out at full length and never be doubled on themselves. If doubled they are easily kinked and broken. When done up, they should be wiped clean and wound on open reels to prevent rusting.

4. A light wire is a cheap and handy substitute for the chain or tape. It is necessary to find its length in some way and then for even lengths of the wire it is capable of as accurate work as the best tape.

5. Marking Pins are used with the chain and tape in measuring. They are usually made of heavy wire about 14 inches in length, with one end sharpened to stick in the ground and a ring turned on the other end for convenience in handling. Strips of cloth are tied in the rings so that they can be seen more readily. The marking pins used in the United States surveys have heavy points, for dropping plumb when chaining on slopes. It is convenient to use eleven pins in chaining. One of them is stuck at the starting point, the leader takes ten, and then there is always one to start from, when the tallies are kept in even tens.

6. Measuring or chaining. Two men are required for this, and a third man can be of great assistance when chaining on slopes and accurate work is to be done. The care and accuracy required will depend on the interests at stake. The surveyor would mistake his calling who should attempt to measure land worth fifty cents an acre with the same care he would use in measuring land worth fifty dollars or more per inch. In making measurements the following things are to be observed, with greater or less care and accuracy of detail, according to the importance of the work in hand.

1st. *Chains are not adapted to great accuracy in measurements.* For the best work *use a steel tape*, of which the exact length at a given temperature, and the rate of expansion are known. Tapes are usually made to be of standard length at a temperature of about 60°, F. The rate of expansion by heat varies with the kind and quality of steel in the tape. It approximates closely to .000007 for each change of a degree in temperature. Thus a tape which is 100 feet long at 60° F. will be 100.014 feet long at 80° F. For very exact measurements take note of the changes in temperature and correct for expansion and contraction. A thermometer is needed for this.

2d. *Measure in straight lines.* In ordinary work, pickets or rods set up along the line, in sufficient numbers for the chainmen to range by, will enable them to secure as great a degree of accuracy as is required in this respect.

3d. *Measure on level lines.* To do this the tape may be brought to a level line and the successive measures transferred to and from the ground by plumb lines. Use a plumb having a fine, strong line and a long, well balanced, sharp pointed bob. Measure down the slope. The rear chainman should hold the tape steadily and firmly at the mark, bracing his hand against his leg near the ground for a support. The leader brings his end of the tape level and in line. If necessary the follower directs him in doing this. He then applies the line to the point or mark on the tape, with the plumb-bob very nearly touching the ground. When he has the proper tension on the tape, and the plumb hangs perfectly still and true, he depresses the line enough to make a slight mark on the ground with the point of the bob, and sticks his marking pin beside it.

Another method of getting the measure on level lines is to drive short stakes or hubs along the line at every change in the slope of the surface. Small headed tacks are driven in the tops of these hubs. The distance between the tackheads is then measured along the surface and each measurement recorded. A level is then taken showing the difference in hights of these points. The length of the level line is found by calculation. Between every two hubs we have a right triangle in which we have the hypothenuse given by the tape, and the altitude given by the level, to find the base. By this method the error may be reduced below 1 in 25,000.

4th. *The tape must be drawn to the proper tension.* Tapes are usually tested under a tension of ten pounds when supported the entire length. They should be further tested to find the amount of additional strain required to overcome the sag, when the tape is not supported between the ends. This varies, in different tapes, from 6 to 12 pounds for a 100 foot tape. The total strain in the unsupported tape in measuring should be from 16 to 22 pounds. The exact amount is to be found for each tape by trial.

7. The following is the *general method of procedure in chaining*, modified as the circumstances require. We will speak of the chainmen as leader and follower. The leader takes his end of the chain or tape and ten marking pins, and steps briskly in the direction of the line to be measured. One pin is stuck at the starting point. Just before the leader has the chain drawn out at full length, the follower calls "halt," and places his end of the chain in the proper position at the starting point. The leader shakes out any kinks there may be in the chain, straightens and levels it in the line brings it to the proper tension and sticks his pin, calling "stuck" when he has done so. When the follower hears this signal, *and not before*, he pulls the marking pin and both move quickly forward, repeating the operation until the leader has stuck his last pin or has reached the end of the line. When the leader has stuck his last pin he calls "tally." The follower drops his end of the chain and brings forward the ten pins which he has, and gives them to the leader, who counts them to be sure none have been lost and then proceeds as before. The follower need not return for his end of the chain. The leader will draw it forward to him. When the end of the line is reached the leader holds his end of the chain at that point while the follower drops his end and comes forward and ascertains the distance, if any, between the last pin that was set and the end of the line.

When *chaining on slopes* which are so steep that the whole length of the chain cannot be leveled at once, the leader first draws it forward the whole length and in the line. He then drops the chain and all his marking pins and returns to a point where he can level a part of the chain and measures the distance, sticking one of the follower's marking pins to mark the point, the follower then drops his end of the chain, comes forward and taking the chain at the same point holds it to the mark while the leader measures a second section, and so on in succession till the end of the chain is reached, where the leader sticks one of his own marking pins. It will not often be necessary to take any note of the lengths of the parts of the chain measured. Observe only to measure to and from the same points in the chain, and take care that the count is not lost by getting the marking pins improperly mixed together.

The follower should see that his end of the chain is correctly and firmly held in its position when measuring. He should, when necessary, direct the leader in keeping the true line. The leader should see that his chain is drawn straight, level, in line, and to a uniform tension. To assist him in keeping the line he should observe objects in the range, both front and rear. He should see that his marking pins are set at the exact point. They should either be set plumb or slanting at right angles with the line, so that the measure may be taken from the point. When a plumb line is used, the latter is the better way. Chainmen should step quickly between points, and in chaining keep up with a man walking at an ordinary gait of three miles an hour. The follower must not stop the leader by a jerk on the chain. The leader must pull steadily when measuring. No jerking on the chain should be permitted.

If there is a difference in the chainmen the best man should take the lead. The chaining should always be uniform. In many surveys uniformity of measure is more important than great exactness.

Tests made by the author have led him to the conclusion, that, in common country surveying with the chain, nothing is gained by leveling the chain where the ground slopes less than five in a hundred. He finds that in field practice, under the ordinary conditions, more is lost by the sag of the chain than is saved by leveling. In one careful field test, six links was lost in a mile by leveling the chain. that being the net difference in favor of surface measurements for that distance.

In that class of work, measurements made along the surface may be corrected on the ground, as follows:

When ground slopes	4	in	100	add	.1	link	per chain
	6	"	"	"	.2	"	" "
	8	"	"	"	.3	"	" "
	9	"	"	"	.4	"	" "
	10	"	"	"	.5	"	" "
	11	"	"	"	.6	"	" "
	12	"	"	"	.7	"	" "

8. The student should practice in the field with the chain and steel tape until he is entirely familiar with their use, and can do accurate and rapid work. He should measure between fixed points over sloping or uneven ground, and repeat the measures until he can secure uniform results. He may be surprised at first to find that he does not measure twice alike. It is well to drive a small wooden stake at every tally or tenth chain, so that in case a marking pin is lost it will not be necessary to go back farther than to the first stake to remeasure. Beware of errors in counting the links less than a full chain. Count from the right end of the chain or tape. When the chain is used do not mistake the tag, as 60 instead of 40 or *vice versa*, or count odd links the wrong way from the tag. Beware of such mistakes as 64 instead of 56, or 48 instead of 52. The tape is generally numbered the whole length from 0 to 100. Nearly the same care is needed to avoid mistakes in reading as with the chain, especially to read the distance from the right end of the tape. Otherwise such mistakes as giving the distance 56 instead of 44 are very liable to occur.

III. Measures of Length and Area.

1. The measures in most general use among surveyors are based on the Gunter chain. The surveyor is however frequently required to express his measurements in units of the old linear and square measure.

Table of Chain Measure.

7.92 inches or .66 foot=1 link.
66 feet=100 links=1 chain.
80 chains=1 mile.

In country surveying the smaller measures are taken in links and parts of a link and distances less than a quarter of a link are not counted In the more exact work in cities, the foot and its subdivisions are in common use, and on account of the greater ease in making computations upon the decimal system, the plan of subdividing the foot decimally is adopted by many surveyors, and is growing in favor

2. Old Linear Measure:

12 inches = 1 foot.
3 feet = 1 yard.
16½ feet = 1 rod.
40 rods = 1 rood or furlong.
320 rods = 1 mile.

Measures for Area

3 Chain Measure:

100,000 square links, or 10 square chains } = 1 acre.
640 acres = 1 sq mile or section.
36 sections = 1 township

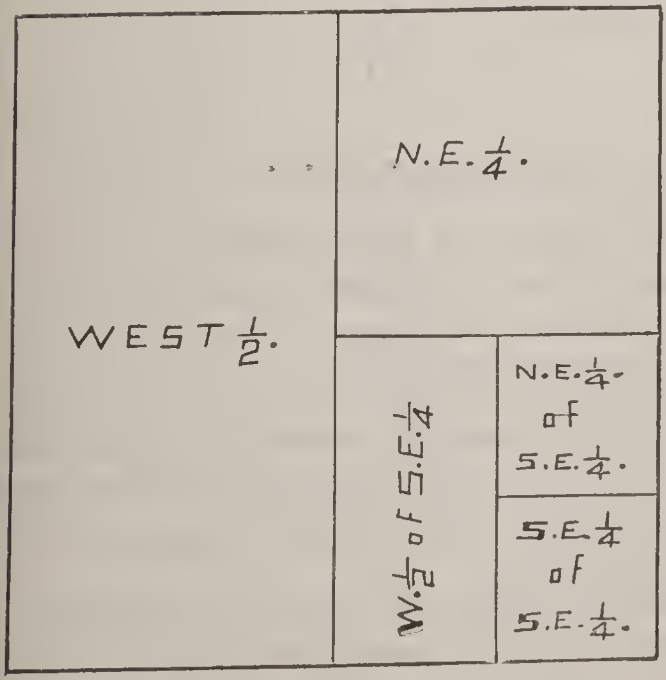


Fig. 1.

In the United States land system, the square mile is known as the Section. It is subdivided into aliquot parts, which are described according to their place in the section. The manner of naming these subdivisions of a section is indicated in Figure 1.

When, because of lakes, rivers, reservations, adjacence to township boundaries, or other causes, any of the parts of a section are increased or diminished from their normal amount, they are known and described as Fractional. That word is used to indicate that the tract to which it is applied is not one of the regular subdivisions of the section. When a fractional lot is small it is the custom of the United States land department to attach it to, and sell it with, an adjacent larger tract which gives the name to the description of the whole tract. The manner of describing fractional lots is indicated in Figure 2. It is also a custom to number the fractional lots on the plats and describe them by numbers, as for example, Lot No. 3 of Section 18. The latter method requires a reference to the plat to know the location of the lot, while the former method does not.

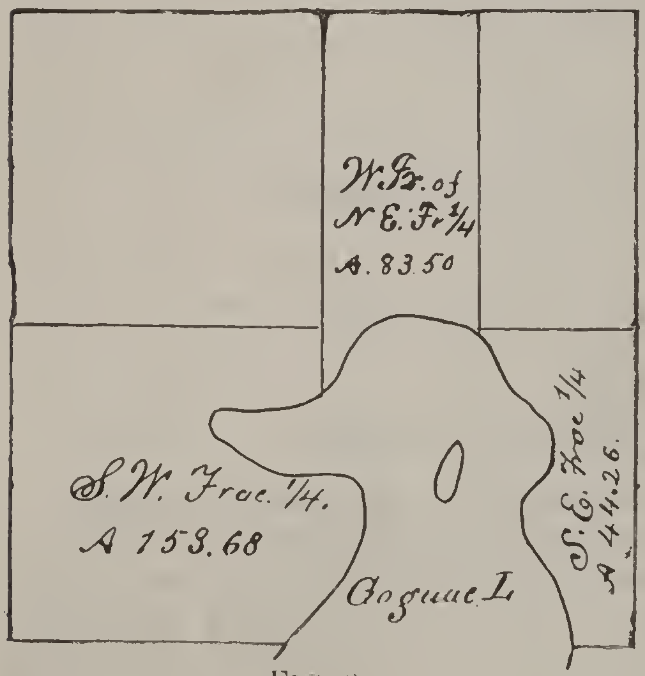


FIG. 2.

4. Old English Land Measure:

144 square inches = 1 square foot.
272¼ square feet = 1 square rod.
40 square rods = 1 rood.
160 square rods = 1 acre.

Square rods and feet are still in common use as subdivisions of the acre. The rood and furlong are very nearly if not quite obsolete in the United States.

5. Spanish Measures. — In Spanish colonies in America, the Spanish system of land measures was used in describing and measuring the land grants, and has continued in use down to the present time in a large extent of country. The principal unit of measure is the "vara," which seems to be a somewhat variable one. In a report of the 14th of November, 1851, from the surveyor-general of California, it is stated that all the grants, etc., of lots or lands in California, made either by the Spanish government or that of Mexico, refer to the "vara" of Mexico as the measure of length; that by common consent, in California, that measure is considered as exactly equivalent to *thirty-three* American inches. That officer enclosed a copy of a document he had obtained as being an extract of a treaty made by the Mexican government, from which it would seem that another length is given to the "vara;" and by J. H. Alexander's (of Baltimore) Dictionary of Weights and Measures, the Mexican vara is stated to be *equal to .92741 of the American yard.* The general land office, however, has sanctioned the recognition, in California, of the Mexican vara as being equivalent to 33 American inches.

Extract of a treaty made with the Mexican government, which accompanied a report dated November 14, 1851, from the U. S. surveyor-general of California, respecting the ratio of land measures between those employed under the Mexican government and those in use in the United States

[From the Mexican ordinance for land and sea.]

Article 20th of the agreement entered into between the minister plenipotentiary of the Mexican government and her agents in London, the 15th of September, 1837, with the holders of Mexican bonds.

20th. In compliance of what is ordered by the seventh article of the preceding law, and in order to carry into effect the stipulation in the preceding agreement in regard to the holders of bonds deferred, it is declared that the act of which mention is made in said agreement answers to 4840 English yards squared, equivalent to 5762.403 Mexican varas square; inasmuch that the "sitio de ganado moyer" contains 4338.464 acres, the Mexican vara having been found by exact measures equal to 837 French millimetres.

Reducing the ratio of 4840 square yards and 5762.403 square varas, the vara will be..............................	32.99312 inches
Reducing the 4338.464 acres.............	32.99311 "

Names of the Measures.	Figures of Measures.	Length of figures expressed in varas.	Breadth in varas.	Areas in square varas.	Areas in caballerias.
Sitio de ganado moyer	Square	5,000	5,000	25,000,000	41.023
Criadero de ganado moyer	do.	2,500	2,500	6,250,000	10.255
Sitio de ganado menor	do.	3,333⅓	3,333⅓	11,111,111	18.232
Criadero de ganado menor	do.	1,666⅔	1,666⅔	$2,777,777\frac{7}{9}$	4.558
Caballeria de tierra	Right angled parall'gram	1,104	552	609,408	1
Media caballeria	Square	552	552	304,704	½
Cuarto caballeria o Suerte de tierra	Right angled parall'gram	552	276	152,352	¼
Fenega de sembraduro de maiz	do.	276	184	50,784	1-12
Sala para casa	Square	50	50	2,500	0.004
Fundo legal para pueblos	do.	1,200	1,200	1,440,000	2.362

The Mexican vara is the unit of all the measures of length, the pattern and size of which are taken from the Castilian vara of the mark of Burgos, and is the legal vara used in the Mexican republic. Fifty Mexican varas make a measure which is called "cordel," which instrument is used in measuring lands.

The legal league contains 100 cordels, or 5,000 varas, which is found by multiplying by 100 the 50 varas contained in a cordel. The league is divided into two halves and four quarters, this being the only division made of it. Half a league contains 2,500 varas, and a quarter of a league 1,250 varas. Anciently, the Mexican league was divided into three miles, the mile into a thousand paces of Solomon, and one of these paces into five-thirds of a Mexican vara; consequently, the league had 3,000 paces of Solomon. This division is recognized in legal affairs but has been a very long time in disuse—the same as the pace of Solomon, which in those days was called vara, and was used for measuring lands. The "mark" was equivalent to two varas and seven-eighths—that is, eight marks containing twenty-three varas—and was used for measuring lands.

In Texas the surveys are made on the vara system. A 20-vara chain is used, the area calculated in varas, and when necessary reduced to acres. The field notes contain no system of measurement except varas. Nearly all the old leagues were laid off in rectangular form, and nearly all the subdivisions since have been by lines parallel with the original league lines.

The following table of comparisons gives the system of land measures in use in that state:

1 vara	= 33⅓ inches.			
1900.8 varas	= 1 mile.			
25,000,000 sq. varas	= 1 league	=	4428.4	acres.
1,000,000 " "	= 1 labor	=	177.136	"
5645.376 " "	=		1	"
1 " "	=		.000177	"

6. **Old French Measures** were used in laying off land in the French colonies, and still find a place in some parts of the country. The unit was the "arpent," of which there were different values, varying from three-fourths of an acre to an acre and a half. The "arpent d'ordonnance" or legal arpent equalled 1.262 acres, and contained 100 square perches of 22 "pieds du roi" on a side.

The old French linear measures were the old Paris foot called "pied du roi" and its sub-multiples—

12 points = 1 ligne.
12 ligne = 1 ponce.
12 ponce = 1 pied du roi = 12.789 inches.

6 pieds du roi = 1 toise,—interesting as being the unit employed in the survey of the great French meridian arc, on which the metre was founded.

Modern French measures are upon the Metric System.

7. Standard Measures.

The constitution of the United States says that congress shall have power to establish a system of weights and measures. It has, however, never done so. In 1832 the secretary of the treasury assumed the authority to adjust and regulate the weights and measures in use in the custom houses, and delegated the construction and adjustment of standards to Mr. Hassler, who was then superintendent of the coast survey.

The standard of length adopted was a yard, as measured between the 27th and 63rd inches of a scale made in London, by Troughton, and brought to this country in 1814. This scale is a copy of the old British Standard, known as the Bird Standard of 1760.

At a temperature of 59.62° F. it is equal in length to the Imperial Standard at 62° F. Although Congress never adopted that yard as a standard, it authorized the transmission of copies thereof to the several states. In many of the states these copies have been legally adopted as the standards. Other states have no legal standards. The Michigan standard is a brass yard, of exact length at a temperature of 58.40° F. It is both a line and an end measure. It is doubtful if these standards in the several states are kept in such a manner as to be reliable for purposes of comparison or if they are so kept, whether the officers in charge of them have the skill and the facilities required for making accurate comparisons. Standard rods are sold by dealers but they are more or less discrepant in length. Surveyors who desire to know the true length of their standard measures can send them to the Superintendent of the Coast and Geodetic Survey, at Washington, who will cause them to be compared and the government stamp placed on them, giving their exact length. The examination and test, for which a fee of fifty cents is charged, secures a sufficient degree of accuracy for ordinary purposes of the surveyor. Where an extra degree of accuracy is called for a higher fee is charged.

Although Congress has not adopted a general standard of measure, it has adopted a standard for the measurement of the public lands, which so far as the resurvey or subdivision of those lands is concerned is final. In section 2395 of the revised statutes of the United States, it is enacted that "all lines shall be measured with chains containing two perches of sixteen and one-half feet, each subdivided into twenty five equal links. In section 2396 it is enacted that "All the corners marked in the surveys returned by the Surveyor General shall be established as the proper corners" &c.; and that "the boundary lines actually run and marked in the surveys returned by the Surveyor General, shall be established as the proper boundary lines of the sections and subdivisions for which they were intended, and *the length of such lines as returned shall be held and considered as the true length thereof.*"

This enactment makes an actual standard of measure between every two adjacent corners of the government survey, which is the only legal standard for measures of that line. The surveyor, in resurveying or subdividing the public lands, has thus a standard laid down for him on every line previously run by the government deputy surveyor and has only to adjust his chain to that standard. This is practically done on the ground by apportioning any difference between the surveyor's measure of a given line and the length of the line as returned in the field notes pro rata between its different parts.

Example.—It is required to locate the half-quarter corner on the line described in the field notes as running, "West on corrected line between Sections 11 and 14 39.72, set qr. sec. post," etc.

Suppose the surveyor on measuring this line finds the distance between the two corners, as actually marked on the ground, to be by his chain 39.84 chains. Then his chain is too short and its legal length for that line is to its nominal length as 39.72 is to 39.84 and the distance to the half-quarter corner is by the new measure 19.92 chains.

IV. Instruments for Running Lines and Their Use.

1. The instruments most commonly used in running lines are *the picket*, *the compass* and *the transit.* There are various modifications of the compass and transit. The methods of running lines with these instruments will be treated of in connection with the description of them.

2. The Picket or Rod is the simplest device for ranging lines. It is simply a straight rod an inch or two in diameter and having a sharp point to stick in the ground. The author prefers to have them sharpened to a long slim point at the top also, and that the pickets shall be of such a length as to be the height of the eye when firmly planted in the ground. Where timber is plenty they may be cut from small straight saplings, or split from body wood as they are wanted, and left standing where they are used, as a guide to the chainmen.

3. *To range a line with pickets.* Set the first picket at the starting point and a second a short distance away in the direction in which the line is to run. Then go ahead and set picket after picket at such distances apart that at least three of them can be distinctly seen at the same time. Set the pickets plumb and align them by sighting over the sharpened points at the top. A plumb line will be of assistance in ranging lines over uneven ground. Set short stakes in the line at uniform distances apart. Then if the line was intended to strike a particular point and missed, it may be corrected by measuring the perpendicular distance from the line to the point, and then moving each intermediate stake its proportional part of that distance according to the distance it is from the starting point.

Example 1.—Commencing at the southwest corner of Mr. B.'s farm, I ran north, setting stakes on the trial line every ten chains. At 40.00 chains, my line intersected the north line of his farm 32 links east of his northwest corner. What correction must be made for each stake?

Solution.—The first stake being set at ¼ the distance between points must be corrected ¼ of 32 = 8 links, and as the trial line came out to the east of the corner, the stakes on that line must be moved to the west. The 2d stake being at ½ the distance between points must be moved west ½ of 32 = 16 links. Similarly the 3d stake must be moved west 24 links.

NOTE.—Sections of the United States survey are tracts of one mile square. Monuments are set at each corner called Section Corners. Others are placed midway between them on the section lines called quarter posts or quarter section corners. Some sections greater or less than these are called Fractional Sections.

Example 2—Commencing at a point 12 links west of the quarter post in the south side of Section 20, I ran north, setting stakes on the trial line every ten chains. At 80 chains my line intersected the north line of the section, 36 links west of the quarter post. What correction must be made to place the intermediate stakes in the true line between the quarter posts, known as the quarter line?

Answer.—Commencing with the first ten chain stake they must be set east, 15, 18, 21, 24, 27, 30, and 33 links respectively.

Example 3.—Commencing at a point 24 links west of the southwest corner of section 16, I ran a trial line north, setting stakes every ten chains. At 80.36 chains, the line intersected the north line of the section, 32 links east of the section corner. What is the correction to be made at each stake to place it in the true section line and at the equidistant points? Answer to be found by the student.

NOTE.—This solution requires corrections both for line and measure. It is a cardinal principle of land law that the original measurements and monuments which were made in the survey in accordance with which the land was sold are in law the true measures and monuments. All subsequent measures for the purpose of locating boundaries must be made to conform with the original measures.

Trial or random lines, as they are usually called, are often run one side of the true line, purposely to avoid obstacles, like fences and hedge rows. The surveyor, by a judicious selection of ground for the random line can often save a great deal of labor and time of the party, by avoiding obstacles which would otherwise have to be removed or offset around. Randoms from which the true line is to be found should be run with as great care as any line.

The student should practice running and measuring trial lines between points until familiar with the processes. He should run various randoms to find the line between the same points and see how they agree when corrected for true line.

4. *To range a true line between points that can not be seen from each other but can both be seen from some intermediate point, as a hill.*

Set up flags at the two points. Two persons then take pickets and station themselves, a short distance apart, at the intermediate position from which the flags can be seen. They face each other and each in turn aligns the other between himself and the flag toward which he faces, until the true line is reached, when the pickets are set in the line.

5. *To pass obstacles in the line.*

From the last two pickets preceding the obstacles, set two other pickets on a line parallel with the true line and at a sufficient distance to pass the obstacle. Prolong the parallel line far enough to set two pickets beyond the obstacle and then regain the original line by measuring back from these two pickets.

6. The methods of running lines with the compass and transit will be given in connection with the descriptions of these instruments.

CHAPTER II.

DESCRIPTION OF INSTRUMENTS.

1. The Surveyor's Compass. The essential features of the surveyor's compass are a *magnetic needle* for finding a meridian line, a *circle* graduated to half degrees known as the *limb*, for laying off angles from the meridian, and *sights* attached for use in prolonging lines on the ground.

When the limb and sights are on separate plates moveable upon each other around a common center through an arc of 15° or 20°, and a vernier is attached, the instrument is known as the Vernier Compass.

The use of the vernier is chiefly for setting the sights of the instrument so that they will be in the true north and south line when the magnetic needle points to zero on the limb. There is only a small portion of the earth's surface in which the needle points to the true north. A line passing through those places where the needle points truly north is called the *agonic* line or line of no variation. This line runs in a northerly course and is constantly changing its position. At all places outside the line of no variation, the needle points to the east or west of true north. This difference between the direction of the needle and the true meridian is spoken of as the *variation*, or, more correctly, the *declination* of the needle. The vernier is used to measure the angle between these two lines.

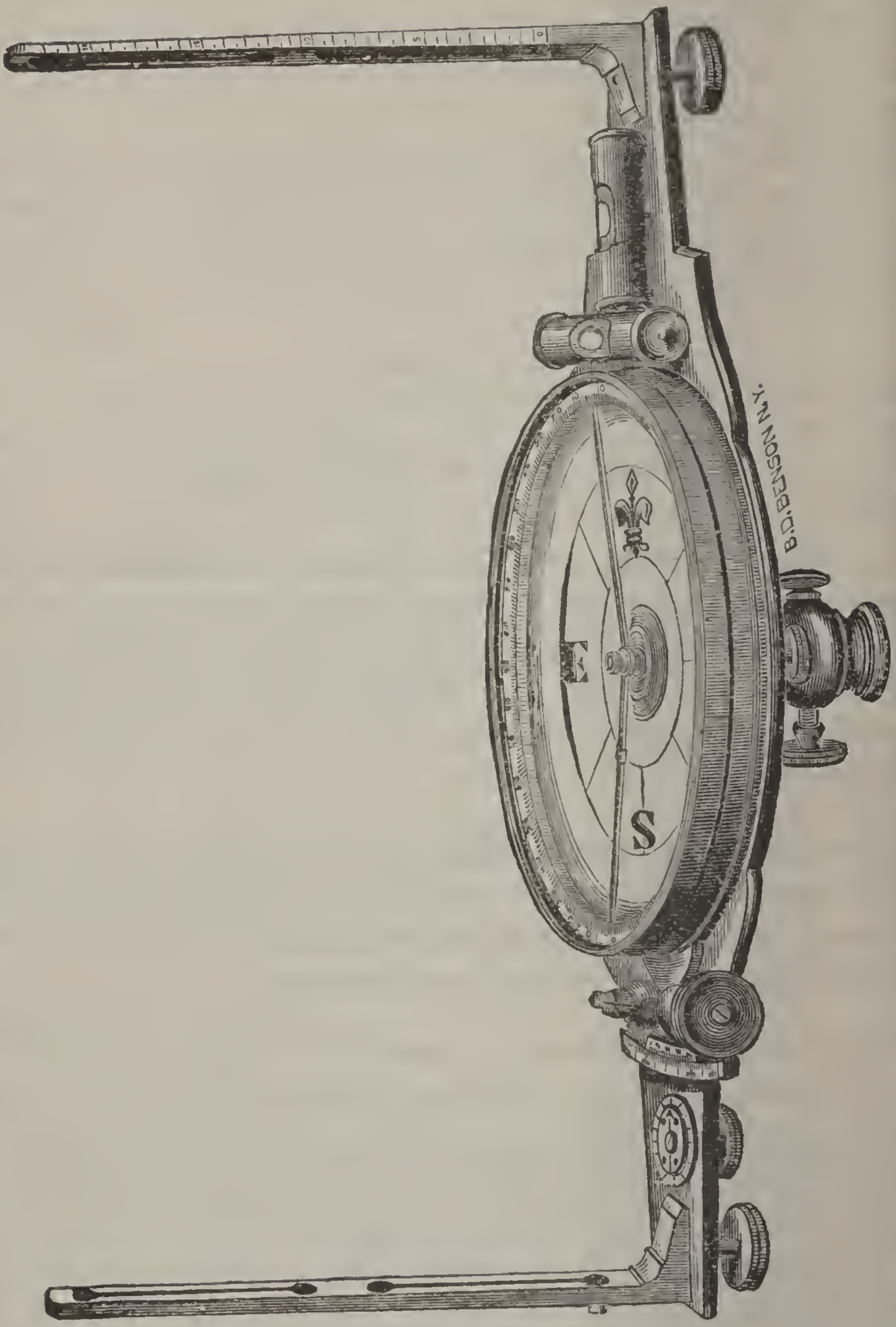


FIG. 3.—VERNIER COMPASS—6-INCH NEEDLE.

Sometimes there is added a divided circle or limb with verniers by which angles can be taken throughout the entire circle independently of the needle. The instrument in this form is called the railroad compass. The addition of leveling screws and a revolving telescope in place of the plain sights makes a surveyor's transit of it.

The Plain Compass consists of a circular box of brass, usually about six inches in diameter, resting upon an arm of the same metal about fourteen inches in length. At the extremities of the arm are vertical attachments through which are fine slits, terminated at intervals by circular apertures, which serve as *sights* in directing the instrument upon any point. At the centre of the box is a small vertical pin upon which is balanced a slender magnetized bar of steel, called the **Needle.**

Turning with a free horizontal motion, the pointed ends of the needle traverse the graduated circumference of the circle. The plane of the sights passes through the center of the circle and cuts the circumference in two points marked N and S, otherwise distinguished as the north and the south points of the instrument. From these points the graduation of the circle runs 90° in each direction to the points marked E and W.

A circle of plate-glass forms the cover of the box. Two small spirit levels are placed at right angles to each other upon the arm, to aid in rendering the plane of the instrument horizontal.

The compass is mounted upon a three-legged support called a **Tripod,** or upon a single staff called a **Jacob Staff,** with which it is so connected as to admit of being turned in any desired direction. In using the compass, the surveyor should keep the south end toward his person, and read the bearings from the north end of the needle. He will observe that the letters E and W on the face of the compass are reversed from their natural position, to correspond with the line of the sights, in order that the direction may be correctly read.

II. Adjustments of the Compass.

The Sights of the compass should be truly at right angles with the plate, so that when set up and leveled ready for use the line of sight will be in a vertical plane.

The needle should cut opposite degrees in any part of the circle, and should have its ends in line with the centre.

The levels should be parallel to the plane of the plate. To adjust the compass to these conditions begin with

The Levels.—First bring the bubbles into the centre, by the pressure of the hand on different parts of the plate, and then turn the compass half-way around; should the bubbles run to the edge of the tubes, it would indicate that those ends were the highest; lower them by tightening the screws immediately under, and loosening those under the lowest ends until, by estimation, the error is half removed; level the plate again, and repeat the first operation until the bubbles will remain in the centre, during an entire revolution of the compass.

The Sights may next be tested by observing through the slits a fine hair or thread, made exactly vertical by a plumb. Should the hair appear on one side of the slit, the sight must be adjusted by filing off its under surface on that side which seems the highest.

The Needle is adjusted in the following manner: Having the eye nearly in the same plane with the graduated rim of the compass-circle, with a small splinter of wood or a slender iron wire, bring one end of the needle in line with any prominent division of the circle, as the zero, or ninety degree mark, and notice if the other end corresponds with the degree on the opposite side; if it does, the needle is said to "cut" opposite degrees; if not, bend the centre-pin by applying the small brass wrench, furnished with the compass, about one-eighth of an inch below the point of the pin, until the ends of the needle are brought into line with the opposite degrees.

Then, holding the needle in the same position, turn the compass half-way around, and note whether it now cuts opposite degrees; if not, correct half the error by bending the needle, and the remainder by bending the centre pin.

The operation should be repeated until perfect reversion is secured in the first position.

This being obtained, it may be tried on another quarter of the circle; if any error is there manifested, the correction must be made in the centre-pin only, the needle being already straightened by the previous operation.

When again made to cut, it should be tried on the other quarters of the circle, and corrections made in the same manner until the error is entirely removed, and the needle will reverse in every point of the divided surface. If the needle has lost its polarity, and needs to be remagnetized, this is effected in the following manner:

The operator being provided with an ordinary permanent magnet, and holding it before him, should pass with a gentle pressure each end of the needle from centre to extremity over the magnetic pole, describing before each pass a circle of about six inches radius, to which the surface of the pole is tangent, drawing the needle towards him and taking care that the north and south ends are applied to the *opposite* poles of the magnet.

Should the needle be returned in a path near the magnetic pole, the current induced by the contact of the needle and magnet, in the pass just described, would be reversed, and thus the magnetic virtue almost entirely neutralized at each operation.

When the needle has been passed about twenty-five times in succession, in the manner just described, it may be considered as fully charged.

A fine brass wire is wound in two or three coils on the south end of the needle, and may be moved back or forth in order to counterpoise the varying weight of the north end.

The Centre-Pin.—This should occasionally be examined, and if much dulled, taken out with the brass wrench, already spoken of, or with a pair of pliers, and sharpened on a hard oil-stone—the operator placing it in the end of a small stem of wood, or a pin-vise, and delicately twirling it with the fingers as he moves it back and forth at an angle of about 30 degrees to the surface of the stone.

When the point is thus made so fine and sharp as to be invisible to the eye, it should be smoothed by rubbing it on the surface of a soft clean piece of leather.

Electricity.—A little caution is necessary in handling the compass that the glass covering be not excited by the friction of cloth, silk, or the hand, so as to attract the needle to its under surface.

When, however, the glass becomes electric, the fluid may be removed by breathing upon it, or touching different parts of its surface with the moistened finger.

III. To Run a Line with the Compass.

Set up the instrument at the point from which the line is to run ; level the plate ; turn the sights in the direction in which the line is to run, which may be ascertained by the needle or otherwise, as is most convenient. An assistant, known as the rodman or flagman, goes ahead with a sharp pointed rod or flag pole to such a distance as is convenient, and, guided by the signals of the compassman, sets his rod in line. When the ground is uneven, the rodman should select his point at the summit of rising ground, when possible to do so, in order to save unnecessary setting of the compass. He should always select the point most favorable for setting up the instrument, both to get a clear spot for the instrument and to get the best point for taking the next sight.

When setting his rod he should face the compass, holding the rod plumb and directly in front of him. He should move steadily in the direction indicated by the signals and not stick the rod down until he receives the signal to do so. After sticking it he should look for further signals, lest a change in its position might be required. After the rod is set the compassman should examine his instrument to see that it is in position, correcting it and resetting the rod when necessary. He then sets up a picket in line near his instrument, to be used for a back sight, and moves his compass forward in the line to the point marked by the rodman, sets it up in the line, with the sights ranging back to the backsight, and continues the line as far as desirable. The needle may or may not be used, according to circumstances. At the beginning of the line the direction will usually be obtained from the needle. If used afterwards on the same line, care should be taken to have it in proper condition and working freely. When being carried the needle should be raised off the pivot, otherwise the point of the pivot will become dulled and the needle will not traverse freely.

IV. To Pass Obstacles in the Line.

1. When the obstacle is a tree, and no great degree of accuracy is required, make a mark on the tree where the line strikes it and set the compass up on the opposite side of the tree, putting it in line by taking a backsight on the tree, and finding the direction of the line by the needle.

2. Make an offset far enough to pass the obstacle on a parallel line, the same as when running a picket line. When it is found that the line strikes a tree too large to be removed, set the rod in line near the tree, and then before moving the compass, set the picket for backsight at one side of it, a sufficient distance to pass the tree. Then move the compass ahead and set it up the same distance, and direction from the rod that the backsight picket was set from the compass. Get the direction of the line by ranging to the backsight. Prolong the parallel line beyond the obstacle and regain the true line in a similar manner. Other methods of passing obstacles in line will be given further on.

V. The Magnetic Needle.

1. The compass, because of its being so convenient for use has been for many years the principal instrument used

in Land Surveying. It is now very generally superseded by other instruments in surveys where accuracy is required. So far as the direction of lines is concerned, all compass surveying is based on the tendency of the magnetic needle to adjust itself to the magnetic meridian when free to do so, in other words to point north and south. It is however constantly changing its direction.

2. *Secular Change.* The line of no variation, as it is commonly called, otherwise known as the *agonic line* seems to have a periodical motion, back and forth, to the east and west, like the swinging of the pendulum. The length of the period is unknown but probably covers several centuries.

In the United States, so far back as known, its motion was to the eastward until the beginning of the present century, since which time it has been moving westward. In Michigan the secular change has been between 3′ and 4′ per year to the westward for the past sixty years. The agonic line is, in 1890, in the vicinity of Lansing.

3. *Diurnal Change.* The needle when undisturbed and free to move, swings back and forth each day through an arc varying from 5′ to 20′ or more in amount. In the northern hemisphere the north end of the needle moves westward from about 8 A. M. until about 1:30 P. M., then returning and reaching its former position at about 8 P. M. The amount of this motion is not uniform from day to day, being least on cloudy days ; nor from month to month, being least in winter. Nor is it the same in different localities. The effect of the diurnal variation is such that if a surveyor were to start a line in the morning and continue running it all day in the same direction, as shown by the needle, he would run a line like a letter S.

4. *Irregular Changes.* The needle is subject to sudden and violent changes in its direction, sometimes coincident with a thunderstorm or an Aurora Borealis,—often without any apparent cause. The writer has observed a

change of half a degree in less than ten seconds of time, for which there was no apparent or discoverable cause. It was supposed to have been occasioned by a magnetic storm.

5. *Local Attraction.* Iron ore in the earth, or iron or steel in the vicinity of the needle will deflect it from its normal direction. High mountains or running streams are also said to deflect the needle more or less. Pocket knives and steel watch chains are prolific sources of error as well as chains and axes.

6. *Difference in Instruments.* It is found by observation that different instruments do not indicate the same declination of the needle when observed at the same time and place. A difference of 15′ is not uncommon. Mr. Gurley made six needles taking great pains to have them as nearly alike as possible. He tried them in succession on the same centre-pin. Three of them gave the same results. The other three differed from 5′ to 10′.

7. *Things to be Observed in Running Compass Lines.* For these reasons it is practically impossible to run a true line and repeat it, relying on the needle alone for direction. Hence in all original surveys, made with the compass, the field notes of the survey should give the date, and state whether the directions of the lines are given according to the magnetic meridian. If not, state what the angle is between the magnetic meridian and the meridian adopted for the survey, or in other words state the declination of the needle, estimated or allowed for in the survey. The meridian adopted will usually be as nearly coincident with the true meridian as known. Backsights should be used whenever the line is prolonged beyond a single sight, both to secure accuracy in the line, and as a check against local disturbances of the needle. They also save time, as a compass can be pointed to a backsight in much less time than it takes a good needle to settle.

8. *Marking Lines.* It is a cardinal principle of common law, as well as the statute law of the United States with reference to the public lands, that the original surveys as marked on the ground, in accordance with which the land was sold, are conclusive as to the corners and boundary lines. When the land is once sold, no change can be made in the marked boundaries without disturbing the vested rights of the owners. Resurveys are made to find the location on the ground of the original survey. The compass is a useful assistant in pointing out where to look for the more certain evidences, such as marked trees, stakes or corner stones, and, in the absence of anything better, may be used to determine the location of the line. A marked tree of the original survey is, however, better evidence of the location of the line than any line afterward run by a compass. It is possible that the line might be exactly retraced by the compass, but it could not be known to be so without the aid of other evidence. Hence the marks on the ground which define boundary lines cannot be made and kept too plain and permanent. The field notes and records which describe these marks should be full, clear and concise.

VI. True Meridians and how to Find them with the Compass.

In a country that has had the first surveys made and boundary lines marked, and subsequent surveys are based on these lines, it is very rarely of any consequence to the surveyor to know where the true meridian is. The original boundary lines are unchangeable, and it is no help to the surveyor to know where the true meridian is unless he also knows that the original surveys were in conformity with it, and that the causes of error heretofore mentioned can be eliminated. That is very rarely the case. His main concern is to know where the lines were and not where they ought to have been. The writer in nearly a quarter century of active practice as a surveyor has never had occasion, except as a matter of curiosity, to know where the true meridian was. In making the first surveys of a country with a compass, it is well to know the position of the true meridian, in order that the lines may be run as nearly in conformity with it as the limitations of the instrument will permit, or that the divergence may be known. Subsequently, a knowledge of the changes in the declination of the needle is all that serves any practical purpose. This can be learned by observations on any line between two permanent points.

To find a true north and south line by means of the north star.

The north star appears to describe a small circle about the true north point or pole as a center. The radius of this circle is called the **Polar Distance** of the star. This polar distance is not a constant quantity, but becomes about ⅓ of a minute of arc less every year. On the first of January, 1890 it was about 1° 16′ 41″.

When in its revolution, the star is farthest from the meridian, it is said to be at its **greatest eastern or western elongation.**

The times of the elongations as given by a correct clock, for latitude from 38° N to 60° N and for the year 1890, are approximately as shown in the following tables:

EASTERN ELONGATIONS.

Day.	Apr.	May.	June.	July.	Aug.	Sept.
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6 37 A.M.	4 39 A.M.	2 37 A.M.	12 39 A.M.	10 37 P.M.	8 36 P.M.
7	6 14 "	4 16 "	2 14 "	12 16 "	10 14 "	8 12 "
13	5 50 "	3 52 "	1 50 "	11 52 P.M.	9 50 "	7 48 "
19	5 26 "	3 28 "	1 26 "	11 29 "	9 27 "	7 25 "
25	5 03 "	3 05 "	1 03 "	11 05 "	9 03 "	7 01 "

WESTERN ELONGATIONS.

Day.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6 27 A.M.	4 25 A.M.	2 28 A.M.	12 26 A.M.	10 24 P.M.	8 30 P.M.
7	6 04 "	4 02 "	2 04 "	12 02 "	10 00 "	8 06 "
13	5 40 "	3 38 "	1 40 "	11 39 P.M.	9 36 "	7 43 "
19	5 17 "	3 15 "	1 17 "	11 15 "	9 13 "	7 19 "
25	4 53 "	2 51 "	12 53 "	10 51 "	8 49 "	6 55 "

To find the meridian of a place by means of an elongation of the north star requires the arrangement of the following preliminaries.

Set two posts firmly in the ground about three feet apart east and west, and saw them off to a level about three feet from the ground.

Lay upon the posts a plank 3 or 4 feet long and 6 or 8 inches wide, planed smooth on the upper surface, and nail or pin it securely to the supports, forming a sort of table.

To the north of the table at a distance of 10 or 12 feet set in the ground a stiff pole 12 or 15 feet high, having a cross bar nailed to its top, in an east and west direction, from which to suspend a plumb-line nearly reaching the ground, and having a bob weighing 1 or 2 pounds, which may be caused to hang in a pail of water, to insure steadiness.

Provide also a block or piece of plank 8 or 10 inches long, and smooth on the under side. Let one of the compass sights be fastened at right angles with the upper surface of the block and even with the side which is to be toward the south.

Everything being in readiness, the observer, a few minutes before the time of an elongation as given in the above Table, should be at his post and begin moving the block, even with the south edge of the table, keeping the plumb-line and star, as seen through the vertical slit, constantly in range with each other. A light will generally be needed near the plumb-line, to render it visible. As the star approaches its elongation, it will appear to move nearly vertical for several minutes, so as to be seen without moving the sight. When it is certain that the star has reached its elongation, confine the block carefully, by sticking a few tacks along its edges. Project the vertical slit to the ground by means of a plumb-line and mark the point by setting a substantial stake with its top a little below the surface of the ground.

Being still careful not to move the block, let an assistant take one of the iron-pointed rods, or a stake, with a light, and go a hundred feet or more toward the star, and having found the point as directed by the observer, in range with the plumb-line as seen through the slit, let him mark it by driving a stake.

Having now two stakes in range of the elongation, the remainder of the operation may be deferred till morning.

To find the angle which the line as above determined makes with the meridian of the point of observation, requires a trigonometrical computation.

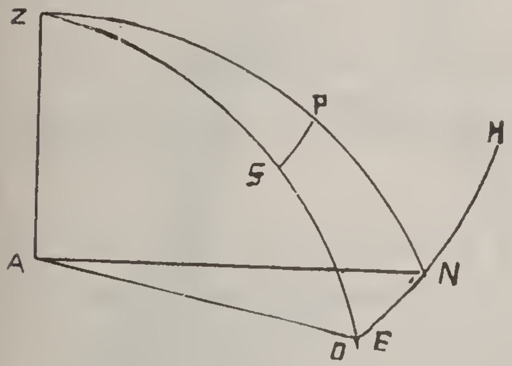


FIG. 4.

Let A be the point of observation, Z, the zenith of that point, HO, an arc of the northern horizon, N, the north point of that arc, S, the north star at its eastern elongation, PS, the polar distance of the star, AN, the meridian of the point of observation, and AE, the line of the two stakes.

The angle sought is $NAE =$ angle $PZS =$ arc NE.

Now, in the spherical triangle PZS, PZ is the co-latitude of the point A, which must be known. Solving this triangle, we have $\sin Z = \frac{\sin PS}{\sin ZP}$, or $\sin Z = \frac{\sin \text{polar dist.}}{\cos \text{lat.}}$

From this, the angle Z becomes known, and, accordingly, it may be formed on the west side of the line AE, and thus the direction of the meridian AN determined.

On AN, thus found, let a substantial stake be set a hundred yards or more from A, and we have a permanent meridian with which we may compare the magnetic meridian at any time, and thus determine the declination of the needle.

The declination of the needle is the angle which the magnetic meridian makes with the astronomical meridian.

For the purpose, simply, of finding the declination of the needle, it is sufficient to lay out on the ground the line of direction of the star at one of its elongations, and then, knowing the bearing of this line as shown by the needle, and the corresponding azimuth of the star, the declination of the needle is readily computed.

Thus, let $\pm a$ = azimuth, $\pm b$ = bearing, and $\pm d$ = declination, accordingly as they are *east* or *west*.

Then $\pm d = \pm a - (\pm b)$.

RULE.—*Subtract the bearing from the azimuth.*

In applying the Rule, due regard is to be had to the algebraic signs.

A near approximation to a true meridian may be had by observing the pole star while it is in the same vertical plane with the star *Delta*, in the constellation Cassiopeia. When both are behind the plumb-line together, they are very nearly in the true meridian. When Delta Cassiopeia passes the meridian above the pole, it is too high in the heavens to serve this purpose. It passes the meridian below the pole at midnight April 10th, and may be used for two months before and after that date. Six months later the star *Zeta*, the last but one in the tail of the Great Bear, takes its place. Fig. 5 shows the relative position of these stars and the pole.

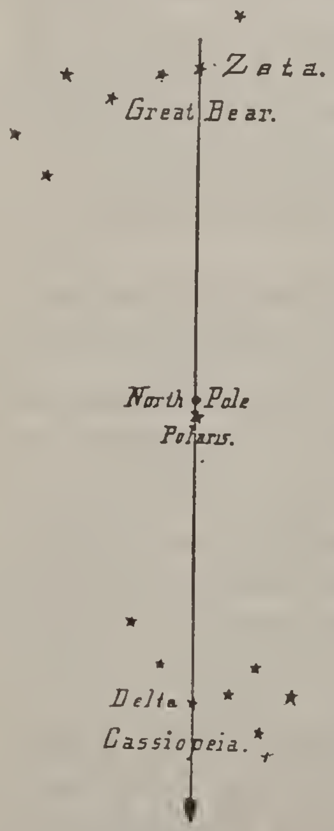


FIG. 5.

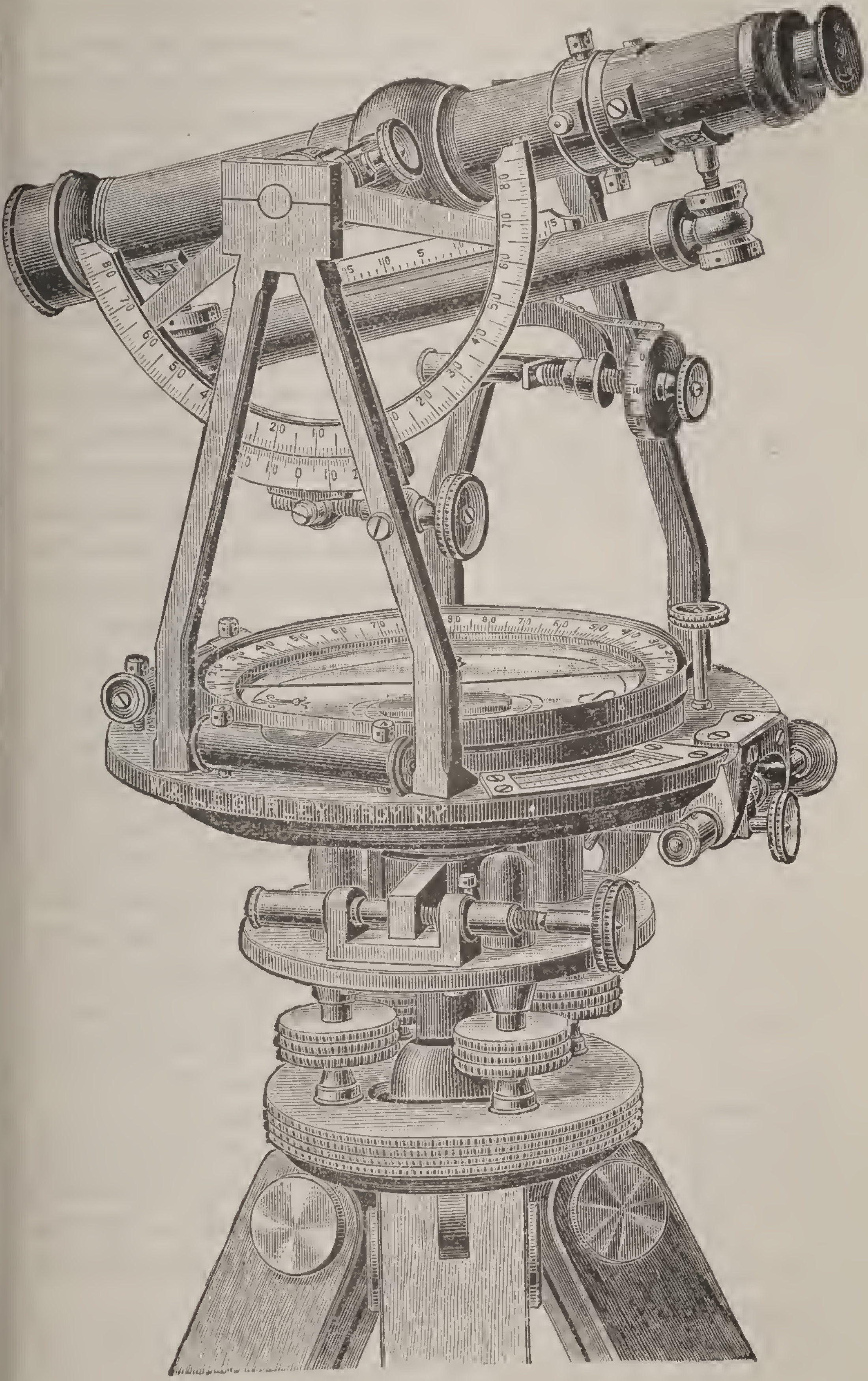


FIG. 6.

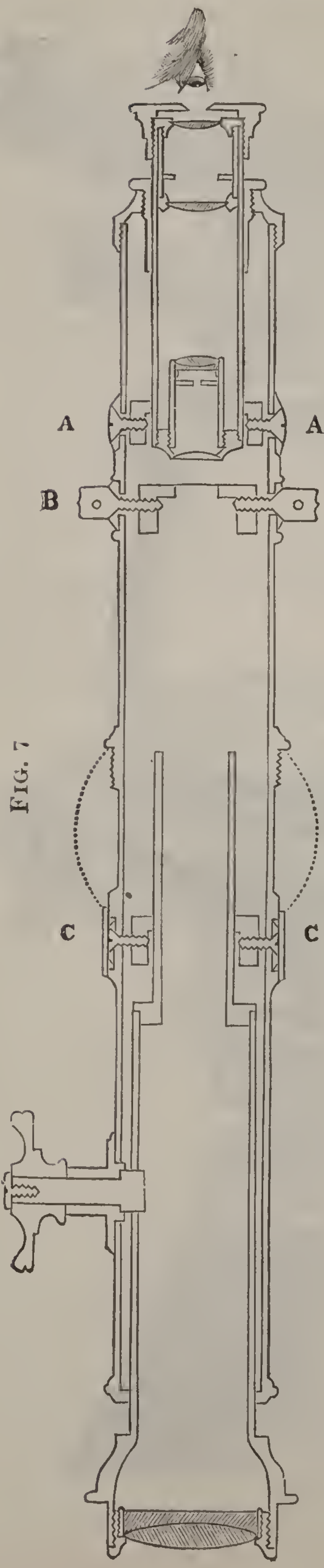


FIG. 7

VII. The essential parts of the Transit, as shown in the cut, are the *telescope* with its axis and two supports, the *circular plates* with their attachments, the *sockets* upon which the plates revolve, the *leveling head*, and the *tripod* on which the whole instrument stands.

The *telescope* is from ten to eleven inches long, firmly secured to an axis having its bearings nicely fitted in the standards, and thus enabling the telescope to be moved in either direction, or turned completely around if desired.

The different parts of the telescope are shown in Fig. 7.

The object-glass, composed of two lenses, so as to show objects without color or distortion, is placed at the end of a slide having two bearings, one at the end of the outer tube, the other in the ring *CC*, suspended within the tube by four screws, only two of which are shown in the cut.

The object-glass is carried out or in by a pinion working in a rack attached to the slide, and thus adjusted to objects either near or remote as desired.

The eye-piece is made up of four plano convex lenses, which, beginning at the eye-end, are called respectively the eye, the field, the amplifying, and the object lenses, the whole forming a compound microscope having its focus in the plane of the cross-wire ring *BB*.

The eye-piece is brought to its proper focus usually by turning its milled end, the spiral movement within carrying the eye-tube out or in as desired; sometimes a pinion, like that which focuses the object-glass, is employed for the same purpose.

1. **The Cross-Wires,** (Fig. 8), are two fibres of spider-web or very fine platinum wire, cemented into the cuts on the surface of a metal ring, at right angles to each other, so as to divide the open space in the center into quadrants.

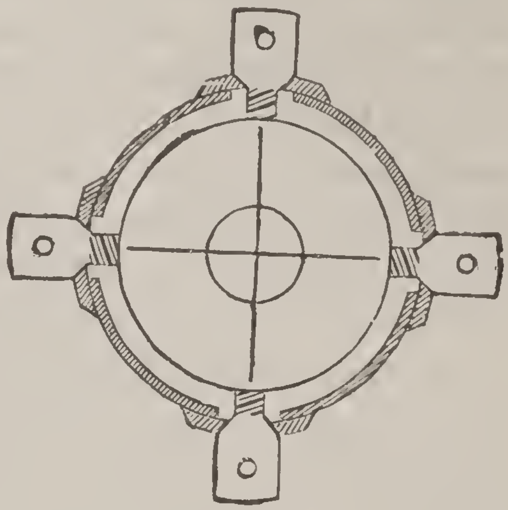


FIG. 8

2. **Optical Axis.**—The intersection of the wires forms a very minute point, which, when they are adjusted, determines the optical axis of the telescope, and enables the surveyor to fix it upon an object with the greatest precision.

The imaginary line passing through the optical axis of the telescope, is termed the **Line of Collimation**, and the operation of bringing the intersection of the wires into the optical axis is called **Adjusting the Line of Collimation.** This will be hereafter described.

3. **The Vertical Circle** firmly secured to the axis of the telescope is 4½ inches diameter, plated with silver, divided to half degrees, and with its vernier enables the surveyor to obtain vertical angles to single minutes.

4. **The Level on Telescope** consists of a brass tube about 6½ inches long, each end of which is held between two capstan-nuts connected with a screw or stem attached to the under side of the telescope tube.

5. The Magnetic Needle is four to five inches long in the different sizes of transits, its brass cup having inserted in it a little socket or center of hardened steel, perfectly polished, and this resting upon the hardened and polished point of the center-pin, allows the needle to play freely in a horizontal direction, and thus take its direction in the magnetic meridian. The needle has its north end designated by a scallop or other mark, and on its south end has a coil of fine brass wire, easily moved, so as to bring both ends of the needle to the same level. The needle is lifted from the pin by a concealed spring underneath the upper plate, actuated by a screw shown above, thus raising the button so as to check the vibrations of the needle, or bring it up against the glass when not in use, to avoid the unnecessary wear of the pivot.

6. The Lower Plate, called the **Limb**, is divided on its upper surface—usually into degrees and half-degrees—and figured in two rows, viz., from 0 to 360, and from 0 to 90 each way; sometimes but a single series is used, and then the figures run from 0 to 360 or from 0 to 180 on each side.

7. The Verniers, of which there are two placed opposite each other against the limb, are auxiliary scales used in measuring smaller portions of the limb than are shown by its graduations. Thirty divisions on the vernier correspond precisely with twenty-nine half degrees on the limb. Hence one division on the limb exceeds one division on the vernier by one-thirtieth of one-half of a a degree, that is, by one minute.

Accordingly, the number of any division of the vernier, *on the side toward which the vernier is moved*, which coincides with a division of the limb is the number of minutes of arc intercepted by the zero of the vernier and the last preceding division of the limb.

Thus, by the device of a vernier we are enabled to measure angles to within one minute, although the limb of the transit is graduated only to half-degrees.

Adjustments.—The principal adjustments of the Transit are—

(1) *The Levels.*
(2) *The Line of Collimation.*
(3) *The Standards.*

8. To Adjust the Levels.—Set up the instrument upon its tripod as nearly level as may be, and having unclamped the plates, bring the two levels above and on a line with the two pairs of leveling screws; then with the thumb and first finger of each hand clasp the heads of two opposite, and, turning both thumbs in or out, as may be needed, bring the bubble of the level directly over the screws, exactly to the centre of the opening. Without moving the instrument proceed in the same manner to bring the other bubble to its centre; after doing this, the level first corrected may be thrown a little out; bring it in again; and when both are in place, turn the instrument half-way around; if the bubbles both come to the centre, they would need no correction, but if not, with the adjusting pin turn the small screws at the end of the levels until the bubbles are moved over half the error; then bring the bubbles again into the centre by the leveling screws, and repeat the operation until the bubbles will remain in the center during a complete revolution of the instrument, and the adjustment will be correct.

9. To Adjust the Line of Collimation.—To make this adjustment—which is, in other words, to bring the intersection of the wires into the optical axis of the telescope, so that the instrument, when placed in the middle of a straight line, will, by the revolution of the telescope, cut its extremities—proceed as follows:

Set the instrument firmly on the ground and level it carefully; and then having brought the wires into the focus of the eye-piece, adjust the object-glass on some well-defined point, as the edge of a chimney or other object, at a distance of from two hundred to five hundred feet; determine if the vertical wire is plumb, by clamping the instrument firmly and applying the wire to the vertical edge of a building, or observing if it will move parallel to a point taken a little to one side; should any deviation be manifested, loosen the cross-wire screws, and by the pressure of hand on the head outside the tube, move the ring around until the error is corrected.

The wires being thus made respectively horizontal and vertical, fix their point of intersection on the object selected; clamp the instrument to the spindle, and having revolved the telescope, find or place some good object in the opposite direction, and at about the same distance from the instrument as the first object assumed.

Great care should always be taken in turning the telescope, that the position of the instrument upon the spindle is not in the slightest degree disturbed.

Now, having found or placed an object which the vertical wire bisects, unclamp the instrument, turn it half way around, and direct the telescope to the first object selected; having bisected this with the wires, again clamp the instrument, revolve the telescope, and note if the vertical wire bisects the second object observed.

Should this happen, it will indicate that the wires are in adjustment, and the points bisected are with that of the centre of the instrument, in the same straight line.

If not, however, the space which separate the wires from the second point observed, will be double the deviation of that point from a true straight line, which may be conceived as drawn through the first point and the centre of the instrument, since the error is the result of

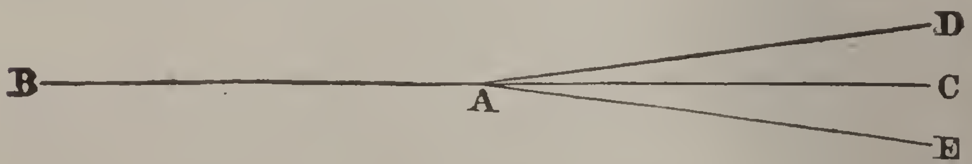


Fig. 9.

two observations, made with the wires when they are out of the optical axis of the telescope.

For, as in the diagram, let *A* represent the centre of the instrument, and *BC* the imaginary straight line, upon the extremities of which the line of collimation is to be adjusted.

B represents the object first selected, and *D* the point which the wires bisected, when the telescope was made to revolve.

When the instrument is turned half around, and the telescope again directed to *B*, and once more revolved, the wires will bisect an object, *E*, situated as far to one side of the true line as the point *D* is on the other side.

The space, *DE*, is therefore the sum of two deviations of the wires from a true straight line, and the error is made very apparent.

In order to correct it, use the two capstan head screws on the sides of the telescope, these being the ones which affect the position of the vertical wire.

Remember that the eye-piece inverts the position of the wires, and therefore that in loosening one of the screws and tightening the other on the opposite side, the operator must proceed as if to increase the error observed. Having in this manner moved back the vertical wire until, by estimation, one-quarter of the space, *DE*, has been passed over, return the instrument to the point *B*, revolve the telescope, and if the correction has been carefully made, the wires will now bisect a point, *C* situated midway between *D* and *E*, and in the prolongation of the imaginary line, passing through the point *B* and the centre of the instrument.

To ascertain if such is the case, turn the instrument half around, fix the telescope upon *B*, clamp to the spindle, and again revolve the telescope toward *C*. If the wires again bisect it, it will prove that they are in adjustment, and that the points, *B*, *A*, *C*, all lie in the same straight line.

Should the vertical wire strike to one side of *C*, the error must be corrected precisely as above described, until it is entirely removed.

10. To Adjust the Standards.—In order that the wires may trace a vertical line as the telescope is moved up or down, it is necessary that both the standards of the telescope should be of precisely the same height.

To ascertain this and make the correction, if needed, proceed as follows:

Having the line of collimation previously adjusted, set up the instrument in a position where points of observation, such as the point and base of a lofty spire, can be selected, giving a long range in a vertical direction.

Level the instrument, fix the wires on the top of the object and clamp to the spindle; then bring the telescope down, until the wires bisect some good point, either found or marked at the base; turn the instrument half around, fix the wires on the lower point, clamp to the spindle, and raise the telescope to the highest object.

If the wires bisect it, the vertical adjustment is effected; if they are thrown to either side this would prove that the standard opposite that side was the highest, the apparent error being double that actually due to this cause.

To correct it, one of the bearings of the axis is made movable, so that by turning a screw underneath the sliding-piece, as well as the screws which hold on the cap of the standard, the adjustment is made with the utmost precision.

11. To Adjust the Vertical Circle.—Having the instrument firmly set up and carefully leveled, bring into line the zeros of the circle and vernier, and with the telescope find or place some well-defined point or line, from one hundred to five hundred feet distant, which is cut by the horizontal wire.

Turn the instrument half way around, revolve the telescope, and fixing the wire upon the same point as before, note if the zeros are again in line.

If not, loosen the capstan-head screws which fasten the vernier, and move the zero of the vernier over half the error; bring the zeros again into coincidence, and proceed precisely as at first, until the error is entirely corrected when the adjustment will be complete.

It is not always convenient to make this adjustment so as entirely to eliminate the index error. In this case, the error should be noted and the proper correction made in measuring a vertical angle.

To find the index error we have the following

RULE, — *Level the instrument and direct the telescope upon some well defined spot. Note the reading of the circle.*

Reverse the telescope and turn the vernier plate 180°. Direct the telescope upon the point and note the reading of the circle.

Subtract the first reading from the second, and divide the remainder by 2.

12. To Run a Line with the Transit.

1. *Setting up the Transit.*—Set the instrument up over the starting point, centreing it by means of the plumb line. While doing so, place it as nearly level as possible, leaving as little as may be, to be done in leveling up the plates by the leveling screws. There is opportunity for the display of a good deal of skill in setting up a transit over a point, quickly, and in proper position. For hill sides, a tripod having adjustable legs, called an extension tripod, is a great convenience. When the legs are not adjustable, set one leg of the tripod down hill and two legs on the upper side of the line. It is important that the instrument should stand firmly on the ground. Some soils are so yielding that it is impossible for the man at the transit to change the weight of his body from one foot to the other, without getting the transit out of position. One remedy is, to not change the centre of gravity of the person, after the transit is in position, until the observation is taken. Another is, to drive stout stakes into the ground, to set the transit legs on. Another is to make a bridge of planks or poles for the transitman to stand on, so as to carry the bearing of his weight

as far as possible away from the instrument. Sometimes the aid of an assistant will need to be called in, so that the transitman need not move around the transit before sighting.

When the transit is set up firmly in place, loosen the lower clamp and turn the instrument on the spindle till the level tubes are each parallel to an opposite pair of the leveling screws.

Turn the parallel pair of screws both inward or outward until the bubble comes to the centre. Each level being treated in this way, the limb of the instrument is caused to be parallel to the horizon.

Unclamp the vernier plate and set the zero of the vernier to coincide with the zero of the limb. Clamp the plates in this adjustment. The leveling screws should be kept bearing equally against the plates.

Do not turn the leveling screws up too tightly. It tends to spring the plate and causes unnecessary wear of the screw threads. Simply bring them to a firm bearing.

2. **Assistants and their Duties. The Rodman.**—A rodman, often called a flagman, using a rod called a color pole, and one or more axemen are needed. The color pole is often carried by the head chainman.

The man who carries the color pole, selects places to set up the instrument, and gets the transit points, is a very important factor in running a line. Nearly as much depends upon him for accuracy and speed as upon the transitman. He should be thoroughly drilled in his duty. He should hold the color pole perpendicularly, clasping it lightly between the thumb and forefinger of both hands, and the hands held above the head. The point should be lifted a little above the ground or hub. He must keep it squarely in front of him, and move his body the same distance that he does the color pole, when getting a point. As soon as the "All Right" signal is given, let go of the pole. It will fall vertically and make the point plain. If the pole is held to one side it is apt to have some uneven pressure given which will make it incline more or less.

A man cannot stand awkwardly and hold a color pole accurately. He must be able to judge of the stability of the ground to set up on. He must select places where the longest sights can be had, and in running through timbered country he should select transit points where the ground begins to ascend or descend. If any deep ravines or gullies are to be crossed, he must select points to get across them with the least possible chopping, and without having to set up on a steep hillside. He should not select a point on the shaded side of a big tree, but where the most light comes in through the leaves. A small limb cut out of the way will often let in a wonderful amount of light, or a white handkerchief spread over the chest, or a light colored straw hat held in the right position, sometimes reflects enough light to show clearly objects which before were indistinct. In fact, he must be a man of gumption and equal to any emergency. But he cannot do good work unless he is provided with a good color pole.

3. **The Color Pole.**—It should be made from a good piece of straight grained timber. White or Norway pine is good. It is fitted at the bottom with a shoe made from gas pipe, with a steel point welded on, and finished by turning down in a machine. The shoe ought to be of sufficient weight to bring the centre of gravity within two feet of the bottom, so that it will have a greater tendency to hang vertically when held up.

The sizes of color poles vary according to the places where they are used. If one is dressed down with planes to a six or eight-sided stick, tapering slightly toward the top, it will keep straight much longer than a stick turned in a lathe. The shoe should be made of sufficient size to receive the stick, without dressing it down to go into the socket. When finished it should be thoroughly tested, to see if the point of the shoe has been set in line with the centre of the pole. Suspend a plumb bob from a point in a ceiling, and mark on the floor the point carried down. Fasten a string in the centre of the top of the color pole and suspend it from the same point. If the point of the shoe covers the mark on the floor it is all right. Prying with a color pole should be prohibited.

4. **Axeman.** The axemen provide pickets for backsights, clear the line of brush and trees, and drive stakes and hubs for transit points. They should keep close to the line, so that in clearing through woods they do no unnecessary cutting. A clear line two feet wide through the brush is generally all that is needed. Hubs for transit points should be cut square on top and driven firmly into the earth, nearly level with the surface.

5. **Projecting the Line.** The flagman selects the point and, facing the transitman, holds the color pole directly in front of him, and guided by the transitman, places it in line and makes a mark in the ground. The axeman then drives a hub at the place and the rodman again holds up his pole and finds the exact point where the line crosses the hub and a tack is driven. For most surveys a line within the limits of a tack head is considered close enough.

The hub for transit point should not be driven near a large tree, in soft ground, as a breeze will cause the tree to sway so as to move the earth for many feet around it. For a backsight it is a good plan to set up a picket, pointed at the top, so that the point shall coincide with the hole in the eye piece of the telescope. Or it may be set far enough from the transit so that the point may be aligned by the instrument. The picket should be set so firmly in the ground that it will retain its place as long as it is needed. A root will sometimes so press against a picket as to throw the point out of line after it is set. It may be necessary to drive the picket with the axe and then insert a wooden point in a cleft in the top of the picket. Several such points set up in the line before the transit is moved help to secure accuracy in the line.

When the backsight is set, the transit is taken forward and set up over the tack point in the hub. The lower clamp is loosened, the telescope reversed and sighted to the backsight and the instrument clamped in that position. The telescope is then righted and the line continued to the next tack point. When two or more backsight points are visible at once, any error in the adjustment of the instrument or in running the line will be readily detected, and the proper correction may be applied.

If the line of collimation is out of adjustment and it is not desirable to stop and adjust it, the lower clamp is loosened, the instrument turned half way round and clamped on the backsight. The telescope is then reversed on its axis and a second point marked beside the first. (See Fig. 9.) A tack is then driven in the true line, which is midway between the two. If the instrument is much out of adjustment it may be necessary to drive three hubs for this purpose. The transit is then set up in the true line, and the line continued as far as necessary, in the same manner. Obstacles in line are passed by offsets to parallel lines, in the same manner as when running lines by pickets or compass. Other methods will be considered in connection with Angular Measurements.

Examples, to be solved by the student in the field:

1. Run a line half a mile and mark four or more points along the line with hubs and tacks.

2. Retrace it in the opposite direction, testing the points to see how they agree.

3. Run a line over a hill, marking points at the top and bottom and along the slopes.

4. Retrace it in the opposite direction, testing the points.

5. Run a line across a valley, marking points, and retrace it in the opposite direction, testing the points.

CHAPTER III.

Description of Instruments, Continued.

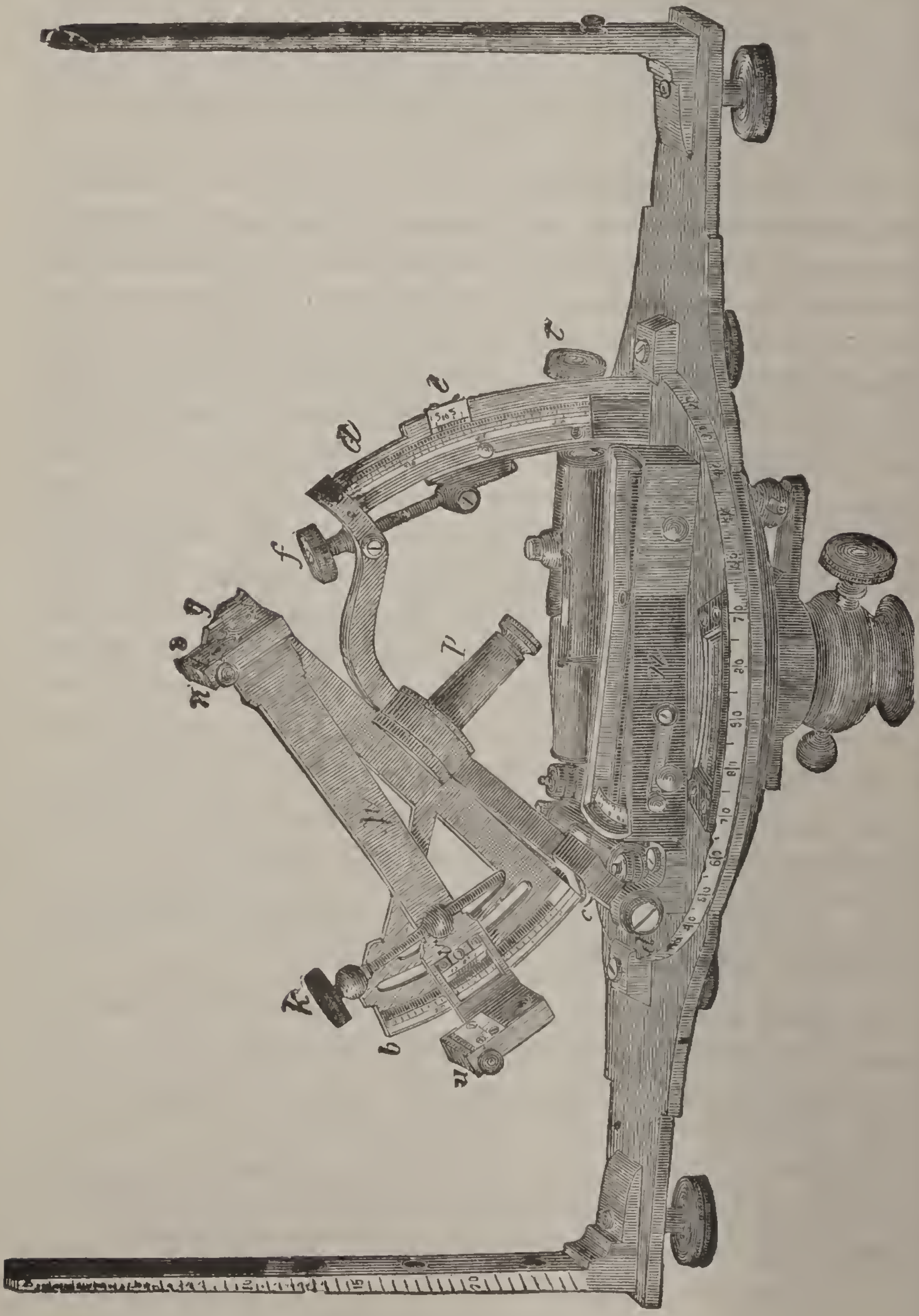


Fig. 10. THE SOLAR COMPASS.

1. This instrument, for readily determining a true meridian, or north and south line, was invented by William A. Burt and John Mullett, of Michigan, and patented by Burt in 1836. It has since come into general use in the surveys of United States public lands, the principal lines of which are required to be run with reference to the true meridian.

The arrangement of its sockets and plates is similar to that of the surveyor's transit, except that the sight vanes are attached to the under plate or limb, and this revolves around the upper or vernier plate on which the solar apparatus is placed.

The limb is divided to half degrees, is figured in two rows, as usual, and reads by the two opposite verniers to single minutes.

2. **The Solar Apparatus** is seen in the place of the needle, and in fact operates as its substitute in the field.

It consists mainly of three arcs of circles, by which can be set off the latitude of a place, the declination of the sun, and the hour of the day.

These arcs, designated in the cut by the letters *a*, *b*, and *c*, are therefore termed the **latitude**, the **declination**, and the **hour** arcs, respectively.

3. **The Latitude Arc,** *a*, has its centre of motion in two pivots, one of which is seen at *d*, the other is concealed in the cut.

It is moved either up or down within a hollow arc, seen in the cut, by a tangent-screw at *f*, and is securely fastened in any position by a clamp-screw.

The latitude arc is graduated to quarter degrees, and reads by its vernier, *e*, to single minutes; it has a range of about thirty-five degrees, so as to be adjustable to the latitude of any place in the United States.

4. **The Declination Arc,** *b*, is also graduated to quarter degrees, and has a range of about twenty-eight degrees.

Its vernier, *v*, reading to single minutes, is fixed to a movable arm, *h*, having its center of motion at the end of the declination arc, at *g;* the arm is moved over the surface of the declination arc, and its vernier set to any reading by turning the head of the tangent-screw, *k*. It is also securely clamped in any position by a screw, concealed in the engraving.

5. Solar Lenses and Lines.—At each end of the arm, *h*, is a rectangular block of brass, in which is set a small convex lens, having its focus on the surface of a little silver plate, *A*, (Fig. 11,) fastened by screws to the inside of the opposite block.

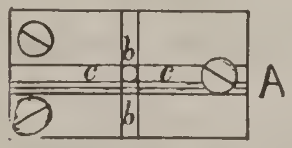


FIG. 11.

On the surface of the plate are marked two sets of lines, intersecting each other at right angles; of these, *bb* are termed the **hour** lines, and *cc* the **equatorial** lines, as having reference respectively to the hour of the day and the position of the sun in relation to the equator.

In Fig. 11 the equatorial lines are those on the lower block, parallel to the surface of the hour arc, *c;* the hour lines are of course those at right angles to the first.

6. Equatorial Sights.—On the top of each of the rectangular blocks is seen a little sighting piece, termed the equatorial sight, fastened to the block by a small milled head-screw, so as to be detached at pleasure.

They are used, as will be explained hereafter, in adjusting the different parts of the solar apparatus.

7. The Hour Arc, *c*, is supported by the two pivots of the latitude arc, already spoken of, and is also connected with that arc by a curved arm, as shown in the figure.

The hour arc has a range of about 120°, is divided to half degrees, and figured in two series, designating both the hours and the degrees, the middle division being marked 12 and 90 on either side of the graduated lines.

8. The Polar Axis.—Through the center of the hour arc passes a hollow socket, *p*, containing the spindle of the declination arc, by means of which this arc can be moved from side to side over the surface of the hour arc, or turned completely round, as may be required.

The hour arc is read by the lower edge of the graduated side of the declination arc.

The axis of the declination arc, or indeed the whole socket *p*, is appropriately termed the **polar axis.**

9. **The Adjuster.**—Besides the parts shown in the cut, there is also an arm used in the adjustment of the instrument as described hereafter, but laid aside in the box when that is effected.

The parts above described constitute properly the solar apparatus.

Beside these, however, are seen the needle-box, *n*, with its arc and tangent-screw, *t*, and the spirit levels, for bringing the whole instrument to a horizontal position.

10. **The Needle Box** has an arc of about 36° in extent, divided to half degrees, and figured from the center or zero mark on either side.

The needle, which is made as in other instruments, except that the arms are of unequal lengths, is raised or lowered by a lever shown in the cut.

The needle-box is attached by a projecting arm to a tangent-screw, *t*, by which it is moved about its center, and its needle set to any variation.

This variation is also read off by the vernier on the end of the projecting arm, reading to three minutes a graduated arc, attached to the plate of the compass.

11. **The Levels** seen with the solar apparatus have ground glass vials, and are adjustable at their ends like those of other instruments.

The edge of the circular plate on which the solar work is placed, is divided and figured at intervals of ten degrees, and numbered, as shown, from 0 to 90 on each side of the line of sight.

These graduations are used in connection with a little brass pin, seen in the center of the plate, to obtain approximate bearings of lines, which are not important enough to require a close observation.

12. Lines of Refraction.—The inside faces of the sights are also graduated and figured, to indicate the amount of refraction to be allowed when the sun is near the horizon. These are not shown in the cut.

13. Principles of the Solar Compass.—The interval between two equatorial lines, *cc*, in Fig. 10, as well as between the hour lines, *bb*, is just sufficient to include the circular image of the sun as formed by the solar lens on the opposite end of the revolving arm, *h*, Fig. 9.

When, therefore, the instrument is made perfectly horizontal, the equatorial lines and the opposite lenses being accurately adjusted to each other by a previous operation, and the sun's image brought within the equatorial lines, his position in the heavens, with reference to the horizon, will be defined with precision.

Suppose the observation to be made at the time of one of the equinoxes; the arm *h*, set at zero on the declination arc *b*, and the polar axis *p*, placed exactly parallel to the axis of the earth.

Then the motion of the arm *h*, if revolved on the spindle of the declination arc around the hour circle *c*, will exactly correspond with the motion of the sun in the heavens, on the given day and at the place of observation; so that if the sun's image were brought between the lines *cc*, in the morning, it would continue in the same position, passing neither above nor below the lines, as the arm was made to revolve in imitation of the motion of the sun about the earth.

In the morning, as the sun rises from the horizon, the arm *h* will be in a position nearly at right angles to that shown in the cut, the lens being turned toward the sun, and the silver plate on which his image is thrown directly opposite.

As the sun ascends, the arm must be moved around, until when he has reached the meridian, the graduated side of the declination arc will indicate 12 on the hour circle, and the arm *h*, the declination arc *b*, and the latitude arc *a*, will be in the same plane.

As the sun declines from the meridian, the arm *h* must be moved in the same direction, until at sunset its position will be the exact reverse of that it occupied in the morning.

14. Allowance for Declination.—Let us now suppose the observation made when the sun has passed the equinoctial point, and when his position is affected by declination.

By referring to the Almanac, and setting off on the arc his declination for the given day and hour, we are still able to determine his position with the same certainty as if he remained on the equator.

When the sun's declination is south, that is, from the 22d of September to the 20th of March in each year, the arc *b* is turned toward the plates of the compass, as shown in the engraving, and the solar lens, *o*, with the silver plate opposite, are made use of in the surveys.

The remainder of the year, the arc is turned from the plates, and the other lens and plate employed.

When the solar compass is accurately adjusted, and its plates made perfectly horizontal, the latitude of the place, and the declination of the sun for the given day and hour, being also set off on the respective arcs, *the image of the sun cannot be brought between the equatorial lines until the polar axis is placed in the plane of the meridian of the place, or in a position parallel to the axis of the earth.* The slightest deviation from this position will cause the image to pass above or below the lines, and thus discover the error.

We thus, from the position of the sun in the solar system, obtain a certain direction absolutely unchangeable, from which to run our lines, and measure the horizontal angles required.

This simple principle is not only the basis of the construction of the solar compass, but the sole cause of its superiority to the ordinary or magnetic instrument. For in a needle instrument, the accuracy of the horizontal angles indicated, and therefore of all the observations made, depends upon the delicacy of the needle, and the constancy with which it assumes a certain direction, termed the magnetic meridian.

The principal causes of error in the needle, briefly stated, are the dulling of the pivot, the loss of polarity in the needle, the influence of local attraction, and the effect of the sun's rays, producing the diurnal variation.

From all these imperfections the solar instrument is free.

The sights and the graduated limb being adjusted to the solar apparatus, and the latitude of the place and the declination of the sun also set off upon the respective arcs, we are able, not only to run the true meridian, or a due east and west course, but also to set off the horizontal angles with minuteness and accuracy from a direction which never changes, and is unaffected by attraction of any kind.

15. **Adjustments.**—The adjustments of this instrument, with which the surveyor will have to do, are simple and few in number, and will now be given in order.

1st. **To Adjust the Levels.**—Proceed precisely as directed in the account of the other instruments we have described, by bringing the bubbles into the centre of the tubes by the leveling screws of the tripod, and then reversing the instrument upon its spindle, and raising or lowering the ends of the tubes, until the bubbles will remain in the centre during a complete revolution of the instrument.

2d. **To Adjust the Equatorial Lines and Solar Lenses.**—First detach the arm *h* from the declination arc, by withdrawing the screws shown in the cut from the ends of the posts of the tangent-screw *k*, and also the clamp-screw, and the conical pivot with its small screws by which the arm and declination arc are connected.

The arm *h*, being thus removed, attach the adjuster in its place by replacing the conical pivot and screws, and insert the clamp-screw so as to clamp the adjuster at any point on the declination arc.

Now level the instrument, place the arm *h* on the adjuster, with the same side resting against the surface of the declination arc as before it was detached. Turn the instrument on its spindle so as to bring the solar lens to be adjusted in the direction of the sun, and raise or lower the adjuster on the declination arc, until it can be clamped in such a position as to bring the sun's image as near as may be between the equatorial lines on the opposite silver plate, and bring the image precisely into position by the tangent of the latitude arc or the leveling-screws of the tripod. Then carefully turn the arm half way over, until it rests upon the adjuster by the opposite faces of the rectangular blocks, and again observe the position of the sun's image.

If it remains between the lines as before, the lens and plate are in adjustment; if not, loosen the three screws which confine the plate to the block, and move the plate under their heads, until one-half the error in the position of the sun's image is removed.

Again bring the image between the lines, and repeat the operation until it will remain in the same situation, in both positions of the arm, when the adjustment will be completed.

To adjust the other lens and plate, reverse the arm, end for end, on the adjuster, and proceed precisely as in the former case, until the same result is attained.

In tightening the screws over the silver plate, care must be taken not to move the plate.

This adjustment now being complete, the adjuster should be removed, and the arm h, with its attachments, replaced as before.

3d. **To Adjust the Vernier of the Declination Arc.**—Having leveled the instrument, and turned its lens in the direction of the sun, clamp to the spindle, and set the vernier v, of the declination arc, at zero, by means of the tangent-screw at k, and clamp to the arc.

See that the spindle moves easily and yet truly in the socket, or polar axis, and raise or lower the latitude arc by turning the tangent-screw f, until the sun's image is brought between the equatorial lines on one of the plates. Clamp the latitude arc by the screw, and bring the image precisely into position by the leveling-screws of the tripod or socket, and without disturbing the instrument, carefully revolve the arm h, until the opposite lens and plate are brought in the direction of the sun, and note if the sun's image comes between the lines as before.

If it does, there is no index error of the declination arc; if not, with the tangent-screw k, move the arm until the sun's image passes over half the error; again bring the image between the lines, and repeat the operation as before, until the image will occupy the same position on both plates.

We shall now find, however, that the zero marks on the arc and the vernier do not correspond, and to remedy this error, the little flat-head screws above the vernier must be loosened until it can be moved so as to make the zeros coincide, when the operation will be completed.

4th. **To Adjust the Solar Apparatus to the Compass Sights.**—First level the instrument, and with the clamp and tangent-screws set the main plate at 90° by the verniers and horizontal limb. Then remove the clamp-screw and raise the latitude arc until the polar axis is by estimation very nearly horizontal, and if necessary, tighten the screws on the pivots of the arc, so as to retain it in this position.

Fix the vernier of the declination arc at zero, and direct the equatorial sights to some distant and well marked object, and observe the same through the compass sights. If the same object is seen through both, and the verniers read to 90° on the limb, the adjustment is complete; if not, the correction must be made by moving the sights or changing the position of the verniers.

16. To Use the Solar Compass.—Before this instrument can be used at any given place, it is necessary to set off upon its arcs both the declination of the sun as affected by its refraction for the given day and hour, and the latitude of the place where the observation is made.

To Set off the Declination.—The declination of the sun, given in the ephemeris of the Nautical Almanac from year to year, is calculated for apparent noon at Greenwich, England.

To determine it for any other hour at a place in the United States, reference must be had, not only to the difference of time arising from the longitude, but also to the change of declination from day to day.

The longitude of the place, and therefore its difference in time, if not given directly in the tables of the Almanac, can be ascertained very nearly by reference to that of other places given, which are situated on, or very nearly on, the same meridian.

It is the practice of surveyors in the states east of the Mississippi, to allow a difference of *six* hours for the difference in the longitude, calling the declination given in the Almanac for 12 M., that of 6 A. M., at the place of observation.

Beyond the meridian of Santa Fe, the allowance would be about *seven* hours, and in California, Oregon, and Washington Territory about *eight* hours.

Having thus the difference of time, we very readily obtain the declination for a certain hour in the morning, which would be earlier or later as the longitude was greater or less, and the same as that of apparent noon at Greenwich on the given day. Thus, suppose the observation made at a place, say, five hours later than Greenwich, then the declination given in the Almanac for the given day at noon, affected by the refraction, would be the declination at the place of observation for 7 o'clock A. M.; this gives us the starting-point.

To obtain the declination for the other hours of the day, take from the Almanac the declination for apparent noon of the given day, and, as the declination is increasing or decreasing, add to or subtract from the declination of the first hour, the difference for one hour as given in the ephemeris, which will give, when affected by the refraction, the declination for the succeeding hour; and proceed thus in making a table of the declination for every hour of the day.

17. Refraction.—By reason of the increasing density of the atmosphere from its upper regions to the earth's surface, the rays of light from the sun are bent out of their course, so as to make his altitude appear greater than is actually the case.

The amount of refraction varies, according to the altitude of the body observed; being 0 when it is in the zenith, about one minute when midway from the horizon to the zenith, and almost 34′ when in the horizon.

18. Allowance for Refraction.—The proper allowance to be made for refraction in setting off the declination of the sun upon the Solar Compass has long been a source of perplexity to the surveyor. Accordingly, a table has been prepared, (Table XI), by which the amount of refraction for any hour of the day throughout the year may be readily obtained. The manner of using the table is shown in the solution of the following

Example.—1. To find the declination for the different hours of April 16, 1883, at Troy, N. Y.

Solution.—Latitude of Troy, about 42° 30′ N. Longitude, 4 hr., 54 min., 40 sec., practically 5 hr.

Apparent noon at Greenwich is 7 A. M. at Troy. Declination of sun at Greenwich at noon of April 16, 1883, as given by Nautical Almanac, N. 10° 6′ 2′′+, and hourly change, 53′′.

Refraction in Lat. 42° 30′, declination 10°, time 5 hr. before noon as given by table, 1′ 58′′.

Whence the following figures:

N. 10° 6′ 2′′ + Ref. 5 hrs. 1′ 58′′ = 10° 8′ 0′′ = Dec. at 7 A. M. Troy.
add hr. dif. 53′′

N. 10° 6′ 55′′ + " 4 " 1′ 11′′ = 10° 8′ 0′′.6 = " 8 "
add hr. dif. 53′′

N. 10° 7′ 48′′ + " 3 " 0′ 52′′ = 10° 8′ 40′′ = " 9 "
add hr. dif. 53′′

N. 10° 8′ 41′′ + " 2 " 0′ 39′′ = 10° 9′ 20′′ = " 10 "
add hr. dif. 53′′

N. 10° 9′ 34′′ + " 1 " 0′ 36′′ = 10° 10′ 10′′ = " 11 "
add hr. dif. 53′′

N. 10° 10′ 27′′ + " 0 " 0′ 36′′ = 10° 11′ 03′′ = " 12 M.
add hr. dif. 53′′

N. 10° 11′ 20′′ + " 1 " 0′ 36′′ = 10° 11′ 56′′ " 1 P. M.
add hr. dif. 53′′

N. 10° 12′ 13′′ + " 2 " 0′ 39′′ = 10° 12′ 52′′ = " 2 "
add hr. dif. 53′′

N. 10° 13′ 06′′ + " 3 " 0′ 52′′ = 10° 13′ 58′′ = " 3 "
add hr. dif 53′

N. 10° 13′ 59′′ + " 4 " 1′ 11′′ = 10° 15′ 10′′ = " 4 "
add hr. dif. 53′

N. 10° 14′ 49′′ + " 5 " 1′ 58′′ = 10° 16′ 50′′ = " 5 "

Example.—2. To find the declination for the different hours of Oct. 16, 1883, at Troy, N. Y.

Solution.—Declination of sun at Greenwich at noon of Oct. 16, 1[illegible], as given by Nautical Almanac S. 8°51′47′′.7. hourly change 55′′.

Refraction 5 hr. before noon, Lat. 42° 30′, Dec. — 9°, is very nearly 9′ 24′′, and operates *to diminish* the declination.

Whence the following:

S. 8° 51′ 47′′.7	—Ref.	5 hr.	9′ 24′′ =	8° 42′ 23′′ =	Dec. at	7 A. M. at Troy.
add hr. diff. 55′′						
S. 8° 52′ 42′′	— "	4 "	2′ 49′′ =	8° 49′ 53′′ =	"	8 "
add hr. diff. 55′′						
S. 8° 53′ 37′′	— "	3 "	1′ 49′′ =	8° 51′ 48′′ =	"	9 "
add hr. diff. 55′′						
S. 8° 54′ 32′′	— "	2 "	1′ 26′′ =	8° 53′ 06′′ =	"	10 "
add hr. diff. 55′′						
S. 8° 55′ 27′′	— "	1 "	1′ 14′′ =	8° 54′ 13′′ =	"	11 "
add hr. diff. 55′′						
S. 8° 56′ 22′′	— "	0 "	1′ 14′′ =	8° 55′ 08′′ =	"	12 M.
add hr. diff. 55′′						
S. 8° 57′ 17′′	— "	1 "	1′ 14′′ =	8° 56′ 03′′ =	"	1 P. M.
add hr. diff. 55′′						
S. 8° 58′ 12′′	— "	2 "	1′ 26′′ =	8° 56′ 46′′ =	"	2 "
add hr. diff. 55′′						
		etc.		etc.	etc.	

19. To Set Off the Latitude.—Find the declination of the sun for the given day at noon, at the place of observation, as just described, and with the tangent-screw set it off upon the declination arc, and clamp the arm firmly to the arc.

Observe in the Almanac the equation of time for the given day, in order to know about the time the sun will reach the meridian.

Then, about fifteen or twenty minutes before this time, set up the instrument, level it carefully, fix the divided surface of the declination arc at 12 on the hour circle, and turn the instrument upon its spindle until the solar lens is brought into the direction of the sun.

Loosen the clamp-screw of the latitude arc, and with the tangent-screw raise or lower this arc until the image of the sun is brought precisely between the equatorial lines, and turn the instrument from time to time so as to keep the image also between the hour lines on the plate

As the sun ascends, its image will move below the lines, and the arc must be moved to follow it. Continue thus, keeping it between the two sets of lines until its image begins to pass above the equatorial lines, which is also the moment of its passing the meridian.

Now read off the vernier of the arc, and we have the latitude of the place, which is always to be set off on the arc when the compass is used at the given place.

It is the practice of surveyors using the solar compass to set off, in the manner just described, the latitude of the point where the survey begins, and to repeat the observation and correction of the latitude arc every day when the weather is favorable, there being also nearly an hour at mid-day when the sun is so near the meridian as not to give the direction of lines with the certainty required.

20. To Run Lines with the Solar Compass.—Having set off in the manner just given, the latitude and declination upon their respective arcs, the instrument being also in adjustment, the surveyor is ready to run lines by the sun.

To do this, the instrument is set over the station and carefully leveled, the plates clamped at zero on the horizontal limb, and the sights directed north and south, the direction being given, when unknown, approximately by the needle.

The solar lens is then turned to the sun, and with one hand on the instrument, and the other on the revolving arm, both are moved from side to side, until the sun's image is made to appear on the silver plate; when by carefully continuing the operation, it may be brought precisely between the equatorial lines.

Allowance being now made for refraction, the line of sights will indicate the true meridian; the observation may now be made, and the flag-man put in position.

When a due east and west line is to be run, the verniers of the horizontal limb are set at 90°, and the sun's image kept between the lines as before.

The solar compass being so constructed that when the sun's image is in position the limb must be clamped at 0 in order to run a true meridian line, it will be evident that the bearing of any line from the meridian may be read by the verniers of the limb precisely as in the ordinary magnetic compass, the bearings of lines are read from the ends of the needle.

21. Use of the Needle.—In running lines, the magnetic needle is always kept with the sun; that is, the point of the needle is made to indicate 0 on the arc of the compass box, by turning the tangent-screw connected with its arm on the opposite side of the plate. By this means, the lines can be run by the needle alone in case of the temporary disappearance of the sun; but, of course, in such cases the surveyor must be sure that no local attraction is exerted.

The variation of the needle, which is noted at every station, is read off in degrees and minutes on the arc, by the edge of which the vernier of the needle-box moves.

22. Allowance for the Earth's Curvature.—When long lines are run by the solar compass, either by the true meridian, or due east and west, allowance must be made for the curvature of the earth.

Thus, in running north or south, the latitude changes about one minute for every distance of 92 chains 30 links, and the side of a township requires a change on the latitude arc of 5′ 12″, the township, of course, being six miles square.

This allowance is of constant use where the surveyor fails to get an observation on the sun at noon, and is a very close approximation to the truth.

In running due east and west, as in tracing the standard parallels of latitude, the sights are set at 90° on the limb, and the line is run at right angles to the meridian.

If no allowance were made for the earth's curvature, these lines would, if sufficiently produced, reach the equator, to which they are constantly tending.

Of course, in running short lines either east or west, the variation from the parallel would be so small as to be of no practical importance; but when long sights are taken, the correction should be made by taking fore and back sights at every station, noting the error on the back sight, and setting off one-half of it on the fore sight on the side toward the pole.

23. Time of Day by the Sun.—The time of day is best ascertained by the solar compass when the sun is on the meridian, as at the time of making the observation for latitude.

The time thus given is that of apparent noon, and can be reduced to mean time by merely applying the equation of time as directed in the Almanac, and adding or subtracting as the sun is slow or fast.

The time, of course, can also be taken before or after noon, by bringing the sun's image between the hour lines, and noticing the position of the divided edge of the revolving arm, with reference to the graduations of the hour circle, allowing four minutes of time for each degree of the arc, and thus obtaining apparent time, which must be corrected by the equation of time as just dedescribed.

24. Caution as to the False Image.—In using the compass upon the sun, if the revolving arm be turned a little one side of its proper position, a false or reflected image of the sun will appear on the silver plate in nearly the same place as that occupied by the true one. It is caused by the reflection of the true image from the surface of the arm, and is a fruitful source of error to the inexperienced surveyor. It can, however, be readily distinguished from the real image by being much less bright, and not so clearly defined.

25. **Approximate Bearings.**—When the bearings of lines, such as the course of a stream, or the boundaries of a forest, are not desired with the certainty given by the verniers and horizontal limb, a rough approximation of the angle they make with the true meridian is obtained by the divisions on the outside of the circular plate.

In this operation, a pencil, or thin straight edge of any sort, is held perpendicularly against the circular edge of the plate, and moved around until it is in range with the eye, the brass center-pin, and the object observed.

The bearing of the line is then read off at the point where the pencil is placed.

Time for Using the Solar Compass.—The solar compass, like the ordinary instrument, can be used at all seasons of the year, the most favorable time being, of course, in the summer, when the declination is north, and the days are long, and more generally fair.

It is best not to take the sun at morning and evening, when it is within half an hour of the horizon, nor, for about the same interval, before and after it passes the meridian.

II. THE SOLAR ATTACHMENT.

1. **The Solar Attachment** is essentially the solar apparatus of Burt placed upon the cross-bar of the ordinary transit, the polar axis only being directed above instead of below, as in the solar compass. A little circular disk of an inch and a half diameter, and having a short round pivot projecting above its upper surface, is first screwed firmly to the axis ofthe telescope.

Upon this pivot rests the enlarged base of the polar axis, which is also firmly connected with the disk by four capstan-head screws passing from the under side of the disk into the base already named.

These screws serve to adjust the polar axis, as will be explained hereafter.

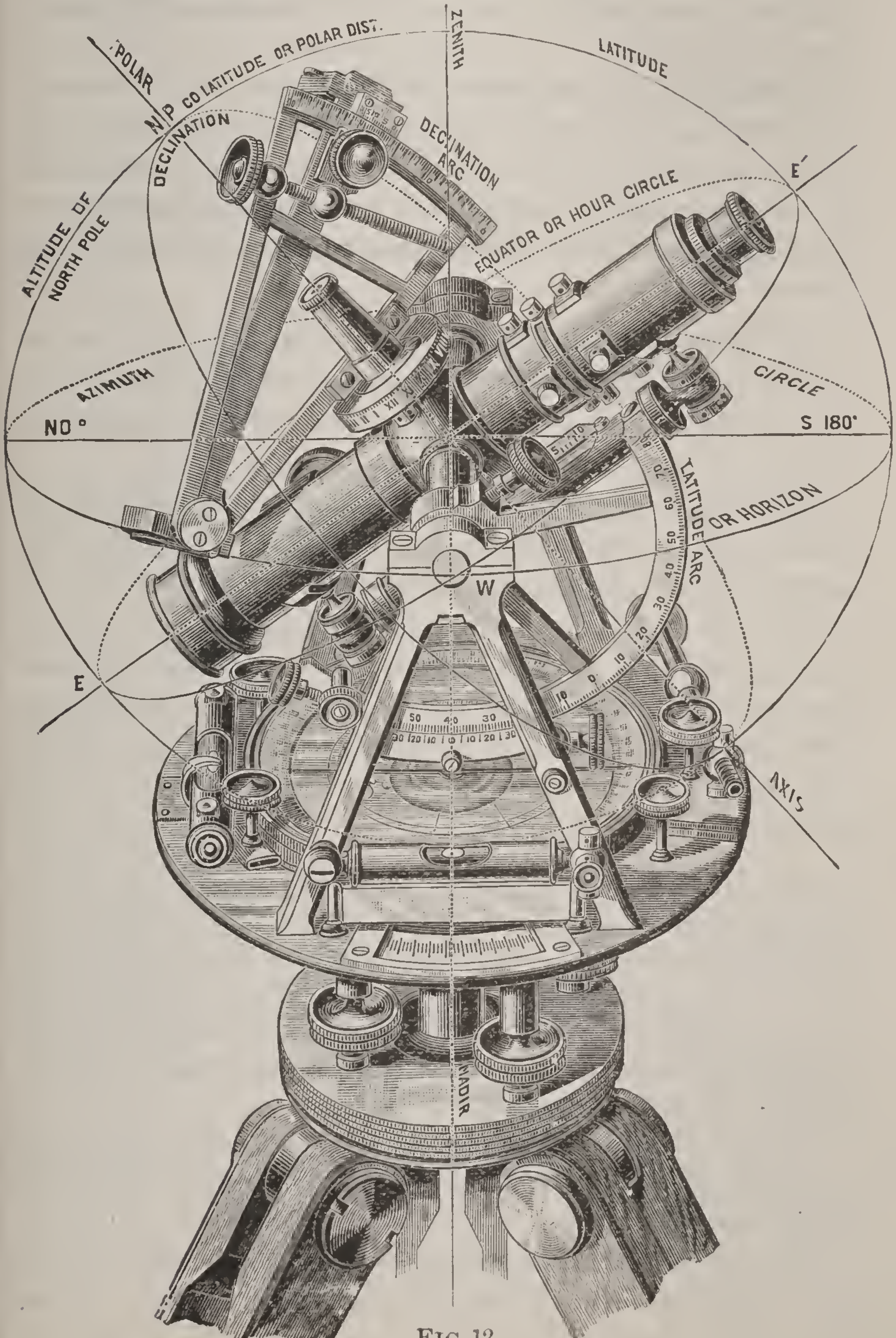


FIG 12.

2. **The hour circle** surrounding the base of the polar axis is easily movable about it, and can be fastened at any point desired by two flat-head screws above. It is divided to five minutes of time; is figured from I. to XII., and is read by a small index fixed to the declination circle, and moving with it.

A hollow cone, or socket, fitting closely to the polar axis and made to move snugly upon it, or clamped at any point desired by a milled-head screw on top, furnishes by its two expanded arms below, a firm support for the declination arc, which is securely fastened to it by two large screws.

3. **The declination arc** is of about five inches radius, is divided to quarter degrees, and reads by its vernier to single minutes of arc, the divisions of both vernier and limb being in the same plane.

The declination arm has the usual lenses and silver plates on the two opposite blocks, made precisely like those of the ordinary solar compass, but its vernier is outside the block, and more easily read.

The declination arm has also a clamp and tangent movement, as shown in the cut. The arc of the declination limb is turned on its axis and one or the other solar lens used, as the sun is north or south of the equator.

4. **The latitude** is set off by means of a large vertical limb having a radius of two and a half inches; the arc is divided to thirty minutes, is figured from the centre, each way, in two rows, viz. from 0 to 80°, and from 90° to 10°, the first series being intended for reading vertical angles; the last series for setting off the latitude, and is read by its vernier to single minutes.

It has also a clamp-screw inserted near its centre, by which it can be set fast to the telescope axis in any desired position.

The vernier of the vertical limb is made movable by the tangent-screw attached, so that its zero and that of the limb are readily made to coincide when, in adjusting the limb to the level of the telescope, the arc is clamped to the axis.

The usual tangent movement to the telescope axis serves, of course, to bring the vertical limb to the proper elevation, as hereafter described.

A level on the under side of the telescope, with ground vial and scale, is indispensable in the use of the Solar attachment.

The divided arcs, vernier, and hour circle are all on silver plate, and are thus easily read and preserved from tarnishing.

5. Adjustments.—These pertain to the solar lenses and lines, the declination arc, the polar axis and hour arc, as follows:

(1) **The solar lenses and lines** are adjusted precisely like those of the ordinary Solar, the declination arm being first detached by removing the clamp and tangent-screws, and the conical centre with its two small screws, by which the arm is attached to the arc.

The adjuster, which is a short bar furnished with every instrument, is then substituted for the declination arm, the conical centre screwed into its place, at one end, and the clamp-screw into the other, being inserted through the hole left by the removal of the tangent-screw, thus securing the adjuster firmly to the arc.

The arm is then turned to the sun, as described in the article on the Solar Compass, and reversed by the opposite faces of the blocks upon the adjuster, until the image will remain in the centre of the equatorial lines.

(2) **The vernier of the declination arc** is adjusted by setting the vernier at zero, and then raising or lowering the telescope by the tangent-screw until the sun's image appears exactly between the equatorial lines.

Having the telescope axis clamped firmly, carefully revolve the arm until the image appears on the other plate.

If precisely between the lines, the adjustment is complete; if not, move the declination arm by its tangent-screw, until the image will come precisely between the lines on the two opposite plates; clamp the arm and remove the index error by loosening two screws that fasten the vernier; place the zeros of the vernier and limb in exact coincidence, tighten the screws, and the adjustment is finished.

(3) **To Adjust the Polar Axis.**—First level the instrument carefully by the long level of the telescope, using in the operation the tangent movement of the telescope axis in connection with the leveling screws of the parallel plates until the bubble will remain in the centre during a complete revolution of the instrument upon its axis.

Place the equatorial sights on the top of the blocks as closely as is practicable with the distinct view of a distant object; and having previously set the declination arm at zero, sight through the interval between the equatorial sights and the blocks at some definite point or object, the declination arm being placed over either pair of the capstan-head screws on the under side of the disk.

Keeping the declination arm upon the object with one hand, with the other turn the instrument half around on its axis, and sight upon the same object as before. If the sight strikes either above or below, move the two capstan-head screws immediately under the arm, loosening one and tightening the other as may be needed until half the error is removed.

Sight again and repeat the operation, if needed, until the sight will strike the same object in both positions of the instrument, when the adjustment of the axis in one direction will be complete.

Now turn the instrument at right angles, keeping the sight still upon the same object as before; if it strikes the same point when sighted through, the axis will be truly vertical in the second position of the instrument.

If not, bring the sight upon the same point by the other pair of capstan-head screws now under the declination arc, reverse as before, and continue the operation until the same object will keep in the sight in all positions, when the polar axis will be made precisely at right angles to the level and to the line of collimation.

It should here be noted that, as this is by far the most delicate and important adjustment of the solar attachment, it should be made with the greatest care, the bubble kept perfectly in the center and frequently inspected in the course of the operation.

(4) **To Adjust the Hour Arc.**—Whenever the instrument is set in the meridian, as will be hereafter described, the index of the hour arc should read apparent time.

If not, loosen the two flat-head screws on the top of the hour circle, and with the hand turn the circle around until it does, fasten the screws again, and the adjustment will be complete.

To obtain mean time, of course the correction of the equation for the given day, as given in the Nautical Almanac, must always be applied.

6. To Find the Latitude.—First level the instrument very carefully, using, as before, the level of the telescope until the bubble will remain in the center during a complete revolution of the instrument, the tangent movement of the telescope being used in connection with the leveling screws of the parallel plates, and the axis of the telescope firmly clamped.

Next clamp the vertical arc, so that its zero and that of its vernier coincide as near as may be, and then bring them into exact line by the tangent screw of the vernier.

Then, having the declination of the sun for 12 o'clock of the given day as affected by the meridianal refraction carefully set off upon the declination arc, note also the equation of time, and fifteen or twenty minutes before noon, the telescope being directed to the north, and the object-end lowered until, by moving the instrument upon its spindle and the declination arc from side to side, the sun's image is brought nearly into position between the equatorial lines. Now bring the declination arc directly in line with the telescope, clamp the axis firmly, and with the tangent screw bring the image precisely between the lines and keep it there with the tangent screw, raising it as long as it runs below the lower equatorial line, or in other words, as long as the sun continues to rise in the heavens.

When the sun reaches the meridian, the image will remain stationary for an instant and then begin to rise on the plate.

The moment the image ceases to run below is of course apparent noon, when the index of the hour arc should indicate XII, and the latitude be determined by the reading of the vertical arc.

It must be remembered, however, that the angle through which the polar axis has moved in the operation just described is measured from the zenith instead of the horizon as in the ordinary solar, so that the angle read on the vertical limb is the complement of the latitude.

The latitude itself is readily found by subtracting this angle from 90°; thus, at Troy, the reading of the limb being found as above directed to be 47° 16′, the latitude will be $90° - 47° 16' = 42° 44'$.

It will be noticed that with this apparatus the latitude of any place can be most easily ascertained without any index error, as in the usual solar compass.

7. To Run Lines with the Solar Attachment.—Having set off the complement of the latitude of the place on the vertical arc, and the declination for the given day and hour, as in the solar, the instrument being also carefully leveled by the telescope bubble, set the horizontal limb at zero and clamp the plates together, loosen the lower clamp so that the transit moves easily upon its lower socket, set the instrument approximately north and south, the object end of the telescope pointing to the north, turn the proper solar lens to the sun, and with one hand on the plates and the other on the revolving arm, move them from side to side until the sun's image is brought between the equatorial lines on the silver plate.

The lower clamp of the instrument should now be fastened and any further lateral movement be made by the tangent screw of the tripod. The necessary allowance being made for refraction, the telescope will be in the true meridian, and being unclamped, may be used like the sights of the ordinary solar compass, but with far greater accuracy and satisfaction in establishing meridian lines. Of course when the upper or vernier plate is unclamped from the limb, any angle read by the verniers is an angle from the meridian, and thus parallels of latitude or any other angles from the true meridian may be established as with the solar compass.

The bearing of the needle, when the telescope is on the meridian, will also give the declination of the needle at the point of observation.

The declination of the needle being set off, the needle kept then at zero, or "with the sun," lines may be run by the needle alone, when the sun is obscured.

The sun, however, must ever be regarded as the most reliable guide, and should, if possible, be taken at every station.

CHAPTER IV.

Measurement of Angles.

1. The instruments already described are used both for running lines and for measuring angles. The *transit* is used where the greatest degree of accuracy is required and where angles are to be measured within 1′ or less.

The compass is used when no great degree of accuracy is required and the measurement of an angle within 5′ is as close as is ordinarily expected.

Professional Surveyors are provided with the compass or transit in some of their various forms.

Students and others may or may not have them. In case of necessity the tape may be used to measure angles, and in connection with the picket, sections of the United States Survey may be subdivided, irregular fields measured, and other similar operations performed, with a rapidity and accuracy equal to, if not superior to work done with a compass, the picket being used to run the lines and the tape to measure both distances and angles.

2. To Measure Angles with the Tape.

This is most conveniently done with the aid of tables of trigonometrical functions with which the student is supposed to be familiar.

Prob. 1. *To lay off a right angle from a point p in a given line AB.*

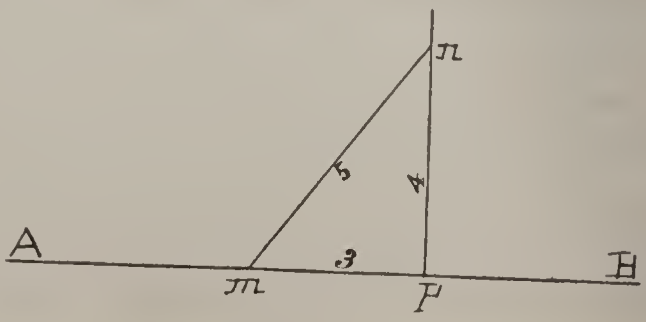


Fig. 13.

When the sides of a triangle are to each other as 3, 4 and 5, the angle between the smaller sides is a right angle. Hence to lay off a right angle with the tape or chain, stick a marking pin at p and then measure along the line $p\ m = 3$ and stick another pin at m. Then from p as a center with a radius 4 and from m as a center with radius 5 strike arcs intersecting at n. Then will mpn be the required angle. If the line pn is to be prolonged as a picket line, it will be better to range from, if longer sides, as 60, 80 and 100 are used.

This is the most useful of the many methods of laying off a right angle with the tape, and can be applied where any method can be. The other methods are, for the most part, more curious than useful. The following is one of the best of them:

2d Method. Measure along the line in opposite directions from p and stick pins in the line at m and m' making $pm = pm'$. Then from m and m' as centres with any radius greater than pm strike two arcs intersecting at n. Mpn is the required angle.

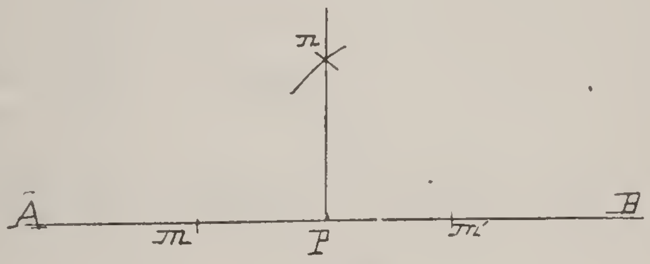


FIG. 14.

Prob. 2. *From a point p in a given line AB to run a line making any required angle with the line AB.*

1st Method. From p measure $p\ m$ equal to the cosine of the required angle and stick a pin in the line at m. Then from m as a centre with a radius equal to the sine of the required angle and from p as a centre and radius r strike arcs intersecting at n. Then mpn will be the required angle and p and n will be points in the required line. If $r = 100$ then the lengths of cosine and sine are used just as taken from the table of natural sines, only changing the decimal point. Otherwise the tabular numbers must first be multiplied by the length adopted for r.

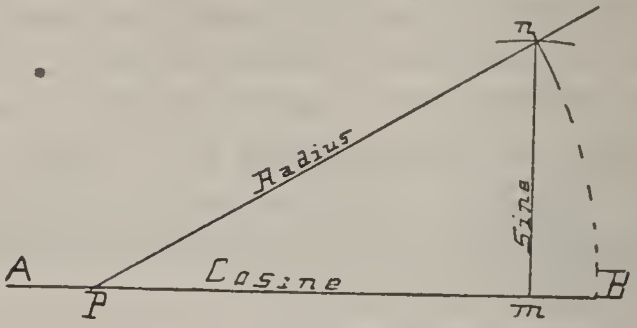


FIG. 15.

2d Method. In a similar manner we may use the natural tangents and secants. From p and m as centres, with the secant and tangent of the required angle as radii, strike arcs intersecting at n. Secants not given in the table may be found from the table of natural sines by the formula $secant = \frac{1}{cosine}$.

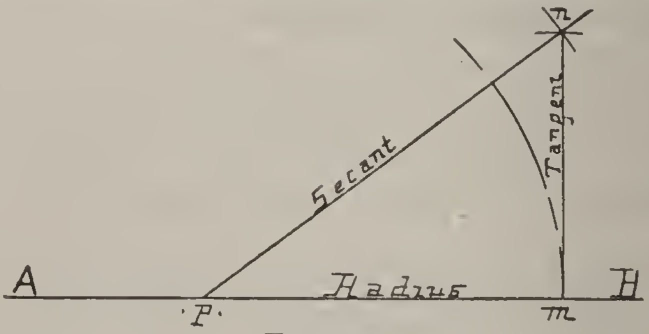


FIG. 16.

Example 1. Lay off, by the use of sines and cosines, an angle of 36° 28′.

Solution.— Let $r = 100 = pn$. Then $mn = 59.44$, $pm = 80.4$.

Ex. 2. Lay off by the use of tangent and secant, an angle of 25° 20′.

Solution.— Let $r = 100 = pm$. Then $mn = 47.34$; $pn = 110.64$.

Ex 4. Lay off by each method, angles of 48° 20′, 63° 15′, 26° 32′, 8° 40′, 18° 23′, 37° 06′, 82° 45′.

3d Method. By chords. From the point p as a centre, with any radius,—preferably 100, strike an arc mx. Find the natural sine of half the angle. Double it for the chord. With this distance as radius, from m as a centre, strike an arc intersecting the arc mx at n. Then p and n are points in the required line and mpn the required angle.

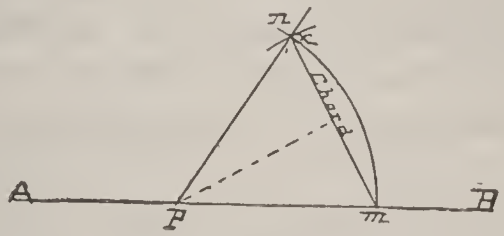


FIG. 17.

Example 1. Having run the line from the east quarter post of section 26 north to the section corner and marked it with a sufficient number of pickets, it is required to locate the centre line of a highway commencing at the quarter post and running north 22½° west.

Solution.—Measure north in the line from the quarter post the full length of the tape = 100, stick a marking pin m carefully in line, and strike an arc to the left around the quarter post as a centre. Find the sine of half the angle and double it. Sine $11° \ 15' \times 2 = .19509 \times 2 = .39018$ or correcting the decimal point 39.018. With this distance as a radius, from m as a centre, locate the intersecting point n which is a point in the required line.

The student should now select a level plat of ground, mark out a line upon it with pickets and solve the preceding examples or similar ones, on the ground, each one by the several different methods and compare results,

Also set pickets at the angles of a field of three or more sides and measure the sides and angles of the field.

3. To Measure Angles with the Compass.

Set the compass up at the intersection of the lines, between which the angle is to be measured. Put the sights in range with one of the lines and note the reading of the needle. Then put them in range with the other line and again note the reading of the needle. Read off from the limb, or calculate the number of degrees passed over by the needle between the two readings. In land surveying, a line traced out upon the ground is termed a *course* and the angle which the line makes with a north and south line is called its *bearing or course.* In compass work the bearings only are taken. The angles between the lines of th survey may be computed therefrom if necessary. They are seldom required. In reading and writing down the bearings it is customary to state first the direction of the line from which the bearing is taken and then the angle to the east or west, which the course makes with that line, *e. g.*, North 60 degrees West. South 5 degrees East. Written N. 60° W; S. 5° E.

It is customary in Land Surveying to refer all lines to a meridian real or assumed. The cosine of a bearing multiplied by the length of its course is called the **Latitude.**

The sine of the bearing multiplied by the length of the course is called the **Departure.**

When desirable to find the angles between two lines from their bearings, they may be computed as follows:

Calling N. and S. *meridianal* letters, we have for the angle between two lines from the same station, the following:

PRINCIPLES.—1. *When the meridianal letters are alike and the others unlike, the angle is the sum of the bearings.*

(2) *When the meridianal letters are unlike and the others alike, the angle is the supplement of the sum of the bearings.*

(3) *When both the meridianal and the other letters are alike, the angle is the difference of the bearings.*

(4) *When both the meridianal and the other letters are unlike, the angle is the supplement of the difference of the bearings.*

Observe that the bearings are given in their proper relative direction with each other and none of them are reversed, as S. E. when it should be N. W.

Examples. 1. The bearings, of two lines are N. 60° W. and N. 3° E. What is the angle between them?

Ans. 63°.

2. Required the angles between lines having the following bearings: N. 37° E. and S. 26° E.; N. 87° E. and S. 86° W.; S. 15° E. and S. 26° E. *Ans.* 117°; 179°; 11°.

3. Stake out a triangle in the field and take the bearings of the sides.

Find the angles of the triangle and compare the sum with 180.

4. Stake out fields having 4, 5 and 6 sides. Take the bearings and find the angles between the sides.

4. To Correct Courses of Random Lines.

CASE 1ST.—*Where the line has but one course.*

Random lines as they are usually called are simply trial lines run to find the true line between two fixed points which are not visible from each other. These lines are usually started from one of the points and run as nearly in the true direction as can be estimated. If the estimate proves correct, and the line strikes the point aimed for, the random becomes the true line. If not, the perpendicular distance from the line to the point is measured, from which the correction for the course may be computed.

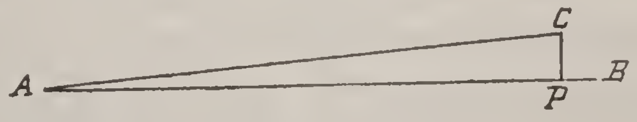


FIG. 18.

If PC is made perpendicular to AB as is generally the case where randoms are run between corners of the United States survey then $\text{Tan. } CAP = \dfrac{CP}{AP}$ whence the angle CAP is found, which is the correction to be applied to the bearing.

The angle CAP, when it is quite small, may be found by multiplying 57.3° by PC, and dividing by AC. This is called the **Fifty-seven and three-tenths rule.** The rule depends upon the fact that for small angles, AP differs insensibly from AC, and CP from the arc subtending the angle CAP.

Whence, angle CAP:360°::CP:2×3.1416×AP,

or angle $CAP = \frac{CP}{AP} \times \frac{360°}{6.2832} = \frac{CP \times 57.3°}{AP}$, or $\frac{CP \times 57.3°}{AC}$

The semi-circumference of a circle, with radius AP, is $3.14159265 \times AP$.

Whence arc $1' = 3.14159265 \times AP \div 10800$.

If $AP = 1$ ch., arc $1' = 0.00029088$ ch. $= 0.029088$ l.

If $AP = 1$ mi. $= 80$ ch., arc $1' = 0.029088$ l. $\times 80 = 2.327$ l. $= 2\frac{1}{3}$ l.

When angle $PAC = 1'$ and AP or $AC = 1$ mi., the perpendicular PC, without perceptible error, is $2\frac{1}{3}$ links. The line PC is called the departure of AC, for the distance AP or AC.

Taking $2\frac{1}{3}$ l, as the departure of 80 ch. at an angle of $1'$, the departure for 40 ch., would be $\frac{1}{2}$ of $2\frac{1}{3}$ l. $= 1\frac{1}{6}$ l. $= 1$ l. $+ \frac{1}{6}$ of 1 l.

For quite small angles, the departure varies directly as the angle. Whence, for 40 ch., the following:

Dep. for $1' = 1$ l. $+ \frac{1}{6}$ of 1 l.;
" " $2' = 2$ l. $+ \frac{1}{6}$ of 2 l.;
" " $3' = 3$ l. $+ \frac{1}{6}$ of 3 l.;

and so on, practically true, to $60'$ or 1°.

For any other distance, at the same angle, the departure varies directly as the distance. Accordingly,

Given minutes of angle, to find links of departure, we have the following:

RULE.—*To the number of minutes, add its one-sixth and multiply the sum by the ratio of the distance to 40 ch.* (Good to sixty minutes.)

On the following:

General Rule.—*Multiply 0.0291 by the number of minutes, and multiply the product by the number of chains in the distance.* (Good to 240 minutes.)

Example.—Given angle = 30′ and distance = 23.20 ch., to find the departure.

Since for 40 ch., 1′ of angle gives $1\frac{1}{6}$ l. of departure, we may say, without sensible error for a small angle that 1 l. of departure gives $\frac{6}{7}$ of 1′ of angle, for the same distance.

Or as it may be written,

Dep. of 1 l. = $1' - \frac{1}{7}$ of 1′.

Similarly, " " 2 l. = $2' - \frac{1}{7}$ of 2′,

" " 3 l. = $3' - \frac{1}{7}$ of 3′,

and so on, practically true to 60′ or 1°.

For any other distance with the same departure, the angle varies inversely as the distance. Accordingly,

Given links of departure, to find minutes of angle, we have the following:

Rule.—*From the number of links of departure, subtract its one-seventh and divide the remainder by the ratio of the distance to 40 ch.* (Good to 60 minutes.)

General Rule.—*Multiply 0.0291 by the number of chains in the distance, and divide the number of links of departure by the product.* (Good to 240 minutes).

In the Table of Departures, the value of *PC* in chains and decimals is given for angles from 1′ to 60′, and for the distances most commonly required in making resurveys and subdivisions of Sections of the United States Survey. To use the Table: *Having measured the outing PC on the ground, find the nearest tabular number in the column for the corresponding distance.*

The angle will be found in the minute column.

Example 1. Commencing at the west quarter post of Section 16, and running north, the random line intersected the north line of the section, 15 links east of the corner. What is the amount of the correction for course?

Solution. In 40 chain column, nearest number .151. Corresponding number of minutes 13.

2. Commencing at the south quarter post of section 16 with declination of needle estimated at 2° 17′ E. set off on the vernier, ran north on random and intersected the north line of the section, 42 links east of the quarter post. What is the declination of the needle as referred to the quarter line?

Solution. Distance 80 chains, correction 18′. As the line came out east of the corner, it is evident that the angle between the magnetic meridian and the quarter line was 18′ greater than was estimated, = 2° 35′.

NOTE.—The North and South lines of the United States Survey are, in a legal sense all true meridians, whatever they may be astronomically, and their locations are fixed by the monuments planted for the section corners and quarter posts. Hence it is a custom among Surveyors to refer the declination of the needle—or the variation as it is more frequently called, to these lines, and to mark on each line on their plats, the declination for that line. Under that custom the line referred to in Example 2 would be marked Var. 2° 35′ E.

3. "East on random between Sections 13 and 24. 79.98 chains intersected east boundary 34 links south of post." What is the bearing of the corrected line running west? *Ans.* S. 89° 45′ W.

CASE 2ND.—*Where the line is a broken one of several courses.*

Surveyors are frequently called on to retrace the lines of angling roads to settle the boundaries of adjacent lands, or to locate meander lines, or to find the boundaries of irregular tracts, where several courses have to be run between the nearest known points of the original survey.

In such cases random lines are run according to the notes of the original survey, and temporary stakes driven at the angles of the random line. It will generally be found that corrections for course or distance or for both will have to be made to place the stakes in their correct location.

PROBLEM.—*To correct a random line of several courses.*

In Fig. 19 let A, B, C, D represent the lines and angles of the original survey between the known points A and D.

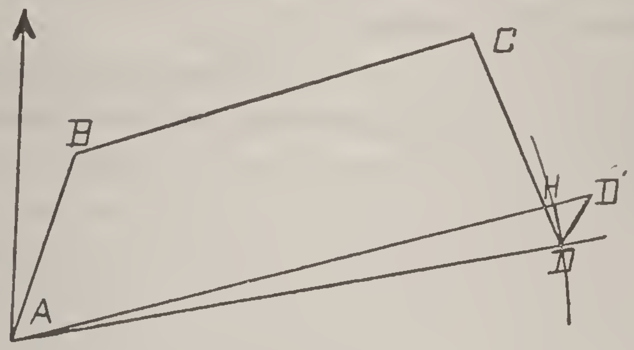


FIG. 19.

Let D' represent the terminus of a random run to retrace these lines, the direction and distance of which from D is known.

From A draw the line AD, producing it indefinitely beyond D; also, from A as a centre, with radius AD, draw an arc through D. Now, if the error in the random was of direction only, then the point D' would be in the arc. If it was an error of the chain only, D' would be in the line AD or AD produced. Hence the position of D' with reference to the arc and the line AD indicates the kind of correction and in what direction it is to be applied.

$\frac{AD}{AD'}$ is the length of the original chain in terms of the chain used on the random. That portion of the arc which is intercepted between the point D and a line joining AD', measures the angle of correction. In the field we may calculate the course and length, and run a sufficient part of the line $D'A$, and then trace the arc from D to its intersection with that line, and thus find the relative length of the lines AD and AD', by which to determine the correction for the chain and also find the chord of the angular correction; or they may be calculated as shown in the following example:

Example 1.—The boundaries of a farm between the nearest known monuments are as follows, (See Fig. 19):

1. N. 16°, E. 12.00 chains.
2. N. 72°, E. 26.00 "
3. S. 22°, E. 14.00 "

A random was run with var. 2° 30′ E. and came out N. 28° E. 32 links from the monument. Required the correction for the variation of needle and for the stakes in the angles of the random line.

We will first find the total latitudes and departures of each station on the random line, and the direction and distance of a line, AD', which will join the termini.

	N. Lat.	S. Lat.	E. Dep.	Tot. Lat.	Tot. Dep.
1. N. 16° E. 12.00	11.54		3.31	11.54	3.31
2. N. 72 E. 26.00	8.03		24.73	19.57	28.04
3. S. 22 E. 14.00		12.98	5,24	6.59	33.28

If we now divide the total departure of the point D' by its total latitude we will have the tangent of the bearing of the line $D'A$.

$$\frac{33.28}{6.59} = 5.050 = \tan\ 78°\ 48' \text{ or S. } 78°\ 48' \text{ W.}$$

The length of the line $D'A = \sqrt{6.59^2 + 33.28^2} = 33.927.$

If we now subtract the bearing of the line $D'D$ from the bearing of the line $D'A$ we shall have the angle $DD'A = 78°\ 48' - 28° = 50°\ 48'$. Let DH be a perpendicular from D to the line AD'; then we have the following equations:

$$DH = D'D \sin AD'D = .32 \times .77494 = .24798+.$$
$$D'H = D'D \cos AD'D = .32 \times .63203 = .20225.$$
$$AH = AD' - D'H = 33.926 - .20225 = 33.7237+.$$
$$\frac{DH}{AH} = \tan DAD' = .24798+ \div 33.7237+ = .00735 =$$

tan 25′ = correction for course.

$$AD = \sqrt{AH^2 + HD^2} = \frac{AH}{\cos DAD'} = 33.7237 \div .99997 =$$

33.724. When the angle DAD' is small, AD and AH may be considered equal, without sensible error.

$$\frac{AD}{AD'} = \frac{33.724}{33.926} = .99404 = \text{length of original chain in}$$

terms of the chain used on the random. As the random came out to the left of the true line the variation, 2° 30′ E., was too great, hence we subtract the 25′, giving 2° 05′ as the variation of the needle from the meridian of the original survey. To find corrections for the stakes it will be better to refer them to the meridian of the random, hence we will now apply the corrections for course and distance to find the courses and distances of the original survey, as they would be according to the meridian and measure of the random. This done, we calculate their total latitudes and departures. The difference between these and the latitudes and departures of the corresponding points of the random is the correction to be applied.

					N. Lat.	S. Lat.	E. Dep.	Tot. Lat.	Tot. Dep.
1.	N. 16° 25′	E.	11.928		11.44		3.37	11.44	3.37
2.	N. 72 25	E.	25.844		7.81		24.64	19.25	28.01
3.	S. 21 35	E.	13.916			12.94	5.12	6.31	33.12

The last course is computed in this table simply as a check on the work, as it was a condition of the problem that the line DD' was N. 28°, E. 32 links; from which it is known that the difference between the two points is: latitude 28 lks., and departure 15 lks. We will now compare the results in the two tables and find the correction at B, C and D.

	B Lat.	*B* Dep.	*C* Lat.	*C* Dep.	*D* Lat.	*D* Dep.
Random Line	11.54	3.31	19.57	28.04	6.59	33.28
Original Line	11.44	3.37	19.25	28.01	6.31	33.13
Correction	S. 10	E. 6	S. 32	W. 3	S. 28	W. 15

Example 2.—Description of a highway between two known points:

1. N. 62° E. 14.00 chains.
2. N. 43½° E. 8.00 "
3. N. 5° W. 12.00 "
4. N. 72½° E. 10.25 "
5. S. 12° W. 6.43 "

A random run with var. 2° 17′ E. came out 62 lks. east of the point. What is the correction for variation of needle, and what change must be made in the position of each stake at the angles of the random?

5. To Measure Angles with the Transit.

1. Set up the transit at the apex of the angle and set the zero of the vernier to coincide with the zero of the limb. Clamp the plates in this adjustment and with the clamp to the spindle loosened, turn the telescope in the direction of one of the lines. Clamp the spindle and bring the wire exactly to centre the line by the slow motion screw to the spindle clamp. Unclamp the vernier and turn the telescope in the direction of the other line. Clamp the vernier in that position and make the final adjustment of the wire to the line by the use of the upper tangent screw. The angle may then be read from the limb.

2. Instead of first setting the verniers at zero they may be clamped in any position on the limb and then the difference in the two readings will be the angle. When great accuracy is required numerous readings of the angle are taken on various parts of the limb and the mean of the several results taken for the final reading.

3. *To find the angle which the parts of a broken line form with any given line.*

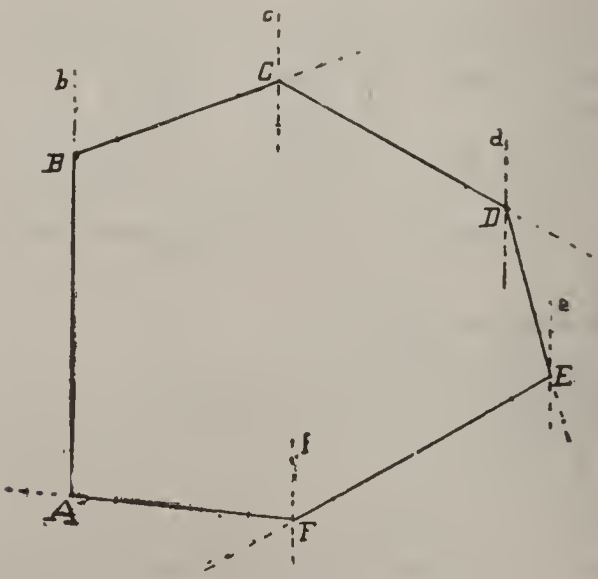


FIG. 20.

SUGGESTIONS.—Let $ABCDEF$ be a broken line, and suppose it is required to find the angles which the parts BC, CD, DE and EF form with the line AB.

Set the transit at B, with the vernier set at zero. Loosen below, reverse the telescope and direct it to A. Clamp the limb, revolve the telescope on its horizontal axis, unclamp the vernier and direct the telescope to C. The reading of the instrument will be the angle bBC the line which BC forms with the line AB.

Remove to C; and, leaving the vernier clamped, unclamp below, reverse the telescope, and direct it to B.

The limb remaining securely clamped, revolve the telescope, unclamp the vernier, and direct to D. The reading will now be the angle cCD which the line CD forms with the line Cc or its parallel AB.

The work goes on in this manner to its close.

Let the student further describe it.

If the broken line enclose a field, the reading of the instrument when set as at A and directed to B, having gone entirely around the field, should be 360°. This constitutes a check against errors occurring anywhere in the work.

4. *To measure an angle of elevation or depression.*

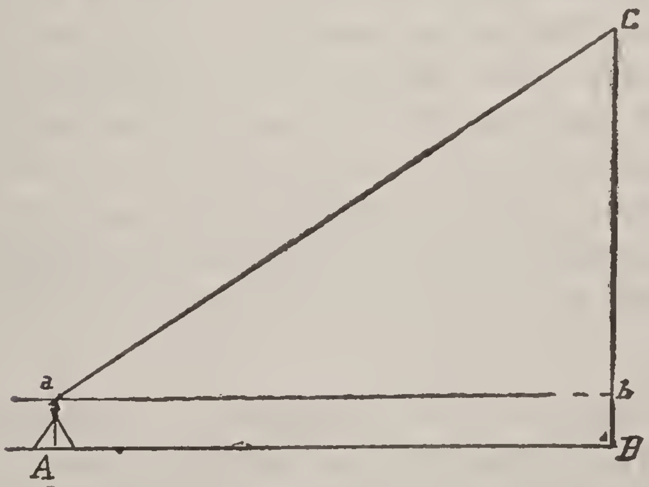


FIG. 21.

SUGGESTIONS.—Set the instrument at the vertex of the angle and level the horizontal limb.

Revolve the telescope upward or downward as the case may require, and adjust the line of sight to the inclined side of the angle. Take the reading of the vertical circle, applying the proper correction for index error.

Otherwise, take the reading of the circle, repeat the observation with the telescope and vernier plate reversed, and find the mean of the two readings for the angle sought.

6. **Verniers** are auxiliary scales for measuring smaller portions of space than those into which the main scale is divided. They are movable beside the main scale and are divided into parts which are either a little shorter or a little longer than the parts into which the main scale is divided. This small difference in length is what we are enabled to measure.

When the limb of a transit is divided to half degrees it is common to make either 29 or 31 divisions of the Vernier Scale equal to 30 on the limb, making each division on the vernier 31′ or 29′ in length.

The zero of the Vernier Scale is the point to which the reading is to be taken. Suppose the zero line of the vernier to make a straight line with some even division of the limb and each division on the vernier scale is 29′ in length. Now if the Vernier be moved 1′, the first line of the Vernier Scale from zero in the direction in which the vernier was moved, will be in a line with the first division on the limb. If moved 2′ the second lines will coincide; if 3′ the third lines; and so on to the end of the scale. Such a vernier is called direct reading. It is the kind most commonly used on surveyors' instruments.

Suppose however that the spaces on the vernier were 31′ long. Then when the vernier was moved forward 1′ the first line *back* of the zero point would coincide with the line in the limb and so on. Such a vernier is called a retrograde vernier.

To read any vernier. If the zero of the vernier coincides with any division of the scale, that will be the correct reading. If not, note the nearest next less division on the limb, and then look along the vernier scale till a line is found which coincides with a line on the limb. The number of this line *on the vernier* tells that so many of the subdivisions which the vernier indicates (usually minutes) are to be added to the reading of the entire divisions on the limb.

If several lines appear to coincide equally well, take the middle line.

CHAPTER V.

Passing Obstacles. Measuring Inaccessible Distances.

Having considered the various methods of running lines and measuring angles we are now prepared to take up some further problems in passing obstacles in the line and measuring inaccessible distances.

These problems may be solved in the field by the use of the picket and tape, the compass, or the transit.

1. *To pass an obstacle in the line and measure the distance.*

1st, by **Parallel Lines.** From a in the line AB run and measure the line ac in any convenient direction, a sufficient distance. From c run cd parallel with AB.

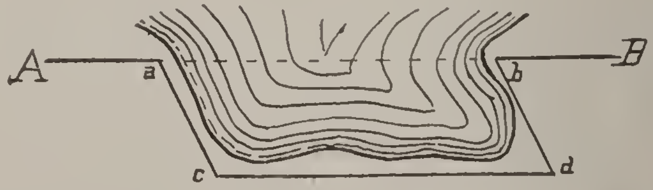


Fig. 22.

From d, run and measure db equal to and parallel with ac. Then $ab = cd$ and b is a point in the line AB. When running through heavy forests or towns it will often be necessary to run several parallel lines before returning to the original line.

2. **By 60° Angles.** From a run and measure ac making the angle $Bac = 60°$. Run and measure $cb = ac$ and the angle $acb = 60°$. Then b is a point in the line

AB and the angle $abc = 60°$, whence the line may be continued; ab will equal ac.

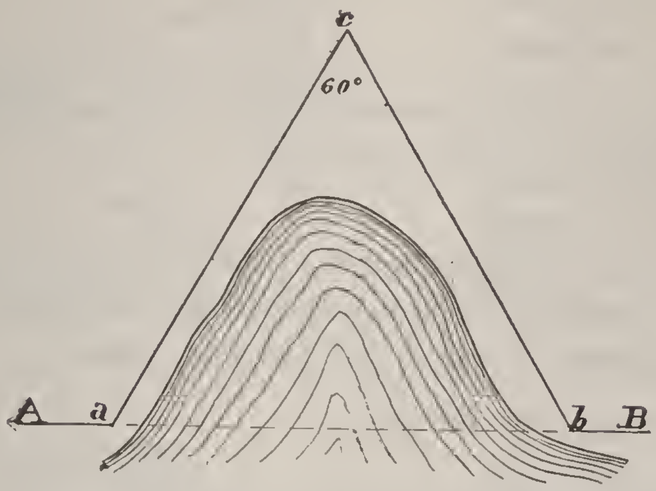


FIG. 23.

2. To Measure Inaccessible Distances.

CASE 1ST. *When the points are visible from each other as over a stream or pond.*

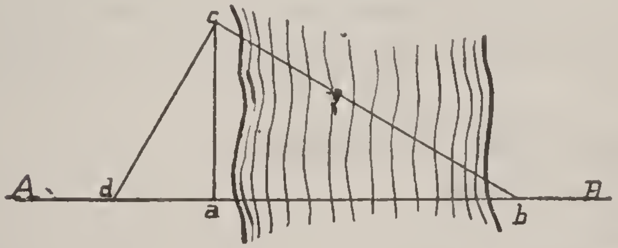


FIG. 24.

I. By Similar Triangles.

From a point a in the line AB, required the distance ab across the stream.

At a erect a perpendicular ac to the line AB. From c run a perpendicular to cb intersecting AB at d. Measure ac and ad. Then as the triangles cad and bcd are similar,

$$ad : ac = ac : ab, \text{ whence } ab = \frac{ac^2}{ad}.$$

There are numerous other devices for obtaining the distance ab by similar triangles on the ground. Let the student work out some of them in the field.

2. Method by Tangents.

Erect a perpendicular to AB at a and run it a sufficient distance ac. Measure the angle acb. Then $ab = ac \times \tan acb$. If ac is made 100 or 1,000, ab may be read directly from the table of natural tangents, observing to put the decimal point in the proper place. If $acb = 45°$ then $ab = ac$.

FIG. 25.

3. Method by Sines.

From a run a line ac as most convenient. Measure the angles acb and cab and the side ac. Compute the angle abc. Then $\sin abc : \sin acb = ac : ab \quad \therefore \quad ab = \dfrac{ac \sin acb}{\sin abc}$.

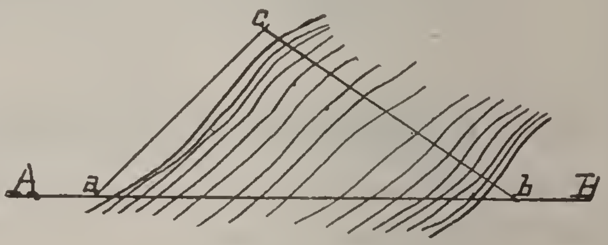


FIG. 26.

4. Method by Cosines.

From a run a line ac to the point c in a line perpendicular to AB at b.

Measure the angle cab and the line ac.

Then $ab = ac \times \cos cab$.

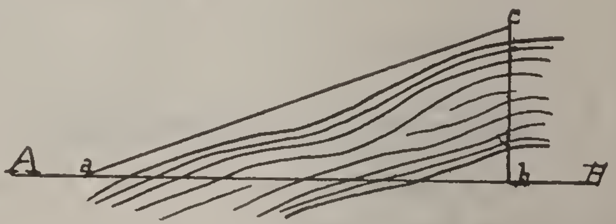


FIG. 27.

5. Method by Secants.

Run ac as before, to a point c from which a perpendicular to ac will strike the the point b. Measure ac and the an-

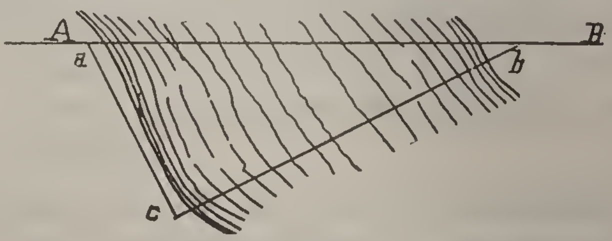


FIG. 28.

gle *bac*. Then $ab = ac \times$ secant *bac*. If $ac = 100$ or 1,000 the distance *ab* is taken directly from the table.

6. By 5° 43′ Angle.

From *a* lay off the angle $bac = 5°\ 43'$, making *bc* perpendicular to *ab*. Measure *bc*. Then $ab = 10bc$. This method gives results too large by 1.07 in 1,000.

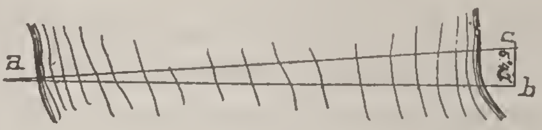


FIG. 29.

CASE 2ND. *Where the points are invisible from each other.*

1. If visible and accessible from a common point *c* outside the line. Measure the lines *ac* and *bc* and the angle *acb*. Subtract this angle from 180° and we have the sum of the remaining angles of the triangle, to find the difference.

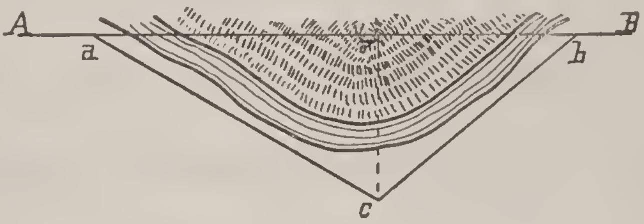


FIG. 30.

Then $ac + bc : ac - bc = \tan \dfrac{abc + bac}{2} : \tan \dfrac{abc - bac}{2}$

And $\dfrac{abc + bac}{2} + \dfrac{abc - bac}{2} = abc.$

Also $\dfrac{abc + bac}{2} - \dfrac{abc - bac}{2} = bac.$

$ab = ac \times \cos bac + bc \times \cos abc.$

If *a* and *b* are inaccessible from *c*, the sides *ac* and *bc* may be measured by any of the preceding methods.

2. If instead of two lines *ac* and *bc*, we have a broken line of any number of courses, as *a b c d e f*, the bear-

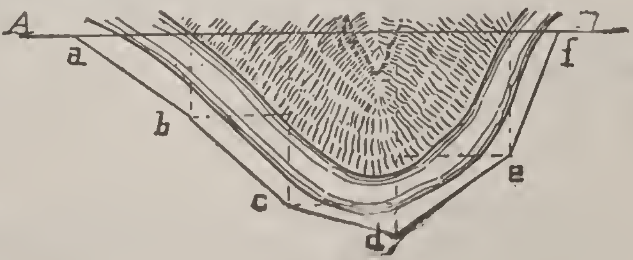


FIG. 31.

ings of which are referred to the line af as a meridian —then the algebraic sum of the products of the cosines of the several bearings into their respective distances will be equal to af.

In the United States Surveys distances across lakes and bends of large streams are frequently computed from the latitudes and departures of the courses around them.

Examples.—1. In Fig. 24 $ac = 100$ $ad = 27$. Required ab. *Ans.* 370.37+

2. Same Figure, $ac = 250$, $ad = 96$. Required ab. *Ans.* 651.04+

3. Fig. 25, $ac = 100$, angle $c = 61° 20'$. Required ab. *Ans.* 182.9.

4. Same Figure, $ac = 250$, angle $c = 61° 10'$. Required ab. *Ans.* 454.1+

5. Fig. 26, $ac = 500$, angle $a = 48° 20'$, angle $c = 118° 10'$. Required ab. *Ans.* 1011.+

6. Same Figure, $ac = 658$, $a = 54° 16'$, $c = 88° 32'$. Required ab. *Ans.* 1087.9+

7. Fig. 27, $ac = 1,000$, angle $a = 28° 35'$. Required ab. *Ans.* 878.12+

8. Same Figure, $ac = 950$, angle $a = 18° 56'$. Required ab. *Ans.* 898.6.

9. Fig. 28, $ac = 100$, angle $a = 76° 40'$. Required ab. *Ans.* 433.6+

10. Same Figure, $ac = 250$, angle $a = 56° 20'$. Required ab. *Ans.* 450.97.

11. Fig. 29, $ac = 900$, $bc = 648$, angle $c = 112°$. Required ab. *Ans.* 1291.

12. Given the following courses and distances along a broken line between the points a and b. Required the distance ab.

1. N. 18° E. 6.25 chains.
2. N. 40° E. 8.00 "
3. N. 5° W. 12.00 "
4. N. 44° W. 8.68 " *Ans.* 30.26+ chains.

13. The field notes of the meanders of a lake in sections 11 and 12 in the township 1, south, range 10 west, meridian of Michigan, — by the government survey, read as follows:

Courses	Chs. Lks.	Began at post in line of sections 11 and 12 on south side of lake: thence in sec. 12.
N. 58 E.	10.00	
N. 11 W.	20.00	
N. 63 W.	5.16	to post in line of sec. 11 and 12, N. side of lake.
N. 63 W.	5.00	in section 11.
S. 60 W.	6.00	
S. 14 E.	10.00	
S. 33 W.	15.00	
S. 51 E.	10.00	
N. 73½ E	7.90	to place of beginning.

Required the distance between the posts on the opposite sides of the lake. Compute the distance by the meanders on each side of the lake. Compare the results together, and also with the distance returned in the field notes which is 27.27 chains.

14. There is a cliff beside a railroad in the Wasatch Mountains known as the Castle Gate. Desiring to know its height above the railroad grade I set up the transit at Station 744 of the railroad survey and took the angle of elevation to the top of the cliff = 38° 42′. Elevation of station 744 = 6573.62 ft.

Height of instrument above station 744 = 4.84 ft.

I next went to station 748 in the line with and 400 ft. farther away from the cliff and again took the angle of elevation to the top of the cliff = 26° 15′.

Elevation of station 748 = 6567.62 ft.

Height of instrument above the station, 4.56 ft.

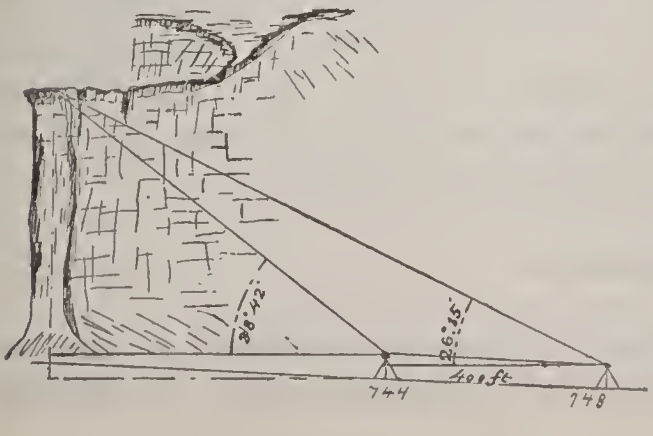


FIG. 32.

Required the height of the Castle Gate above the station 744 and its horizontal distance.

Answer.

Height 501.54.
Distance 620.

15. On Christmas 1881 a party of surveyors climbed a mountain peak, erected a monument on its summit and named it Christmas Peak. Observations from the line of the railroad survey were made as follows, the stakes of that survey being 100 feet apart:

From station 933 + 49.6 P. T.
Angle of elevation of summit, 23° 42′.
Angle to right from railroad line ahead, 76° 10′.
Elevation of station, 5005.28 ft.
Instrument above station, 4.82 ft.
From station, 940 + 31.4 P. C.
Angle to left from railroad line back = 82° 18′. Required the height of the peak and its distance from station 933 + 49.6.

3. Other Methods of Measuring Distances.

1. *To cross a stream or pond.*

Set up the transit at a convenient point, *a*. Set up a rod at *b* in the line, at a convenient distance, as 100 feet, from *a*. Set up a second rod in line at *c*, over the stream. Any plain, straight rods will answer. Leveling rods with targets are convenient. They should be set up plumb. Mark points *d* and *e*, in line, on the rods where the horizontal wire of the telescope cuts them. Raise or lower the telescope and mark two other points, *f* and *g*, in line on the rods where the wire cuts them. Measure *df* and *eg*. Then *adf* and *aeg* are similar triangles, and $df : af :: eg : ag$. If $df = 1$ and $af = 100$, $eg = 6.25$; then $ag = 625$.

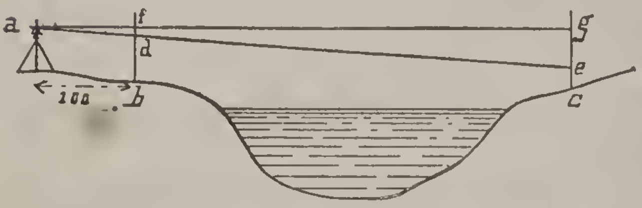


FIG. 33.

2. Stadia Measures.

1. Instead of using two rods as described in the last paragraph, two wires are sometimes placed in the dia-

phragm of the telescope and adjusted at such a distance apart that they will cover a specified space on a rod, as 1 foot when the rod is 100, 200 or any other specified distance away. These wires are one on each side of and parallel with the horizontal wire of the telescope. They may be either fixed on the diaphragm or attached to slides by which their distance apart may be adjusted. When the wires are adjusted to cover a certain space, as one foot on a rod placed 100 feet away, they will cover two feet on a rod 200 feet away, or .5 foot on a rod 50 feet away. This proportion is strictly true only when the measures are taken from a point in front of the instrument at a horizontal distance from the object glass equal to its focal length. The focal length may be found nearly enough by measuring from the plane of the object glass to the capstan-headed screws which carry the diaphragm. When the telescope is focused on some very distant object, as the moon or a star, the horizontal distance from the plumb line to the point mentioned forms a constant which is to be added to all the distances as taken from the rod.

2. It is more convenient, though less accurate, to adjust the wires so that they will cover the required space on the rod at a specified distance measured from the center of the instrument. This method is usually adopted on the government surveys, where stadia measures are taken, the length of the base being taken at about a mean of the distances which the stadia is intended to measure. For all shorter distances the reading is too small. For longer distances it is too large. The error is neglected as of no consequence in the class of work for which the stadia is used.

When the stadia wires are not adjustable the rod is graduated to conform to the wires. A rod is set up at the selected distance from the transit. The space intercepted on it by the wires is subdivided decimally, and the stadia rod graduated to that scale.

Where the wires are adjusted to cover a foot on a rod

100 or 200 feet away, the ordinary leveling rod answers the purpose of a stadia rod.

3. In case the measures are not on horizontal lines it will be necessary to apply a correction to the stadia readings to reduce them to the horizontal. *If the rod has been held perpendicular to the line of sight, the horizontal distance is found by multiplying the distance to the rod by the cosine of the angle of elevation or depression.*

The position of the rod is determined either by a right-angle sight applied to the rod, or by the rodman slowly moving the top of the rod back and forth until the smallest intercept is obtained. On hillsides it will be found quite as easy to hold the rod perpendicular to the line of sight as to hold it plumb.

When the rod is held plumb and the base is measured from the point in front of the transit the reduction to horizontal is made as follows:

Let f = focal distance of the telescope,

r = space intercepted on the rod as held vertically,

s = image of the same intercepted by the stadia wires,

CO' = line of sight at an angle e with the horizon.

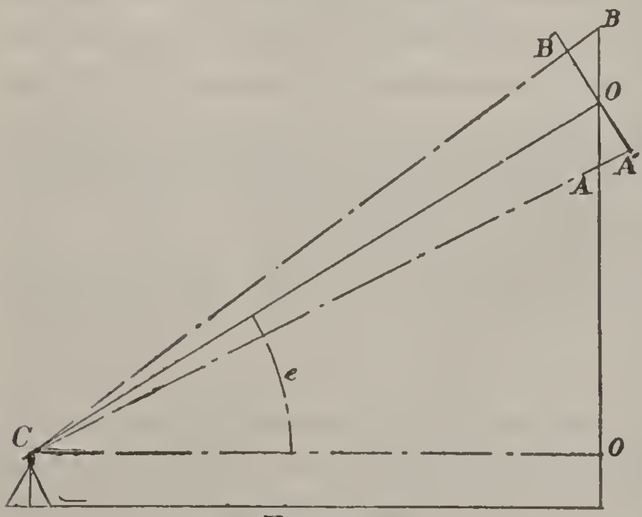


FIG. 34.

Let $A'B' = r'$ be the intercept on the rod as inclined at an angle e with the vertical; and let $b' = f\dfrac{r'}{s}$ be the corresponding base. Let the angle $O'CB$ or $O'CA = v$. We shall then have:

Angle $OCB = e + v$, and angle $OCA = e - v$, whence angle $OBC = 90° - (e + v)$, and angle $OAC = 90° - (e - v)$. The angle $O'B'B = 90° + v$, and angle $O'A'A = 90° - v$.

In the triangle $O'B'B$ we have

$$\frac{O'B'}{O'B} = \frac{\sin [90° - (e + v)]}{\sin (90° + v)} \quad \text{or,} \quad \frac{r'}{2O'B} = \frac{\cos (e + v)}{\cos v} \quad (a)$$

In the triangle $O'A'A$ we have

$$\frac{O'A'}{O'A} = \frac{\sin [90° - (e - v)]}{\sin (90° - v)} \quad \text{or,} \quad \frac{r'}{2O'A} = \frac{\cos (e - v)}{\cos v} \quad (b)$$

Adding (a) and (b), we obtain

$$\frac{r' r}{2O'B \times O'A} = 2 \cos e \quad (c).$$

Multiplying (a) and (b) together, we obtain

$$\frac{r' r'}{4\, O'B \times O'A} = \frac{\cos^2 e \cos^2 v - \sin^2 e \sin^2 v}{\cos^2 v} \quad (d)$$

Dividing (c) by (d), we have, after a little reduction,

$$\frac{r}{r'} = \frac{\cos e}{\cos^2 e - \sin^2 e \tan^2 v}, \quad (e)$$

which is an expression of the relation sought.

Cor.—With the wires adjusted to one foot on the rod for a base of 100 feet, we should have

$$\tan v = 0.005 \text{ ft., or } \tan^2 v = 0.000025 \text{ ft.}$$

Thus, $\tan^2 v = 0$, without material error.

Whence formula (e) becomes $r' = r \cos e$.

To find the distance CO' we have

$$CO' = d' = f\frac{r'}{s} + f + c = b' + f + c.$$

Whence, $CO = d = (b' + f + c) \cos e$.

For vertical rod we have, $b' = b \cos e$.

Whence, $d = b \cos^2 e + (f + c) \cos e$. (f)

The height $OO' = h = \frac{1}{2}b \sin 2e + (f + c) \sin e$. (g)

Example.—Given $e = 10° 30'$, $r = 5.36$ ft., and $f + c = 1$ ft., to find d and h.

Solution.—Suppose the wires adjusted to give 1 ft. on the rod to the 100 ft., whence $b = 536$ ft.

Cos $e = 0.983$ and $\cos^2 e = 0.9668$.
Whence, $d = 536 \times 0.9668 + 0.98 = 519.18$ ft.
Sin $e = 0.182$, and $\frac{1}{2} \sin 2e = 0.1792$.
Whence, $h = 536 \times 0.1792 + 0.18 = 96.23$ ft.

Formula (f) may be put in the form
$d = b \cos^2 e + (f + c) \cos^2 e + (f + c) \cos e (1 - \cos e)$.
Dropping the last term, we have

$$d = (b + f + c) \cos^2 e. \quad (h)$$

Assuming $f + c = 1$ ft. as a mean value in different instruments, the omission of the term $(f + c) \cos e (1 - \cos e)$ introduces an error for ordinary elevations of less than 0.01 ft. in a base of 1000 ft.

Moreover, the use of formula (h) operates to diminish the very minute error introduced by use of formula (f)

For slight elevations, as from 1° to 2°, the reduction to horizontal may be omitted. For 5° 44′ the amount of the reduction is about one per cent. The correction for horizontal measurement is sometimes made by omitting to add $f + c$ to the base.

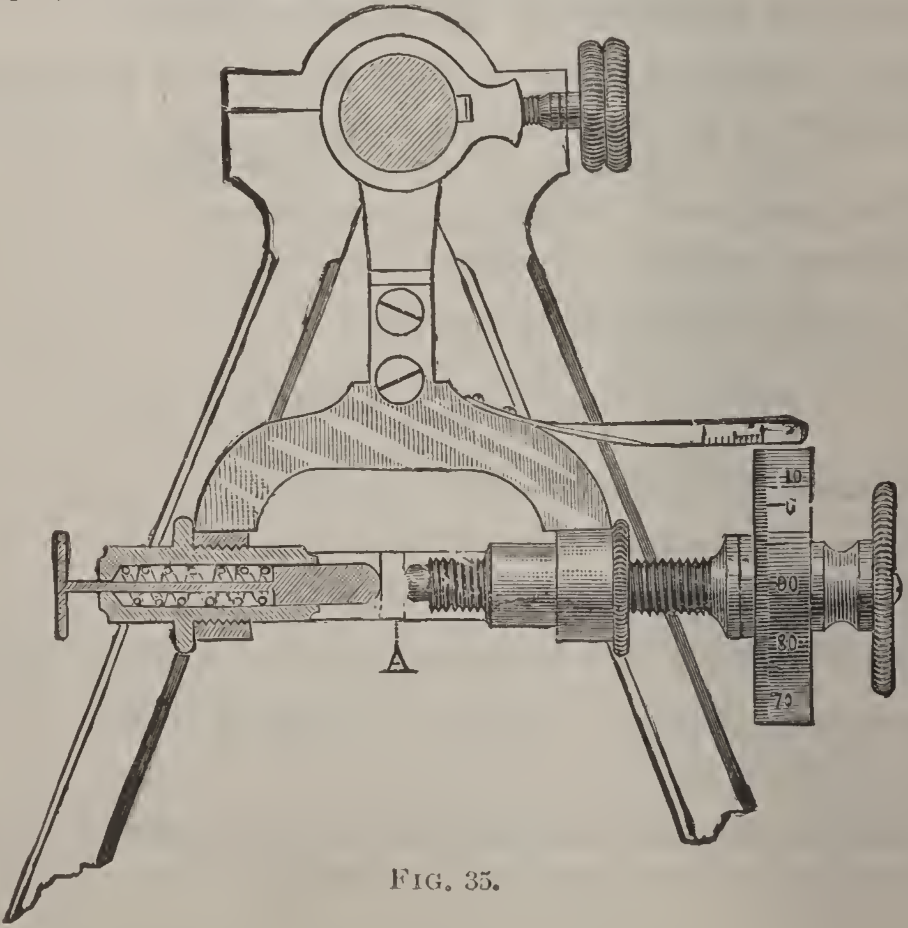


FIG. 35.

4. **The Gradienter** is an attachment to the transit for fixing grades and determining distances.

As made by Gurley, it consists of a screw attached to the semicircular expanded arm of the ordinary clamp of the telescope axis; the screw is accurately cut to a given number of threads, and passing through a nut in one side of the arm, presses against a little stud, *A*, fixed to the inside surface of the right-hand standard.

In the other side of the semicircular arm is inserted a hollow cylinder containing a pin actuated by a strong spiral spring, the end of the pin pressing against the side of the stud opposite that in contact with the screw.

Near the other end of the screw, and turning with it, is a wheel, or micrometer, the rim of which is plated with silver, and divided into 100 equal parts.

A small silver scale, attached to the arm and just above the micrometer wheel, is divided into spaces, each of which is just equal to one revolution of the screw; so that by comparing the edge of the wheel with the divisions of the scale, the number of complete revolutions of the screw can be easily counted.

It will be seen that when the clamp is made fast to the axis of the clamp-screw, and the gradienter-screw turned, it will move the telescope vertically, precisely like the tangent-screw ordinarily used.

And as the value of a thread is such that a complete revolution of the screw will move the horizontal cross-wire of the telescope over a space of one foot on a rod at a distance of one hundred feet, it is clear that when the screw is turned through fifty spaces on the graduated head, the wire will pass over fifty one-hundredths, or one-half a foot on the rod, and so on in the same proportion.

In this way, the gradienter can be used in the measurement of distances, precisely like the stadia.

Grades can also be established with great facility, as follows: Level the instrument; bring the telescope level to its centre by the clamp and gradienter screw; move the graduated head until its zero is brought to the edge of the scale, and then turn off as many spaces on the head as there are hundredths of feet to the hundred in the grade to be established.

Having a transit with gradienter attachment, let the student solve the following problems in the field:

Prob. 1. *To find the grade between two points.*

SUGGESTIONS.—Set the instrument over one of the points, level the plates and the telescope, and bring the zero of the screw to the edge of the scale.

Set the target of the leveling rod at height of instrument.

With the rod held upon the other point, note the number of revolutions of the screw required in bringing the cross-wire upon the center of the target. That number, as so many feet, is the grade.

Prob. 2. *To find the distance between two points.*

SUGGESTIONS.—Set up and adjust the parts of the instrument as in Prob. 1. On a leveling rod held upon the other point, note the number of feet covered by one revolution of the screw, and multiply that number by 100.

If, in order to cover r feet on a rod at a distance of d feet, n revolutions of the screw are required, then we should have: $d : 100 :: r : n$; whence $d = 100\, r \div n$.

Example.—Given $n = 2.30$ and $r = 5$ ft., to find d.

Result, $d = 217.39$ ft.

On inclined ground the horizontal sight line may be above or below the rod. In such cases, as in stadia measurement, a formula of reduction to a horizontal is employed, which may be deduced as follows:

Let $CO = d$ (Fig. 34), be a horizontal sight line;

Angle $OCO' = e$, the elevation of telescope to foot of rod;

Angle $O'CB = v$, the angle described by n revolutions of the screw;

$O'B' = r'$, the space on a rod perpendicular to CO', subtending angle v, and

$O'B = r$, the corresponding space on a vertical rod.

We shall then have, [Formula (a)],

$$\frac{r'}{r} = \frac{\sin [90° - (e + v)]}{\sin (90° + v)} = \frac{\cos e \cos v - \sin e \sin v}{\cos v}.$$

Whence, $r' = r (\cos e - \sin e \tan v)$.

Let $CO' = d'$. Then, $\tan v = \frac{r'}{d'} = \frac{n}{100}$.

Whence, $d' = \frac{100\, r'}{n} = \frac{100\, r}{n} \left\{ \cos e - \sin e \times \frac{n}{100} \right\}$

or $\quad d' = r \left\{ \frac{100 \cos e}{n} - \sin e \right\}. \quad (1)$

Now, $d = d' \cos e$.

Whence, $d = r \left\{ \frac{100}{n} \cos^2 e - \frac{1}{2} \sin 2e \right\}. \quad (2)$

Cor.—If $n = 1$, we have,

$$d' = r\,(100 \cos e - \sin e), \qquad (3)$$

and $d = r\,(100 \cos^2 e - \frac{1}{2} \sin 2e), \quad (4)$

in which r is the space on a vertical rod included by *one* revolution of the screw.

The numbers by which this value of r must be thus multiplied for various elevations are given in Table IX.

Examples.—1. Given $e = 15° 20'$, and $r = 5.42$ for one revolution of the screw, to find d' and d.

Solution.—We find in Table IX,

factor for inclined distance for				$15° =$	96.33
"	"	"	"	$15° 30' =$	96.09
		Difference for		$30' =$	0.24
whence,		"	"	$20' =$	0.16

Whence, factor for inclined distance for $15° 20' = 96.17$.
Accordingly, $d = 5.42 \times 96.17 = 521.24$ ft.

Again, in Table IX we have

$$\begin{array}{rr} \text{factor for horizontal distance for } 15° = & 93.05 \\ \text{“ “ “ “ } 15° 30' = & 92.59 \\ \hline \text{Difference for } 30' = & 0.46 \\ \text{whence, “ “ } 20' = & 0.31 \end{array}$$

Whence, factor for horizontal dist. for $15° 20' = 92.74$.

Hence, $d = 5.42 \times 92.74 = 502.65$ ft.

2. Given $e = 10.35$ rev. to foot of rod, and $r = 6.25$, to find d' and d.

Suggestion.—From Table X find the angle e, and solve as above.

When e is an angle of depression, the point O' is the upper end of the rod. The application of the formula is, however, the same in this case as in the one considered.

Stadia and Gradienter Measurements are found very convenient in solving some of the problems in land surveying, but are almost useless in others. They save time and trouble in measuring across streams, bogs and other places inaccessible to the chain or tape. They furnish a quick and easy means of determining how far it is to an object, but a slow one of locating points at any desired distance, such as setting stakes for a town plat, a ditch line, or a railroad.

CHAPTER VI.

Platting and Computing Areas.

1. **A Plat** or **Plot** is a representation, upon a small scale, of the lines of a survey. Platting is simply surveying on paper. The instruments used are analogous to those used in the field.

Lines are marked upon the paper with pencil or pen and ink. Generally they will first be drawn lightly in pencil; afterward the permanent lines will be inked, and all erroneous or superfluous lines erased. Pencils hard enough to hold a fine point without breaking are the best for this use.

The right line pen is used for drawing straight lines. It is made in various sizes and forms. One of the best is shown at *b*, in Figure 36.

The scale of equal parts is the counterpart of the chain or tape. A great variety of scales are made. One of the most useful is the triangular scale (Fig. 36, *e*). It has six different graduations, all brought to the edge, so that the scale may be laid down on the paper and the distance marked off directly from the scale. The scale in which the inch is divided into 10, 20, 30, 40, 50 and 60 equal parts is the one most useful to the surveyor. Paper scales are made on fine Bristol board, with any graduation desired. They are cheap, and as good as any scale as long as they last. The student may make his own scales on paper.

The protractor (Fig. 36, *a*) takes the place of the compass or transit. It is simply the whole or part of a graduated circle or limb. Protractors are made in a great variety of forms. One of the cheapest and best has the

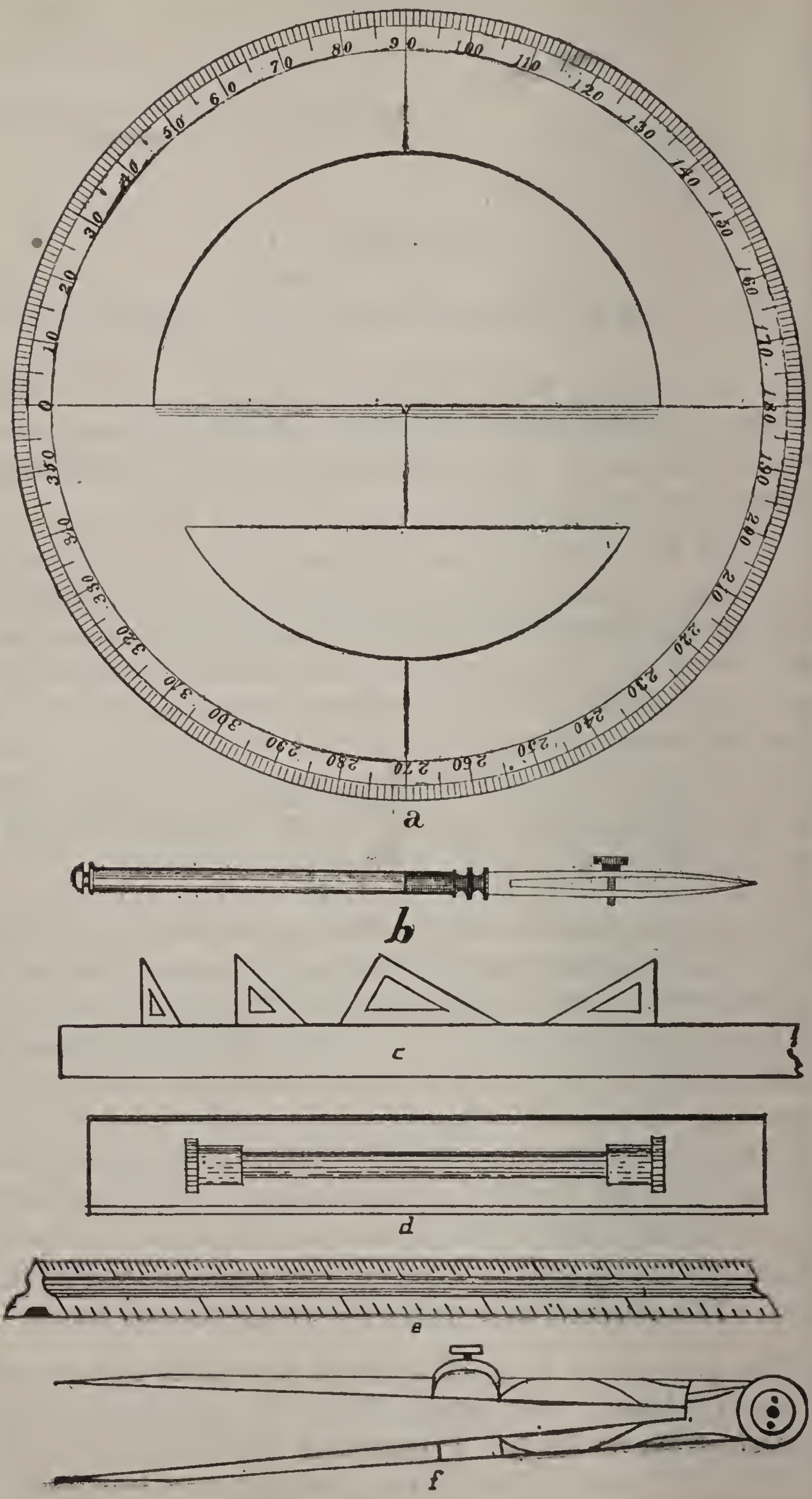


FIG. 36.

entire circle graduated to quarter degrees. It is made of paper, has the middle part cut out, and fine threads or wires crossing at the centre of the circle. A paper protractor 14 inches in diameter, graduated to quarter degrees, costs from 30 to 40 cents.

Dividers, (Fig. 36, *f*) are used to space off distances on the plat, or transfer distances from the scale to the plat or the reverse. When provided with pen or pencil points they are used to strike circles and arcs. When they are used for the latter purpose they should have a needle point on the stationary leg.

Parallel rulers, as the name indicates, are used in drawing parallel lines. When a paper protractor is used in platting, it is found convenient to fasten it at some point outside the plat and transfer the bearing of the lines from the protractor to the plat by means of the parallel rule. The best rule for this purpose moves upon rollers, (Fig. 36, *d*.)

The straight-edge ruler and *triangle* are also used to mark parallel lines, as well as to lay off angles. Many other articles will be found convenient in platting. A drawing board, made of the softest wood, planed smooth and true, and thumb-tacks to fasten the paper to the board, may almost be considered as necessaries. Neither the student nor surveyor needs many instruments for platting, but those he has should be perfect in their kind. It is not deemed necessary at this point to give further details of these instruments and their uses, any suggestion which the student may need being left to the teacher to make.

EXERCISES.

The first seven exercises are the elementary problems of Geometry, and are designed to be solved on paper by use of the dividers and ruler.

2. 1. To draw a straight line equal to a given straight line.

2. To make an angle equal to a given angle.

3. To draw through a given point a line parallel to a given line.

4. To draw through a given point a line perpendicular to a given line. Two cases.

5. To bisect a given line; a given angle.

6. To construct lines proportional to given lines.

7. To construct a polygon similar to a given polygon.

8. Plat the following lines:

(1) 8 chains, to scale of 2 chains to the inch.
(2) 10 chains, to scale of 5 chains to the inch.
(3) 10 chains, to scale of 4 chains to the inch.
(4) 17.25 chains, to scale of 3 chains to the inch.
(5) 25.40 chains, to scale of 4 chains to the inch.

9. Plat a triangle whose sides are 13.50 ch., 14.25 ch. and 16.20 ch., on a scale of 5 chains to an inch; on a scale of 3 chains to an inch.

10. Plat a rectangle whose adjacent sides are 9.24 ch. and 13.78 ch., on a scale of 4 chains to the inch.

11. Plat a quadrilateral the sides of which are 22.60 ch., 14.35 ch., 12.20 ch. and 9.80 ch., on a scale of 4 chains to the inch, and having one angle of 83° 30′.

12. Measure the remaining angles and find their sum.

13. Plat any figure having five equal sides; measure the interior angles and find their sum.

14. Plat a right triangle having a base of 16.25 ch. and a perpendicular of 8.60 ch. Find the remaining side and angles of the triangle.

II. Computing Areas.

In land surveying the areas are computed in triangles and quadrangles. If a field has more than four sides, in making the computation it is parted off into triangles and rectangles or trapezoids, the area of which is computed and their sum taken.

1. Area of Triangles.

1. To find the area of a right angled triangle.

Multiply the base by one-half the perpendicular.

2. *To find the area of an oblique angled triangle.*

CASE 1ST.—*When the sides are given.*

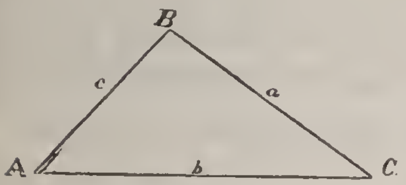


FIG. 37.

Let A, B, C represent the angles, and a, b, c the sides opposite them.

Let $\frac{a+b+c}{2} = s$. Let $x =$ area.

Then $x = \sqrt{s(s-a)(s-b)(s-c)}$.

CASE 2ND.—*Having two sides and the included angle.*

Let a, b be the sides, C the given angle, and $x =$ area. From B drop a perpendicular, d, to the side b. This divides the triangle into two right triangles, the area of each of which equals its base multiplied by half the perpendicular, d, and the sum of their areas equals the sum of their bases multiplied by half the perpendicular; that is, $x = \frac{bd}{2}$. But $d = a \sin C$. Hence, $x = \frac{ab \sin C}{2}$.

CASE 3D.—*Given two angles and the included side.*

Let A and B be the angles, and c the side given. Find $C = 180° - (A + B)$. Find b.

$\sin C : \sin B :: c : b \quad \therefore\ b = \frac{c \sin B}{\sin C} \qquad x = \frac{bc \sin A}{2}$.

CASE 4TH.—*Given two angles and a side opposite*, (A, B and a.)

Find $C = 180° - (A + B)$. Find $c = \frac{a \sin C}{\sin A}$.

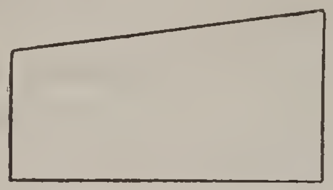
Find $b = \frac{a \sin B}{\sin A}$. Then $x = \frac{bc \sin A}{2}$.

2. Areas of Quadrangles.

CASE 1ST.—*Squares and rectangles.*

Multiply the base by the perpendicular.

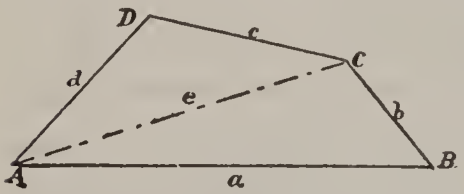
CASE 2ND.—*Trapezoids.* A trapezoid is a figure having four sides, only two of which are parallel.



Trapezoid. FIG. 38.

Its area is equal to the half sum of the parallel sides, multiplied by the perpendicular distance between them.

CASE 3RD.—*Trapeziums* have no two sides parallel.



Trapezium. FIG. 39.

The area is found by parting off into triangles and computing their areas.

1. *Having the sides and angles given.*

Let A, B, C, D represent the angles, and a, b, c, d the sides of the trapezium. Let AC be a diagonal dividing the trapezium into the triangles ABC and ADC. In each of these we have two sides and an included angle given;

hence, $x = \frac{ab \sin B}{2} + \frac{cd \sin D}{2}$.

2. *Given the diagonals of a quadrilateral and an angle formed by their intersection, to find the area.*

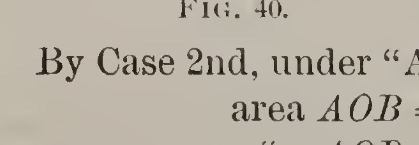


FIG. 40.

Solution.—Let $ABCD$ be the quadrilateral, m and n its diagonals, and O an angle at which the diagonals intersect.

By Case 2nd, under "Area of Triangles,"

$$\text{area } AOB = \tfrac{1}{2} AO \times BO \sin O$$
$$\text{" } AOD = \tfrac{1}{2} AO \times DO \sin O$$
$$\text{" } DOC = \tfrac{1}{2} CO \times DO \sin O$$
$$\text{" } BOC = \tfrac{1}{2} CO \times BO \sin O.$$

Whence, by addition, area $ABCD = \frac{1}{2}(AO + CO) \times (BO + DO) \sin O$,

or, area $ABCD = \frac{mn \sin O}{2}$

Example.—The diagonals of a four-sided field were found to measure 18 ch. and 24 ch. Setting a compass at their intersection, the bearings of two adjacent corners of the field were found to be N. $30\frac{1}{2}°$ E. and S. 50° E. Required the area of the field.

Solution.—Applying logarithms in the above formula, having found $O = 99\frac{1}{2}°$, we have

$m = 18$	log	1.255273
$n = 24$	log	1.380211
$O = 99\frac{1}{2}°$	log sin	9.994003
2	ar. co. log	9.698970
Area = 213.03	log	2.328457

or, Area = 21.303 A.

3. *Given three sides, a, b, d, and the included angles, A and D.* (See Fig. 39.)

Let $AC = e$, be a diagonal. Let the angle $BCA = E$, $BAC = F$, and $CAD = G$. In the triangle ABC the sides a, b and angle B are known. In the triangle CAD the side d only is known. It is required to find the side e and the angle G. To find G: $E + F = 180° - B$. By trigonometry, $a + b : a - b :: \tan\dfrac{E+F}{2} : \tan\dfrac{E-F}{2}$, by which we find the sum and the difference of the angles E and F. $\dfrac{E+F}{2} - \dfrac{E-F}{2} = F$, and $G = A - F$.

To find e: $\text{Sin}\, F : \sin B :: b : e \quad \therefore e = \dfrac{b \sin B}{\sin F}$.

$$x = \frac{ab \sin B}{2} + \frac{cd \sin D}{2}.$$

4. This method of finding the area of a trapezium may be applied to polygons of any number of sides, when the sides and angles are given. The polygon is divided into triangles two less in number than the number of sides Each triangle has two sides and the included angle given or readily found.

Take for example the irregular polygon of eleven sides shown in Fig. 41, which is divided into nine triangles.

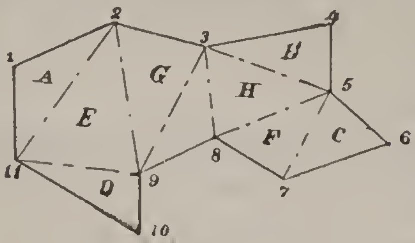


FIG. 41.

In the triangles A, B, C and D two sides and the included angle of each are given. From the remaining sides and angles we find two sides and the included angle of the triangles E and F, and so each triangle in turn furnishes the data for computing the adjacent triangle, till all are complete.

3. Offsets.—When it is desired to find the area of a field having irregular sides, such as along a stream or lake, it is well to run a straight line where most convenient to do so, and then run and measure perpendiculars to the margin of the field. These are called offsets. They divide the space between the straight line and the margin of the field into triangles and trapezoids, whose areas may be computed separately and the sum taken.

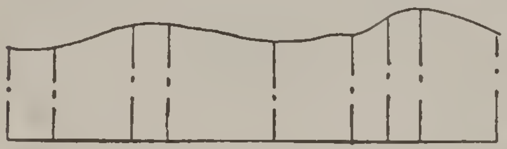


FIG. 42.

If the offsets are equidistant the area may be found by the following

RULE.—*From the sum of the offsets, subtract the half sum of the extreme ones, and multiply the remainder by the common distance between them.*

4. What is the area in acres of the following right angled triangles?

1. Base = 23.20 ch., perpendicular = 14.60 ch.? *Ans.* 16.936 *A*.
2. Base = 19.46 ch., perpendicular = 12.18 ch.?

What is the area in acres of the following oblique angled triangles: (See Fig 37.)

3. $a = 14.26$ ch., $b = 19.40$ ch., $c = 12.18$ ch.? *Ans.* 8.666 *A*.
4. $a = 9.43$ " $b = 11.61$ " $c = 8.42$ "

5. $a = 6.23$ " $b = 14.26$ " $C = 22° 40'$? *Ans.* 1.71+ *A.*
6. $a = 12.20$ " $b = 20.00$ " $C = 36° 15'$?
7. $A = 16° 45'$, $B = 82° 30'$, $c = 21.16$ ch.? *Ans.* 6.458+ *A.*
8. $A = 35°$, $B = 62° 42'$, $c = 18.20$ "
9. $A = 46°$, $B = 58° 15'$, $a = 26.50$ " *Ans.* 40.264 *A.*
10. $A = 37° 20'$, $B = 72° 40'$, $a = 19.36$ "

11. A square field is 6.25 chains on a side. Required its area.

12. A square field contains 20 acres. What is the length of its sides? *Ans.* 14.142 *ch.*

13. What is the area of a rectangle whose sides are 16.41 and 8.26 chains?

14. A rectangular field containing 16 acres measures 12.50 chains on the base. What is the perpendicular? *Ans.* 12.80 *ch.*

15. Commencing on the margin of a river a line was run across a bend 20.00 chains to the margin. Commencing at the end of the second chain, offsets were taken every two chains, to the margin of the river, as follows: 1.61 ch., 2.27 ch., 3.72 ch., 1.96 ch., 4.23 ch., 2.92 ch., 3.26 ch., 2.50 ch. and 1.25 ch. Required the area between the line and the river. *Ans.* 4.744 *acres.*

16. Required the area of a field bounded as follows:

1st. North 17.65 ch.
2nd. S. 36° 12′ W. 8.20 ch.
3rd. S. 12° 34′ W. 7.26 "
4th. S. 58° 26′ E. 7.53½ "

Suggestion.—First: Change bearings into angles between the lines and compute as two triangles.

Second: Take the first line as a base, divide the figure into two right angled triangles and a trapezoid, and compute the area. Compare the two methods as to number of figures required for the solution.

17. The sides of a pentagon measure 6.25 chains each. What is its area?

Suggestion.—Part the figure into three triangles and compute. Also part into five isosceles triangles. Compute and compare the two methods.

5. 1. Rectangular Coordinates.—Let XX' and YY' be two lines intersecting each other at right angles, as at O.

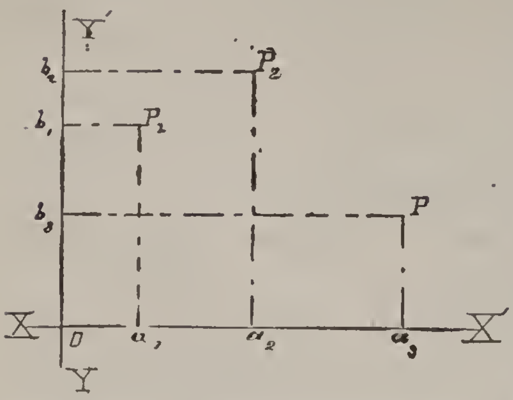


FIG. 43.

Let P_1, P_2, P_3 be any points in the plane of the lines.

Let P_1a_1, P_2a_2, P_3a_3 be perpendiculars from the points upon the axis XX', and P_1b_1, P_2b_2, P_3b_3 be perpendiculars from the points upon the axis YY'.

The distances Oa_1, Oa_2, Oa_3 are called **Abscissas** of the points P_1, P_2, P_3; and the distances Ob_1, Ob_2, Ob_3 are called **Ordinates** of the points.

The point O is called the **Origin.**

The abscissa and ordinate of a point are together called **Coordinates** of the points.

Coordinates at right angles with each other are called **Rectangular Coordinates.**

It is customary to denote abscissas by x and ordinates by y, coordinates of different points in connection with each other being distinguished by use of subscripts.

Thus, of the point P_1, the coordinates Oa_1 and Ob_1 or a_1P_1 may be denoted by x_1 and y_1; of the point P_2, the coordinates Oa_2 and Ob_2 or a_2P_2 may be denoted by x_2 and y_2; and so on.

It will be seen that the coordinates of a point afford the means of locating it with respect to the axes.

The use of longitude and latitude in Geography is an illustration.

By use of the signs + and —, the coordinates of any point in the plane of the axes are readily expressed.

EXERCISES.

2.—1. Construct the point of which $x = 4$ and $y = 7$.

2. Given $x = -5$ and $y = 3$, to construct the point.
3. Given $x = -3$ and $y = -6$, to construct the point.
4. Given $x = 6$ and $y = -4$, to construct the point.
5. Given $x = 0$, $y = 2$; $x = -5$, $y = 0$; $x = 0$, $y = 0$. Required the points.

3. **Application to Area.**—Let it be required to find the area of a series of trapezoids included between perpendiculars from the points of a broken line upon a straight line. Suppose the straight line, as OX', to be an axis of abscissas, and the first perpendicular at the left, as OA, to be an axis of ordinates.

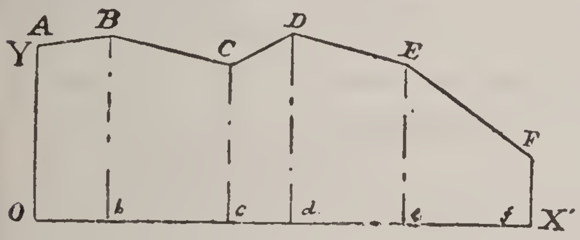


FIG. 44.

Let x_1, x_2, x_3, etc., be the abscissas of the points A, B, C, etc., and y_1, y_2, y_3, etc., the corresponding ordinates.

Accordingly, the area of the several trapezoids is

$$\tfrac{1}{2}\,[x_2\,(y_1 + y_2) + (x_3 - x_2)\,(y_2 + y_3) + (x_4 - x_3)\,(y_3 + y_4) + \text{- - -}\ (x_n - x_{n-1})\,(y_{n-1} + y_n)],$$

in which n is the number of trapezoids plus one.

The above formula may be changed to the form

$$\tfrac{1}{2}\,[x_2\,(y_1 - y_3) + x_3\,(y_2 - y_4) + x_4\,(y_3 - y_5) + \text{- - - - - -}\ x_{n-1}\,(y_{n-2} - y_n) + x_n\,(y_{n-1} + y_n)]. \quad (a)$$

Whence, for the area included between a straight line, as a base, and a broken line whose points are given by their coordinates upon the base, we have the following

RULE.—*From each ordinate subtract the second succeeding one and multiply the remainder by the abscissa corresponding to the intervening ordinate.*

Also, multiply the sum of the last two ordinates by the last abscissa.

Divide the algebraic sum of the products by 2.

The above formula and Rule have been deduced independently of any supposition as to the relative directions of the parts of the broken line. They are therefore true whatever may be the form of the broken line. That is, whether any part should be perpendicular to the base, either toward or from it, or whether any part should be turned backward respecting the preceding one.

SUGGESTION.—Let the student verify the rule in a case, for example, like the following, in which BC is represented as being parallel to the base, CD as perpendicular toward it, and FG as being turned backward from EF.

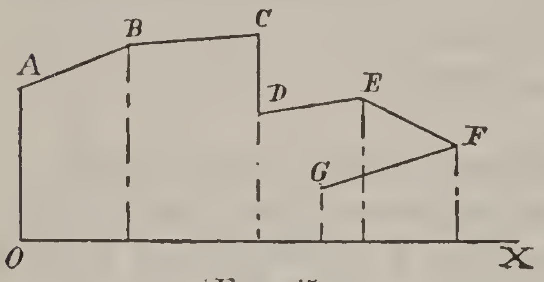


FIG. 45.

Find how it would be, if one or more of the ordinates were zero; if one or more were negative.

EXERCISES.

4.—1. Given $y_1 = 12$, $y_2 = 12$, $y_3 = 16$, $y_4 = 8$ and $y_5 = 6$, also $x_1 = 10$, $x_2 = 18$, $x_3 = 24$, $x_4 = 30$ and $x_5 = 20$, to find area.

Given the following, to find area:

(2)

140	1000
435	812
250	725
200	500
360	450
320	000

(3)

000	950
240	844
306	530
640	325
415	200
000	000

(4)

1000	200
1150	317
828	420
650	305
460	524
000	250

5. As a second example of the application of coordinates in finding area, let there be taken an ordinary polygon, as $ABCDEF$. (Fig. 46.)

Let x_1, x_2, x_3, etc., be the abscissas of the points A, B, C, etc., and y_1, y_2, y_3, etc., the corresponding ordinates.

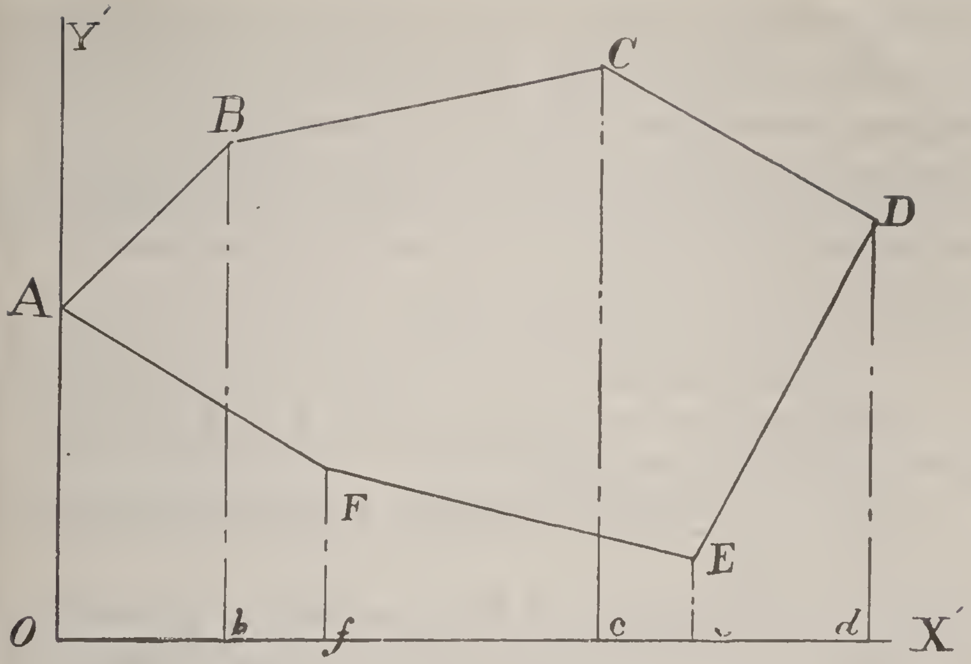


FIG. 46.

Now, since formula (a) is true for any broken line, it holds for the case in which the broken line beginning, as at A, returns to the same point, forming thus a polygon, as $ABCDEFA$.

In this case, the last term of (a) vanishes, and we have as the area a polygon of n sides,

$$\tfrac{1}{2}\left[x_1(y_n - y_2) + x_2(y_1 - y_3) + x_3(y_2 - y_4) + x_4(y_3 - y_5)\right.$$
$$\left.+ \text{ etc., to } n \text{ terms}\right]. \qquad (b)$$

or, factoring with respect to y, we have the form

$$-\tfrac{1}{2}\left[y_1(x_n - x_2) + y_2(x_1 - x_3) + y_3(x_2 - x_4) + y_4(x_3 - x_5)\right.$$
$$\left.+ \text{ etc., to } n \text{ terms}\right]. \qquad (c)$$

Whence, for the area of a polygon whose vertices are given by their coordinates, we have the following

RULE.—*From the ordinate of each vertex subtract the second succeeding one, and multiply the remainder by the abscissa of the intervening vertex;* or, *from the abscissa of each vertex subtract the second succeeding one, and multiply the remainder by the ordinate of the intervening vertex.*

Divide the sum of the products by 2

SCH.—Formulas (b) and (c) will be seen to be in accordance with any situation of the coordinate axes, agreeably with convenience of field work. In particular cases, one or more terms will be found to disappear. Due attention to algebraic signs is important.

The formulas are easy to remember, and simple of application. With an instrument adapted to laying off right angles, they afford a practical means of computing the contents of irregular tracts.

EXERCISES.

6. Required the area and a plat of a field the coordinates of whose corners are

$x_n = x_6 = 0$, $x_1 = 7$ ch., $x_2 = 12\frac{1}{2}$ ch., $x_3 = 18$ ch., $x_4 = 15$ ch., $x_5 = 10$ ch.; and

$y_n = y_6 = 6$ ch., $y_1 = 12$ ch., $y_2 = 20$ ch., $y_3 = 15$ ch., $y_4 = 8\frac{1}{4}$ ch., $y_5 = 0$ ch. Area, 16.175 *acres*.

Find the area, supposing a different situation of the axes.

7. *Given the lengths and bearings of the sides of a polygonal field, to find the area.*

Solution. — Let $ABCDE$ represent the field. Let NS denote the meridian of the most westerly station. This line, which may be assumed as passing through any station at pleasure, but more conveniently the extreme western or the eastern one, is called the **Principal Meridian.**

To the principal meridian let there be drawn from the several stations the per-

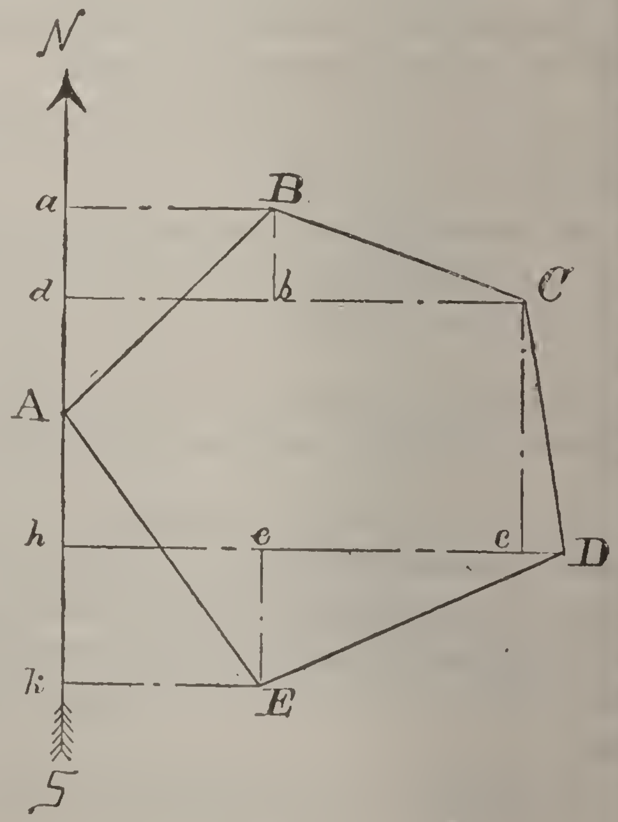


FIG. 47.

pendiculars Ba, Cd, Dh and Ek, and upon Cd and Dh let there be drawn the perpendiculars Bb, Cc and Ee.

These perpendiculars are, respectively, the bases and the altitudes of trapezoids composing a portion of the field.

Now, if from the sum of the areas of the trapezoids the sum of the areas of the triangles ABa and AEk be subtracted, the remainder will be the area sought.

That is, clearing of fractions,

$$2 \times \text{area pol.} = (aB + Cd)\,Bb + (dC + Dh)\,Cc + (hD + Ek) \times Ee - (aB \times aA) - (Ek \times kA.)$$

It is now to be considered how the dimensions of the trapezoids and triangles depend upon the lengths and bearings of the sides of the field.

8. **Latitude and Departure.**—For convenience of description, let it be supposed that a survey of the field above represented was made " with the land on the right," beginning at A.

In going from A to B, there was made a distance Aa, *north*, and a distance aB, *east;* in going from B to C there was made a distance Bb, *south*, and a distance bC, *east.* Finally, in going from E to A, there was made a distance kA, *north*, and a distance Ek, *west.* Distances made *north* are called **Northings**, and *south*, **Southings**; distances made *east* are called **Eastings**, and *west*, **Westings.** Northings and southings are together called **Latitudes,** and eastings and westings are called **Departures.**

It will be seen that the length of a course is the hypotenuse of a right triangle of which the latitude of the course is the side adjacent to the bearings, and the departure, the side opposite the bearing. Whence,

Latitude = length of course $\times$ cosine of bearing, and
Departure = length of course $\times$ sine of bearing.

From these fundamental formulas, several others expressing relations of either of the four quantities to two others are easily derived.

Thus, denoting the latitude by l, the departure by d, the length of course by c, and the bearing by b, is obtained the following

TABLE OF CASES.

No.	Given.	Required.	Formulas.	
1	b, c	l, d	$l = c \cos b$	$d = c \sin b.$
2	b, l	c, d	$c = \dfrac{l}{\cos b}$	$d = l \tan b.$
3	b, d	c, l	$c = \dfrac{d}{\sin b}$	$l = \dfrac{d}{\tan b}.$
4	c, l	b, d	$\cos b = \dfrac{l}{c}$	$d = \sqrt{c^2 - l^2}.$
5	c, d	b, l	$\sin b = \dfrac{d}{c}$	$l = \sqrt{c^2 - d^2}.$
6	l, d	b, c	$\tan b = \dfrac{d}{l}$	$c = \sqrt{l^2 + d^2}.$

The Traverse Table.—This table, which is given with others in the back part of the book, shows the latitude and departure for any bearing to each quarter degree for any distance from 1 to 10. For other distances, the latitude or departure is found by adding the latitudes or the departures of the partial distances, as shown in the following

EXERCISES.

9.—1. Find the latitude and the departure for a bearing of 24°, for a distance of 7 ch.; for a distance of 5 ch.; for a distance of 10 ch.

2. Find the latitude and the departure on a bearing of $37\frac{1}{4}°$, for a distance of 12 ch.

OPERATIONS.

For $37\frac{1}{4}°$,	distance	10,	lat.	= 7.9600	dep.	= 6.0529
"	"	"	2,	"	= 1.5920	"	= 1.2106
"	"	"	12,	"	= 9.5520;	"	= 7.2635.

3. Find the latitude and departure on a bearing of $40\frac{3}{4}°$ for a distance of 17.23 ch.

OPERATIONS.

For $40\frac{3}{4}°$,	distance	10,	lat.	= 7.5756,	dep.	= 6.5276
"	"	"	7,	"	= 5.3030,	"	= 4.5693
"	"	"	.2	"	= 0.15151,	"	= 0.13055
"	"	"	.03	"	= 0.022727,	"	= 0.019583
"	"	"	17.23	"	= 13.053,	"	= 11.247

ANOTHER FORM OF WORK.

Bearing.	*Distances.*	*Latitudes.*	*Departures.*
$40\frac{3}{4}°$	1000	07576	06528
	700	53030	45693
	20	15151	13055
	3	22727	19583
	1723	1305.3237	1124.7433

We take the distance in links, and write the latitude and departure for the first figure of the number, omitting the decimal point; we write under them the latitude and departure for the second figure, setting them down one place farther toward the right; under them, the latitude and departure for the third figure, setting them one place farther toward the right, and so on.

We then add the separate latitudes and separate departures, and point off four figures from the right. The results thus obtained are the latitude and departure sought, as expressed in links.

Notice that bearings from 45° upward are found in the right hand column of the table, and the columns of latitude and departure are denoted at the foot of the page. Care needs to be taken here to avoid mistakes of latitudes for departures and departures for latitudes.

Find the latitudes and departures for the following bearings and distances:

(1) Bearing $52\frac{1}{2}°$, Distance 437.
(2) Bearing $65\frac{1}{4}°$, Distance 3669.
(3) Bearing $21\frac{3}{4}°$, Distance 2030.
(4) Bearing 40°, Distance 506.
(5) Bearing $81\frac{1}{2}°$, Distance 12.34 ch.

10. Meridian Distance.—The distance of a station or any point from the principal meridian is called its **Meridian Distance.** The meridian distance of a line is the meridian distance of its middle point. If the meridian passing through the extreme easterly or westerly station of a survey around a tract of land be taken as a base and perpendiculars be drawn from it to each station of the survey, the tract and the space between it and the meridian will be divided into triangles and trapezoids whose areas are readily computed.

Beginning with the station through which the meridian passes which we call Sta. 0, then the meridian distance of Sta. 1 will equal the departure of the first course.

The meridian distance of any station will equal the algebraic sum of the departures of all the preceding courses up to that point.

The meridian distance of any course or line will equal the half sum of the meridian distances of the stations at the two ends of that course or line.

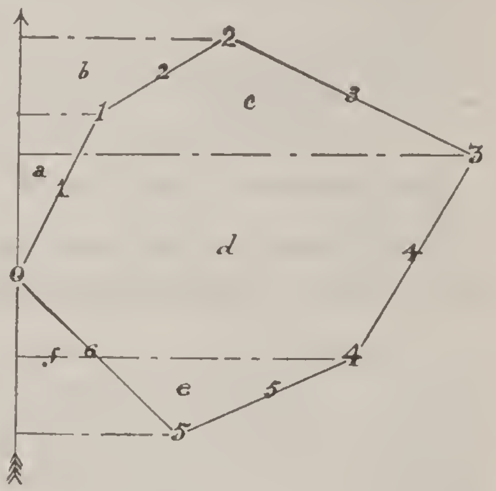
The area of any triangle or trapezoid thus formed will equal the product of the latitude of the line or course on which it is based multiplied by the meridian distance of that line.

The area of the tract is equal to the sum of the areas of all the triangles and trapezoids thus formed minus the sum of the areas of those triangles and trapezoids which lie outside the lines of the survey.

The area of the tract is also equal to the difference between the sums of those areas found from latitudes which are northings and of those where they are southings.

We will now apply the foregoing principles to find the area of the tracts described in the following Field Notes and shown in the figure. On the figure each station is numbered to correspond with the field notes and each line is also numbered in its order as run. The several triangles and trapezoids formed by perpendiculars from the stations to the meridian are lettered in their order.

Station	Bearing	Distance
0	N. 26½° E.	12.00 ch.
1	N. 59° E.	9.80 "
2	S. 66° E.	19.60 "
3	S. 35° W.	15.68 "
4	S. 66° W.	13.12 "
5	N. 46° W.	14.72 "



Finding from the Traverse Table the latitudes and departures to the nearest link, we have

Bearing 26½°	Dist. 12.00	Lat. 10.74 N.	Dep. 5.35 E.
" 59°	" 9.80	" 5.05 N.	" 8.40 E.
" 66°	" 19.60	" 7.97 S.	" 17.91 E.
" 35°	" 15.68	" 12.85 S.	" 8.99 W.
" 66°	" 13.12	" 5.34 S.	" 11.99 W.
" 46°	" 14.72	" 10.23 N.	" 10.59 W.

Obviously, in going entirely around a field there should be made the same southing as northing, and the same westing as easting. But from unavoidable lack of precision in the use of instruments, this is practically seldom found to have been done, according to the figures used. The error, however, can usually be made very small. Finding it large, the entire field work should be reviewed.

It is not a settled point among surveyors how great an error of latitude or departure may be allowed without resurveying the lot. Some would admit a difference of one link for every three chains in the sum of the distances, others for every five chains, and again others would require it to be within one link for every ten chains.

As a check against errors of bearing, a back sight should be taken at every station, and the reverse bearing compared with the corresponding direct bearing of that station. If the two are found to differ considerably, both should be reviewed. Let us now see how small an error of latitude and of departure we have in the present case.

Sum of northings = 10.74 + 5.05 + 10.23 = 26.02.
" " southings = 7.97 + 12.85 + 5.34 = 26.16.

Difference of latitudes = 00.14 = error of latitude.

Sum of eastings = 5.35 + 8.40 + 17.91 = 31.66.
" " westings = 8.99 + 11.99 + 10.59 = 31.57.

Difference of departures = 00.09 = error of departure.

The above errors may be considered reasonably small for a field of the size of the present one.

In practice, some of the courses may have been measured over rough or uneven ground, and, accordingly, such courses should bear a larger proportion of the error.

Some of the bearings may have been taken with an indistinct sight, which would dictate the allotment of more than a proportionate amount of the error to them.

Distances as measured over uneven ground are liable to be too long. In such cases, the length of a course may be diminished when such change would favor the balancing. Similarly, a doubtful bearing may be changed, if the error should appear to be attributable to it.

It is a common mistake to reverse the position of the latitude and departure in the columns. If the bearing is greater than 45° the departure is greater than the latitude, and it is less when the bearing is less than 45°. Scan the columns for such errors.

11. Balancing.—The next work is to distribute the errors among the several courses in proportion to their lengths, in accordance with the following

PRINCIPLE.—*As the sum of the lengths of all the courses is to the length of each course, so is the total error to the error of that course.*

This operation is called **Balancing.**

Applying the above principle, we divide the errors by the sum of the lengths of all the courses and multiply the quotients by the length of each course, indicating the products as positive or negative, accordingly as they are to be added or subtracted in making the required correction.

Thus, 00.14÷84.92=00.00165; and 00.09÷84.92=00.00106; 00.00165×12=00.0198 or +00.02; and 00.00106×12=00.01272 or —00.01, to the nearest link.

In the same manner, by multiplying the above quotients by the lengths of the other courses, the correction for them is readily obtained.

Collecting results thus found, we have the following

TABLE I.

Sta.	Latitude.		Departure.		Cor. L	Cor D	Balanced.			
	N.	S.	E.	W.			N.	S.	E.	W.
1	10.74		5.35		+.02	—.01	10.76		5.34	
2	5.05		8.40		+.02	—.01	5.07		8.39	
3		7.97	17.91		—.03	—.02		7.94	17.89	
4		12.85		8.99	—.03	+.02		12.82		9.01
5		5.34		11.99	—.02	+.01		5.32		12.00
6	10.23			10.59	+.02	+.02	10.25			10.61

We next find the Meridian Distance of the several stations.

M. D. of Sta. 1=Dep. of Course 1=5.34.
M. D of Sta. 2=M. D. of Sta. 1+Dep. of C. 2
=5.34+8.39=13.73.
M. D. of Sta. 3=M. D. of Sta. 2+Dep. of C. 3
=13.73+17.89=31.62.
M. D. of Sta. 4=M. D. of Sta. 3—Dep. of C. 4
=31.62—9.01=22.61.
M. D. of Sta. 5=M. D. of Sta. 4—Dep. of C. 5
=22.61—12.00=10.61.
M. D. of Sta. 0=M. D. of Sta. 5—Dep. of C. 6
=10.61—10.61=0.00

M. D. of C. 1= $\frac{\text{M. D. Sta. 0+M. D. Sta. 1}}{2} = \frac{0+5.34}{2} = 2.670.$

" " 2= $\frac{\text{M. D. Sta. 1+M. D. Sta. 2}}{2} = \frac{5.34+13.73}{2} = 9.535.$

" " 3= $\frac{\text{M. D. Sta. 2+M. D. Sta. 3}}{2} = \frac{13.73+31.62}{2} = 22.675.$

$$\text{M. D. of C. } 4 = \frac{\text{M. D. Sta. } 3 + \text{M. D. Sta. } 4}{2} = \frac{31.62 + 22.61}{2} = 27.115.$$

$$\text{" " } 5 = \frac{\text{M. D. Sta. } 4 + \text{M. D. Sta. } 5}{2} = \frac{22.61 + 10.61}{2} = 16.610.$$

$$\text{" " } 6 = \frac{\text{M. D. Sta. } 5 + \text{M. D. Sta. } 0}{2} = \frac{10.61 + 0.00}{2} = 5.305.$$

We may now put the whole matter in compact tabular form as follows.

Sta.	Bearing.	Distance.	Corrected Latitude.		Corrected Departures.		M. D. of Sta.	M. D. of Course.	N. Area.	S. Area.
			N.	S.	E.	W.				
0	N. $26\frac{1}{2}$° E.	12.00	10.76		5.34			2.67	28.7292	
1	N. 59° E.	9.80	5.07		8.39		13.73	9.535	48.34245	
2	S. 66° E.	19.60		7.94	17.89		31.62	22.675		180.0395
3	S. 35° W.	15.68		12.82		9.01	22.61	27.115		347.6143
4	S. 66° W.	13.12		5.32		12.00	10 61	16.61		88.3652
5	N. 46° W.	14.72	10.25			10.61	0.00	5.305	54.37625	
			26.08	26.08	31.62	31.62			131.4479	616 0190
										131.4479
										484.5711
									= Acres	48.45711

In this example the area of the tract is evidently equal to the sum of the areas of the trapezoids *c d* and *e* based on courses 3, 4. and 5 minus the sum of the areas of the triangles and trapezoid *a b* and *f* based on courses 1, 2, and 6.

The area of the triangle *a* equals the M. D. of course or line 1 multiplied by its latitude $= 2.67 \times 10.76$.

The area of the trapezoid *b* equals the M. D. of course 2 multiplied by its latitude $= 9.535 \times 5.07$.

In a similar manner we find the area of each triangle and trapezoid.

Examples for Solution :

The following examples are taken from the field notes of the original United States Surveys in Michigan and are fair samples of the average work done on the government land surveys. The meanders of lakes and streams are run for the purpose of finding how much dry or uncovered land is contained in the adjacent tract to be paid for by the purchaser.

Ex. 1. Meanders of a Lake in Section 5.

Began at post corner to Sections 4, 5, 8, and 9, thence in Section 5, N. 60° W. 6.50 ch. to S. E. Margin of Lake, thence in Sec. 5. N. 25° E. 4.00 ch., thence N. 51° W. 5.00 ch., thence N. 18° W. 7.00 ch., thence N. 3° W. 7.00 ch., thence N. 63° W. 10.00 ch., thence S. 79° W. 6.00 ch., thence S. 7° W. 13.00 ch., thence S. 20° E. 6.00 ch., thence S. 6° W. 5.00 ch., thence N. 78° E. 14.00 ch., thence S. 27° E. 5.00 ch., thence N. 71° E. 3.87 ch. to place of beginning on margin of Lake.

Find the area of the lake. Also find the areas of the North and South halves respectively of the quarter section in which the lake lies, on the supposition that the quarter section is just 40 chains square and that the lines are run with the same variation of the needle as was used in meandering the lake. These areas are given in the official plat as follows: North ½, A. 66.18. South ½, A. 55.92.

2. Find the area of the lake described in the example 13, page 91, also the area of each of the quarter-quarter sections adjoining the lake in the south half of Sections 11 and 12. These areas are marked in the official plat as follows: In Section 11, S. E. ¼ of S. E. ¼ A. 31.50, N. E. ¼ of S. E. ¼ A. 20.40. In Section 12, S. W. ¼ of S. W. ¼ A. 37.61, N. W. ¼ of S. W. ¼ A. 27.10. The meander post at the beginning of the survey is 14.00 chains North from the Section Corner.

3. Meander of a Lake in section 2.

Began at quarter post in line of Sections 2 and 11. thence North 10.00 ch., to S. margin of Lake, thence in Sec. 2, thence S. 57° E. 13.00 ch., thence E. 3.00 ch., thence N. 45° E. 5.00 ch., thence N. 4° W. 6.00 ch., thence N. 70° W. 15.00 ch., thence S. 80° W. 6.00 ch., thence S. 24½ E. 7.17 ch., to place of beginning in margin of Lake.

Find the area of the Lake also the area of the W. ½ of S. E. ¼ of Section 2 and of the S. E. ¼ of the S. W. ¼ of the Section. The first is given on the official plat as A. 62.88 and the latter as A. 38.95.

13. Problem.—*Given the bearings of the sides of a field, to find the bearings when the field is supposed to be revolved so as to cause one of the sides to coincide with a meridian.*

EXAMPLES.

1. The bearings of the sides of a field are, 1st, N. 12° E., 2d, N. 83¼° E., 3d, S. 21° W., and 4th, N. 47° W. What will the bearings be, if the field be supposed to be revolved so as to cause the first side to be on a meridian?

Ans.—1st, N., 2d, N. 71¼° E., 3d, S. 9° W., and 4th, N. 59° W.

SUGGESTION.—Suppose the field to be revolved toward the left, through an angle of 12°. Accordingly, each bearing would be changed by that amount. The readings of the new bearings are readily determined by inspection.

2. The bearings of the sides of a field are 1st S. 3½° W., 2d N. 86½° W., 3d N. 16½° E., and 4th E. Required the new bearings when the first side is made to coincide with the meridian.

Ans.—1st S., 2d W., 3d N. 13° E., and 4th N. 86½° E.

3. The bearings of the sides of a field are 1st S. 20° W., 2d S. 70° W., 3d N. 31° W., 4th N. 45° E., and 5th S. 60° E. Required the new bearings when the third side is made to coincide with the meridian.

Ans.—1st S. 51° W., 2d N. 79° W., 3d N., 4th N. 76° E., and 5th S. 29° E.

4. The bearings of the sides of a field are, 1st N. 45° E., 2d S. 30° W., 3d S. 5° E., 4th W., and 5th N. 20° E. What will the bearings become, if the field be revolved so as to bring the third side to the meridian?

Ans.—1st N. 50° E., 2d S. 35° W., 3d S., 4th N. 85° W., 5th N. 25° E.

5. The bearings of the sides of a field are, 1st E., 2d N. 9° E., 3d S. 69° E., 4th S. 66° E., 5th S. 42° W., 6th S. 75° W., 7th N. 39° W., and 8th N. 42° E. What will the bearings become, if the field be revolved so as to cause the fourth side to coincide with the meridian?

Ans.—1st S. 24° E., 2d N. 75° E., 3d S. 3° E., 4th S., 5th N. 72° W., etc.

Additional exercises may be formed from the above by requiring different sides to be brought to coincide with the meridian.

RULE.—*Change each bearing agreeably with the direction in which the field is supposed to be revolved by an amount equal to the bearing of the side which is brought to the meridian, and express the result in accordance with the proper form of denoting bearings.*

6. What were the bearings of the sides of a field which are now N. $16\frac{1}{2}°$ E., E., S. $3\frac{1}{2}°$ W., and N. $86\frac{1}{2}°$ W., the variation of the needle having changed $2\frac{1}{2}°$ toward the west since the former survey?

Supplying Omissions.—From inaccessibility of lines and sometimes from accident, omissions may occur in the field notes of a survey. In a closed survey, any two omissions may, in general, be supplied by computation. It is, however, desirable to avoid as far as possible the necessity of supplying omissions in this manner, since it infringes upon the tests which otherwise serve to verify the work.

The several cases which may occur are presented in the following problems:

14. Prob. 1. *To find an omitted bearing and distance.*

CASE 1.—*When the omissions pertain to the same course.*

In a closed survey, the sum of the northings should equal the sum of the southings; and the sum of the eastings should equal the sum of the westings. The defect of these equalities in the present case must be on the one hand the latitude and on the other the departure of the omitted course.

Example.—

Sta.	Bearing.	Dist.	Lat.	Dep.
A	N. 31° W.	9.40	+8.057	− 4.841
B	N. 45° E.	9.30	+6.576	+ 6.576
C	Omit	ted.		
E	S. 20° W.	5.30	−4.980	− 1.813
F	S. 70° W.	10.90	−3.728	−10.243

Solution.—Sum of

$$\begin{aligned} \text{northings} &= 14.633 \\ \text{of southings} &= 8.708 \\ \hline \text{Diff.} = CG &= 5.925 \end{aligned}$$

Sum of

$$\begin{aligned} \text{westings} &= 16.897 \\ \text{of eastings} &= 6.576 \\ \hline \text{Diff.} = GE &= 10.321 \end{aligned}$$

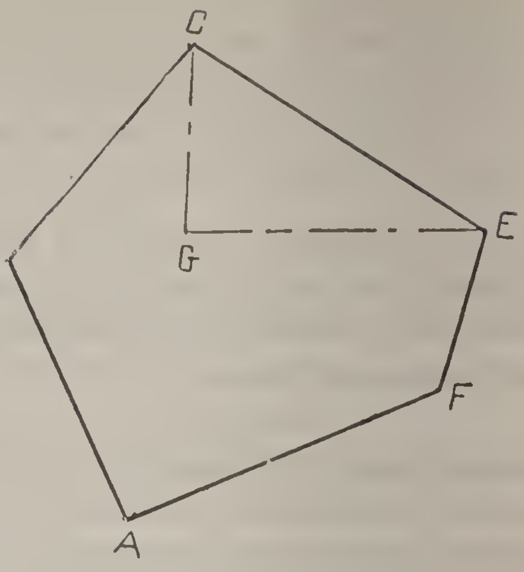


FIG. 48.

The latitude of the omitted course is thus a southing and its departure, an easting. Its bearing is therefore S. —° E.

To find the bearing or angle GCE, we have

$$\tan GCE = \frac{GE}{CG} = \frac{10.321}{5.925} = 1.74194.$$

Whence, $GCE = 60°\ 8'$; or the required bearing is S. 60° 8′ E.

To find the distance CE, we have

$$CE = (5.925^2 + 10.321^2)^{\frac{1}{2}} = 12.00.$$

REMARK.—It will be noticed that a plat of the field may be made, and the area found without supplying the omissions.

CASE 2.—*When the omissions pertain to different courses.*

If the field be supposed to be revolved until the side whose length is omitted becomes a meridian, the given bearings being changed accordingly (Art. 13, Prob.), then, since the departure of the side made a meridian is 0, the difference between the sums of the eastings and westings of the other courses is the departure, in its new position, of the side whose bearing is omitted.

Knowing the length and the departure of this side, its latitude and bearing may be found, (Art. 8).

The difference between the sums of the northings and southings of the courses in their new positions, is the length of the side which was made a meridian.

Example.—

Sta.	Bearing.	Changed Bearing.	Distance.	Lat.	Dep.
A	N. 20° E.	North.	Omitted.		0.0000
B	N. 45° E.	N. 25° E.	8.00	+7.2505	+3.3809
C	S. 30° W.	S. 10° W.	5.00	—4.9240	—0.8682
D	Omitted.		7.20		
E	West.	S. 70° W.	5.92	—2.0248	—5.5630

Solution.—Sum of eastings $= 3.3809$

" " westings $= 6.4312$

Difference $= 3.0503$ (an easting).

Latitude of $DE = (7.20^2 - 3.0503^2)^{\frac{1}{2}} = 6.5219$ (a southing).

Sine of changed bearing of $DE = 3.0503 \div 7.20 = 0.42365$.

Whence " " " DE is S. 25° 4′ E.

Whence original " " DE was S. 5° 4′ E.

Sum of northings $= 7.2505$

" " southings $= 13.4707$

Difference $= 6.22$ $=$ length of AB.

REMARK.—It is sometimes doubtful whether the latitude of the course whose bearing is omitted is a northing or a southing.

In the present case, the question is determined by a simple inspection of the latitudes, since the sum of the southings is less than the sum of the northings, without considering the northing of the first course.

In other cases, there may be two sets of values of the omitted parts, with either of which the problem is satisfied.

Practically, however, the ambiguity is removed by a general knowledge which the surveyor has of the directions of the lines.

15. Prob. 2. *To find the omitted lengths of two courses.*

CASE 1.—*When the courses are consecutive.*

The bearing and length of a line which would close a survey, leaving out the unknown sides, may be found by Prob. 1, Case 1. This line and the unknown sides form a triangle in which the angles, as found from the given bearings, and the length of one side are known: The lengths of the other sides may therefore be computed.

The procedure will be readily worked out by the student, without illustration.

CASE 2.—*When the courses are not consecutive.*

This case may be treated in the same manner as the preceding.

Or, we may suppose the field to be revolved so as to make one of the sides whose length is omitted, a meridian, the bearings of the other sides being changed accordingly.

We may then find the difference of the sums of the eastings and westings, which will be the departure, in its new position, of the other side whose length is wanting.

Having the bearing of that side and its departure, its length and latitude may be found. Finding the difference between the sums of the northings and southings, we obtain the length of the side which was made a meridian.

Example.—

Sta.	Bearing.	Changed Bearings.	Distance.	Lat.	Dep.
A	N. 15° E.	N. 30° W.	5.00	+ 4.33	−2.50
B	N. 45° E.	North.	Omitted.		0.00
C	S. 55° E.	N. 80° E.	10.05	+ 1.75	+9.90
D	S. 15° W.	S. 30° E.	12.25	−10.61	+6.12
E	S. 75° W.	S. 30° W.	Omitted.		
F	N. 33¾° W.	N. 78¾° W.	9.96	+ 1.95	−9.77

Sum of eastings $= 16.02$
" " westings $= 12.27$

Difference $= \ \ 3.75 = \text{Dist.} \times \sin 30°$.

Whence, length of $EF = 3.75 \div 0.5 = 7.50$.

Lat. $EF = 3.75 \div \tan 30° = 6.50$.

Sum of northings $= \ \ 8.03$
" " southings $= 17.11$

Difference $= \ \ 9.08 =$ length of BC.

REMARK.—If the sides whose lengths are omitted are parallel, the problem is indeterminate.

16. Prob. 3. *To find the omitted bearings of two courses.*

We find, (Prob. 1, Case 1) the bearing and length of a line which would close a survey, having the lines whose bearings are given as the other sides.

The line thus found and the two lines whose bearings are omitted form a triangle. The lengths of the sides of the triangle being known, its angles may be found; and from the angles and the bearing of one of the sides the bearings of the other sides may be found.

The closing line and the triangle are illustrated by the diagram accompanying the following

Example.—

Sta.	Bearing.	Dist.	Lat.	Dep.
A	N. 15° E.	5.00	+ 4.8296	+1.2941
B	Omitted.	9.08		
C	S. 55° E.	10.05	− 5.7645	+8.2325
D	S. 15° W.	12.25	−11.8327	−3.1705
E	Omitted.	7.50		
F	N. $33\frac{3}{4}$° W.	9.96	+ 8.2814	−5.5334

10

The side EF, without change of bearing, is represented by CG. BG is the closing line of the field $ABGHF$, in which we have

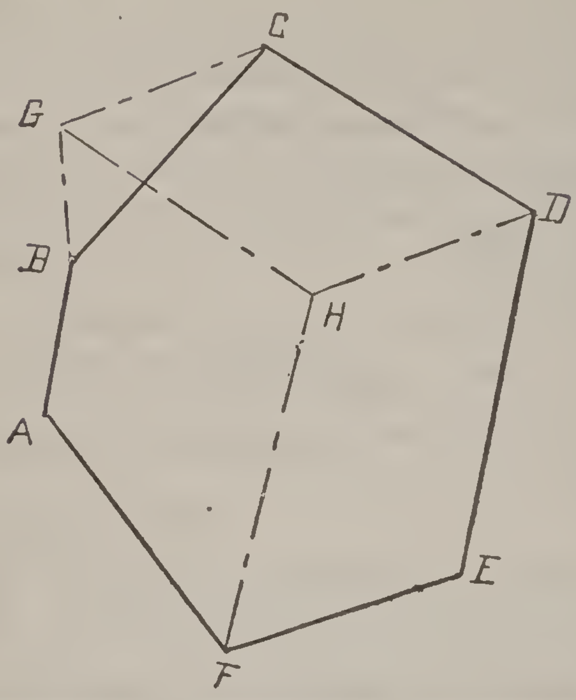


FIG. 49.

Sum of
northings = 13.1110
southings = 17.5972

Difference = 4.4862
(a northing).

Sum of
eastings = 9.5266
westings = 8.7039

Difference = 0.8227
(a westing.)

Whence (Prob. 1), bearing BG is N. 10° 23′ 30″ W., and length BG is 4.56.

In the triangle BGC, $BC = 9.08$ and $CG = EF = 7.50$.

Solving the triangle, we find
angle $GBC = 55° 25′ 40″$, and angle $BGC = 94° 31′ 49″$.

Whence, bearing BC is N. 45° 2′ 10″ E., and bearing EF is S. 75° 4′ 41″ W.

REMARK.—The problem may possibly have two solutions, accordingly as the triangle may fall on either side of the closing line. The ambiguity is, however, practically unimportant.

Exercises.—To be made by the student in the field.

17. Most of the foregoing problems for finding areas may be simplified and much labor saved in calculation, by reducing the irregular polygons and oblique triangles to right triangles and trapezoids on the plat, and taking their dimensions by direct measurements from the plat, instead of calculating them. If the plat is made on a large enough scale—showing not more than four chains to the inch—and the drafting is carefully done, the measures on the plat will be very nearly if not quite as good as those taken on the ground, and will give results sufficiently close for most purposes.

1st Method.—*Draw a diagonal between two distant angles of the figure, and perpendiculars to it from the other angles.*

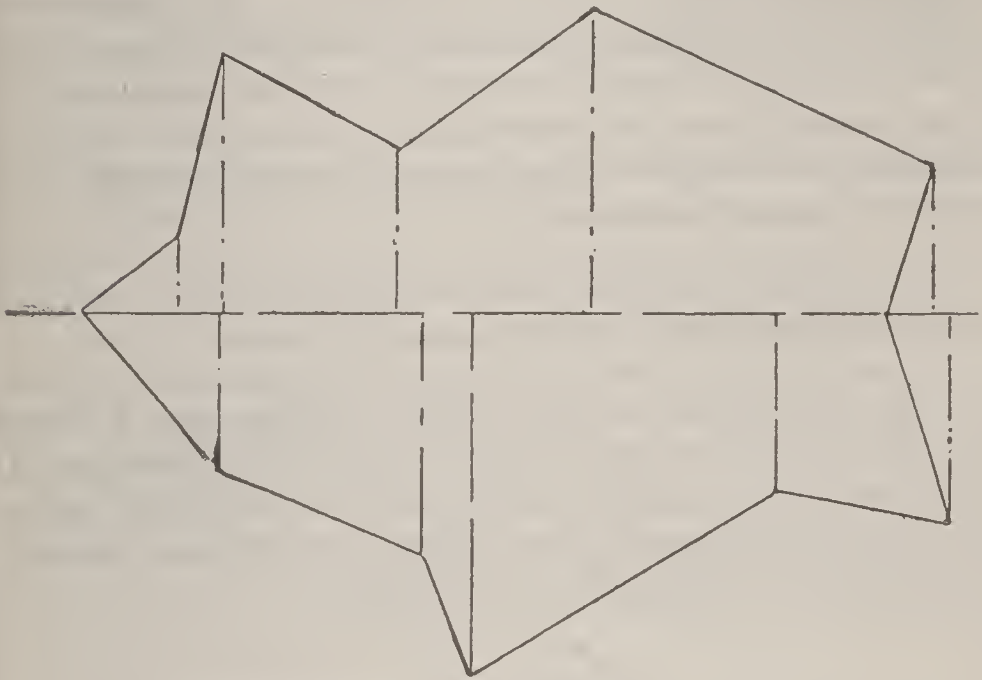


Fig. 50.

2nd Method.—*Reduce the figure to a single equivalent triangle.*

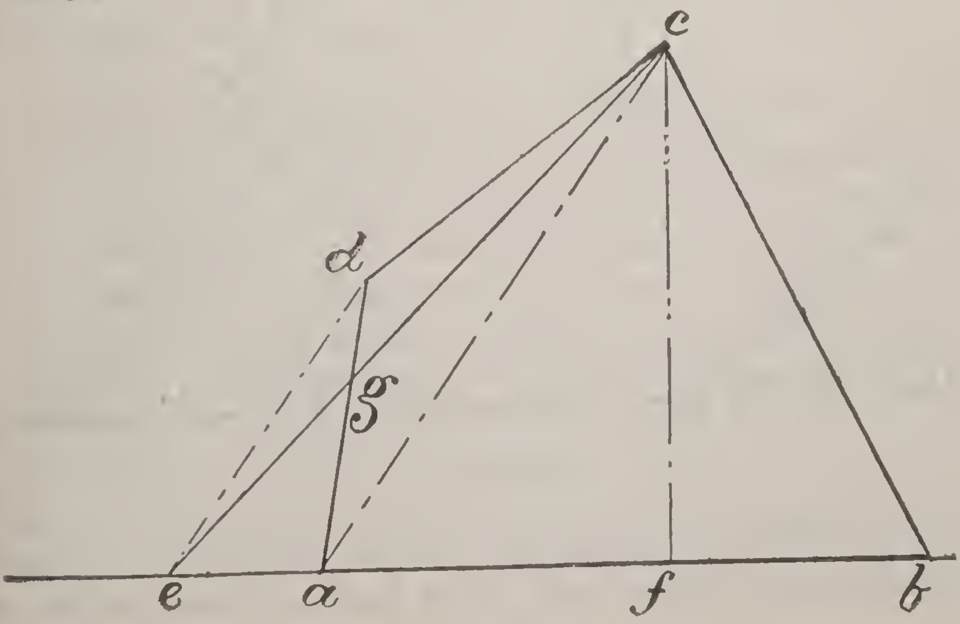


Fig. 51.

1. To reduce the trapezium *abcd* (Fig. 51) to its equivalent triangle.

Produce the line *ab* an indefinite distance. With the parallel ruler, or straight edge and triangle, find the point *e*, where a line through *d* parallel to *ca* intersects the line *ab*. Draw the line *ec*, intersecting *ad* at *g*.

Then the triangle *ecb* is equivalent to the trapezium *abcd*, for the triangles *acd* and *ace*, having the same base *ac* and equal altitudes, are equal; and the triangle *acg* being taken from both leaves the triangle *eag*, which is added to the original figure, equal to the triangle *cdg*, which is taken from it.

The perpendicular may now be drawn from *c*, and the base *eb* and altitude *fc* measured on the plat.

2. By an extension of the same process, any polygon may be reduced to one or more equivalent triangles. It will frequently be found convenient to divide the figure into two or more parts, and reduce the sides separately. The process is indicated in Figure 52.

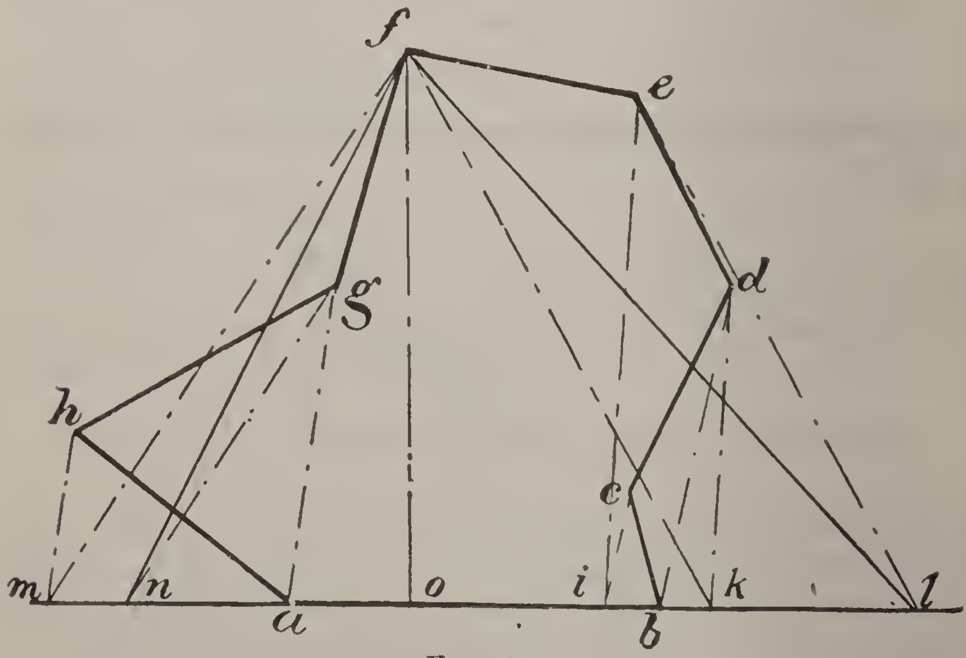


FIG. 52.

Let *abcdefgh* be the polygon to be reduced. Extend one side, as *ab*, indefinitely for a base. From *c* draw *ci* parallel to *bd*. From *d* draw *dk* parallel to *ei*. From *e* draw *el* parallel to *fk*. Having selected *f* as the vertex of the triangle, we next draw *fl* for one of its sides.

Next, from *h* draw *hm* parallel to *ga*.

From *g* draw *gn* parallel to *fm*.

From *f* draw *fn* for the third side of the triangle, and *fo*, its altitude.

The triangle *fln* is equivalent to the polygon *abcdefgh*. It is best to draw all these lines lightly on the plat, to avoid errors.

If we consider each point, *i*, *k*, *l*, marked in succession on the base as an angle of the polygon, which it is until its successor is located, we have the following

General Rule.—*Extend one side indefinitely as a base. Commencing at the first angle from the base, draw from it to the base a line parallel to a line joining the two adjacent angles of the polygon. Continue drawing lines to the base from each angle in succession as far as required. Join the last angle from which a parallel was taken, with the last point of intersection on the base, for a side of the final triangle.*

It is sometimes more convenient not to produce one of the lines of the figure for a base, but to draw a perpendicular to it from one end or from the end produced. The same rule applies.

18. The preceding methods of taking measurements from the plat are found very convenient in estimating the area of land benefited by drainage, under the drain laws. Surveyors are frequently called on to make surveys and maps of drainage districts, showing the location of the drains and the location and area of the lands, belonging to the various owners, which will be benefited by the drainage. In most, if not all these cases, no man can tell, either before or after the drainage has been executed, just exactly where the dividing line is, between land which is benefited and that which is not benefited. For this reason a rapid survey of the approximate line, by stadia measures, is just as good as the most elaborate work with the chain or tape. The one is likely to get as near the true dividing line as the other.

The writer has found the following method to work well in his practice. Suppose a tract of marsh or swamp is to be measured and mapped, having more or less cleared upland around it:

Assume some line as a base. A section line or quarter line of the United States Survey answers well for this purpose. From this base run a broken line around the swamp wherever it is most convenient to do so. Set a stake at each angle in the line. Note the length of each course and the angle which it makes with the common base, as described on page 83.

When the circuit of the swamp has been made, and the transit again set up at the starting point, the work will prove itself. After taking a back sight on the last station and pointing the telescope along the base line, if the work has all been correctly done, the vernier should give the same reading as it did to start with, showing that just 360° have been passed around.

In passing around the swamp an assistant with the stadia rod follows its margin, setting up his rod at every point where it changes its general direction. The transit-man notes down the direction of each point at which the rod is set up, by its angle from the base line and its distance from the transit as read off from the rod.

When as many points are taken as are convenient from one station, the transit is moved up to the next one, and the operation continued till all the desirable points are located. This being done in the field, they are reproduced on the plat on a scale large enough to permit measurements on the plat with a reasonable degree of accuracy. The points along the margin of the swamp having been laid down on the plat, are connected by straight lines, and all intersecting farm lines or other points of interest are also laid down.

We now have a map, showing as correctly as it is possible to do so, the location of the swamp on each man's land. The areas of the several tracts are found by taking the parallel rule and needle point and reducing these irregular polygons to their equivalent triangles and rectangles, making the necessary measures on the plat and computing the areas from these measures.

19. Division and Partition of Land.—The surveyor is sometimes called on to divide areas into portions having a specified relation to each other, or to part off from a field a given number of acres by a line fulfilling some specified condition with respect to the field divided.

There is a great variety of these problems, most of which occur very rarely in the surveyor's practice. A few of those which occur most frequently are given.

Prob. 1.—*To divide a triangle into parts having a given ratio.*

CASE 1.—*By lines from an angle.*

Solution.—Let ABC be any triangle, and suppose it is required to divide it by a line from B, into two parts having the ratio of m to n.

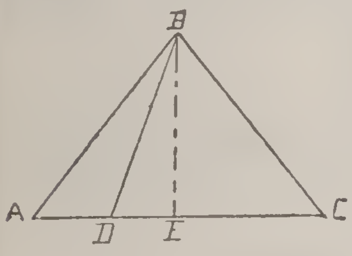


FIG. 53.

Let BD be the line of division, so that $ABD : DBC :: m : n$ (1)
But $ABD : DBC :: AD : DC$ (2)

Combining (1) and (2), we have

$$AD : DC :: m : n,$$

whence, $AD : AC :: m : m+n$,

whence, $AD = \dfrac{m \times AC}{m + n}$. Similarly, $DC = \dfrac{n \times AC}{m + n}$.

Measure the distance AD thus found, and run the line BD.

If the triangle were to be divided into three parts in the ratio of $m : n : p$, we should have

$$AD = \frac{m \times AC}{m+n+p} \text{ and } DE = \frac{n \times AC}{m+n+p}.$$

Cor.—To part off by a line, as BD, a given area a, we have $AD : AC :: a :$ area ABC, whence $AD = \dfrac{a \times AC}{\text{area } ABC}$.

Examples.—1. Find the measurements required to divide a trianglar field by lines from an angle to a side whose length is 12.30 ch., into parts to each other as 2, 3 and 4.

2. Find the measurement required to part off 3.5 acres from a triangular field a side of which is 18.50 ch., and a perpendicular thereupon from the opposite angle is 10.40 ch.

CASE 2.—*By lines parallel to a side.*

Solution.—Let D be the point in the side AB from which a line parallel to BC shall divide ABC so that $ADE : DECB :: m : n$. Then

$$ADE : ABC :: m : m + n.$$

But $ADE : ABC :: AD^2 : AB^2$, whence, $AD^2 : AB^2 :: m : m + n$,

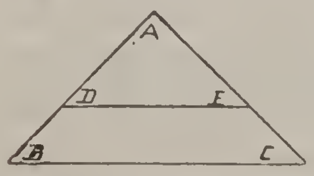


FIG. 54.

$$\text{giving } AD = AB \left\{ \frac{m}{m + n} \right\}^{\frac{1}{2}}$$

Measure the distance AD thus found, and run DE parallel to BC.

If the triangle is required to be divided into three parts in the ratio of $m : n : p$, we should have

$$AD = AB \left\{ \frac{m}{m + n + p} \right\}^{\frac{1}{2}} \text{ and } AF = AB \left\{ \frac{m + n}{m + n + p} \right\}^{\frac{1}{2}}$$

Cor. 1.—To part off a triangle, as ADE, of given area a

we have $AD = AB\left\{\dfrac{a}{\text{area } ABC}\right\}^{\frac{1}{2}}$.

Cor. 2.—To part off a quadrilateral, as $DECB$, of given area, a', we may find by Cor. 1 the distance AD required to part off a triangle of the area $ABC - a'$ and measure $BD = BA - AD$.

Examples.—1. Find the measurement for dividing a triangular field of 12 A. into parts in the ratio of 4 to 5 by a parallel run from a point in a side whose length is 10.35 ch.

2. Find measurements for dividing by parallels, the above field into three equivalent parts.

3. Find measurement for parting off from the same field by a parallel, a triangle of 5 A.; a quadrilateral of 7½ A.

CASE 3.—*By lines perpendicular to a side.*

Solution.—Let ABC be a triangle required to be divided by a perpendicular to AC, into parts having the ratio of m to n.

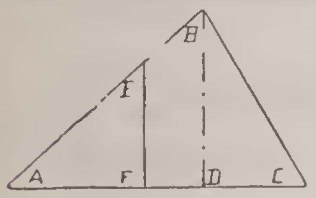


FIG. 55.

Let EF be the line of division, so that $AEF : EBCF :: m : n$, or $AEF : ABC :: m : m+n$. (1) Let BD be a perpendicular upon AC

Then $AEF : ABC :: AF \times EF : AC \times BD :: m : m+n$. (2)

From similar triangles, $AF : EF :: AD : BD$,

$$\text{whence, } EF = \frac{AF \times BD}{AD}.$$

Substituting this value of EF in (2), we have

$$\frac{AF^2 \times BD}{AD} : AC \times BD :: m : m+n,$$

$$\text{or } AF^2 : AC \times AD :: m : m+n$$

$$\text{whence, } AF = \left\{\frac{AC \times AD \times m}{m+n}\right\}^{\frac{1}{2}}.$$

Find AD and then AF. Measure the distance AF and run FE perpendicular to AC.

Similarly, may be found the distances to perpendiculars dividing the triangle into three or more parts having a given ratio.

Cor.—To part off a triangle, as AEF, of given area, a, we have $AF = \left\{\frac{AC \times AD \times a}{\text{area } ABCD}\right\}^{1/2}.$

The distance AF to a perpendicular which shall part off a triangle $AEF = a$, may be found otherwise, as follows: triangle $AEF = \frac{1}{2} AF \times EF = a$, and $EF = AF \times \tan A$. Whence, $AF = \left\{\frac{2a}{\tan A}\right\}^{1/2}.$

Examples.—1. The bearings and lengths of two sides of a triangular field from the same corner are N. 20° E., 15 ch., and N 50° E., 20 ch. Required the measurement from that corner to a perpendicular upon the longer side which shall divide the field into two parts having the ratio of 2 to 3.

2. Required the measurement to a perpendicular which shall divide the above field into two equivalent parts; into three equivalent parts.

3. Required the measurement to a perpendicular which shall part off from the same field a triangle of 4 A.; a quadrilateral of 5 A.

20. Prob. 2. *To divide a trapezoid into parts having a given ratio.*

CASE 1.—*By lines dividing the bases proportionally.*

Solution.—Let $ABCD$ be any trapezoid required to be divided into parts having the ratio of $m : n : p$.

This is done in the easiest manner by dividing each base into parts having the ratio to each other as m, n and p, and joining the corresponding points of division. The measurements necessary to find the points of division are:

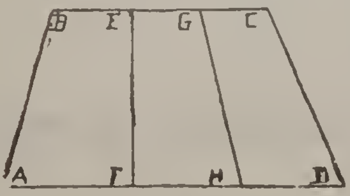


FIG. 56.

$$BE = \frac{m \times BC}{m+n+p},\quad EG = \frac{n \times BC}{m+n+p},\quad AF = \frac{m \times AD}{m+n+p},$$

and $FH = \dfrac{n \times AD}{m+n+p}$.

Cor.—To part off a given area a by a line, as EF, which shall divide the bases proportionately, we have

$$BE = \frac{a \times BC}{\text{area } ABCD} \text{ and } AF = \frac{a \times AD}{\text{area } ABCD}.$$

Examples.—1. Given AD, N. 80° E., 12.60 ch., AB, N. 10½° E., 8.12 ch., and BC, N. 80° E., 10.34 ch., to find the measurements required in dividing the field into parts having the ratio of 4 to 7, by a line dividing the parallel sides proportionally.

2. Find the measurements for parting off from the above field an area of 5 A., by a line dividing the parallel sides proportionally.

CASE 2.—*By lines parallel to the bases.*

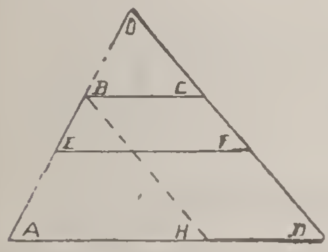


FIG. 57.

Solution.—Let $ABCD$ be a trapezoid to be divided into parts in the ratio of m to n, by a line parallel to BC.

Suppose EF to be the required line of division, so that

$$EBCF : AEFD :: m : n.$$

Regarding the sides AB and DC as prolonged to meet at O, we have $OAD : OBC :: AD^2 : BC^2$,

whence, $OAD - OBC$,

or $\quad ABCD : OBC :: AD^2 - BC^2 : BC^2. \qquad (1)$

Similarly, we have $EBCF : OBC :: EF^2 - BC^2 : BC^2$. (2)

Combining (1) and (2), $ABCD : EBCF :: AD^2 - BC^2 : EF^2 - BC^2$,

or $m + n : m :: AD^2 - BC^2 : EF^2 - BC^2$.

$$\text{whence } EF = \left\{ \frac{m \times AD^2 + n \times BC^2}{m+n} \right\}^{\frac{1}{2}} \qquad (a)$$

Supposing BH to be parallel to CD, the triangles ABH and EBG give $AB : AH :: EB : EG$,

$$\text{or} \quad AB : AD - BC :: EB : EF - BC.$$

$$\text{Whence, } EB = \frac{AB\,(EF - BC)}{AD - BC}. \quad (b).$$

Thus, first finding EB by formula (a), we can then find EB by formula (b), and measuring that distance from B, we may run EF parallel to BC, dividing the trapezoid as required.

Similarly, a trapezoid may be divided in three or more parts having a given ratio. Indeed, the above formulas may be directly applied to that purpose by making a simple substitution.

Cor.—To part off a trapezoid of given area a, adjoining BC, we obtain from formula (a)

$$EF = \left\{ \frac{a \times AD^2 + (\text{area } ABCD - a)\, BC^2}{\text{area } ABCD} \right\}^{\frac{1}{2}}$$

The distance BE is then found from formula (b).

Examples.—1. Given a trapezoidal field $ABCD$ in which AB is an east and west line, 9 ch., BC a north and south line, 5.19 ch., and AD a north and south line, 8 ch., it is required to run a north and south line dividing the field so that the parts on BC and AD shall have the ratio of 2 to 3.

2. Find the measurement from A to part off from the above field by a north and south line an area of 3 A. adjoining AD.

CASE 3.—*By lines perpendicular to the bases*

Solution.—Let $ABCD$ be a trapezoid to be divided into parts in the ratio of m to n by a line perpendicular to AD.

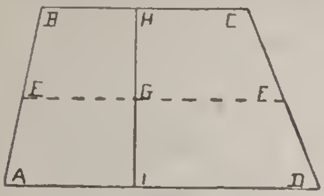


FIG. 57.

Let EF be the line joining the middle points of the non-parallel sides AB and CD. We divide EF, as at G, into two parts having the ratio of m to n, and through G run HI perpendicular to AD.

To find the point G on the ground, we have the formula $EG = \dfrac{m(BC + AD)}{2(m+n)}$. Whence, measuring from E the distance EG on the bearing of BC, we have the point sought.

Cor.—To part off a given area a, by a line perpendicular to the bases, we have $EG = \dfrac{a(BC + AD)}{2 \times \text{area } ABCD}$,

Or, denoting the altitude of the trapezoid by h, we have $EG = \dfrac{a}{h} = \dfrac{a}{AB \times \sin A}$.

The point I or H may be found by the formula $AI = EG + AE \times \cos A$, or $BH = EG - EB \times \cos A$.

Examples.—1. Given AD, E. 20 ch., AB, N. 15° E., 9.50 ch., and BC, E. 12 ch., required the measurement for dividing the field by a perpendicular to AD into two parts having the ratio of m to n.

2. Required the measurement for parting off from the above field, by a perpendicular to AD, an area of 4 A. adjoining AB.

21. Prob. 3.—*To divide a trapezium into parts having a given ratio.*

CASE 1.—*By lines from an angle.*

Solution.—Let $ABCD$ be a trapezium to be divided into two parts having the ratio of m to n, by a line from C.

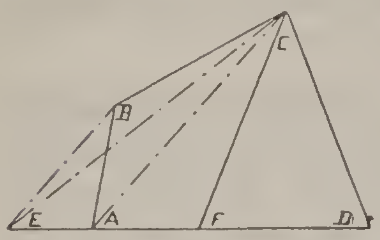


FIG. 59.

We draw AC, and from B draw a line parallel to AC, meeting DA produced at E. We then divide ED, as at F, into the parts EF and FD, having the ratio of m to n. The line CF divides the trapezium as required. That is, $ABCF : FCD :: m : n$, or $ABCF : ABCD :: m : m+n$.

SCH.—The above solution is readily executed on the ground.

In a similar manner a trapezium may be divided into any number of parts having a given ratio.

The point F may be otherwise found as follows:

The triangle $DCF = \dfrac{n \times ABCD}{m+n}$,

and again, $DCF = \dfrac{DC \times \sin D \times DF}{2}$.

Whence, $DF = \dfrac{2n \times ABCD}{DC\,(m+n) \sin D}$.

Cor.—To part off a triangle, as DCF, of given area a, we have $DF = \dfrac{2a}{DC \times \sin D}$.

Examples.—1. Given AB, N. 8° W., 7.60 ch., BC N. 76½° E., 10.21 ch., CD, S., 11.40 ch., and DA, N. 80½° W. 9.00 ch. Required the measurement for locating a line CF which shall divide $ABCD$ into the parts $ABCF$ and FCD, to each other, respectively, as 2 to 3.

2. Required the measurement for parting off from the above field a triangle DCF of 10 A.

CASE 2.—*By lines parallel to a side.*

Solution.—Let it be required to divide a trapezium, as $ABCD$, by a line, as EF, parallel to AD, into two parts, $EBCF$ and $AEFD$, to each other as m to n.

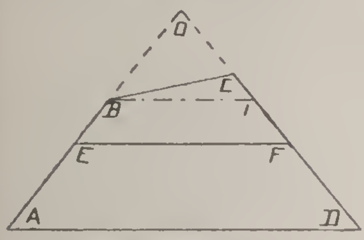


FIG. 60.

Suppose the sides including the parallel to be produced to meet at O. The triangle BOC may be regarded as known. Call its area a. The trapezium $EBCF$ is known as to area, being $\frac{m \times ABCD}{m + n}$. Call this area b.

The area of the triangle AOD is known. Call it c. Its side AO is also known.

Now, (Art. 19, Prob. 1, Case 2), $OE = AO\left\{\frac{(a + b)}{c}\right\}^{\frac{1}{2}}$

Whence, $BE = OE - OB$.

Measure this distance and run EF parallel to AD.

Another procedure is to draw BI parallel to AD, forming the triangle BCI, whose area and side BI may be found; whence the ratio of the trapezoid $EBIF$ to the trapezoid $ABID$ is obtainable, and accordingly the distance BE.

SCH.—The problem of parting off a given area from a trapezium by a line parallel to a side, is essentially the same as the above.

Examples.—1. The field being as given in Ex. 1, Case 1, it is required to find the measurement for locating a parallel to BC that shall divide the field into two parts in the ratio of 3 to 4.

2. Find the measurement required to part off from the same field an area of 10 A., by a line parallel to BC.

22. Prob. 4. *Two men own land situated between a road XX' and a line YY', and divided by a line BA'.*

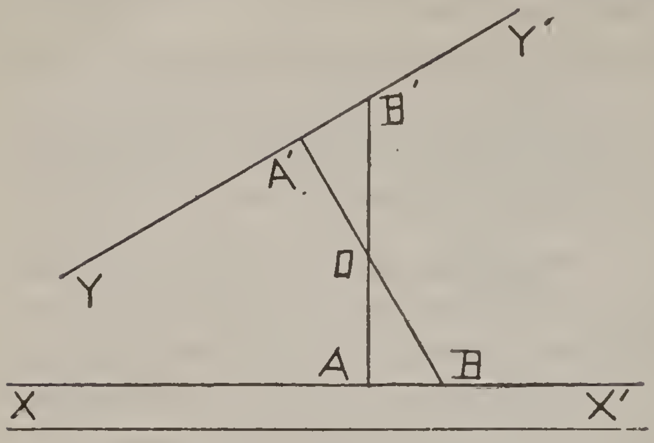


FIG. 61.

It is required to run a line AB', at right angles with the road, which shall part off areas of equal value from the two portions.

Solution.—Let T be the triangle AOB, and T' the triangle $A'OB'$.

Let v = value per acre of T, and v' = value per acre of T'.

Let angle $OBA = B$, and angle $OA'B' = A'$ be known; and let $AB = x$, $BA' = c$, and $BO = z$.

We shall then have

$$\text{area } T = \frac{z^2 \sin B \cos B}{2}, \qquad (1)$$

$$\text{and area } T' = \frac{(c - z)^2 \sin A' \cos B}{2 \cos (A' - B)}, \qquad (2)$$

By conditions of the problem, $Tv = T'v'$.

Whence, $T : T' :: v' : v$. Let the ratio $v' : v = r$.

Then $T = T'r$. Whence, from (1) and (2),

$$\frac{z^2}{(c - z)^2} = \frac{r \sin A'}{\sin B \cos (A' - B)}$$

$$\text{or } \frac{z}{c - z} = \left\{ \frac{r \sin A'}{\sin B \cos (A' - B)} \right\}^{\frac{1}{2}} = n.$$

$$\text{Whence, } z = \frac{cn}{n + 1}, \text{ and } x = z \cos A = \frac{cn \cos B}{n + 1}.$$

23. Many problems which the surveyor meets with may be readily solved by trial lines and successive approximations. A line is run or assumed to meet the required conditions as nearly as can be judged. The area parted off by the line is computed and the amount of error found. A new line is assumed to correct the error, and thus successive approximations to the true line are made until the error disappears. If good judgment is used, it is sometimes the quickest and easiest method to solve the problem.

Example.—The northwest quarter of Section 30 is divided by an angling road. The owner wishes it laid off into five acre lots, commencing at the south end, the lot lines to be parallel with the quarter line, and running from the center of the road west to the section line. Required the number of lots, the area of the fractional lot, if any, at the north end, and the dimensions of the several lots. The total dimensions are given on the figure.

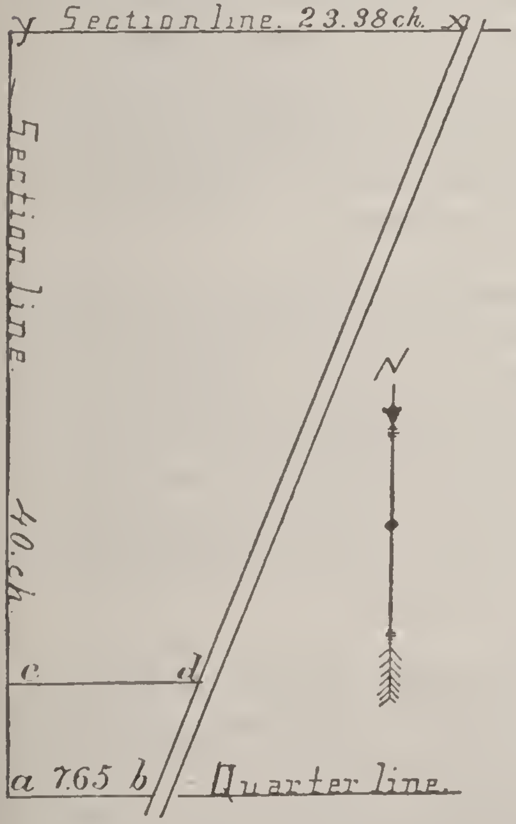


FIG. 62.

Solution, (First Lot).—Length of south line, 7.65 ch. If the lot were a rectangle of 9.00 chains base, the perpendicular ac would be $\frac{50.0000}{9.00} = 5.555+$ chains. Assume that $ac = 5.60$, to find cd. The line bx diverges from ay at the rate of $\frac{23.38 - 7.65}{40.00}$ or .39325 ch. per chain. Then $cd = 7.65 + (560 \times .39325) = 9.852$ chains. Area $abcd = \frac{9.852 + 7.65}{2} \times 5.60 = 4.900056$ acres. This is too small by .1 acre, which must be added.

For the next approximation we observe that the addition of 1 link along the line *cd* adds nearly .01 A. to the area. So we will add 10 links for the trial. $5.60 + .10 = 5.70$, and $5.70 \times .39325 = 2.2415$ = divergence of lines. $\frac{2.2415 + 7.65 + 7.65}{2} \times 5.70 = 4.99933$ A. The result is still a trifle short, but in ordinary surveying would be sufficiently correct.

To find the remaining side of the lot, *bd*, we have a right triangle, with a base equal to *ac* and perpendicular equal to *cd* — *ab*.

The method is now sufficiently described so that the student may finish the computations and make a plat of the example.

Field Notes.—Nearly every surveyor has a method of his own for keeping the field notes of his surveys. For general purposes probably no better plan has been devised than that employed in the United States land surveys. This method gives, in a condensed narrative form, each item in the survey, in the order in which it was executed, and affords opportunity for explaining all the details as fully as may be necessary.

It is a common fault among surveyors to condense their notes into the least possible space by omitting many things of importance and by the use of arbitrary signs, which may or may not be understood by any one else who may have need to refer to them. The notes are thus deprived of much of their value, and in case it were desired to use them as evidence in the courts, they might be excluded altogether.

The field notes should be full and explicit, and, especially in re-surveys, should state in plain, concise words every material fact in regard to the work done. Starting points should be described and identified; the direction of lines, how determined, whether from the true meridian, the magnetic meridian, or from an arbitrary meridian adopted for the line, should be shown. It is not enough to say that the survey started from a certain corner. That may be disputed, and the notes should give the evidence by which it is known to be the corner. Tell what was found to mark the corner. If a bearing tree of a former survey is found, give its direction and distance from the corner. Make everything so clear and plain that the average citizen can understand it and judge of the trustworthiness of the survey. The following is a sample of the field notes of the United States survey. It is an extract from the

FIELD NOTES

OF THE SURVEY OF THE

SUBDIVISION AND MEANDER LINES

OF

Township No. 6 North, Range No. 34 East

OF THE

PRINCIPAL BASE AND MERIDIAN

OF

MONTANA TERRITORY,

as surveyed by

WALTER W. DE LACY,

U. S. Deputy Surveyor,

Under his Contract,

No. 87,

Dated July 3, 1880

T. 6 N., R. 34 E.

Chains.	
	Preliminary to commeneing this survey, I ran west on a blank line on the south boundary of Sec. 36, and at 39.97 chs. found the ¼ sec. cor. and at 80.01 chs. found the see. cor. As the east boundary of See. 31 erosses the Yellowstone River it was not re-run. My compass will therefore run the same line as the exterior boundaries, and the ehaining praetically agrees.
	Survey commenced August 6th, 1879, with a Burt's improved solar compass.
	I commenced at the eor. to Sees. 1, 2, 35, and 36, on the south boundary, which is a sandstone 30x8x2½ ins. firmly set in the ground, with one notch on E. and 5 notehes on W. edges, and pits 18x18x12 ins. in each sec. 5½ ft. dist. with mound of earth 2 ft. high, 4½ ft. base alongside. Thenee I run North bet. Secs. 35 and 36. Va. 18° 30′ E.
20.00	Enter scattering timber. Alexander's house bears N. 31° W.
31.00	Leave scattering timber.
40.00	Set a post 3 ft. long, 3 ins. square, with marked stone, 12 ins. in the ground, for ¼ sec. cor., marked ¼ S. on W. side, dug pits 18x18x12 ins. N. and S. of post 5½ ft. dist., and raised a mound of earth 1½ ft. high, 3½ ft. base, around post.
	Alexander's house bears S. 53¾° W.
52.70	Enter brush.
53.82	Right bank of the Yellowstone River. Set a post 4 ft. long, 4 ins. square, with marked stone, 12 ins. in the ground, for meander cor. to fractional secs. 35 and 36, marked M. C., and T. 6 N. on S., R. 34 E. S. 36 on E., and S. 35 on W faces, dug pit 3 ft. square, 12 ins. deep, 8 lks. S. of post, and raised mound of earth 2 ft. high, 4½ ft. base, around post.
	There being an island on line on N. side of ehannel, I send a flag across, and set it on line bet. sees. 35 and 36, on bar S. of island. I then go across to flag and run a base line W. 11.14 chs., to a point from which meander cor. on right bank bears S. 37° 50′ E., which gives for distance across the river to edge of bar 14.34 chs. I then run north from flag 66 lks. to south bank of island, making the whole distance 53.82 + 14.34 + 0.66 chs., or
68.82	To south bank of island, which point I established by setting a post 4 ft. long, 4 ins. square, with marked stone, 12 ins. in the ground, for meander cor. to fractional secs. 35 and 36 on S. bank of island, marked M. C., and T. 6 N. on N., R. 34 E. S. 36 on E., and S. 35 on W. faces, dug pit 3 ft. square, 12 ins. deep, 8 lks. N. of post, and raised a mound of earth 2 ft. high, 4½ ft. base, around post.
	Thence continue on line across island, enter brush.
72 50	Leave brush, enter timber.
80 00	Set a post 4 ft. long, 4 ins. square, with marked stone, 12 ins. in the ground, for cor. to sees. 25, 26, 35, and 36, marked T. 6 N. S. 25 on N. E., R. 34 E. S. 36 on S. E., S. 35 on S. W., and S. 26 on N. W. faces, with 1 notch on S. and E. edges, from which A cottonwood, 12 ins. diam., bears N. 12¾° E., 180 lks. dist., marked T. 6 N., R. 34 E., S. 25 B. T. A eottonwood, 18 ins. diam., bears S. 82° E., 154 lks. dist., marked T. 6 N , R. 34 E., S. 36 B. T. A cottonwood, 10 ins. diam., bears S. 29½° W., 56 lks. dist., marked T. 6 N., R. 34 E., S. 35 B. T. A cottonwood, 10 ins. diam., bears N. 46½° W., 119 lks. dist., marked T. 6 N., R. 34 E., S. 26 B. T.
	Land, level.
	Soil, rich loam—1st rate.
	Timber, cottonwood and willow, undergrowth same, 12.30 ch.

The following is a sample of **Field Notes of a Resurvey,** kept upon the same plan:

SURVEY ON SECTION 14, TOWNSHIP 2 SOUTH, RANGE 10 WEST,
For J. R. Comings and H. Rowland.
May 22, 1874.
S. F. Kingsley, } *Chainmen.* S. Comings, *Flagman.*
C. Rowland,

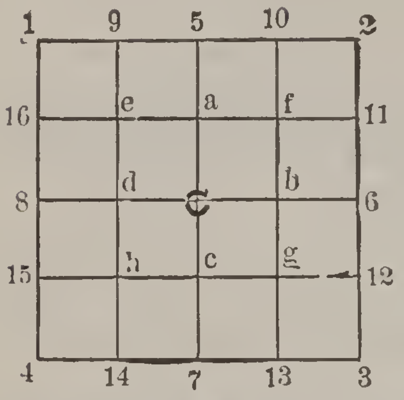
Commenced at the S. E. corner of Section 14. Found a piece of strap railroad iron driven for the corner, which Hugh Shafter says he knows to have been kept in the same place, unquestioned, as the corner for over 30 years. Marked
a maple, 8 in. diam., S. 45° W., 77 lks. dist.
a burr oak, 12 " " N. 43° W., 123 " "

Chains.	
	I set up a tall flag on the corner and then ran west on random Va. 2° 15′ E., setting temporary stakes every 10 chains in line.
40.00	Quarter section corner lost.
80.24	Intersected the west line of Sec. 14, 42 links south of the corner. Found rotten stake at correct point, N. 26° E., 104 lks. from stump of wh. oak 24 in. diam., bearing tree of U. S. Survey, having surveyor's mark distinct on it. Set a piece of steel T rail 28 inches long for corner. Marked locust, 16 in. diam., S. 28° W., 116 lks. distant, burr oak, 18 " " N. 78° E., 152 " "
	Ran thence east on corrected line at single sight with transit, from corner to corner. Va. 2° 33′ E. 10:30 A. M.
40.12	Found cedar stake 3 feet below surface of road crossing and 2½ links south of line. No other evidence of corner to be found. Put a piece of T rail 24 inches long on top of the stake for quar. sec. cor., 55 links south of south rail of M. C. R. R. No tree near.
60.18	Planted granite boulder 20x12x6 inches, with cross + mark, for ½ quarter corner, in true line between qr. post and section corner and marked maple, 12 in. diam., S. 16° E., 55 lks. distant, burr oak, 16 " " N. 54° E., 118 " "

In some surveys, such as laying out additions to cities or villages, or any similar work, it is better to make a rough sketch or plat of the work in the field book and mark the dimensions and directions of lines on the plat. Field books which are ruled in small cross sections are best adapted to this use.

ABBREVIATIONS.— Where the work of the land surveyor consists in re-surveys and sub-dividing sections of the United States Surveys, the field notes may be made more concise and liability of error reduced by always using a definite symbol to refer to each corner of the section or sub-division. The symbols should be simple and adopted upon some system by which they may be easily remembered and located in the mind.

The system shown in the figure has been used many years by surveyors in Michigan and found satisfactory.



All the corners lying in the exterior lines of the section are numbered in a definite order of rotation in accordance with their relative importance. Letters are used for the interior corners, the first letters being used for the corners lying in the quarter lines and the others for the centers of the quarter sections.

The following is a sample of the manner of using the symbols in keeping notes upon the U. S. System when sub-dividing a section.

	Began at 7. Found stake in place and both bearing trees standing. Planted stone 25″ × 8″ × 6″ marked + for corner. Thence north on random. Var. 2° 30′ E. setting temporary stakes every 10 chains
80.22	Intersected Section line 26 links west of 5.
	At 5 found rotten stake at correct point. S. 28° W. 66 lks from stump of W. Oak bearing tree of U. S. Survey.
	Drove stake for corner and put broken earthenware and glass around it and marked
	Wh Oak 12″ d; N. 66° E. 42 lks.
	Wh Oak 18 N 34 W 63 lks.
	From 5 ran east on random, setting temporary stakes every 10 chains.
39.92	Intersected Sec. line 12 lks. North of 2. Found earthen post in correct position and bearing trees of resurvey standing. Thence West on corrected line.
9.98	Set stake on true line.
19.96	At 11 set stake with stones around it and marked
	Pine 12 N. 46° W. 79 lks. dist.
	Red Oak 24 S. 19½° W. 72 dist.
29.94	Set stake on true line.
	From 11 ran south on random Var. 2° 19′ E. and set temporary stakes at 20 and 40 chains.
	Then went to 6. Found post and bearing trees of resurvey standing. Ran thence West on random Var. 2° 20′ E.
20.02	Intersected random from North 6 links South of temp. stake.
40.18	Intersected random ¼ line 8 links North of temp. stake.
80.04	Int. Sec. line 10 links South of 8. Corner post dug out in road. Set iron plow beam for corner S. 29 W. 76 lks. from bearing tree of U. S. Survey.
	Thence East Corrected line.
39.99	At intersection of quarter lines set post.

CHAPTER VII.

CURVELINEAR SURVEYING.

1. As land surveyors have occasion in laying out streets in villages, parks, cemeteries, race courses, drains, etc., sometimes to make use of curved lines, it has been deemed proper to include in this work a short discussion of the manner of locating the simpler curves, and add such tables as are needed for this use. For a more complete exposition of the subject, consult the field books of Henck, Trautwine, Shunk, or Searles.

The curve most commonly used is the circular curve, simple or compound.

The simple circular curve, as its name indicates, is a circle or an arc. When an arc is used to connect two straight lines, these lines, from their relation to the circle, are termed tangents.

The compound circular curve is a combination of arcs having different radii. At the point of junction of any two of these arcs their radii lie in the same straight line.

Of the several geometrical propositions on which the theory of running curved lines depends, it will not be necessary for our purpose to recall more than the following

PRELIMINARY PROPOSITIONS.

1. If a circle be drawn touching each of two intersecting lines at but a single point, then the exterior angle made by the intersection of these lines is equal to the angle at the center of the circle which is measured by the arc intercepted by the two lines at their points of tangency.

2. The angle which either line makes with the chord of the intercepted arc equals one-half the angle at the centre of the circle which is subtended by that chord.

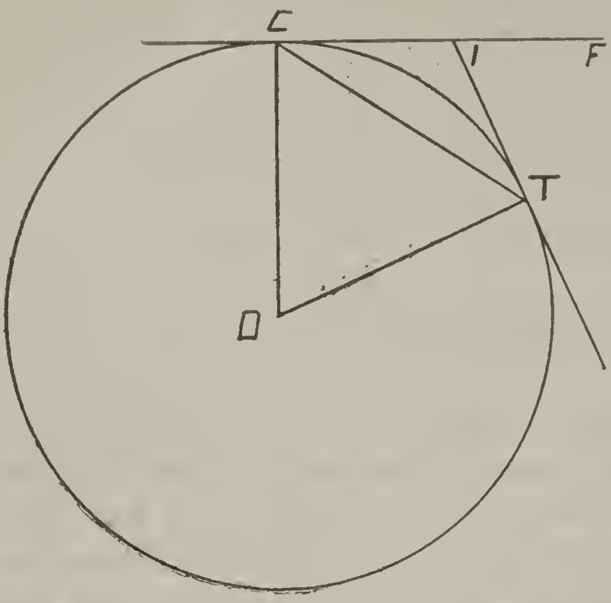


FIG. 63.

In Fig. 63 *CF* and *TI* represent the two lines tangent to the circle at *C* and *T*, and intersecting at *I*. The angle $FIT = COT$, and the angle $FCT = \frac{1}{2} COT$.

The angle *FIT* is called the deflection angle, and the angle *FCT* the tangential angle.

Curves are named from the angle which is subtended by a chord 100 feet long. Thus, if the 100 foot chord subtends an angle of 1 degree, the curve is spoken of as a 1° curve; if of 5°, as a 5° curve, and so on. Tables have been prepared giving the various functions of a 1° curve, which are of great assistance in running curved lines, saving nearly all the trouble of calculation. The foot is taken as the primary unit of these tables and is most commonly used, but any other unit using the decimal notation, as a link or metre, is just as readily applied.

Curves are run on the ground by successive deflections of chords. The amount of each deflection may be measured on the ground with the tape or turned off on the transit.

2. To run a Curve with Pickets and Tape. —First, determine the radius of the curve and the length of chord to be used. The latter is usually 100. From these data the amount of deflection for each chord is determined as follows:

$$\text{Defl. dist.} = \frac{\text{chord}^2}{\text{radius}} \qquad \text{Tangential dist.} = \tfrac{1}{2} \text{ defl. dist.}$$

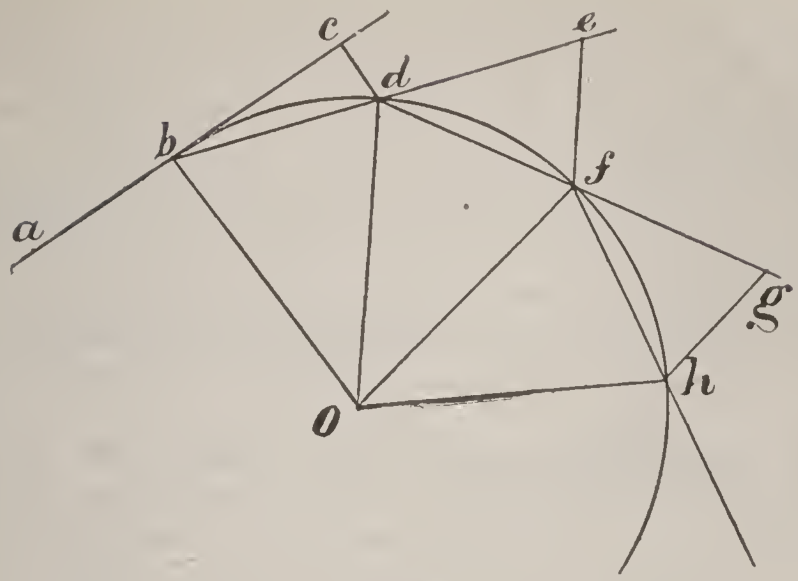


FIG. 64.

Example 1.—Let ab be the straight line or tangent which is to be continued from b by a curve having a radius of 1,433 feet, using chords of 100 feet.

Extend the line ab to c, making $bc = \sqrt{bd^2 - cd^2}$. Extend the chord bd to e, making $de = bd = df$. Extend the chord df in a similar manner. cbd is the tangential angle, and cd the amount of the deflection to be measured from the tangent to find the line of the curve. edf is the deflection angle, and ef is the amount of deflection to be measured off from the extension of the chord bd to find the line of the curve.

To find the distance ef.—The triangles edf and dof being similar, $ef : df :: df : do$. $\therefore ef = \frac{df^2}{do} = \frac{100^2}{1433} = 6.98$ nearly. The tangential deflection being one-half the chord deflection, $cd = \frac{1}{2}ef = 3.49$. The triangle bcd is right-angled at c, hence $bc = \sqrt{bd^2 - cd^2} = \sqrt{100^2 - 3.49^2} = 99.94$. The difference between bc and bd is so small that in all curves of large radius it may be neglected on the ground and bc be measured off $= bd$.

These lines may be run with pickets, the chords measured with the tape, and the deflections when not too large measured off by a graduated rod or a rod cut to the exact length.

Example 2.—Lay off on the ground a curve having a radius of 2,640 feet, using chords of 50 feet.

Ex. 3.—Lay off a curve having a radius of 819 feet and chord of 50 feet.

Ex. 4.—Lay off a curve with radius 2,865 feet, chord 100 feet.

Ex. 5.—Lay off a curve with radius 1,910, chord 100.

Ex. 6.—Lay off a curve with radius 882, chord 50.

Ex. 7.—Lay off a curve with radius 1,042, chord 100.

3. Keeping the Field Notes of Transit Lines.—The field notes of transit work where long lines are being run, as for railroads, drains, etc., are usually kept in a different manner from those of other surveys. The notes proper are kept on the left-hand page of the field book. The opposite page is used for explanatory matter, sketches of topography along the line, such as road and stream crossings and obstacles in line, in greater or less minuteness of detail according to circumstances. The line is marked by stakes driven at regular intervals, usually 100 feet or 100 links, and numbered from 0 upwards. The corresponding numbers are kept on the left-hand column of the note book, commencing at the bottom of the page and running upwards.

If the topography is sketched on the right-hand page, the number of every stake is put down in its regular order, and the ruling of the book forms a scale by which the sketches are made. A book ruled in cross-sections is very convenient for this work. If the topography is not taken, the important stations are noted down and the intermediates are omitted. The following abbreviations are used: P. I., point of intersection; P. C., point of curve, or point where the curve begins; P. C. C., point of compound curve; P. R. C., point of reverse curve; P. T., point tangent, or point where the curve ends; T. P., turning point, indicating where the transit was set up, also indicated by ○ or △.

The direction of the tangents is kept as shown by the magnetic needle. This serves as a check on the angles of deflection, and assists in locating errors.

SPECIMEN OF ABRIDGED NOTES.

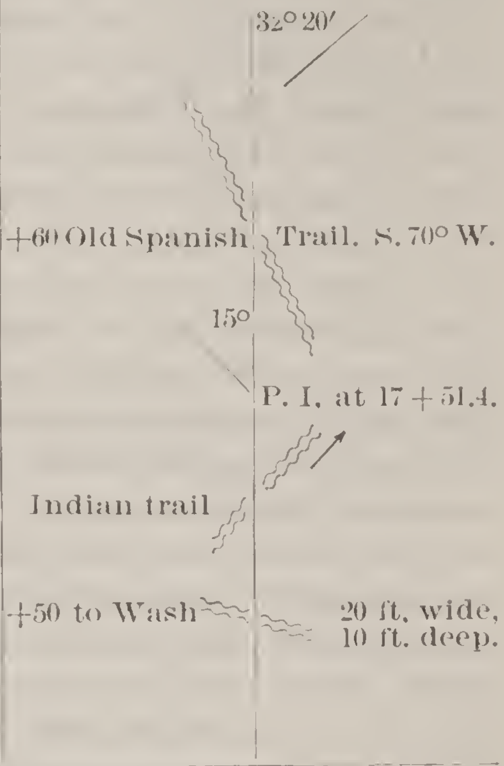
[LEFT PAGE.]

Notes of Line "B," D. & R. G. mile above the Dead Horse satch Co., Utah.

Sta.			
30 o	P. C. 4° C	urve r't.	Def 32° 20′
			15° 00′
20 o	P. T.	S. 80° E.	7° 30′
19			6° 00′
18			4° 30′
17			3° 00′
16			1° 30′
15 o	P. C. 3° C	urve l'ft	
10 o			
4			
3			
2			
1			
0 o	S. 65° E.		

[RIGHT PAGE.]

W. R. R., commencing about a Crossing of Price River, Wah-



4. **To Run a Curve with the Transit.**—The transit is set up on the point in the tangent from which the curve is to commence. The limb is clamped with the verniers at zero, the telescope ranged along the line of the tangent, and the instrument clamped in that position. The tangential angle, = ½ the deflection angle, is then turned off on the limb. The leading chain-man draws out the chain or tape in the desired direction, and when out at full length, places his rod in line as directed by the signals of the transit-man. He then carefully measures the length of the chord, marking the distance with his rod, which is then aligned the second time. A stake is driven to mark the point, and the chain-men go ahead and measure the second chord, being aligned by the transit-man as before, and thus continue as far as necessary or convenient. The transit-man turns off equal

angles on the transit for each successive chord as it is measured. At the end of the last chord which is run from any one setting of the transit, a short stake is driven firmly into the ground and a tack driven in the top of the stake, to mark the exact point. If the curve is to be continued, the transit is moved up to this point, and with the limb clamped as it was used at the last observation, the telescope is ranged back to the point from which the observation was taken, and the instrument clamped in that position. As the angles have all been turned off from a point in the circumference of the circle, they are only half as great as the angle at the center subtended by the same chords. Hence the transitman now unclamps the limb and turns off as much more angle as he had previously laid off. This gives him a new line, tangent to the curve, from which he may continue to lay off chords as before.

Some transit men, instead of doubling the angle after the back-sight is taken, turn off an equal amount in the opposite direction on the limb before taking the back-sight. Then, after getting the back-sight, the verniers are brought to zero on the limb, when the telescope will give the line of the new tangent, as before.

Curves are usually run to connect two straight lines which have been previously located. In such a case, preliminary to running the curve, it is necessary to find—

1st. The deflection angle between the lines.

2nd. The radius of the curve to be used.

3d. The P. C. and P. T.

4th. The length of the curve.

The manner of procedure in such a case is indicated in the following:

Example 1.—To join two straight lines having a deflection angle of 48° 16′, by a curve the middle point (f) of which shall be at a distance of 112 feet from the point of intersection.

Assume that the line *abc* has been marked with stakes 100 feet apart, and that the point of intersection is found to be at stake No. 116, + 43.7; in other words, that the P. I. is 43.7 feet beyond stake No. 116.

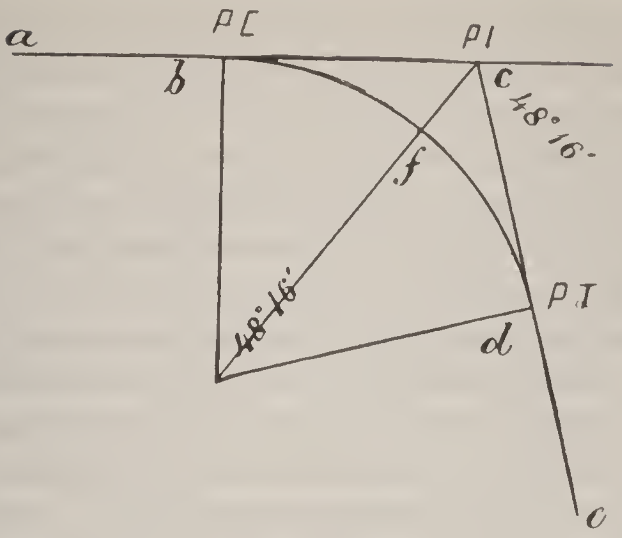


FIG. 65.

The transit is set up over the point of intersection, the verniers clamped at zero, the telescope reversed and ranged along the line *ab*, and the instrument clamped in that position. The telescope is then righted, the upper clamp loosened, the telescope turned and the limb again clamped with the telescope pointing along the line *cde*, and the angle read = 48° 16′. Before proceeding further, it is necessary to determine the degree of curve to be used. By the conditions of the example, the middle point of the curve is to be 112 feet from the P. I. Turning to the table of functions of a 1° curve, we find its external secant, *cf*, to be 548.8 feet for an angle of 48° 16′. Dividing this by 112, we find $\frac{548.8}{112} = 4.9$, or 4° 54′, to be the degree of curvature to be used. Next we find the distance $bc = cd$, which is to be measured along the lines to find the beginning and end of the curve, the P. C. and P. T. Referring again to our table, we find that the tangent of a 1° degree curve for a deflection of 48° 16′ is 2567.1, which divided by 4.9, the degree of curvature, gives 523.9.

We now measure from the P. I. 523.9 feet along the line *cde*, and set a hub and drive a tack in it for the P. T. In a similar manner we next locate the beginning of the curve, which, subtracting 5 + 23.9 from 116 + 43.7, we find to be at Station 111, + 19.8. If the ground be clear and open, so that the whole curve may be seen at once, the transit may now be set up on the P. T., and the whole curve and as much of the next tangent *de* as desired run at one setting of the instrument, at the same time avoiding most of the errors usually made in running the curve from the P. C. If this cannot be done, the transit is set up at the P. C. with verniers at zero and a foresight on the P. I., or back-sight to some point along the line *ab*. The P. C. being at Sta. 111, + 19.8, the first deflection will be for the partial chord found by subtracting 19.8 from 100 = 80.2, or .802 of the full deflection. The tangential deflection for a full chord being 2° 27′, for the partial chord would be .802 of 2° 27′ = 1° 58′ for the first deflection. For each subsequent full chord 2° 27′ additional is turned off on the transit as far as the line can be seen. Say that the line cannot be seen farther than Sta. 116; the several deflections up to that point would be, for Sta. 112, 1° 58′; Sta. 113, 4° 25′; Sta. 114, 6° 52′; Sta. 115, 9° 19′; Sta. 116, 11° 46′. A hub and tack are driven at Sta. 116, and the transit moved up to that point or, what is better, to the P. T., if the station is visible from there. If the transit is set up at Sta. 116, the back-sight is taken on the P. C., with the limb clamped at 11° 46′, as at the last observation. The telescope is then righted, and an additional 11 °46′ turned off for the new tangent, from which the subsequent deflections are turned off. For Station 117 the deflection would be 11° 46′ + 11° 46′ + 2° 27′ = 25° 59′; for Sta. 118, 28° 26′; for Sta. 119, 30° 53′; for Sta. 120, 33° 20′; for Sta. 121, 35° 47′.

Before passing this point, we must know the length of the curve. As there are 48° 16′ total deflection, and each chord cuts off 4° 54′ of it, it is evident there are as many 100 foot chords as 4° 54′ is contained in 48° 16′. Reducing the minutes to decimals and dividing, we have $\frac{48.266}{4.9} =$ 9.85 chords for the length of the curve. This added to 111 + 19.8 gives us 121 + 04.8 for the end of the curve, and 04.8 feet for the last partial chord. We find the deflection for this distance to be .07′, giving for the last deflection 35° 47′ + .07′ = 35° 54′.

The work should now prove itself, by coming out at the stake which was previously set for the end of the curve, and we may further test it by setting the transit up at the P. T., back-sight to Sta. 116, with the instrument clamped at 35° 54′, as last used. Unclamp the limb and turn off as much more as has been turned from Sta. 116, 35° 54′ — 23° 32′ = 12° 22′, which added to 35° 54′ = 48° 16′, the total deflection. If the work has been accurately performed, a back-sight through the telescope should strike the P. I. It is very seldom that curves run in this way will come out just right, hence it is better to never set up the transit at points in the curve between the P. C. and P. T. when it can readily be avoided. Still it is the ordinary and sometimes the only way the curves can be run.

Let the student make the necessary calculations to locate curves from the following data:

Ex. 2.—Total deflection, 26° 50′. External (*cf*, Fig. 65), 120.87 feet. P. C. at Sta. 112, + 40. Transit moved every 550 feet.

Ex. 3.—Total deflection, 35° 15′. External, 126.2 feet. P. I. at Sta. 262, + 07.3. T. P. at Sta. 263.

Ex. 4.—Total deflection, 18° 36′. Curve, 1° 2′, P. I. at 96, + 42.6. T. P. at Sta. 93 and 100.

The starting point of a curve is sometimes so situated that it is not convenient to set up the transit at that point, or to run the line from it if it were, as in streams, gullies, etc., and it then becomes convenient to set up the transit first at some intermediate point in the curve which has to be found.

5. To Locate a Curve from the Middle Point.—Set the transit up at the P. I. Bisect the interior angle *bcd* (Fig. 65). Find the external *cf* of the desired curve and measure it off on the line of bisection. This gives the middle point of the curve. The transit is then set up at this point and a back-sight taken either on the P. C. or P. I., and the curve run in. Let the student make the necessary calculations and give the various deflections which would be used on the transit to locate from the middle point the curve required in Ex. 1, Fig. 133, the first back-sight to be taken from the P. C. Give the same, the back-sight being taken from the P. I. Also, solve the following curves, to be run from the middle points, back-sights from P. C., also from P. I.:

Examples.—1. Total deflection, 16° 24′. Curve, 1° 32′. P. I. at 96, + 27.

2· Total deflection, 26° 18′. Curve, 2° 24′. P. I. at 13, + 62.7.

3. Total deflection, 35° 40′. Curve, 3° 16′. P. I. at 97, + 62.6.

It is sometimes convenient, from various reasons—

6. To Locate the Curve with the Transit at some other Intermediate Point on the Curve than the middle. Such points may be located by ordinates from the tangent. This is usually done to avoid obstacles in the line of the curve. To find approximately on the ground at what point the transit may be set up, the following formula may be used:

Let x = length of the ordinate,
d = distance along the tangent from the P. C.,
t = nat. tangent of ½ the deflection angle of the curve,

Then $x = d^2t$.

Example.—To find whether the transit can be set up at a point on a 4° curve opposite a point on the tangent 400 feet from the P. C.

t = nat. tang., 2° = 03.5. $d^2 = 16$. $\therefore x = 56$. A measure of 56 feet from the tangent will show whether the transit can be set up at this point or not.

It will be fonnd the most convenient in running the curve to select the point at a regular station at the end of a full chord, which may be located as follows:

Example 1.—Total deflection, 48° 48′. P. I. at 62, + 36. Curve, 4° . To find the 4th full station on the line of the curve, and locate the remainder of the curve from that point.

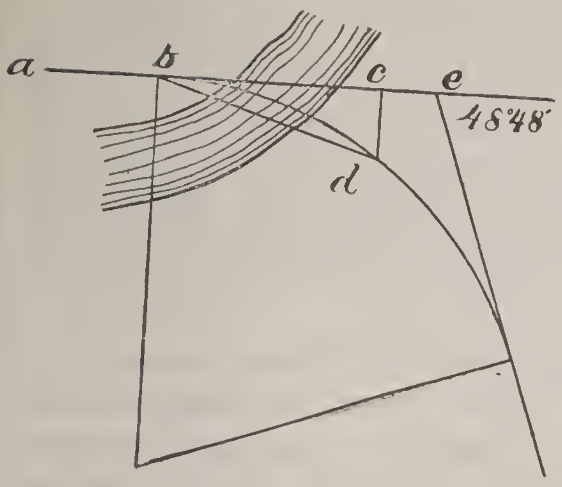


FIG. 66.

First find the number of the station at the P. C. $be =$ tangt. of 1° $\div 4 = \frac{2599.2}{4} = 649.8$ or $6 + 49.8$. This taken from $62 + 36 = 55 + 86.2$, which is the number of the station at the P. C. From here to the 4th full station there is then a short chord of 13.8 feet and four full chords. The tangential angle *cbd* is therefore $4.138 \times 2° = 8° 16\frac{1}{2}'$; whence the deflection angle $= 16° 33'$, the chord of which, *bd*, $= 413.4$. In the right-angled triangle *bcd*, we now have the side *bd* $= 413.4$, and the angle *cbd* $= 8° 16\frac{1}{2}'$, to find the sides *bc* and *cd*, from which we find that *bc* $= 409.1$ and *cd* $= 58.5$. The point *c* may be found by measuring from the P. I. $649.8 - 409 = 240.7 = ec$. Having thus located the point *d*, which is Station 60 on the curve, the transit is set up at that point, with the vernier clamped at 90° , and a back-sight taken to the point *c*. The upper clamp is then loosened and the limb brought to 16° 33′, which gives the tangent from which the remainder of the curve is located.

Let the student calculate the following curves:

2. Total deflection, 36 20′. P. I. at 26, + 44.6. Curve, 2° 30′, to be located from the 3rd full station on the curve.

3. Total deflection, 61° 18′. P. I. at 42, + 28.5. Curve, 4° 40′, to be located from 6th full station on the curve.

4. Total deflection, 42° 50′. P. I. at 112, + 72. Curve, 3° 18′, to be located from Station 114 + 50 on the curve.

7. Short Curves.—When the deflections between the lines are but small, and it is not important that any particular degree of curvature be used, it will be found convenient to make the curve an even two or four stations in length. In case this is done, the curve may be marked out before the transit is moved from the P. I., after observing the deflection angle, and it will not be necessary to set it up on the curve at all. The middle of the curve will be located by laying off the external secant as before directed. The P. C. and P. I. are also located as usual. If four stations are used, the intermediate stations may be determined from the P. I., the same as if the transit were at the P. C. or P. T., the error being so small that it may usually be neglected.

8. Passing Obstructions in the Line.—One method of doing this, by offset from the tangent, has already been sufficiently explained. Another method, which is very generally applicable, is by parallel offsets from the curve. An offset is made in any convenient direction far enough to pass the obstruction. The curve is continued from this point till the obstacle is passed, when the true line is regained by an inset equal to and parallel with the offset. If the lines are run in the manner indicated on page 82, (3), this will be a very simple matter, as the telescope will always point in the same direction when the verniers mark the same point on the limb.

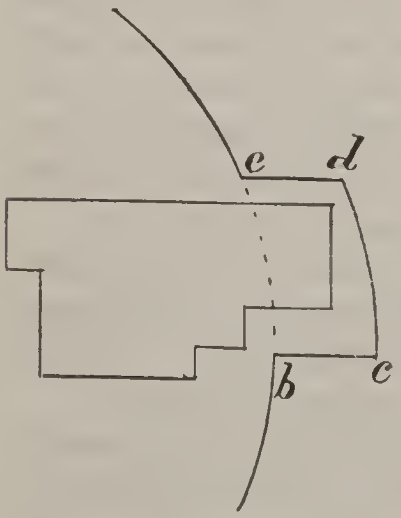


Fig. 67 illustrates this method of passing obstacles. *bc* and *de* are equal and parallel.

FIG. 67.

9. Compound Curves, being a combination of simple curves, have their several components located in the same manner. They are usually run to fit the topography of the country through which they are laid, in order to get uniform gradients on street or railroad lines, or save labor and expense in construction.

Having the several straight lines determined which are to form the tangents of the curve, it is only necessary to find the degrees of curvature of the several component curves, which are then located in the manner already described. Usually there will be found on the ground special reasons for selecting a particular radius for one of the component curves, which will thus dictate the radii of the rest.

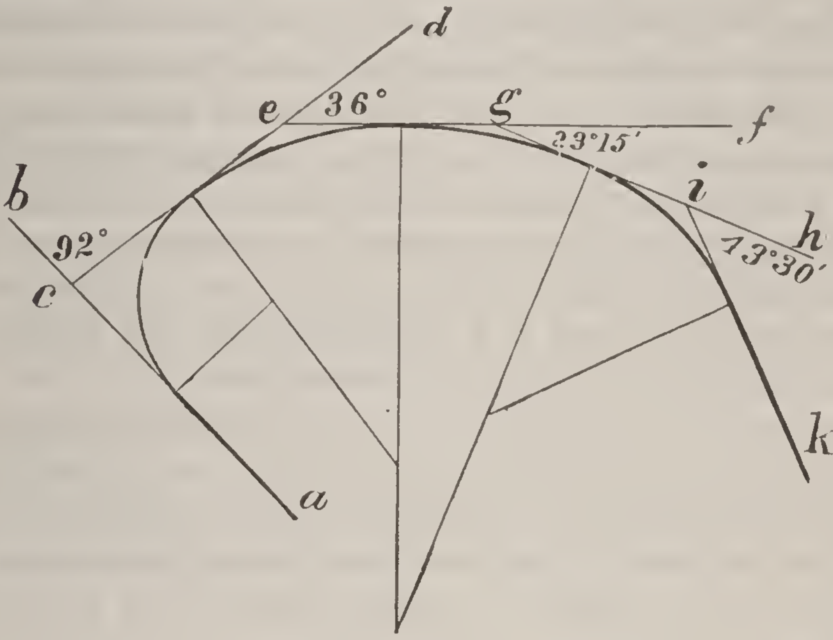


FIG. 68.

Example 1.—Let *ac*, *ce*, *eg*, *gi* and *ik* repre ent tangents of the curve, and *bcd*, *def*, *fgh* and *hik* the angles of deflection.

Let $ce = 1370$, $eg = 1200$, $gi = 1000$.

Let $bcd = 92°$, $def = 36°$, $fgh = 23° \ 15'$, and $hik = 43° \ 30'$, the corresponding curves of which we will number 1, 2, 3 and 4.

Let the tangents *ce* and *eg* be united by a 3° curve.

Required the radii or degrees of curvature of the remaining components of the curve, and the length of the curve.

SUGGESTIONS.—First find the tangent of a 3° curve for an angle of 36°. Tangent of 1° curve for 36° = 1861.8; ∴ for 3° curve = 620.6. This leaves 1370 — 620.6 = 749.4, length of tangent of curve No. 1. Tangent of 1° curve for 92° = 5933.2, which divided by 749.4 = 7.917° or 7° 55′, the degree of curvature. Radius, 724.3. Length of curve $= \dfrac{92}{7.917} = 1162$ feet. We find the tangent of curve No. 3

by subtracting the tangent of curve No. 2, 620.6, from the length of the line *eg*, 1200, = 579.4. The tangent of a 1° curve for a deflection of 23° 15′ we find from the table to be 1178.8, which divided by 579.4 gives the degree of curve to be used, 2.034° = 2° 02′. The calculations for the remainder of the curve are made in a similar manner.

It is customary in running long lines for drains, railways, etc., to run preliminary lines by angles, omitting the curves, till the location of the tangents is definitely determined. Stakes are set and numbered the same as on the final location. Both the staking and measuring are sometimes omitted, the lines being run as simple picket lines. In such case, when the final location is made, the line is staked out to the point of intersection of the tangents and afterward, as the curve is run in, the stakes between the P. C. and P. I. are taken up and moved to their proper place in the line of the curve.

Examples for solution.—1. Let the student calculate the curves and plat the line from the following notes of a preliminary angle line, making all the calculations that would be required in the field, and giving the corrected numbers of the stations at the several P. C.'s, P. T.'s, P. C. C.'s and P. R. C.'s:—

176 o	
165 o	P. I. Compound Curve. Angle right, 14°.
163 + 20 o	P. I. Reverse Curve. Angle right, 36°.
153 o	P. I. Reverse Curve. Ang'e left, 17° 26′.
144 + 26 o	P. I. 2° Curve. Angle right, 16° 30′
129 o	
118 o	
108 o	
98 + 15 o	P. I. of Reverse Curve. Angle right, 53° 12′.
85 + 60 o	P. I. Compound Curve. Angle left, 16°.
76 + 48 o	P. I. Compound Curve. Angle left, 7° 8′.
67 o	P. I. ———— Curve. Angle left, 12° 23′.
58 o	
48 o	
41 o	P. I. 2° 16′ Curve. Angle right, 14°.
30 o	
21 o	P. I. 3° Curve. Angle left, 26° 32′.
14 o	
0 o	N. 45° E.

2. The following are notes of the north side of a street in Park Beidler. The measures are taken with a 66 foot tape of 100 links. The street is one chain wide. A tier of lots two chains deep is laid out on each side of the street. The lots are one chain wide on the street, and are marked by stakes set and numbered at regular intervals of one chain. The lines for the south side of the street and for the back ends of the two tiers of lots are to be run with the transit and tape. Required the details of these lines and the widths of lots at the back end, the lot lines being at right angles with the street and on the radii of the curves.

48	Intersect west line of Dawn Street. Course N. and S.
45	P. T.
40	P. R. C. 10° Curve right.
36	P. C. C. 8° Curve left.
32	P. C. C. 5° Curve left.
26	P.C. 2° Curve left.
24	P. T.
21	P. C. C. 6° Curve right.
18	P. R. C. 4° Curve right.
12	P. C. C. 4° Curve left.
8 + 50	P. R. C. 8° Curve left.
6 + 20	P. C. C. 4° 30′ Curve right.
2	P. C. 3° Curve right.
0	East at right angles with Sylvan St. Course N. and S.

The following formula has been found very useful in solving many problems in the location of curves. Like the formula $x = d^2t$ in Art. 6, it is designed to express the length of an ordinate from the tangent to the curve:

Let $x =$ length of the ordinate,

$n =$ length of the curve in chords of 100 feet each,

$d =$ degree of curvature.

Then $x = \frac{7}{8}n^2d$. Thus a 6° curve will have diverged from its tangent at the end of 500 feet, $\frac{7}{8} \times 5^2 \times 6 = 131.25$ feet.

By making d equal the difference of the degree of curvature of two curves of different radii but having a common origin, x will be their divergence from each other at the end of n stations. This formula is not mathematically exact, and therefore gives only approximate results; but it is sufficiently correct for all ordinary cases. It is easily remembered; it requires no tables; and with its aid, with such modifications as a little ingenuity will suggest, and a table of actual tangents for a 1° curve, the surveyor can solve almost any case that will ordinarily arise in the field. For example: Suppose a 5° curve to the right 8 stations long has been located, and its extremity falls 28 feet too far to the right to throw the tangent on the best ground. Making $x = 28$, we obtain $d = \frac{1}{2}$, showing that a 4° 30′ curve starting from the same origin would pass through the required spot. Again: Suppose that in this same case the new curve is to commence 200 feet back of the first one; then the required divergence from the tangent will be $\frac{7}{8} \times 8^2 \times 5 - 28 = 252$. Substituting this value for x, and making $n = 8 + 2$, we have $d = 2.88 = 2° 53′$.

CHAPTER VIII.

ORIGINAL SURVEYS.

1. In land surveying, the surveyor has two distinct classes of problems to deal with. In the first class, he is called upon

(*a*) To lay down upon the ground the corners and boundary lines of tracts of land of specified dimensions; and

(*b*) To find the areas of tracts which are already defined by natural or artificial boundaries.

In this class is included the original marking out upon the ground of the boundaries of every tract of land however great or small. Hence we call surveys of this nature **Original Surveys.**

2. When the boundaries have once been laid down upon the ground and marked by persons having authority to do so, then the surveyor, who is afterward called upon, has a different class of problems to deal with. He then has

(*a*) To find the corner posts and monuments;

(*b*) To re-locate them when lost; and

(*c*) To retrace old boundary lines.

Surveys of this nature we shall call **Resurveys.**

3. Original Surveys include: *First.* The rectangular surveys of the United States, known as the government survey; similar surveys in Canada and other countries by government authority, and the subdivision of sections. *Second.* Surveys made by the proprietors in those regions where the government surveys do not extend, including in the United States the surveys of all land not granted by the original states of the Union to the general government; and surveys for town plats, highways and like purposes.

4 **United States Survey.**—The territory embraced within the present States of Ohio, Indiana, Illinois, Michigan, Wisconsin, and Tennessee, that part of Minnesota lying east of the Mississippi River, and all of Alabama and Mississippi lying north of the thirty-first parallel, was held by Massachusetts, Connecticut, New York, Virginia, North Carolina, South Carolina, and Georgia, under grants from Great Britain, during their colonial condition. These territorial interests were surrendered to the General Government of the Union by the last named States at different times hereinafter set forth, and constituted the nucleus of our public domain with some reservations as to former grants, and was the remainder of the territory conceded to the United States under the definitive treaty of 1783, and consisted of 404,955.91 square miles, or 259,171,787 acres. This was the public domain of the United States on April 30, 1803, the date of the Louisiana purchase, and for which the original survey and disposition laws were made.

The United States were recognized by the Crown in the definitive treaty of peace with Great Britain as "free sovereign and independent States, and that he treats with them as such, and for himself, his heirs, and successors relinquishes all claims to the government, proprietary and territorial rights of the same, and every part thereof."

The Government of the United States acquired as custodian for the Nation, lands known as the public domain as follows:

From States (colonies prior to July 4, 1776) ceded under the Confederation and under the Constitution.

This was in pursuance of a resolution of the Congress of the Confederation passed Tuesday, October 10, 1780, providing for the reception and care of such unappropriated lands as might be ceded by States to the United States, and for the disposition of the same for the common benefit of the United States.

The dates of cession of these lands to the United States were as follows:

Colony.	State.	Date of Cession.
New Hampshire	New Hampshire.	No cession.
New York	New York.	March 1, 1781.
Rhode Island and Providence Plantations	Rhode Island.	No cession.
New Jersey	New Jersey.	Do.
New Castle, Kent and Sussex, on Delaware	Delaware.	Do.
Pennsylvania	Pennsylvania.	Do.
Virginia	Virginia.	March 1, 1784, and December 30, 1788.*
Maryland	Maryland.	No cession.
Massachusetts Bay	Massachusetts.	April 19, 1785.
Connecticut	Connecticut.	September 13, 1786; confirmed May 30, 1800.
South Carolina	South Carolina.	August 9, 1787.
North Carolina	North Carolina.	February 25, 1790.
Georgia	Georgia.	April 24, 1802.

*An act to change the conditions of the cession of March 1, 1784, only so far as to ratify the fifth article of the compact of the ordinance of 1787.

AREA OF CESSIONS.

	Sq. miles.	Acres.
Massachusetts (disputed) claimed (estimated)*	54,000.00	34,560,000
Connecticu (disputed) and Western Reserve and Fire-lands (estimated)*	40,000.00	25,600,000
From New York and Massachusetts cession, actual	315.91	202,187
From Virginia (disputed and undisputed) to the United States (exclusive of Kentucky and including area of Western Reserve and the Fire-lands)†	265,562.00	169,959,680
South Carolina cession	4,900.00	3,136,000
North Carolina cession, nominal, because the area of Tennessee was almost covered with reservations	45,600.00	29,184,000
Georgia cession	88,578.00	56.689,920
Total actual State cessions to the United States for public domain	404,955.91	259,171,787

*The area above was also claimed by Virginia and included in her cession.

†Connecticut's jurisdictional cession of the Western Reserve and Fire-lands, containing about 3,800,000, included under Virginia cession.

AREA OF PURCHASES—PUBLIC AND NATIONAL DOMAIN.

	Sq. miles.	Acres.
Louisiana purchase, April 30, 1803	1,182,752	756,961,280
East and West Florida, Feb. 22, 1819	59,268	37,931,520
Gaudalupe Hidalgo, February 2, 1848	522,568	334,443,520
State of Texas, November 25, 1850	96,707	61,892,480
Gadsden purchase, December 30, 1853	45,535	29,142,400
Alaska purchase, March 30, 1867	577,390	369,529,600
	2,484,220	1,589,900,800

At a total cost of $88,157,389.98.

The Texas annexation of 1845 added to the national domain the area of the present State of Texas, viz., 274,356 square miles, or 175,587,840 acres, included in the national domain, besides the purchase of 1850 from the State, now public domain.

The total area of purchased and annexed territory, included in the national and public domain since 1803, is 2,758,576 square miles, or 1,765,488,640 acres, at a total cost of $88,157,389.98 for the purchase, and including the Georgia cession of 1802, $6,200,000.

5. **The present system of survey** of the public lands was inaugurated by a committee appointed by the Continental Congress, and consisting of the following delegates: Hon. Thomas Jefferson, chairman, Virginia; Hon. Hugh Williamson, North Carolina; Hon. David Howell, Rhode Island, Hon. Elbridge Gerry, Massachusetts; Hon. Jacob Read, South Carolina.

On the 7th of May, 1784, this committee reported "An ordinance for ascertaining the mode of locating and disposing of lands in the western territory, and for other purposes therein mentioned." This ordinance required the public lands to be divided into "hundreds" of ten geographical miles square, and those again to be subdivided into lots of one mile square each, to be numbered from 1 to 100, commencing in the *north-western* corner. and continuing from west to east and from east to west consecutively. This ordinance was considered, debated, and amended, and reported to Congress April 26, 1785, and required the surveyors "to divide the said territory into townships of 7 miles square, by lines running due north and south, and others crossing these at right angles. * * * The plats of the townships, respectively, shall be marked by subdivisions into sections of 1 mile square, or 640 acres, in the same direction as the external lines, and numbered from 1 to 49. * * * And these sections shall be subdivided into lots of 320 acres."

This is the first record of the use of the terms "township" and "section."

May 3, 1785, on motion of Hon. William Grayson, of Virginia, seconded by Hon. James Monroe, of Virginia, the section respecting the extent of townships was amended by striking out the words "seven miles square" and substituting the words "six miles square." The records of these early sessions of Congress are not very full or complete; but it does not seem to have occurred to the members until the 6th of May, 1785, that a township six miles square could not contain 49 sections of 1 mile square. At that date a motion to amend was made, which provided, among other changes, that a township should contain 36 sections; and the amendment was *lost*. The ordinance as finally passed, however, on the 20th of May, 1785, provided for townships 6 miles square, containing 36 sections of 1 mile square. The first public surveys were made under this ordinance. The townships, 6 miles square, were laid out in ranges, extending northward from the Ohio River, the townships being numbered from south to north, and the ranges from east to west. The region embraced by the surveys under this law forms a part of the present State of Ohio, and is usually styled "The Seven Ranges." In these initial surveys only the *exterior lines* of the townships were surveyed, but the plats were marked by subdivisions into sections of 1 mile square, and mile corners were established on the township lines. The sections were numbered from 1 to 36, commencing with No. 1 in the *southeast* corner of the township, and running from *south* to *north* in each tier to No. 36 in the *northwest* corner of the township, as shown in the following diagram:

36	30	24	18	12	6
35	29	23	17	11	5
34	28	22	16	10	4
33	27	21	15	9	3
32	26	20	14	8	2
31	25	19	13	7	1

The surveys were made under the direction of the Geographer of the United States.

The act of Congress approved May 18, 1796, provided for the appointment of a surveyor-general, and directed the survey of the lands northwest of the Ohio River, and above the mouth of the Kentucky River, "in which the titles of the Indian tribes have been extinguished." Under this law *one-half* of the townships surveyed were subdivided into sections "by running through the same, each way, parallel lines at the end of every two miles, and by making a corner on each of said lines at the end of every mile," and it further provided that "the sections shall be numbered, respectively, beginning with the number one in the northeast section and proceeding west and east alternately, through the township, with progressive numbers till the thirty-sixth be completed." This method of numbering sections, as shown by the following diagram, is still in use:

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

The act of Congress approved May 10, 1800, required the "townships west of the Muskingum, which * * * are directed to be sold in quarter townships, to be subdivided into half sections of three hundred and twenty acres each, as nearly as may be, by running parallel lines through the same from east to west, and from south to north, at the distance of one mile from each other, and marking corners, at the distance of each half mile on the lines running from east to west, and at the distance of each mile on those running from south to north. * * * And the interior lines of townships intersected by the Muskingum, and of all the townships lying east of that river, which have not been heretofore actually subdivided into sections, shall also be run and marked. * * * And in all cases where the exterior lines of the townships thus to be subdivided into sections or half sections shall exceed, or shall not extend, six miles, the excess or deficiency shall be specially noted, and added to or deducted from the western and northern ranges of sections or half sections in such township, according as the error may be in running the lines from east to west or from south to north."

6. The acts of Congress defining the system of public land surveys, and the principles to be employed in carrying them out, are to be found in the United States Statutes as follows:

Act of	May	18, 1796,	Volume	1,	Chap.	29.
"	"	10, 1800,	"	2,	"	55.
"	Feb.	11, 1805,	"	2,	"	14.
"	April	24, 1820,	"	3,	"	51.
"	"	5, 1832,	"	4,	"	55.
"	May	30, 1862,	"	12,	"	86.
"	March	3, 1875,	"	18,	"	130.
"	"	3, 1875,	"	19,	"	105.

Such portions of the various acts as are now in force are published by the government in a volume entitled "Existing Land Laws." Those Sections which refer directly to the surveys are as follows:

7. United States Laws relating to Surveys and Surveyors.—Sec. 77. There shall be appointed by the President, by and with the advice and consent of the Senate, a surveyor-general for the States and Territories herein named, embracing, respectively, one surveying district, namely: Louisiana, Florida, Minnesota, Kansas, California, Nevada, Oregon, Nebraska and Iowa, Dakota, Colorado, New Mexico, Idaho, Washington, Montana, Utah, Wyoming, Arizona.

3 Stat. 755; 4 *id.* 492; 9 *id.* 496; 10 *id.* 244, 306, 308, 309, 611; 11 *id.* 212; 12 *id.* 176, 211, 214; 14 *id.* 77, 85. 314, 542; 15 *id.* 91; 16 *id.* 65, 240; 17 *id.* 76; 18 *id.* 18, 34, 121, 122, 123, 201, 303; 19 *id.* 126, 207: R. S. 2207.

Sec. 84, Every surveyor-general shall, before entering on the duties of his office, execute and deliver to the Secretary of the Interior a bond, with good and sufficient security, for the penal sum of thirty thousand dollars, conditioned for the faithful disbursement, according to law, of all public money placed in his hands, and for the faithful performance of the duties of his office; and the President has discretionary authority to require a new bond and additional security, under the direction of the Secretary of the Interior, for the lawful disbursement of public moneys.

3 Stat. 697; R. S. 2215, 2216. U. S. *v.* Vanzandt, 11 Wheat, 184; U. S. *v.* Tingey, 5 Pet. 115; Farrar and Brown *v.* U. S,, 5 *id.* 373; U. S. *v.* Bradley, 10 *id.* 343; U. S. *vs.* Linn, 15 *id.* 290; U. S. *v.* Prescott, 3 How. 578; U. S. *v.* Boyd, 5 *id.* 29; Bryan *v.* U. S., 1 Black, 140; Boyden *v.* United States, 13 Wall. 17; Bevans *v.* U. S , 13 *id.* 56; U. S. *v.* Thomas, 15 *id.* 337; U. S. *v.* Stephenson, 1 McClean, C. C. 462; U. S. *v.* Linn, 2 *id.* 501; U. S. *v.* Ward, 3 *id.* 179. 8 Op. Att. Gen. 7. Cir. G. L. O., July 1, 1871; *id.* May 14, 1879. Treasury Cir., July 13, 1871 (Copp's L. L. 783; 1 Lester's L. L. 312, 314).

Sec. 85. The commission of each surveyor-general shall cease and expire in four years from the date thereof, unless sooner vacated by death, resignation, or removal from office.

3 Stat. 697; R. S. 2217. Best *v.* Polk, 18 Wall. 112. Decision Com. G. L. O., Feb. 20, 1858 (1 Lester's L. L. 340).

Sec. 86. Every surveyor-general, except where the President sees cause otherwise to determine, is authorized to continue in the uninterrupted discharge of his regular official duties after the day of expiration of his commission and until a new commission is issued to him for the same office, or until the day when a successor enters upon the duties of such office; and the existing official bond of any officer so acting shall be deemed good and sufficient and in force until the date of the approval of a new bond to be given by him, if recommissioned, or otherwise, for the additional time he may so continue officially to act, pursuant to the authority of this section.

10 Stat. 247; 18 *id*, 62; R. S. 2222.

Sec. 87. Whenever the surveys and records of any surveying district are completed, the surveyor-general thereof shall be required to deliver over to the Secretary of State of the respective states, including such surveys, or to such other officer as may be authorized to receive them, all the field-notes, maps, records, and other papers appertaining to land titles within the same; and the office of surveyor-general in every such district shall thereafter cease and be discontinued.

5 Stat. 384; 19 *id.* 121; R. S. 2218.

SEC. 88. In all cases of discontinuance, as provided in the preceding section, the authority, powers, and duties of the surveyor-general in relation to the survey, resurvey, or subdivision of the lands therein, and all matters and things connected therewith, shall be vested in and devolved upon the Commissioner of the General Land Office.

10 Stat. 152; R. S. 2219.

SEC. 89. Under the authority and direction of the Commissioner of the General Land Office, any deputy surveyor or other agent of the United States shall have free access to any such field-notes, maps, records, and other papers for the purpose of taking extracts therefrom or making copies thereof without charge of any kind; but no transfer of such public records shall be made to the authorities of any State until such State has provided by law for the reception and safe-keeping of such public records and for the allowance of free access thereto by the authorities of the United States.

10 Stat. 152; 18 *id.* 62; R. S. 2220, 2221.

SEC. 90. Every surveyor-general shall engage a sufficient number of skillful surveyors as his deputies, to whom he is authorized to administer the necessary oaths upon their appointments. He shall have authority to frame regulations for their direction, not inconsistent with law or the instructions of the General Land Office, and to remove them for negligence or misconduct in office.

Taylor and Quarlls *v.* Brown, 5 Cranch, 234; Craig *et al. v.* Braxford, 3 Wheat, 594; Ellicott *et al. v.* Pearl, 10 Pet. 412; Brown's Lessee *v.* Clements, 3 How. 650. Reed *v.* Conway 20 Mo. 22; same case, 26 *id.* 13; Hamil *v.* Carr, 21 Ohio St. 258; Doe *v.* Hildreth, 2 Ind. 274; McClintock *v.* Rodgers, 11 Ills. 279. Cir. G. L. O., June 26, 1880.

Second. He shall cause to be surveyed, measured, and marked, without delay, all base and meridian lines through such points and perpetuated by such monuments, and such other correction parallels and meridians as may be prescribed by law or by instructions from the General Land Office in respect to the public lands within his surveying district, to which the Indian title has been or may be hereafter extinguished.

Gazzen *v.* Phillips' Lessee, 20 How. 372. 3 Op. Att. Gen., 281, 284. Atshire *v.* Hulse, 1 Ohio, 170; Hastings *v.* Stevenson, 2 *d.* 9; McKinney *v.* McKinney, 8 *id.* 423; Hamil *v.* Carr, 21 Ohio St. 258; Hendrick *v.* Eno, 42 Iowa 411; Saint Louis *v.* Walker, 40 Mo. 383; Jordan *v.* Barrett, 13 La. 24; Fowler *v.* Duval, 11 *id.* 561; Cox *v.* Jones, 47 Cal. 412. Cir. G. L. O., June 26, 1880.

Third. He shall cause to be surveyed all private land claims within his district after they have been confirmed by authority of Congress, so far as may be necessary to complete the survey of the public lands.

Menard's Heirs *v.* Massey, 8 How. 293; Kissell *v.* St. Louis Public Schools, 18 *id.* 19; Stanford *v.* Taylor, 18 *id.* 409; Ballance *v.* Forsyth, 24 *id.* 183; U. S. *v.* Fossat, 25 *id.* 445; Carondelet *v.* St. Louis, 1 Black, 179; U. S. *v.* Sepulveda, 1 Wall. 104; U. S. *v.* Halleck, 1 *id.* 439; U. S. *v.* Billings, 2 *id.* 444; Sutter's case, 2 *id.* 562; U. S. *v.* Pacheco, 2 *id.* 587; Fossat case, 2 *id.* 649; Dehon *v.* Bernal, 2 *id.* 774; U. S. *v.* Armijo, 5 *id.* 444; Higueras *v.* U. S. 5 *id.* 827; Maguire *v.* Tyler, 8 *id.* 650; Lynch *v.* Bernal 9 *id.* 315; Henshaw *v.* Bissell, 18 *id.* 255; Shepley *et al.* *v.* Cowan *et al.*, 1 Otto, 330; Miller *et al.* *v.* Dale *et al.*, 2 *id.* 473; Van Reynegand *v.* Bolton, 5 *id.* 33; U. S. *v.* Throckmorton, 8 *id.* 61; Snyder *v*, Sickles, 8 *id.* 203; Scull *v.* U. S., 8 *id*, 410. Bissell *v.* Henshaw, 1 Saw. C. C. 553; Leroy *v.* Jamison, 3 *id.* 369. Gibson *v.* Chouteau, 39 Mo. 536; Milburn *v.* Hardy, 28 *id.* 514; Funkhouser *v.* Hantz, 29 *id.* 540; Dent *v.* Legesson, 29 *id.* 489; Carondelet *v.* St. Louis, 29 *id.* 527; Maguire *v.* Tyler, 30 *id.* 202; Robins *v.* Eckler, 36 *id.* 494; Clark *v.* Heammerle, 36 *id.* 620; Gibson *v.* Chouteau, 39 *id.* 536; Vasquez *v.* Ewing, 42 *id.* 247; Glasgow *v.* Lindell, 50 *id.* 60; Rector *v.* Gaines, 19 Ark. 70; Ashley *v.* Rector, 20 *id.* 359; Meaux *v.* Breaux, 10 Martin (La.) 364; Moon *v.* Wilkinson, 13 Cal. 478; Boggs *v.* Mining Co., 14 *id.* 279; Mott *v.* Smith, 16 *id.* 534; Johnson *v.* Van Dyke, 20 *id.* 225; McGarrahan *v.* Maxwell, 27 *id.* 75; Treadway *v.* Semple, 28 *id.* 652; Searle *v.* Ford, 29 *id.* 104; Mahoney *v.* Van Winkle, 33 *id.* 448; Morrill *v.* Chapman, 35 *id.* 85; Yates *v.* Smith, 38 *id.* 60; San Diego *v.* Allison, 46 *id.* 163. Decisions Sec. Int., July 16, 1872; Aug. 8, 1876; Aug. 17, 1876; March 16, 1877. Decisions Com. G. L. O., Aug. 18, 1860; Sept. 18, 1874; Nov. 3, 1874; Sept. 18, 1875; Oct. 28, 1875; June 26, 1879. Cir. G. L. O., June 26, 1880.

Fourth. He shall transmit to the register of the respective land offices within his district general and particular plats of all lands surveyed by him for each land district; and he shall forward copies of such plats to the Commissioner of the General Land Office.

Barnard *v.* Ashley, 18 How. 43; Water and Mining Co. *v.* Bugbee, 6 Otto. 165; Hamil *v.* Carr, 21 Ohio St. 258; Doe *v.* Hildreth, 2 Ind 274; Pope *v.* Athearn, 42 Cal. 606; Com. G. L. O. Instructions to Surveyor-General, April 17, 1879.

Fifth. He shall, so far as is compatible with the desk duties of his office, occasionally inspect the surveying operations while in progress in the field, sufficiently to satisfy himself of the fidelity of the execution of the work according to contract, and the actual and necessary expenses incurred by him while so engaged shall be allowed; and where it is incompatible with his other duties for a surveyor-general to devote the time necessary to make a personal inspection of the work in progress, then he is authorized to depute a confidential agent to make such examination, and the actual and necessary expenses of such person shall be allowed and paid for that service, and five dollars a day during the examination in the field; but such examination shall not be protracted beyond thirty days, and in no case longer than is actually necessary; and when a surveyor-general, or any person employed in his office at a regular salary, is engaged in such special service he shall receive only his necessary expenses in addition to his regular salary.

1 Stat. 464; 13 *id.* 325; 4 *id.* 492; 10 *id.* 245, 247; 18 *id.* 34; 19 *id.* 126; R. S. 2223. Sec. Int. Instructions, July 1, 1874; Sept. 21, 1874. Cir. G. L. O., June 26, 1880.

SEC. 91. Every deputy surveyor shall enter into a bond, with sufficient security, for the faithful performance of all surveying contracts confided to him: and the penalty of the bond, in each case, shall be double the estimated amount of money accruing under such contracts, at the rate per mile stipulated to be paid therein. The sufficiency of the sureties to all such bonds shall be approved and certified by the proper surveyor-general.

4 Stat. 493; 10 *id.* 247; R. S. 2230. U. S. *v.* Vanzandt, 11 Wheat. 184; U. S. *v.* Tingey, 5 Pet. 115; Farrar *et al.* *v.* U. S., 5 *id.* 373; U. S. *v.* Bradley, 10 *id.* 343; U. S. *v.* Linn, 15 *id.* 290. U. S. *v.* Stephenson, 1 McLean, C C. 462.

SEC. 92. The surveyors-general, in addition to the oath now authorized by law to be administered to deputies on their appointment to office, shall require each of their deputies, on the return of his surveys, to take and subscribe an oath that those surveys have been faithfully and correctly executed according to law and the instructions of the surveyor-general.

9 Stat. 79; R. S. 2231. Ellicott and Meredith *v.* Pearle, 10 Pet. 412; U. S. *v.* Hanson, 16 *id.* 196; Bollard *et al.* *v.* Dwight *et al.*, 4 Cranch, 421; Taylor *et al.* *v.* Brown, 5 *id.* 234. Cir. G. L. O., June 26, 1880.

SEC. 93. The district attorney of the United States, in whose district any false, erroneous, or fraudulent surveys have been executed, shall, upon the application of the proper surveyor-general, immediately institute suit upon the bond of such deputy, and the institution of such suit shall act as a lien upon any property owned or held by such deputy or his sureties at the time such suit was instituted.

9 Stat. 79; R. S. 2232.

SEC. 99. The public lands shall be divided by north and south lines run according to the true meridian, and by others crossing them at right angles, so as to form townships of six miles square, unless where the line of an Indian reservation, or of tracts of land heretofore surveyed or patented, or the course of navigable rivers, may render this impracticable; and in that case this rule must be departed from no further than such particular circumstances require.

McKinney *v.* McKinney, 8 Ohio, 423; Hamil *v.* Carr, 21 Ohio St. 258, Decision Sec. Int , Jan. 24, 1880. Cir. G. L. O , June 26, 1880.

Second. The corners of the townships must be marked with progressive numbers from the beginning, each distance of a mile between such corners must be also distinctly marked with marks different from those of the corners.

Third. The township shall be subdivided into sections, containing, as nearly as may be, six hundred and forty acres each, by running through the same, each way, parallel lines at the end of every two miles; and by making a corner on each of such lines, at the end of every mile. The sections shall be numbered, respectively, beginning with the number one in the northeast section and proceeding west and east alternately through the township with progressive numbers till the thirty-six be completed.

Grogan *v.* Knight, 27 Cal. 516. Decision Sec. Int., April 14, 1879. Cir. G. L. O., June 26, 1880.

Fourth. The deputy surveyors, respectively, shall cause to be marked on a tree near each corner established in the manner described, and within the section, the number of such section, and over it the number of the township within which such section may be; and the deputy surveyors shall carefully note, in their respective field-books, the names of the corner-trees marked and the numbers so made.

Cir. G. L. O., June 26, 1880.

Fifth. Where the exterior lines of the townships which may be subdivided into sections or half-sections exceed, or do not extend six miles, the excess or deficiency shall be specially noted, and added to or deducted from the western and northern ranges of sections or half-sections in such townships, according as the error may be in running the lines from east to west, or from north to south; the sections and half-sections bounded on the northern and western lines of such townships shall be sold as containing only the quantity expressed in the returns and plats respectively, and all others as containing the complete legal quantity.

Knight *v.* Elliott, 57 Mo. 317; Vaughn *v.* Tate, 64 *id.* 491; Walters *v.* Commons, 2 Port. (Ala.) 38; Lewen *v.* Smith, 7 *id* 428. Decision Sec. Int., April 14, 1879. Cir. G. L. O., June 26, 1880.

Sixth. All lines shall be plainly marked upon trees, and measured with chains, containing two perches of sixteen and one-half feet each, subdivided into twenty-five equal links; and the chain shall be adjusted to a standard to be kept for that purpose.

Bradley *v.* Taylor, 5 Cranch, 191; McIvers *v.* Walker, 9 *id.* 173; Shipp *v.* Miller's Heirs, 2 Wheat. 316; Holmes *v.* Trout, 7 Pet. 171; Brown *v.* Huger, 21 How. 305; Meron *v.* Whitney, 5 Otto, 551; Robinson *v.* Moon, 4 McLean, C. C. 279. Oakley *v.* Stuart, 52 Cal. 521. Cir. G. L. o., June 26, 1880.

Seventh. Every surveyor shall note in his field-book the true situations of all mines, salt licks, salt springs, and and mill-seats which come to his knowledge; all water courses over which the line he runs may pass; and also the quality of the lands.

Newsom *v.* Pryor's Lessee, 7 Wheat. 7; Preston *v.* Bowman, 6 *id.* 580; Patterson *v* Jenks, 2 Pet. 216.

Eighth. These field-books shall be returned to the surveyor-general, who shall cause therefrom a description of the whole lands surveyed to be made out and transmitted to the officers who may superintend the sales. He shall also cause a fair plat to be made of the townships and fractional parts of townships contained in the lands, describing the subdivisions thereof and the marks of the corners. This plat shall be recorded in books to be kept for that purpose; and a copy thereof shall be kept open at the surveyor-general's office for public information, and other copies shall be sent to the places of the sale and to the General Land Office.

1 Stat. 465; 2 *id.* 73; 19 *id.* 348; R. S. 2395. Taylor *et al.* *v.* Brown, 5 Cranch, 234; Barnard *v.* Ashley, 18 How. 43; Water and Mining Co. *v.* Bugbee, 6 Otto, 165. Rector *v.* Gaines, 19 Ark. 70; Lewen *v.* Smith, 5 Port. (Ala.) 428; Mott *v.* Smith, 16 Cal. 534; Hamil *v.* Carr, 21 Ohio St. 258; Doe *v.* Hildreth, 2 Ind. 274; McClintock *v.* Rodgers, 11 Ills. 279. Decision Sec. Int., Jan. 15, 1878 Decision Com. G. L. O., April 17, 1879.

SEC. 100. The boundaries and contents of the several sections, half-sections, and quarter-sections of the public lands shall be ascertained in conformity with the following principles:

First. All the corners marked in the surveys, returned by the surveyor-general, shall be established as the proper corners of sections, or subdivisions of sections, which they were intended to designate; and the corners of half and quarter sections, not marked on the surveys, shall be placed as nearly as possible equidistant from those two corners which stand on the same line.

Second. The boundary lines, actually run and marked in the surveys returned by the surveyor-general, shall be established as the proper boundary lines of the sections, or subdivisions, for which they were intended, and the length of such lines, as returned, shall be held and considered as the true length thereof. And the boundary lines which have not been actually run and marked shall be ascertained by running straight lines from the established corners to the opposite corresponding corners; but in those portions of the fractional townships where no such opposite corresponding corners have been or can be fixed, the boundary lines shall be ascertained by running from the established corners due north and south or east and west lines, as the case may be, to the water-course, Indian boundary line, or other external boundary of such fractional township.

Mott *v.* Smith, 16 Cal. 534; Guin *v.* Brandon, 29 Ohio St. 656; McClintock *v.* Rodgers, 11 Ills. 279; Goodman *v*, Myrick, 5 Oreg. 65. Cir. G. L. O., June 26, 1880.

Third. Each section or subdivision of section, the contents whereof have been returned by the surveyor-general, shall be held and considered as containing the exact quantity expressed in such return; and the half-sections and quarter-sections, the contents whereof shall not have been thus returned, shall be held and considered as containing the one-half or the one-fourth part, respectively, of the returned contents of the section of which they make part.

2 Stat. 313; R. S. 2396. Lindsey *v.* Hawes, 2 Black, 554; U. S. *v.* Pacheco, 2 Wall. 587; Railway Co. *v.* Schurmier, 7 *id.* 272; County of Saint Clair *v.* Livingston, 23 *id.* 46; Heidekoper *v.* Brooms, 1 Wash. C. C. 109; Coon *v.* Pen, 1 Pet. C. C. 496. 2 Op. Att. Gen. 578. Knight *v.* Elliott, 57 Mo. 317; Vaughn *v.* Tate, 64 *id.* 491; Waters *v.* Commons, 2 Port. (Ala.) 38; Lewen *v.* Smith, 7 *id.* 428; Billingsly *v.* Bates, 30 Ala. 376; Doe *v.* Hildreth, 2 Ind. 274; Grogan *v.* Knight, 27 Cal. 516. Decision Com. G. L. O., May 17, 1875. Cir. G. L. O., June 26, 1880.

Sec. 101. In every case of the division of a quarter-section the line for the division thereof shall run north and south, and the corners and contents of half quarter-sections which may thereafter be sold shall be ascertained in the manner and on the principles directed and prescribed by the section preceding, and fractional sections containing one hundred and sixty acres or upwards shall in like manner, as nearly as practicable, be subdivided into half quarter-sections, under such rules and regulations as may be prescribed by the Secretary of the Interior, and in every case of a division of a half quarter-section, the line for the division thereof shall run east and west, and the corners and contents of quarter quarter-section, which may thereafter be sold, shall be ascertained, as nearly as may be, in the manner and on the principles directed and prescribed by the section preceding; and fractional sections containing fewer or more than one hundred and sixty acres shall in like manner, as nearly as may be practicable, be subdivided into quarter quarter-sections, under such rules and regulations as may be prescribed by the Secretary of the Interior.

3 Stat. 566; 4 *id.* 503; R. S. 2397. Gazzam *v.* Phillips' Lessee, 20 How. 372; Railway Co. *v.* Schurmier, 7 Wall. 272. Buel *v.* Tuley, 4 McLean, C. C. 268. Wharton *v.* Littlefield, 30 Ala. 245. 3 Op. Att. Gen. 281, 284. Decision Sec. Int., April 14, 1879. Decision Com. G. L. O., May 17, 1875. Cir. G. L. O., June 26, 1880.

Sec. 102. Whenever, in the opinion of the President, a departure from the ordinary method of surveying land on any river, lake, bayou, or water-course would promote the public interest, he may direct the surveyor-general, in whose district such land is situated, and where the change is intended to be made, to cause the lands thus situated to be surveyed in tracts of two acres in width, fronting on any river, bayou, lake, or water-course, and running back the depth of forty acres; which tracts of land so surveyed shall be offered for sale entire, instead of in half quarter-sections, and in the usual manner, and on the same terms in all respects as the other public lands of the United States.

4 Stat. 34; R. S. 2407.

SEC. 103. In extending the surveys of the public lands in the State of Nevada, the Secretary of the Interior may vary the lines of the subdivisions from a rectangular form, to suit the circumstances of the country.

14 Stat. 86; R. S. 2408. Heydenfeldt *v.* Mining Co., 3 Otto, 634.

SEC. 104. The Secretary of the Interior, if he deems it advisable, is authorized to continue the surveys in Oregon and California, to be made after what is known as the geodetic method, under such regulations and upon such terms as have been or may hereafter be prescribed by the Commissioner of the General Land Office; but none other than township lines shall be run where the land is unfit for cultivation; nor shall any deputy surveyor charge for any line except such as may be actually run and marked or for any line not necessary to be run.

9 Stat. 496; 10 *id.* 245; R. S. 2409.

SEC. 105. Whenever, in the opinion of the Secretary of the Interior, a departure from the rectangular mode of surveying and subdividing the public lands in California would promote the public interests, he may direct such change to be made in the mode of surveying and designating such lands as he deems proper, with reference to the existence of mountains, mineral deposits, and the advantages derived from timber and water privileges; but such lands shall not be surveyed into less than one hundred and sixty acres or subdivided into less than forty acres.

10 Stat. 245: R. S. 2410. Cir. G. L. O., June 26, 1880.

SEC. 106. The public surveys shall extend over all mineral lands, and all subdividing of surveyed lands into lots less than one hundred and sixty acres may be done by county and local surveyors at the expense of claimants; but nothing contained in this section shall require the survey of waste or useless lands.

10 Stat. 15, 21; 16 *id.* 218; R. S. 2406.

SEC. 107. The printed manual of instructions relating to the public surveys, prepared at the General Land Office, and bearing date June thirtieth, eighteen hundred and ninety-four, the instructions of the Commissioner of the General Land Office, and the special instructions of the surveyor-general, when not in conflict with such printed manual or the instructions of the Commissioner, shall be taken and deemed to be a part of every contract for surveying the public lands.

12 Stat. 409; R. S. 2399. Cir. G. L. O., June 26, 1880.

SEC. 108. Legal subdivisions of forty acres of placer lands may be subdivided into ten-acre lots.

16 Stat. 213; R. S. 2330.

SEC. 2320. Mining claims upon veins or lodes of quartz or other rock in place bearing gold, silver, cinnabar, lead, tin, copper, or other valuable deposits, heretofore located, shall be governed as to length along the vein or lode by the customs, regulations, and laws in force at the date of their location. A mining-claim located after the tenth day of May, eighteen hundred and seventy-two, whether located by one or more persons, may equal, but shall not exceed, one thousand five hundred feet in length along the vein or lode; but no location of a mining-claim shall be made until the discovery of the vein or lode within the limits of the claim located. No claim shall extend more than three hundred feet on each side of the middle of the vein at the surface, nor shall any claim be limited by any mining regulation to less than twenty-five feet on each side of the middle of the vein at the surface, except where adverse rights existing on the tenth day of May, eighteen hundred and seventy-two, render such limitation necessary. The end-lines of each claim shall be parallel to each other.

10 May, 1872, c. 152, s. 2, v. 17, p. 91.

Sec. 2322. The locators of all mining locations heretofore made or which shall hereafter be made, on any mineral vein, lode, or ledge, situated on the public domain, their heirs and assigns, where no adverse claim exists on the tenth day of May, eighteen hundred and seventy-two, so long as they comply with the laws of the United States, and with State, Territorial and local regulations not in conflict with the laws of the United States governing their possessory title, shall have the exclusive right of possession and enjoyment of all the surface included within the lines of their locations, and of all veins, lodes, and ledges throughout their entire depth, the top or apex of which lies inside of such surface-lines extended downward vertically, although such veins, lodes, or ledges may so far depart from a perpendicular in their course downward as to extend outside the vertical side-lines of such surface locations. But their right of possession to such outside parts of such veins or ledges shall be confined to such portions thereof as lie between vertical planes drawn downward as above described, through the end-lines of their locations, so continued in their own direction that such planes will intersect such exterior parts of such veins or ledges. And nothing in this section shall authorize the locator or possessor of a vein or lode which extends in its downward course beyond the vertical lines of his claim to enter upon the surface of a claim owned or possessed by another.

10 May, 1872, c. 152, s. 3, v. 17, p. 91.

Sec. 2323. Where a tunnel is run for the development of a vein or lode, or for the discovery of mines, the owners of such tunnel shall have the right of possession of all veins or lodes within three thousand feet from the face of such tunnel on the line thereof, not previously known to exist, discovered in such tunnel, to the same extent as if discovered from the surface; and locations on the line of such tunnel of veins or lodes not appearing on the surface, made by other parties after the commencement of the tunnel, and while the same is being prosecuted with reasonable diligence, shall be invalid; but failure to prosecute the work on the tunnel for six months shall be considered as an abandonment of the right to all undiscovered veins on the line of such tunnel.

10 May, 1872, c. 152, s. 4, v. 17, p. 92.

SEC. 2324. The miners of each mining-district may make regulations not in conflict with the laws of the United States, or with the laws of the State or Territory in which the district is situated, governing the location, manner of recording, amount of work necessary to hold possession of a mining-claim, subject to the following requirements: The location must be distinctly marked on the ground so that its boundaries can be readily traced. All records of mining-claims hereafter made shall contain the name or names of the locators, the date of the location, and such a description of the claim or claims located by reference to some natural object or permanent monument as will identify the claim.

10 May, 1872, c. 152, s. 5, v. 17, p. 92.

SEC. 109. The surveyor-general of the United States may appoint in each land district containing mineral lands as many competent surveyors as shall apply for appointment to survey mining claims. The expenses of the survey of vein or lode claims, and the survey and subdivision of placer claims into smaller quantities than one hundred and sixty acres, shall be paid by the applicants, and they shall be at liberty to obtain the same at the most reasonable rates, and they shall also be at liberty to employ any United States deputy surveyor to make the survey. The Commissioner of the General Land Office shall have power to establish the maximum charges for such surveys; and to the end that he may be fully informed on the subject, each applicant shall file with the register a sworn statement of all charges and fees paid by such applicant for surveys, which statement shall be transmitted to the Commissioner of the General Land Office.

17 Stat. 95; 19 *id.* 52; R. S. 2334. Decision Com. G. L. O., April 20, 1877.

SEC. 110. The surveyor-general of the United States shall prepare or cause to be prepared a plat and field-notes of all mining surveys made by authority of law, which shall show accurately the boundaries of such claims; and, when warranted by the facts, he shall give to the claimant his certificate that five hundred dollars' worth of labor has been expended or improvements made upon the claim by the claimant or his grantors, and that the plat is correct, with such further description by such reference to natural objects or permanent monuments as shall identify the claim, and furnish an accurate description to be incorporated in the patent.

17 Stat. 92 R. S. 2325

SEC. 111. Contracts for the survey of the public lands shall not become binding upon the United States until approved by the Commissioner of the General Land Office, except in such cases as the Commissioner may otherwise specially order.

12 Stat. 409; R. S. 2398. Maguire *v.* Tyler, 1 Black, 201; Parks *v.* Ross, 11 How. 362; Spencer *v.* Lapsley, 20 *id* 264. Reed *v.* Conway, 26 Mo. 13. Decision Sec. Int., Feb. 27, 1878.

SEC. 112. The Commissioner of the General Land Office has power, and it shall be his duty, to fix the prices per mile for public surveys, which shall in no case exceed the maximum established by law; and, under instructions to be prepared by the Commissioner, an accurate account shall be kept by each surveyor-general of the cost of surveying and platting private land claims, to be reported to the General Land Office, with the map of such claim; and patents shall not issue for any such private claim, nor shall any copy of such survey be furnished, until the cost of survey and platting has been paid into the Treasury by the claimant or other party; and before any land granted to any railroad company by the United States shall be conveyed to such company or any persons entitled thereto, under any of the acts incorporating or relating to said company, unless such company is exempted by law from the payment of such cost, there shall first be paid into the Treasury of the United States the cost of surveying, selecting, and conveying the same by the said company or persons in interest.

12 Stat. 409; 18 *id.* 384; 19 *id.* 122; R. S. 2400. Railway Co. *v.* Prescott, 16 Wall. 603; Railway Co. *v.* McShane, 22 *id.* 444; Hannewell *v.* Cass Co., 22 *id.* 464; Colorado Co. *v.* Commissioners, 5 Otto, 259. Decisions Sec. Int., Dec. 17, 1874; Feb. 27, 1878; Feb. 20, 1879; March 5, 1879; April 2, 1879. Decisions Com. G. L. O., April 18, 1867; August 18, 1867; Feb. 17, 1869; March 26, 1870. Cir. G. L. O., June 26, 1880.

SEC. 113. The Commissioner of the General Land Office may authorize, in his discretion, public lands in Oregon densely covered with forests or thick undergrowth, to be surveyed at augmented rates, not exceeding eighteen dollars per mile for standard parallels, fifteen dollars for townships, and twelve dollars for section lines; and under like conditions he may allow augmented rates in California, and in Washington Territory, not exceeding eighteen dollars per linear mile for standard parallels, sixteen dollars for township, and fourteen dollars for section lines.

16 Stat. 304, 305; 17 *id.* 358; R. S. 2404, 2405. Decision Sec. Int., June 16, 1879. Cir. G. L. O., June 26, 1880.

SEC. 114. Whenever the public surveys, or any portion of them, in the States of Oregon and California, are so required to be made as to render it expedient to make, compensation for the surveying thereof by the day instead of by the mile, it shall be lawful for the Commissioner of the General Land Office, under the direction of the Secretary of the Interior, to make such fair and reasonable allowance, as, in his judgment, may be necessary to insure the accurate and faithful execution of the work.

10 Stat. 247; R. S. 2411. Decision Sec. Int., June 16, 1879. Cir. G. L. O., June 26, 1880.

Sec. 118. Each surveyor-general, when thereunto duly authorized by law, shall cause all confirmed private land claims within his district to be accurately surveyed, and shall transmit plats and field-notes thereof to the Commissioner of the General Land Office for his approval. When publication of such surveys is authorized by law, the proof thereof, together with any objections properly filed and all evidence submitted either in support of or in opposition to the approval of any such survey, shall also be transmitted to said Commissioner.

2 Stat. 326, 352; 3 *id.* 325; 5 *id.* 740; 9 *id.* 242, 633; 10 *id.* 244, 308, 599; 11 *id.* 294; 12 *id.* 172, 209, 369, 409; 13 *id.* 332, 344; 14 *id.* 218; 16 *id.* 64, 304; 18 *id.* 305; 19 *id.* 121, 202: R. S. 2447. Bissell *v.* Penrose, 8 How. 317; Villalobus *v.* U. S., 10 *id.* 541; Ledoux *v.* Black, 18 *id.* 473; U. S. *v.* Fossat, 20 *id.* 413; Brown *v.* Huger, 21 *id.* 305; U. S. *v.* Fossat, 21 *id.* 445; Castro *v.* Hendricks, 23 *id.* 438; Ballance *v.* Forsyth, 24 *id.* 183; U. S. *v.* Sepulveda, 1 Wall. 104; U. S. *v.* Halleck, 1 *id.* 439; U. S. *v.* Vallejo, 1 *id.* 658; Sutter's case 2 *id.* 562; Fossat case, 2 *id.* 649; Higueras *v.* U. S , 5 *id.* 827; Alviso *v.* U. S., 8 *id.* 337. 12 Op. Att. Gen. 116, 250; 14 *id*, 74, 601. U. S, *v.* Garcia, 1 Saw. C.C. 383; Russell *v.* Henshaw, 1 *id.* 553; Leroy *v.* Jamison, 3 *id.* 369; U. S. *v.* Flint, 4 *id.* 42. Dent *v.* Sergerson, 29 Mo. 480; Fowler *v.* Duvall, 11 La. Ann. 561; Waterman *v.* Smith, 13 Cal. 373; Moore *v* Wilkerson, 13 *id.* 478; Merrit *v.* Judd, 14 *id.* 60; Mott *v.* Smith, 16 *id.* 534; Johnson *v.* Van Dyke, 20 *id.* 225; McGarraghan *v.* Maxwell, 27 *id.* 75; Seale *v.* Ford, 29 *id.* 104. Cir. G. L. O., June 26, 1880.

Sec. 120. Every person who in any manner, by threat or force, interrupts, hinders, or prevents the surveying of the public lands, or of any private land claim which has been or may be confirmed by the United States, by the persons authorized to survey the same, in conformity with the instructions of the Commissioner of the General

Land Office, shall be fined not less than fifty dollars nor more than three thousand dollars, and be imprisoned not less than one nor more than three years.

4 Stat. 417; R. S. 2412.

Sec. 121. Whenever the President is satisfied that forcible opposition has been offered, or is likely to be offered, to any surveyor or deputy surveyor in the discharge of his duties in surveying the public lands, it may be lawful for the President to order the marshal of the State or district, by himself or deputy, to attend such surveyor or deputy surveyor with sufficient force to protect such officer in the execution of his duty, and to remove force should any be offered.

4 Stat. 417; R. S. 2413.

Sec. 122. The President is authorized to appoint surveyors of public lands, who shall explore such vacant and unappropriated lands of the United States as produce the live-oak and red-cedar timbers, and shall select such tracts or portions thereof, where the principal growth is of either of such timbers, as in the judgment of the Secretary of the Navy may be necessary to furnish for the Navy a sufficient supply of the same. Such surveyors shall report to the President the tracts by them selected, with the boundaries ascertained and accurately designated by actual survey or water-courses.

3 Stat. 347; R. S. 2459. U. S. *v.* Briggs, 9 How. 351.

Sec. 123. The director of the geological survey shall, under the Interior Department, have the direction of the geological survey and the classification of the public lands and examination of the geological structure, mineral resources, and products of the national domain.

20 Stat. 394.

8. Manner of Field Work and Changes that have been Made.—In accordance with these laws, instructions have been issued from time to time, by the

Commissioners of the General Land Office, directing the manner in which the field work should be performed.

In the earlier surveys under the act of 1796 (Sec. 2395 R. S. See p. 180, Sec. 99, Third,) the township was subdivided by parallel lines two miles apart. The mile posts were planted on these lines, but no half mile (or quarter-section) corners set.

The act of 1800 provided that the townships west of the Muskingum River should be subdivided into half sections of 320 acres each, as near as may be, by parallel lines run through them from east to west and from north to south at distances of a mile apart. Half-mile posts were to be set on the east and west lines, but not on the lines running north and south.

The act of 1805 (Sec. 2396 R. S. P. 181, Sec. 100) covers in its provisions the two classes of surveys above noted, as well as the principles governing all subsequent surveys of the public lands.

Since that time, few changes have been made in the manner of carrying on the surveys.

The principal change has been in the manner of closing the subdivision lines on the exterior line of the township.

In some of the earlier surveys, three sets of corners were marked in the range lines. The first set was marked when the range line was run, and were not really corners of the subdivisions.

The other two sets were marked at the points where the subdivision lines of the townships, both east and west, intersected the range line—those lines not being required to close on the corners previously set on the range line.

Later the surveyors were required to close their subdivision lines upon the corners previously set on the east line of the township, but not on the north or west. Double corners were thus produced on all the exterior lines of the township.

Most of the surveys before 1846 were made under this system, which is thus laid down in the Instructions of 1815:

"Each side of a section must be made one mile in measure by the chain, and quarter-section corners are to be established at every half mile, except when in the closing of a section if the measure of the closing side should vary from 80 chains or one mile, you are in that case to place the quarter-section corners equidistant, or at an average distance from the corners of the section; but in running out the sectional lines on the west or north side of the township, you will establish your quarter-section posts or corners at the distance of half a mile from the last corner, and leave the remaining excess or defect on the west or north tier of quarter-sections, which balance or remainder you will carefully measure and put down in your field-notes in order to calculate the remaining or fractional quarter-section on the north and west side of the township: also in running to the western or northern boundary, unless your sectional lines fall in with the posts established there for the corners of sections in the adjacent townships, you must set post and mark bearing trees at the points of intersection of your lines with the town boundaries, and take the distance of your corners from the corners of the sections of the adjacent townships, and note that and the side on which it varies in chains or links, or both.

The sections must be made to close by running a random line from one corner to another, except on the north and west ranges of sections, and the true line between them is to be established by means of offsets."

Under the present system, which has been in use in some parts of the country since 1846, the section lines are required to close on the corners previously set on the north and west boundaries, the same as on the east, thus doing away with the system of double section corners.

The practice in the several surveying districts in the United States does not seem to have been uniform at any time previous to 1860, and perhaps not always since that date. For instance, in the Instructions of the Commissioner of the General Land Office to surveyors-general, dated Feb. 22, 1855, which is stated to be a revision of the manual of surveying instructions prepared for Oregon in 1851, it is expressly ordered that "double corners are to be nowhere except on the base and standard lines;" while in the instructions to deputy surveyors of the United States for the district of Illinois and Missouri, published in 1856, P. 9, the deputy surveyors were directed to plant their closing corners at the intersection of their lines with the north and west boundary and return their direction and distance from the corners of the corresponding sections on the north and west of these boundaries," the surveyor-general of that district thus giving different instructions from those of the Commissioner of the General Land Office.

9. Fractional Areas.—It has been a puzzle to many surveyors to know how the area of the fractional quarter-sections adjoining the north and west boundaries of the township were calculated. It has been just as much of a puzzle to the surveyors-general and Commissioners of the General Land Office.

Edward Tiffin, surveyor-general of the Northwest Territory, in 1815 issued instructions how to do it, which instructions were made applicable to the surveys in Ohio, Michigan, Arkansas and Missouri. Under these instructions, the calculations of the areas of these fractions were to be made on the assumption that the quarter-posts on the township and range lines were common to the sections on both sides of these lines, thus making the lengths of the fractions more or less unequal where there were double section corners. This plan does not seem to have been in force long, or to have been very generally followed. Another plan quite extensively adopted was to make the calculations on the theory that all the north and south quarter-lines of these fractional sections were to be parallel with the east line of the sections, and all east and west quarter-lines parallel with the south line of the sections. Neither plan was in harmony with the law of 1805, which required "the corners of half and quarter-sections not marked on the surveys to be placed as nearly as possible equidistant from those two corners which stand on the same line."

The plan under which most if not all the fractional areas of Michigan were calculated was on the theory that the quarter-posts on the township and range lines were to be placed midway between their respective section corners.

Previous to 1828, the deputy surveyors were required to return with their field notes plats of all the townships which they surveyed, and to calculate the area of the fractions. These plats were rudely constructed, and in many cases the areas put down on them were erroneous. If this was found out before the land was sold, the areas were re calculated in the surveyor-general's office. In making the calculations of the areas of the fractions along the township and range lines, some of the deputies considered the quarter-section corners along those lines as common to the sections on both sides, some adopted the second method described above, while the areas of many of the fractions appear to have been put down without any calculation whatever.

In the U. S. Surveying Instructions of June 30, 1894, the following rules are given: —

In the north tier of Sections the fractional lots along the boundary are numbered 1 to 4 from east to west. In the west tier they are numbered from north to south. In Section 6 they are numbered from 1 to 7 from the N. E. corner of the Section along the boundary to the S. W. corner.

1. In regular townships, the tracts of land in each section adjoining the north and west boundaries of such townships, in excess of the regularly subdivided 480 acres (except in section 6), will, in general, be in the form of trapezoids, 80.00 chains in length by about 20 chains in width.

On the plats of such townships, each of said tracts will be divided into four lots, by drawing broken lines at intervals of 20.00 chains, parallel to the ends of the tracts, which will be regarded as parallel to each other.

With the exception of section 6, the south boundaries of sections of the north tier, when within prescribed limits, will be called 80.00 chains.

When the above-named conditions obtain, the areas of the lots in any one tract (except in section 6) may be determined, as follows:—

Divide the *difference* between the widths of the ends of the tract by 4; if 3 remains, increase the hundredth figure of the quotient by a unit; in *all other cases* disregard the fraction; call the quotient thus obtained, "d;" then, taking the end widths of the tract in *chains and decimals of a chain*, the areas of the lots, in *acres*, will be:—

Of the *smallest* lot: *twice* the width of the *lesser* end, *plus* "d;"

Of the *largest* lot: *twice* the width of the *greater* end, *minus* "d;"

Of the *smaller middle* lot: *sum* of the widths of the ends, *minus* "d;"

Of the *larger middle* lot: *sum* of the widths of the ends, *plus* "d."

A check on the computation may be had by multiplying the *sum* of the widths of the ends of the tract by 4; the product should agree *exactly* with the total area of the four lots.

The proper application of the above rules will always give areas correct *to the nearest hundredth of an acre;* and, as the use of fractions is entirely avoided, the method is recommended for its simplicity and accuracy.

Example 1.

The $\frac{1}{4}$ difference of latitudinal boundaries is $0.03\frac{3}{4}$ chains: consequently, "d" is .04 chains; then,

$18.35 \times 2 + .04 = 36.74$ acres, the area of lot 1;
$18.50 \times 2 - .04 = 36.96$ acres, the area of lot 4;
$18.50 + 18.35 - .04 = 36.81$ acres, the area of lot 2;
$18.50 + 18.35 + .04 = 36.89$ acres, the area of lot 3;
Check: $[18.35 + 18.50] \times 4 = 147.40$ acres, the area of the four lots.

The arithmetical operations are here written in detail, for the purpose of illustration; but the practical computer will perform all the work mentally.

2. Section 6. The areas of lots 5, 6, and 7 may be obtained by the foregoing rules in all cases, *except* when the township closes on a base line or standard parallel; also, the area of lot 4, *provided* both meridional boundaries are 80.00 chains in length; when the last condition obtains, the areas of lots 1, 2, and 3 will be equal, and each will contain 40.00 acres.

In any case where the *west* boundary of sec. 6, is 80.00 chains, and the *east* boundary either *greater or less than 80.00* chains, the areas of lots 1, 2, 3, and 4 will be computed as follows:—

Determine the difference, "q," between the *east* boundaries of lots 1 and 4 by the following proportion:—

N. bdy. sec. 6.: diff. of meridional bdrs. sec. 6.::60 chs.: q: then will E. bdy. lot 4=E. bdy. lot 1±q; in which, "q" will be *added* when the *east* boundary of sec. 6 is *less than 80.00 chains;* but *subtracted* when said east boundary is *greater than 80.00 chains.*

Now take *one third* of "q," and add it to the *shorter east boundary* of lots 1 or 4, as conditions may require, and thereby determine the length of one of the *meridional* boundaries of lot 2; to which again add "one third of q," and thus obtain the length of the opposite side of lot 2. The areas of lots 1, 2, and 3, in *acres*, will be found by taking the sum of their respective *meridional boundaries*, expressed in *chains and decimals of a chain.*

The area of lot 4 may be had by multiplying its *mean width* by its *mean length.*

Finally, to test the entire work, multiply the sum of the *latitudinal* boundaries by 4, and to the product *add* the area of the small triangle C A B, if the east boundary is *greater* than 80.00 chains; but *subtract* the area of said small triangle if the east boundary is *less* than 80.00 chains. These operations, correctly performed, will give *the true area of the section*, which should agree *exactly* with the total area of its legal subdivisions, obtained as directed in the preceding paragraphs.

Example 2.

Compute areas of lots 5, 6, and 7 of sec. 6, as directed in paragraph 1, and illustrated by the example; then write:—

$$\overset{\text{chs.}}{77.75} : \overset{\text{chs.}}{0.05} :: \overset{\text{chs.}}{60.00} : \overset{\text{chs.}}{0.0386}=q;\ \tfrac{1}{3}\,q=\overset{\text{chs.}}{0.0129}$$

chs.	chs.	chs.	
20.0500—0.0386=20.01, the E. bdy. of lot 4;			
20.0114+0.0129=20.02, the E. bdy. of lot 3;			
20.0243+0.0129=20.04, the E. bdy. of lot 2.			

Then, for the areas of lots 1, 2, 3, and 4, we have:—

chs. chs. acres.

20.05+20.04........= 40.09, the area of lot 1;
20.04+20.02........= 40.06, the area of lot 2;
20.02+20.01 ....= 40.03, the area of lot 3;

$$\frac{20.00+20.01}{2}\times\frac{17.75+17.78}{2} = 35.54, \text{ the area of lot 4.}$$

Also [17.78+17.87]×3 =106.95. the area of lots 5, 6, and 7.
Area of regular subdivisions=360.00

Total........=622.67, the area of Sec. 6.

chs. chs.
Check: [77.87+77.75]×4=622.48
77.75× 0.025 = 0.19, the area of triangle C A B.

Total........=622.67, which agrees with the area of section 6, before determined.

3. The area in acres of a tract 40.00 chains long, adjoining north or west township boundaries (except in N. W. ¼ sec. 6), is equal to the sum of its *parallel boundaries* (expressed in chains and decimals thereof) multiplied by 2; (*e. g.*) the area of lots 6 and 7, is [17.87+17.81] ×2=71.36 acres.

The area in acres of a tract 60.00 chains long, situated as above described (excluding lot 4, of sec. 6), may be found by multiplying the sum of its *parallel boundaries* (expressed in chains and decimals of a chain) by 3; (*e. g.*) Fig. 6; south boundary lot 4=17.78 chs.; area of lots 5, 6, and 7 is [17.78+17.87]×3=106.95 acres. (See example 2.)

The area in acres of quarter sections adjoining north and west township boundaries (excluding N. W. ¼ sec. 6), may be obtained by multiplying the sum of their *parallel boundaries* (taken in chains and decimals of a chain), by 2; (*e. g.*) the area of S. W. ¼ sec. 6 (Fig. 6), is [37.87+37.81]×2=151.36 acres.

The area in acres of any section along the north and west boundaries of regular townships (except sec. 6) may be had by multiplying the sum of its *parallel boundaries* (expressed in chains and decimals of a chain) by 4; (*e. g.*) the area of sec. 1 (Plate IV) is [80.00+79.77]×4=639.08 acres.

The area in acres of a *theoretical township* may be obtained by multiplying the sum of its *latitudinal boundaries* (expressed in chains and decimals of a chain) by 24 (*e. g.*) the area of a township is [480.00+479.34]×24=23, 024.16 acres.

10. Instructions of 1894.—The U. S. Manual of Surveying Instructions for 1894, is a large volume of 236 pages and contains minute instructions in regard to all the operations of the survey of the public lands and private land claims. It is furnished to Deputy U. S. Surveyors and may be had by others who apply for it to the Commissioner of the General Land Office at Washington. The following extracts are made from it:—

SYSTEM OF RECTANGULAR SURVEYING.

1. Existing law requires that in general the public lands of the United States "shall be divided by north and south lines run according to the true meridian, and by others crossing them at right angles so as to form townships six miles square," and that the corners of the townships thus surveyed "must be marked with progressive numbers from the beginning."

Also, that the townships shall be subdivided into thirty-six sections, each of which shall contain six hundred and forty acres, as nearly as may be, by a system of two sets of parallel lines, one governed by true meridians and the other by parallels of latitude, the latter intersecting the former at right angles, at intervals of a mile.

2. In the execution of the public surveys under existing law, it is apparent that the requirements that the lines of survey shall conform to true meridians, and that the townships shall be 6 miles square, taken together, involve a mathematical impossibility due to the convergency of the meridians.

Therefore, to conform the meridianal township lines to the true meridians produces townships of a trapezoidal form which do not contain the precise area of 23,040 acres required by law, and which discrepancy increases with the increase in the convergency of the meridians, as the surveys attain the higher latitudes.

In view of these facts, and under the provisions of section 2 of the act of May 18, 1796, that sections of a mile square shall contain 640 acres, *as nearly as may be*, and also under those of section 3 of the act of May 10, 1800, that "in all cases where the exterior lines of the townships, thus to be subdivided into sections and half sections, shall exceed, or shall not extend 6 miles, the excess or deficiency shall be specially noted, and added to or deducted from the western or northern ranges of sections or half sections in such township, according as the error may be in running lines from east to west, or from south to north; the sections and half sections bounded on the northern and western lines of such townships shall be sold as containing only the quantity expressed in the returns and plats, respectively, and all others as containing the complete legal quantity." the public lands of the United States shall be surveyed under the methods of the system of rectangular surveying, which harmonizes the incompatibilities of the requirements of law and practice, as follows:—

First. The establishment of a principal meridian conforming to the true meridian, and, at right angles to it, a base line conforming to a parallel of latitude.

Second. The establishment of standard parallels conforming to parallels of latitude, initiated from the principal meridian at intervals of 24 miles and extended east and west of the same.

Third. The establishment of guide meridians conforming to true meridians, initiated upon the base line and successive standard parallels at intervals of 24 miles, resulting in tracts of land 24 miles square, *as nearly as may be*, which shall be subsequently divided into tracts of land 6 miles square by two sets of lines, one conforming to true meridians, crossed by others conforming to parallels of latitude at intervals of 6 miles, containing 23,040 acres, *as nearly as may be*, and designated *townships.*

Such townships shall be subdivided into thirty-six tracts, called sections, each of which shall contain 640 acres, as nearly as may be, by two sets of parallel lines, one set *parallel to a true meridian* and the other *conforming to parallels of latitude*, mutually intersecting at intervals of 1 mile and at right angles, *as nearly as may be.*

Any series of contiguous townships situated north and south of each other constitutes a *range*, while such a series situated in an east and west direction constitutes a *tier.*

The section lines are surveyed from *south* to north and from *east* to west, in order to throw the excess or deficiency in measurement on the north and west sides of the township, as required by law. In case where a township has been partially surveyed, and it is necessary to complete the survey of the same, or where the character of the land is such that only the north or west portions of the township can be surveyed, this rule cannot be strictly adhered to, but, in such cases, it will be departed from only so far as is absolutely necessary. It will also be necessary to depart from this rule where surveys close upon State or Territorial boundaries, or upon surveys extending from different meridians.

3. The tiers of townships will be numbered, to the north or south, commencing with No. 1, at the base line; and the ranges of the townships, to the east or west, beginning with No. 1, at the principal meridian of the system.

4. The thirty-six sections into which a township is subdivided are numbered, commencing with number *one* at the *northeast* angle of the township, and proceeding west to number six, and thence proceeding east to number twelve, and so on, alternately, to number thirty-six in the southeast angle. In all cases of surveys of fractional townships, the sections will bear the same numbers they would have if the township was full.

5. Standard parallels shall be established at intervals of every 24 miles, north and south of the base line, and

guide meridians at intervals of every 24 miles, east and west of the principal meridian; thus confining the errors resulting from convergence of meridians and inaccuracies in measurement within comparatively small areas.

Instruments.—6. The surveys of the public lands of the United States, embracing the establishment of base lines, principal meridians, standard parallels, meander lines, and the subdivisions of townships, will be made with instruments provided with the accessories necessary to determine a direction with reference to the true meridian, independently of the magnetic needle.

Burt's improved solar compass, or a transit of approved construction, with or without solar attachment, will be used in *all* cases. When a transit *without* solar attachment is employed, *Polaris observations* and the *retracements* necessary to execute the work in accordance with existing law and the requirements of these instructions will be insisted upon.

7. Deputies using instruments with solar apparatus will be required to make observations on the star Polaris at the beginning of every survey, and, *whenever necessary, to test the accuracy of the solar apparatus.*

The observations required to test the adjustments of the solar apparatus will be made at the corner where the survey begins, or at the camp of the deputy surveyor nearest said corner; and in all cases the deputy will fully state in the field notes the exact location of the observing station.

Deputy surveyors will examine the adjustments of their instruments, and *take the latitude daily*, weather permitting, while running *all lines of the public surveys.* They will make complete records in their field notes, under proper dates, of the making of all observations in compliance with these instructions, showing the character and condition of the instrument in use, and the precision attained in the survey, by comparing the direction of the line run with the meridian determined by observation.

On every survey executed with solar instruments, the deputy will, *at least once on each working day*, record in his field notes the proper reading of the latitude arc; the declination of the sun, corrected for refraction, set off on the declination arc; and note the correct *local mean time* of his observation, which, for the record, will be taken *at least two hours from apparent noon.*

8. The construction and adjustments of all surveying instruments used in surveying the public lands of the United States will be tested at least once a year, and oftener, if necessary, on the true meridian, established under the direction of the surveyor general of the district; and if found defective, the instruments shall undergo such repairs or modifications as may be found necessary to secure the closest possible approximation to accuracy and uniformity in all field work controlled by such instruments.

A record will be made of such examinations, showing the number and character of the instrument, name of the maker, the quantity of instrumental error discovered by comparison, in either solar or magnetic apparatus, or both, and means taken to correct the same. The surveyor general will allow no surveys to be made until the instruments to be used therefor have been approved by him.

9. The township and subdivision lines will usually be measured by a two-pole chain 33 feet in length, consisting of 50 links, each link being seven and ninety-two hundredths inches long. On uniform and level ground, however, the four-pole chain may be used. The measurements will, however, always be expressed in terms of the four-pole chain of 100 links. The deputy surveyor shall provide himself with a measure of the standard chain kept at the office of the surveyor general, to be used by him as a field standard. The chain in use will be compared and adjusted with this field standard each working day, and such field standard will be returned to the surveyor general's office for examination when the work is completed.

Deputy surveyors will use eleven tally pins made of steel, not exceeding 14 inches in length, weighty enough toward the point to make them drop perpendicularly, and having a ring at the top, in which will be fixed a piece of red cloth, or something else of conspicuous color, to make them readily seen when stuck in the ground.

Process of Chaining.—In measuring lines with a two-pole chain, *five* chains are called a "*tally;*" and in measuring lines with a four-pole chain, *ten* chains are called a "*tally,*" because at that distance the last of the ten tally pins with which the forward chainman sets out will have been stuck. He then cries "tally," which cry is repeated by the other chainman, and each registers the distance by slipping a thimble, button, or ring of leather, or something of the kind, on a belt worn for that purpose, or by some other convenient method. The hind chainman then comes up, and having counted in the presence of his fellow, the tally pins which he has taken up, so that both may be assured that none of the pins have been lost, he then takes the forward end of the chain, and proceeds to set the pins. Thus the chainmen alternately change places, each setting the pins that he has taken up, so that one is forward in all the odd, and the other in all the even tallies. Such procedure, it is believed, tends to insure accuracy in measurement, facilitates the recollection of the distances to objects on the line, and renders a mistally almost impossible.

Leveling the Chain and Plumbing the Pins.—1. The length of every surveyed line will be ascertained by precise horizontal measurement, as nearly approximating to an air line as is possible in practice on the earth's surface. This all-important object can only be attained by a rigid adherence to the three following observances:—

First. Ever keeping the chain drawn to its utmost degree of tension on even ground.

Second. On uneven ground, keeping the chain not only stretched as aforesaid, but *leveled.* And when ascending and descending steep ground, hills, or mountains, the chain will have to be *shortened* enough to accurately obtain the true horizontal measure.

Third. The careful plumbing of the tally pins, so as to attain precisely *the spot* where they should be stuck. The more uneven the surface, the greater the caution needed to set the pins.

Marking Lines.—1. All lines on which are to be established the legal corner boundaries will be marked after this method, viz.: Those trees which may be intersected by the line, will have two chops or notches cut on the sides facing the line, without any other marks whatever. These are called "*sight trees*" or "*line trees.*" A sufficient number of other trees standing within 50 links of the line, on either side of it, will be *blazed* on two sides diagonally or quartering toward the line, in order to render the line conspicuous, and readily to be traced; the blazes to be opposite each other, coinciding in direction with the line where the trees stand very near it, and to approach nearer each other toward the line, the farther the line passes from the blazed trees. Due care will ever be taken to have the lines so well marked as to be readily followed, and to cut the blazes deep enough to leave recognizable scars as long as the trees stand.

Where trees 2 inches or more in diameter are found, the required blazes will not be omitted.

Bushes on or near the line should be bent at right angles therewith, and receive a blow of the ax at about the usual height of blazes from the ground, sufficient to leave them in a bent position, but not to prevent their growth.

2. On trial or random lines, the trees will not be blazed, unless occasionally, from indispensable necessity, and then it will be done so guardedly as to prevent the possibility of confounding the marks of the trial line with the *true*. But bushes and limbs of trees may be lopped, and *stakes set* on the trial or random line, at every ten chains, to enable the surveyor on his return to follow and correct the trial line and establish there-

from the *true line.* To prevent confusion, the temporary stakes set on the trial or random line will be *pulled up* when the surveyor returns to establish the true line.

Insuperable Objects on Line — Witness Points.— 1. Under circumstances where the survey of a line is obstructed by an impassable obstacle, such as a pond, swamp, or marsh (not meanderable), the line will be prolonged across such obstruction by making the necessary right-angle offsets; or, if such proceeding is impracticable, a traverse line will be run, or some proper trigonometical operation will be employed to locate the line on the opposite side of the obstruction; and in case the line, either *meridional* or *latitudinal*, thus regained, is recovered beyond the intervening obstacle, said line will be surveyed back to the margin of the obstruction and *all the particulars, in relation to the field operations, will be fully stated in the field notes.*

2. As a guide in alinement and measurement, at each point where the line intersects the margin of an obstacle, a *witness point* will be established, *except* when such point is *less than 20 chains* distant from the true point for a legal corner which falls in the obstruction, in which case a *witness corner* will be established at the intersection.

3. In a case where all the points of intersection with the obstacle to measurement fall *more than 20 chains* from the proper place for a legal corner in the obstruction, and a witness corner can be placed on the offset line *within 20 chains* of the inaccessible corner point, such "witness corner" will be established.

Establishing Corners.— 1. To procure the faithful execution of this part of a surveyor's duty, is a matter of the utmost importance. After true coursing and most exact measurements, the establishment of corners is the consummation of the field work. Therefore, if the corners be not perpetuated in a permanent and workmanlike manner, the *principal object* of surveying operations will not have been attained.

2. The points at which corners will be established are

fully stated in the several articles: "Base Lines," "Principal Meridians," "Standard Parallels," etc., following the title "Initial Points."

3. The best marking tools adapted to the purpose will be provided for marking *neatly*, *distinctly*, and *durably*, all the letters and figures required to be made at corners, *arabic* figures being used exclusively; and the deputy will always have at hand the necessary implements for keeping his marking irons in perfect order.

Descriptions of Corners.—1. The form and language used in the following articles, in describing, for each one of the thirteen classes of corners, eight specific constructions and markings, with the stated modifications in certain cases, will be carefully followed by deputy surveyors in their *field notes;* and their *field work* will strictly comply with the requirements of the descriptions.

2. When pits and mounds of earth are made accessories to corners, the pits will always have a *rectangular* plan; while the mounds will have a *conical* form, with *circular* base; and in all cases both pits and mounds will have dimensions *at least* as great as those specified in the descriptions. Deputy surveyors will strictly adhere to these provisions, and no departure from the stated requirements will be permitted, either in *instructions* or *practice in the field.*

3. Referring to the numbered paragraphs, the corners described in "3" will be preferred to those described in either "1" or "2," when corners are established in loose, sandy soil, and good bearing trees are available: under similar conditions, the corners described in "5" and "8" will be preferred to those described in "4" and "7," respectively.

4. The selection of the particular construction to be adopted in any case will be left, as a matter of course, to the judgment and discretion of the deputy, who will assign the greatest weight to the *durability* of the corner materials and *permanency* of the finished corners.

5. The following abbreviations and contractions will be used in the descriptions of corners, viz.:—

A. M. C.	for auxiliary meander cor.	N.	for north.
bdy.	for boundary.	¼ sec. cor.	for quarter section corner.
bdrs.	for boundaries.	R.	for range.
bet.	for between.	Rs.	for ranges.
C. C.	for closing corner.	sec . secs.	for section, sections.
cor., cors.	for corner, corners.	S. M. C.	for special meander cor.
dist.	for distance.	S. C.	for standard corner.
E.	for east.	sq.	for square.
ft.	for foot or feet.	S.	for south.
fracl.	for fractional.	T. or Tp.	for township.
ins.	for inches.	Ts. or Tps.	for townships.
diam.	for diameter.	W.	for west.
lks.	for links.	W. C.	for witness corner.
M. C.	for meander corner.	W. P.	for witness point.

For "18 inches long, 7 inches wide, 6 inches thick," in describing a corner stone, write "18×7×6 ins.," being particular to always preserve the same order of length, width, and thickness (or depth), and use a similar form when describing *pits*.

STANDARD TOWNSHIP CORNERS.

METHOD OF MARKING.

When more than one half of all the standard township and section corners on any 6 miles of a base line or standard parallel are stone corners, the descriptions in paragraphs 1 and 2, if the corners therein described are established, will be modified as follows: Strike out "S. C., on N." After "marked," insert the words:—

"S. C., 13 N. on N.,
22 E. on E., and
21 E. on W. faces;"

When under the conditions above specified, the corner described in paragraph 1 is established, a stake may be driven in the east pit and marked instead of the stone, and described as exemplified in the last clause of paragraph 6.

1. *Stone, with Pits and Mound of Earth.*

Set a — stone, —×—×— ins.,— ins. in the ground, for standard cor. of (*e. g.*) Tps. 13 N., Rs. 21 and 22 E., marked S. C. on N.; with 6 grooves on N., E., and W. faces; dug pits, 30×24×12 ins., crosswise on each line, E. and W., 4 ft., and N. of stone, 8 ft. dist.; and raised a mound of earth, 5 ft. base, 2½ ft. high, N. of cor. The *direction* of the mound, from the corner, will be stated wherever a mound is built.

2. *Stone, with Mound of Stone.*

Set a — stone, —×—×— ins., — ins. in the ground, for standard cor. of (*e. g*) Tps. 13 N., Rs. 21 and 22 E., marked S. C., on N.; with 6 grooves on N., E., and W. faces; and raised a mound of stone, 2 ft. base, 1½ ft. high, N. of cor. Pits impracticable. Mound of stone will consist of not less than *four* stones, and will be *at least* 1½ ft. high, with 2 ft. base.

3. *Stone, with Bearing Trees.*

Set a — stone, —×—×— ins., — ins. in the ground, for standard cor. of (*e. g.*) Tps. 13 N., Rs. 21 and 22 E., marked S. C., on N.; with 6 grooves on N., E., and W. faces; from which

A —, — ins. diam., bears N. — ° E., — lks. dist., marked

T. 13 N., R. 22 E., S. 31, B. T.

A —, — ins. diam., bears N. —° W., — lks. dist., marked

T. 13 N., R. 21 E., S. 36, B. T.

All bearing trees, except those referring to quarter section corners, will be marked with the *township, range, and section in which they stand.*

4. *Post, with Pits and Mound of Earth.*

Set a — post, 3 ft. long, 4 ins. sq., with marked stone (charred stake or quart of charcoal), 24 ins. in the ground, for standard cor. of (*e. g.*) Tps. 13 N., Rs. 22 and 23 E., marked

S. C., T. 13 N. on N.

R. 23 E., S. 31 on E., and

R. 22 E., S. 36 on W. faces; with 6 grooves on N., E., and W. faces; dug pits, 30×24×12 ins., crosswise on each line, E. and W., 4 ft., and N. of post, 8 ft. dist.; and raised a mound of earth, 5 ft. base, 2½ ft. high, N. of cor.

5. *Post, with Bearing Trees.*

Set a — post, 3 ft. long, 4 ins. sq., 24 ins. in the ground, for standard cor. of (*e. g.*) Tps. 18 N., Rs. 22 and 23 E., marked

S. C., T. 13 N. on N.,
R. 23 E. S. 31 on E., and
R. 22 E., S. 36 on W. faces; with 6 grooves on N., E., and W. faces, from which
A —, — ins. diam., bears N. —° E., — lks. dist., marked
T. 13 N., R. 23 E., S. 31, B. T.
A —, — ins. diam., bears N. —° W., — lks. dist., marked
T. 13 N., R. 22 E., S. 36, B. T.

6. *Mound of Earth, with Deposit, and Stake in Pit.*

Deposited a marked stone (charred stake or quart of charcoal) 12 ins. in the ground, for standard cor. of (*e. g.*) Tps. 13 N., Rs. 22 and 23 E.; dug pits, 30×24×12 ins., crosswise on each line, N., E., and W. of cor., 5 ft. dist.; and raised a mound of earth, 5 ft. base, 2½ ft. high, over deposit.

In E. pit drove a — stake, 2 ft. long, 2 ins. sq., 12 ins. in the ground marked
S. C., T. 13 N. on N.,
R. 23 E., S. 31 on E., and
R. 22 E., S. 36 on W. faces; with 6 grooves on N., E., and W. faces.

7 *Tree Corner, with Pits and Mound of Earth.*

A —, — ins. diam., for standard cor. of (*e. g.*) Tps. 13 N., Rs. 22 and 23 E., I marked
S. C., T. 13 N. on N.,
R. 23 E., S. 31 on E., and
R. 22 E., S. 36 on W. sides; with 6 notches on N., E., and W. sides; dug pits, 24×18×12 ins., crosswise on each line, N., E., and W. of cor., 5 ft. dist.; and raised a mound of earth around tree.

8. *Tree Corner, with Bearing Trees.*

A —, — ins. diam., for standard cor. of (*e. g.*) Tps. 13 N., Rs. 22 and 23 E., I marked
S. C., T. 13 N. on N.,
R. 23 E., S. 31 on E., and
R. 22 E., S. 36 on W. sides; with 6 notches on N., E., and W. sides; from which

A —, — ins. diam., bears N. —° E., — lks. dist., marked

T. 13 N., R. 23 E., S. 31, B. T.

A —, — ins. diam., bears N. —° W., — lks. dist., marked

T. 13 N., R. 22 E., S. 36, B. T.

Witness Corners.—1. When the true point for any corner described in these instructions falls where prevailing conditions would insure its destruction by natural causes, a witness corner will be established in a secure position, *on a surveyed line if possible*, and *within twenty chains* of the corner point thus witnessed.

2. *Markings on Witness Corners.*

A witness corner will bear the same marks that would be placed upon the corner for which it is a witness, and in addition, will have the letters "W. C." (for witness corner), *conspicuously* displayed above the regular markings; such witness corners will be established, *in all other respects*, like a regular corner.

3. *Markings on Bearing Trees of Witness Corners.*

When bearing trees are described as accessories to a witness corner, the prescribed markings on each tree will be preceded by the letters "W. C.," *distinctly* cut into the wood.

The true bearing and distance of witness corners, from the true point for the corner, will always be clearly stated in the field notes.

4. *Witness Corners to Corner Points Falling in Roads, etc.*

The point for a corner falling on a railroad, street, or wagon road, will be perpetuated by a marked stone charred stake or quart of charcoal, deposited 24 inches in the ground, and *witnessed by two witness corners*, one of which will be established on each limiting line of the highway. The deposit will not be practicable in the case of railroads; but the witness corners will be established on the lines limiting the right of way.

In case the point for any regular corner falls at the *intersection* of two or more streets or roads, it will be perpetuated by a marked stone (charred stake or quart of charcoal), deposited 24 inches in the ground, and *witnessed by two witness corners* established on opposite sides of the corner point, and at the mutual intersections of the lines limiting the roads or streets, as the case may be.

Witness Points will be perpetuated by corners similar to those described for quarter section corners, with the marking "W. P." (for witness point), in place of "$\frac{1}{4}$," or "$\frac{1}{4}$ s.", as the case may be.

If bearing trees are available as accessories to *witness points*, each tree will be marked W. P. B. T. (See "Insuperable objects on line — Witness Points."

Miscellaneous.—1. *Corners on Rock in Place, or on Boulders.*

When a corner falls on rock *in place*, or on a *boulder*, a cross (×), will be made at the exact corner point, and witnessed by the proper number of bearing trees, if they are available; in the absence of suitable trees, a mound of earth will be raised, if size of the boulder or form of the rock in place permits the excavation of pits. As a last resort, a mound of stone will be built to attract attention to the point, if loose rock can be obtained in the vicinity.

2. *Location of Mounds.*

When mounds of earth or other material are raised as accessories to corners, they will be placed as specified in the foregoing Description of Corners, and in every case the *direction* of the mound from the corner will be carefully stated. The use of the indefinite description "alongside" will be discontinued.

In case the character of the land is such that the mound cannot be placed as hereinbefore described, the deputy will state in his notes, by bearing and distance, exactly where the mound is located with reference to the corner, and will give his reasons for placing it as described.

3. *Mounds of Stone, Covered with Earth.*

In a case where pits are practicable and the deputy prefers raising a mound of stone, or a mound of stone covered with earth, he will use the form given for "*Stone with mound of stone,*" when the corner thus described is established; but when the corner "*Stone, with mound of stone covered with earth,*" is constructed, the description will be modified as follows: Strike out the words "Pits impracticable;" in place of "Mound of stone, 2 ft. base, 1½ ft. high," write "Mound of stone covered with earth, — ft. base, — ft. high," inserting in the blank spaces the dimensions of the mound given in paragraph 1, following the designation of each class of corners.

4. *Bearing Trees.*

Bearing trees marked as accessories to standard corners, either township, section, or quarter section, will be selected on the *north* side of base lines or standard parallels, and bearing trees referring to the closing corners on said lines, will be located on the *south* side; in general, the bearing trees referring to any particular closing corner, together with one pit and the mound belonging to such corner, will be located on *the same side of the line closed upon*, and on the side *from which the surveys have been closed.*

When the requisite number of trees can be found within 300 links of the corner point, two (2) bearing trees will be marked and described for every standard or closing township or section corner, or corner common to two townships or sections, only; four (4) for every corner common to four townships or four sections; one (1) for a corner referring to one township or one section, only; two (2) for every quarter section corner or meander corner, and four (4) for each mile or half mile corner, or corner monument on a reservation or other boundary, not conforming to the system of rectangular surveying.

In case the prescribed number of trees cannot be found within limits, the deputy will state in his field notes, after describing those marked, "No other trees within limits," and add "Dug pits —×—×— ins.," etc., or "Raised a mound of stone, — ft. base, — ft. high, — of cor.," as prevailing conditions may require.

Bearing trees, being the most important accessories to the corners, will have their exact bearings from the true meridian taken with the instrument used in running the lines of survey; and the distance *from the middle of each bearing tree to the middle point of the corner* will be carefully measured, and recorded in the field notes.

A *plain blaze* will be made at the usual or most convenient height, on each bearing tree, on the side facing the corner. The height of all other markings on the tree will in no case exceed the limit of *two and one half feet* above the ground.

5. *Stones for Corners.*

Stones 18 ins. long, or less, will be set with two thirds of their length in the ground, and those more than 18 ins. long will have three fourths of their length in the ground.

No stones measuring less than 504 cubic inches, or less than 12 ins. in length, will be used for corners.

6. *Objects to be Noted.*

Particular attention is directed to the "Summary of objects and data required to be noted," and the deputy will thoroughly comply with the same in his work and field notes.

7. *Lines Discontinued at Legal Corners.*

No mountainous lands, or lands not classed as surveyable, will be meandered, and all lines approaching such lands will be discontinued at the section or quarter-section corner nearest the unsurveyed land.

8. *Marks to be cut.*

All letters and figures on posts, trees, or stones, etc., will be *cut into* the object upon which they are placed. Arabic figures and plain letters will be used for all markings.

9. *Orientation of Corners.*

Corners referring to one, two, or four townships or sections, not identical with standard or closing corners, will be set with their faces *directed* NE. and SW., and NW. and SE., while all other corners will be set with their sides *facing the cardinal points;* except corners on boundaries of reservations and private land claims, which will be set squarely on line.

10. *Size of Posts, Mounds, etc.*

The sizes of wooden posts, mounds, and pits, noted in the foregoing descriptions, will be regarded as *minimum*, and their dimensions will be increased whenever practicable.

11. *Corner Materials.*

In establishing corners, *durable* stones will be used when obtainable; then, posts; and lastly, mounds, with stake in pit.

Wood of a perishable nature will not be used for posts or stakes.

12. *Instructions will be Examined.*

Deputy surveyors will carefully read, study, and familiarize themselves with all instructions contained in this volume, and will instruct their assistants as to their duties before commencing work. An extra copy of this Manual may be furnished each deputy, for the use of his assistants.

Initial Points.—Initial points from which the lines of the public surveys are to be extended will be established whenever necessary, under such special instructions as may be prescribed in each case by the Commissioner of the General Land Office. The locus of such initial points will be selected with great care and due consideration for their prominence and easy identification, and must be established astronomically.

The lines of the public surveys are classified as follows:—

Class 1. Base lines and standard parallels.

Class 2. Principal and guide meridians.

Class 3. Township exteriors (or meridional and latitudinal township boundaries).

Class 4. Subdivision and meander lines.

The initial point having been established, the line of the public surveys will be extended therefrom, as follows:—

Base Line.—1. From the initial point the base line will be extended east and west on a parallel of latitude, by the use of transit or solar instruments, as may be directed by the surveyor general in his written special instructions. The *transit* should be designated for the alinement of all important lines.

2. The direction of base lines will conform to parallels of latitude and will be controlled by true meridians; consequently the correct determination of true meridians by *observations on Polaris at Elongation* is a matter of prime importance.

3. When transits are employed, certain reference lines having a known position and relation to the required parallel of latitude will be prolonged as straight lines, by two back and two fore sights at each setting of the instrument, the horizontal limb being revolved 180° in azimuth between the observations.

4. Where solar apparatus is used, the deputy will test the instrument, whenever practicable, by comparing its indications with a meridian determined by Polaris observations; and in all cases where error is discovered, he will make the necessary corrections of his line before proceeding with the survey. All operations will be fully described in the field notes.

5. The proper township, section, and quarter section corners will be established at lawful intervals, and meander corners at the intersection of the line with all meanderable streams, lakes, or bayous.

6. In order to detect errors and insure accuracy in measurement, two sets of chainmen will be employed; one to note distances to intermediate points and to locate topographical features, the other to act as a check. Each will measure 40 chains, and the proper corner will be placed midway between the ending points of the two measurements.

The deputy will be present when said corner is thus established, and will record in the body of his field notes the distances to the same, according to the measurement by each set of chainmen.

To obviate collusion between the sets of chainmen, the second set should commence at a point in advance of the beginning corner of the first set, the initial difference in measurement thus obtained being known only to the deputy.

Principal Meridian.—1. This line shall *conform to a true meridian* and will be extended from the initial point, either north or south, or in both directions, as the conditions may require, by the use of transit or solar instruments, as may be directed by the surveyor general in his special written instructions.

2. The methods used for determination of directions, and the precautions to be observed to secure accuracy in measurement, are fully stated above under the title "Base Line," and will be complied with in every particular.

3. In addition to the above general instructions, it is required that in all cases where the establishment of a new principal meridian seems to be necessary to the surveyor general, he shall submit the matter, together with his reasons therefor, to the commissioner of the General Land Office, and the survey of such principal meridian shall not be commenced until written authority, together with such special instructions as he may deem necessary, shall have been received from the commissioner.

Standard Parallels.—1. Standard parallels, which are also called correction lines, shall be extended east and west from the principal meridian, at intervals of every 24 miles north and south of the base line, in the manner prescribed for running said line, and all requirements under the title "Base Line" will be carefully observed.

2. Where standard parallels have been placed at intervals of 30 or 36 miles, regardless of existing instructions, and where gross irregularities require additional standard lines, from which to initiate new, or upon which to close old surveys, an intermediate correction line should be established to which a *local* name may be given, *e. g.*, "Cedar Creek Correction Line;" and the same will be run, in all respects, like the regular standard parallels.

Guide Meridians.—1. Guide meridians shall be extended north from the base line, or standard parallels, at intervals of every 24 miles east and west from the principal meridian, in the manner prescribed for running the principal meridian, and all the provisions for securing accuracy of alinement and measurement found, or referred to under the title "Principal Meridian," will apply to the survey of said guide meridians.

2. When existing conditions require that such guide meridians shall be run *south* from the base or correction lines, they will be initiated at properly established closing corners on such lines.

3. Where guide meridians have been improperly placed at intervals greatly exceeding the authorized distance of 24 miles, and standard lines are required to limit errors of old, or govern new surveys, a new guide meridian may be run from a standard, or properly established closing corner, and a local name may be assigned to the same, *e. g.*, "Grass Valley Guide Meridian." These additional guide meridians will be surveyed in all respects like the regular guide meridians.

Township Exteriors.—1. Whenever practicable, the township exteriors in a tract of land 24 miles square, bounded by standard lines, will be surveyed successively through the block, beginning with those of the *southwestern* township.

2. The *meridional* boundaries of townships will have precedence in the *order* of survey and will be run from south to north on *true meridians*, with permanent corners at lawful distances; the latitudinal boundaries will be run *from east to west* on *random* or trial lines, and corrected back on true lines.

The falling of a *random*, north or south of the township corner to be closed upon, will be carefully measured, and, with the resulting true return course, will be duly recorded in the field notes.

Should it happen, however, that such *random* intersects the meridian of the objective corner, north or south of said corner, or falls short of, or overruns the length of the south boundary of the township by more than *three chains* (due allowance being made for convergency), said random, and, if necessary, all the exterior boundaries of the township, will be retraced and remeasured to discover and correct the error

When running random lines from east to west, temporary corners will be set at intervals of 40.00 chains, and proper *permanent* corners will be established upon the true line, corrected back in accordance with these instructions, thereby throwing the excess or deficiency against the west boundary of the township, as required by law.

3. Whenever practicable, the exterior boundaries of townships belonging to the *west* range, in a tract or block 24 miles square, will first be surveyed in succession, through the range, from south to north; and in a similar manner, the other three ranges will be surveyed in regular sequence.

4. In cases *where impassable objects occur* and the foregoing rules *cannot be complied with*, township corners will be established as follows:—

In extending the *south* or *north* boundaries of a township to the *west*, where the *southwest* or *northwest* corners cannot be established in the regular way by running a north and south line, such boundaries will be run *west on a true* line, allowing for convergency on the west half mile; and from the township corner established at the end of such boundary, the west boundary will be run *north* or *south*, as the case may be. In extending *south* or *north* boundaries of a township to the *east*, where the *southeast* or *northeast* corner cannot be established in the regular way, the same rule will be observed, except that such boundaries will be run *east on a true line*, and the *east* boundary run *north* or *south*, as the case may be.

5. Allowance for the convergency of meridians will be made whenever necessary.

Method of Subdividing.— 1. The exterior boundaries of a full township having been properly established, the subdivision thereof will be made as follows:—

At or near the *southeast* corner of the township, a *true meridian* will be determined by Polaris or solar observations, and the deputy's instrument will be tested thereon; then from said corner the first mile of the east and south boundaries will be retraced, if subdivisions and survey of the exteriors have been provided for in *separate* contracts; but, if the survey of the exterior and subdivisional lines are included in the *same* contract, the retracements referred to will be omitted. All discrepancies resulting from disagreement of bearings or measurements will be carefully stated in the field notes.

2. After testing his instrument on the true meridian thus determined, the deputy will commence at the corner to sections 35 and 36, on the south boundary, and run a line *parallel to the range line*, establishing at 40.00 chains, the quarter section corner between sections 35 and 36, and at 80.00 chains the corner for sections 25, 26, 35, and 36.

3. From the last-named corner, a random line will be run eastward, without blazing, *parallel to the south boundary of section* 36, to its intersection with the east boundary of the township, placing at 40.00 chains from the point of beginning, a post for temporary quarter section corner. If the random line intersects said township boundary exactly at the corner for sections 25 and 36, it will be blazed back and established as the true line, the permanent quarter section corner being established thereon, *midway* between the initial and terminal section corners.

If, however, the random intersects said township boundary to the north or south of said corner, the falling will be carefully measured, and from the data thus obtained, the true return course will be calculated, and the true line blazed and established, and the position of the quarter section corner determined, as directed above.

The details of the entire operation will be recorded in the field notes.

4. Having thus established the line between sections 25 and 36; from the corner for sections 25, 26, 35 and 36, the *west* and *north* boundaries of sections 25, 24, 13, and 12, will be established as directed for those of section 36; with the exception that the random lines of said

north boundaries will be run *parallel to the established south boundaries of the sections to which they belong*, instead of the south boundary of section 36; e. g. the random line between sections 24 and 25 will be run parallel to the established south boundary of section 25, etc.

5. Then, from the last established section corner, i. e. the corner for sections 1, 2, 11, and 12, the line between sections 1 and 2, will be projected northward, on a random line, *parallel* to the east boundry of the township, setting a post for temporary quarter section corner at 40.00 chains, to its intersection with the north boundary of the township. If the random intersects said north boundary exactly at corner for sections 1 and 2, it will be blazed back and established as the true line, the temporary quarter section corner being established permanently in its original position, and the fractional measurement thrown into that portion of the line between said corner and the north boundary of the township.

If, however, said random intersects the north boundary of the township, to the east or west of the corner for sections 1 and 2, the consequent falling will be carefully measured, and from the data thus obtained, the true return course will be calculated, and the true line established; the permanent quarter section corner being placed upon the same at 40.00 chains from the initial corner of the random line, thereby throwing the fractional measurement in that portion lying between the quarter section corner and the north boundary of the township.

When the north boundary of a township is a base line or standard parallel, the line between sections 1 and 2 will be run *parallel to the range line* as a *true line*, the quarter section corner will be placed at 40.00 chains, and a *closing corner* will be established at the point of intersection with such base or standard line; and in such case, the distance from said closing corner, to the nearest standard corner on such base or standard line, will be carefully measured and noted as a *connection line.*

6. Each successive range of sections progressing to the west, until the fifth range is attained, will be surveyed in a similar manner; then, from the section corners established on the west boundary of said range of sections, random lines will be projected to their intersection with the west boundary of the township, and the true return lines established as prescribed for the survey of the first or most eastern range of sections, with the exception that on the true lines thus established, the quarter-section corners will be established at 40.00 chains from the initial corners of the randoms, the fractional measurements being thereby thrown into those portions of the lines situated between said quarter-section corners and the west boundary of the township.

7. The following general requirements are reiterated for emphasis:—

The *random* of a latitudinal section line will always be run parallel to the south boundary of the section to which it belongs, and with the true bearing of said boundary; and when a section has no linear south boundary, the random will be run parallel to the south boundary of the range of sections in which it is situated, and fractional true lines will be run in a similar manner.

8. The deputy is not required to complete the survey of the first range of sections from south to north before commencing the survey of the second or any subsequent range of sections, but the corner on which any random line closes shall have been previously established by running the line which determines its position, except as follows: Where it is impracticable to establish such section corner in the regular manner, it will be established by running the latitudinal section line as *a true line*, with a *true bearing*, determined as above directed for *random* lines, setting the quarter-section corner at 40.00 chains and the section corner at 80.00 chains.

9. Quarter-section corners, both upon meridional and latitudinal section lines, will be established at points *equidistant* from the corresponding section corners, *except*

upon the lines closing on the north and west boundaries of the township, and in those situations the quarter-section corners will always be established at precisely *forty chains* to the north or west (as the case may be) of the respective section corners from which those lines respectively *start*, by which procedure the excess or deficiency in the measurements will be thrown, according to law, on the extreme tier or range of quarter sections, as the case may be.

10. Where by reason of impassable objects only a portion of the south boundary of a township can be established, an auxiliary base line (or lines, as the case may require) will be run through the portion which has no linear south boundary, first random, then corrected, connecting properly-established corresponding section corners (either interior or exterior) and as far south as possible, and from such line or lines, the section lines will be extended northwardly in the usual manner, and any fraction *south* of said line will be surveyed in the opposite direction from the section corners on the auxiliary base thus established.

11. Where by reason of impassable objects *no portion of the south boundary* of a township can be regularly established, the subdivision thereof will proceed from *north to south* and from east to west, thereby throwing all fractional measurements and areas against the west boundary, and the meanderable stream or other boundary limiting the township on the south.

If the *east* boundary is without regular section corners and the north boundary has been run eastwardly as a true line, with section corners at regular intervals of 80.00 chains, the subdivision of the township will be made from *west to east*, and fractional measurements and areas will be thrown against the irregular east boundary.

12. When the proper point for the establishment of a township or section corner is inaccessible, and a witness corner can be erected upon each of the two lines which approach the same, at distances not exceeding twenty chains therefrom, said witness corners will be properly

established, and the half miles upon which they stand will be recognized as *surveyed lines.*

The witness corner will be marked as conspicuously as a section corner, and bearing trees will be used wherever possible.

The deputy will be required to furnish good evidence that the section corner is actually inaccessible.

Meandering.—1. Proceeding *down* stream, the bank on the *left* hand is termed the *left bank* and that on the *right* hand the *right bank.* These terms will be universally used to distinguish the two banks of a river or stream.

2. Navigable rivers, as well as all rivers not embraced in the class denominated "navigable," the right-angle width of which is three chains and upward, will be meandered on both banks, at the ordinary *mean high water mark*, by taking the general courses and distances of their sinuosities, and the same will be entered in the field book. Rivers not classed as navigable will not be meandered above the point where the average right-angle width is less than three chains. Shallow streams, without any well-defined channel or permanent banks, *will not be meandered*, except tide-water streams, whether more or less than three chains wide, which should be meandered at ordinary high-water mark, as far as tide-water extends.

At every point where either standard, township, or section lines intersect the bank of a navigable stream, or any meanderable line, corners will be established at the time of running these lines. Such corners are called meander corners, and the deputy will commence at one of these corners, follow the bank or boundary line, and measure the length of each course from the beginning corner to the next "meander corner." Compass courses, by the needle or solar, will be used in meanders. Transit angles are not allowed.

The crossing distance between meander corners on same line and the true bearing and distance between corresponding meander corners will be ascertained by triangulation, or direct measurement, in order that the river may be protracted with entire accuracy. The particulars will be given in the field notes.

In meandering water courses or lakes, where a distance is more than *ten chains* between successive stations, whole chains only should be taken; but if the distance is *less* than ten chains, and it is found convenient to employ chains and links, the number of links should be *a multiple of ten*, thereby saving time and labor in testing the closings, both in the field and office.

3. The meanders of all lakes, navigable bayous, and deep ponds, of the area of twenty-five acres and upwards, will be commenced at a meander corner and continued, as above directed for navigable streams; from said corner, the courses and distances of the entire margin of the same, and the intersections with all meander corners established thereon will be noted.

All streams falling into the river, lake, or bayou will be noted, and the width at their mouths stated; also, the position, size, and depth of springs, whether the water be pure or mineral; also the heads and mouths of all bayous; all islands, rapids, and bars will be noted, with intersections, to their upper and lower ends, to establish their exact situation. The elevation of the banks of lakes, bayous, and streams, the height of falls and cascades, and the length and fall of rapids will be recorded in the field notes.

To meander a lake or deep pond lying entirely within the boundaries of a section, two lines will be run from the two nearest corners on different sides of such lake or pond, the courses and length of which will be recorded, and if coincident with unsurveyed lines of legal subdivisions, that fact will also be stated in the field notes, and at each of the points where said lines intersect the margin of the pond or lake, a *special* meander corner will be established as above directed.

The relative position of these points being thus definitely fixed in the section, the meandering will commence at one of them and be continued to the other, noting the intersection, and thence to the beginning. The proceedings are to be fully entered in the field notes.

4. Meander lines will not be established at the segregation line between dry and swamp or overflowed land, but at the *ordinary high-water mark* of the actual margin of the rivers or lakes on which such swamp or overflowed lands border.

5. The precise relative position of an island, in a town-

ship made fractional by a river or lake in which the island is situated, will be determined by triangulation from a special and carefully measured base line, initiated from the surveyed lines, on or near the lake or river bank on the main land, so as to connect by course and distance on a direct line, the meander corner on the mainland with the corresponding point on the island, where the proper meander corner will be established.

6. In making the connection of an island lying entirely within a section, with the mainland, a special base will be measured from the most convenient meander corner, and from such base, the location of an *auxiliary* meander corner will be determined by triangulation, at which the meanders of the island will be initiated.

7. In the survey of lands bordering on *tide water*, "meander corners" will be established at the points where surveyed lines intersect *high-water mark*, and the meanders will follow the *high-water line*.

8. The field notes of meanders will show the dates on which the work was performed, as illustrated in the specimen notes. The field notes of meanders will state and describe the corner from which the meanders commenced, and upon which they closed, and will exhibit the meanders of each fractional section separately; following, and composing a part of such notes, will be given a description of the land, timber, depth of inundation to which the bottom is subject, and the banks, current, and bottom of the stream or body of water meandered. The utmost care will be taken to pass no object of topography, *or change therein*, without giving a particular description thereof in its proper place in the notes of the meanders.

Summary of Objects and Data Required to be Noted.—1. The precise length of every line run, noting all necessary offsets therefrom, with the reason for making them, and method employed.

2. The kind and diameter of all bearing trees, with the course and distance of the same from their respective corners; and the precise relative position of witness corners to the true corners.

3. The kind of materials of which corners are constructed.

4. *Trees on line.* The name, diameter, and distance on line to all trees which it intersects.

5. Intersections by line of *land objects.* The distance at which the line intersects the *boundary lines* of every reservation, mining claim, settler's claim, improvement, or rancho; prairie, bottom land, swamp, marsh, grove,

and windfall, with the course of the same at all points of intersection; also, the distances at which the line begins to ascend, arrives at the top, begins to descend, and reaches the foot of all *remarkable* hills and ridges, with their courses, and *estimated height in feet*, above the level land of the surrounding country, or above the bottom lands, ravines, or waters near which they are situated. Also, distance to and across *large* ravines, their depth and course.

6. Intersections by line of *water objects.* All rivers, creeks, and smaller streams of water which the line crosses; the distances measured on the *true* line to the bank *first arrived at*, the course *down* stream at *points of intersection*, and their *widths on line.* In cases of *navigable* streams, their width will be ascertained between the *meander corners*, as set forth under the proper head.

7. The land's *surface*—whether level, rolling, broken, hilly, or mountainous.

8. The *soil*—whether first, second, third, or fourth rate.

9. *Timber*—the several kinds of timber and undergrowth, in the order in which they predominate.

10. *Bottom lands*—to be described as wet or dry, and if subject to inundation, state to what depth.

11. *Springs of water*—whether fresh, saline, or mineral, with the course of the stream flowing from them.

12. *Lakes and ponds*—describing their banks and giving their height, and also depth of water, and whether it be pure or stagnant.

13. *Improvements.*—Towns and villages; houses or cabins, fields, or other improvements with owner's names; mill sites, forges, and factories, mineral monuments, and all corners not belonging to the system of rectangular surveying; will be located by bearing and distance, or by intersecting bearings from given points.

14. *Coal* banks or beds; *peat* or turf grounds; *minerals* and ores; with particular description of the same as to quality and extent, and all *diggings* therefor; also *salt* springs and licks. All reliable information that can be obtained respecting these objects, whether they be on the line or not, will appear in the general description to be given at the end of the notes.

15. *Roads* and *trails*, with their directions, whence and whither.

16. Rapids, cataracts, cascades, or falls of water, with the estimated height of their fall in feet.

17. Precipices, caves, sink holes, ravines, stone quarries, ledges of rocks, with the kind of stone they afford.

18. *Natural curiosities*, interesting fossils, petrifactions,

organic remains, etc.; also all ancient works of art, such as mounds, fortifications, embankments, ditches, or objects of like nature.

19. The *magnetic declination* will be *incidentally* noted at all points of the lines being surveyed, where any *material change* in the same indicates the probable presence of iron ores; and the position of such points will be perfectly identified in the field notes.

Prescribed Limits for Closings and Lengths of Lines.—1. If in running a *random* township exterior, such random falls short of or exceeds its proper lengths by more than *three chains*, or falls more than *three chains* north or south of its objective corner, it will be re-run, and if found correct, so much of the remaining boundaries of the township will be retraced or resurveyed, as may be found *necessary* to locate the error.

2. Every meridional section line, except those terminating in the north boundary of the township, shall be *eighty chains* in length.

3. The *random* meridional section lines through the north tier of sections shall fall within *fifty links* east or west of the section corners established on the north boundary of the township, *except* when closing on a base line or standard parallel.

4. Every *meridional* section line through the north tier of sections, shall be within *fifty links* of the actual distance established on the east boundary line of the township for the width of said tier of sections.

5. All *random* latitudinal section lines shall fall within *fifty links* north or south of their objective section corners.

The *latitudinal* section lines. except those terminating in the west boundary of the township, shall be within *fifty links* of the actual distance established on the south boundary line of the township for the width of the range of sections to which they belong.

6. The north boundary and the south boundary of *any one section*, except in the extreme western range of sections, shall be within *fifty links* of equal length.

7. The meanders within each fractional section, or between any two successive meander corners, or of an island in the interior of a section, should close within a limit to be determined by allowing *five eighths of a link* for *each chain* of said meander line. Where the meander corners marking the ends of a meander line in a fractional section are located on standard, township, or section lines, the above limit, increased by *one fourth of the regular perimeter of the fractional section, expressed in miles, multiplied by 71 links, will be allowed.*

The extreme limit, however, will in no case be permitted to exceed *one hundred and fifty links.*

Field Notes.—1. The proper blank books for *original field notes* will be furnished by the surveyor general, and in such books the deputy surveyor will make a faithful, distinct, and minute record of everything done and observed by himself and his assistants, pursuant to instructions, in relation to running, measuring, and marking lines, establishing corners, etc., and present, as far as possible, full and complete topographical sketches of all standard and exterior lines, drawn to the usual scale for township exteriors. These "original field notes" are not necessarily the entries made in the field, in the deputy's pocket note books called tablets; but they are to be fully and correctly written out in ink, from such tablets, for the permanent record of the work. Tablets should be so fully written as to verify the "original field notes" whenever the surveyor-general requires them for inspection.

2. A full description of all corners belonging to *old* surveys, from which the lines of *new* surveys *start*, or upon which they *close*, will in all cases be furnished the deputy from the surveyor general's office, when authority is given for commencing work; then, if the old corners are found to agree with said descriptions, the deputy will describe any one of them in this form, "which is a stone firmly set, marked, and witnessed, as described by the surveyor general;" but should a corner *not* answer the description supplied, the deputy will give a *full description* of such corner and its accessories, following the proper approved form given in these instructions.

A full description of each corner established under any one contract will be given *once* only; subsequent reference to such corner will be made in the form, "heretofore described," or (*e. g.*) "the corner for sections 2, 3, 10, and 11," as the case may require.

In all cases where a corner is *re-established*, the *original field notes* will describe fully the manner in which it is done.

3. The *original field notes* of the survey of base, standard, and meridian lines will describe all corners established thereon, how established, the crossings of streams, ravines, hills, and mountains; character of soil, timber, minerals, etc.; and after the description of each township corner established in running such lines, the deputy will note particularly in the "general description" the character of townships on each side of the lines run.

4. The *original field notes* of the survey of exterior

boundaries of townships will describe the corners and topography, as above required, and the "general description" at the end of such notes will describe the townships as fully as possible, and also state whether or not they should be subdivided.

5. The *original field notes* of the subdivisional survey of townships will describe the corners and topography as above required, and the "general description" at the end of such notes will state minutely the character of the land, soil, timber, etc., found in such townships.

The topography will be given on the *true line* in all cases, and will be taken correctly, not estimated or approximated.

6. With the *original field notes* of the survey of base lines and standard parallels, and principal and guide meridians forming a tract 24 miles square, including those of the township exteriors therein, the deputy will submit a diagram of the lines surveyed, drawn to a scale of half an inch to one mile, upon which will be written the *true bearings and lengths of all surveyed lines*, except the lengths of those which are actually 40.00 or 80.00 chains. These diagrams will exhibit all water courses, with the direction of each indicated by an arrow head pointing *down stream;* also, the intersection of the lines with all prairies, marshes, swamps, ravines, lakes, ponds, mountains, hills, and all other natural or artificial topographical features mentioned in the *original field notes*, to the fullest extent possible.

7. With the *special instructions* for making subdivisional surveys of townships into sections, the deputy will be furnished by the surveyor general with *blank township diagrams* drawn to a scale of *one inch to forty chains*, upon which the true bearings and lengths of the township and section lines, from which the surveys are to be projected, or upon which they are to close, will be carefully marked; and on such diagrams the deputy who subdivides will make appropriate sketches of the various objects of topography as they occur on his lines, so as to exhibit not only the points of intersection therewith, but also the directions and relative positions of such objects between the lines, or within each section, as far as practicable, so that every topographical feature may be properly completed and connected in the showing.

8. Triangulations, offsets, or traverses, made to determine distances that cannot be directly measured, such as those over (*e. g.*) deep streams, lakes, impassable swamps, cañons, etc., will be made on the *random lines*, when random lines are run. All particulars will be fully stated in the field notes.

CHAPTER IX.

SUBDIVISION OF SECTIONS.

1. **Subdivisions of sections** are original surveys to be made in the following manner:

1. Section and quarter-section corners set by the government surveyors, and the boundaries actually run by them, as well as the length of all lines as returned in their field notes, are to be taken as correct. (See Sec. 2396 R. S., First and Second. P. 182, Sec. 100.)

2. The corners of half and quarter sections which were not marked on the government surveys, must be placed as nearly as possible equidistant from those two corners which stand on the same line. (Sec. 2396, First. P. 182, Sec. 100.)

This applies to the quarter-posts on the north and west lines of the township which were surveyed previous to 1846; also to those townships which, under the act of 1796, were surveyed into blocks of two miles square (P. 180, Sec. 99, Third), and to those surveyed under the act of 1800,* where no quarter-section corners were planted on the lines running from south to north.

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*No. 21.—An Act to amend the act entitled "An act providing for the sale of the lands of the United States, in the territory northwest of the Ohio, and above the mouth of the Kentucky River."

SEC. 3. *And be it further enacted*, That the surveyor-general shall cause the townships west of the Muskingum, which by the above-mentioned act are directed to be sold in quarter townships, to be subdivided into half sections of three hundred and twenty acres each, as nearly as may be, by running parallel lines through the same from east to west, and from south to north, at the distance of one mile from each

3. The boundary lines of sections, (see Page 180, Sec. 99, Third), and of half and quarter sections, which were not actually run and marked, are to be ascertained by running straight lines from the established corners to the opposite corresponding corners. Where no such opposite corners have been or can be fixed, the line should be run from the established corner due north and south or east and west, as the case may be, to the water-course or other external boundary. (P. 182, Sec. 100, Second.) These due lines are to be found by trial of the boundary lines of the section, as actually run by the government surveyor, and the subdivision line, run on a course intermediate between the courses of the section lines which lie parallel with it.

The following figure illustrates the manner of subdividing sections. It shows sections 5, 6, 7, and 8, repre-

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other, and marking corners, at the distance of each half mile on the lines running from east to west, and at the distance of each mile on those running from south to north, and making the marks, notes, and descriptions prescribed to surveyors by the above-mentioned act: And the interior lines of townships intersected by the Muskingum, and of all the townships lying east of that river, which have not been heretofore actually subdivided into sections, shall also be run and marked in the manner prescribed by the said act for running and marking the interior lines of townships directed to be sold in sections of six hundred and forty acres each. And in all cases where the exterior lines of the townships, thus to be subdivided into sections or half-sections, shall exceed or shall not extend six miles, the excess or deficiency shall be specially noted, and added to or deducted from the western and northern ranges of sections or half-sections in such township, according as the error may be in running the lines from east to west, or from south to north; the sections and half-sections bounded on the northern and western lines of such townships shall be sold as containing only the quantity expressed in the returns and plats, respectively, and all others as containing the complete legal quantity. And the President of the United States shall fix the compensation of the deputy surveyors, chain-carriers, and axemen: *Provided*, The whole expense of surveying and marking the lines shall not exceed three dollars for every mile that shall be actually run, surveyed, and marked.

senting the four different cases which occur in a township surveyed previous to 1846. In the later surveys, the details would differ a little, owing to the fact that the section and quarter-section corners on the township and range lines are common to the townships on each side of and adjoining those lines. The principle of subdivision is, however, the same.

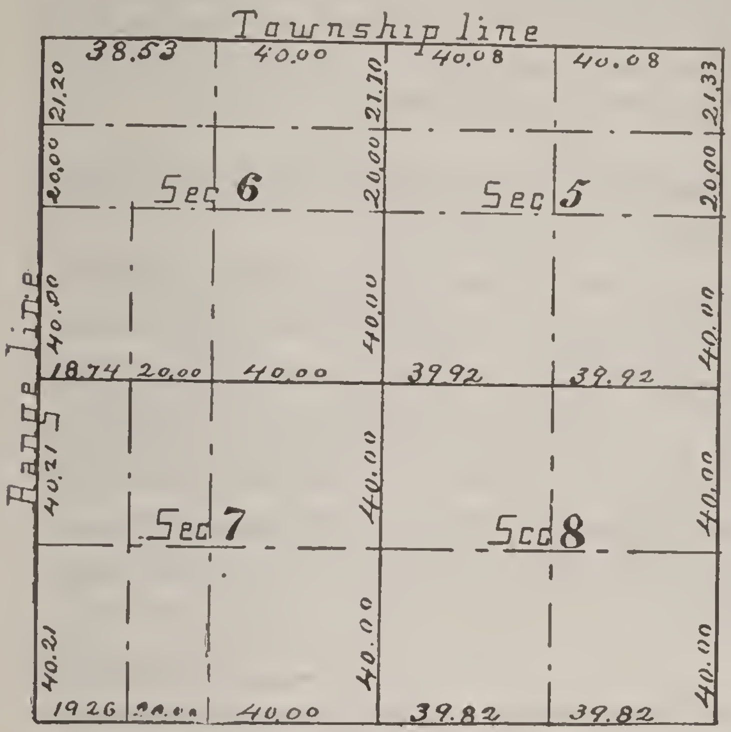

FIG. 69.

CASE 1.—Section 8. All the quarter posts are at equidistant points from the section corners which are on the same line.

CASE 2.—Section 5. Quarter posts on the north and the south are at equidistant points. Those on the east and the west are 40 chains from the south line of the section. The fraction is on the north half of the section.

CASE 3.—Section 7. Quarter posts on the north and the south are placed at 40 chains from the east line of the section. Those on the east and the west are at equidistant points. The west half of the section is fractional.

CASE 4.—Section 6. The quarter posts on the north and the south are placed at 40 chains from the east line of the section. Those on the east and the west are 40 chains from the south line of the section. Fractional both on the north and west.

NOTE.—In 1856, Thomas A. Hendricks, then Commissioner of the General Land Office, gave the following rule for locating the center of a section: "Run a true line from the quarter-section corner on the east boundary, to that in the west boundary, and at the equidistance between them establish the corner for the center of the section."

This was in harmony with an opinion previously given by the Surveyor General of Missouri and Illinois, and was very generally followed by the surveyors in those States. This rule has not been sustained by the courts, nor by any other ruling of the Land Office, so far as we can learn. It was expressly overruled by the Secretary of the Interior in 1868.

Quarter-sections are to be subdivided into half-quarters by lines running north and south.

The corners which were not marked are to be placed as nearly as possible equidistant between the two corners of the quarter-section which stand on the same line. Then run straight lines from the established corners to the opposite corresponding corners, (Page 183, Sec. 101.)

Half-quarter sections are to be subdivided into quarter-quarters in a similar manner, by east and west lines. (P. 183, Sec. 101.)

It may be well to remark here, that the instructions from the General Land Office have not been uniform in regard to the proper manner of subdividing quarter-sections, and, as might be expected, the practice is not uniform among good surveyors. Commissioners Wilson and Edmunds held that half-quarter and quarter-quarter lines should be "straight lines running through the section" to points on the section line. (See Hawes's Manual, p. 142, and Dunn's Land Laws, p, 19.)

The foregoing rules are those of the statute, and are endorsed by Commissioners Drummond, Williamson, and McFarland.

Commissioner Drummond's instructions are as follows:

"In the subdivision of quarter-sections, the quarter-quarter posts are to be placed at points equidistant, and on straight lines between the section and quarter-section corners, *and between the quarter-corners and the common center of the section,*" etc. The difference in the two methods occurs when, as very often happens, the quarter-posts are not in line between the section corners.

2. **Fractional sections** are to be subdivided according to the Fifth paragraph of Sec. 2395 of the Revised Statutes, under such rules and regulations as may be prescribed by the Secretary of the Interior. (Sec. 99, Ex. Land Laws, and U. S. Instructions, 1881, p. 39.)

Under these regulations,* the fractional quarter-sections lying next to the north line of the township are divided

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* Note.—"*Circular to Surveyors-General, Nov. 9, 1821.*—Sir: By the first section of the act of April 24, 1820, all the public lands of the United States shall be offered at public sale in half-quarter sections; and

into half-quarters by lines running east and west, parallel with and twenty chains distant from the quarter-section line. (See Keasling v. Truitt, 30 Ind. 506.)

The quarter-sections lying next to the west line of the township are divided into half-quarters by lines running north and south, parallel with and twenty chains distant from the quarter-section line.

3. Section 6 adjoins both the north and the west lines of the township, and is subject to both rules. The north half is divided into half-quarters by an east and west line, and the south half by north and south lines.

The quarter-post on the north side of section six should be placed on the township line at a point 40 chains of original measure west from the northeast corner of the section.

The quarter-post on the west line of section six should be placed at a point on the range line 40 chains of original measure north from the southwest corner of the section. By original measure is meant such measure as was actually laid down on the ground by the deputy surveyors who made the original survey.

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fractional sections containing one hundred and sixty acres and upward shall, as nearly as practicable, be divided into half-quarter sections, under such rules and regulations as may be prescribed by the Secretary of the Treasury; but fractional sections containing less than one hundred and sixty acres shall not be divided, etc. By the act of May 10, 1800, section 3, the excess or deficiency of regular sections or quarter-sections in any township is to be thrown on the north and west sides of the township, making fractional sections more or less than one hundred and sixty acres. In subdividing such fractional sections to form a half-quarter section, viz., 80 acres, the Secretary of the Treasury directs that the subdividing line for such fractions as lie on the north side of a township shall be an east and west line, forming the half-quarter section on the south side of the fraction; and for such fractions as lie on the west side, the subdividing line shall be a meridian, forming the half-quarter section on the east side of the fraction. This mode of subdivision will preserve the compactness of the tracts with the general divisions, and will not interfere with the rule adopted relative to fractions formed by a stream, a river, etc."

In further subdividing the northwest quarter of Section 6 into quarter-quarters, it is done by a line parallel with and 20 chains west of the north and south quarter-section line.

The foregoing is the general plan adopted for the subdivision of sections of the United States Survey. There have, however, been many exceptions in the earlier official plats, in accordance with which the land was sold. To meet all such cases the rule has been adopted to subdivide in such a way as to suit the calculation of the areas on the official plat. This is sometimes difficult, the areas in some cases seeming to have been put down without any calculation.

Sections made fractional by waters, reservations, etc., should be subdivided in such a manner as to produce the same result as would have been produced had the section been full. This may sometimes be done by extending and by measuring the lines on the ice, or over the reservation.

FIG. 70.

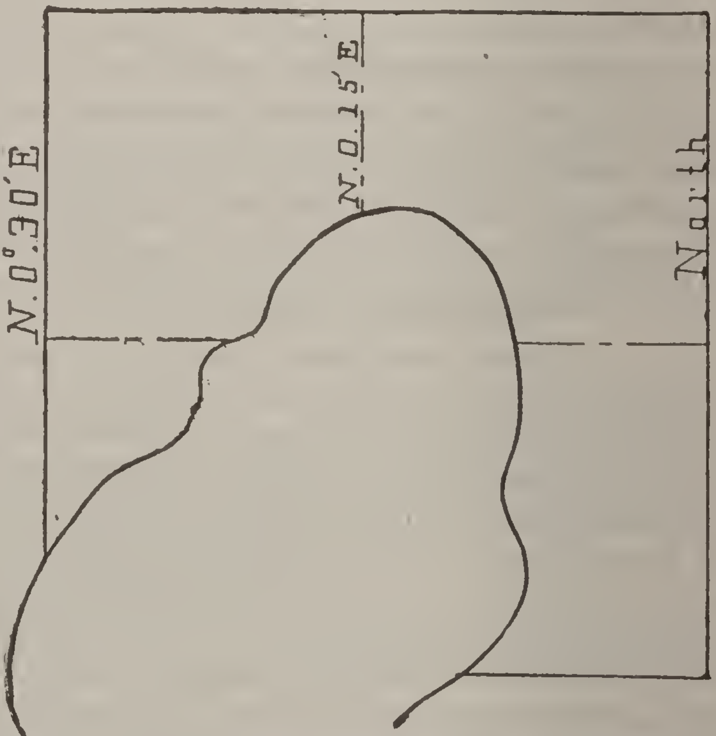

Figure illustrating the Subdivision of a Section fractional on waters.

Commissioner Drummond says (see Copp's Land Laws, p. 761): "In the subdivision of fractional sections, where no opposite corners have been or can be fixed, the subdivision lines should be ascertained by running lines from the established corners due north, south, east or west, as the case may be, to the water-course, Indian boundary line, or other external boundary line of such fractional section. The law presupposes the section lines surveyed and marked in the field by the United States deputy surveyors to be due north and south or east and west lines. But in actual experience, this is not always the case. Hence, in order to carry out the spirit of the law, it will be necessary in the running of subdivisional lines through fractional sections to adopt mean courses where the lines are not due lines, or to run the subdivisional line parallel with the section line when there is no opposite section line."

4. Irregular Subdivisions of Fractional Sections.—In making irregular subdivisions of fractions bounded on streams or lakes, there seems to have been no rule laid down by the authorities.

It has been decided by the Supreme Court of the United States that "the meander lines run in surveying fractional portions of the public lands bordering upon navigable rivers are run not as boundaries of the tract but for the purpose of defining the sinuosities of the stream and as the means of ascertaining the quantity of land in the fraction, and which is to be paid for by the purchaser."

R. R. Co. *v*, Schurmier, 7th Wallace (U.S.) 272.

It is fair to infer that the same lines are to be used in ascertaining the quantity of land in any portion of the fraction. Thus, as often happens, if a deed calls for so many acres off the end of the fraction, the surveyor in making his computations to determine at what point to locate the dividing line, should in the absence of anything showing to the contrary, use the meander line for the purpose of estimating the area of the tract, and lay down the dividing line accordingly. Otherwise there could be no common basis of calculation and as many different results would be arrived at as there were different surveyors to run the line, or different times of survey.

This is especially true of fractions bordering on lakes whose shore lines are subject to great change from natural causes or artificial drainage.

The common law rule for calculating the quantity of land bordering on a non-navigable stream is that no reference is had to what lies between low water mark and the centre of the stream. On navigable waters, high water mark is the line.

Lamb *v.* Rickett, 11 Ohio 311.

5. **Exceptional Cases.**—In the United States surveys made previous to 1815, there was much irregularity in the practice of the surveyors in carrying on the surveys. The fractional sections were frequently thrown upon the south or east tiers of sections in the township; the surveys being carried on from the north to the south and from the west to the east. Where the township was made fractional by large rivers or lakes, they were frequently so laid off as to throw all the fractions into the sections bordering on the water.

There was even greater irregularity in the manner of subdividing the fractional sections into the lesser tracts. Many of them had no quarter section corners. In some, the government plats show no subdivision; some are subdivided in one way and some in another.

In making resurveys and subdivisions of these and all other exceptional cases, the surveyor must always make his resurvey conform to the plan as shown by the field-notes and plats of the original survey.

6. **Other Original Surveys.**—In a considerable portion of the United States, the general government never had any ownership of the land.

The surveys were there made by the proprietors upon such system or plan as suited themselves.

The further subdivision of these tracts is original surveying. It is sufficient to say of this work that it should be done with great care, and that *the marks upon the ground which indicate the boundary lines should be of* *the plainest and most permanent character which the circumstances of the case permit,*

These marks are intended to fix for all time the boundaries of the tract laid off and *they cannot be too plain or permanent.* Want of due care and precaution in making permanent land marks upon the ground, at the time of the original survey, is the fruitful cause from which arises most of the litigation about boundary lines.

7. **Highway surveys,** like other surveys, lose much of their value if their corners and lines are not so thoroughly marked as to be readily found at any future time. The centre line of a highway is very commonly used as a boundary line. Good permanent landmarks, well guarded by bearings and distances to the most permanent objects in the vicinity should be planted at the starting and closing points of the survey, at each angle in its course, and at every crossing of a section line. The distance of the crossing points should be given from the nearest government corners each way on the section line.

8. **Surveys for town plats** are made upon any system to suit the circumstances of the case, or the views of the owners of the land platted.

In making these surveys, it is important that the work be in every respect carefully done; that full and complete notes be taken, so that the plat when finished shall show every material fact which may be of use to the public or to the future surveyor.

The relation which the lines of the plat bear to the lines of the original boundaries, whether of the government survey or otherwise, should be shown on the plat, and, what is most important of all, *the location of the lines upon the ground should be marked by a sufficient number of permanent monuments so that there may never arise any difficulty in determining the exact position those lines occupy.*

Such monuments should be placed at the corners and angles of the tract platted, and if included in the United

States survey, they should be placed at the corners of the legal subdivisions of a section which are included in the plat. Monuments should also be placed so as to define the lines and termini of all streets.

For this purpose, they may be placed either along the centre lines and angles of the streets or along their margins at the corners and angles of blocks. Each method has its advantages and disadvantages. The surveyor should consider the special circumstances of each case, and so locate the monuments that, while effecting the purpose for which they were intended, they shall be the most likely to remain in position and the easiest to refer to.

9. In Michigan town plats are required by law (Session Laws of 1885) to be made and recorded in the following manner:

The plats must be made on sheets of good muslin backed paper, 18 inches by 24 inches in size, on a scale showing not more than 200 feet to an inch.

The plat must have upon it a full, detailed written description of the land embraced in it, showing the township, range, section and subdivision of section of the land platted. If the premises platted are not included in the legal subdivisions of the government survey, then the boundaries are to be defined by metes and bounds and courses.

The plat must contain the full name of the town, city, village or addition platted; the names of the proprietors and of the person making the plat, and the date.

It must be signed by the proprietors and by the person making it, and be witnessed and acknowledged in the same manner as deeds.

The sections and parts of sections must also be designated on the plat by lines with appropriate letters and figures.

There must be a plain designation of the cardinal points of the compass and a correct scale.

When complete and before any copies are made from it, the plat must be submitted to the Auditor General for his approval,

10. **The Record.**—An exact duplicate of the original plat must be filed in the office of the Register of Deeds for the county in which the land is situated. It must contain all the matter in the original plat and the certificate of the Register of Deeds and the person who made the original plat, that they have separately carefully compared the duplicate with the original plat, and that it is an exact duplicate thereof and of the whole of such plat,

A third copy must be filed in the office of the Auditor General. This copy must contain the certificate of the Register of Deeds and of the person who made the plat, that they have separately compared it with the duplicate plat on record, and that it is a true transcript therefrom and of the whole of such duplicate plat so recorded.

The Register of Deeds receives a fee of $2.00 for recording the plat, and the sum of $1.00 must accompany the plat filed in the Auditor General's office.

The law was amended in 1887 so as to require the surveyor to plant permanent monuments at all angles in the boundaries of the land platted, and at all the intersections of streets, or streets and alleys, as shown on the plat; and when there are permanent objects in the vicinity of such monuments, the bearings and distances of such objects to be noted. The character of the monuments and the bearings and distances of such objects or witness points must be given in the most convenient manner on the plat. The surveyor must certify that the plat is a correct one, and that the monuments described in it have been planted as therein described. The new provisions of the law are very important. The monuments are the crowning work of the survey, without which all else is of little value. They mark out the standard of measure on the grouud, to which all subsequent surveys must conform. President Steele, of the Michigan Engineering Society, says: "Place more monuments instead of less. Place them everywhere, no matter whether at the intersections of streets at the side lines, the centre lines, or any other lines. Put down all you can. Plant them in exact relation one to another. Put the bearing on every line, the angle at every intersection. Put it all on your plat, and the more you have the better. Leave nothing to guess at. Have it so plain that a man who never knew anything about the ground can go there and find all the points."

II. Monuments.—It is more important to a man to know precisely where his boundary lines are and that they are unchangeable without his consent, than it is that he shall have the precise quantity of land; hence one of the most important duties the surveyor has to perform, is to fix the most permanent and unmistakable monuments to define and preserve boundary lines. This is equally true of all original surveys, whether in country or town. Mathematical accuracy in measuring distances or running lines, fails of its purpose unless there be some means of securing an unvarying starting point; while if the landmarks of the original survey, in accordance with which the land was conveyed, be preserved intact, no measurements, good or bad, are needed to define the boundaries.

Monuments for landmarks should be durable and easily distinguishable from other objects in the vicinity.

They should be accessible, not liable to be moved, and their position located by bearings and distances to the most permanent objects in the vicinity.

Various things are used for landmarks—according to the nature of the soil and the materials at hand; chiefly wood, stone, earthenware, or iron, in some of their forms.

Wood. A wooden post, if of suitable size and kind and properly planted, makes an excellent landmark, where very precise definition of the boundary is not required. It should be from 2½ to 4 inches in diameter, sound and straight and planted vertically in the ground to a depth of at least three feet, for permanent purposes. Red cedar black-walnut, cherry or white oak hearts make very durable posts. When the post has decayed the rotten wood and cavity in the earth preserve the point better than the sound post, as they cannot be pulled up nor moved from place without moving the surrounding earth with them.

Stone. If a rough stone or boulder is used for a monument, it should either be so large as not to be moved by any ordinary accident or so firmly imbedded in the earth as to defy the plow or the road maker. If of a kind common in the vicinity, it should be very plainly marked and have some foreign material like brick, iron, glass, or crockery imbedded around it, to identify it by.

If cut stone is used, it should be of the best quality and be long enough and set deeply enough to insure permanency. If the stone is a soft one it should be protected from injury. A stone 36x8x8 inches dressed down at the top to 6x6 inches is the size in use in many of the large cities for landmarks. It is common to cut a cross or drill a hole in the top of the stone to indicate more precisely the corner or line. If still greater precision is required a piece of metal is set in the stone and the point indicated by lines cut as finely as desirable.

Iron. Monuments of cast iron have been used and are excellent. A hollow cone 18 to 36 inches in length with a broad flange at the bottom, when set in the ground holds its position very firmly and will last indefinitely. Iron rods, and pieces of gas pipe are also used. They need to be well packed about the top with brick or stone to keep them in position.

Other Materials. Some monuments are made of the same material as the earthenware sewer pipe, and burned and glazed in a similar manner. They are solid, cylindrical, three inches in diameter and of various lengths. The ends are suitably marked before burning. They are very convenient to use and durable, but need to be well protected. Brick set on end two and two to a depth of three feet and packed about the top to prevent moving make an excellent monument. Another excellent device is to make a deep hole in the earth, one or two inches in diameter, and fill it with a paste of quick lime, plaster of paris, or portland cement.

Protection. A good plan for protecting monuments in the streets of a town, is to place them in shallow pits a foot or more in diameter. Set the monument in the pit so that the top of it shall be several inches above the bottom of the pit and as much below the street pavement. Protect it with a cast iron cylinder set about it, having a slightly conical cover which is level with and forms part of the pavement. The summit of the cover answers some of the purposes of the monument, while by removing it the monument itself is brought to view.

CHAPTER X.

RESURVEYS.

1. In an old settled country, the principal work of the surveyor is to retrace old boundary lines, find old corners, and relocate them when lost. In performing this duty, he exercises, to a certain extent, judicial functions. He usually takes the place of both judge and jury, and acting as arbiter between adjoining proprietors, decides both the law and the facts in regard to their boundary lines. He does this not because of any right or authority he may possess, but because the interested parties voluntarily submit their differences to him as an expert in such matters, preferring to abide by his decision rather than go to law about it.

In making resurveys the surveyor is called upon—

1. To construe descriptions in deeds;
2. To find the location of corners and boundary lines;
3. To renew corner monuments and to mark anew boundary lines.

2. **In construing the descriptions** the following rules have been laid down by the courts:

RULE 1. The description of boundaries in a deed is to be taken most strongly against the grantor.

Marshall *v.* Niles, 8 Conn. 369.
Ryan *v.* Wilson, 9 Mich. 262.

2. A deed must be construed according to the condition of things at the date thereof.

Crogan *v.* Burling Mills, 124 Mass. 390.

Written descriptions of property are to be interpreted in the light of the facts known to and in the minds of the parties at the time.

Wiley *v.* Sanders, 36 Mich. 60.
McConnell *v.* Rathbun, 46 Mich. 305.

And should be construed with reference to any plats, facts, and monuments on the ground referred to in the instrument.

Anderson *v.* Baughman, 7 Mich. 77.
Bowen *v.* Earl, 28 Mich. 538.

3. Where the description of the boundaries in a deed are indefinite or uncertain, the construction given by the parties, and manifested by their acts on the ground, is deemed the true one unless the contrary is clearly shown

Reed *v.* Prop. Locks and Canals, 8 How. (U. S.) 274.

4. Every call in the description of the premises in a deed must be answered if it can be done, and none is to be rejected if all the parts can stand consistently together.

Herrick *v.* Hopkins, 10 Shep. (Me.) 217.

5. Where the boundaries mentioned are inconsistent with each other, those are to be retained which best subserve the prevailing intention manifested on the face of the deed.

Gates *v.* Lewis, 7 Vt. 511.

6. The certain description must prevail over the uncertain, in absence of controlling circumstances.

Richer *v.* Barry, 34 Me., 116.
Tewksbury *v.* French, 44 Mich. 102.
See also 35 N. H. 121, and 11 Conn. 335.

7. When one part of the description in a deed is false and impossible, but by rejecting that a perfect description remains, such false and impossible part should be rejected and the deed held good.

Anderson *v.* Baughman, 7 Mich. 79.
Johnston *v* Scott, 11 Mich. 232.

8. A deed is to be construed so as to make it effectual rather than void. (*Ibid.*)

9. Where the description in a deed calls for land "owned and occupied," the actual line of occupation is a material

call to be considered in locating the lines of the land bounded therein.

Fahey *v.* Marsh, 40 Mich. 239.
Cronin *v.* Gore, 38 Mich. 386.

10. Where land is described as running a certain distance by measure to a known line, that line will control the measure and determine the extent of the grant.

Flagg *v.* Thurston, 13 Pick. (N. Y.) 145.
See also 13 Wend. (N. Y.) 300, and 7 Iredell. (N. C.) 169 and 310.

11. Not so if the line is obscure, not definitely fixed, marked or known, and therefore likely to be looked upon by the parties as less certain than the measure given.

Howell *v.* Merrill, 30 Mich. 282.

12. In the case of Land Co. *v.* Saunders in 5th Otto (U. S.), the Supreme Court of the United States held the west line of Hart's location to be the boundary of a grant. It was in a mountainous country and had never been surveyed or marked—although capable of being marked—the line being simply marked on the plat of the location. This line is held to be such a monument as would control course and distance.

13. Where land is conveyed "beginning at" and bounding land of "B," the point of beginning and boundary is the true line of B's land, and not the line of occupation as shown by a fence set up and maintained by B before and after the conveyance, with the consent of the owner of the lot conveyed, under the mistaken belief that such was the true line.

Cleveland *v.* Flagg. 4 Cushing (Mass.) 76.

14. A course from corner to corner means *prima facie* a right line; but this may be explained, by other matters in the case, to be a crooked or curved line, as following a ditch, or hedge, or stream.

Baker *v.* Talbott, 6 Mont. (Ky.) 182.

15. "Northward" or "northerly" means due north when nothing is mentioned to show the deflection of the course to the east or west.

Jackson *v.* Reeves, 3 Caines, N. Y. 293.
Brandt *v.* Ogden, 1 Johns. N. Y. 156.

16. The use of the term "about" indicates that exact precision is not intended; but where nothing more certain can be found to control the course and distance, the grantee is limited to the exact course and distance given.

Cutts *v.* King, 3 Greenl. Me. 482.[1]

17. Where a given quantity of land is to be laid off on a given base, it must be included in four lines, so that the lines proceeding from the base shall be at right angles with it, and the line opposite the base shall be parallel with it, unless this form is repugnant to the entry.

Massie *v.* Watts, 6 Cranch. (U. S.) 148.
Ker *v.* Watts, 6 Wheat. (U. S.) 550.

18. Seventy acres lying and being in the southwest corner of a section, is a good description, and the land will be in a square.

Walsh *v.* Ringer, 2 Ham. (Ohio) 327.

19. Where lines are laid down on a map or plan, and are referred to in a conveyance of land, the courses, distances, and other particulars appearing on such plan are to be as much considered the true description of the land conveyed as they would if expressly recited in the deed.

Davis *v.* Rainsford, 17 Mass. 211.
See also 14 Mass. 149, and 1st Greenl. Me. 219.

20. A conveyance by metes and bounds will carry all the land included within them, although it be more or less than is stated in the deed.

Butler *v.* Widger, 7 Cow. (N. Y.) 723.
Bratton *v.* Clawson, 3 Strobh. S. C. 127.
Gillman *v.* Riopelle, 18 Mich. 164.

21. A grant of land bounded by a highway takes to the center of the highway. If it be designed to exclude the highway, it must be so stated in explicit terms.

Champlin *v.* Pendleton, 13 Conn. 23.
See also 7 N. H. 275; 6 Shep. Me. 276.
Purkiss *v.* Benson, 28 Mich. 538.

A deed of land lying east of a certain street, and explicitly bounded by the east line of the street, conveys no title to the soil in the street.

G. R. & I. R. R. Co. *v.* Mary Heisel, 38 Mich. 62.

22. The mention of quantity of acres after a definite description by metes and bounds, or by the aliquot part of the section, is a matter of description only, and quantity being the least certain, does not control.

Amich v. Holman, 13 Strobh. S. C. 132.
McClintock v. Rogers, 11 Ills. 279.
Martin v. Carlin, 19 Wis. 454.

23. Where boundaries are doubtful, then quantity often becomes a controlling consideration.

Winans v. Cheney, 55 Cal. 567.

24. Grants by government are to be construed according to the common law, unless it has done some act to exclude that construction.

Middleton v. Pritchard, 3 Scam. Ills. 510.

The references in the following recent decisions are to the several "Law Reporters," published by the West Publishing Co., of St. Paul, Minn.

25. A reference in a description to the government patent, makes the patent description and the government survey a part of the deed.

Miller v. Topeka Land Co., (Kan.) 24 P. 420.

26. Where a survey is referred to in a deed for greater certainty, it legally forms a part of it and both should be construed together.

Heffleman v. Otsego Water-Power Co., (Mich.) 43 N. W. 1096.

27. Extrinsic evidence is always admissible to explain the calls of a deed for the purpose of applying them to the subject-matter, and thus to give effect to the deed.

Thompson v. Southern Cal. M. R. Co., (Cal.) 23 P. 130.

28. An exception in a deed which reads, "Except the dower of fifty acres, as fully described in the deed given the C. B. Co.," is not void, though the boundaries of the excepted land are not defined in any way, as reference may be had to the deed to the C. B. Co. to ascertain them.

McAffee v. Arline, (Ga.) 10 S. E. 441.

29. A deed conveying property by lot numbers is not void for uncertainty, though the recorded plat shows no division of the blocks into lots; it being shown that the proprietors had always treated the blocks as divided into lots, and that for many years the property had been assessed, conveyed, and generally known by the lot numbers.

Marvin v. Elliott, (Mo.) 12 S. W. 899.

30. A deed describing the granted premises as "subdivision of lot No. 4 of division No. 16," etc., followed by the total number of acres contained in lot 4, and then excepting land previously sold, is not void for indefiniteness, though lot 4 was never subdivided, as it evinces a clear intent to convey the balance of whatever land the grantor owned in lot 4; and the deed will be construed as though the word "of" after the word "subdivision" had been omitted.

Weeks v. Martin, 10 N. Y. S. 656.

31. A deed to a railroad company of a right of way "along the line as surveyed and laid out" by the company's engineer is not void for uncertainty where it appears that when the deed was executed the line of the road had been surveyed and distinctly marked by stakes stuck in the ground, and that subsequently the road was constructed following the exact line of the survey.

Thompson v. Southern Cal. M. R. Co., (Cal.) 23 P. 130.

32. In a deed of land by metes and bounds, an exception, of "lot 6, block 36, heretofore conveyed to B," excepts a lot so numbered on a plat made by the grantor and grantee, but not then recorded, there being no other lot 6 block 36, within the land granted. The recital of a conveyance to B. may be rejected as a *falsa demonstratio*.

Ambs v. Chicago, St. P., M. & O. R'y Co., (Minn.) 46 N. W. 321.

33. Though a plat be incomplete as respects the location of monuments, or in respect to measurements and distances, yet where land so surveyed has been conveyed

by reference thereto, and the location of the lots so conveyed and designated is well known by all parties interested, and susceptible of identification according to the actual survey on the ground, the description is sufficient to pass the title.

Bohrer v. Lange (Minn.) 46 N. W. 358.

34. The description in a deed was: "Beginning at * * * ; running thence northeasterly, along Grove street, 25 feet; and thence northwesterly, and parallel with Woodruff avenue, 108 feet 9 inches, to lot No. 80, on said map; thence southwesterly, along lot No. 80, 25 feet; and thence southeasterly, and parallel with Woodruff avenue, 108 feet 9 in., to the westerly side of Grove street, the point or place of beginning." Lines drawn from Grove street, 108 feet 9 inches, parallel to Woodruff avenue, would not reach lot 80 by 5 inches. *Held*, that there was a mistake in describing the length of the lines parallel to Woodruff avenue, and that it was intended that they should extend 109 feet 2 inches, and not that they should run in such a direction that they would reach lot 80 at the distance of 108 feet 9 inches from Grove street.

Casey v. Dunn, 8 N. Y. S. 305.

35. It being stated with certainty in the deed that such lines were parallel to Woodruff avenue, it is immaterial, in construing the description, that the corresponding lines in the conveyances of neighboring property were at right angles to Grove street, instead of being parallel to Woodruff avenue.

Casey v. Dunn, 8 N. Y. S. 305.

36. The description in a deed was certain as to the northern and western boundaries. The course of the eastern boundary was south for a distance of 8 rods. The southern boundary was "then west, in a line parallel to, and eight rods south of," the northern boundary, "one hundred and sixty-two feet, to" the western boundary. By reference to another deed, it was made certain that the north 6 rods of the eastern boundary was a straight wall. The course of the other 2 rods was uncertain.

Extending the line of the wall 2 rods south, and from the end of this line drawing a line parallel to the northern boundary, to the western boundary, a southern boundary 165 feet in length would be obtained. *Held*, that from the southern end of the wall the eastern line should be deflected towards the west at such an angle that at the distance of 2 rods it would intersect a line parallel with the northern boundary at the distance of 162 feet from the western boundary.

Ladies' Seamen's Friend Soc. v. Halstead, (Conn.) 19 A. 658.

37. A city, by its president and trustees, conveyed to defendants' grantor "that lot of land containing 60 acres, lying in block No. 1111, according to the official map of said city made by * * * A. D. 1856." The deed referred to a resolution of the trustees, under which the lands were sold, which provided that all surveys should be made by the purchaser. At the time of the deed there had been no survey or subdivision of the block. *Held*, that the deed conveyed an undivided 60 acres of the block.

Cullen v. Sprigg, (Cal.) 23 P. 222.

38. Where a description by metes and bounds is supplemented by a reference to a particular subdivision of land to indicate the tract intended to be conveyed, the former will not necessarily be controlling, when it would leave a strip 13 feet front by 100 deep in the grantor, which clearly appears to have been intended to be conveyed by the latter description.

Cannon v. Emmons, (Minn.) 46 N. W. 356.

39. Ordinarily, calls for natural or artificial monuments will control courses and distances; but a call for course and distance will not be subordinated to a call for an unmarked line in a prairie, which cannot itself be ascertained except by running the boundaries of another survey according to course and distance.

Johnson v. Archibald, (Tex.) 14 S. W. 266.

40. A complaint was filed to quiet title to 150 acres of land lying on the south side of a fractional section. A surveyor was ordered to survey that quantity, to be taken the full length of the section from the east side thereof to a river as the western boundary, and extending far enough north to include 150 acres. The surveyor executed the order, and reported a survey, which was accepted, and the court entered judgment, wherein the land was doubly described by inconsistent descriptions. The first described it as in the order of survey, and the second by metes and bounds, by which, after beginning at the southeast corner of the section, and following the south line to the river, it ran up the river, with the meanders thereof, to a stake placed by said surveyor $19\frac{1}{3}$ chains north of the south line of the section; thence running westerly, parallel with the south line, 53.04 chains, to a stake in the east line of the section; and thence southerly with said line $9\frac{1}{3}$ chains, to the beginning. The stakes were gone, but were shown to have been placed at points $19\frac{1}{3}$ chains from the south line, thereby including 150 acres. *Held*, that the first description should govern.

Caspar v. Jamison, (Ind.) 21 N. E. 743.

41. Under a deed of land bounded by a street, according to a map referred to, the line of the street as actually surveyed is the boundary of the land conveyed.

Andreu v. Watkins, (Fla.) 7 So. 876.

42. A deed described the land conveyed as "commencing on the S. road at the north-east corner of the land owned by S.; running south, to the south-east corner of said S.'s land, two acres; from thence, easterly and parallel with said S. road, two acres; thence running northerly two acres, until it strikes said road; and thence westerly, along said road, two acres, to the beginning; containing four acres of land, neither more nor less." *Held*, that as the description by quantity so clearly shows the intention to limit the grant to four acres in rectangular form, and as the length of the west line is given, the intention must control distances.

Rioux v. Cormier, (Wis.) 44 N. W. 654.

A similar construction is to be given the United States statute providing for the survey in certain cases of tracts of land two acres in width and running back a depth of forty acres. R. S. 2407.

43. A city condemned a strip of land for railroad and sewer purposes, and, after constructing a road-bed along this, it conveyed to a railroad company "its title to the road bed, bridges, and right of way" along the entire route, and "all the land belonging to the city," between certain streets, "for depot purposes." The company had formerly occupied a right of way for a double track on other streets, and the city, in consideration of the change of the railway to the street forming the line of the road in the conveyance, agreed to furnish the company a road-bed. *Held*, that outside of the part conveyed for depot purposes nothing but the road-bed was conveyed.

Long v. Louisville & N. R. Co., (Ky.) 13 S. W. 3.

44. The deed of a city lot, and plat with reference to which it was made, called for the south line of Cherry street as the northern boundary of the lot. The line referred to had been established by the City Surveyor 37 years before and ever since acquiesced in. The other lots in the block had been bought, fenced and built upon on the assumption that this survey was correct. A more recent survey tended to show that the line was three feet too far north. *Held*, that the presumption of correctness was with the older survey, and as the lot owner had got all he bargained for, and the later survey would cause the lines of the other lots to cut into the buildings, the older survey must prevail.

Wilmarth v. Woodcock, Mich. 33 N. W. Rep. 401.

45. A description in a deed reads: The east ½ of the east ½ of the northwest ¼, and the east ½ of the east ½ of the southwest frac. ¼," etc., "containing 50 acres of land; being the east half of 100 acres conveyed by A. and B. to E. The south line of the tract is irregular on a lake, and a line north and south through the center of the tract would give one parcel nine acres more land than the other. *Held*, that the language is apt and proper to

divide the tract by a north and south line which would give to each 50 acres, or one-half of the whole.

A description of the half of the parcel of land, according to the United States survey, would have excluded the idea of equal quantities and fixed the dividing line in accordance with the Act of Congress. If any other line had been agreed upon between the owners as the boundary line, it would govern the case.

Jones v. Pashby, Mich. 29 N. W. Rep. 376.
Dart v. Barbour, 32 Mich. 276.
Heyer v. Lee, 40 Mich. 353.

46. A description in a deed, if otherwise good, is not vitiated by the omission of the word "rods" to avoid tautology, when the meaning is plain.

Taber v. Shattuck, 55 Mich. 370.

1. **Adverse Possession.**—When the boundary line between the lands owned by adjoining land-owners is unknown, they may by parol fix a line between each party, each party mutually agreeing thereto and acting thereon, which is binding between them; but if the line is known, then the transfer of any portion of the land on one side of the line from the one to the other must be in writing, to be valid.

Jinkins v. Trager, 40 F. 726.

2. The adverse possession of land by a grantor cannot avail his grantee, beyond the boundary line described in the deed.

Jenkins v. Trager, 40 F. 726.

3. Possession as owner is an essential condition by which the ownership of immovables can be acquired without title, or possession in good faith.

Stille v. Schull, (La.) 6 So. 634.

4. Continuous possession of land for more than 30 years under claim of ownership, though without color of title, constitutes title in fee.

Bowen v. Swander, (Ind.) 22 N. E. 725.

5. One cannot acquire title to land by adverse possession where he claims title under a deed which in fact does not include such land in its description.

Casey v. Dunn, 8 N. Y. S. 305.

6. Where title is claimed by adverse possession, if the possession is by actual occupation of the possessor under claim of title, it is visible, open, notorious, distinct, and will be presumed to be hostile.

Green v. Anglemire, (Mich.) 43 N. W. 772.

7. Where the line between adjoining owners is in doubt, but they only claim ownership to the true line, wherever that may be, no title by adverse possession can arise in either, as against the other.

Krider v. Milner, (Mo.) 12 S. W. 461.

3. In construing deeds conveying title to **lands bordering on waters**, it will be necessary for the surveyor to inquire into the local laws of the State in which the premises lie, as different States by their laws and courts give different constructions to the word "navigable" as applied to streams and the smaller lakes. The statute of the United States provides that

"SEC. 440. All navigable rivers, within the territory occupied by the public lands, shall remain and be deemed public highways; and, in all cases where the opposite banks of any streams not navigable belong to different persons, the stream and the bed thereof shall become common to both." (1 Stat. 468; 2 *id.* 235; R. S. 2476.)

It is a universal rule that grants of land bordering on navigable streams take only to high-water mark, while grants on non-navigable streams take to the center of the stream, or the *filum aquæ*, as it is termed.

Now, whether the proprietor in any given case owns the land under water to the center of the stream, or only takes to high-water mark, depends on the local construction given to this word navigable.

Under the *Common Law*, a navigable stream is one in which the tide ebbs and flows. Some exceptions to the rule are made in England.

Under the *Civil Law*, a navigable stream is one capable of being used as a highway of commerce. In the case of the Railroad Co. *v.* Schurmier, (7 Wallace, 272), the Supreme Court of the United States says that "the words navigable and non-navigable were applied by Congress without respect to the ebb and flow of the tide," and in the case of Bowman and Bumley *v.* Wathieu and others, (2d McLean, 276), they say that "the common law doctrine as to the navigableness of streams can have no application in this country, and the fact of navigableness does in no respect depend on the ebb and flow of the tide."

The courts of Pennsylvania, North Carolina, South Carolina and Alabama hold the same view. On the contrary, in Maine, New Hampshire, Massachusetts, Connecticut, New York, Maryland, Virginia, Ohio, Illinois, Indiana, and Michigan, the common law doctrine is held to prevail. (See Angell on Tide Waters, pp. 77 and 78.)

Hence, in applying the principles laid down by the courts in the following decisions, the surveyor will bear in mind the locality in which they are to be applied.

1. Proprietors of lands bordering on navigable rivers, under titles derived from the United States, hold only to the stream, as the express provision is, that all such rivers shall be deemed to be and remain public highways.

R. R. Co. *v.* Schurmeir, 7 Wallace (U. S.) 272.

2. Where a sea or bay is named as a boundary, the line of ordinary high-water mark is always the line, where the common law prevails.

U. S. *v.* Pacheco, 2 Wallace (U. S.) 587.

3. A boundary *on* a stream or *by* or *to* a stream includes flats at least to low-water mark, and in many cases to the middle thread of the river.

Thomas *v.* Hatch, 3 Sumner (U. S.) 170.

4. A boundary on the bank of a river referring to fixed monuments on the bank, limits the grant to the bank and excludes the flats. (*Ibid.*)

See also Hopkins *v.* Kent, 9 Ohio, 13.

5. The words "along the bank" are strong and definite enough to exclude the idea that any part of the river or its bed was granted in the navigable or unnavigable parts of the river.

Howard *v.* Ingersoll, 13 How. (U. S.) 341, 416.

A deed describing the land by a boundary running to a stream, and thence along its bank, and reserving the right to use the river front a specified time, conveys the land to the water's edge and covers the riparian rights to the middle of the stream.

Cole *v.* Wells, 49 Mich. 450.

6. Congress, in making a distinction between streams *navigable* and those *not so*, in the acts relating to the sale of the public lands bordering thereon, intended to provide that the common law rules of riparian ownership should apply to the lands bordering on the *latter*, but that the title to lands bordering on the *former* should stop at the stream.

R. R. Co. *v.* Schurmeir, 7 Wall. (U. S.) 272.

7. In streams which are not navigable, adjacent proprietors own to the center of the stream measured from low-water mark.

Clark *v.* Caupau, 19 Mich. 325.
Moore *v.* Sanborn, 2 Mich. 519.
Lorman *v.* Benson, 8 Mich. 18.
Bay City Gas Light Co. *v.* Ind. Wks 8 Mich. 182.
Lamb *v.* Ricketts, 11 Ohio, 311.

8. The same principle is applied to Lake Muskegon, in Michigan, (Rice *v.* Ruddeman, 10 Mich. 125), but not applied to a similar lake in Wisconsin, where the court says, (Deidrich *v.* N. W. U. Ry. Co., 42 Wis. 271): "Riparian owners upon a natural lake or pond take only to the shore."

In the case of the State of Indiana *v.* Milk, Circuit Court of the United States, April term, 1882, 11th Bissell, page 197, the court rejects the theory of riparian ownership in the lake, and after presenting its reasons at some length, concludes with the following: "That while a general grant of land on a river or stream non-navigable extends the line of the grantee to the middle or thread of the current, a grant on a natural pond or lake extends only to the water's edge."

9. Islands in rivers fall under the same rule as to ownership as the soil under water does. If not otherwise lawfully appropriated, they belong to the proprietors on either side of the stream, according to the original dividing line or *filum aquæ* as it would run if the islands were under water. The *filum aquæ* is midway between the lines of ordinary low-water mark, without regard to the channel or depth of water. When the island is appropriated, the boundary is then midway between it and the mainland.

McCullough *v.* Wall, 4 Rich. (S. C.) 68.
Kimball *v.* Schaff, 40 N. H. 190.

10. The grant includes any land between the meander line and the water, in an unnavigable stream.

The same principle applies to unnavigable lakes.

Forsyth *v.* Smale, 7 Biss. (U. S.) 201.

11. High-water mark in the Mississippi River is to be determined from the river bed, and that only is river bed which the river occupies long enough to wrest it from vegetation.

Houghton *v.* Railway Co., 47 Iowa, 370.

12. A bank is the continuous margin where vegetation ceases. The shore is the sandy space between it and low-water mark.

McCullough *v.* Wainwright, 14 Penn. St. 59.

13. Where a levee was shown to have been judiciously located by a competent engineer and agents of the State acting under authority conferred by the State Legislature, it was held that such levee became the boundary line of high water, and that no private ownership could be acquired to land lying between that and the bed of the stream.

Musser *v.* Hershey, 42 Iowa, 356.

14. Grant of a city lot bounded on a river, takes to the center of the stream.

Watson v. Peters, 26 Mich. 508.

Riparian rights, unless expressly limited, extend to the middle of the navigable channel, and cover any shallows or middle ground not shown in the government surveys, but lying between such shallows and the shore, and it makes no difference that the deed conveying the premises to which the rights attach describes them according to a city plat instead of the government entry.

Fletcher v. Thunder Bay Boom Co., 51 Mich. 277.

15. But if the plat plainly indicates the proprietor's ntent to reserve the space between the shore and the thread or main channel, the case would be different.

Watson v. Peters, 26 Mich. 508.

16. Riparian rights extend laterally into the stream. Rocks and shoals along the margin of navigable rivers above tide-water belong to the riparian owner.

Moore v. Willamette T. and L. Co., 7 Oregon R. 355.

17. When a navigable stream is meandered in making the public surveys, and the United States has granted to the meander line, the grantee takes to the river. The stream, and not the meander line, is the true boundary of the riparian owner.

Minto v. Delaney, *id.*, 337.

18. Lands patented by the United States on a tide-water stream extend to the meandered line of the stream, which is the line of ordinary high water.

Parker v. Taylor, *id.*, 435.

19. A boundary by the shore of a mill pond takes to low water mark.

Stevens v. King, 76 Maine 197.

20. N. conveyed a lot according to a certain plat. The plat represented the lot as bounded north by a street south by a stream; on the east and west by lines running from the street to the stream, with figures purporting to give the length of these lines. In fact, the distance to the stream was greater than indicated by these figures.

Held, that the conveyance of the lot according to the plat included all the land between the street and the stream.

Nicolin v. Schneiderham, Minn. 33, N. W Rep. 33.

In Turner vs. Holland, the Supreme Court of Michigan gives riparian rights to owners of lots bounded by a bayou of Saginaw river, described by plat similar to the above.

33 N. W. Rep. 283.

21. In a navigable stream, as the DesMoines river in Iowa, high water mark is the boundary line. When, by action of the water, the river bed changes, high water mark changes and ownership of adjoining land changes with it. The location of meander lines does not affect the question. Meander lines are not boundary lines.

Steele v. Sanchez, 33 N. W. Rep. 367.
Krant v. Crawford, 10 Iowa 549.
Lockwood v. R. R. Co., 37 Conn. 387.

22. A boundary stated in a deed as a line forty feet above the border of a river at high water mark, is not ambiguous, and if disputed is to be fixed like any other fact, by testimony and an examination of the ground.

Bresler v. Pitts, 59 Mich. 348.

Recent decisions from "Law Reporters:"

23. A patent for a fractional quarter section, which is bounded by a meandered stream, passes title to all land within the lines of said quarter section between the meandered line and the water's edge.

Sphung v. Moore, (Ind.) 22 N. E. 319.

24. The owner of land on the margin of a navigable stream, holding under a grant from the United States, does not take to the middle of the stream, but to high-water mark, which is determined by the change in the vegetation and the character of the soil, and the beds of all navigable streams, though the tide does not ebb and flow in them, belong to the state.

St. Louis, I. M. & S. Ry. Co. v. Ramsey, (Ark.) 13 S. W. 931.

25. The owner of land on a bay conveyed an acre at the end of the tract nearest the bay, described as follows: "Beginning * * * by the beach, running * * *

along the beach to," etc. In the general description of the tract it was bounded "easterly by the said beach." The grantee was given the privilege of a road from the middle of the front of the lot to the bay, and also half the drift coming on shore in front of the lot, and all the other privileges of the beach were reserved by the grantor, who bound himself not to build any house in front of the lot. The courses and distances would not carry the boundary to high-water mark. *Held*, that the beach did not pass by the deed.

Benson v. Townsend, 7 N. Y. S. 162.

26. Where two deeds in plaintiff's chain of title respectively define the boundary of the land "by the edge of the mill-pond" and as "the bank of said mill-pond," and defendant is entitled to pond as much land as the pond flowed at the time of his purchase, defendant may enter on land originally covered by the pond, but which has subsequently become dry land by the receding of the water, though plaintiff's deed on its face shows his line to be the center of the pond.

Holden v. Chandler, (Vt.) 18 A. 310.

27. Where the patentee of "the north half of the southeast quarter, and that part of the northeast fractional quarter, of Section 36," etc., "which lies north of the Kankakee river, containing in all 122.70 acres," conveys "the northeast quarter of Section 36," etc., "containing 122.70 acres," the deed passes title to all of the land in said northeast fractional quarter lying south of said river.

Sphung v. Moore, (Ind.) 22 N. E. 319.

28. Where one who owns a tract of land that surrounds and underlies a non-navigable lake, the length of which is distinguishably greater than its breadth, conveys a parcel thereof that borders on the lake, by a description which makes the lake one of its boundaries, the presumption is that the parties do not intend that the grantor should retain the title to the land between the edge of the water and the center of the lake, and the title of the purchaser, therefore, will extend to the center thereof.

Lembeck v. Nye, (Ohio) 24 N. E. 686.

29. A patent from the United States of a surveyed fractional government subdivision, bounded on a meandered lake, conveys the land to the lake, although the meander line of the survey be found to be not coincident with the shore line.

Everson v. City of Waseca, (Minn.) 46 N. W. 405.

30. Where the description is by metes and bounds, no reference being made therein to the lake, then only the land included within the lines as fixed by the terms used by the parties to the deed will pass to the grantee.

Lembeck v. Nye, (Ohio) 24 N. E. 686.

31. If, however, the call in the description be to and thence along the margin of the lake, no such presumption arises, and the title of the purchaser will extend to low-water mark only.

Lembeck v. Nye, (Ohio) 24 N. E. 686.

32. Where a deed conveys land "bounded and described according to" a certain survey, does not call for a river, but calls for a line run between certain points, designated by the surveyor as on the bank of a navigable river, and it appears that the lines of such survey exclude flats between high and low water marks, evidence *aliunde* is admissible that the bank referred to was an artificial dike; that the grantee had notice that the grantors reserved the flats; that the grantors refused to execute a deed expressly conveying the flats; and that the sale was expressly subject to the survey, as tending to show that the flats were excluded, whatever may be the presumption from the deed.

Palmer v. Farrell, (Pa.) 18 A. 761.

For further rulings, see *Boundary Lines.*

Second.

4. In locating the corners and boundary lines on the ground, we will consider:

1. General rules which apply to all resurveys;

2. Special applications of these rules to the rectangular system of United States surveys.

GENERAL RULES.

RULE 1.—In locating a deed on the ground, we are to rely—

(1) On the actual lines originally surveyed;

(2) On lines run from acknowledged calls and corners.

(3) On lines run according to the course and distance in the deed.

Avery *v.* Baum, Wright's Ohio, 576.
1 Rich. (S. C.) 491.

2. When the boundaries of lands are fixed, known and unquestionable monuments, though neither courses, distances, nor computed contents correspond, the monuments must govern.

Pernam *v.* Wead, 6 Mass. 131.
Nelson *v.* Hall, 1 McLean (U. S.) 518.

3. Marked lines and corners control courses and distances. Surplus lands do not vitiate a survey nor does a deficiency of acres called for in a survey operate against it. Wherever the boundaries can be established, they must prevail.

Robinson *v.* Moore, 4 McLean (U. S. C. C.) 279.
Morrow *v.* Whitney, 5 Otto (U. S.) 551.

4. A deed called for posts as corners. The survey was made and the posts set prior to the execution of the deed. It was afterward found that there was a shortage of several acres. Held that proof that posts were set up as corners between adjoining owners controls the call for course and distance.

Alseire *v.* Hulse, 5 Ohio, 534.

5. The rule that courses, distances and quantities must yield to monuments, is not inflexible, especially when the distances are very short, and the monuments artificial ones, as here, a mill-race, etc.

Higinbotham *v.* Stoddard, 72 N. Y. 94.
Ga. R. R. Co. *v.* Hamilton 59 Ga. 171.

In a case where no mistake could be reasonably supposed in the courses and distances, the reasons of the rule were held to fail, and the rule was not applied.

Davis *v.* Rainsford, 17 Mass. 207.

6. The rule that natural or artificial boundaries will control distances or courses, authorizes no other departure from the course or distance than such as is necessary to effectuate the apparent intent of the grantor.

Distances may be increased and courses departed from in order to preserve the boundary, but the rule authorizes no other departure from the course and distance than such as is necessary to preserve the boundary.

Johnson *v.* McMillan, 1 Strobh. (S, C.) 143.

7. If the courses and distances cannot be otherwise reconciled with the monuments in a description, a line in a survey which has evidently been omitted will be supplied to prevent the obvious intent of the grantor from being frustrated.

Serrano *v.* Rawson, 47 Cal. 52.

See also Schultz *v.* Young, 3 Iredell, N. C. 385,

where two lines must be run instead of the one called for, to best conform with the whole description in the deed.

8. A survey must be closed in some way or other. If this can only be done by following the course the proper distance, then it would seem that distance should prevail; but when the distance falls short of closing, and the course will do it, the reason for observing distance fails,

Doe *v*, King, 3 How. Miss. 125.

9. It is a universal rule that course and distance yield to natural and ascertained objects. But where these objects are wanting, and the course and distance can not be reconciled, there is no universal rule that obliges us to prefer the one to the other. Cases may exist in which either one may be preferred, according to the circumstances.

Preston's Heirs *v.* Bowman, 6 Wall. (U. S.) 580.

10. If no principle of location be violated by closing from either of two points, that may be closed from which will be more against the grantor and include the greater quantity of land.

Johnson *v.* McMillan, 1 Strobh, S. C. 143.

11. The boundary line is to be ascertained by running direct lines from one monument to the other.

Melcher *v.* Merryman, 4 Me. 601.

12. A line actually marked must be adhered to, though not a right line from corner to corner. Where a line has been marked only part of the way, the remainder of the line must run direct to the corner called for.

Cowan *v.* Fauntleroy, 2 Bibb (Ky.) 261.

13. A marked line of another tract, when called for in a conveyance, must be run disregarding distance; but where such line can not be established, the distance run must govern.

Gause *v.* Perkins, 2 Jones Law Rep. (N. Y.) 222.

14, Where a line is described as running a certain distance to a particular monument, and that monument has disappeared and its place cannot be ascertained, the course and distance, in the absence of other controlling words, must govern.

Budd *v.* Brooke, 3 Gill (S. C) 198.
See also, Bruekner *v.* Lawrence, 1 Douglass (Mich.) 19.

15. Course and distance yield to known, visible and definite objects; but they do not yield unless to calls more material and equally certain.

Shipp *et al.* *v.* Miller's Heirs, 2 Wheat. (U. S.) 316.

Courses and distances in the deed are not to be controlled by monuments or objects variant therefrom and not called for in the description, but they must yield to such objects and monuments as are referred to.

Bruckner's Lessee *v.* Lawrence, 1 Doug., Mich., 29.
Moore *v.* People, 2 Doug., Mich., 424.
Bower *v.* Earle, 18 Mich. 165.

16. Wherever it can be proved that the line was actually run, was marked, and the corners made, the party claiming under the deed will hold accordingly, although there is a mistake in the description in the deed.

Cherry *v.* Slade, 3 Murph. (N. C.) 82.

A sold to B lot 7, informing B, at the time of the sale, that it was four rods wide, and marking it out upon the ground. He subsequently sold to C lot 8 and a vacated alley one rod in width between lots 7 and 8, informing C, at the time, that lot 8 was four rods wide, and the alley one rod wide, making five rods in all, and pointing out to C the marks previously made by him for the boundary of lot 7, sold to B, as being also the boundary of the alley sold to C. The premises were occupied by B and C in accordance therewith, without dispute. It was subsequently found, by reference to the plat, that lot 7 was five rods wide, and that there was no alley between the lots; whereupon B claimed the additional rod. Held, that to allow B to hold the rod in width of land which she did not purchase or pay for, and to deprive C of land which he did purchase and pay for, would be both bad law and bad morals.

Bolton *v.* Eggleston, Iowa.
N. W. Rep., Vol. 16, p. 62.

17. Boundary may be proved by any evidence which is admissible to establish any other fact.

Smith *v.* Prewitt, 2 A. K. Marsh. (Ky.) 158.

18. Where no bounds were established, the dividing line must be run by aid of the measurements in the deeds, the oldest title receiving its full measure first.

Talbott *v.* Copeland, 38 Me. 333.

19. A long established fence is better evidence of actual boundaries, settled by practical location, than any survey made after the monuments of the original survey have disappeared. A resurvey made after the monuments of the original survey have disappeared, is for the purpose of determining *where they were.* and *not where they ought to have been.*

Diehl *v.* Zauger, 39 Mich. 601.
Hunt's Lessee *v*, McHenry and Williams, Wright's (Ohio) 599.

20. Where between the plan and the original survey there is a difference in the location of the lines and monuments, the lines and monuments originally marked as

such are to govern, however much they may differ from those represented on the plan.

Ripley *v.* Barry, 5 Greenl. (Me.) 24.

See also 2 Greenl. (Me.) 214, and 3 Gr. (Me.) 126.

21. But no such rule has obtained where the survey was subsequent to the plan.

Thomas *v.* Patten, 1 Shep. (Me.) 329.

22. Purchasers of town lots have a right to locate them according to the stakes which they find planted and recognized, and no subsequent survey can be allowed to unsettle them. The question afterwards is not where they should have been, in order to make them correspond with the lot lines as they should be if the platting were done with mathematical accuracy, but it is whether they were planted by authority, and the lots were purchased and taken possession of in reliance on them. If such was the case, they must govern, notwithstanding any errors in locating them.

Flynn *v.* Glenny, 51 Mich. 580.

23. Where two surveys call for each other, there can be no vacancy unless the lines marked on the ground contradict the call; and in such case the marked lines must govern.

McGinnis *v.* Porter, 20 Penn. 80.

24. Where two surveys made twenty-three years apart are found to disagree, the probabilities favor the earlier survey when the original corners and witnesses are gone at the time of the last survey, especially if the line of the first survey has remained unquestioned for many years.

Case *v.* Trapp, 49 Mich. 61.

25. When the same grantor conveyed to two persons, to each one a lot of land, limiting each to a certain number of rods from opposite known bounds running in a direction to meet if extended far enough, and by measure the lots do not join—when it appears from the same deeds that it was the intention that they should join, a rule

should be applied which will divide the surplus between the grantees in proportion to the length of the respective lines as stated in their deeds.

Lincoln *v.* Edgecomb, 28 Maine, 275.

26. Where original surveys have been made, and returned as a block into the land office, the location of each tract therein may be proved by proving the location of the block. In ascertaining the location of a tract, the inquiry is not where it should or might have been located, but where it actually was located.

Every mark on the ground tending to show the location of any tract in the block, is some evidence of the location of the whole block, and therefore of each tract therein.

Coal Co. *v.* Clement, 95 Pa. St. 126.

27. Where lots are conveyed by number according to a plat which is made from an actual survey, the corners and lines fixed by that survey are to be respected.

Pyke *v.* Dyke, 2 Greenl., Me., 214.

28. Streets which are well defined, and designated by some natural or artificial monument, must govern course and distance in fixing boundaries of lands; but streets which are not thus defined, and themselves require to be located, would furnish very uncertain guides in arriving at the boundaries of other lands.

Saltenstall *v.* Riley, 28 Ala. 164.

29. When streets have been opened and long acquiesced in, in supposed conformity to the plat, they should be accepted as fixed monuments in locating lots or blocks contiguous thereto or fronting thereon.

Van den Brooks *v.* Correon, 48 Mich. 283.

30. Lands have been laid off into lots and blocks, and platted, before being cleared, when, by reason of inequalities of the surface, logs, and other obstructions, strictly accurate surveys were not and could not be made. Where the blocks and streets were staked out at the time, such

monuments would be fixed and permanent, leaving the excess or shortage to be dealt with by itself.

So where the streets, although not so designated, have by the parties interested or by the public authorities been opened, used, and acquiesced in, they thereby become permanent boundaries and form new starting points in subsequent surveys of the premises.

Twogood *v.* Hoyt, 42 Mich. 609.

31. Ancient reputation and possession in regard to streets in a town are entitled to more respect in deciding on the boundaries of lots than any experimental survey that may be afterwards made.

Ralston *v.* Miller, 3 Rand. (Va.) 44.

32. Where lots are sold by numbers and a plat, any variance in the distance between known and fixed points as found by actual measure on the ground, and the distance between the same points as laid down on the plat, is to be divided between the lots in proportion to the respective lengths as laid down on the plat.

Marsh *v.* Stephenson, 7 Ohio, N. 3, 264.
Quinnin *v.* Reimers, 46 Mich. 605.

Surplus or shortage in a block is to be divided *pro rata* between the lots.

Newcomb *v.* Lewis, 31 Iowa, 488.
O'Brien *v.* McGraw, 27 Wis. 446

33. Where the accuracy of the starting points taken for test surveys is merely matter of speculation, they cannot be used to fix a disputed boundary between two lots when the dispute arises from a discrepancy which affects all the lots in a block, and must therefore be apportioned among them.

Reimers *v*, Quinnin, 49 Mich. 449.

34. A resurvey is inadmissible in evidence to show that a private boundary is incorrect, if its starting point is outside of and does not belong to the immediate plan or local system by which the original survey was controlled.

Burns *v.* Martin, 45 Mich. 22.

35. If in running the lines of a grant, one line be found which is admitted or proved to be a line of the grant, which will run with a variation from the calls of the grant, if no other marked lines be found, the other calls should be run with the same variation as that found on the marked line.

Sevier v. Wilson, Peck. 146.

36. Where a deed conveys lots in a town, and refers to a plat to identify them, and, in describing their lines, calls the points of compass as designated on the plat by its lines and angles, a correct survey cannot be based on any other system; and although the lines there delineated are not conformable to the true meridian, the plat and not the compass should govern.

Bower v. Earl, 18 Mich. 367.

The remaining decisions under this head, except the last four, are of recent issue, and are from the "Law Reporters," St. Paul, Minn.:

37. An instruction that, in arriving at a boundary line as originally run, natural objects are controlling calls; artificial objects, second in importance; course, third, and distance, fourth; and that, where there is still uncertainty, that rule should be adopted most consistent with the intent of the grant, is correct.

Luckett v. Scruggs, (Tex.) 11 S. W. 529.

38. An instruction that the beginning corner of a survey is of no higher dignity or importance than any other corner, and that, "if there are well-known and undisputed original corners established upon the ground around the survey, they would control the other calls of the survey, which are conflicting and contradictory, if there are any such," is correct.

Luckett v. Scruggs, (Tex.) 11 S. W. 529.

39. Where the beginning corner of a survey is the southwest, but the southeast corner is equally well identified, a charge limiting the jury to finding the unidentified northeast corner by the first and second lines from the

southwest corner, is erroneous, as the southeast corner is of equal importance, unless the line from the former corner was actually run and measured, and that from the latter not.

Scott v. Pettigrew, (Tex.) 12 S. W. 161.
Lancaster v. Ayres, Id. 163.

40. An instruction making the importance of an established northeast corner, in locating the north and west lines of a survey, dependent upon the jury's belief that such western line was not run, is erroneous, as such corner has the same weight for the purpose in question, whether the western line was run or not.

Scott v. Pettigrew, (Tex.) 12 S. W. 161.

41. In the description of lands, as to questions of boundaries the rule is settled in Virginia and West Virginia that natural land-marks, marked lines and reputed boundaries will control mere courses and distances, or mistaken descriptions in surveys and conveyances.

Gwynn v. Schwartz, (W. Va.) 9 S. E. 880.

42. The course of the eastern line of the H. tract, as given in the original survey made in 1745, was 14 deg. east. The course of the western line of the B. tract, lying immediately east of the H. tract, as given in the original survey made in 1813, was 17 deg. and 15 min. east. The western line of the B. tract was made of exactly the same length as the eastern line of the H. tract, and the beginning point of the two lines was the same. The difference in the course of the two lines could be satisfactorily explained by the change in the position of the magnetic needle which had taken place in the time intervening between 1745 and 1813. *Held*, that the two lines must be considered as coincident.

Scott v. Yard, (N. J.) 18 A. 359.

43. Where neither the corners of plaintiffs' nor defendants' land are satisfactorily established, and there is a well-established and identified corner of another survey, from which, by following course and distance, defendants' survey can be constructed, such course should be

followed, though the boundaries thus established include land within the boundaries of plaintiffs' junior survey.

Griffith v. Rife, (Tex.) 12 S. W. 168.

44. A county surveyor, employed to restore the lines and corners of adjoining tracts of land according to the original government survey, found township corners only, then (the other quarter and section corners being missing) ran a straight line from one township corner to the other, and on this line placed the quarter and section corners, but did not take any testimony to ascertain the lines or corners of the original survey, did not attempt to prove his lines or corners by re-establishing the missing corners from all the nearest known original corners, in all directions, did not sufficiently regard the field notes, and did not, where the original monuments had disappeared, regard the boundary lines long recognized and acquiesced in. *Held*, that such a survey is incomplete, and cannot be approved as the true and correct determination of the boundaries and corners as originally established by the government.

Reinert v. Brunt, (Kan.) 21 P. 807.

45. Upon an issue as to the location of a line of the government survey, evidence of the location of monuments is not overcome by field-notes of the original survey, taken at the time of the erection of said monuments or subsequent thereto.

Hubbard v. Dusy, (Cal.) 22 P. 214.

46. As between complicated descriptions of a line dividing two sections or quarter sections, that one is to be adopted which is most in conformity with the monument established by the government survey.

Hubbard v. Dusy, (Cal.) 22 P. 214.

47. As between different monuments, those best identified should prevail, independent of anything in the field-notes of the original or any subsequent survey.

Hubbard v. Dusy, (Cal.) 22 P. 214.

48. Where it is doubtful which of two lines of monuments is the true government line, other things being equal, that one is to be so considered which most nearly conforms to the field-notes.

Hubbard v. Dusy, (Cal.) 22 P. 214.

49. On a question as to the true location of a land patent, boundaries fixed by reversing the courses and distances must govern when found to coincide with the natural calls of the patent.

Ellinwood v. Stancliff, 42 F. 316.

50. When the points fixed by reversing the courses and distances do not coincide with the natural calls of the patent, or the natural calls cannot be identified, then the regular courses and distances must govern.

Ellinwood v. Stancliff, 42 F. 316.

51. When a survey calls for the "Dougherty" survey as one of its adjoiners, an instruction that if the jury find that the "King" is the survey intended by the call for "Dougherty," the former being located, the call would furnish "some evidence" of the location of the survey in question, is insufficient, as such a finding would locate the survey in the absence of marks upon the ground.

Tyrone Min. & Manuf'g Co. v. Cross, (Pa.) 18 A. 519.

52. Where no marks are found on the boundaries of a survey, and it cannot be located on the ground, evidence of the location of junior surveys which call for the lines of the elder as adjoiners is admissible, as showing where the surveyors upon the ground located such lines.

Tyrone Min. & Manuf'g Co. v. Cross, (Pa.) 18 A. 519.

53. Where the court, in an action of ejectment, instructs the jury that, "after a survey of blocks had been returned and had remained in the land-office 21 years, it was conclusively presumed that it was run upon the ground, whether marks were found upon the ground or not," but in other portions of his charge repeatedly states the law to be that marks made by the surveyor on the ground are the first and highest evidence of the true survey, the instruction cannot, on the whole, be said to be misleading,

as he will be reasonably understood to have charged that the presumption in favor of returns of surveys on file for 21 years is only applicable to such surveys where no monuments or marks on the ground are found to contradict them.

Grier v. Pennsylvania Coal Co., (Pa.) 18 A. 480.

54. The exterior of two adjoining interior surveys were undisputed. The boundary line between them had never been surveyed, but its southern end was marked by an oak. North of these surveys were two others. These four surveys were originally returned as being of equal size, and having one common corner. The northern end of the line between these two latter surveys was marked by a sugar-maple; which was not directly opposite the oak, and it was proved that the northern line of these surveys was shorter than the southern line of the others. *Held*, that the boundary line between the two southern surveys should run from the oak parallel to the end lines, and not diagonally from the oak to the maple.

Bloom v. Ferguson, (Pa.) 18 A. 488.

55. Where a dividing line is established between tracts of land owned by a county, before purchases are made of land on each side of it, and the deeds under which parties claim have been made, and are known by the parties to have been made with reference to that line, they, and all the persons claiming through them, are bound by it.

Briscoe v. Puckett, (Tex.) 12 S. W. 978.

56. The northwest corner of a survey was plainly marked, and part of the west line was also marked. The rest of the survey had apparently not been run on the ground, but the southeast corner was ascertainable from the field-notes, being located on an established line of another survey and at a given distance from an established point. The lines of survey as called for in the field-notes were correct as to courses, but were too short to reach from one of said corners to the other. *Held*, that the survey included all the land between the corners bounded by the lines as extended so as to reach from one corner to the other.

Randall v. Gill, (Tex). 14 S. W. 134.

57. Where a deed describes a lot conveyed as of a certain width, and a party-wall stands on the south line, the north line may be found by measuring the given distance north from the middle of such wall.

Warfel v. Knott, (Pa.) 18 A. 390.

58. The statement of the quantity of land supposed to be conveyed, and inserted in deeds by way of description, must not only yield to natural land-marks and marked lines, but also to descriptions in deeds by courses and distances.

Gwynn v. Schwartz, (W. Va.) 9 S. E. 880.

59. A call for a lot by the name or number which it bears on a plat of the land will prevail over courses and distances, and ordinarily over calls for monuments.

O'Herrin v. Brooks, (Miss.) 6 So. 844.

60. Where the descriptions in a deed refer to a survey and a map based thereon, making both a part of the deed, and there is a discrepancy between the map and the survey, the latter will prevail.

Whiting v. Gardner, (Cal.) 32 P. 71.

61. The owner of a lot in the city of Rochester, of the area of about one-half acre, rectangular in form, fronting 274 feet on a street, and abutting on the rear for the same distance on a canal, the location of both, as well as the other lines, being undisputed, conveyed a portion, by description, of "137 feet front and rear, measuring from G. H.'s north line on G. street, and also 137 feet from G. H.'s south line on the canal; being the piece of land occupied as a garden by the grantor." The lot was divided by a fence, one side being used as a garden; the fence starting on G. street midway, but striking the back line at the canal at a point 19½ feet from the middle of the lot. That fence was not mentioned in the deed. *Held*, that the reference to the garden was too indefinite to control the calls for exact distances from known bounds, and the divisional point on the canal should be located 137 feet from G. H.'s line.

Harris v. Oakley, 7 N. Y. S. 232.

62. Plaintiff owned a village lot, No. 124, and a tract of land lying adjacent thereto on the south and east sides. River street, which lay along a river's edge, was the westerly front of both the lot and the tract. He conveyed the tract to defendant, reserving a part thereof, beginning at the S. W. corner of the lot; thence southeasterly, along River street, 32 feet; thence northeasterly, "on a line with the southeast corner of lot No. 124," 10 rods and 23 links; thence N. to M. street; thence W. to the N. E. corner of the lot; thence southwesterly, to the S. E. corner; thence to the beginning. Locating the beginning point at the S. W. corner of the lot as appeared by the village plat on the easterly side of the street, the line passed directly through the S. E. corner of lot 124, taking no part of the lot, and thus making the reservation wholly within the tract conveyed; but by beginning at the river's edge, on the westerly side of the street, on the theory that plaintiff's property extended to the river, subject only to the easement of the street, the line would pass through and take part of lot 124. *Held* that the former location of the corner was correct.

Anderson v. Scott, (Mich.) 42 N. W. 991.

63. In an action to recover a tract of land lying between a slough and a river, plaintiff claimed title by virtue of a grant which bounded the land granted by the river, and the defendant introduced evidence that the surveyor who surveyed the grant meandered the slough instead of the river. *Held*, that, in determining the true boundaries of the grant, the sole question was to ascertain exactly where the surveyor ran his lines, and, if the jury found that he ran the line along the slough, they should find for the defendant.

Allen v. Koepsel, (Tex.) 14 S. W. 151.

64. Where, in ejectment, a surveyor testified that he ran the boundary line in dispute about 1868; that he found the original stake of the government survey at the section corner, and used it as a starting point; and it appeared that about the same time defendant built a fence upon this line, which he has ever since maintained

—this line must prevail over one surveyed 20 years later, when the corner mark was gone, by one who testified that he located the section corner by measurements from various lines and points, and then by digging found a stump which he took to be the original witness, and based his survey upon it.

Carpenter v. Monks, (Mich.) 45 N. W. 477.

65. The monuments or marks of the surveyor on the ground determine the true survey as against calls for adjoinders or courses and distances as returned; but, each block of surveys being separate and complete of itself, the call of a tract in one block for an adjoinder in another does not make the monument of the adjoinder the monument of the later block.

Grier v. Pennsylvania Coal Co., (Pa.) 18 A. 480.

66. Where a boundary line is assented to by the owner of a tract of land at a time when there is no dispute concerning such line, and on the supposition that it is the true boundary, he is not estopped, on discovering that such is not the case, from claiming title to the real boundary.

Schraeder Min. & Manuf'g Co. v. Packer, 9 S. Ct. 385.

67. Continuous and uninterrupted possession, under claim of ownership, to the line of a division fence, will not bar title, where it appears that such occupation was under a belief that the fence was on a true line, and without intention of claiming beyond the true line, as described in the deeds.

Skinker v. Haagsma. (Mo.) 12 S. W. 659.

68. Lands are not surveyed lands by the United States until a certified copy of the official plat of survey has been filed in the local land office.

United States v. Curtner, 38 F. I.

69. One who receives deeds of lots, and conveys to others, according to an unacknowledged plat of a town, is thereby estopped from denying the sufficiency of the dedication for want of the acknowledgment.

Giffen v. City of Olathe, (Kan.) 24 P. 470.

70. Testimony of declarations of a grantor, before the execution of a deed, tending to establish a boundary other than that made by the deed as construed by the court on appeal, is inadmissible, as its effect would be to convey land by parole in contravention of the statute of frauds.

Harris v. Oakley, 7 N. Y. S. 232.

71. Where a town site was surveyed and laid out in lots, blocks, streets and alleys, and a plat thereof made and lithographed, and distributed among the occupants of the town site, and one of the lithographed copies was afterwards recorded in the office of the register of deeds, but the same was not acknowledged, and the town site was pre-empted by the president of the town site company, and a patent was obtained by him for the benefit of the occupants, under the town-site act (5 U. S. St. 657), there was a sufficient dedication of the streets and alleys of said town, despite the want of acknowledgment of the recorded plat.

Giffen v. City of Olathe, (Kan.) 24 P. 470.

72. A deed conveying land in a town, but "reserving streets and alleys according to recorded plat of the town," passes the fee in such streets when such fee was at the time held by the grantor subject to the easement of the public therein.

Gould v. Howe, (Ill.) 23 N. E. 602.

73. Where surveys of 1837 and 1856 do not agree the former holds.

Palmer v. Montgomery, 26 N. Y. Rep. 536.

74. The boundary lines of water lots fronting on a river extend into the river at right angles with the thread of the stream, without reference to the shape of the shore.

Clark v. Campau, 19 Mich. 328.
Bay City Gas Light Co. v. Ind. Works, 28 Mich. 182.
Twogood v. Hoyt, 42 Mich. 609.
Norris v. Hill, 1 Mich. 202.

75. Where a certain distance is called for from a *given point* on a navigable stream to another point on the stream to be ascertained by measurement, such measurement must be made by its meanders, and not in a straight

line. The same rule prevails when distance is called for along a traveled highway. A different rule is sometimes adopted when the stream is not navigable.

When a *tract of land* is bounded upon a navigable stream, the distance upon the stream will be ascertained, in the absence of other controlling facts, by measuring in a straight line from the opposite boundaries.

People v. Henderson, 40 Cal. 29.

76. In computing the number of acres in a survey, "from," "to," and "with" the bank of a stream mean to low-water mark.

Lamb v. Ricketts, 11 Ohio 311.

1. **Alluvium** means an addition to riparian land gradually and imperceptibly made through causes either natural or artificial by the water to which the land is contiguous. It matters not whether the addition be on streams which overflow their banks, or on those which do not. In each case it is alluvium.

County of St. Clair v. Livingston, 23 Wall. (U. S.) 46.

2. Land formed by alluvium in a river is in general to be divided among the several riparian owners entitled to it, according to the following rule: Measure the whole extent of their ancient line on the river, and ascertain how many feet each proprietor owned on this line. Divide the newly formed river line into an equal number of parts, and appropriate to each owner as many of these parts as he owned feet on the old line; and then draw lines from the points at which the proprietors respectively bounded on the old, to the points thus determined as points of division on the newly formed shore.

This rule is to be modified under particular circumstances; for instance, if the ancient margin has deep indentations or sharp projections, the general available line of the river ought to be taken, and not the actual length of the margin as thus changed by the indentations or projections.

Deerfield v. Arms, 17 Pick. Mass. 41.
Jones *et al.* v. Johnston, 18 How. (U. S.) 100.

3. Under Rev. Stat. U. S. § 2396. *Held*, that in surveying a lot bordering on a river the water-course becomes the boundary, and continues so, no matter how much it shifts by accretion, and conveyances of the lot pass all, including such accretion to that line.

East Omaha Land Co. v. Jeffries, 40 F. 386.

4. The facts that rapid changes in the banks of the Missouri River are constantly going on, and that 40 acres have been added to adjoining land, do not overthrow an averment of a bill to quiet title to such addition, on the ground of accretion, that it was by an imperceptible increase, where it was nearly 20 years in forming.

East Omaha Land Co. v. Jeffries, 40 F. 386.

5. The rule that owners of land bounded by streams are entitled to additions to their land formed by accretion is applicable to the Missouri river, notwithstanding the peculiar character of that stream, and of the soil through which it flows, whereby changes in its banks are great and rapid.

Jeffries v. East Omaha Land Co., 10 S. Ct. 518.

6. Where the official plat of the survey of government lands shows a river as one boundary of a certain lot, in accordance with Rev. St. U. S. § 2395, et seq., a subsequent patent for the lot, describing it by number, and referring to the plat, on which it is marked as containing a certain amount, and deeds, describing the lot by number, pass all accretion to the lot up to their respective dates.

Jeffries v. East Omaha Land Co., 10 S. Ct. 518.

5. Rules applicable to the United States Surveys.—"All the corners marked in the surveys returned by the surveyor-general *shall be established as the proper corners* of the sections or subdivisions of sections which they were intended to designate."

"The boundary lines actually run and marked in the surveys returned by the surveyor-general *shall be established as the proper boundary lines* of the sections or subdivisions for which they were intended; and the length of such lines as returned *shall be held and considered as the true length* thereof."

The preceding quotation from section 2396 of the Revised Statutes of the United States, settles all questions in regard to any change in the corners, lines or measures of the government survey. They are thereby made unchangeable, the statute thus emphasizing the common law, which holds the same doctrine to be true of all original surveys after the land has been conveyed in accordance with them. Hence, in making resurveys, the surveyor must find, if possible, the original corners, and make his courses and distances agree with those of the United States survey.

The following points have been decided by the courts with reference to these surveys :

RULE 1.—The original surveys by which the government sold its land and conveyed it to the purchaser establish the rights of the parties as to the boundaries No line which will vary the rights thus acquired can afterwards be established without the consent of all parties.

May *v.* Baskins, 12 S. and M. (Miss.) 428.

2. Land sold under the United States surveys pass according to the description of the legal subdivisions, whether those subdivisions contain the legal quantity or not, more or less.

Fulton *v.* Doe, 6 Miss. 751.

3. Each section or a subdivision of a section is independent of any other section in the township and must be governed by its marked and established boundaries. Should they be obliterated, a last recource must be had to the best evidence that can be obtained showing their former situation and place.

Lewen *v.* Smith, 7 Port (Ala.) 428.

4. Field notes must yield to actual monuments erected by the original surveyor. They are only to be relied on as evidence to assist in finding the exact situation of the monuments.

McClintock *v.* Rogers, 11 Ill. 279.

5 Monuments found at the two extremes of a township line are entitled to no more controlling influence in determining the actual location of an intermediate line than the section corners established along the line. All original monuments established in connection with the field notes and plats must be referred to in order to define the locality of the line.

McClintock *v.* Rogers, 11 Ill. 279.

6. The corners established by the original surveyors of public lands by authority of the United States are conclusive as to the boundaries of sections and divisions thereof; and no error in placing them can be corrected by any survey made by individuals or a state surveyor.

Arnier *v.* Wallace, 28 Miss. 556.

In ascertaining the lost corner of a section, recourse must be had to the unobliterated marks of the original survey, the field notes and plats and subsequent surveys made under their guidance. If only a portion of one of the boundary lines leading to the lost corner on a township line has been obliterated, the remaining portion must be considered established as marked, and the corner must be presumed, in the absence of evidence to the contrary, to be at the point where the marked line if continued would intersect the township line But if the lost corner is proved to have been at another point, the lost portion of the boundary must be ascertained by running a straight line from the point where the marks disappear to that corner.

Billingley *v.* Bates, 30 Ala. 378.

7. In determining the line between the quarters of a section, the quarter post established by the government surveyors must govern in all cases where its location can be ascertained

Vroman *v.* Dewey, 23 Wis. 530.
Britton *v.* Ferry, 14 Mich. 53.

8. In re-establishing a lost quarter post on a section line, any difference in the length of such line by actual

measure as compared with that indicated by the government survey should be divided between the parts in proportion to their respective lengths as shown by that survey.

Jones *v.* Kimble, 19 Wis. 429.

9. If the distance between recognized government corners as originally established overruns or underruns that given in the field notes, it should be divided pro rata between the intervening sections. The original field notes should be the main guide. Section lines being frequently deflected, the true corners must be tested by east and west distances from the recognized government corners yet standing in the same township as well as by north and south distances.

Martz *v.* Williams, 67 Ill. 306.

10. Unknown corners must be found by the corroborative testimony of all known corners with as little departure as may be from the system adopted on the original survey, without giving preponderance to the testimony of any one monument above another.

In re-establishing lost corners between remote corners of the same survey, when the whole length of the line is found to vary from the length called for; we are not permitted to presume that the variance arose from the defective survey of any part, but must conclude in the absence of circumstances showing the contrary that it arose from the imperfect measurement of the whole line, and distribute such variance between the several subdivisions of the whole line in proportion to their respective lengths.

Moreland *v.* Page, 2 Clarkes, Iowa, 139.

11. Quarter posts of the government survey are to be as much respected as the corners of townships or sections however distant from the center line.

Campbell *v.* Clark, 8 Mo. 558.

12. There was a mistake in the government survey of a section by which the quarter section line and the meander

line of a river were shown on the official plat to be one and the same line, being the boundary line of the fractional lots. As a matter of fact they were a considerable distance apart. There was no question as to the location of the quarter section corners. In a suit to determine the ownership of the land between the quarter section line and the river, it was held that the quarter section line should be adhered to as the more certain call, and that where the lines of a survey can be run from well ascertained and established monuments, they are to control and govern a description delineated on a plat, although the quantity in the fraction fell short of the amount laid down in the plat about as much as there was land contained between the quarter line and the river.

Martin *v.* Carlin, 19 Wis. 454.

13. When a deed designates the land conveyed as one of the subdivisions known in the United States survey, as, for instance, a quarter, half-quarter or quarter-quarter section, the presumption is that the parties intend that the tract shall be ascertained in the same manner as is done in the government surveys.

Not so, where the deed conveys a tract of land not known in that system of surveys, as, for instance, the east half of a lot, or of a quarter-quarter section.

Cogan *v.* Cook, 22 Minn. 142.

14. The defendant sold the north half of a lot which is bounded on the west side by the Au Gres river. But the river is not straight at this point, and the north line of the lot is longer than the south line.

The bill demands the north half of the lot, and the north half must mean the north half in quantity divided from the remainder by an east and west line.

Au Gres Boom Co. *v.* Whitney, 26 Mich. 44.

15. It is a question of fact to be determined by all the surrounding circumstances whether the land between the

meander line and the shore of the lake or water course is included in the survey.

Shoemaker *v.* Hatch, 13 Nev. 267.

16. The lines run to divide sections into halves and quarters, if erroneous, may be corrected, for they are subdivided by law; and if the officer in running the subdivision line makes a mistake, it can be corrected by running the line according to law.

Nolin *v.* Palmer, 21 Ala. 66.

17. An original township was divided into sections "by running through the same, each way, parallel lines at the end of every two miles, and making a corner at the end of every mile," and afterward a supplemental survey was made under a subsequent statute, which directed that these two mile blocks should be subdivided by running straight lines from the corners thus marked to the opposite corresponding corners. Held, that where the original mile corners in a certain block can be clearly identified, the courses of lines of subdivision within the block cannot be determined by proof of monuments, blazes, or other witness marks found in other blocks in the township.

Ginn *v.* Brandon, 29 Ohio St. 656.

18. When a navigable stream intervenes in running the lines of a section, the surveyor stops at that point, and does not continue across the river. The fraction thus made is complete, and its contents can be ascertained.

Therefore, when there is a discrepancy between the *corners* of the *section* as established by the United States, and the *lines* as run and marked, the latter do not yield to the former.

Lewen *v.* Smith, 7 Port. (Ala.) 428.

19. In government surveys, the line actually run by the government surveyors is the true line.

Goodman *v.* Myrick, 5 Oregon, 65.

20. In a case where the township lines had been run and marked by the United States survey, but the field

notes of the subdivision lines were fraudulent and rejected by the surveyor-general, because incorrect, no proper survey of them having been made, it was held that the line between sections *one* and *two* must be ascertained by running a straight line from the corner of the sections established on the exterior line of the township to the corresponding corner on the opposite side of the township.

Hamil *v.* Carr, 21 Ohio St. 258.

21. Where the initial point in the description of premises in a deed is the southeast corner of the north half of the southeast quarter, fractional, of a section, and the quarter-section is made fractional by a meandered lake so situated as to cover the eastern and central portions thereof; and the parcel described was carved out of the north half within a year after the same was patented, the southeast corner in question is construed to be the point which constituted the southeast corner of the land as it was surveyed out and platted by the government, which located it on the meandered line of the lake. The fact that the waters of the lake have since receded cannot change the boundaries as previously located.

Verplanck *v.* Hall, 27 Mich. 79.

22. *Extending fractional lots beyond quarter lines:* Etheridge and Stone were the original settlers, pre-emptors, and purchasers of fractional section 22. Etheridge's patent called for "the S. W. ¼ of Sec. 22, containing 92.67 acres." Stone's patent called for "S. E. subdiv. Qr. Sec. 22, containing 110.50 acres." These two descriptions were in controversy in

Brown's lessees *v.* Clements, 3d How. 650.

In the figure (page 287) the full lines show the fractional section as it was returned on the official plat. The dotted lines show the quarter lines as they would have been if the section had been full.

On the part of the grantees of Etheridge two claims were set up. One was that under the pre-emption laws Etheridge was entitled to a full quarter section of land. The other was that, as his deed called for the S. W. ¼ and the fractional section was of such size and shape that a regular southwest quarter could be laid out from it, he was entitled to it, and that the action of the Surveyor General in returning irregular subdivisions of the section, when he could have made one regular quarter section out of it, was contrary to law, and therefore void. The Supreme Court by a bare majority upheld these claims and decided the case on those grounds.

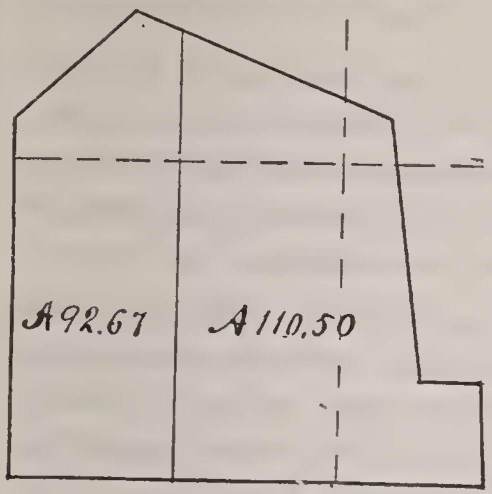

FIG. 71

The case of Brown's lessees *v.* Clements was decided in 1845, several of the judges strongly dissenting from the decision. In 1858 the same tract of land came in question again.

Gazzam *v.* Phillips' lessee and others, 20th Howard 372.

Speaking of the sales to Stone and Etheridge, the Court says:

"The sales in each case were made in conformity with the plat of the survey then on file in his office," etc.

"We deny altogether the right of the court in this action to go beyond these terms thus explicit and specific and under a supposed equity in favor of Etheridge, arising out of the pre-emption laws, to the whole of the southwest quarter—enlarge the description in the grant, or more accurately speaking, determine the tract and quantity of the land granted by this supposed equity instead of by the description of the patent.

"We are not satisfied that there was any want of power in the surveyor general in making subdivisions of this section according to the plat and in conformity with which the sales of the lands in dispute were made.

"The Act of 1820 provides that fractional sections containing 160 acres and upwards shall in like manner, as nearly as practicable, be subdivided into half quarter sections under such rules and regulations as may be prescribed by the secretary of the treasury.

"The secretary of the treasury, on the 10th of June following the passage of the act, issued regulations through the commissioner of the land office, directing fractional sections containing more than 160 acres to be divided by north and south or east and west lines, so as to preserve the most compact and convenient form. This section was divided by a north and south line according to these instructions. The question came before the secretary of the treasury and before us in 1837, and the construction first given and the practice of the surveyor general under it confirmed. Attorney General Butler in a well considered opinion observed: 'If congress had intended that fractional sections should at all events be divided into half quarter sections when their shape permitted the formation of such a subdivision, I think they would have said so in explicit terms, and that the discretionary power entrusted to the secretary would have been plainly confined to the residuary parts of the section. And further that the clause in the first section of the act of 1820, concerning fractional sections containing less than 160 acres (which are not to be divided at all) is decisive to show that congress * * did not deem it indispensable that regular half quarter sections should in all practicable cases be formed by the surveyors. On the contrary, it shows that they preferred a single tract though containing more than 80 acres to small inconvenient fractions.'"

The court adds: "We entirely concur in this construction of the act," and further goes on to say: "The only difficulty we have had in this case arises from the circumstance that a different opinion was expressed by a majority of this court in the case of Brown's lessees *v.* Clement, 3 How. 650.

"It is possible some rights may be disturbed by refusing to follow the opinion expressed in that case, but we are satisfied that far less inconvenience will result from this dissent than by adhering to a principle which we think unsound and which in its practical operation will unsettle the surveys and subdivisions of fractional sections of the public land running through a period of some 38 years. We cannot adopt that decision or apply its principles in rendering the judgment in this case."

10. *Quarter posts on section lines where there are double sets of section corners:* "Quarter section corners are not required to be established on the *west* boundary of the western tier of sections in a township, nor on the north boundary of the north tier of sections in a township south of and bordering on a standard parallel. The resurvey of *township*, standard, or base lines, by the deputy surveyor for the purpose of establishing such quarter-posts, is unnecessary and will not be paid for."

Instructions to surveyors-general by Commissioner Edmunds, p. 9.

11. "Range lines are run north or south from the base line, and corners for sections and quarter sections are established thereon at every mile and half mile for the sections and quarter sections on the *west* side of the line, *but not for those on the east side.*" On township lines "the corners of sections and quarter sections are established at every 80 and 40 chains for the sections and quarter sections on the *north* side of the line, *but not for those on the south side.*"

Instructions to Deputy Surveyors of the United States for the district of Illinois and Missouri, 1856, p. 50.

6. Decisions of the General Land Office with reference to Mineral Surveys.—*Plats and field notes:* Of surveys of mining claims, required to disclose all conflicts with prior surveys, giving areas of all conflicts.

In future, surveyor-general will use no coloring on plats.

Com'r. (N.) Nov. 16, 1882. Circular.

Location (of mine): Must be marked on the ground so that its boundaries can be readily traced.

N. Noonday M'g Co. *v.* Orient M'g Co., 6 Saw., C. C., 299; Myers *et al.* *v.* Spooner *et al.*, 55 Cal. R. 257; Gleason *v.* N. White M'g Co., 13 Nev. R., 443; Southern Cross G. and S. M'g Co. *v.* Europa M'g Co., 15 *id.*, 383.

Surface line: Agreement by adjoining claimants, fixing surface boundary line between them, must be construed as extending such line downward, through the dips of the vein or lode, to the earth's centre.

Richmond M'g Co. *v.* Eureka M'g Co., 103 S. C., 389.

Bearings and distances must be given in a survey, from the respective survey corners to the location corners, and the same must be shown on the plat.

Survey: Of a mining claim should show location of all improvements of a municipal nature, as blocks, alleys, etc.

Sec'y Dec. 18, 1880, and Feb. 3, 1881. Little Nettie Lode.

7. Descriptions in Deeds. — Surveyors are frequently required to make surveys for the purpose of furnishing a description of the land to be conveyed. Every surveyor of experience is familiar with the many difficulties encountered in correctly locating boundary lines, caused by defective, false or impossible descriptions in the deeds. The description is the controlling guide to the surveyor in locating a man's possessions on the ground, hence it is important that it should be clear, distinct and harmonious in its terms.

Where land is conveyed in the regular subdivisions of the United States survey, little difficulty will be met in writing a correct description. The main caution to be observed is to avoid the common clerical error of using the wrong letter or word, such as north instead of south, or east instead of west, thereby locating the deed in a different place from which it was intended. Scrutinize the description closely to see that no such error is made, and write plainly, so that no one need make a mistake in reading or copying the description. A great many of these mistakes are caused by bad penmanship.

Similar remarks apply to the description of land by plat, where only clerical errors are likely to be made.

It is in the description "by metes and bounds" and by courses and distances, that greatest care should be taken.

Do not use two descriptions if one will clearly describe the land. Avoid surplusage and conflicting descriptions. If after writing a description it is found necessary to explain it, lay it aside and if possible write a description that does not need explanation.

Let the starting point be well defined and permanent, so that there need be no difficulty in locating it at any time in the future. A striking example of a disregard of this principle was brought to the attention of the writer when he was called to locate the boundary lines of several lots in a village. The descriptions all referred back to a small cherry tree as a starting point. The lines had never been marked on the ground even by fences, and the cherry tree had been gone so long that no one could be found who could remember that there ever was such a tree.

Not only the starting point but as many of the angles in the boundary as possible should be described by something permanent and definite on the ground. This is of prime importance. Let it be the plainest and most permanent that the nature of the case permits.

If the courses are given by compass bearings, state whether they refer to the magnetic or some other meridian. This is put in the form of a statement of the declination of the needle, written for example, *Var.* 4° 20′ *E.* By this it is understood that the magnetic meridian makes an angle of 4° 20′ to the east of the meridian of the survey. It was formerly a custom to refer all lines to the magnetic meridian. Since the adoption of the system of the United States Land Surveys it has become a custom, especially in that part of the country surveyed under that system, to refer all surveys to the true meridian, or what was supposed to be so. As time has passed and old descriptions have been retained in the deeds conveying the land from owner to owner, it has become impossible in thousands of cases to tell what meridian controls the description. Hence we see the prime importance of permanent monuments describing the boundaries, and of describing the meridian of the survey. If we must needs figure out courses from the change in direction of the needle, let us have something definite to start from.

Do not describe a boundary solely by reference to the boundary of the adjoining tract, if it can be avoided without error. Such a description requires the finding of the description of the adjoining tract whenever a survey is made, and may cause great delay and trouble before the correct definite description can be found. The writer knows of a case where the only description of the boundary line between two village lots in either deed is by a reference to the other: A.'s land is bounded on the east by B.'s land, and B.'s land is bounded on the west by A.'s land—nothing more.

If a boundary line is not intended to be a straight line, but to follow a fence, a wall, a hedge or a stream, say so in the description. Make everything clear, definite, concise and consistent throughout, so that a surveyor having the description in the deed can locate the boundaries on the ground, without having to hunt up descriptions from other deeds.

8. Illustrations.—1. "*The east half of the northeast quarter of Section 16, Township 2 south, Range 10 west.*"

The United States land department in selling land in regular subdivisions of non-fractional sections does not state the quantity in the patent. It is quite customary in later conveyances to add something like the following: "containing 80 acres, more or less, according to the United States survey." Nothing is gained by the addition. There is a good deal of useless verbiage and repetition in deeds, the only effect of which is to add to the expense of making out and recording them.

2. "*The north fractional half of the northeast fractional quarter of Section 3, Township 3 south, Range 9 west, containing 98.72 acres, according to the official plat of the United States Survey.*"

The area of fractional lots is stated in the United States patents. The word fractional is used and the area given to show that the land is conveyed according to the system of the United States survey. Without them the description would convey the aliquot part of the entire area of the section in the same manner as Description No. 1.

3. "*The south fraction of the southeast quarter of Section 28, Township 6 north, Range 3 west, containing 117.85 acres.*"

Sections are made fractional by streams, lakes and reservations, making fractional lots of all manner of sizes and shapes. The land department attaches small outlying fractions to the adjacent larger ones, and sells the whole under one description, which takes its name from the larger lot. The above description might contain land attached from the southwest quarter. Such descriptions do sometimes contain land attached from other sections, and even from other townships. The official plat of the section shows precisely what land is included in the description.

4. "*A piece of land twenty feet wide off from the east side of Lot 99 of the lithographed plat of the village of Kalamazoo.*"

A description like the above sometimes leads to controversy. Suppose the original survey by which the lots were laid out, was made with a long chain, as it was in Kalamazoo, and that there was a surplus in the lot. The purchaser might claim that he was entitled under the common law to his proportional share of the surplus, while the seller, if he owned the balance of the lot, might claim it all as his own. Such questions do frequently arise, and it is better to settle them at the outset, by putting it definitely in the description what is meant. In the above case suppose the recorded width of the lot to be sixty feet; then a description calling for the "east one-third of Lot 99" would show clearly that any surplus or shortage in the lot was to be divided, while a description reading "20 feet off the east side of Lot 99, etc., as surveyed by F. H., May 22nd, 1883," would show that the later surveyor's measure was to govern. The care and accuracy of measurement of land in cities keeps pace with its increase in value, and as a careful, accurate measure cannot be expected to agree with a careless, inaccurate one, it is best to settle such questions in advance, as far as possible.

5. "*Commencing at a stone with a hole drilled in it, set in the east and west quarter line of Section 18, Township 4 south, Range 10 west, 22 chains east of the range line, from which stone a*

White oak 16 inches diameter, bears S. 28° W., 62 links distant, and running thence (Var. 2° 40′ E., at 10 A. M., June 12th, 1880), north 22° east 12.00 chains to a stone marked with a cross, set in an angle of a hedge;

Thence east along the hedge 8.00 chains to an iron stake of 1½ inch gas pipe, driven on west bank of a ditch;

Thence south along the bank of the ditch 5.00 chains to an iron stake of gas pipe driven in the bank where the ditch turns east;

Thence south 22° west 6.61 chains to a stake set in the quarter line, from which a

Burr Oak 12 in. di. bears N. 16° E., 26 lks. distant,

Burr Oak 18 in. di. bears S. 46° E., 51 lks. distant;

Thence west along the quarter line 10.21 chains to the place of beginning."

This is given as a sample of a description by metes and bounds such as a surveyor may furnish under the ordinary circumstances when called on to make a survey for that purpose, and such as he or any other surveyor would have no trouble in locating on the ground at any future time so long as any of the monuments or bearing trees could be found.

CHAPTER XI.

RE-LOCATION OF LOST CORNERS.

The general principles to be observed in re-locating lost corners are laid down in the Supreme Court decisions which have already been quoted.

A corner is not lost so long as its position can be determined by evidence of any kind without resorting to surveys from distant corners of the same or other surveys. Often after making a survey from a distant corner, the surveyor will come upon some traces or evidence which will enable him to determine the true position of the corner he is seeking. It is an uncertain way at the best to locate corners by running lines and measuring from distant corners, and should only be resorted to in absence of better proof of the original location of the corner sought.

It will sometimes happen that the exact spot where a lost corner stood cannot be found or shown by evidence, but it can be proved that it stood within certain limits. In these cases, which are not rare, there is no question but that the corner should be placed at that point within the known limits which best agrees with all the evidence in the case.

Failing of better evidence by which to determine the location of a lost corner, we may next resort to the following methods:

General Rule.—Retrace the known lines of the description and find how the lengths and directions of these lines by your survey agree with those of the same lines as laid down in the original description. Then run the unknown lines and place the lost corners so that they will bear the same relation to the known lines and corners as they are required to do by the description of the original survey.

Example.—The four lines of a description are as follows:

1. North 7° east 12.00 chains.
2. South 83° east 6.00 "
3. South 7° west 12.00 "
4. North 83° west 6.00 "

The first line and its termini are known. We retrace that line and find by our survey that it runs north 7° 30′ east and 12.24 chains.

We would then run the remaining lines, making them as follows:

2. South 82° 30′ east 6.12 chains.
3. South 7° 30′ west 12.24 "
4. North 82° 30′ west 6.12 "

Or the compass may be set on the known line and the vernier so adjusted that the reading of the needle shall be the same as that given in the original description and the remaining lines run accordingly.

2. Re-location of Lost Corners of the United States Survey.

RULE 1.—On *base lines, correction parallels, township* and *range lines.* Restore the lost corner in line between the nearest known corners on the same line and at distances from them proportional to those laid down in the field notes of the government survey.

This rule supposes the original line to have been a straight line. As a matter of fact this is frequently not the case. If there is reason to suspect the line to have angles in its course, measures from known corners to the right and left of the line will aid in determining its true position.

RULE 2.—**Lost closing section corners** upon a township or range line, where the closing distance from the adjacent corners is not given in the field notes should be restored by prolonging the known portion of the line to its intersection with the township or range line.

RULE 3. **Lost interior section corners** should be restored at distances from the nearest known corners, north, south, east and west, proportional to those laid down in the field notes of the original survey.

This rule supposes that the measurements of the original survey were uniform on the several adjacent sections. This is frequently not the case, and it will be well for the surveyor to compare his chaining on each section with the original measure between known corners of the same sections, choosing by preference those lines which on the government survey were measured next previous to the portion of the line closing on the lost corner.

RULE 4.—**Lost township corners**, when common to four townships, are to be restored in a similar manner to interior section corners, Rule 3. When common to only two townships, they are to be restored according to Rule 1.

RULE 5.—**Lost quarter section corners** are to be restored in line between the section corners which stand on the same line and at distances between them proportional to those returned in the field notes of the government survey.

RULE 6.—**Lost meander corners** are to be restored by running the line from the nearest known corner the direction and distance called for by the notes of the original survey. When a portion of the line leading to the meander corner is known, it should be prolonged in the same direction. When no portion of the line is known' the surveyor will have to use his own judgment as to what method under the circumstances of the case will most nearly retrace the original line to the corner.

There is no rule which will rigidly and inflexibly apply to all cases for restoring lost corners and boundary lines except this—that the aim of the surveyor should always be to find the exact spot where the original corner or line was located. *The thing to find out is not where the corner or line ought to have been, but where it actually was.*

There are many cases in which other methods for restoring any of the corners mentioned will prove more satisfactory than the rules heretofore given.

For instance, a half-quarter post properly planted at a time when both the section and quarter-section corners adjacent were known, may be used in restoring either of these corners when lost, by prolonging the line over the known corners and doubling the distance. Any other intermediate corner whose location is definitely known may be used in a similar manner. On a similar principle, the Supreme Court of Illinois decided in the case of Noble *v*, Chrisman (88 Ill. 186) that the northwest corner of section 19 could, in that instance, be better determined by tracing the section lines from known corners east and west of the range line to their intersection with that line, and measuring the jog between the corners, than it could by prorating six miles of the range line.

Most of the difficulties which the surveyor has to contend with in restoring lost corners arise from errors made in the original survey, or in the field notes thereof. He should bear in mind that errors in the original survey cannot be corrected by him. In any case of a lost corner, find as many of the adjacent corners of the original survey as possible, according to the best evidence that can be had to prove their exact location. Having done this, the others may be found according to the rules already laid down. *But do not give up a corner as lost while any means of finding its exact location are left untried.* There is great virtue in a pick and shovel intelligently applied to the finding of corner posts and monuments. This is very important, as it is very difficult, if not impossible, in many cases, to re-locate a lost corner in the exact position it originally occupied, by surveys from distant corners. The following extracts from a paper read by the author before the Michigan Association of Surveyors and Engineers, treat more fully of the application of the foregoing principles to finding corners of the United States survey in those regions where wooden posts were planted for corner monuments:

"It often happens that one surveyor will fail utterly in finding the marks of an original corner, while another, more apt in discovering the evidences, will strike upon it readily. These evidences are of various kinds, some of which it is the principal aim of this paper to discuss.

I take it that the best possible evidence of the location of an original corner is the monument fixed at that corner when the survey was made. (*Vide* McClintock *v.* Rogers, 11 Ill. 279; also Gratz *v.* Hoover, 16 Penn. State Rep. 232; 16 Ga. 141.) After this come witness trees, fences, distant corners of the same survey, and the testimony of persons.

All these latter kinds of evidence only go to corroborate the first, and may take the place of the first only so far as they may any of them seem to have weight in any particular case.

Many of the corners of the United States survey were marked by planting a post or stake in the ground. These stakes had notches cut in them, were squared at the top, and set in certain regular positions in the ground. These marks tended to distinguish them from other stakes that might chance to be driven in the ground for any purpose. When trees stood conveniently near, two of them were marked, and their directions and distances from the corner were given in the field notes. When no trees were near, a mound was sometimes raised about the post.

Some of the posts have been entirely destroyed, but the bottoms of a great many of them still remain, much decayed, but plainly visible when the surface earth is removed from about them.

To find them, careful manipulation is required. The surveyor first determines as nearly as he can, from extrinsic evidence, the point where the corner post should be looked for. He then, with a shovel, spade or hoe, carefully removes the surface earth, a little at a time, being particular not to strike deep at first into the earth at the level as it was when the stake was set. The best and sometimes the sole evidence of a corner has often been destroyed by an ignorant person striking deep into the ground, expecting to find a sound stake, and casting away the decayed wood and filling up the hole of a rotten one without observing it. If the surveyor is looking in the right place, and the earth has not been previously removed, he will soon come upon the object of his search; but he must be careful lest he mistake it. If the soil is a stiff clay, packed hard, as in a road, or covered with a sward, he will presently find a hole of the size and shape of the stake which made it. This hole will contain the decayed wood of the stake, and a marking pin may be readily thrust to the bottom. By carefully scraping or cutting away the earth from the top, or cutting down at one side of the hole, its size, shape and direction may be readily discovered. Thus it often happens that the position of a corner is as well and satisfactorily marked by the decayed stake as it was by the sound one. It sometimes happens that new stakes have been driven beside the original stake, so that several different ones will be found by the surveyor. He will seldom have any difficulty in deciding which is the true corner by its appearance, for the first stake will be more completely decayed and of a darker color.

As a rule, it will be driven deeper and straighter down than the newer stakes. Then, too, the original stakes were generally round, being cut from whole timber, while the later ones were often cut from rails or other split timber, the sharp corners of which can be readily seen in the holes made by them.

There is thus in the appearance of the stakes of the United States survey such peculiarities and such likeness to each other, even when far gone in decay, that the experienced surveyor will be impressed with the appearance of truthfulness pervading them, and will seldom be deceived. This appearance of truthfulness about a stake, which to a surveyor is one of the most valuable parts of the testimony of these silent witnesses, is something that courts and juries can seldom take cognizance of, because, *first*, they speak in a language that courts and juries do not understand, and *secondly*, the evidence is itself destroyed by the surveyor in the taking, and does not come before court or jury in all its freshness, truth and purity. These decayed stakes may be best observed in the light-colored subsoil after the black surface mould has been removed. In sandy soil, the cavity made by the stake is gradually filled by the falling sand as the wood decays, but rotten wood discolors the sand so that where it has not been disturbed the position, size and shape of the stake may be readily traced. In the black muck of our marshes and river bottoms it is more difficult to distinguish the stake near the surface, but as the ground is soft and wet the stakes were driven deep, and we may sometimes find in the wet, peaty subsoil the bottom of the stake so perfectly preserved that even the scratches made in the wood by nicks in the axe are plainly to be seen. When the stakes are constantly wet, they do not decay.

Next we consider the bearing or witness trees. These are marked and their directions and distances noted, in order to assist in finding the corner posts set on the survey. These bearing trees are marked with a blaze and a notch near the ground on the side facing the corner. The measures were taken from this notch. At this time most of the living witness trees have grown to such an extent that only a scar remains in sight, to indicate the point where the notch was cut. In order to get at the notch, the superincumbent wood, which is in some cases a foot in thickness, will have to be cut away. It will not often be necessary to do this, as we can come sufficiently near the correct point to find the stake without it. But if the stake has been destroyed, or there are several stakes near, we shall need to be exact, and measure from the notch. If the tree has been cut down, and a sound stump remains, the marks will be easily exposed. Sometimes the mark is gone, but a part of the stump is left. At others the stump is gone, but a dish-like cavity remains in the earth to show where the tree once stood. We can almost always find under and around these cavities places where the large roots have penetrated the subsoil, and thus be able to locate within a foot or so the position of the bole of the tree when standing. In looking for a corner post, we may frequently assume for the time being that a certain stump or a cavity where a tree had stood was the stump of or the place occupied by a bearing tree. If we then measure the required direction and distance, and find a stake, we may reasonably conclude that our assumption was correct. Such assumptions are frequently of great assistance in finding corners. There may be, and I know there are cases, where the original corner stakes have been destroyed, and can be more nearly restored to their original position by measurements from old stump bottoms or holes in the ground than in any other way. But bearing trees, however good their condition, are by no means infalible witnesses as to the location of a corner. Mistakes in laying down their direction or distance, or both, are not rare. (See McClintock *v.* Rogers, 11 Ills, 279.) A direction may be given as north instead of south, east instead of west, or *vice versa.* The limb may have been wrongly read 64° for 56°. The figures denoting the bearing may have been transposed in setting down, as 53 for 35. So, too, the chain may have been wrongly read, as 48 for 52, the links having been counted from the wrong end. Or they may have counted from the wrong tag, as 48 for 38. Mistakes of the nature of these mentioned are common, so that in working from a bearing tree to find a corner, and not finding the stake at the place indicated in the notes, it will be well to test all these sources of error before giving up the search, for as I have said before, the *post planted at the time of the original survey is the best evidence* of the corner it was intended to indicate.

I next consider fences in their relations to corners. (Potts *v* Everhart, 26 Penn. St. Rep., 493.) Whether any particular fence may be depended on to indicate the true line will depend on the particular circumstances attending that case. In a general and rough way, a fence will indicate to the surveyor where to begin looking for his corner. But the practice has been, and still is common, for the first settlers on a section to clear and fence beyond the line in order to have a clear place on which to set their permanent fence when they get ready to build it. Afterward they forget where the line is and set the new fence where the old one stood. Many fences, too, were set without any survey or any accurate knowledge where the line was and left there to await a convenient time to have the line established. So, too, where the land has been long settled and occupied, it is a common custom for adjoining land owners by consent to set the fence on one side of the true line, there to remain until they are ready to rebuild, the one party to have the use of the land for that time in consideration of clearing out and subduing the old fence row. The original parties frequently sell out or die, and the new owners have no knowledge of the agreement and suppose the fence to be on the true line. For these reasons, fences should be looked on with suspicion, unless corroborated by other evidence, and the surveyor should enquire pretty closely into the history of a fence before placing any great reliance on it to determine the position of a corner. It may be the best of evidence, or it may be utterly worthless.

It not unfrequently happens that there are no trustworthy marks near a corner to direct the surveyor in his search for the post or from which to replace it if it be destroyed. In these cases, he must visit the nearest corners he can find in each direction (varying with the circumstances whether it be section corner or quarter post he wishes to find or restore), go through the process of identification with each of them, and then make his point so that it will bear the same relation to these corners as did the original corner post. Many very intelligent gentlemen suppose that if the surveyor can but find one of the corners of the original United States survey he can readily determine the position of all the rest from it. They were never more mistaken in their lives. The continual change in the direction of the magnetic needle, the uncertainty as to what its direction was when any particular line was run, the difference in the lengths of chains, and the difference in the men who use them, introduce so many elements of uncertainty into the operation as to render it one of little value, and not to be resorted to except in the absence of trustworthy evidence nearer at hand.

If it be a section corner you desire to find or replace, and have adjacent quarter posts in each direction to work from, you will not be likely on the one hand to fall more than a rod or two out of the way, and on the other hand will not be likely to come within a foot or two of the right place. This method will assist you in seaching for the original stake, and if that be destroyed, and no better evidence presents itself, may be used to determine the point where the corner stake shall be placed. The chief difficulty in applying this method to determine corners arises from the fact that the measurements made on the original surveys were not uniform in length on different sections, and frequently not on different parts of the same section. I have measured sections 22 and 23 on a level prairie, along the line of highways, where no obstacles of any kind interfered to prevent accurate work. I took the greatest possible care in the chaining to have it as accurate as chain work can be done. On the north line of section 22 my chaining tallied exactly with that of the United States survey, viz., 79.60. On the north line of section 23, my measure was 80.96, that of the United States survey, 80.40—a difference of 56 links. Fortunately, all the corners of the original survey on this two miles of line were well preserved, and the distance between quarter post and section corners was uniform on the same section in both sections. But suppose that a part of them had been lost, and it was required to restore the middle section corner (n. e. of 22) from the remaining ones. Omit all consideration of corners, north or south, and there remain four different solutions of the problem, depending on which corners were lost and which preserved. Of these different solutions, one would place the corner 9⅓ links, one 14 links, one 18⅔ links, and one 28 links, all east of the true corner. This is not by any means an extreme instance, as I have observed discrepancies twice as great. It is given simply to show how unreliable is the evidence drawn from distant corners of the United States survey.

Lastly, I shall consider the evidence of living persons. [Weaver *v.* Robinett, 17 Mo., 459; Chapman *v.* Twitchell, 37 Maine, 59; Dagget *v.* Wiley, 6 Florida, 482: Lewen *v.* Smith, 7 Port. (Ala), 428; McCoy *v.* Galloway, 3 Han. (Ohio), 283; and Stover *v.* Freeman, 6 Mass., 441.] Conceding all men to be equally honest in their evidence, there is a vast deal of difference among them with regard to their habits of observation and their ability to determine localities. Some have an exceedingly acute sense of locality, if we may so call it, and can determine very accurately the position of any object which they have been accustomed to see; while others seem to have little or no capacity of that sort. I have found many men who would describe accurately the sort of monument used to perpetuate a corner, and who would tell you that they could put their foot on the very spot to look for it; but when the trial came I have found but few of them who could locate the point within several feet, unless they had some object near at hand to assist the memory, and even then they would frequently fail.

It may happen where a corner post has been destroyed, that its location can be more nearly determined by the testimony of persons who were familiar with it when standing and can testify to its relations to other objects in its vicinity, than in any other way. But the surveyor in receiving this testimony should ascertain as far as possible what are the habits of accurate observation and the memory of localities possessed by the person testifying, in order to know how much weight to give his testimony."

CHAPTER XII.

MISCELLANEOUS.

1. **Questions of Practice.**—Answers to most if not all questions which arise in the surveyor's practice will be found in the Supreme Court decisions which have been quoted. The following questions which have been raised in several surveyors' associations, are given with the answers adopted in each case, or a reference to the law decision or principle which governs it.

1. An interior section has its quarter posts out of line and not at equidistant points between the section corners. How shall the centre be determined?

Ans. At the intersection of straight lines from each quarter section corner to its opposite corresponding corner.

See page 182, Sec. 100, Second.

2. How shall the quarter posts on the north and west lines of the township which were not established by the U. S. survey be located?

Ans. The corners of half and quarter sections, not marked on the surveys, shall be placed as nearly as possible equidistant from those two corners which stand on the same line.

See page 182, Sec. 100, First.

Section 6 is an exception to this rule.

See page 235.

3. Posts for lines closing on the north and west boundaries of townships are often off the boundary line to one side or the other. Shall the boundary line be deflected to pass through these posts?

Ans. No. The posts serve to show the position of the section line, but the line itself stops at the township boundary.*

Mich. Surv. Rep., 1881.

4. Are the station or line trees marked on the government surveys and returned in the field notes, monuments of the lines?

Ans. Yes.

See page 182, Sec. 100, Second.

Billingsley *v.* Bates, 30 Ala. 378.

5. How shall the east and west quarter line of section 30 be located, there having been no quarter post set on the east side of the section by the U. S. survey, because of a lake?

Ans. Locate the west quarter post as directed in the answer to question 2. Then run the quarter line east on a course which is intermediate between the courses of the north and the south lines of the section.

See page 232.

6. A closing corner on the north or west boundary of the township is lost. The field notes do not give the distance between the closing corner and the adjacent corner on the boundary. How shall it be restored?

Ans. Prolong the known portion of the line to its intersection with the boundary and there set the corner.

See Billingsley *v.* Bates, 30 Ala. 378; see p. 282.

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* Note.—The author has not met with any judicial determination of this question. Some very able surveyors hold a different view from that expressed above. But suppose that the deputy surveyor, not finding the standard corner, as frequently happens, ran his line directly over it, and planted his closing corner in the section line beyond. It would then be impossible to deflect the township line so as to pass through both corners.

It would seem to be a safe way for the surveyor, in making a survey on a section, to locate his lines with reference to the corners established for that section only; and leave any question of title, raised by overlapping or non-closing lines, to be settled by the courts.

7. Should section lines running north and south be run in a straight line between known corners to locate lost corners on interior sections?

Ans. Not unless the original lines were actually straight lines between the known points, which they seldom are.

See Moreland *v.* Page, 2 Clarkes, Iowa, 139; see p. 283;
Martz *v.* Williams, 67 Ill., 306; see p. 283.

8. How shall the half-quarter corner on the quarter line be located on those quarter sections which adjoin the north and west lines of the township?

Ans. Measure the distance from the centre of the section to the quarter post on the township line.

Then place the corner on the quarter line at a distance of twenty chains proportionate measurement from the centre of the section. In order to prorate the distance, your own measure should be compared with a distance which is a mean between the distances given in the field notes as the length of the corresponding lines of the section on either side. For example, on section 3 the distance by U. S. survey from the east ¼ post to township line is 42.18; from the west ¼ post to township line is 43.20; which gives a mean distance of 42.69.

Commissioner McFarland gives the following reply to a similar question·

DEPARTMENT OF THE INTERIOR,
GENERAL LAND OFFICE,
Washington, D. C., February 11, 1882.

Isaac Teller, Esq., Webberville, Ingham County, Michigan:

SIR—I am in receipt of your letter of the 5th instant requesting information in regard to the proper method of locating the quarter-quarter corners north of the legal centres of the northern tier of sections in a township when the present measurement of the east and west boundaries of the section differs from the original measurement.

In reply, I have to state that the length of the quarter line from the south quarter corner to the township line is to be considered as the mean of the east and west boundaries of the section as given in the field notes, and where the present measurement of the section lines differs from the original measurement, the rule of proportionate measurement applies to the quarter line as well as to the section lines in the establishment of quarter-quarter corners on the half mile closing on the township boundary. See enclosed circular dated November 1, 1879.

The mean width of the north half of the section in the case stated by you is 40.18 chains, while by your chaining it is 42.42 chains (calling the distance to the east and west quarter line 40.00 chains), therefore the proportion will be as 40.18 : 42.42 :: 20.00 : 21.11 chains, the distance north of the centre of the section at which by your chaining the quarter-quarter corner should be located.

Very respectfully,

N. C. McFARLAND, Commissioner.

9. In surveying sections fractional on the township line to restore lost quarter section corners, should the lines be divided pro rata according to the U. S. field notes, or should the south or east quarters be made full and the entire excess or deficiency be thrown into the fraction?

Ans. Any difference between your measure and the government measure must be distributed proportionally between the different parts of the section.

See p. 246, Sec. 100, Second.
Moreland *v.* Page, 2 Clarkes, Iowa 139.
Jones *v.* Kimble, 19 Wis. 429.
Martz *v.* Williams, 67 Ill. 306,

In Missouri, the Supreme Court holds (Knight *v.* Elliott, 57 Mo. 317) a different view, viz., that the difference in measure is all to be thrown into the fraction.

It is difficult to see upon what grounds this decision can be upheld in view of the fact that all rights to the land were acquired and held under the law of Congress, which expressly states that the length of such lines as returned by the surveyor-general shall be held and considered as the true length thereof.

Northwest Quarter, Sec. 18.

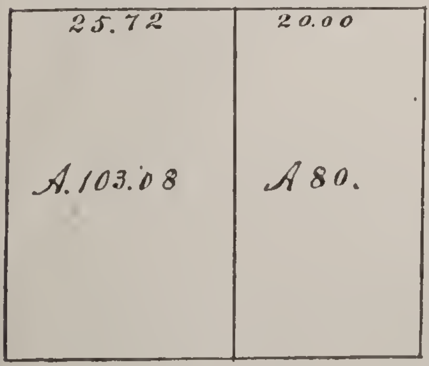

FIG. 72.

10. The accompanying figure is a copy of the plat of the U.S. survey of this quarter section.

A owns the whole quarter. He sells to B the W. $\frac{1}{2}$ of the N. W. $\frac{1}{4}$ of section 18, containing $91\frac{54}{100}$ acres. At about the same time he sells to C the E. $\frac{1}{2}$ of the N. W. $\frac{1}{4}$ of section 18, containing $91\frac{54}{100}$ acres.

Where shall the surveyor run the dividing line between B and C?

Ans. The language of the deed clearly shows the intention of A to sell and of B and C to purchase each the half of the area of the quarter section. The surveyor should so locate the line as to carry out the evident intent of the parties. See rule 2, p. 244 and rule 14, p. 284. The fact that the quarter is differently subdivided on the government plat has no bearing on this case.

11. Certain early surveyed townships had three sets of corners on the range lines. (1) Those set when the range lines were run; (2) Those set as closing corners running east; (3) Those set as closing corners running west. What use is made of each set of corners?

Ans. The first corners set determine the location of the range line. The second and third sets of corners determine the location of their respective section lines which close on and terminate at the range line.

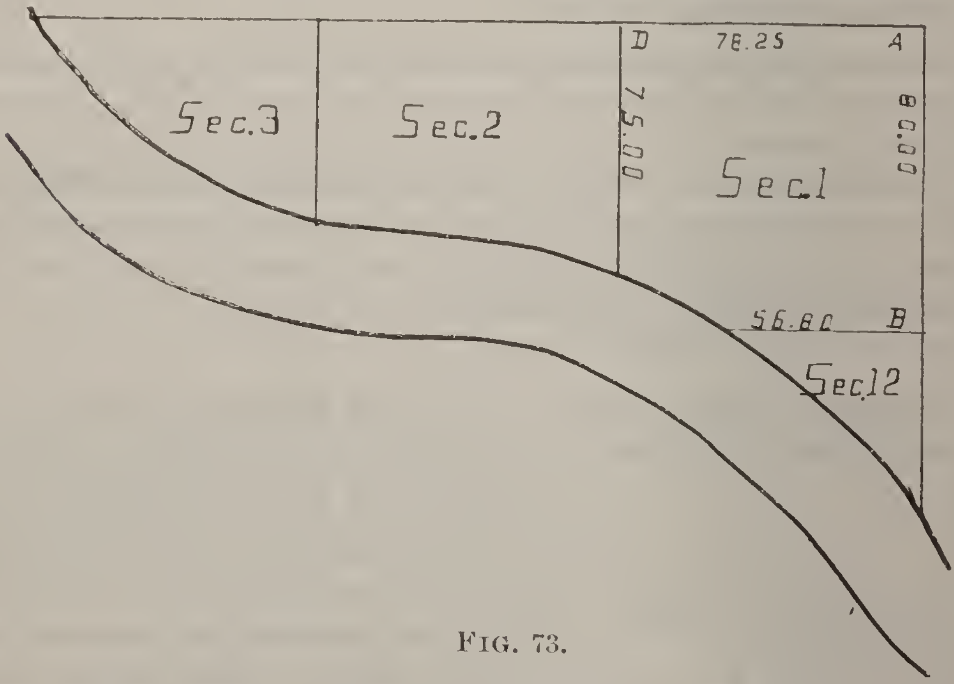

FIG. 73.

12. This figure shows a fractional township on the Ohio River. The figures show the dimensions of section 1, as shown by the field notes of the United States survey. By a subsequent measure,

$$AB = 82.25 \text{ chains, and } AD = 79.50 \text{ chains.}$$

How shall the northeast quarter of section 1 be laid off, no quarter-posts having been planted?

Ans. Place the quarter-section corners on the north and east sides of the section in line with and midway between their respective section corners. Make the east and west quarter-line parallel with the south line of the section, placing the west quarter-post at the point where the quarter-line thus run intersects the section line. From the north quarter-post run the quarter-line south on a course which is a mean between the courses of the east and the west lines of the section, placing the south quarter post at the intersection of the section and quarter-section lines.

The exceptional features of this case are that no quarter-posts were set on the United States survey, and that the east line of the section is just 80 chains in length, having been run from the north to the south.

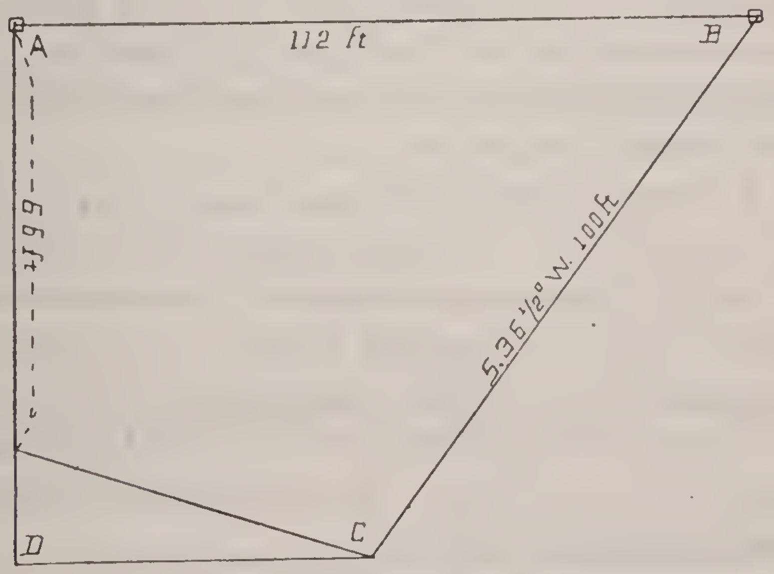

FIG. 74.

13. The description in the deed runs: "Beginning at a stone (A), at the N.W. corner of lot 401; thence east 112 ft. to a stone (B); thence S. $36\frac{1}{2}°$ W. 100 ft.; thence west parallel with AB to the west line of said lot 401; thence north on west line of said lot, 66 ft., to the place of beginning." The points A and B, and angle ABC, are fixed. C, by

construction and in fact, is $80\frac{13}{100}$ ft. distant, at right angles from the line *AB*.

1. Shall I locate *CD* parallel with *AB*, or locate *D* 66 ft. from *A*?

2. Have I any right to consider any apparent intention to locate 66 ft. or $80\frac{13}{100}$ ft. from *A*?

3. Have I, if I know it, any authority to consider the actual intention of the grantor to locate *CD*?

4. If the distance *AB* should actually measure 114 ft. am I to use it, or shall I make *B* 112 ft. from *A*?

Ans. 1. The answer to this question will depend upon the state of facts brought out in answer to questions 2 and 3. If there be evidence showing what the intention and understanding of the parties to the conveyance was as to which of the two lines should be taken, that evidence would settle the question. If not, that construction may be given to the deed which will operate most strongly against the grantor and give the grantee the greater amount of land. So far as anything is shown in the question, the deeds to the adjacent land might furnish the necessary evidence.

2 and 3. Yes. Judge Cooley says, (see "Judicial Functions of Surveyors"): "The surveyor must inquire into all the facts, giving due prominence to the acts of parties concerned, and always keeping in mind * * * * that courts and juries may be required to follow after the surveyor over the same ground, and that it is exceedingly desirable that he govern his action by the same lights and the same rules that will govern theirs."

4. The monument controls the distance.

14. A piece of land is sold, and described as commencing at the north quarter-post of section 15, and running thence east 100 rods; thence south 160 rods; thence west 100 rods; thence north 160 rods, to the place of beginning; containing 100 acres, according to the United States survey.

Ques. How shall it be set off?

Ans. The deed clearly indicates the understanding of the parties to the conveyance to be that the land should pass according to the rules that govern the United States survey. One of these rules is, that "the length of the boundary lines as returned by the surveyor-general shall be held and considered as the true length thereof." Hence in this case, measure east from the quarter-post along the section line 25 chains of just such measure as the United States surveyors gave; or in other words, of *pro rata* measurement. Suppose the distance by the field notes to be 40.32 chains from quarter-post to section corner. Then lay off $\frac{25.00}{40.32}$ of that distance. Proceed in a similar manner, running east on the quarter-line from the center of the section, and the two points thus located will be the corners of the 100 acres. To get the length of the south line of the N. E. $\frac{1}{4}$ of the section by the United States survey, take the half sum of the measure given on the north and the south lines of the section. Supposing it to be 40.32 on the north, and 40.18 on the south, then the distance on the quarter-line would be equal to

$$\frac{40.32 + 40.18}{2} = 40.25,$$

and you should measure off $\frac{25.00}{40.25}$ of this distance for the corner.

2. The Rights, Duties and Responsibilities of Surveyors.—Surveyors, by the consent and acquiescence of the parties concerned, are usually the arbiters of disputed boundaries, and their decisions, when thus acquiesced in by the parties, become in time as binding, and as much respected by the authorities, as the decisions of juries and courts of law. It is probable that at least ninety-nine per cent. of all questions of disputed boundaries are thus settled by the interested parties themselves, in accordance with the decision of the surveyor.

Surveyors, from constantly exercising this seeming authority, come at last in many cases to believe it to be absolute and final, something which must be respected, overlooking the fact that the only force their decisions have comes from the consent of the parties. When that consent is withheld, the case goes to the courts for settlement; and thus the courts have in some cases felt called upon to define the surveyor's standing before the law. They say:

1. "Surveyors have no more authority than other men to determine boundaries, of their own motion. All bounds and starting points are questions of fact to be determined by testimony. Surveyors may or may not have in certain cases means of judgment not possessed by others, but the law can not and does not make them arbiters of private rights.

Cronin *v.* Gore, 38 Mich. 381.

2. The law recognizes surveyors as useful assistants in doing the mechanical work of measurement, and calculation, and also allows such credit to their judgment as belongs to any experience which may give it value in cases where better means of information do not exist. But the determination of facts belongs exclusively to courts and juries. Where a section line or other starting point actually exists, is always a question of fact, and cannot be left to the opinion of an expert for final decision. And where, as is generally the case in an old community, boundaries have been fixed by long use and acquiescence, it would be contrary to all reason to have them interfered with on any abstract notion of science.

Stewart *v.* Carleton, 31 Mich. 273.

Gregory *v.* Knight, 50 Mich. 61.

3. New surveys disturbing old boundaries are not to be encouraged.

Toby *v.* Secor, Wisconsin. N. W. Reporter, Vol. 19, p. 79.

4. Lines long unquestioned ought not to be disturbed upon a mere disagreement among surveyors, especially when the last survey is made under the unfavorable circumstances of corner posts and witness trees being gone, which it is probable to suppose were in existence at the time of the first survey.

Case *v.* Trapp, 49 Mich. 59.

5. County surveyors' certificate are not admissible in evidence unless they contain all the particulars required by the statute to be entered in the surveyor's record.

Smith *v.* Rich, 37 Mich. 549.

The statute of Michigan required the length of all lines run, the area of lands surveyed, and other particulars, to be entered in the county surveyor's record. In the above case the survey was solely to find the location of a corner post. As the surveyor's certificate did not show any area of land surveyed, it was not admitted in evidence.

6. A surveyor was called on to survey the line of a highway. He performed the work so unskillfully as to render a new survey necessary. A large amount of road constructed at great expense, on the line designated by the surveyor before the mistake was discovered, had to be abandoned. Action was brought to recover damages. Held, that whether the defendant was a professional or official surveyor, or represented himself as such, his undertaking was that he should bring to the work the necessary knowledge and skill to perform the same properly and correctly; and if he failed to do so, and the plaintiff suffered damage in consequence of such failure, the plaintiff will be entitled to recover.

Commissioner of Highways *v.* Beebe, Mich. Sup. Court. N. W. Rep., Vol. 20, No. 16.

The following paper, by Chief Justice Cooley, of the Supreme Court of Michigan, discusses more fully the surveyor's functions :

3. The Judicial Functions of Surveyors.—When a man has had a training in one of the exact sciences, where every problem within its purview is supposed to be susceptible of accurate solution, he is likely to be not a little impatient when he is told that, under some circumstances, he must recognize inaccuracies, and govern his action by facts which lead him away from the results which theoretically he ought to reach. Observation warrants us in saying that this remark may frequently be made of surveyors.

In the State of Michigan, all our lands are supposed to have been surveyed once or more, and permanent monuments fixed to determine the boundaries of those who should become proprietors. The United States, as original owner, caused them all to be surveyed once by sworn officers, and as the plan of subdivision was simple, and was uniform over a large extent of territory, there should have been, with due care, few or no mistakes; and long rows of monuments should have been perfect guides to the place of any one that chanced to be missing. The truth unfortunately is, that the lines were very carelessly run, the monuments inaccurately placed; and, as the recorded witnesses to these were many times wanting in permanency, it is often the case that when the monument was not correctly placed, it is impossible to determine by the record, by the aid of anything on the ground, where it was located. The incorrect record of course becomes worse than useless when the witnesses it refers to have disappeared.

It is, perhaps, generally supposed that our town plats were more accurately surveyed, as indeed they should have been, for in general there can have been no difficulty in making them sufficiently perfect for all practical purposes. Many of them, however, were laid out in the woods; some of them by proprietors themselves, without either chain or compass, and some by imperfectly trained surveyors, who, when land was cheap, did not appreciate the importance of having correct lines to determine boundaries when land should become dear. The fact probably is, that town surveys are quite as inaccurate as those made under authority of the general government.

It is now upwards of fifty years since a major part of the public surveys in what is now the State of Michigan were made under authority of the United States. Of the lands south of Lansing, it is now forty years since the major part were sold, and the work of improvement began. A generation has passed away since they were converted into cultivated farms, and few if any of the original corner and quarter stakes now remain.

The corner and quarter stakes were often nothing but green sticks driven into the ground. Stones might be put around or over these if they were handy, but often they were not, and the witness trees must be relied upon after the stake was gone. Too often the first settlers were careless in fixing their lines with accuracy while monuments remained, and an irregular brush fence, or something equally untrustworthy, may have been relied upon to keep in mind where the blazed line once was. A fire running through this might sweep it away, and if nothing was substituted in its place, the adjoining proprietors might in a few years be found disputing over their lines, and perhaps rushing into litigation, as soon as they had occasion to cultivate the land along the boundary.

If now the disputing parties call in a surveyor, it is not likely that any one summoned would doubt or question that his duty was to find, if possible, the place of the original stakes which determined the boundary line between the proprietors. However erroneous may have been the original survey, the monuments that were set must nevertheless govern, even though the effect be to make one half-quarter section ninety acres and the one adjoining seventy; for parties buy, or are supposed to buy, in reference to these monuments, and are entitled to what is within their lines, and no more, be it more or less.

While the witness trees remain, there can generally be no difficulty in determining the locality of the stakes.

When the witness-trees are gone, so that there is no longer record evidence of the monuments, it is remarkable how many there are who mistake altogether the duty that now devolves upon the surveyor. It is by no means uncommon that we find men, whose theoretical education is thought to make them experts, who think that when the monuments are gone, the only thing to be done is to place new monuments where the old ones should have been, and would have been, if placed correctly. This is a serious mistake. The problem is now the same that it was before: To ascertain by the best lights of which the case admits, where the original lines were. The mistake above alluded to, is supposed to have found expression in our legislation; though it is possible that the real intent of the act to which we shall refer is not what is commonly supposed.

An act passed in 1869, (Compiled Laws, § 593), amending the laws respecting the duties and powers of county surveyors, after providing for the case of corners which can be identified by the original field notes or other unquestionable testimony, directs as follows:

"*Second.* Extinct interior section corners must be re-established at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it.

"*Third.* Any extinct quarter-section corner, except on fractional lines, must be re-established equidistant and in a right line between the section corners; in all other cases at its proportionate distance between the nearest original corners on the same line."

The corners thus determined, the surveyors are required to perpetuate by noting bearing trees when timber is near.

To estimate properly this legislation, we must start with the admitted and unquestionable fact that each purchaser from government bought such land as was within the original boundaries, and unquestionably owned it up to the time when the monuments became extinct. If the monument was set for an interior section corner, but did not happen to be "at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it," it nevertheless determined the extent of his possessions, and he gained or lost according as the mistake did or did not favor him.

It will probably be admitted that no man loses title to his land or any part thereof merely because the evidences become lost or uncertain. It may become more difficult for him to establish it as against an adverse claimant, but theoretically the right remains; and it remains as a potential fact so long as he can present better evidence than any other person. And it may often happen that notwithstanding the loss of all trace of a section corner or quarter stake, there will still be evidence from which any surveyor will be able to determine with almost absolute certainty where the original boundary was between the government subdivisions.

There are two senses in which the word extinct may be used in this connection: One, the sense of physical disappearance; the other, the sense of loss of all reliable evidence. If the statute speaks of extinct corners in the former sense, it is plain that a serious mistake was made in supposing that surveyors could be clothed with authority to establish new corners by an arbitrary rule in such cases. As well might the statute declare that if a man loses his deed, he shall lose his land altogether.

But if by extinct corner is meant one in respect to the actual location of which all reliable evidence is lost, then the following remarks are pertinent:

1. There would undoubtedly be a presumption in such a case that the corner was correctly fixed by the government surveyor where the field notes indicated it to be.

2. But this is only a presumption, and may be overcome by any satisfactory evidence showing that in fact it was placed elsewhere.

3. No statute can confer upon a county surveyor the power to "establish" corners, and thereby bind the parties concerned. Nor is this a question merely of conflict between State and federal law; it is a question of property right. The original surveys must govern, and the laws under which they were made must govern, because the land was bought in reference to them; and any legislation, whether State or federal, that should have the effect to change these, would be inoperative, because disturbing vested rights.

4. In any case of disputed lines, unless the parties concerned settle the controversy by agreement, the determination of it is necessarily a judicial act, and it must proceed upon evidence, and give full opportunity for a hearing. No arbitrary rules of survey or of evidence can be laid down whereby it can be adjudged.

The general duty of a surveyor in such a case is plain enough. He is not to assume that a monument is lost until after he has thoroughly sifted the evidence and found himself unable to trace it. Even then he should hesitate long before doing anything to the disturbance of settled possessions. Occupation, especially if long continued, often affords very satisfactory evidence of the original boundary when no other is attainable; and the surveyor should inquire when it originated, how, and why the lines were then located as they were, and whether a claim of title has always accompanied the possession, and give all the facts due force as evidence. Unfortunately, it is known that surveyors sometimes, in supposed obedience to the State statute, disregard all evidences of occupation and claim of title, and plunge whole neighborhoods into quarrels and litigation by assuming to "establish" corners at points with which the previous occupation cannot harmonize. It is often the case that where one or more corners are found to be extinct, all parties concerned have acquiesced in lines which were traced by the guidance of some other corner or landmark, which may or may not have been trustworthy; but to bring these lines into discredit when the people concerned do not question them, not only breeds trouble in the neighborhood, but it must often subject the surveyor himself to annoyance and perhaps discredit, since in a legal controversy the law as well as common sense must declare that a supposed boundary line long acquiesced in is better evidence of where the real line should be than any survey made after the original monuments have disappeared. (Stewart *v.* Carleton, 31 Mich. Reports, 270; Diehl *v.* Zanger, 39 Mich. Reports, 601.) And county surveyors, no more than any others, can conclude parties by their surveys.

The mischiefs of overlooking the facts of possession most often appear in cities and villages. In towns the block and lot stakes soon disappear; there are no witness trees, and no monuments to govern except such as have been put in their places, or where their places were supposed to be. The streets are likely to be soon marked off by fences, and the lots in a block will be measured off from these, without looking farther. Now it may perhaps be known in a particular case that a certain monument still remaining was the starting point in the original survey of the town plat; or a surveyor settling in the town may take some central point as the point of departure in his surveys, and assuming the original plat to be accurate, he will then undertake to find all streets and all lots by course and distance according to the plat, measuring and estimating from his point of departure. This procedure might unsettle every line and every monument existing by acquiescence in the town; it would be very likely to change the lines of streets, and raise controversies everywhere. Yet this is what is sometimes done; the surveyor himself being the first person to raise the disturbing questions.

Suppose, for example, a particular village street has been located by acquiescence and used for many years, and the proprietors in a certain block have laid off their lots in reference to this practical location. Two lot owners quarrel, and one of them calls in a surveyor, that he may make sure his neighbor shall not get an inch of land from him. This surveyor undertakes to make his survey accurate, whether the original was so or not, and the first result is, he notifies the lot owners that there is error in the street line, and that all fences should be moved, say one foot to the east. Perhaps he goes on to drive stakes through the block according to this conclusion. Of course, if he is right in doing this, all lines in the village will be unsettled; but we will limit our attention to the single block. It is not likely that the lot owners generally will allow the new survey to unsettle their possessions, but there is always a probability of finding some one disposed to do so. We shall then have a lawsuit; and with what result?

It is a common error that lines do not become fixed by acquiescence in a less time than twenty years. In fact, by statute, road lines may become conclusively fixed in ten years; and there is no particular time that shall be required to conclude private owners, where it appears that they have accepted a particular line as their boundary, and all concerned have cultivated and claimed up to it. Public policy requires that such lines be not lightly disturbed, or disturbed at all after the lapse of any considerable time. The litigant, therefore, who in such a case pins his faith on the surveyor, is likely to suffer for his reliance, and the surveyor himself to be mortified by a result that seems to impeach his judgment.

Of course nothing in what has been said can require a surveyor to conceal his own judgment, or to report the facts one way when he believes them to be another. He has no right to mislead, and he may rightfully express his opinion that an original monument was at one place, when at the same time he is satisfied that acquiescence has fixed the rights of parties as if it were at another. But he would do mischief if he were to attempt to "establish" monuments which he knew would tend to disturb settled rights; the farthest he has a right to go, as an officer of the law, is to express his opinion where the monument should be, at the same time that he imparts the information to those who employ him, and who might otherwise be misled, that the same authority that makes him an officer and entrusts him to make surveys, also allows parties to settle their own boundary lines, and considers acquiescence in a particular line or monument, for any considerable period, as strong if not conclusive evidence of such settlement. The peace of the community absolutely requires this rule. It is not long since, that in one of the leading cities of the State an attempt was made to move houses two or three rods into a street, on the ground that a survey under which the street had been located for many years, had been found on a more recent survey to be erroneous.

From the foregoing, it will appear that the duty of the surveyor where boundaries are in dispute must be varied by the circumstances. 1. He is to search for original monuments, or for the places where they were originally located, and allow these to control if he finds them, unless he has reason to believe that agreements of the parties, express or implied, have rendered them unimportant. By monuments in the case of government surveys we mean of course the corner and quarter-stakes; blazed lines or marked trees on the lines are not monuments: they are merely guides or finger posts, if we may use the expression, to inform us with more or less accuracy where the monuments may be found. 2. If the original monuments are no longer discoverable, the question of location becomes one of evidence merely. It is merely idle for any State statute to direct a surveyor to locate or "establish" a corner, as the place of the original monument, according to some inflexible rule. The surveyor, on the other hand, must inquire into all the facts: giving due prominence to the acts of parties concerned, and always keeping in mind, *first*, that neither his opinion nor his survey can be conclusive upon parties concerned; and, *second*, that courts and juries may be required to follow after the surveyor over the same ground, and that it is exceedingly desirable that he govern his action by the same lights and the same rules that will govern theirs.

It is always possible, when corners are extinct, that the surveyor may usefully act as a mediator between parties, and assist in preventing legal controversies by settling doubtful lines. Unless he is made for this purpose an arbitrator by legal submission, the parties, of course, even if they consent to follow his judgment, cannot, on the basis of mere consent, be compelled to do so; but if he brings about an agreement, and they carry it into effect by actually conforming their occupation to his lines, the action will conclude them. Of course, it is desirable that all such agreements be reduced to writing; but this is not absolutely indispensable if they are carried into effect without.

Meander Lines.—The subject to which allusion will now be made, is taken up with some reluctance, because it is believed the general rules are familiar. Nevertheless, it is often found that surveyors misapprehend them, or err in their application; and as other interesting topics are somewhat connected with this, a little time devoted to it will probably not be altogether lost. The subject is that of meander lines. These are lines traced along the shores of lakes, ponds, and considerable rivers, as the measures of quantity when sections are made fractional by such waters. These have determined the price to be paid when government lands were bought, and perhaps the impression still lingers in some minds that the meander lines are boundary lines, and that all in front of them remains unsold. Of course this is erroneous. There was never any doubt that, except on the large navigable rivers, the boundary of the owners of the banks is the middle line of the river; and while some courts have held that this was the rule on all fresh-water streams, large and small, others have held to the doctrine that the title to the bed of the stream below low-water mark is in the State, while conceding to the owners of the banks all riparian rights. The practical difference is not very important. In this State, the rule that the center line is the boundary line, is applied to all our great rivers, including the Detroit, varied somewhat by the circumstance of there being a distinct channel for navigation, in some cases, with the stream in the main shallow, and also sometimes by the existence of islands.

The troublesome questions for surveyors present themselves when the boundary line between two contiguous estates is to be continued from the meander line to the center line of the river. Of course, the original survey supposes that each purchaser of land on the stream has a water front of the length shown by the field notes; and it is presumable that he bought this particular land because of that fact. In many cases it now happens that the meander line is left some distance from the shore by the gradual change of course of the stream, or diminution of the flow of water. Now the dividing line between two government subdivisions might strike the meander line at right angles, or obliquely; and, in some cases, if it were continued in the same direction to the center line of the river, might cut off from the water one of the subdivisions entirely, or at least cut it off from any privilege of navigation, or other valuable use of the water, while the other might have a water front much greater than the length of a line crossing it at right angles to its side lines. The effect might be that, of two government subdivisions of equal size and cost, one would be of very great value as water-front property, and the other comparatively valueless. A rule which would produce this result would not be just, and it has not been recognized in the law.

Nevertheless it is not easy to determine what ought to be the correct rule for every case. If the river has a straight course, or one nearly so, every man's equities will be preserved by this rule: Extend the line of division between the two parcels from the meander line to the center line of the river, as nearly as possible at right angles to the general course of the river at that point. This will preserve to each man the water front which the field notes indicated, except as changes in the water may have affected it, and the only inconvenience will be that the division line between different subdivisions is likely to be more or less deflected where it strikes the meander line.

This is the legal rule, and is not limited to government surveys, but applies as well to water lots which appear as such on town plats. (Bay City Gas Light Co. *v.* The Industrial Works, 28 Mich. Reports, 182.) It often happens, therefore, that the lines of city lots bounded on navigable streams are deflected as they strike the bank, or the line where the bank was when the town was first laid out.

When the stream is very crooked, and especially if there are short bends, so that the foregoing rule is incapable of strict application, it is sometimes very difficult to determine what shall be done; and in many cases the surveyor may be under the necessity of working out a rule for himself. Of course his action cannot be conclusive; but if he adopts one that follows as nearly as the circumstances will admit, the general rule above indicated, so as to divide as near as may be the bed of the stream among the adjoining owners in proportion to their lines upon the shore, his division, being that of an expert, made upon the ground and with all available lights, is likely to be adopted as law for the case. Judicial decisions, into which the surveyor would find it prudent to look under such circumstances, will throw light upon his duties and may constitute a sufficient guide when peculiar cases arise. Each riparian lot owner ought to have a line on the legal boundary, namely, the center line of the stream proportioned to the length of his line on the shore and the problem in each case is, how this is to be given him. Alluvion, when a river imperceptibly changes its course, will be apportioned by the same rules.

The existence of islands in a stream when the middle line constitutes a boundary, will not affect the apportionment unless the islands were surveyed out as government subdivisions in the original admeasurement. Wherever that was the case, the purchaser of the island divides the bed of the stream on each side with the owner of the bank, and his rights also extend above and below the solid ground, and are limited by the peculiarities of the bed and the channel. If an island was not surveyed as a government subdivision previous to the sale of the bank, it is of course impossible to do this for the purposes of government sale afterward, for the reason that the rights of the bank owners are fixed by their purchase; when making that they have a right to understand that all land between the meander lines, not separately surveyed and sold, will pass with the shore in the government sale: and having this right, anything which their purchase would include under it cannot afterward be taken from them. It is believed, however that the federal courts would not recognize the applicability of this rule to large navigable rivers, such as those uniting the great lakes.

On all the little lakes of the state which are mere expansions near their mouths of the rivers passing through them—such as the Muskegon, Pere Marquette and Manistee—the same rule of bed ownership has been judicially applied that is applied to the rivers themselves; and the division lines are extended under the water in the same way. (Rice *v.* Ruddiman, 10 Mich., 125.) If such a lake were circular, the lines would converge to the center; if oblong or irregular, there might be a line in the middle on which they would terminate, whose course would bear some relation to that of the shore. But it can seldom be important to follow the division line very far under the water, since all private rights are subject to the public rights of navigation and other use, and any private use of the lands inconsistent with these would be a nuisance, and punishable as such. It is sometimes important, however, to run the lines out for considerable distance, in order to determine where one may lawfully moor vessels or rafts, for the winter, or cut ice. The ice crop that forms over a man's land of course belongs to him. (Lorman *v.* Benson, 8 Mich., 18; People's Ice Co. *v.* Steamer Excelsior, recently decided.)

What is said above will show how unfounded is the notion, which is sometimes advanced, that a riparian proprietor on a meandered river may lawfully raise the water in the stream without liability to the proprietors above, provided he does not raise it so that it overflows the meander line. The real fact is that the meander line has nothing to do with such a case, and an action will lie whenever he sets back the water upon the proprietor above, whether the overflow be below the meander lines or above them.

As regards the lakes and ponds of the state, one may easily raise questions that it would be impossible for him to settle. Let us suggest a few questions, some of which are easily answered, and some not:

1. To whom belongs the land under these bodies of water, where they are not mere expansions of a stream flowing through them?

2. What public rights exist in them?

3. If there are islands in them which were not surveyed out and sold by the United States, can this be done now?

Others will be suggested by the answers given to these.

It seems obvious that the rules of private ownership which are applied to rivers cannot be applied to the great lakes. Perhaps it should be held that the boundary is at low water mark, but improvements beyond this would only become unlawful when they became nuisances Islands in the great lakes would belong to the United States until sold, and might be surveyed and measured for sale at any time. The right to take fish in the lakes, or to cut ice, is public like the right of navigation, but is to be exercised in such manner as not to interfere with the rights of shore owners. But so far as these public rights can be the subject of ownership, they belong to the state, not to the United States; and so, it is believed, does the bed of a lake also. (Pollord *v.* Hagan, 3 Howard's U. S. Reports.) But such rights are not generally considered proper subjects of sale, but like the right to make use of the public highways, they are held by the state in trust for all the people.

What is said of the large lakes may perhaps be said also of many of the interior lakes of the state; such, for example, as Houghton, Higgins, Cheboygan, Burt's, Mullet, Whitmore, and many others. But there are many little lakes or ponds which are gradually disappearing, and the shore proprietorship advances *pari passu* as the waters recede. If these are of any considerable size—say, even a mile across—there may be questions of conflicting rights which no adjudication hitherto made could settle. Let any surveyor, for example, take the case of a pond of irregular form, occupying a mile square or more of territory, and undertake to determine the rights of the shore proprietors to its bed when it shall totally disappear, and he will find he is in the midst of problems such as probably he has never grappled with, or reflected upon before. But the general rules for the extension of shore lines, which have already been laid down, should govern such cases, or at least should serve as guides in their settlement.

Where a pond is so small as to be included within the lines of a private purchase from the government, it is not believed the public have have any rights in it whatever. Where it is not so included, it is believed they have rights of fishery, rights to take ice and water, and rights of navigation for business or pleasure. This is the common belief, and probably the just one. Shore rights must not be so exercised as to disturb these, and the states may pass all proper laws for their protection. It would be easy with suitable legislation to preserve these little bodies of water as permanent places of resort for the pleasure and recreation of the people, and there ought to be such legislation.

If the state should be recognized as owner of the beds of these small lakes and ponds, it would not be owner for the purpose of selling. It would be owner only as trustee for the public use; and a sale would be inconsistent with the right of the bank owners to make use of the water in its natural condition in connection with their estates. Some of them might be made salable lands by draining; but the state could not drain, even for this purpose, against the will of the shore owners, unless their rights were appropriated and paid for.

Upon many questions that might arise between the state as owner of the bed of a little lake and the shore owners, it would be presumptuous to express an opinion now, and fortunately the occasion does not require it.

I have thus indicated a few of the questions with which surveyors may now and then have occasion to deal, and to which they should bring good sense and sound judgment. Surveyors are not and cannot be judicial officers, but in a great many cases they act in a *quasi* judicial capacity with the acquiescence of parties concerned; and it is important for them to know by what rules they are to be guided in the discharge of their judicial functions. What I have said cannot contribute much to their enlightenment, but I trust will not be wholly without value.

CHAPTER XIII.

MAP DRAWING AND LETTERING.

BY C. S. DENISON.

Materials.—Without entering too minutely into detail, let us briefly consider the materials to be used, their quality and adaptability to the purpose in hand; and, first, as to

Paper.—The essential quality of a paper for map drawing is a hard, firm surface of uniform texture, which will take ink or water color smoothly and readily, and which is sufficiently tough to bear the use of instruments and rubber without sensible injury to its surface. Perhaps no paper fulfills these requirements better than Whatman's. It comes in sheets of various sizes up to "antiquarian," 31x53 inches. These papers are either hot or cold pressed, the former with a fine, smooth surface, particularly adapted for pen work; the latter has a somewhat coarse-grained surface, and is more especially designed for water color drawing, or plats on which the brush is to be used more or less. Of course there is a large variety of paper, and for various purposes, but within the limits of the size above given, it would be difficult to find anything superior to the product of "Whatman's Turkey Mills," which words can usually be read in water-line on each sheet. This paper also improves with age, and an old stock is consequently rather more valuable.

Pens.—Probably the best pens for mapping purposes are Gillott's. His lithographic pen, mapping pen, and crow-quill are all valuable in free-hand work, but generally, for ordinary use, his writing pen No. 303 will be found very serviceable, while for filling in the bodies of letters of moderate size nothing is more useful than a common stub-pen, though some draughtsmen prefer a regular old goose-quill.

Ink.—It is well to be somewhat critical in the matter of ink. There are a number of brands of liquid ink on the market, some of which are good, some admirable for special purposes, but after all no sufficient substitute can be found for the best quality of India ink, and when ground with a few drops of water, on a Keuffel & Esser ink slab, the time occupied is so slight that it is more than compensated by the satisfaction experienced in its use. Most emphatically India ink should be of a fine quality, and as a rule the higher the price the better the ink. True economy would pronounce in favor of an expensive cake, a fragment of which would last a draughtsman through the natural term of an active professional life. The best ink works up smoothly with water, forming a perfectly black mixture, and after drying upon the paper forms a fine, glossy surface. For use it should be *just black*, as pale ink will make the boldest drawing look weak, and, on the other hand, if too thick it will constantly annoy the draughtsman by clogging the pen. After once grinding up a quantity of ink, a drop or two of pure glycerine will prevent its drying away for some time, and does not injure the mixture. If it is preferred, a liquid ink can be prepared by pulverizing a portion of the cake, and boiling it in a small quantity of water till the proper consistency is reached. It can then be kept in a vial and used as desired. Such a preparation, however, is apt to become offensive in the course of a week or two, and should then be re-boiled after the addition of fresh water. A common test-tube is the best vessel in which to make the solution. After grinding ink from a cake of fine quality, it is well always to wipe it off carefully with a soft cloth, as this prevents waste by cracking and consequent disintegration of the cake.

Instruments.—As to instruments for map drawing—in addition, of course, to drawing-board, T square and triangle—but few are needed, but these few should be of good, not to say of the best quality, and ought to consist in general of at least one large right-line pen, for drawing heavy lines, such as borders; one small right-line pen, for the finer lines; a pair of spacing dividers, and a large pair of dividers, with pen, pencil and needle-points. These instruments should be kept in perfect condition, or they cannot be made to do good work. The pens should be frequently sharpened by gently passing the outer surfaces of the blades over a fine-grained stone, used dry. A fine knife-edged stone is sometimes applied to the inner surfaces, but this must be done with great care. When properly sharpened the points will be exactly even, and sharp enough to make, with slight pressure, a fine cut on the finger nail. A magnifying glass will aid in determining when the points are smooth and even. The best test, however, is to fill the pen and draw a series of lines, when, if they are all sharp and clear and unbroken, the pen is in good condition. The best implement for charging a drawing pen with ink is a common quill toothpick, using the long pointed end.

Execution.—Having touched briefly upon the materials used by the draughtsman in his work, let us now consider, somewhat more carefully, the manner and methods of execution. While no one will gainsay the fact that the first and supreme requirement in a map is accuracy, or correctness, that quality of reliability without which it is not what it pretends to be—a map—nevertheless there are other qualities which it is by no means wise or judicious to neglect. Among these may be mentioned clearness, precision, legibility, neatness and sharpness of execution, and a certain prepossessing appearance which inspires the observer with confidence in the skill and powers of the maker. In fact, no map or technical drawing is above suspicion, or safe from the shadow of a doubt, when it bears upon its face traces of weakness which mark a want of knowledge and ability even in so simple a matter as, for instance, the form and proportions of the letters of the alphabet.

Furthermore, as a map or plan addresses itself directly to the eye as a product of skilled labor in one department of the art of design, it is not only right and proper, but essential to its complete success, that it should produce a pleasing impression, satisfying the demands of good taste, which can only be the result of the proper application of the laws of design.

Principles.—We may pretty safely assume that the demands of a cultivated taste will be met by a reasonably close adherence to the following simple principles: First—Construction may be decorated, but decoration should never be purposely and obviously the object of such construction. Second—Generally ornamentation should be based upon geometrical construction, and should make no attempt at relief or pictorial representation, and "natural objects should not be used as ornament, but conventional representations founded upon them, sufficiently suggestive to convey the intended image to the mind without destroying the unity of the object they are employed to decorate." Third—"Harmony of form consists in the proper balancing and contrast of the straight, the inclined and the curved." These are fundamental propositions from one of the highest authorities on decorative design. While these principles are safe guides, and doubtless there are powerful motives for the introduction and use of geometrical forms for decorative purposes, founded upon their inherent beauty and almost universal employment, still, if any authority can be claimed for precedent, custom seems to have sanctioned the use for map decoration, to a limited extent, of vignettes, free-hand sketches, and symbolic devices, as are sometimes seen in finely executed work, wrought into the corner piece or into the title. In fact, the value of a map, as a means of conveying information of a topographical nature, may often be greatly enhanced, especially to the non-professional eye, by fine pen-and ink sketches in perspective of portions of scenery or objects of special interest. But these, to be admissible, must be executed with that spirit and decision which mark the eye and hand of an artist. Of course there are maps, and maps, and the foregoing remarks are intended to apply mostly to such as are of large dimensions and intended for general inspection. For those of a strictly professional nature, it is safe to say that the less exclusively ornamental work upon them, the more satisfactory will their appearance be. The dignity of an almost severe simplicity should pertain to a drawing addressed only to the professional eye, depending for its pleasing effect upon the sharpness and clearness of its execution, the completeness and beauty of the lettering, and its evident fitness to the purpose for which it was made, namely, *business*. This character of work is perfectly illustrated in many of the maps issued from the government departments at Washington, and particularly in the charts of the United States Coast Survey.

Border.—Having taken this general glance at the subject, let us enter upon the consideration of the design and execution of the more important features of a properly finished map, and first, let us examine the border. Strictly speaking, this is a purely decorative feature as its entire omission would not affect the utility of the drawing. Still it aids materially in producing a happy effect, by limiting the eye at once to the consideration of those parts having a distinct claim upon its notice. The border bears a certain relation to the enclosed drawing, and is a sort of tribute or compliment to that which it encloses, and should in general reflect the character of the drawing, Thus a line drawing should have a border of geometrical design, executed with the same material as the drawing itself, and of greater or less complexity according to the extent and elaborateness of the work enclosed. It may be rectangular, or elliptical, or composed of both right lines and curves, according to the nature of the space enclosed, and the draughtsman should never hesitate to omit or break the border in order to accommodate some projecting point or small portion of the plat with which its continuance would interfere. Should the drawing contain much free-hand work, the border might with propriety be partly free-hand also, but free-hand decorations representing garlands, vines, tassels, etc., and all rambling, sprawling decoration is always weak and in wretched taste. In all map drawing, color should be rigidly excluded from the border, as it gives a cheap and offensive conspicuousness to a subordinate feature. The most appropriate border for a map of moderate dimensions, and the one generally adopted by the best practice, consists simply of two lines, one heavy and the other, or inner one, light, with a white space between them about as wide as the heavy black line. The rule commonly followed is to make the total width of the border about the one-hundredth part of the shortest side of the map, supposing it to be a rectangle.

The Meridian.—The next feature of a rightly constructed map, and one which is by no means simply decorative but of very great practical importance, is the meridian. A true meridian is a necessary adjunct of all rightly constructed maps, as it is directionally their common line of comparison, and without it no just notion of the situation of the territory represented by the map, or of the bearings of its lines, can be obtained It is, in fact, one of the co-ordinates to which reference is made for the solution of all problems of position on the drawing, and as such is entitled to consideration. This line should therefore be a somewhat conspicuous object, and the object of its existence demands that it should not be so obscured by ornament as to defeat its use as a sharp, clear line of reference for all north and south lines. Nevertheless, the draughtsman is warranted in giving to its construction more than a hasty or careless consideration. It is usual to ornament the northern end of this meridian with some neatly drawn and characteristic device, such as an arrow-head, a *fleur-de-lis*, the head of a mediæval lance, etc. At its southern extremity is sometimes placed the feather end of an arrow or a crescent. Near the middle of the line may be drawn an east and west line, or four or eight pointed star, or radiating lines marking convenient points of graduation of the circle. It is well, also, to draw the magnetic meridian at the time of the survey, through the middle point of the true meridian, and mark the declination. This magnetic meridian should be even less ornamental than the true one, and when both are used it is generally agreed to draw a complete arrow-head on the latter, while the magnetic line is subordinated by giving it only half a head, drawn on the right or left-hand side, as the declination is east or west.

The construction of a meridian affords considerable opportunity for the display of skill and taste in the draughtsman. It may easily be made an attractive, simple and elegant feature, reflecting the intelligence and spirit of an accomplished workman; or, by its awkward design and slovenly execution, shake one's confidence in the mental capacity of one upon whom we should have a right to rely. Perhaps it would not be inappropriate to say that the meridian line should be sufficiently long, on most maps, to serve conveniently the purpose of transferring its direction to other parts of the drawing by means of a triangle and straight-edge. The arrow-head at the vertex should be a sharply pointed figure, entirely different from the obtuse, nondescript object which too often offends the eye in that position. And, to avoid all possibility of mistake, it is well to place the letter N some slight distance above or below the arrow point. When a star is used to give the various points of the compass, its radiating arms should be narrow and slender, with sharp points, avoiding all appearance or suggestion of dullness. In short, the entire figure should be constructed in the spirit of lightness and radiation, in harmony with its office, which is simply that of indicating *direction.*

Scale.—Of course it is entirely unnecessary to state that no map is complete, or prepared to accomplish the end and aim of its existence, without a scale. The scale is here mentioned merely for the purpose of directing attention to its paramount importance as furnishing one of the co-ordinates of a map. What is said later, with reference to the title, applies equally to the treatment of scales.

Lettering.—In no department of his work is the responsibility of the draughtsman greater, both in the matter of correctness of detail and beauty of appearance, than in that of lettering. Nothing more readily and strongly recommends a drawing to a favorable consideration than an appropriate and handsome style of lettering; while if poor, weak, uncertain and badly executed, it is apt to at once arouse in the mind an uncomfortable feeling of distrust and aversion, which surely detracts from the artistic value which the finished work should possess.

There is a certain undefinable pleasure, a mental gratification, which we experience in looking upon anything *well done;* and this is particularly the case with all lettering upon maps, charts and drawings, as it conveys to the mind not only the sense of the words written, but beyond this there is indicated something of the mental character and practical experience of him to whom the work is due If it be strong, free and graceful in style, and sharp and clean in execution, it at once commands our confidence and respect, and saves us continual annoyance and waste of time in the endeavor to decipher some blurred or illegible letter or figure, the failure to do which may put one to great inconvenience or delay. Thus, more perhaps than anything else, the lettering reflects the distinguishing qualities of the maker, and should be of such a nature as to clearly indicate a man completely and easily up to his work.

Our alphabet is composed of a certain number of distinct characters, or letters, and we also have certain additional characters for expressing numbers. It is not proposed to trace the origin and development of these characters, their evolution or history; it is sufficient to say that, so far as we are concerned now, they are arbitrary signs which we are bound to adopt and follow. This is true, however, only as to their *essential elements.*

Preserving these in their integrity, we are at liberty to alter and modify form and proportion very largely, according to individual fancy. By "essential elements" are meant those lines which are necessary to the recognition of the letter. Thus, every capital A, whatever else it may have, must have the three right-line elements which characterize it in its most simple form.

Classification of Letters.—If one examines a book of specimen letters, he may conclude, from the bewildering variety of alphabets, that they are absolutely dissimilar, and subject to no system or order of classification. A little closer inspection, however, will reveal the fact that each letter of these alphabets, which apparently differ so widely, is constructed upon the common characteristic elements of that letter, a sufficient acquaintance with which, and their methods of variation, will enable one readily to design with consistency and uniformity the letters of any style of alphabet, which gives a most desirable independence of the mere devotion to copy.

Roman Letters.—Probably the most difficult letter to draw accurately and properly is the Roman, from which as a model we derive most if not all the alphabets now in use. This being so, possibly it will be well to enter somewhat minutely into the discussion of its construction. Let us not, even here, entirely abandon our independence of judgment, but bear in mind that this style of letter in its present beauty of development is simply the resultant of the taste and practice of centuries; and that, while it is generally accepted as a standard, authorities differ as to its exact proportions and minor details, which relieves it from the undesirable rigidity of precise rule.

Let us then, examine the Roman capitals. We can separate this alphabet into three groups. Let the first embrace all letters whose essential elements are either horizontal or vertical right lines, such as I, E and H, six letters in all. The second group contains all letters having oblique right-line elements, as A, W, X and Y—nine; and the third group will comprise all letters into which curved elements enter, such as P, R, S, etc., eleven in number.

An examination of one or two members of each group will be sufficient for our purpose.

Let us take I, for instance, and by assigning it the proportions given in a set of letters designed by Prof. Warren, we make its height equal unity, its width one-fourth the height; the caps at top and bottom project beyond the body of the letter a distance equal to half the width of the column of the letter. The complete proportions would then be: Height, unity; width of column, 4-16ths; total width, including caps, 8-16ths; projection of caps, 2-16ths; thickness of cap, 1-16th—these proportions to be preserved in all vertical columns of all letters. The letter L consists of an I with the addition of the arm at the bottom; this arm is 7-16ths in height, and the total width of the letter is 14-16ths. From these proportions the first group can readily be drawn.

In the second group we meet with inclined elements, both heavy and light. Let it be noticed that in all letters having slanting heavy elements, with the exception of Z, all heavy elements slant downward to the *right*, and that these elements are 4-16ths perpendicular width; and all light elements have the same width as the caps, namely, 1-16th. By properly proportioning the total widths of the letters, those of this section can be easily constructed. Observe that N is never drawn with a cap at the lower right-hand corner.

In the third group, the widest part of curved elements equals the thickness of the columns, and the narrowest part equals the thickness of a cap.

The letter just described is rather too heavy in its appearance, and would be much improved by reducing the width of all heavy elements to one-sixth of the total height, even one-seventh being sometimes used.

The following table, taken from the last edition of Lieut. Smith's "Topographical Drawing," will serve as a guide in proportioning capital letters:

"Taking the extreme width of H, measured across the middle, or exclusive of the caps, as a unit, the widths of the other letters, or of their characteristic parts, may be expressed approximately by the numbers in the third column of the table. In case of letters having oblique lines, these widths are to be taken at the intersections of the outer oblique lines with the upper or lower limit of each letter. The caps are in all cases excluded."

Letter.	Measured at	H = 1.	Letter.	Measured at	H = 1.
A	Bottom ..	1 1-16th.	N	....	7-8ths.
B	Top......	15-16ths.	O	Middle.....	1 1-8th.
	Bottom ..	1	P	Top........	15-16ths.
C	Top......	1	Q		1 1-8th.
	Bottom ..	1 1-16th.	R	Top........	15-16ths.
D	Middle...	1 1-16th.		Bottom*...	15-16ths.
E	Top......	15-16ths.	S	Top........	15-16ths.
	Middle...	5-8ths.		Bottom	1
	Bottom ..	1	T	Top........	1 1-16th.
F	Top......	15-16ths.	U	....	7-8ths.
	Middle...	5-8ths.	V	Top........	15-16ths.
G	Top*.....	1	W	Top	1½.
	Bottom ..	1 1-16th.	X	Top........	15-16ths.
J	Bottom ..	3-4ths.		Bottom	1 3-32nds.
K	Top......	15-16ths.	Y	Top........	15-16ths.
	Bottom ..	1 1-16th.	Z	Top........	15-16ths.
L	Bottom ..	1		Bottom	1
M	....	1 3-32nds.			

*Measurement is from vertical tangent to the curve.

The horizontal bars of H, E and F are at the middle of each letter; those of B, P and R are very slightly above it; while the horizontal bar of A is from one-eighth to one-sixth the height of the letter below the middle. It improves the appearance of the E, C, G and S to make the lower half a trifle wider than the upper, as is indicated by the proportions given in the table, and the ends of the arms of these letters should come nearly together, the extremities of the S being nearly on a horizontal line.

Of course, no practical draughtsman, engaged in his work, stops to pay attention to these refinements of proportion. They serve simply as guides to the inexperienced, and should be employed only in the study of the letter on a large scale. Letters of the size ordinarily employed should be proportioned by the eye, and sketched in free-hand in pencil and afterwards inked in, instrumentally if preferred. In drawing the small Roman letters, the same free-hand practice is required, and they are finally put in by a bold pressure of the pen for the heavy parts, the height of the small letters, such as a, e and o, being equal to three-fifths of the height of capitals.

The general rule for *spacing* letters, is to make the areas between them approximately equal, and of course these areas are only estimated by the eye.

Other Letters.—Having conquered the Roman, other alphabets will offer but few difficulties to the draughtsman. In the order of their importance for mapping purposes the letters are ranked thus: First, the upright Roman CAPITAL; second, the inclined *CAPITAL;* third, Roman, or ordinary small type; fourth, the *small italic*, or *stump print*, to be followed by the block, the skeleton, and an infinite variety of styles, fanciful and otherwise, not forgetting Old English. Inclined letters are usually regulated by a slope of three horizontal to eight vertical, and in the stub print the height of the smaller letters is three-fifths the height of capitals. This stub print somewhat resembles careful writing with the pen, and with some practice can be done very rapidly and made to present a very neat appearance. It is most perfectly adapted for statements, explanatory notes, etc. Free-hand random letters, of vigorous and graceful style, may be easily designed, and sometimes relieve a drawing of undue stiffness. Good models in great variety can be easily obtained, and can with profit be carefully studied and practiced.

Height.—The adjustment of the height of letters to the scale of the map is worth considering, and the following proportions have been suggested by Prof. McMillan, C. E.:

SCALE.	Height of Largest Upright Capitals.	Height of Small Letters for Explanatory Notes, etc.
1-600th, or 1 in. to 50 feet.	6-10ths inch.	12-100ths inch.
1-2620th, or 2 ft. to 1 mile.	5-10ths "	10-100ths "
1-5280th, or 1 ft. to 1 "	4-10ths "	8-100ths "
1-10560th, or 4 in. to 1 "	3-10ths "	6-100ths "

The size and character of the letter used depends upon the nature and importance of the object to which it is applied, as capitals to large cities, important bodies of water, etc. On the United States Coast Survey charts upright letters are used for land objects, such as islands, points, etc., and for bodies of water, rivers, bays, and so on, the inclined letter is employed.

When practicable the lines of lettering should be parallel to the base of the drawing; if the lines incline toward the upper right-hand corner, the letters should be arranged to read from the bottom upward; if they incline from the bottom upward to the left, they should read from the top downward.

Title.—The last feature to which I shall direct attention is the title, the execution of which affords a suitable opportunity for enhancing the beauty of the map by a choice selection of the letters used, an appropriate arrangement of the words, and the indulgence in a fitting amount of ornamentation.

If the title be brief, and takes but a single line, it may be placed outside of and just below the border, otherwise it must be placed within it, its location depending upon the configuration of the map, preferably one of the corners. The letters of the wording are varied in size according to the importance of the words. Their order of prominence is usually determined by answers to the following questions: First—What does the map represent? Second—Where is the locality? Third—For whom have the survey and map been made? Fourth—By whom and when?

The title should be symmetrically arranged, with reference to a middle vertical line, placing the most important words, if possible, about midway between the top and bottom of this arrangement; and the height of the letters composing this principal line should not exceed one thirty-third part of the shortest side of the map.

In the case of an elaborate title it is a good plan to make a preliminary drawing, in which alterations can be made in the style and arrangement until the proper effect is produced.

The title is the introduction of the drawing to the observer, and should be marked by a somewhat formal and dignified grace. Anything of the nature of over-decoration should be carefully avoided, and I cannot close without expressing cordial antagonism toward the too frequent practice of employing elaborate pen-flourishes, "rustic letters," fluttering ribbons, and numerous devices of a like nature, for map decorations, with the laudable intention, no doubt, of adding enrichment to the drawing, but with the unfortunate results of cheapening the effect and violating good taste."

CHAPTER XIV.

LEVELING.

1. **Leveling** is the operation of measuring the difference in height of two or more points.

The surface of water at rest is an example of a level surface.

If the earth was a perfect sphere, **a line of true level** would be an arc or a circle having its centre at the centre of gravity of the earth. So far as common leveling is concerned it may be so considered, as the error arising therefrom is so small as to be of no practical consequence.

The **line of apparent level** is a straight horizontal line passing through the point of observation, tangent to the line of true level.

In **precise leveling** the difference between true and apparent level is measured, the instruments used are of the best, and all the operations are performed so as to reduce the error to the smallest possible amount. In **common leveling** for streets, railroads, drains, water powers and the like operations, a lower degree of accuracy is required and the refinements of precise leveling are dispensed with. No attention is paid to the difference between true and apparent level, it being too small to affect the practical result.

2. *The deviation of the true from the apparent level between two points is equal to the square of the distance between the points, divided by the diameter of the earth.*

Also, *The deviations for different distances are proportional to the squares of the distances.*

Calling the diameter of the earth 7920 miles and taking points one mile apart, we find the deviation $= 0.000126$ miles $= 0.665$ ft. $= 7.98$ inches. For m miles, deviation $= 7.98\,m^2$ inches.

The effect of the refraction of light is to apparently increase the difference between true and apparent level.

For considerable distances the correction for curvature as above found is sometimes diminished by about one-sixth of itself.

If the instrument is placed midway between the points whose difference in height is required, the errors are balanced and eliminate each other, giving a correct result.

3. In leveling, two instruments are required, one to find a horizontal line, and the other to measure vertical distances. These instruments are called a **Level** and a **Leveling rod.**

Level lines, for many common purposes, on a limited area, when no instruments are at hand, can be obtained by the following method:

Suspend from some fixed point of support P by stout cords as indicated, a pole of any shape A B, having the longer end sharpened to a fine point. From this pole hang a heavy weight R as shown. Set two stakes SS so that the point of the pole when swung around will just touch them. Smooth a place on each stake to receive marks

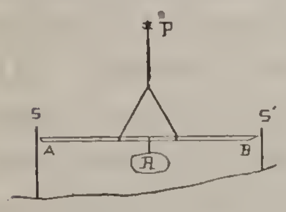

Fig. 75.

After taking the twist out of the supporting cord, carefully swing the pole around and mark the exact place where the point of the pole touches each stake. Repeat this, and take the most satisfactory points. They will determine a level line of sight.

A cheap instrument which almost anyone can make, having a more extended range, is made as follows: Take

Fig. 76.

two pieces of glass tubing three or four inches long and connect them with a rubber tube two or three feet long, so as to make a continuous water tight tube, with glass ends. Pass the ends of the tube through holes in a cross bar made of a piece of board of suitable size, as shown in the cut, and fasten them with the tops projecting an inch or more above the bar. The cross bar may be fastened with a bolt and nut to a staff so that it may be set up and adjusted to a level line. Colored fluid is poured into the tube. The surface of the fluid in the glass tubes determines the level line. Sights of horse hair or fine wire may be attached close to the glass tubes and the cross bar adjusted to bring them into a level line.

An instrument can thus be made at the expense of a few cents in money and a few minutes labor that will do very satisfactory work.

4. If a tube be nearly filled with any liquid, as water, alcohol or ether, and closed, the liquid will seek the lowest part, and the vacant space or bubble, as it is called, will be found at the highest part of the tube. If the tube is of glass, and very truly ground on the inside to a segment of a circle, it furnishes the best known means for determining a level line. Such tubes are made and nearly filled with ether or alcohol, leaving a small space or bubble. When such a tube is placed convex side uppermost, the bubble seeks the highest point. Then a vertical line passing through the centre of the bubble will coincide with the radius of the arc to which the tube is ground. A perpendicular to this vertical line is a line of apparent level. Such a tube is the most essential part of the level. It is encased in a brass tube, having an opening so that the bubble and as much of the glass tube as necessary can be seen. A graduated scale is attached to it, or marked on the tube, by means of which the bubble is measured and its position with relation to other parts of the instrument is determined. The tube thus prepared is attached to a telescope in such a manner that it can be adjusted so as to bring the radius of the ground glass perpendicular to the line of sight in the telescope.

The telescope is mounted in such a manner as to permit it to revolve freely in a horizontal plane and to be readily adjusted to the line of apparent level.

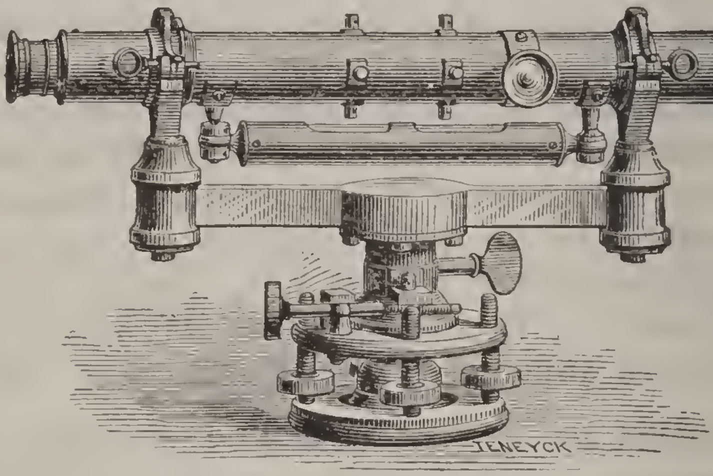

Fig. 77.

The plan of mounting the telescope most in favor in the United States is by a **horizontal bar** with forked arms called **wyes.** The telescope rests upon the wyes and is held in place by clips which may be loosened, permitting the telescope to be rolled over in the wyes. The bar is connected by a spindle to the tripod socket and leveling head similar to that used upon the transit. By permission of Messrs. Buff & Berger, of Boston, the following quotation is taken from their catalogue :

5. "**The Adjustments.**—In a theoretically perfect level the following points are established :

1. The object and eye-glasses are perpendicular to the optical axis at all distances apart.

2. The optical axis coincides with the axis of rotation in the wyes.

3. The axis of collimation coincides with the optical axis.

4. The axis of collimation is parallel to the telescope level.

5. The collars resting in the wyes are circles of the same diameter and concentric with the line of collimation of the telescope.

6. The wyes are exactly similar, and similarly placed with reference to the line of collimation of the telescope.

7. The level bubble moves over equal spaces for equal displacements of the telescope in altitude.

8. The level bubble expands or contracts equally from the center in both directions, during changes of temperature.

9. The vertical axis of revolution is perpendicular to the line of collimation of the telescope.

Of the above, the maker establishes points numbered 1, 2, 5, 6, 7 and 8. The remaining points, 3, 4 and 9, are established when the instrument leaves the shop, but being liable to derangement from rough usage, they are made adjustable in the field.

Adjusting. After the engineer has set up the instrument and adjusted the eye-piece for parallax, the horizontal cross-line had better be made to lie in the plane of the azimuthal rotation of the instrument. This may be accomplished by rotating the reticule, after loosening the capstan-headed screws, until a point remains bisected throughout the length of the line when the telescope is moved in azimuth. In making this adjustment, the level tube is to be kept directly beneath the telescope tube. When made, the small set screw attached to one of the wyes may be set so that by simply bringing the projecting pin from the telescope against it, the cross-lines will be respectively parallel and perpendicular to the motion of the telescope in azimuth.

The first collimating of the instrument may be made using an edge of some building, or any profile which is vertical. Make the vertical cross-line tangent to any such profile, and then turn the telescope half-way round in its wyes. If the vertical cross-line is still tangent to the edge selected, the vertical cross-line is collimated.

Select some horizontal line, and cause the horizontal cross-line to be brought tangent to it. Again rotate the telescope half way round in its wyes, and if the horizontal cross-line is still tangent to the edge selected, the horizontal cross-line is collimated.

Having adjusted the two wires separately in this manner, select some well defined point which the cross-lines are made to bi-sect. Now rotate the telescope half way round in its wyes. If the point is still bi-sected, the telescope is collimated. A very excellent mark to use is the intersection of the cross-lines of a transit instrument.

Center the eye-piece by the four capstan-headed screws nearest the eye end. This is done by moving the opposite screws in the same direction until a distant object under observation is without the appearance of a raise or fall throughout an entire rotation of the telescope in its wyes. The telescope is now adjusted.

Next, bring the level bar over two of the leveling screws, focus the telescope upon some object about 300 feet distant, and put on the sun-shade. These precautions are necessary to a nice adjustment of the level tube. Throw open the two arms which hold the telescope down in its wyes, and carefully level the instrument over the two level screws parallel to the telescope. Lift the telescope out of its wyes, turn it end for end and carefully replace it. If the level tube is adjusted, the level will indicate the same reading as before. If it does not, correct half the deviation by the two leveling screws and the remainder by moving the level tube vertically by means of the two cylinder nuts which secure the level tube to the telescope tube at its eye-piece end. Loosen the upper nut with an adjusting pin, and then raise or lower the lower nut as the case requires, and finally clamp that end of the level tube by bringing home the upper nut. This adjustment may require several repetitions before it is perfect.

The level is now to be adjusted so that its axis may be parallel to the axis of the telescope. Rotate the telescope about 20° in its wyes, and note whether the level bubble has the same reading as when the bubble was *under* the telescope. If it has, this adjustment is made. If it has not the same reading, move the end of the level tube nearest the object-glass in a horizontal direction, when the telescope is in its proper position, by means of the two small capstan-headed screws which secure that end of the level to the telescope tube. If the level bubble goes to the object-glass end when that end is to the engineer's right hand, upon rotating the telescope level toward him, then these screws are to be turned in the direction of a left-handed screw, as the engineer sees them, and *vice versa*. Having completed this adjustment, the level bar itself must now be made parallel to the axis of the level.

To do this, level the instrument carefully over two of its leveling screws, the other two being set as nearly leve as may be; turn the instrument 180° in azimuth, and if the level indicates the same inclination, the level bar is adjusted. If the level bubble indicates a change of inclination of the telescope in turning 180°, correct half the amount of the change by the two level screws, and the remainder by the two capstan-headed nuts at the end of the lever bar, which is to the engineer's left hand when he can read the firm's name. Turn both nuts in the same direction, an equal part of a revolution, starting that nut first which is in the direction of the desired movement of the level bar. Many engineers consider this adjustment of little importance, preferring to bring the level bubble in the middle of its tube at each sight by means of the levelling screws alone, rather than to give any consideration to this adjustment, should it require to be made."

6. Leveling rods are made in a variety of styles and are of two principal classes, viz: *target rods* and *speaking or self reading rods.*

Target rods are made of hard wood in two or more parts, which are grooved and tongued to slide upon each other, by which means they are lengthened out to 12 or more feet. They are graduated to feet, tenths and hundredths, the decimal notation being more convenient for computation than the division into inches and fractions of an inch. The target is a disc of brass made to slide up and down on the rod and to be clamped fast to the rod at any desired place. It is divided into quadrants painted alternately white and red. When used in leveling the target is moved up and down on the rod until the horizontal line between these divisions is brought to coincide with the line of sight in the level. The target has a vernier attached by which the distance on the rod is read to the nearest $\frac{1}{1000}$ part of a foot. In common leveling it is a useless refinement to carry the reading to thousandths of a foot, as it is out of harmony with the other conditions of the rod and the work to be done. The target on the rod, as a rule, is not capable of being set as closely and accurately to the level line as the vernier will read, nor will the rod be held so truly plumb as to justify so close a reading. Generally the line between the quadrants of the target is not perpendicular to the rod and does not coincide with the zero of the vernier within several thousandths.

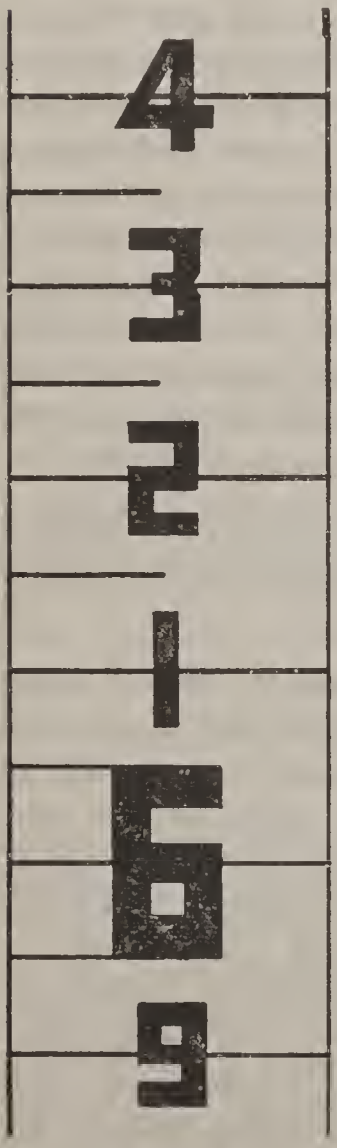

Fig. 78.

Speaking rods are plain, straight rods. having the graduations marked on them so boldly and distinctly that they can be read from the instrument. No targets are used with them, although some rods, like the Philadelphia rod, are made so as to be used either as target or speaking rods. There are many devices for marking the speaking rod, all of which are intended to facilitate accurate reading by the observer A simple form of graduation and lettering which gives excellent results in actual service is shown on a reduced scale in the cut. The graduations are to tenths and half tenths of a foot. Distances less than half a tenth are estimated by the eye. This is facilitated by having the figures for tenths made either .04 or .06 feet in length, and accurately spaced on the rod.

The student having a level and a rod for use in practice may now solve the following problems in the field :

7. Prob. 1. *To find the difference of level of two points.*

Case 1.—*When the difference of level may be found by one setting of the instrument.*

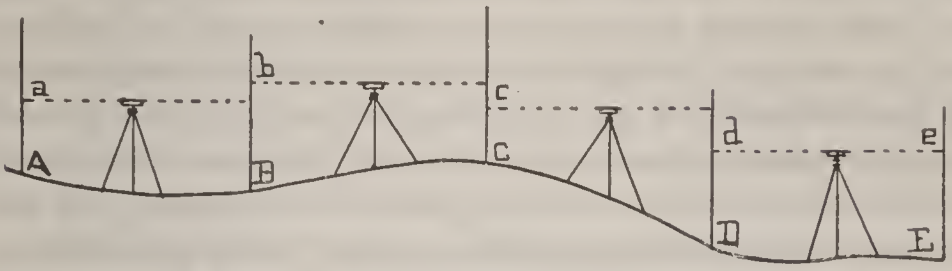

FIG. 79.

Suppose A and B to be the points. Set up the level at a point about equidistant from A and B, though not necessarily in a line between them. Plant it firmly on the ground, placing the legs so as to bring the instrument nearly level, leaving as little as possible to be done with the leveling screws. If set up on yielding ground constant care will be required to be sure that the instrument is level at the instant the observation is taken. When the level is set up on ice or frozen ground, the legs will settle into the frost. It is well to set the instrument in the shade whenever convenient, as the rays of the sun, a passing cloud or a sudden breeze will throw the instrument out of level by causing unequal expansion and contraction of the metal. In precise leveling the instrument must be shaded. Having the instrument firmly planted, bring the telescope in line with one pair of the leveling screws and turn them in or out till the bubble is brought to the middle. Then bring the level in line with the other pair and again level it. Repeat until the bubble will remain in the middle of the tube through an entire revolution of the telescope around the spindle.

The rod-man holds the rod at A, and its reading, Aa is taken. This is called a **Back Sight.** All observations on other points taken at the same setting of the instrument are termed **Fore Sights.** The distance Aa shows how much the line of collimation of the level is above the point A and is called the **height of instrument.** The rod-man now holds the rod on the point B and its reading is taken. The difference between the back sight and the fore sight is the difference in height of the points A and B. If the back sight is 9.20 and the fore sight 6.40, then B is 2.80 *higher* than A. If the fore sight were 11.45 instead of 6.40, then B is 2.25 *lower* than A. The rod-man should stand square behind his rod and hold it plumb. Sometimes small levels are attached to the rod to plumb it by. If they are not used the leveler when necessary directs the rod-man to move the top of the rod to the right or left to plumb it that way, and the rod-man also moves it gently back and forth towards the level, until the smallest reading of the rod is obtained. It is manifest that as many points may be taken as can be reached from the instrument and that their relative heights will be shown by the distances they are below the horizontal plane of the instrument, which is told by the readings on the rod.

CASE 2.—*When the difference of level cannot be found by one setting of the instrument.*

Suppose A and E to be the points, and that it is necessary to set the instrument four times to find the difference between them. We find by the first setting the difference between the points A and B, as already described. We then go forward and find successively the differences between the points B and C, C and D, and D and E. The algebraic sum of these differences is the difference in height of the points A and E.

A convenient form of field notes in cases like the above consists of three columns as shown in the following

Example.—Required the difference of level between the points A and E from the accompanying notes:

Sta.	Back Sights.	Fore Sights.
A	3.28	
B	2.14	7.15
C	3.25	8.50
D	4.70	3.45
E		2.75

Which point is the higher and how much?

A Bench Mark or **Bench** is a fixed point used for reference in finding the heights of other points. It is indicated in the notes by the letters *B. M.* It is customary to establish bench marks at convenient distances along a line of levels by which the work may be reviewed, or at which it may be resumed after temporary cessation. The most convenient permanent objects are selected for the location of these bench marks, such as foundation stones in buildings, rocks or large boulders, or shoulders cut in the roots of large trees, so situated that the rod can be set up on them and the level readily taken.

Where a line of levels is run taking a number of points it is customary to refer the heights to an assumed level plane called a **datum.** This is generally assumed to be far enough below the first or principal bench mark so that it shall be below the lowest station likely to be found in any part of the survey for which it is used.

Negative heights are thus avoided:

A line of levels is usually marked by stakes set at uniform distances apart, marked and numbered consecutively from zero upwards. 100 feet is the distance most usually adopted between stations, although in levels for country drains it is sometimes found more convenient to space the stations by chains to correspond with the measures of the land surveys. Intermediate stakes are usually referred to as *plus stations*, and are so marked on the stakes and in the notes. For instance, a stake set between stakes No. 6 and 7 at 40 feet from No. 6, is marked $6+40$ or simply $+40$.

8. Prob. 2. *To find the heights above a datum plane, of several stations on a given line.*

Suggestions.—Let AB (Fig. 80, page 354) be the given line and DP the datum plane assumed at any convenient distance, say 10 ft., below a bench near A.

Set up the level at some convenient point, for example between stations 2 and 3.

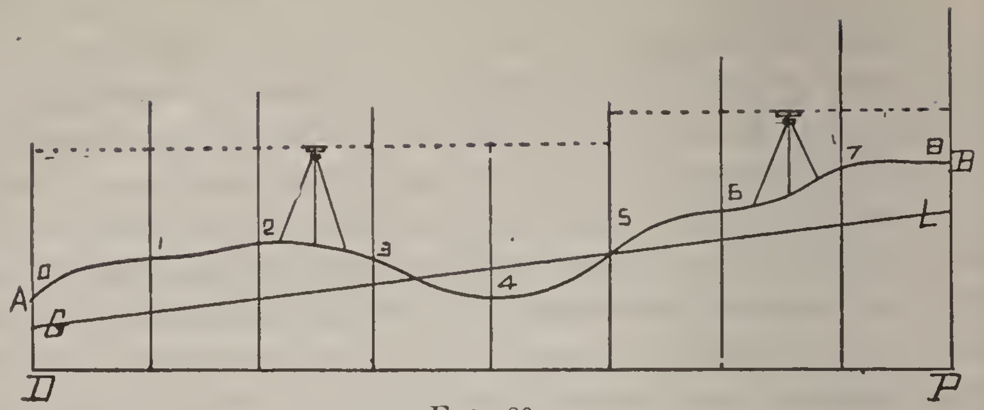

FIG. 80.

Take the reading of the rod upon the bench and add it to the assumed height of the bench above the datum. The sum is the height of the instrument.

Take the readings upon stations 0, 1, 2, 3, 4 and 5 in succession, and subtract each from the height of the instrument. The remainders are the heights, respectively, of those stations above the datum.

Carry the instrument forward to another position, as between stations 6 and 7.

Take the reading of the rod a second time on station 5, and add it to the height of station 5 as before found. The sum is the new height of instrument, with which proceed as before.

A point used as station 5, as above indicated, is called a **Turning Point.** In practice, a bench is often adopted as a turning point.

The reading of the rod upon a turning point or bench-mark is usually taken with somewhat greater precision than upon other points.

A reading upon a bench or turning point is added to the height of the point above the datum in finding height of instrument; and a reading upon any point is subtracted from the height of instrument in finding the height of the point.

Accordingly, an observation for the former is called a **Plus Sight,** denoted by +S, and for the latter, a **Minus Sight,** denoted by —S.

The height of instrument is denoted by H. In., and the height of any point above the datum, by H.

The following is an example of the notes made in solving the above problem :

Sta.	+S.	H. In.	−S.	H.	Remarks.
B. M.	3.426	13.426		10.000	A stone 20 ft. S. E. of 0.
0			5.45	7.976	
1			7.30	6.126	
2			5.35	8.08	
3			5.40	8.03	
4			6.23	7.20	
5	8.274		3.76	9.666	
6		17.940	5.25	12.69	
7			5.10	12.84	
8			5.00	12.94	

9. Prob. 3. *To find the cut or fill, to grade, at points between two given points.*

SUGGESTIONS.—Let A and B, (Fig. 80), denote the given points. Beginning at A, for example, measure the distance AB, at the same time marking it off into convenient divisions of equal length, as 33 ft., 50 ft., 66 ft., or 100 ft., for example, by driving pegs down to the surface of the ground. The last division will usually be fractional. Number the divisions, 0, 1, 2, 3, etc., beginning at A.

Find now, (Prob. 2), the heights of the points, 0, 1, 2, 3, etc., above some convenient datum.

For illustration, suppose the heights to be as given in the above Table (Prob. 2). Also suppose the height of the grade line at A to be 5 ft., and at B, 9 ft.

The distance from A to B consisting of 8 equal parts, say of 50 ft., we should then have

$$(9 \text{ ft.} - 5 \text{ ft.}) \div 8 = 0.5 \text{ ft.} = \text{rise per station.}$$

Beginning at A or station 0, we have

7.98 — 5.	=	2.98	= cut	at	0
6.13 — 5.50	=	0.63	= "	"	1
8.08 — 6.00	=	2.08	= "	"	2
8.03 — 6.50	=	1.53	= "	"	3
6.20 — 7.00	=	— 0.80	= fill	"	4
9.67 — 7.50	=	2.17	= cut	"	5
etc.		etc.	etc.		

Observe that we take the difference in height between the grade line and the station at each station; and since we have here proceeded from lower to higher points of the grade, we have *added* the rise of the grade per station to the height of the grade at the last preceding station.

Let the student find the cut at each station, beginning at B, all other things being as above.

Again, supposing the heights of the stations to be as above, let the student find the depths of cut and fill under the supposition that the height of the grade at A is 6 ft., and at B, 8.4 ft.

10. Drawing Profile. Fig. 80 represents a section formed by a vertical plane passing through the points A and B, and meeting the datum plane in the line DP. The irregular line AB represents the intersection of the vertical plane with the surface of the ground, and is called the **Profile.**

The manner of drawing the profile is as follows:

Draw a horizontal line to represent the datum line, on which lay off to a convenient scale the distance between the stations.

At the points of division of the datum line, erect perpendiculars, on which lay off the surface heights of the several stations, in their order, but to a scale usually ten times greater than that used for the horizontal distances.

A line drawn through the points thus located forms the profile.

The use of a larger scale in drawing the vertical distances serves to render the irregularities of the surface more apparent to the eye than they would be if drawn to the same scale with the horizontal measurements.

The grade line is drawn through any two points at the proper distances from the datum line. The position and inclination of the grade line depend upon certain conditions required to be fulfilled by the work, such as the flowage of water, ease of travel, economy of construction, etc.

In road work the grade is often adopted with reference to an equalization of "cut" and "fill," so that the material furnished by excavations shall make the embankments. The required position of the grade line, in order to fulfill this condition most advantageously, is conveniently got by stretching a thread across the profile, varying the position of the thread until the areas intercepted by it and the profile on opposite sides appear to be equal.

EXERCISES.

11. 1. Find depths of cut or fill, and draw profile and grade line from the following notes :

Sta.	+ S.	H. In.	− S.	H.	H. Gr.	Cut.	Fill.
0	4.26	14.26		10.00	8.00		
1			6.30				
2			8.45				
3	4.12		3.23				
3^{40}		15.15	8.20				
4			4.63				
5			5.53				
5^{25}			5.75		9.575		

Distance between stations, 100 ft.

2—5. Examples made by the student in the "Field."

II. DRAINAGE SURVEYING.

12. Of the many applications of leveling, the most common, perhaps, in the province of the ordinary surveyor, is that relating to drainage. Almost every neighborhood offers occasions for work of this kind.

13. Drains are of two forms: the **Open Drain** or **Ditch,** and the **Under Drain.**

The former is adapted to the case of water lying upon he surface of the ground, and the latter to water underlying the surface. Under drains are usually discharged into open drains, which are thus rendered an essential auxiliary to thorough drainage.

14. **Making the Survey.**—This will be, in the first place, a careful reconnoissance of the locality respecting the general "lay of the land," natural water courses, etc. In this will be determined the proper commencement, route and terminus of the drain. The term commencement will be here understood to mean the upper end of the drain, and terminus the outlet. The word commencement in connection with open drains will also be taken as significant of the proper place to begin the survey.

Preliminaries having been settled, a stake marked 0 is driven at the point of commencement, and the survey, proper, begins by setting the transit over the stake and taking the bearings and distances of two convenient objects near by as witnesses of the point of commencement. The location of the commencement should be described also by distances and direction from some neighboring monument or line of original survey. Thus, 10 ch. E. and 7.15 ch. N. of $\frac{1}{4}$ post bet. Secs. 11 and 14, T. 2 N. R. 5 E.

These items are to be entered in the column of remarks in the Transit book, opposite the station 0.

The instrument is then turned upon the first angle in the line of the drain and its bearing entered in the column of bearings opposite station 0.

Ax-men are required in clearing away bushes, making and driving stakes, etc. Two chain-men, the forward one carrying a transit-rod, now begin to measure at 0 in the direction of the first angle, and stakes marked 1, 2, 3, etc., are driven at uniform distances from each other.

A 100-ft. tape is a convenient measure, and locates the stations at ordinarily suitable distances.

A stake should be set also at each angle of the drain, and its distance from the last preceding station entered in the notes. The points of meeting of any land-lines, roads, etc., should be noted by distances in a similar manner.

The number of acres in farms whose lines are met may, very properly, be made a matter of memorandum

The following is a specimen of the form of notes which are taken, in accordance with the above suggestions:

TRANSIT NOTES.

Sta.	Bearing.	Distance of Course.	Remarks.
0	S. 70° E.		0. A point 10 ch. E. and 7.15 ch. N. of ¼ post on line bet. Secs. 11 and 14, T——, R——
1	"		
2	"		W. Oak 15, N. 23½° E., 57 ft.; Hickory 12, S. 40° E., 34 ft.
3	"		Land owned by John Doe, 80 A.; about 6 A. wet.
4	"		
5	"		
5^{28}	S. 28½° E.	528 ft.	5^{28}. 1st Angle.
6	"		
7	"		
8	"		
8^{40}	"		8^{40} Line bet. Secs. 13 and 24.
9	"		B. Oak 10, S. 35¼° W., 10 ft.; W. Oak 18, N. 63° W., 28 ft.
10	"		Richard Rowe, 160 A. on south, 30 A. swamp.
11	"		
11^{80}	East.	652 ft.	11^{80}. 2d Angle.
12	"		
13	"		
.	"		
.	"		
.	"		
23	"		
23^{43}	"	1163 ft.	23^{43}. Terminus in drain by road side on Township line.
			Marked Boulder, N. 20° E., 15 ft., Ash 14, S. 27° W., 10 ft.

15. Taking the Levels.—The line of the drain having been established, the next thing is to take the levels. This is done in the manner previously described. Beside the engineer or principal surveyor, two men are required—a rod-man, and an ax-man to make and drive pegs.

The pegs should be driven down even with the surface of the ground and at such a distance from the stakes marking the stations that they may be used without disturbance in excavating. Some practice driving them, say six inches, in front of the stakes; others set them opposite and at such a uniform distance from the record stakes as not to be disturbed by the digging.

Bench marks should be made at convenient distances, for example at every tenth station, and far enough from the line not to be disturbed.

16. Platting.—The field work having been completed, the next thing is to make a plat of the line and also of the sections or tracts of land which will be affected by the drain, writing the owner's name and number of acres on each. On some convenient part of the plat, the courses and their corresponding distances should be noted, also the number of linear feet of drain on each separate tract.

Next comes the drawing of the profile. This is most conveniently done by use of paper, called **Profile** paper, prepared specially for the purpose. Taking a piece of the proper width and of sufficient length to contain also the title and necessary explanatory notes, at the left hand, we begin on the edge next to us and write the numbers of all the stations in their order toward the right, upon the vertical lines. We then mark with the point of a sharp pencil the point of elevation of each station as taken from the column of elevations in the level notes. Connecting the points thus marked, by an ink line, we have the profile of the surface of the ground on the line of the drain. We then take a black thread and stretch it on the profile between the points assumed *as grade*, at the first and the last station. From this inspection, it will be seen whether it is necessary or desirable to introduce one or more changes of grade between the extreme points in order to avoid objectionable cuts.

Having determined the situation of the grade lines, we then draw them in their places, preferably with red ink.

Under the grade lines and upon the vertical lines of the several stations should be written in red ink the elevations of the grade, and below that, in black ink, the elevations of the surface. In a similar manner, above the profile may be written first, in red ink, the depths of the cuts, and, second, the widths of the ditch at bottom and top.

The names of the land owners through whose land the ditch passes, with the number of linear feet on each, may be conveniently written upon the datum line.

17. The writer has saved himself and assistants a great many miles of tramping and wading through swamps and morasses in drainage surveys by running the transit and level lines for the drains both at one operation. It was found by repeated tests on long lines that the level on the transit gave very nearly if not quite as accurate results in leveling as the wye level. Hence the wye level was left at home and the transit line and levels were both run at the same time with the transit. A condensed form of keeping the notes was used, of which the following is a sample extract:

Commencing at a point in the Section line 4.53 chains east of the quarter post between Sections 11 and 14, and running thence S. 16° W. Stations 2.00 chains apart.

Sta.	Obs.	Ht. Inst.	Elev.	Grade Ht.	Cut.	Remarks.
B. S. on B. M.	4.96	104.96	100.00			On Elm 40′ to rt. of Sta. 1.
0	5.21		99.75	96.00	3.75	Elm and Black Ash.
1	5.30		99.66	95.90	3.76	
2	5.28		99.68	95.80	3.88	+50, enter thick Willows.
3	5.46		99.50	95.70	3.80	
4	5.72		99.24	95.60	3.64	
5	5.83		99.13	95.50	3.63	
+60	Angle rt. 12° 24′ = S. 28° 24′ W.					Cross line fence between Smith and Jones.
C	5.84		99.12			
B. S.	2.91	102.03				
6	2.95		99.08	95.40	3.68	Open marsh. Saw grass.
7	3.06		98.97	95.30	3.67	

All the rod readings are kept in one column. The back or plus sights, to be added to the elevation for height of instrument, are marked "B. S." The others are all to be subtracted from "Ht. Inst." for elevation of stations.

18. Depth and Width.—The depth of a drain obviously depends upon the situation of the grade line with respect to the surface. In adjusting the grade line it is more important to guard against the drain being too shallow rather than too deep; most open drains are too shallow.

Again, it should be taken into account, if the drain is to run through soft marshes and hard ridges, that the soft ground, on the withdrawal of the water, will settle; and so the drain may need to be dug deeper in some places than would otherwise be necessary.

The necessary width of a drain of given depth and grade depends upon the quantity of water it is required to discharge in a given time.

The width at the top is determined from the width at the bottom and the slope or inclination given the sides, which is usually from one to one and one-half feet on the horizontal to each foot in depth.

19. Quantity of Discharge.—The amount of water which a drain may discharge in a given time obviously depends upon the area of the water-way or cross-section of the drain and the velocity of the stream.

Thus, denoting by Q the quantity of discharge, by a the area of the water-way, and by v the mean velocity of discharge, we should have

$$Q = av \quad (1)$$

As an approximate formula for computing the mean velocity of water flowing in an open canal of uniform cross section and fall, Trautwine gives the formula

$$V = \left\{\frac{af \times 8975}{p}\right\}^{\frac{1}{2}} - .1089 \quad (2)$$

in which $V =$ mean velocity in feet per second, $a =$ area of water-way in square feet, $f =$ fall in feet per foot, and $p =$ wet perimeter or the water border of the channel.

REMARK.—In applying the above formula, it is customary to use 9000 for 8975 and .11 for .1089.

Example.—Required the velocity and the capacity of a drain 5 ft. wide at the bottom, the sides having a slope of 1 to 1, depth of water 3 ft., and the fall 2 ft. to 1000 ft.

Solution.—Width at top $= 5$ ft. $+ 2 \times 3$ ft. $= 11$ ft.
Area of water-way $= 1\frac{1}{2}$ (11 ft. + 5 ft.) $= 24$ sq. ft.
Wet perimeter $= 5$ ft. $+ 6\sqrt{2}$ ft. $= 13.5$ ft.
Fall per foot $= 0.002$ ft.
Substituting in (2), $V = \left\{\frac{24 \times 0.002 \times 9000}{13.5}\right\}^{\frac{1}{2}} - 0.11$
$= 5.55.$

Substituting in (1), $Q = 24 \times 5.55 = 133.2$ cu. ft. per second, or 11,508,480 cu. ft. per day.

Trautwine gives also the following formula, with the remark that it is applicable also to sewers:

$$V = \left\{ \frac{a}{p} \times 2\,F \right\}^{\frac{1}{2}} \qquad (3)$$

in which a and p are as above described, and F is the fall in feet per mile.

REMARK.—In connection with the above formulas, as well as with others of similar import, Trautwine repeats again and again the caution that they are to be regarded only as approximately true.

Table XII shows approximately the number of acres served by drains having bottom widths of 1 to 10 ft., with side slopes of 1 to 1, and various rates of fall per station, on the supposition of 1 inch rain-fall in 24 hours, one-half of which reaches the drain.

20. Amount of Rainfall.—All calculations of requisite capacity of drains must be based upon the probable amount or number of inches of rainfall in a given time. The soil, however, acts as a reservoir up to the point of saturation, depending upon its texture, keeping from the drains altogether a portion of the rainfall, which passes off by evaporation or is absorbed by plants.

The average annual rainfall in Michigan, Indiana, Illinois and Missouri is about 35 inches. In Ohio, for a period of ten years, it was reported to be 37.86 inches.

In the matter of rainfall in Michigan, we are indebted to Prof. Carpenter for the following data:

"By a consultation of the meteorological records of the Agricultural College we learn that, although large showers in which the rainfall exceeds one inch occur comparatively seldom (on the average only four times a year), yet they bring with them twenty-eight per cent. of our total rainfall during that period, and consequently they must be fully provided for in any works for thorough drainage. The following table is compiled from the meteorological records kept at the college, and shows the comparative depth and number of showers from the months of March to December for five years. The last column shows the total percentage of rainfall in all the showers of a given depth. The last column but one shows the total percentage of the number of showers compared with the whole number. Although this table is not extended sufficiently far back to give very accurate results, it is thought (since one year's rainfall does not differ greatly from that of another year) to be sufficiently reliable to produce data for any ordinary case of farm drainage in this part of the United States

TABLE OF SHOWERS FROM MARCH TO DECEMBER.

Depth of Rainfall in Inches.	Number of Showers.						Percentage of Total.	
	1872	1873	1874	1875	1876	Total	No. of Showers.	Am't of Rainfall.
.00 to .25	19	40	28	35	43	165	54.2	17
.25 to .50	20	14	13	9	11	67	22.0	21
.50 to .75	6	8	6	10	5	35	11.5	21
.75 to 1.00	2	6	5	2	3	18	06.0	13
1.00 to 1.25	-----	-----	-----	-----	2	2	00.7	2
1.25 to 1.50	3	2	-----	-----	3	8	02.6	9
1.50 to 1.75	1	-----	1	1	-----	3	01.0	4
1.75 to 2.00	-----	1	-----	1	-----	2	00.7	3
2.00 to 2.25	-----	-----	-----	-----	-----	-----	-----	-----
2.25 to 2.50	-----	1	-----	-----	-----	1	00.3	2
2.50 to 2.75	1	-----	-----	-----	-----	1	00.3	2
2.75 to 3.00	-----	-----	-----	-----	-----	-----	-----	-----
3.00 to 3.25	-----	-----	-----	1	1	2	00.7	6
Totals	-----	-----	-----	-----	-----	304	100.00	100

"The amount of discharge of drains as compared with the rainfall is usually estimated at about 50 per cent. So that in order to produce thorough drainage it is necessary to assume that the capacity of the drains shall be sufficient to carry off during twenty-four hours one-half the water that fell the previous twenty-four hours. The probability of the rainfall in any day exceeding one inch is so slight that we shall be safe in assuming as the necessary carrying capacity of drains one-half of 3,630 cu. ft., or 1,815 cu. ft. of water for each acre drained."

21. Under Drains are formed in various ways; sometimes of brush, rails or loose stone trenched in, sometimes of tubes made of logs or of iron, sometimes of plank or of brick or stone laid in cement, and again of earthen tubes, of which there are various forms, called **Tiles.**

The prevailing method of under-drainage for agricultural purposes consists in the use of cylindrical tiles, which are made of different sizes and usually about a foot in length.

It is of this form of under drain, only, that we propose to write briefly.

22. Surveying for Under Drains.—Very much of what has been said upon surveying for the ditch or open drain applies also to the tile drain. The same preliminary inspection is required to determine the best location of the outlet and the proper directions of trunk and branch lines. Indications as to source of water, whether from springs on the premises or on lands situated above, whether from rainfall, merely, upon the particular tract or also as flowing off from neighboring areas; the directions of slopes, whether of surface or of underlying strata; the character of the soil, etc., all have to be carefully observed and their bearing duly considered.

23. Location of Drains.—As above intimated, any well conducted survey for under drains contemplates the execution of a *system* of drains working together and depending upon each other. This will include usually a principal drain, called a **Main,** and lateral drains, called **Minors,** which discharge into the main. In an extended system, auxiliary mains called **Sub-Mains** are also introduced.

Since it is the direct office of the minors to remove the surplus water from the ground, it is of the first importance that they be so located as successfully to perform their functions. To do this requires the exercise of careful judgment on the part of the engineer, respecting the proper directions of the minors and also their distances from each other. Equal care is requisite also in regard to the location of the main, so as properly to receive the water from the minors and discharge it at the principal outlet.

As a rule, the main should be located at the foot of the regular slopes, or along the valleys of the field; and, in general, the minors should run directly down the slopes, discharging themselves obliquely into the main.

Cases, however, will sometimes occur that require departure from the above rules, but these are to be regarded as "exceptions which prove the rule."

The distances of the minors from each other will be governed largely by the character of the soil as to permeability, and to some extent by the depth of the drains. In a porous soil, as a general rule, the deeper the drain the further it will draw.

Circumstances are infinitely varied. Every situation is a new one and must be treated on its own merits. None but the most general instruction on this point can be given in any treatise. About as practical a suggestion as may be afforded the student is, *Go into the field and there mix plenty of brains with your work.*

24. Running the Lines.—Having settled the question of the proper system of drains to be adopted, the next thing to be done is to lay out and measure the lines. This is perhaps most conveniently done in the case of under drains, by beginning at the outlet, measuring and staking out, first, the main lines of the system and then the branches.

A distance of 50 ft. between stations is a convenient one in tile draining. In some instances, as where the fall is very slight, a less distance may be desirable; in others a greater one may give equally good results. In addition to the stakes driven at the uniform distances of the stations, a stake should mark the entrance of each minor, and the distance to it should be entered in the notes, in the usual manner. Such stakes mark the points of beginning in running out the minors.

To facilitate examinations for "faults," the points of entrance of the branches in the main drain should be established by witnesses.

25. Taking the Levels.—This is done in the same manner as in the case of open drains, but, perhaps, with a somewhat greater degree of care and precision. The point assumed for the outlet must, of course, be sufficiently low to receive all the water of the field; and at the same time the outlet ought to be high enough to be at all times above the back water of the stream into which the drain empties. A drain is of little more use under a violation of the latter condition than under a disregard of the former.

In assuming the grade, due consideration must be had for proper depth consistently with required fall.

The depth of an under drain should be, at the least, two feet; all the better if three or four feet in most soils.

Henry F. French, author of "Farm Drainage," says: "We cannot, however, against the overwhelming weight of authority, and against the reasons for deeper drainage, which to us seem so satisfactory, conclude that even three feet is, in general, deep enough for under drains. Three-foot drains will produce striking results on almost any wet lands, but four-foot drains will be more secure and durable, will give wider feeding-ground to the roots, better filter percolating water, warm and dry the land earlier in Spring, furnish a larger reservoir for heavy rains, and, indeed, more effectually perform every office of drains."

Accordingly, the rule should be to approximate as closely as possible to what are thus regarded as desirable depths, admitting depths very much below the standard only when we *must*, in order to have any drains at all.

Upon the question of necessary amount of fall, with which the surveyor is so often confronted in connection with the requirement of desirable depths, it is to be observed in the first place that large, deep streams require less fall than small ones; and, again, the form and the condition of the channel have much to do with the movement of water.

"It has been found in practice that a water-course thirty feet wide and six feet deep will flow at the rate of one mile per hour, with a fall of no more than *six inches per mile*."

Examples are cited of successful operation of drains with three inches or even two and one-half inches fall to one hundred feet.

These, however, are to be regarded, probably, as exceptional cases or as presenting, perhaps, the lowest limit that, even under the most favorable conditions of ordinary drainage, ought to be attempted.

A very excellent authority says: "As to the fall necessary in tile draining, I consider one foot in one hundred yards the least fall to work upon with safety."

The above considerations will be perceived to bear upon the situation of the grade line, in order, on the one hand, to avoid too shallow drains, and, on the other, to secure the requisite fall for the proper movement of the water.

Changes of grade, though undesirable, are admissible when not easily avoided. If possible, the heaviest grades should be in the direction of the outlet. When this cannot be, it may be desirable to introduce silt-wells at points of any considerable change of grade.

The heights of the outlets of minor drains into the main are usually the heights of grade in the main drain for the same points.

26. Constructing the Drain.—The principal point is the method of opening the trench and laying the tiles on the grade line.

To do this systematically requires a measuring rod six or eight feet in length divided into feet, tenths, and hundredths of feet, the larger divisions being numbered upward, as in the ordinary leveling rod. A cord or wire, also, is needed, which is to be stretched above the line of the drain and adjusted to a position parallel to the grade line. This is done by inverting the measuring rod on the grade peg and bringing the cord or wire to the division of the rod indicating the cut at that point. The cord is thus placed at the full length of the measuring rod from the grade line or intended bottom of the trench.

The cord may be held each fifty or one hundred feet by two slats, each about seven feet long, and movable about a bolt passing through a little distance from the upper end. These are called **Shears.** The cord or wire is prevented from slipping by a couple of turns, and is tied to a stake eight or ten feet from the shears.

Another device consists in the use of stakes or posts driven on opposite sides of the ditch, and connected with a cross-bar arranged so that either end may be raised or lowered to a level, and fastened to the posts by a clamp and thumb-screw. The cross-bars being adjusted to the proper height, as above described, the cord or wire is drawn tightly across them, directly over the center line of the drain.

Again, single stakes or posts, driven on one side of the ditch, each having attached at right angles an arm which may be raised or lowered, and secured in place by a clamp and screw, are sometimes employed.

By such means as the above, the ditch is readily dug to just the proper depth, and the tile laid to grade with exceeding accuracy and with great rapidity. The proper distance from the top of the tile to the cord may be indicated by an arm attached to the measuring rod.

27. Size of Tile.—The size of tile required in a given case will depend upon the quantity of water to be removed and the fall available to remove it. Formulas are given in works upon hydraulics, to express the velocity and discharge of water flowing in pipes, but the conditions are so different in case of tiles that such formulas, at best, give only the most roughly approximate results.

Thus, for example, the following, which is Poncelet's formula:

$$V = 48 \left\{ \frac{D \times H}{L + 54\, D} \right\}^{\frac{1}{2}}$$

in which, $V =$ approximate velocity in feet per second, $D =$ diameter of pipe in feet, $H =$ total head in feet, and $L =$ total length of pipe in feet.

Having found the velocity, we have

Discharge in cu. ft. = vel. × cross-section of pipe.

Tables XII and XIII are used for the above purpose, the latter quite extensively by drainage engineers and has been found to give good results.

As regards size of tile for main and sub-main drains a good authority says, "*that* can be regulated only by the person in charge of the drainage at any particular place, after seeing the land opened up and the minor drains discharging, As a rule, a circular pipe of three inches internal diameter will discharge the *ordinary* drainage of six statute acres, and give sufficient space for the circulation of the air."

This estimate is based upon an amount of annual rainfall of from twenty-six to thirty inches, which differs but slightly from that of Michigan and adjoining states.

In addition to the above, it may be remarked that if the fall in the main is slight, a larger size of tile would be required than if the fall was considerable.

And, again in order to provide suitably for the accumulation of water which occurs toward the outlet, a larger size may be there required than that used in the upper part of the main.

28. Protection at Outlets.—The outlets of underdrains should be protected by some construction to prevent the earth from falling down in front of the drain. A retaining wall of masonry laid in hydraulic cement is the best provision for the purpose. The outlets should be protected also by a coarse grating of some sort in front of the tile to prevent muskrats and other creatures from getting in.

A common practice is to introduce at the outlet a box made of plank a few feet in length, into which the tile is made to discharge.

29. Silt Well.—This is a well sunk below the level of the tile for catching the silt gathered by the drains above it. It serves also the purpose of affording a means of inspecting the working of the drains. Silt wells may be constructed with a view, chiefly, to facilitating the movement of the water at an abrupt bend in the drain. And again, they may be constructed somewhat with reference to convenience of obtaining a pail of water for any purpose, in the field.

A B C D E F G H I J K L
M N O P Q R S T U V W
X Y Z

a b c d e f g h i j k l m n
o p q r s t u v w x y z

1 2 3 4 5 6 7 8 9 0 . , -

A B C D E F G H I J K
L M N O P Q R S T U V
W X Y Z

a b c d e f g h i j k l m n
o p q r s t u v w x y z

1 2 3 4 5 6 7 8 9 0 . , -

A B C D E F G H I J K L M

N O P Q R S T U V W X Y Z

a b c d e f g h i j k l m n o p q

r s t u v w x y z

1 2 3 4 5 6 7 8 9 0 . ,

A B C D E F G H I J K

L M N O P Q R S T U

V W X Y Z

a b c d e f g h i j k l m n

o p q r s t u v w x y z

1 2 3 4 5 6 7 8 9 0 . , -

A B C D E F G H I J K L M

N O P Q R S T U V W X Y Z

a b c d e f g h i j k l m n o p q r

s t u v w x y z

1 2 3 4 5 6 7 8 9 0 . , -

TABLES.

SUGGESTIONS TO YOUNG SURVEYORS ON THE USES OF THE TABLES.

Traverse Table.—The table calculated to quarter degrees is adapted to the simplest work of compass surveying, where great accuracy is neither required nor expected. When the transit is used, and the angles are taken to minutes or less, the author prefers the tables of logarithms and logarithmic sines and cosines to any traverse table yet made. They are capable of any required degree of accuracy, and require the use of no more figures than the ordinary traverse table. In transit work, where latitudes and departures are to be calculated, it is well to refer the angles of all lines to a common base, just as in compass surveying all lines are referred to the meridian as a base. Then, in any course,

Latitude = co-sine of angle × length of the course.

Departure = sine of angle × length of the course.

Using the logarithmic tables, this is a short and simple computation.

Example 1.—Angle, 36° 22′. Distance, 47.63. Required the latitude and departure.

Log. of 47.63 = 1.677881 to which add
log. sine, 36° 22′ = 9.773018

11.450899 the log of 28.24 + = departure.

Log. of 47.63 = 1.677881 to which add
log. cos., 36° 22′ = 9.905925

11.583806 the log. of 38.35 + = latitude.

2. Course N. 57° 21′ 20″ E. 34.36½ chains. Required the latitude and departure.

1. **The Table of Tangents** is convenient in estimating courses of lines to be run.

Example 1.—From the quarter post on the east side of Section 2 I wish to run a line for a road straight to a point 80 rods north of the southwest corner of Section 30. What course shall I run?

Solution.—Distance west, 5 miles; distance south, 4.25 miles, which divided by 5 equals the natural tangent of the angle which the course makes with an east and west line, = .850. Find this number in the table of natural tangents and take out the corresponding angle, = 40° 22′, which is the same as S. 49° 38′ W.

2. What is the course from the village of Climax, at the east quarter post of Section 3, Township 3 south, Range 9 west, to the village of Richland, at the southwest corner of Section 14, Township 1 south, Range 10 west? To the village of Schoolcraft, at the southeast corner of Section 19, T. 4 S., R. 11 W., from Climax? What to Schoolcraft from Richland?

2. **The Table of Secants** is convenient for finding the hypothenuse of a triangle, thus simplifying many computations in the field. Secants not given in the table may be found by interpolation or by the formula:

$$\text{Secant} = \frac{1}{\text{cosine}}.$$

The following example indicates one of the practical applications in the field:

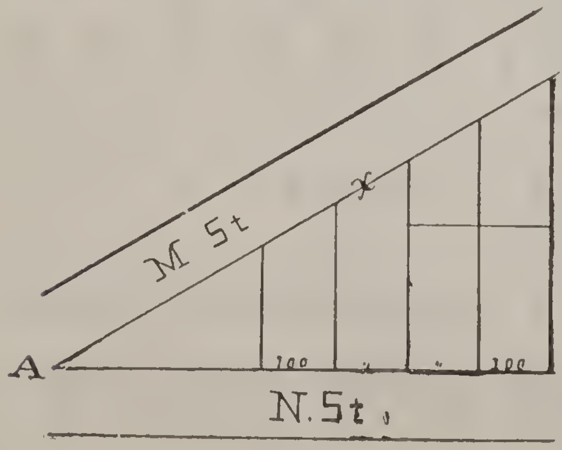

FIG. 81.

Example.—Lots in a city are laid out with their lines perpendicular to N Street and running through to M Street. Required the width (x) of the lots on M Street.

Call the width of the lots on N Street r. Measure the angle A.

Then $x = r$, sec. A. If $r = 100$, as is common, x may be taken directly from the table. If $r = 100$, $A = 21° 40'$, then $x = 107.6$. In laying out such lots it is generally easier and quicker to measure this distance on the street line than it is to set up the transit for each lot line and run it in.

3. **Table of Departures.**—This table has many convenient uses, of which a few examples are given.

Examples.—1. I wish to stake out a line along an old hedge row from quarter-post to section corner. On one side is a clear field. I go to the section corner, and make an offset of 25 links and set up a flag. I then go to the quarter-post, and, making an equal offset, find that I cannot see the flag; so I offset until I can see it—say 37 links more. I sight to the flag, find from the table of departures the angle corresponding to 37 links at a distance of 40 chains = 32′, turn off the angle on the transit, and run the line back parallel with the section line, setting stakes on the true line, by 62 link offsets, as often as required.

2. To run a true half-quarter-line when one end is inaccessible.

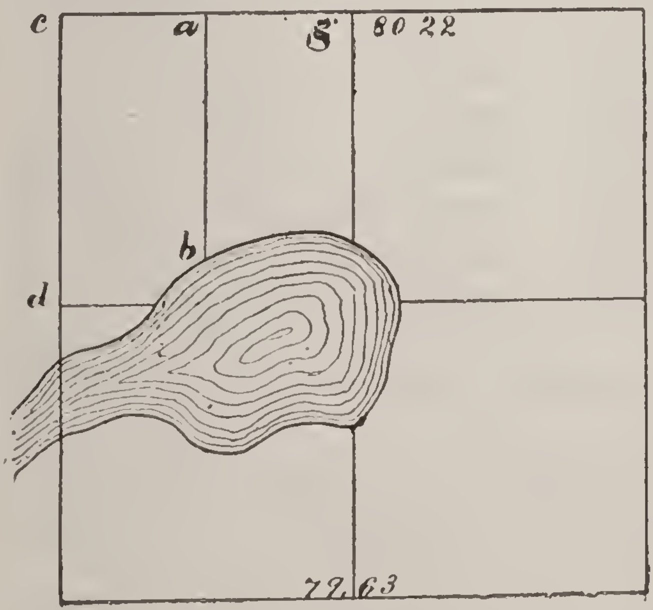

FIG. 82.

Fig. 82 represents the whole section, and ab the line to be run.

Bisect cg, setting stake at a. Measure the angle acd, which we will call 89° 24′. By the field notes the north line of the section measures 80.22, hence $ac = 20.05\frac{1}{2}$. The south line measures

79.63, one-fourth of which is $19.90\frac{3}{4}$. Hence the section line and half-quarter-line converge at the rate of 20.055 — 19.9075 = .1475 chains per mile. From the table of departures we find the corresponding angle to be a little more than 6′. Hence we make the angle *gab* 6′ greater than *acd* = 89° 30+′, and run the line accordingly.

The foregoing are given as samples of many labor-saving uses of the tables, which the young surveyor should study out and be prompt to avail himself of when the occasion requires.

TRIGONOMETRIC FUNCTIONS AND FORMULÆ.

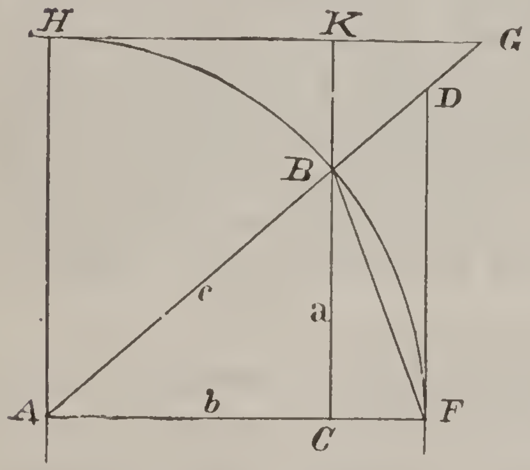

Fig. 83.

Let Fig. 83 represent the various trigonometric functions. Let *ABC* represent the angles, and *abc* the sides opposite in the right triangle formed by the radius, sine and cosine. Other parts as shown in the figure.

Then $\sin A = BC$ $\cos A = AC$

$\tan A = DF$ $\cot A = HG$

$\sec A = AD$ $\operatorname{cosec} A = AG$

$\operatorname{versin} A = CF$ $\operatorname{coversin} A = BK$

$\operatorname{exsec} A = BD$ $\operatorname{coexsec} A = BG$

$\operatorname{chord} A = BF$ $\operatorname{chord} 2A = 2\,BC.$

Tables of these functions are calculated with radius $AH = 1$.

$$\operatorname{Sin} A = \frac{a}{c} = \cos B \qquad \cos A = \frac{b}{c} = \sin B$$

$$\operatorname{Tan} A = \frac{a}{b} = \cot B \qquad \cot A = \frac{b}{a} = \tan B$$

$$\text{Sec } A = \frac{c}{b} = \text{cosec } B \qquad \text{cosec } A = \frac{c}{a} = \text{sec } B$$

$$\text{Vers } A = \frac{c-b}{c} = \text{covers } B \qquad \text{coversin } A = \frac{c-a}{c} = \text{vers } B$$

$$\text{Exsec } A = \frac{c-b}{b} = \text{coexsec } B \qquad \text{coexsec } A = \frac{c-a}{a} = \text{exsec } B$$

$$a = \begin{cases} c \sin A = b \tan A \\ c \cos B = b \cot B \\ \sqrt{(c+b)(c-b)} \end{cases} \qquad b = \begin{cases} c \cos A = a \cot A \\ c \sin B = a \tan B \\ \sqrt{(c+a)(c-a)} \end{cases}$$

$$c = \begin{cases} \dfrac{a}{\sin A} = \dfrac{b}{\cos A} \\ \dfrac{a}{\cos B} = \dfrac{b}{\sin B} \\ \sqrt{a^2 + b^2} \end{cases} \qquad C = 90° = A + B. \qquad \text{Area} = \frac{ab}{2}.$$

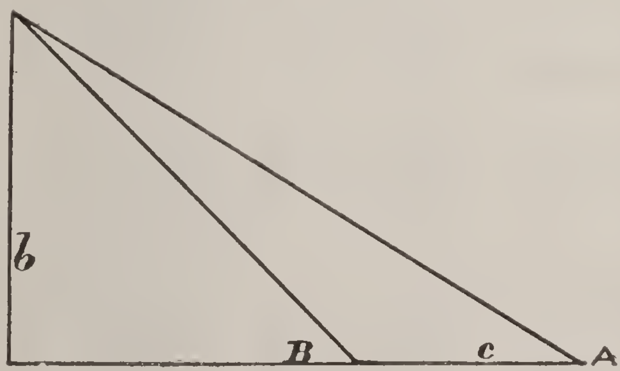

FIG. 84.

$$b = \frac{c}{\cot A - \cot B}.$$

Useful in measuring heights of objects or passing obstacles in line.

SOLUTION OF OBLIQUE TRIANGLES.

Let ABC represent the angles, and abc the opposite sides, of any oblique triangle.

Given.	Sought.	
A, B, a	$C, b,$	$C = 180° - (A + B)$. $b = \dfrac{a}{\sin A} \sin B$.
	c	$c = \dfrac{a}{\sin A} \sin A + B$.
A, a, b	$B, C,$	$B = \dfrac{\sin A}{a} b$. $C = 180 - (A + B)$.
	c	$c = \dfrac{a}{\sin A} \sin C$.
C, a, b	$\frac{1}{2}(A+B)$	$\frac{1}{2}(A + B) = 90° - \frac{1}{2} C$.
	$\frac{1}{2}(A-B)$	$\tan \frac{1}{2}(A - B) = \dfrac{a-b}{a+b} \tan \frac{1}{2}(A + B)$.
	A	$A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B)$.
	B	$B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B)$.
	c	$c = (a + b) \dfrac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} = (a - b) \dfrac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)}$.
	Area	Area $= K = \frac{1}{2} ab \sin C$.
a, b, c	A	Let $s = \frac{1}{2}(a + b + c)$, then $\sin \frac{1}{2} A = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$. $\cos \frac{1}{2} A = \sqrt{\dfrac{s(s-a)}{bc}}$. $\tan \frac{1}{2} A = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$. $\sin A = \dfrac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$. $\text{versin } A = \dfrac{2(s-b)(s-c}{bc}$
	Area	Area $= \sqrt{s(s-a)(s-b)(s-c)}$.
A, B, C, a	Area	Area $= \dfrac{a^2 \sin B \sin C}{2 \sin A}$.

TABLES.

LOGARITHMS OF NUMBERS

FROM

1 TO 10000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0 000000	26	1 414973	51	1 707570	76	1 880814
2	0 301030	27	1 431364	52	1 716003	77	1 886491
3	0 477121	28	1 447158	53	1 724276	78	1 892095
4	0 602060	29	1 462398	54	1 732394	79	1 897627
5	0 698970	30	1 477121	55	1 740363	80	1 903090
6	0 778151	31	1 491362	56	1 748188	81	1 908485
7	0 845098	32	1 505150	57	1 755875	82	1 913814
8	0 903090	33	1 518514	58	1 763428	83	1 919078
9	0 954243	34	1 531479	59	1 770852	84	1 924279
10	1 000000	35	1 544068	60	1 778151	85	1 929419
11	1 041393	36	1 556303	61	1 785330	86	1 934498
12	1 079181	37	1 568202	62	1 792392	87	1 939519
13	1 113943	38	1 579784	63	1 799341	88	1 944483
14	1 146128	39	1 591065	64	1 806180	89	1 949390
15	1 176091	40	1 602060	65	1 812913	90	1 954243
16	1 204120	41	1 612784	66	1 819544	91	1 959041
17	1 230449	42	1 623249	67	1 826075	92	1 963788
18	1 255273	43	1 633468	68	1 832509	93	1 968483
19	1 278754	44	1 643453	69	1 838849	94	1 973128
20	1 301030	45	1 653213	70	1 845098	95	1 977724
21	1 322219	46	1 662758	71	1 851258	96	1 982271
22	1 342423	47	1 672098	72	1 857332	97	1 986772
23	1 361728	48	1 681241	73	1 863323	88	1 991226
24	1 380211	49	1 690196	74	1 869232	99	1 995635
25	1 397940	50	1 698970	75	1 875061	100	2 000000

No.	0	1	2	3	4	5	6	7	8	9	Diff.
100	000000	000434	000868	001301	001734	002166	002598	003029	003461	003891	432
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
2	8600	9026	9451	9876	010300	010724	011147	011570	011993	012415	424
3	012837	013259	013680	014100	4521	4940	5360	5779	6197	6616	419
4	7033	7451	7868	8284	8700	9116	9532	9947	020361	020775	416
5	021189	021603	022016	022428	022841	023252	023664	024075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789	030195	030600	031004	031408	031812	032216	032619	033021	404
8	033424	033826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811	040207	040602	040998	396
110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
2	9218	9606	9993	050380	050766	051153	051538	051924	052309	052694	386
3	053078	053463	053846	4230	4613	4996	5378	5760	6142	6524	382
4	6905	7286	7666	8046	8426	8805	9185	9563	9942	060320	379
5	060698	061075	061452	061829	062206	062582	062958	063333	063709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8186	8557	8928	9298	9668	070038	070407	070776	071145	071514	369
8	071882	072250	072617	072985	073352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426	360
1	082785	083144	083503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905	090258	090611	090963	091315	091667	092018	092370	092721	093071	351
4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
5	6910	7257	7604	7951	8298	8644	8990	9335	9681	100026	346
6	100371	100715	101059	101403	101747	102091	102434	102777	103119	3462	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916	110253	338
9	110590	110926	111263	111599	111934	112270	112605	112940	113275	3609	335
130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940	333
1	7271	7603	7934	8265	8595	8926	9256	9586	9915	120245	330
2	120574	120903	121231	121560	121888	122216	122544	122871	123198	3525	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7105	7429	7753	8076	8399	8722	9045	9368	9690	130012	323
5	130334	130655	130977	131298	131619	131939	132260	132580	132900	3219	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	315
8	9879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911	309
1	9219	9527	9835	150142	150449	150756	151063	151370	151676	151982	307
2	152288	152594	152900	3205	3510	3815	4120	4424	4728	5032	305
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
4	8362	8664	8965	9266	9567	9868	160168	160469	160769	161068	301
5	161368	161667	161967	162266	162564	162863	3161	3460	3758	4055	299
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	176091	176381	176670	176959	177248	177536	177825	178113	178401	178689	289
1	8977	9264	9552	9839	180126	180413	180699	180986	181272	181558	287
2	181844	182129	182415	182700	2985	3270	3555	3839	4123	4407	285
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
4	7521	7803	8084	8366	8647	8928	9209	9490	9771	190051	281
5	190332	190612	190892	191171	191451	191730	192010	192289	192567	2846	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755	200029	200303	200577	200850	201124	274
9	201397	201670	201943	202216	202488	2761	3033	3305	3577	3848	272
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
160	204120	204391	204663	204934	205204	205475	205746	206016	206286	206556	271
1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
2	9515	9783	210051	210319	210586	210853	211121	211388	211654	211921	267
3	212188	212454	2720	2986	3252	3518	3783	4049	4314	4579	266
4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
6	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
9	7887	8144	8400	8657	8913	9170	9426	9682	9938	230193	256
170	230449	230704	230960	231215	231470	231724	231979	232234	232488	232742	254
1	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
3	8046	8297	8548	8799	9049	9299	9550	9800	240050	240300	250
4	240549	240799	241048	241297	241546	241795	242044	242293	2541	2790	249
5	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
7	7973	8219	8464	8709	8954	9198	9443	9687	9932	250176	245
8	250420	250664	250908	251151	251395	251638	251881	252125	252368	2610	243
9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	255273	255514	255755	255996	256237	256477	256718	256958	257198	257439	241
1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
2	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
6	9513	9746	9980	270213	270446	270679	270912	271144	271377	271609	233
7	271842	272074	272306	2538	2770	3001	3233	3464	3696	3927	232
8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	278754	278982	279211	279439	279667	279895	280123	280351	280578	280806	228
1	281033	281261	281488	281715	281942	282169	2396	2622	2849	3075	227
2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
5	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	300161	300378	300595	300813	218
200	301030	301247	301464	301681	301898	302114	302331	302547	302764	302980	217
1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	214
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	310056	310268	310481	310693	310906	311118	311330	311542	212
5	311754	311966	2177	2389	2600	2812	3023	3234	3445	3656	211
6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
9	320146	320354	320562	320769	320977	321184	321391	321598	321805	322012	207
210	322219	322426	322633	322839	323046	323252	323458	323665	323871	324077	206
1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
3	8380	8583	8787	8991	9194	9398	9601	9805	330008	330211	203
4	330414	330617	330819	331022	331225	331427	331630	331832	2034	2236	202
5	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
8	8456	8656	8855	9054	9253	9451	9650	9849	340047	340246	199
9	340444	340642	340841	341039	341237	341435	341632	341830	2028	2225	198
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220	342423	342620	342817	343014	343212	343409	343606	343802	343999	344196	197
1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
3	8305	8500	8694	8889	9083	9278	9472	9666	9860	350054	194
4	350248	350442	350636	350829	351023	351216	351410	351603	351796	1989	193
5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
9	9835	360025	360215	360404	360593	360783	360972	361161	361350	361539	189
230	361728	361917	362105	362294	362482	362671	362859	363048	363236	363424	188
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
4	9216	9401	9587	9772	9958	370143	370328	370513	370698	370883	185
5	371068	371253	371437	371622	371806	1991	2175	2360	2544	2728	184
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
9	8398	8580	8761	8943	9124	9306	9487	9668	9849	380030	181
240	380211	380392	380573	380754	380934	381115	381296	381476	381656	381837	181
1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
4	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
5	9166	9343	9520	9698	9875	390051	390228	390405	390582	390759	177
6	390935	391112	391288	391464	391641	1817	1993	2169	2345	2521	176
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	397940	398114	398287	398461	398634	398808	398981	399154	399328	399501	173
1	9674	9847	400020	400192	400365	400538	400711	400883	401056	401228	173
2	401401	401573	1745	1917	2089	2261	2433	2605	2777	2949	172
3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
7	9933	410102	410271	410440	410609	410777	410946	411114	411283	411451	169
8	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	414973	415140	415307	415474	415641	415808	415974	416141	416308	416474	167
1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
3	9956	420121	420286	420451	420616	420781	420945	421110	421275	421439	165
4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
9	9752	9914	430075	430236	430398	430559	430720	430881	431042	431203	161
270	431364	431525	431685	431846	432007	432167	432328	432488	432649	32809	161
1	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
5	9333	9491	9648	9806	9964	440122	440279	440437	440594	440752	158
6	440909	441066	441224	441381	441538	1695	1852	2009	2166	2323	157
7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
9	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
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No.	0	1	2	3	4	5	6	7	8	9	Diff.
280	447158	447313	447468	447623	447778	447933	448088	448242	448397	448552	155
1	8706	8861	9015	9170	9324	9478	9633	9787	9941	450095	154
2	450249	450403	450557	450711	450865	451018	451172	451326	451479	1633	154
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
7	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
8	9392	9543	9694	9845	9995	460146	460296	460447	460597	460748	151
9	460898	461048	461198	461348	461499	1649	1799	1948	2098	2248	150
290	462398	462548	462697	462847	462997	463146	463296	463445	463594	463744	150
1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
3	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
5	9822	9969	470116	470263	470410	470557	470704	470851	470998	471145	147
6	471292	471438	1585	1732	1878	2025	2171	2318	2464	2610	147
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	477121	477266	477411	477555	477700	477844	477989	478133	478278	478422	145
1	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
2	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958	490099	490239	490380	490520	490661	490801	490941	491081	491222	140
310	491362	491502	491642	491782	491922	492062	492201	492341	492481	492621	140
1	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962	500099	500236	500374	500511	500648	500785	500922	137
7	501059	501196	501333	1470	1607	1744	1880	2017	2154	2291	137
8	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	505150	505286	505421	505557	505693	505828	505964	506099	506234	506370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874	510009	510143	510277	510411	134
4	510545	510679	510813	510947	511081	511215	1349	1482	1616	1750	134
5	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	518514	518646	518777	518909	519040	519171	519303	519434	519566	519697	131
1	9828	9959	520090	520221	520353	520484	520615	520745	520876	521007	131
2	521138	521269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
5	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
8	8917	9045	9174	9302	9430	9559	9687	9815	9943	530072	128
9	530200	530328	530456	530584	530712	530840	530968	531096	531223	1351	128
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340	531479	531607	531734	531862	531990	532117	532245	532372	532500	532627	128
1	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
2	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
4	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
5	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
6	9076	9202	9327	9452	9578	9703	9829	9954	540079	540204	125
7	540329	540455	540580	540705	540830	540955	541080	541205	1330	1454	125
8	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
9	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	544068	544192	544316	544440	544564	544688	544812	544936	545060	545183	124
1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
4	9003	9126	9249	9371	9494	9616	9739	9861	9984	550106	123
5	550228	550351	550473	550595	550717	550840	550962	551084	551206	1328	122
6	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
7	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
8	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
9	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	556303	556423	556544	556664	556785	556905	557026	557146	557267	557387	120
1	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
2	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
3	9907	560026	560146	560265	560385	560504	560624	560743	560863	560982	120
4	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
5	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
7	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568202	568319	568436	568554	568671	568788	568905	569023	569140	569257	117
1	9374	9491	9608	9725	9842	9959	570076	570193	570309	570426	117
2	570543	570660	570776	570893	571010	571126	1243	1359	1476	1592	117
3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
4	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
5	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
6	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
7	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
8	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
9	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	579784	579898	580012	580126	580241	580355	580469	580583	580697	580811	114
1	580925	581039	1153	1267	1381	1495	1608	1722	1836	1950	114
2	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
9	9950	590061	590173	590284	590396	590507	590619	590730	590842	590953	112
390	591065	591176	591287	591399	591510	591621	591732	591843	591955	592066	111
1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
5	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
6	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
8	9883	9992	600101	600210	600319	600428	600537	600646	600755	600864	109
9	600973	601082	1191	1299	1408	1517	1625	1734	1843	1951	109
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No.	0	1	2	3	4	5	6	7	8	9	Diff.
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1	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
2	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
5	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
7	9594	9701	9808	9914	610021	610128	610234	610341	610447	610554	107
8	610660	610767	610873	610979	1086	1192	1298	1405	1511	1617	106
9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736	106
1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
5	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
6	9093	9198	9302	9406	9511	9615	9719	9824	9928	620032	104
7	620136	620240	620344	620448	620552	620656	620760	620864	620968	1072	104
8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
9	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	623249	623353	623456	623559	623663	623766	623869	623973	624076	624179	103
1	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
2	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
4	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
5	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
6	9410	9512	9613	9715	9817	9919	630021	630123	630224	630326	102
7	630428	630530	630631	630733	630835	630936	1038	1139	1241	1342	102
8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	633468	633569	633670	633771	633872	633973	634074	634175	634276	634376	101
1	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	101
2	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
3	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
4	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
5	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	99
6	9486	9586	9686	9785	9885	9984	640084	640183	640283	640382	99
7	640481	640581	640680	640779	640879	640978	1077	1177	1276	1375	99
8	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	643453	643551	643650	643749	643847	643946	644044	644143	644242	644340	98
1	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
5	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
6	9335	9432	9530	9627	9724	9821	9919	650016	650113	650210	97
7	650308	650405	650502	650599	650696	650793	650890	0987	1084	1181	97
8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	653213	653309	653405	653502	653598	653695	653791	653888	653984	654080	96
1	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
2	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
3	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
5	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
7	9916	660011	660106	660201	660296	660391	660486	660581	660676	660771	95
8	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
460	662758	662852	662947	663041	663135	663230	663324	663418	663512	663607	94
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
7	9317	9410	9503	9596	9689	9782	9875	9967	670060	670153	93
8	670246	670339	670431	670524	670617	670710	670802	670895	0988	1080	93
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
8	9428	9519	9610	9700	9791	9882	9973	680063	680154	680245	91
9	680336	680426	680517	680607	680698	680789	680879	0970	1060	1151	91
480	681241	681332	681422	681513	681603	681693	681784	681874	681964	682055	90
1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
9	9309	9398	9486	9575	9664	9753	9841	9930	690019	690107	89
490	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	89
1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
9	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	698970	699057	699144	699231	699317	699404	699491	699578	699664	699751	87
1	9838	9924	700011	700098	700184	700271	700358	700444	700531	700617	87
2	700704	700790	0877	0963	1050	1136	1222	1309	1395	1482	86
3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
510	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336	85
1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
2	9270	9355	9440	9524	9609	9694	9779	9863	9948	710033	85
3	710117	710202	710287	710371	710456	710540	710625	710710	710794	0879	85
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
520	716003	716087	716170	716254	716337	716421	716504	716588	716671	716754	83
1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
4	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
5	720159	720242	720325	720407	720490	720573	720655	720738	720821	0903	83
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	724358	724440	724522	724604	724685	724767	724849	724931	725013	82
1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
7	9974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
8	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
5	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
9	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	748266	748343	748421	748498	748576	748653	748731	748808	748885	77
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
2	9736	9814	9891	9968	750045	750123	750200	750277	750354	750431	77
3	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755875	755951	756027	756103	756180	756256	756332	756408	756484	756560	76
1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
5	9668	9743	9819	9894	9970	760045	760121	760196	760272	760347	75
6	760422	760498	760573	760649	760724	0799	0875	0950	1025	1101	75
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff
580	763428	763503	763578	763653	763727	763802	763877	763952	764027	764101	75
1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
5	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
8	9377	9451	9525	9599	9673	9746	9820	9894	9968	770042	74
9	770115	770189	770263	770336	770410	770484	770557	770631	770705	0778	74
590	770852	770926	770999	771073	771146	771220	771293	771367	771440	771514	74
1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	778151	778224	778296	778368	778441	778513	778585	778658	778730	778802	72
1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
2	9596	9669	9741	9813	9885	9957	780029	780101	780173	780245	72
3	780317	780389	780461	780533	780605	780677	0749	0821	0893	0965	72
4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	785330	785401	785472	785543	785615	785686	785757	785828	785899	785970	71
1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
5	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
6	9581	9651	9722	9792	9863	9933	790004	790074	790144	790215	70
7	790285	790356	790426	790496	790567	790637	0707	0778	0848	0918	70
8	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	792392	792462	792532	792602	792672	792742	792812	792882	792952	793022	70
1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
3	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
5	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	799341	799409	799478	799547	799616	799685	799754	799823	799892	799961	69
1	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	69
2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
640	806180	806248	806316	806384	806451	806519	806587	806655	806723	806790	68
1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
5	9560	9627	9694	9762	9829	9896	9964	810031	810098	810165	67
6	810233	810300	810367	810434	810501	810569	810636	0703	0770	0837	67
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	812913	812980	813047	813114	813181	813247	813314	813381	813448	813514	67
1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	819544	819610	819676	819741	819807	819873	819939	820004	820070	820136	66
1	820201	820267	820333	820399	820464	820530	820595	0661	0727	0792	66
2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	66
4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	826075	826140	826204	826269	826334	826399	826464	826528	826593	826658	65
1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
5	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
6	9947	830011	830075	830139	830204	830268	830332	830396	830460	830525	64
7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	832509	832573	832637	832700	832764	832828	832892	832956	833020	833083	64
1	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	838849	838912	838975	839038	839101	839164	839227	839289	839352	839415	63
1	9478	9541	9604	9667	9729	9792	9855	9918	9981	840043	63
2	840106	840169	840232	840294	840357	840420	840482	840545	840608	0671	63
3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
700	845098	845160	845222	845284	845346	845408	845470	845532	845594	845656	62
1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
8	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585	61
9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851809	61
1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	857332	857393	857453	857513	857574	857634	857694	857755	857815	857875	60
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978	860038	860098	860158	860218	860278	60
5	860338	860398	860458	860518	860578	0637	0697	0757	0817	0877	60
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	863323	863382	863442	863501	863561	863620	863680	863739	863799	863858	59
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	869232	869290	869349	869408	869466	869525	869584	869642	869701	869760	59
1	9818	9877	9935	9994	870053	870111	870170	870228	870287	870345	59
2	870404	870462	870521	870579	0638	0696	0755	0813	0872	0930	58
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	875061	875119	875177	875235	875293	875351	875409	875466	875524	875582	58
1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
8	9669	9726	9784	9841	9898	9956	880013	880070	880127	880185	57
9	880242	880299	880356	880413	880471	880528	0585	0642	0699	0756	57
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
760	880814	880871	880928	880985	881042	881099	881156	881213	881271	881328	57
1	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
5	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	886491	886547	886604	886660	886716	886773	886829	886885	886942	886998	56
1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	9918	9974	890030	890086	890141	890197	890253	890309	890365	56
7	890421	890477	890533	0589	0645	0700	0756	0812	0868	0924	56
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	892095	892150	892206	892262	892317	892373	892429	892484	892540	892595	56
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122	55
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
4	9821	9875	9930	9985	900039	900094	900149	900203	900258	900312	55
5	900367	900422	900476	900531	0586	0640	0695	0749	0804	0859	55
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	903090	903144	903199	903253	903307	903361	903416	903470	903524	903578	54
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	908485	908539	908592	908646	908699	908753	908807	908860	908914	908967	54
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	910037	53
3	910091	910144	910197	910251	910304	910358	910411	910464	910518	0571	53
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
820	913814	913867	913920	913973	914026	914079	914132	914184	914237	914290	53
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549	52
1	9601	9653	9706	9758	9810	9862	9914	9967	920019	920071	52
2	920123	920176	920228	920280	920332	920384	920436	920489	0541	0593	52
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	924279	924331	924383	924434	924486	924538	924589	924641	924693	924744	52
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	929419	929470	929521	929572	929623	929674	929725	929776	929827	929879	51
1	9930	9981	930032	930083	930134	930185	930236	930287	930338	930389	51
2	930440	930491	0542	0592	0643	0694	0745	0796	0847	0898	51
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
5	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	934498	934549	934599	934650	934700	934751	934801	934852	934902	934953	50
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
5	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	939519	939569	939619	939669	939719	939769	939819	939869	939918	939968	50
1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	50
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
880	944483	944532	944581	944631	944680	944729	944779	944828	944877	944927	49
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
5	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
8	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829	49
1	9878	9926	9975	950024	950073	950121	950170	950219	950267	950316	49
2	950365	950414	950462	0511	0560	0608	0657	0706	0754	0803	49
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	954243	954291	954339	954387	954435	954484	954532	954580	954628	954677	48
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959041	959089	959137	959185	959232	959280	959328	959375	959423	959471	48
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
2	9995	960042	960090	960138	960185	960233	960280	960328	960376	960423	48
3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903	47
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
3	9882	9928	9975	970021	970068	970114	970161	970207	970254	970300	47
4	970347	970393	970440	0486	0533	0579	0626	0672	0719	0765	46
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543	46
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
5	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135	46
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
5	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	45
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982678	45
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	986817	986861	986906	986951	986996	987040	987085	987130	987175	45
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
7	9895	9939	9983	990028	990072	990117	990161	990206	990250	990294	44
8	990339	990383	990428	0472	0516	0561	0605	0650	0694	0738	44
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625	44
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
No.	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE II.

NATURAL SINES AND COSINES.

	0°		1°		2°		3°		4°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	89°		88°		87°		86°		85°		

	5°		6°		7°		8°		9°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	84°		83°		82°		81°		80°		

M.	10°		11°		12°		13°		14°		M.
	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24953	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	79°		78°		77°		76°		75°		

	15°		16°		17°		18°		19°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	74°		73°		72°		71°		70°		

	20°		21°		22°		23°		24°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine,	Cos.	Sine.	M.
	69°		68°		67°		66°		65°		

	25°		26°		27°		28°		29°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	64°		63°		62°		61°		60°		

	30°		31°		32°		33°		34°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82748	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84345	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	59°		58°		57°		56°		55°		

	35°		36°		37°		38°		39°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79015	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	54°		53°		52°		51°		50°		

M.	40°		41°		42°		43°		44°		M.
	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	49°		48°		47°		46°		45°		

TABLE III.

NATURAL TANGENTS

AND

COTANGENTS.

	0°		1°		2°		3°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.00000	Infinite	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	. 0349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180 932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.96087	16.4283	31
30	.00873	114.589	.02619	38 1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	. 1425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	89°		88°		87°		86°		

	4°		5°		6°		7°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10 9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8 84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9·78817	.11983	8.34496	.13758	7 26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9 70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	85°		84°		83°		82°		

	8°		9°		10°		11°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	7.01174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4 81471	16
45	.15391	6.49710	.17183	5 81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	81°		80°		79°		78°		

M.	12° Tang.	12° Cotang.	13° Tang.	13° Cotang.	14° Tang.	14° Cotang.	15° Tang.	15° Cotang.	M.
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3 92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	77°		76°		75°		74°		

	16°		17°		18°		19°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	73°		72°		71°		70°		

	20°		21°		22°		23°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2 35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56486	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2 38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2 48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	69°		68°		67°		66°		

	24°		25°		26°		27°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	65°		64°		63°		62°		

	28°		29°		30°		31°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	8
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	61°		60°		59°		58°		

	32°		33°		34°		35°		
M.	Tang.	Cotang	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	57°		56°		55°		54°		

	36°		37°		38°		39°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
M.	Cotang.	Tang.	Cotang.	Tang,	Cotang.	Tang.	Cotang.	Tang.	M.
	53°		52°		51°		50°		

M.	40° Tang.	40° Cotang.	41° Tang.	41° Cotang.	42° Tang.	42° Cotang.	43° Tang.	43° Cotang.	M.
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
M.	49° Cotang.	49° Tang.	48° Cotang.	48° Tang.	47° Cotang.	47° Tang.	46° Cotang.	46° Tang.	M.

	44°				44°				44°		
M.	Tang.	Cotang.	M.	M.	Tang.	Cotang.	M.	M.	Tang.	Cotang.	M.
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
M.	Cotang.	Tang.	M.	M.	Cotang.	Tang.	M.	M.	Cotang.	Tang.	M
	45°				45°				45°		

TABLE IV.

LOGARITHMIC SINES, COSINES,

TANGENTS,

AND

COTANGENTS.

0° | 179°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	Inf. neg.		0.000000	.00	Inf. neg.		Infinite.	60
1	6.463726	5017.17	.000000	.00	6.463726	5017.17	13.536274	59
2	.764756	2934.85	.000000	.00	.764756	2934.83	.235244	58
3	.940847	2082.31	.000000	.00	.940847	2082.31	.059153	57
4	7.065786	1615.17	.000000	.00	7.065786	1615.17	12.934214	56
5	.162696	1319.68	.000000	.00	.162696	1319.69	.837304	55
6	.241877	1115.75	9.999999	.01	.241878	1115.78	.758122	54
7	.308824	966.53	.999999	.01	.308825	996.53	.691175	53
8	.366816	852.54	.999999	.01	.366817	852.54	.633183	52
9	.417968	762.63	.999999	.01	.417970	762.63	.582030	51
10	7.463726	689.88	9.999998	.01	7.463727	689.88	12.536273	50
11	.505118	629.81	.999998	.01	.505120	629.81	.494880	49
12	.542906	579.36	.999997	.01	.542909	579.33	.457091	48
13	.577668	536.41	.999997	.01	.577672	536.42	.422328	47
14	.609853	499.38	.999996	.01	.609857	499.39	.390143	46
15	.639816	467.14	.999996	.01	.639820	467.15	.360180	45
16	.667845	438.81	.999995	.01	.667849	438.82	.332151	44
17	.694173	413.72	.999995	.01	.694179	413.73	.305821	43
18	.718997	391.35	9.999994	.01	.719003	391.36	.280997	42
19	.742477	371.27	.999993	.01	.742484	371.28	.257516	41
20	7.764754	353.15	9.999993	.01	7.764761	351.36	12.235239	40
21	.785943	336.72	.999992	.01	.785951	336.73	.214049	39
22	.806146	321.75	.999991	.01	.806155	321.76	.193845	38
23	.825451	308.05	.999990	.01	.825460	308.06	.174540	37
24	.843934	295.47	.999989	.02	.843944	295.49	.156056	36
25	.861662	283.88	.999988	.02	.861674	283.90	.138326	35
26	.878695	273.17	.999988	.02	.878708	273.18	.121292	34
27	.895085	263.23	.999987	.02	.895099	263.25	.104901	33
28	.910879	253.99	.999986	.02	.910894	254.01	.089106	32
29	.926119	245.38	.999985	.02	.926134	245.40	.073866	31
30	7.940842	237.33	9.999983	.02	7.940858	237.35	12.059142	30
31	.955082	229.80	.999982	.02	.955100	229.81	.044900	29
32	.968870	222.73	.999981	.02	.968889	222.75	.031111	28
33	.982233	216.08	.999980	.02	.982253	216.10	.017747	27
34	.995198	209.81	.999979	.02	.995219	209.83	.004781	26
35	8.007787	203.90	.999977	.02	8.007809	203.92	11.992191	25
36	.020021	198.31	.999976	.02	.020045	198.33	.979955	24
37	.031919	193.02	.999975	.02	.031945	193.05	.968055	23
38	.043501	188.01	.999973	.02	.043527	188.03	.956473	22
39	.054781	183.25	.999972	.02	.054809	183.27	.945191	21
40	8.065776	178.72	9.999971	.02	8.065806	178.74	11.934194	20
41	.076500	174.41	.999969	.02	.076531	174.44	.923469	19
42	.086965	170.31	.999968	.02	.086997	170.34	.913003	18
43	.097183	166.39	.999966	.02	.097217	166.42	.902783	17
44	.107167	162.65	.999964	.03	.107202	162.68	.892797	16
45	.116926	159.08	.999963	.03	.116963	159.10	.883037	15
46	.126471	155.66	.999961	.03	.126510	155.68	.873490	14
47	.135810	152.38	.999959	.03	.135851	152.41	.864149	13
48	.144953	149.24	.999958	.03	.144996	149.27	.855004	12
49	.153907	146.22	.999956	.03	.153952	146.27	.846048	11
50	8.162681	143.33	9.999954	.03	8.162727	143.36	11.837273	10
51	.171280	140.54	.999952	.03	.171328	140.57	.828672	9
52	.179713	137.86	.999950	.03	.179763	137.90	.820237	8
53	.187985	135.29	.999948	.03	.188036	135.32	.811964	7
54	.196102	132.80	.999946	.03	.196156	132.84	.803844	6
55	.204070	130.41	.999944	.03	.204126	130.44	.795874	5
56	.211895	128.10	.999942	.04	.211953	128.14	.788047	4
57	.219581	125.87	.999940	.04	.219641	125.90	.780359	3
58	.227134	123.72	.999938	.04	.227195	123.76	.772805	2
59	.234557	121.64	.999936	.04	.234621	121.68	.765379	1
60	.241855		.999934		.241921		.758079	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

90° | 89°

1° **178°**

M.	Sine.	D.1″.	Cosine.	D.1″	Tang.	D.1″.	Cotang.	M.
0	8.241855	119.63	9.999934	.04	8.241921	119.67	11.758079	60
1	.249033	117.68	.999932	.04	.249102	117.72	.750898	59
2	.256094	115.80	.999929	.04	.256165	115.84	.743835	58
3	.263042	113.98	.999927	.04	.263115	114.02	.736885	57
4	.269881	112.21	.999925	.04	.269956	112.25	.730044	56
5	.276614	110.50	.999922	.04	.276691	110.54	.723309	55
6	.283243	108.83	.999920	.04	.283323	108.87	.716677	54
7	.289773	107.21	.999918	.04	.289856	107.26	.710144	53
8	.296207	105.65	.999915	.04	.296292	105.70	.703708	52
9	.302546	104.13	.999913	.04	.302634	104.18	.697366	51
10	8.308794	102.66	9.999910	.04	8.308884	102.70	11.691116	50
11	.314954	101.22	.999907	.04	.315046	101.26	.684954	49
12	.321027	99.82	.999905	.04	.321122	99.87	.678878	48
13	.327016	98.47	.999902	.04	.327114	98.51	.672886	47
14	.332924	97.14	.999899	.05	.333025	97.19	.666975	46
15	.338753	95.86	.999897	.05	.338856	95.90	.661144	45
16	.344504	94.60	.999894	.05	.344610	94.65	.655390	44
17	.350181	93.38	.999891	.05	.350289	93.43	.649711	43
18	.355783	92.19	.999888	.05	.355895	92.24	.644105	42
19	.361315	91.03	.999885	.05	.361430	91.08	.638570	41
20	8.366777	89.90	9.999882	.05	8.366895	89.95	11.633105	40
21	.372171	88.80	.999879	.05	.372292	88.85	.627708	39
22	.377499	87.72	.999876	.05	.377622	87.77	.622378	38
23	.382762	86.67	.999873	.05	.382889	86.72	.617111	37
24	.387962	85.64	.999870	.05	.388092	85.70	.611908	36
25	.393101	84.64	.999867	.05	.393234	84.70	.606766	35
26	.398179	83.66	.999864	.05	.398315	83.71	.601685	34
27	.403199	82.71	.999861	.05	.403338	82.76	.596662	33
28	.408161	81.77	.999858	.05	.408304	81.82	.591696	32
29	.413068	80.86	.999854	.05	.413213	80.91	.586787	31
30	8.417919	79.96	9.999851	.06	8.418068	80.02	11.581932	30
31	.422717	79.09	.999848	.06	.422869	79.14	.577131	29
32	.427462	78.23	.999844	.06	.427618	78.30	.572382	28
33	.432156	77.40	.999841	.06	.432315	77.45	.567685	27
34	.436800	76.57	.999838	.06	.436962	76.63	.563038	26
35	.441394	75.77	.999834	.06	.441560	75.83	.558440	25
36	.445941	74.99	.999831	.06	.446110	75.05	.553890	24
37	.450440	74.22	.999827	.06	.450613	74.28	.549387	23
38	.454893	73.46	.999823	.06	.455070	73.52	.544930	22
39	.459301	72.73	.999820	.06	.459481	72.79	.540519	21
40	8.463665	72.00	9.999816	.06	8.463849	72.06	11.536151	20
41	.467985	71.29	.999812	.06	.468172	71.35	.531828	19
42	.472263	70.60	.999809	.06	.472454	70.66	.527546	18
43	.476498	69.91	.999805	.06	.476693	69.98	.523307	17
44	.480693	69.24	.999801	.06	.480892	69.31	.519108	16
45	.484848	68.59	.999797	.07	.485050	68.65	.514950	15
46	.488963	67.94	.999793	.07	.489170	68.01	.510830	14
47	.493040	67.31	.999790	.07	.493250	67.38	.506750	13
48	.497078	66.69	.999786	.07	.497293	66.76	.502707	12
49	.501080	66.08	.999782	.07	.501298	66.15	.498702	11
50	8.505045	65.48	9.999778	.07	8.505267	65.55	11.494733	10
51	.508974	64.89	.999774	.07	.509200	64.96	.490800	9
52	.512867	64.31	.999769	.07	.513098	64.39	.486902	8
53	.516726	63.75	.999765	.07	.516961	63.82	.483039	7
54	.520551	63.19	.999761	.07	.520790	63.26	.479210	6
55	.524343	62.64	.999757	.07	.524586	62.72	.475414	5
56	.528102	62.11	.999753	.07	.528349	62.18	.471651	4
57	.531828	61.58	.999748	.07	.532080	61.65	.467920	3
58	.535523	61.06	.999744	.07	.535779	61.13	.464221	2
59	.539186	60.55	.999740	.07	.539447	60.62	.460553	1
60	.542819		.999735		.543084		.456916	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

91° **88°**

M.	Sine.	D.1".	Cosine.	D.1".	Tang.	D.1".	Cotang.	M.
0	8.542819	60.04	9.999735	.07	8.543084	60.12	11.456916	60
1	.546422	59.55	.999731	.07	.546691	59.62	.453309	59
2	.549995	59.06	.999726	.07	.550268	59.14	.449732	58
3	.553539	58.58	.999722	.08	.553817	58.66	.446183	57
4	.557054	58.11	.999717	.08	.557336	58.19	.442664	56
5	.560540	57.65	.999713	.08	.560828	57.73	.439172	55
6	.563999	57.19	.999708	.08	.564291	57.27	.435709	54
7	.567431	56.74	.999704	.08	.567727	56.82	.432273	53
8	.570836	56.30	.999699	.08	.571137	56.38	.428863	52
9	.574214	55.87	.999694	.08	.574520	55.95	.425480	51
10	8.577566	55.44	9.999689	.08	8.577877	55.52	11.422123	50
11	.580892	55.02	.999685	.08	.581208	55.10	.418792	49
12	.584193	54.60	.999680	.08	.584514	54.68	.415486	48
13	.587469	54.19	.999675	.08	.587795	54.27	.412205	47
14	.590721	53.79	.999670	.08	.591051	53.87	.408949	46
15	.593948	53.39	.999665	.08	.594283	53.47	.405717	45
16	.597152	53.00	.999660	.08	.597492	53.08	.402508	44
17	.600332	52.61	.999655	.08	.600677	52.70	.399323	43
18	.603489	52.23	.999650	.08	.603839	52.32	.396161	42
19	.606623	51.86	.999645	.09	.606978	51.94	.393022	41
20	8.609734	51.49	9.999640	.09	8.610094	51.58	11.389906	40
21	.612823	51.12	.999635	.09	.613189	51.21	.386811	39
22	.615891	50.76	.999629	.09	.616262	50.85	.383738	38
23	.618937	50.41	.999624	.09	.619313	50.50	.380687	37
24	.621962	50.06	.999619	.09	.622343	50.15	.377657	36
25	.624965	49.72	.999614	.09	.625352	49.81	.374648	35
26	.627948	49.38	.999608	.09	.628340	49.47	.371660	34
27	.630911	49.04	.999603	.09	.631308	49.13	.368692	33
28	.633854	48.71	.999597	.09	.634256	48.80	.365744	32
29	.636776	48.39	.999592	.09	.637184	48.48	.362816	31
30	8.639680	48.06	9.999586	.09	8.640093	48.16	11.359907	30
31	.642563	47.75	.999581	.09	.642982	47.84	.357018	29
32	.645428	47.43	.999575	.09	.645853	47.53	.354147	28
33	.648274	47.12	.999570	.09	.648704	47.22	.351296	27
34	.651102	46.82	.999564	.09	.651537	46.91	.348463	26
35	.653911	46.52	.999558	.10	.654352	46.61	.345648	25
36	.656702	46.22	.999553	.10	.657149	46.31	.342851	24
37	.659475	45.92	.999547	.10	.659928	46.02	.340072	23
38	.662230	45.63	.999541	.10	.662689	45.73	.337311	22
39	.664968	45.35	.999535	.10	.665433	45.44	.334567	21
40	8.667689	45.06	9.999529	.10	8.668160	45.16	11.331840	20
41	.670393	44.79	.999524	.10	.670870	44.88	.329130	19
42	.673080	44.51	.999518	.10	.673563	44.61	.326437	18
43	.675751	44.24	.999512	.10	.676239	44.34	.323761	17
44	.678405	43.97	.999506	.10	.678900	44.07	.321100	16
45	.681043	43.70	.999500	.10	.681544	43.80	.318456	15
46	.683665	43.44	.999493	.10	.684172	43.54	.315828	14
47	.686272	43.18	.999487	.10	.686784	43.28	.313216	13
48	.688863	42.92	.999481	.10	.689381	43.03	.310619	12
49	.691438	42.67	.999475	.10	.691963	42.77	.308037	11
50	8.693998	42.42	9.999469	.10	8.694529	42.52	11.305471	10
51	.696543	42.17	.999463	.11	.697081	42.28	.302919	9
52	.699073	41.92	.999456	.11	.699617	42.03	.300383	8
53	.701589	41.68	.999450	.11	.702139	41.79	.297861	7
54	.704090	41.44	.999443	.11	.704646	41.55	.295354	6
55	.706577	41.21	.999437	.11	.707140	41.32	.292860	5
56	.709049	40.97	.999431	.11	.709618	41.08	.290382	4
57	.711507	40.74	.999424	.11	.712083	40.85	.287917	3
58	.713952	40.51	.999418	.11	.714534	40.62	.285465	2
59	.716383	40.29	.999411	.11	.716972	40.40	.283028	1
60	.718800		.999404		.719396		.280604	0
M.	Cosine.	D.1".	Sine.	D.1".	Cotang.	D.1".	Tang.	M.

3° **176°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	8.718800	40.06	9.999404	.11	8.719396	40.17	11.280604	60
1	.721204	39.84	.999398	.11	.721806	39.95	.278194	59
2	.723595	39.62	.999391	.11	.724204	39.74	.275796	58
3	.725972	39.41	.999384	.11	.726588	39.52	.273412	57
4	.728337	39.19	.999378	.11	.728959	39.30	.271041	56
5	.730688	38.98	.999371	.11	.731317	39.09	.268683	55
6	.733027	38.77	.999364	.12	.733663	38.89	.266337	54
7	.735354	38.57	.999357	.12	.735996	38.68	.264004	53
8	.737667	38.36	.999350	.12	.738317	38.48	.261683	52
9	.739969	38.16	.999343	.12	.740626	38.27	.259374	51
10	8.742259	37.96	9.999336	.12	8.742922	38.07	11.257078	50
11	.744536	37.76	.999329	.12	.745207	37 87	.254793	49
12	.746802	37.56	.999322	.12	.747479	37.68	.252521	48
13	.749055	37.37	.999315	.12	.749740	37.49	.250260	47
14	.751297	37.17	.999308	.12	.751989	37.29	.248011	46
15	.753528	36.98	.999301	.12	.754227	37.10	.245773	45
16	.755747	36.79	.999294	.12	.756453	36.92	.243547	44
17	.757955	36.61	.999286	.12	.758668	36.73	.241332	43
18	.760151	36.42	.999279	.12	.760872	36.55	.239128	42
19	.762337	36.24	.999272	.12	.763065	36.36	.236935	41
20	8.764511	36.06	9.999265	.12	8.765246	36.18	11.234754	40
21	.766675	35.88	.999257	.12	.767417	36.00	.232583	39
22	.768828	35.70	.999250	.13	.769578	35.83	.230422	38
23	.770970	35.53	.999242	.13	.771727	35.65	.228273	37
24	.773101	35.35	.999235	.13	.773866	35.48	.226134	36
25	.775223	35.18	.999227	.13	.775995	35.31	.224005	35
26	.777333	35.01	.999220	.13	.778114	35.14	.221886	34
27	.779434	34.84	.999212	.13	.780222	34.97	.219778	33
28	.781524	34.67	.999205	.13	.782320	34.80	.217680	32
29	.783605	34.51	.999197	.13	.784408	34.64	.215592	31
30	8.785675	34.31	9.999189	.13	8.786486	34.47	11.213514	30
31	.787736	34.18	.999181	.13	.788554	34.31	.211446	29
32	.789787	34.02	.999174	.13	.790613	34.15	.209387	28
33	.791828	33.86	.999166	.13	.792662	33.99	.207338	27
34	.793859	33.70	.999158	.13	.794701	33.83	.205299	26
35	.795881	33.54	.999150	.13	.796731	33.68	.203269	25
36	.797894	33.39	.999142	.13	.798752	33.52	.201248	24
37	.799897	33.23	.999134	.13	.800763	33.37	.199237	23
38	.801892	33.08	.999126	.13	.802765	33.22	.197235	32
39	.803876	32.93	.999118	.13	.804758	33.07	.195242	21
40	8.805852	32.78	9.999110	.13	8.806742	32.92	11.193258	20
41	.807819	32.63	.999102	.13	.808717	32.78	.191283	19
42	.809777	32.49	.999094	.14	.810683	32.62	.189317	18
43	.811726	32.34	.999086	.14	.812641	32.48	.187359	17
44	.813667	32.19	.999077	.14	.814589	32.33	.185411	16
45	.815599	32.05	.999069	.14	.816529	32.19	.183471	15
46	.817522	31.91	.999061	.14	.818461	32.05	.181539	14
47	.819436	31.77	.999053	.14	.820384	31.91	.179616	13
48	.821343	31.63	.999044	.14	.822298	31.77	.177702	12
49	.823240	31.49	.999036	.14	.824205	31.63	.175795	11
50	8.825130	31.35	9.999027	.14	8.826103	31.50	11.173897	10
51	.827011	31.22	.999019	.14	.827992	31.36	.172008	9
52	.828884	31.08	.999010	.14	.829874	31.23	.170126	8
53	.830749	30.95	.999002	.14	.831748	31.10	.168252	7
54	.832607	30.82	.998993	.14	.833613	30.96	.166387	6
55	.834456	30.69	.998984	.14	.835471	30.83	.164529	5
56	.836297	30.56	.998976	.14	.837321	30.70	.162679	4
57	.838130	30.43	.998967	.15	.839163	30.57	.160837	3
58	.839956	30.30	.998958	.15	.840998	30.45	.159002	2
59	.841774	30.17	.998950	.15	.842825	30.32	.157175	1
60	.843585		.998941		.844644		.155356	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

93° **86°**

4° 175°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	8.843585	30.05	9.998941	.15	8.844644	30.19	11.155356	60
1	.845387	29.92	.998932	.15	.846455	30.07	.153545	59
2	.847183	29.80	.998923	.15	.848260	29.95	.151740	58
3	.848971	29.67	.998914	.15	.850057	29.82	.149943	67
4	.850751	29.55	.998905	.15	.851846	29.70	.148154	56
5	.852525	29.43	.998896	.15	.853628	29.58	.146372	55
6	.854291	29.31	.998887	.15	.855403	29.46	.144597	54
7	.856049	29.19	.998878	.15	.857171	29.35	.142829	53
8	.857801	29.07	.998869	.15	.858932	29.23	.141068	52
9	.859546	28.96	.998860	.15	.860686	29.11	.139314	51
10	8.861283	28.84	9.998851	.15	8.862433	29.00	11.137567	50
11	.863014	28.73	.998841	.15	.864173	28.88	.135827	49
12	.864738	28.61	.998832	.15	.865906	28.77	.134094	48
13	.866455	28.50	.998823	.16	.867632	28.66	.132368	47
14	.868165	28.39	.998813	.16	.869351	28.54	.130649	46
15	.869868	28.28	.998804	.16	.871064	28.43	.128936	45
16	.871565	28.17	.998795	.16	.872770	28.32	.127230	44
17	.873255	28.06	.998785	.16	.874469	28.21	.125531	43
18	.874938	27.95	.998776	.16	.876162	28.11	.123838	42
19	.876615	27.86	.998766	.16	.877849	28.00	.122151	41
20	8.878285	27.73	9.998757	.16	8.879529	27.89	11.120471	40
21	.879949	27.63	.998747	.16	.881202	27.79	.118798	39
22	.881607	27.52	.998738	.16	.882869	27.68	.117131	38
23	.883258	27.42	.998728	.16	.884530	27.58	.115470	37
24	.884903	27.31	.998718	.16	.886185	27.47	.113815	36
25	.886542	27.21	.998708	.16	.887833	27.37	.112167	25
26	.888174	27.11	.998699	.16	.889476	27.27	.110524	34
27	.889801	27.00	.998689	.16	.891112	27.17	.108888	33
28	.891421	26.90	.998679	.16	.892742	27.07	.107258	32
29	.893035	26.89	.998669	.17	.894366	26.97	.105634	31
30	8.894643	26.70	9.998659	.17	8.895984	26.87	11.104016	30
31	.896246	26.60	.998649	.17	.897596	26.77	.102404	29
32	.897842	26.51	.998639	.17	.899203	26.67	.100797	28
33	.899432	26.41	.998629	.17	.900803	26.58	.099197	27
34	.901017	26.31	.998619	.17	.902398	26.48	.097602	26
35	.902596	26.22	.998609	.17	.903987	26.38	.096013	25
36	.904169	26.12	.998599	.17	.905570	26.29	.094430	24
37	.905736	26.03	.998589	.17	.907147	26.20	.092853	23
38	.907297	25.93	.998578	.17	.908719	26.10	.091281	22
39	.908853	25.84	.998568	.17	.910285	26.01	.089715	21
40	8.910404	25.75	9.998558	.17	8.911846	25.92	11.088154	20
41	.911949	25.66	.998548	.17	.913401	25.83	.086599	19
42	.913488	25.56	.998537	.17	.914951	25.74	.085049	18
43	.915022	25.47	.998527	.17	.916495	25.65	.083505	17
44	.916550	25.38	.998516	.18	.918034	26.56	.081966	16
45	.918073	25.29	.998506	.18	.919568	25.47	.080432	15
46	.919591	25.20	.998495	.18	.921096	25.38	.078904	14
47	.921103	25.12	.998485	.18	.922619	25.30	.077381	13
48	.922610	25.03	.998474	.18	.924136	25.21	.075864	12
49	.924112	24.94	.998464	.18	.925649	25.12	.074351	11
50	8.925609	24.86	9.998453	.18	8.927156	25.03	11.072844	10
51	.927100	24.77	.998442	.18	.928658	24.95	.071342	9
52	.928587	24.69	.998431	.18	.930155	24.86	.069845	8
53	.930068	24.60	.998421	.18	.931647	24.78	.068353	7
54	.931544	24.52	.998410	.18	.933134	24.70	.066866	6
55	.933015	24.43	.998399	.18	.934616	24.61	.065384	5
56	.934481	24.35	.998388	.18	.936093	24.53	.063907	4
57	.935942	24.27	.998377	.18	.937565	24.45	.062435	2
58	.937398	24.19	.998366	.18	.939032	24.37	.060968	2
59	.938850	24.11	.998355	.18	.940494	24.30	.059506	1
60	.940296		.998344		.941952		.058048	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

94° 85°

5° **174°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	8.940296	24.03	9.998344	.19	8.941952	24.21	11.058048	60
1	.941738	23.94	.998333	.19	.943404	24.13	.056596	59
2	.943174	23.87	.998322	.19	.944852	24.05	.055148	58
3	.944606	23.79	.998311	.19	.946295	23.97	.053705	57
4	.946034	23.71	.998300	.19	.947734	23.90	.052266	56
5	.947456	23.63	.998289	.19	.949168	23.82	.050832	55
6	.948874	23.55	.998277	.19	.950597	23.74	.049403	54
7	.950287	23.48	.998266	.19	.952021	23.67	.047979	53
8	.951696	23.40	.998255	.19	.953441	23.60	.046559	52
9	.953100	23.32	.998243	.19	.954856	23.51	.045144	51
10	8.954499	23.25	9.998232	.19	8.956267	23.44	11.043733	50
11	.955894	23.17	.998220	.19	.957674	23.37	.042326	49
12	.957284	23.10	.998209	.19	.959075	23.29	.040925	48
13	.958670	23.02	.998197	.19	.960473	23.22	.039527	47
14	.960052	22.95	.998186	.19	.961866	23.14	.038134	46
15	.961429	22.88	.998174	.19	.963255	23.07	.036745	45
16	.962801	22.80	.998163	.19	.964639	23.00	.035361	44
17	.964170	22.73	.998151	.19	.966019	22.93	.033981	43
18	.965534	22.66	.998139	.20	.967394	22.86	.032606	42
19	.966893	22.59	.998128	.20	.968766	22.79	.031234	41
20	8.968249	22.52	9.998116	.20	8.970133	22.72	11.029867	40
21	.969600	22.45	.998104	.20	.971496	22.65	.028504	39
22	.970947	22.38	.998092	.20	.972855	22.57	.027145	38
23	.972289	22.31	.998080	.20	.974209	22.51	.025791	37
24	.973628	22.24	.998068	.20	.975560	22.44	.024440	36
25	.974962	22.17	.998056	.20	.976906	22.37	.023094	35
26	.976293	22.10	.998044	.20	.978248	22.30	.021752	34
27	.977619	22.03	.998032	.20	.979586	22.23	.020414	33
28	.978941	21.97	.998020	.20	.980921	22.17	.019079	32
29	.980259	21.90	.998008	.20	.982251	22.10	.017749	31
30	8.981573	21.83	9.997996	.20	8.983577	22.04	11.016423	30
31	.982883	21.77	.997984	.20	.984899	21.97	.015101	29
32	.984189	21.70	.997972	.20	.986217	21.91	.013783	28
33	.985491	21.63	.997959	.20	.987532	21.84	.012468	27
34	.986789	21.57	.997947	.20	.988842	21.78	.011158	26
35	.988083	21.50	.997935	.21	.990149	21.71	.009851	25
36	.989374	21.44	.997922	.21	.991451	21.65	.008549	24
37	.990660	21.38	.997910	.21	.992750	21.58	.007250	23
38	.991943	21.31	.997897	.21	.994045	21.52	.005955	22
39	.993222	21.25	.997885	.21	.995337	21.46	.004663	21
40	8.994497	21.19	9.997872	.21	8.996624	21.40	11.003376	20
41	.995768	21.12	.997860	.21	.997908	21.34	.002092	19
42	.997036	21.06	.997847	.21	.999188	21.27	.000812	18
43	.998299	21.00	.997835	.21	9.000465	21.21	10.999535	17
44	.999560	20.94	.997822	.21	.001738	21.15	.998262	16
45	9.000816	20.88	.997809	.21	.003007	21.09	.996993	15
46	.002069	20.82	.997797	.21	.004272	21.03	.995728	14
47	.003318	20.76	.997784	.21	.005534	20.97	.994466	13
48	.004563	20.70	.997771	.21	.006792	20.91	.993208	12
49	.005805	20.64	.997758	.21	.008047	20.85	.991953	11
50	9.007044	20.58	9.997745	.21	9.009298	20.80	10.990702	10
51	.008278	20.52	.997732	.21	.010546	20.74	.989454	9
52	.009510	20.46	.997719	.21	.011790	20.68	.988210	8
53	.010737	20.40	.997706	.21	.013031	20.62	.986969	7
54	.011962	20.34	.997693	.22	.014268	20.56	.985732	6
55	.013182	20.29	.997680	.22	.015502	20.51	.984498	5
56	.014400	20.23	.997667	.22	.016732	20.45	.983268	4
57	.015613	20.17	.997654	.22	.017959	20.40	.982041	3
58	.016824	20.12	.997641	.22	.019183	20.33	.980817	2
59	.018031	20.06	.997628	.22	.020403	20.28	.979597	1
60	.019235		.997614		.021620		.978380	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

95° **84°**

6° 173°

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.019235	20.00	9.997614	.22	9.021620	20.23	10.978380	60
1	.020435	19.95	.997601	.22	.022834	20.17	.977166	59
2	.021632	19.89	.997588	.22	.024044	20.12	.975956	58
3	.022825	19.84	.997574	.22	.025251	20.06	.974749	57
4	.024016	19.78	.997561	.22	.026455	20.01	.973545	56
5	.025203	19.73	.997547	.22	.027655	19.95	.972345	55
6	.026386	19.67	.997534	.23	.028852	19.90	.971148	54
7	.027567	19.62	.997520	.23	.030046	19.85	.969954	53
8	.028744	19.57	.997507	.23	.031237	19.79	.968763	52
9	.029918	19.52	.997493	.23	.032425	19.74	.967575	51
10	9.031089	19.47	9.997480	.23	9.033609	19.69	10.966391	50
11	.032257	19.41	.997466	.23	.034791	19.64	.965209	49
12	.033421	19.36	.997452	.23	.035969	19.58	.964031	48
13	.034582	19.30	.997439	.23	.037144	19.53	.962856	47
14	.035741	19.25	.997425	.23	.038316	19.48	.961684	46
15	.036896	19.20	.997411	.23	.039485	19.43	.960515	45
16	.038048	19.15	.997397	.23	.040651	19.38	.959349	44
17	.039197	19.10	.997383	.23	.041813	19.33	.958187	43
18	.040342	19.05	.997369	.23	.042973	19.28	.957027	42
19	.041485	19.00	.997355	.23	.044130	19.23	.955870	41
20	9.042625	18.95	9.997341	.23	9.045284	19.18	10.954716	40
21	.043762	18.90	.997327	.24	.046434	19.13	.953566	39
22	.044895	18.85	.997313	.24	.047582	19.08	.952418	38
23	.046026	18.80	.997299	.24	.048727	19.03	.951273	37
24	.047154	18.75	.997285	.24	.049869	18.98	.950131	36
25	.048279	18.70	.997271	.24	.051008	18.93	.948992	35
26	.049400	18.65	.997257	.24	.052144	18.89	.947856	34
27	.050519	18.60	.997242	.24	.053277	18.84	.946723	33
28	.051635	18.55	.997228	.24	.054407	18.79	.945593	32
29	.052749	18.50	.997214	.24	.055535	18.74	.944465	31
30	9.053859	18.46	9.997199	.24	9.056659	18.70	10.943341	30
31	.054966	18.41	.997185	.24	.057781	18.65	.942219	29
32	.056071	18.36	.997170	.24	.058900	18.60	.941100	28
33	.057172	18.31	.997156	.24	.060016	18.56	.939984	27
34	.058271	18.27	.997141	.24	.061130	18.51	.938870	26
35	.059367	18.22	.997127	.24	.062240	18.46	.937760	25
36	.060460	18.17	.997112	.24	.063348	18.42	.936652	24
37	.061551	18.13	.997098	.24	.064453	18.37	.935547	23
38	.062639	18.08	.997083	.25	.065556	18.33	.934444	22
39	.063724	18.04	.997068	.25	.066655	18.28	.933345	21
40	9.064806	17.99	9.997053	.25	9.067752	18.24	10.932248	20
41	.065885	17.95	.997039	.25	.068846	18.19	.931154	19
42	.066962	17.90	.997024	.25	.069938	18.15	.930062	18
43	.068036	17.86	.997009	.25	.071027	18.10	.928973	17
44	.069107	17.81	.996994	.25	.072113	18.06	.927887	16
45	.070176	17.77	.996979	.25	.073197	18.02	.926803	15
46	.071242	17.72	.996964	.25	.074278	17.97	.925722	14
47	.072306	17.68	.996949	.25	.075356	17.93	.924644	13
48	.073366	17.64	.996934	.25	.076432	17.89	.923568	12
49	.074424	17.59	.996919	.25	.077505	17.84	.922495	11
50	9.075480	17.55	9.996904	.25	9.078576	17.80	10.921424	10
51	.076533	17.51	.996889	.25	.079644	17.76	.920356	9
52	.077583	17.46	.996874	.25	.080710	17.72	.919290	8
53	.078631	17.42	.996858	.25	.081773	17.67	.918227	7
54	.079676	17.38	.996843	.25	.082833	17.63	.917167	6
55	.080719	17.34	.996828	.25	.083891	17.59	.916109	5
56	.081759	17.29	.996812	.26	.084947	17.55	.915053	4
57	.082797	17.25	.996797	.26	.086000	17.51	.914000	3
58	.083832	17.21	.996782	.26	.087050	17.47	.912950	2
59	.084864	17.17	.996766	.26	.088098	17.43	.911902	1
60	.085894		.996751		.089144		.910856	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

96° 83°

7° 172°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.085894	17.13	9.996751	.26	9.089144	17.39	10.910856	60
1	.086922	17.09	.996735	.26	.090187	17.35	.909813	59
2	.087947	17.05	.996720	.26	.091228	17.31	.908772	58
3	.088970	17.00	.996704	.26	.092266	17.27	.907734	57
4	.089990	16.96	.996688	.26	.093302	17.23	.906698	56
5	.091008	16.92	.996673	.26	.094336	17.19	.905664	55
6	.092024	16.88	.996657	.26	.095367	17.15	.904633	54
7	.093037	16.84	.996641	.26	.096395	17.11	.903605	53
8	.094047	16.80	.996625	.26	.097422	17.07	.902578	52
9	.095056	16.76	.996610	.26	.098446	17.03	.901554	51
10	9.096062	16.73	9.996594	.26	9.099468	16.99	10.900532	50
11	.097065	16.68	.996578	.27	.100487	16.95	.899513	49
12	.098066	16.65	.996562	.27	.101504	16.91	.898496	48
13	.099065	16.61	.996546	.27	.102519	16.88	.897481	47
14	.100062	16.57	.996530	.27	.103532	16.84	.896468	46
15	.101056	16.53	.996514	.27	.104542	16.80	.895458	45
16	.102048	16.49	.996498	.27	.105550	16.76	.894450	44
17	.103037	16.46	.996482	.27	.106556	16.72	.893444	43
18	.104025	16.43	.996465	.27	.107559	16.69	.892441	42
19	.105010	16.38	.996449	.27	.108560	16.65	.891440	41
20	9.105992	16.34	9.996433	.27	9.109559	16.61	10.890441	40
21	.106973	16.30	.996417	.27	.110556	16.58	.889444	39
22	.107951	16.27	.996400	.27	.111551	16.54	.888449	38
23	.108927	16.23	.996384	.27	.112543	16.50	.887457	37
24	.109901	16.19	.996368	.27	.113533	16.47	.886467	36
25	.110873	16.16	.996351	.27	.114521	16.43	.885479	35
26	.111842	16.12	.996335	.27	.115507	16.39	.884493	34
27	.112809	16.08	.996318	.27	.116491	16.36	.883509	33
28	.113774	16.05	.996302	.28	.117472	16.32	.882528	32
29	.114737	16.01	.996285	.28	.118452	16.29	.881548	31
30	9.115698	15.98	9.996269	.28	9.119429	16.25	10.880571	30
31	.116656	15.94	.996252	.28	.120404	16.22	.879596	29
32	.117613	15.90	.996235	.28	.121377	16.18	.878623	28
33	.118567	15.87	.996219	.28	.122348	16.15	.877652	27
34	.119519	15.83	.996202	.28	.123317	16.11	.876683	26
35	.120469	15.80	.996185	.28	.124284	16.08	.875716	25
36	.121417	15.76	.996168	.28	.125249	16.04	.874751	24
37	.122362	15.73	.996151	.28	.126211	16.01	.873789	23
38	.123306	15.69	.996134	.28	.127172	15.98	.872828	22
39	.124248	15.66	.996117	.28	.128130	15.94	.871870	21
40	9.125187	15.62	9.996100	.29	9.129087	15.91	10.870913	20
41	.126125	15.59	.996083	.29	.130041	15.87	.869959	19
42	.127060	15.56	.996066	.29	.130994	15.84	.869006	18
43	.127993	15.52	.996049	.29	.131944	15.81	.868056	17
44	.128925	15.49	.996032	.29	.132893	15.77	.867107	16
45	.129854	15.45	.996015	.29	.133839	15.74	.866161	15
46	.130781	15.42	.995998	.29	.134784	15.71	.865216	14
47	.131706	15.39	.995980	.29	.135726	15.68	.864274	13
48	.132630	15.35	.995963	.29	.136667	15.64	.863333	12
49	.133551	15.32	.995946	.29	.137605	15.61	.862395	11
50	9.134470	15.29	9.995928	.29	9.138542	15.58	10.861458	10
51	.135387	15.26	.995911	.29	.139476	15.55	.860524	9
52	.136303	15.22	.995894	.29	.140409	15.51	.859591	8
53	.137216	15.19	.995876	.29	.141340	15.48	.858660	7
54	.138128	15.16	.995859	.29	.142269	15.45	.857731	6
55	.139037	15.13	.995841	.29	.143196	15.42	.856804	5
56	.139944	15.09	.995823	.29	.144121	15.39	.855879	4
57	.140850	15.06	.995806	.29	.145044	15.36	.854956	3
58	.141754	15.03	.995788	.29	.145966	15.32	.854034	2
59	.142655	15.00	.995771	.29	.146885	15.29	.853115	1
60	.143555		.995753		.147803		.852197	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

97° 82°

8° 171°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.143555	14.97	9.995753	.30	9.147803	15.26	10.852197	60
1	.144453	14.93	.995735	.30	.148718	15.23	.851282	59
2	.145349	14.90	.995717	.30	.149632	15.20	.850368	58
3	.146243	14.87	.995699	.30	.150544	15.17	.849456	57
4	.147136	14.84	.995681	.30	.151454	15.14	.848546	56
5	.148026	14.81	.995664	.30	.152363	15.11	.847637	55
6	.148915	14.78	.995646	.30	.153269	15.08	.846731	54
7	.149802	14.75	.995628	.30	.154174	15.05	.845826	53
8	.150686	14.72	.995610	.30	.155077	15.02	.844923	52
9	.151569	14.69	.995591	.30	.155978	14.99	.844022	51
10	9.152451	14.66	9.995573	.30	9.156877	14.96	10.843123	50
11	.153330	14.63	.995555	.30	.157775	14.93	.842225	49
12	.154208	14.60	.995537	.30	.158671	14.90	.841329	48
13	.155083	14.57	.995519	.30	.159565	14.87	.840435	47
14	.155957	14.54	.995501	.31	.160457	14.84	.839543	46
15	.156830	14.51	.995482	.31	.161347	14.81	.838653	45
16	.157700	14.48	.995464	.31	.162236	14.78	.837764	44
17	.158569	14.45	.995446	.31	.163123	14.75	.836877	43
18	.159435	14.42	.995427	.31	.164008	14.73	.835992	42
19	.160301	14.39	.995409	.31	.164892	14.70	.835108	41
20	9.161164	14.36	9.995390	.31	9.165774	14.67	10.834226	40
21	.162025	14.33	.995372	.31	.166654	14.64	.833346	39
22	.162885	14.30	.995353	.31	.167532	14.61	.832468	38
23	.163743	14.27	.995334	.31	.168409	14.58	.831591	37
24	.164600	14.24	.995316	.31	.169284	14.56	.830716	36
25	.165454	14.22	.995297	.31	.170157	14.53	.829843	35
26	.166307	14.19	.995278	.31	.171029	14.50	.828971	34
27	.167159	14.16	.995260	.31	.171899	14.47	.828101	33
28	.168008	14.13	.995241	.32	.172767	14.44	.827233	32
29	.168856	14.10	.995222	.32	.173634	14.42	.826366	31
30	9.169702	14.07	9.995203	.32	9.174499	14.39	10.825501	30
31	.170547	14.05	.995184	.32	.175362	14.36	.824638	29
32	.171389	14.02	.995165	.32	.176224	14.33	.823776	28
33	.172230	13.99	.995146	.32	.177084	14.31	.822916	27
34	.173070	13.96	.995127	.32	.177942	14.28	.822058	26
35	.173908	13.94	.995108	.32	.178799	14.25	.821201	25
36	.174744	13.91	.995089	.32	.179655	14.23	.820345	24
37	.175578	13.88	.995070	.32	.180508	14.20	.819492	23
38	.176411	13.85	.995051	.32	.181360	14.17	.818640	22
39	.177242	13.83	.995032	.32	.182211	14.15	.817789	21
40	9.178072	13.80	9.995013	.32	9.183059	14.12	10.816941	20
41	.178900	13.77	.994993	.32	.183907	14.09	.816093	19
42	.179726	13.75	.994974	.32	.184752	14.07	.815248	18
43	.180551	13.72	.994955	.32	.185597	14.04	.814403	17
44	.181374	13.69	.994935	.32	.186439	14.02	.813561	16
45	.182196	13.67	.994916	.33	.187280	13.99	.812720	15
46	.183016	13.64	.994896	.33	.188120	13.97	.811880	14
47	.183834	13.61	.994877	.33	.188958	13.94	.811042	13
48	.184651	13.59	.994857	.33	.189794	13.91	.810206	12
49	.185466	13.56	.994838	.33	.190629	13.89	.809371	11
50	9.186280	13.54	9.994818	.33	9.191462	13.86	10.808538	10
51	.187092	13.51	.994798	.33	.192294	13.84	.807706	9
52	.187903	13.48	.994779	.33	.193124	13.81	.806876	8
53	.188712	13.46	.994759	.33	.193953	13.79	.806047	7
54	.189519	13.43	.994739	.33	.194780	13.76	.805220	6
55	.190325	13.41	.994719	.33	.195606	13.74	.804394	5
56	.191130	13.38	.994700	.33	.196430	13.71	.803570	4
57	.191933	13.36	.994680	.33	.197253	13.69	.802747	3
58	.192734	13.33	.994660	.33	.198074	13.66	.801926	2
59	.193534	13.31	.994640	.33	.198894	13.64	.801106	1
60	.194332		.994620		.199713		.800287	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

98° 81°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.194332	13.28	9.994620	.33	9.199713	13.62	10.800287	60
1	.195129	13.26	.994600	.33	.200529	13.59	.799471	59
2	.195925	13.23	.994580	.33	.201345	13.57	.798655	58
3	.196719	13.21	.994560	.34	.202159	13.54	.797841	57
4	.197511	13.18	.994540	.34	.202971	13.52	.797029	56
5	.198302	13.16	.994519	.34	.203782	13.49	.796218	55
6	.199091	13.13	.994499	.34	.204592	13.47	.795408	54
7	.199879	13.11	.994479	.34	.205400	13.45	.794600	53
8	.200666	13.08	.994459	.34	.206207	13.42	.793793	52
9	.201451	13.06	.994438	.34	.207013	13.40	.792987	51
10	9.202234	13.04	9.994418	.34	9.207817	13.38	10.792183	50
11	.203017	13.01	.994398	.34	.208619	13.35	.791381	49
12	.203797	12.99	.994377	.34	.209420	13.33	.790580	48
13	.204577	12.96	.994357	.34	.210220	13.31	.789780	47
14	.205354	12.94	.994336	.34	.211018	13.28	.788982	46
15	.206131	12.92	.994316	.34	.211815	13.26	.788185	45
16	.206906	12.89	.994295	.34	.212611	13.24	.787389	44
17	.207679	12.87	.994274	.35	.213405	13.21	.786595	43
18	.208452	12.85	.994254	.35	.214198	13.19	.785802	42
19	.209222	12.82	.994233	.35	.214989	13.17	.785011	41
20	9.209992	12.80	9.994212	.35	9.215780	13.15	10.784220	40
21	.210760	12.78	.994191	.35	.216568	13.12	.783432	39
22	.211526	12.75	.994171	.35	.217356	13.10	.782644	38
23	.212291	12.73	.994150	.35	.218142	13.08	.781858	37
24	.213055	12.71	.994129	.35	.218926	13.06	.781074	36
25	.213818	12.68	.994108	.35	.219710	13.03	.780290	35
26	.214579	12.66	.994087	.35	.220492	13.01	.779508	34
27	.215338	12.64	.994066	.35	.221272	12.99	.778728	33
28	.216097	12.62	.994045	.35	.222052	12.97	.777948	32
29	.216854	12.59	.994024	.35	.222830	12.95	.777170	31
30	9.217609	12.57	9.994003	.35	9.223607	12.92	10.776393	30
31	.218363	12.55	.993982	.35	.224382	12.90	.775618	29
32	.219116	12.53	.993960	.35	.225156	12.88	.774844	28
33	.219868	12.50	.993939	.35	.225929	12.86	.774071	27
34	.220618	12.48	.993918	.36	.226700	12.84	.773300	26
35	.221367	12.46	.993897	.36	.227471	12.82	.772529	25
36	.222115	12.44	.993875	.36	.228239	12.79	.771761	24
37	.222861	12.42	.993854	.36	.229007	12.77	.770993	23
38	.223606	12.39	.993832	.36	.229773	12.75	.770227	22
39	.224349	12.37	.993811	.36	.230539	12.73	.769461	21
40	9.225092	12.35	9.993789	.36	9.231302	12.71	10.768698	20
41	.225833	12.33	.993768	.36	.232065	12.69	.767935	19
42	.226573	12.31	.993746	.36	.232826	12.67	.767174	18
43	.227311	12.29	.993725	.36	.233586	12.65	.766414	17
44	.228048	12.26	.993703	.36	.234345	12.63	.765655	16
45	.228784	12.24	.993681	.36	.235103	12.60	.764897	15
46	.229518	12.22	.993660	.36	.235859	12.58	.764141	14
47	.230252	12.20	.993638	.36	.236614	12.56	.763386	13
48	.230984	12.18	.993616	.36	.237368	12.54	.762632	12
49	.231715	12.16	.993594	.36	.238120	12.52	.761880	11
50	9.232444	12.14	9.993572	.36	9.238872	12.50	10.761128	10
51	.233172	12.12	.993550	.37	.239622	12.48	.760378	9
52	.233899	12.10	.993528	.37	.240371	12.46	.759629	8
53	.234625	12.07	.993506	.37	.241118	12.44	.758882	7
54	.235349	12.05	.993484	.37	.241865	12.42	.758135	6
55	.236073	12.03	.993462	.37	.242610	12.40	.757390	5
56	.236795	12.01	.993440	.37	.243354	12.38	.756646	4
57	.237515	11.99	.993418	.37	.244097	12.36	.755903	3
58	.238235	11.97	.993396	.37	.244839	12.34	.755161	2
59	.238953	11.95	.993374	.37	.245579	12.32	.754421	1
60	.239670		.993351		.246319		.753681	0
M	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M

10° **169°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.239670	11.93	9.993351	.37	9.246319	12.30	10.753681	60
1	.240386	11.91	.993329	.37	.247057	12.28	.752943	59
2	.241101	11.89	.993307	.37	.247794	12.26	.752206	58
3	.241814	11.87	.993284	.37	.248530	12.24	.751470	57
4	.242526	11.85	.993262	.37	.249264	12.22	.750736	56
5	.243237	11.83	.993240	.37	.249998	12.20	.750002	55
6	.243947	11.81	.993217	.38	.250730	12.18	.749270	54
7	.244656	11.79	.993195	.38	.251461	12.17	.748539	53
8	.245363	11.77	.993172	.38	.252191	12.15	.747809	52
9	.246069	11.75	.993149	.38	.252920	12.13	.747080	51
10	9.246775	11.73	9.993127	.38	9.253648	12.11	10.746352	50
11	.247478	11.71	.993104	.38	.254374	12.09	.745626	49
12	.248181	11.69	.993081	.38	.255100	12.07	.744900	48
13	.248883	11.67	.993059	.38	.255824	12.05	.744176	47
14	.249583	11.65	.993036	.38	.256547	12.03	.743453	46
15	.250282	11.63	.993013	.38	.257269	12.01	.742731	45
16	.250980	11.61	.992990	.38	.257990	12.00	.742010	44
17	.251677	11.59	.992967	.38	.258710	11.98	.741290	43
18	.252373	11.58	.992944	.38	.259429	11.96	.740571	42
19	.253067	11.56	.992921	.38	.260146	11.94	.739854	41
20	9.253761	11.54	9.992898	.38	9.260863	11.92	10.739137	40
21	.254453	11.52	.992875	.38	.261578	11.90	.738422	39
22	.255144	11.50	.992852	.38	.262292	11.89	.737708	38
23	.255834	11.48	.992829	.39	.263005	11.87	.736995	37
24	.256523	11.46	.992806	.39	.263717	11.85	.736283	36
25	.257211	11.44	.992783	.39	.264428	11.83	.735572	35
26	.257898	11.42	.992759	.39	.265138	11.81	.734862	34
27	.258583	11.41	.992736	.39	.265847	11.79	.734153	33
28	.259268	11.39	.992713	.39	.266555	11.78	.733445	32
29	.259951	11.37	.992690	.39	.267261	11.76	.732739	31
30	9.260633	11.35	9.992666	.39	9.267967	11.74	10.732033	30
31	.261314	11.33	.992643	.39	.268671	11.72	.731329	29
32	.261994	11.31	.992619	.39	.269375	11.70	.730625	28
33	.262673	11.30	.992596	.39	.270077	11.69	.729923	27
34	.263351	11.28	.992572	.39	.270779	11.67	.729221	26
35	.264027	11.26	.992549	.39	.271479	11.65	.728521	25
36	.264703	11.24	.992525	.39	.272178	11.64	.727822	24
37	.265377	11.22	.992501	.39	.272876	11.62	.727124	23
38	.266051	11.20	.992478	.40	.273573	11.60	.726427	22
39	.266723	11.19	.992454	.40	.274269	11.58	.725731	21
40	9.267395	11.17	9.992430	.40	9.274964	11.57	10.725036	20
41	.268065	11.15	.992406	.40	.275658	11.55	.724342	19
42	.268734	11.13	.992382	.40	.276351	11.53	.723649	18
43	.269402	11.12	.992359	.40	.277043	11.51	.722957	17
44	.270069	11.11	.992335	.40	.277734	11.50	.722266	16
45	.270735	11.08	.992311	.40	.278424	11.48	.721576	15
46	.271400	11.06	.992287	.40	.279113	11.47	.720887	14
47	.272064	11.05	.992263	.40	.279801	11.45	.720199	13
48	.272726	11.03	.992239	.40	.280488	11.43	.719512	12
49	.273388	11.01	.992214	.40	.281174	11.41	.718826	11
50	9.274049	10.99	9.992190	.40	9.281858	11.40	10.718142	10
51	.274708	10.98	.992166	.40	.282542	11.38	.717458	9
52	.275367	10.96	.992142	.40	.283225	11.36	.716775	8
53	.276025	10.94	.992118	.41	.283907	11.35	.716093	7
54	.276681	10.92	.992093	.41	.284588	11.33	.715412	6
55	.277337	10.91	.992069	.41	.285268	11.31	.714732	5
56	.277991	10.89	.992044	.41	.285947	11.30	.714053	4
57	.278644	10.87	.992020	.41	.286624	11.28	.713376	3
58	.279297	10.86	.991996	.41	.287301	11.26	.712699	2
59	.279948	10.84	.991971	.41	.287977	11.25	.712023	1
60	.280599		991947		288652		.711348	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

100° **79°**

11° **168°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.280599	10.82	9.991947	.41	9.288652	11.23	10.711348	60
1	.281248	10.81	.991922	.41	.289326	11.22	.710674	59
2	.281897	10.79	.991897	.41	.289999	11.20	.710001	58
3	.282544	10.77	.991873	.41	.290671	11.18	.709329	57
4	.283190	10.76	.991848	.41	.291342	11.17	.708658	56
5	.283836	10.74	.991823	.41	.292013	11.15	.707987	55
6	.284480	10.72	.991799	.41	.292682	11.14	.707318	54
7	.285124	10.71	.991774	.42	.293350	11.12	.706650	53
8	.285766	10.69	.991749	.42	.294017	11.11	.705983	52
9	.286408	10.67	.991724	.42	.294684	11.09	.705316	51
10	9.287048	10.66	9.991699	.42	9.295349	11.07	10.704651	50
11	.287688	10.64	.991674	.42	.296013	11.06	.703987	49
12	.288326	10.63	.991649	.42	.296677	11.04	.703323	48
13	.288964	10.61	.991624	.42	.297339	11.03	.702661	47
14	.289600	10.59	.991599	.42	.298001	11.01	.701999	46
15	.290236	10.58	.991574	.42	.298662	11.00	.701338	45
16	.290870	10.56	.991549	.42	.299322	10.98	.700678	44
17	.291504	10.55	.991524	.42	.299980	10.97	.700020	43
18	.292137	10.53	.991498	.42	.300638	10.95	.699362	42
19	.292768	10.51	.991473	.42	.301295	10.93	.698705	41
20	9.293399	10.50	9.991448	.42	9.301951	10.92	10.698049	40
21	.294029	10.48	.991422	.42	.302607	10.90	.697393	39
22	.294658	10.47	.991397	.42	.303261	10.89	.696739	38
23	.295286	10.45	.991372	.43	.303914	10.87	.696086	37
24	.295913	10.43	.991346	.43	.304567	10.86	.695433	36
25	.296539	10.42	.991321	.43	.305218	10.84	.694782	35
26	.297164	10.40	.991295	.43	.305869	10.83	.694131	34
27	.297788	10.39	.991270	.43	.306519	10.81	.693481	33
28	.298412	10.37	.991244	.43	.307168	10.80	.692832	32
29	.299034	10.36	.991218	.43	.307816	10.78	.692184	31
30	9.299655	10.34	9.991193	.43	9.308463	10.77	10.691537	30
31	.300276	10.33	.991167	.43	.309109	10.76	.690891	29
32	.300895	10.31	.991141	.43	.309754	10.74	.690246	28
33	.301514	10.30	.991115	.43	.310399	10.73	.689601	27
34	.302132	10.28	.991090	.43	.311042	10.71	.688958	26
35	.302748	10.26	.991064	.43	.311685	10.70	.688315	25
36	.303364	10.25	.991038	.43	.312327	10.68	.687673	24
37	.303979	10.23	.991012	.43	.312968	10.67	.687032	23
38	.304593	10.22	.990986	.43	.313608	10.65	.686392	22
39	.305207	10.20	.990960	.43	.314247	10.64	.685753	21
40	9.305819	10.19	9.990934	.44	9.314885	10.62	10.685115	20
41	.306430	10.17	.990908	.44	.315523	10.61	.684477	19
42	.307041	10.16	.990882	.44	.316159	10.60	.683841	18
43	.307650	10.14	.990855	.44	.316795	10.58	.683205	17
44	.308259	10.13	.990829	.44	.317430	10.57	.682570	16
45	.308867	10.12	.990803	.44	.318064	10.55	.681936	15
46	.309474	10.10	.990777	.44	.318697	10.54	.681303	14
47	.310080	10.09	.990750	.44	.319329	10.53	.680671	13
48	.310685	10.07	.990724	.44	.319961	10.51	.680039	12
49	.311289	10.06	.990697	.44	.320592	10.50	.679408	11
50	9.311893	10.04	9.990671	.44	9.321222	10.48	10.678778	10
51	.312495	10.03	.990645	.44	.321851	10.47	.678149	9
52	.313097	10.01	.990618	.44	.322479	10.46	.677521	8
53	.313698	10.00	.990591	.44	.323106	10.44	.676894	7
54	.314297	9.98	.990565	.44	.323733	10.43	.676267	6
55	.314897	9.97	.990538	.44	.324358	10.41	.675642	5
56	.315495	9.96	.990511	.45	.324983	10.40	.675017	4
57	.316092	9.94	.990485	.45	.325607	10.39	.674393	3
58	.316689	9.93	.990458	.45	.326231	10.37	.673769	2
59	.317284	9.91	.990431	.45	.326853	10.36	.673147	1
60	.317879		.990404		.327475		.672525	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

101° **78°**

12° 167°

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.317879	9.90	9.990404	.45	9.327475	10.35	10.672525	60
1	.318473	9.88	.990378	.45	.328095	10.33	.671905	59
2	.319066	9.87	.990351	.45	.328715	10.32	.671285	58
3	.319658	9.86	.990324	.45	.329334	10.31	.670666	57
4	.320249	9.84	.990297	.45	.329953	10.29	.670047	56
5	.320840	9.83	.990270	.45	.330570	10.28	.669430	55
6	.321430	9.82	.990243	.45	.331187	10.27	.668813	54
7	.322019	9.80	.990215	.45	.331803	10.25	.668197	53
8	.322607	9.79	.990188	.45	.332418	10.24	.667582	52
9	.323194	9.77	.990161	.45	.333033	10.23	.666967	51
10	9.323780	9.76	9.990134	.45	9.333646	10.21	10.666354	50
11	.324366	9.75	.990107	.46	.334259	10.20	.665741	49
12	.324950	9.73	.990079	.46	.334871	10.19	.665129	48
13	.325534	9.72	.990052	.46	.335482	10.17	.664518	47
14	.326117	9.70	.990025	.46	.336093	10.16	.663907	46
15	.326700	9.69	.989997	.46	.336702	10.15	.663298	45
16	.327281	9.68	.989970	.46	.337311	10.14	.662689	44
17	.327862	9.66	.989942	.46	.337919	10.12	.662081	43
18	.328442	9.65	.989915	.46	.338527	10.11	.661473	42
19	.329021	9.64	.989887	.46	.339133	10.10	.660867	41
20	9.329599	9.62	9.989860	.46	9.339739	10.08	10.660261	40
21	.330176	9.61	.989832	.46	.340344	10.07	.659656	39
22	.330753	9.60	.989804	.46	.340948	10.06	.659052	38
23	.331329	9.58	.989777	.46	.341552	10.05	.658448	37
24	.331903	9.57	.989749	.47	.342155	10.03	.657845	36
25	.332478	9.56	.989721	.47	.342757	10.02	.657243	35
26	.333051	9.54	.989693	.47	.343358	10.01	.656642	34
27	.333624	9.53	.989665	.47	.343958	10.00	.656042	33
28	.334195	9.52	.989637	.47	.344558	9.98	.655442	32
29	.334767	9.50	.989610	.47	.345157	9.97	.654843	31
30	9.335337	9.49	9.989582	.47	9.345755	9.96	10.654245	30
31	.335906	9.48	.989553	.47	.346353	9.95	.653647	29
32	.336475	9.46	.989525	.47	.346949	9.93	.653051	28
33	.337043	9.45	.989497	.47	.347545	9.92	.652455	27
34	.337610	9.44	.989469	.47	.348141	9.91	.651859	26
35	.338176	9.43	.989441	.47	.348735	9.90	.651265	25
36	.338742	9.41	.989413	.47	.349329	9.88	.650671	24
37	.339307	9.40	.989385	.47	.349922	9.87	.650078	23
38	.339871	9.39	.989356	.47	.350514	9.86	.649486	22
39	.340434	9.37	.989328	.47	.351106	9.85	.648894	21
40	9.340996	9.36	9.989300	.47	9.351697	9.84	10.648303	20
41	.341558	9.35	.989271	.47	.352287	9.82	.647713	19
42	.342119	9.34	.989243	.47	.352876	9.81	.647124	18
43	.342679	9.32	.989214	.47	.353465	9.80	.646535	17
44	.343239	9.31	.989186	.47	.354053	9 79	.645947	16
45	.343797	9.30	.989157	.47	.354640	9.78	.645360	15
46	.344355	9.29	.989128	.48	.355227	9.76	.644773	14
47	.344912	9.27	.989100	.48	.355813	9.75	.644187	13
48	.345469	9.26	.989071	.48	.356398	9.74	.643602	12
49	.346024	9.25	.989042	.48	.356982	9.73	.643018	11
50	9.346579	9.24	9.989014	.48	9.357566	9.72	10.642434	10
51	.347134	9.22	.988985	.48	.358149	9.70	.641851	9
52	.347687	9.21	.988956	.48	.358731	9.69	.641269	8
53	.348240	9.20	.988927	.48	.359313	9.68	.640687	7
54	.348792	9.19	.988898	.48	.359893	9.67	.640107	6
55	.349343	9.17	.988869	.48	.360474	9.66	.639526	5
56	.349893	9.16	.988840	.48	.361053	9.65	.638947	4
57	.350443	9.15	.988811	.49	.361632	9.63	.638368	3
58	.350992	9.14	.988782	.49	.362210	9.62	.637790	2
59	.351540	9.13	.988753	.49	.362787	9.61	.637213	1
60	.352088		.988724		.363364		.636636	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

102° 77°

13° **166°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.352088	9.11	9.988724	.49	9.363364	9.60	10.636636	60
1	.352635	9.10	.988695	.49	.363940	9.59	.636060	59
2	.353181	9.09	.988666	.49	.364515	9.58	.635485	58
3	.353726	9.08	.988636	.49	.365090	9.57	.634910	57
4	.354271	9.07	.988607	.49	.365664	9.55	.634336	56
5	.354815	9.05	.988578	.49	.366237	9.54	.633763	55
6	.355358	9.04	.988548	.49	.366810	9.53	.633190	54
7	.355901	9.03	.988519	.49	.367382	9.52	.632618	53
8	.356443	9.02	.988489	.49	.367953	9.51	.632047	52
9	.356984	9.01	.988460	.49	.368524	9.50	.631476	51
10	9.357524	8.99	9.988430	.49	9.369094	9.49	10.630906	50
11	.358064	8.98	.988401	.49	.369663	9.48	.630337	49
12	.358603	8.97	.988371	.49	.370232	9.47	.629768	48
13	.359141	8.96	.988342	.49	.370799	9.45	.629201	47
14	.359678	8.95	.988312	.50	.371367	9.44	.628633	46
15	.360215	8.94	.988282	.50	.371933	9.43	.628067	45
16	.360752	8.92	.988252	.50	.372499	9.42	.627501	44
17	.361287	8.91	.988223	.50	.373064	9.41	.626936	43
18	.361822	8.90	.988193	.50	.373629	9.40	.626371	42
19	.362356	8.89	.988163	.50	.374193	9.39	.625807	41
20	9.362889	8.88	9.988133	.50	9.374756	9.38	10.625244	40
21	.363422	8.87	.988103	.50	.375319	9.37	.624681	39
22	.363954	8.86	.988073	.50	.375881	9.36	.624119	38
23	.364485	8.84	.988043	.50	.376442	9.35	.623558	37
24	.365016	8.83	.988013	.50	.377003	9.33	.622997	36
25	.365546	8.82	.987983	.50	.377563	9.32	.622437	35
26	.366075	8.81	.987953	.50	.378122	9.31	.621878	34
27	.366604	8.80	.987922	.50	.378681	9.30	.621319	33
28	.367131	8.79	.987892	.50	.379239	9.29	.620761	32
29	.367659	8.78	.987862	.50	.379797	9.28	.620203	31
30	9.368185	8.76	9.987832	.51	9.380354	9.27	10.619646	30
31	.368711	8.75	.987801	.51	.380910	9.26	.619090	29
32	.369236	8.74	.987771	.51	.381466	9.25	.618534	28
33	.369761	8.73	.987740	.51	.382020	9.24	.617980	27
34	.370285	8.72	.987710	.51	.382575	9.23	.617425	26
35	.370808	8.71	.987679	.51	.383129	9.22	.616871	25
36	.371330	8.70	.987649	.51	.383682	9.21	.616318	24
37	.371852	8.69	.987618	.51	.384234	9.20	.615766	23
38	.372373	8.68	.987588	.51	.384786	9.19	.615214	22
39	.372894	8.66	.987557	.51	.385337	9.18	.614663	21
40	9.373414	8.65	9.987526	.51	9.385888	9.17	10.614112	20
41	.373933	8.64	.987496	.51	.386438	9.16	.613562	19
42	.374452	8.63	.987465	.51	.386987	9.15	.613013	18
43	.374970	8.62	.987434	.51	.387536	9.14	.612464	17
44	.375487	8.61	.987403	.52	.388084	9.12	.611916	16
45	.376003	8.60	.987372	.52	.388631	9.11	.611369	15
46	.376519	8.59	.987341	.52	.389178	9.10	.610822	14
47	.377035	8.58	.987310	.52	.389724	9.09	.610276	13
48	.377549	8.57	.987279	.52	.390270	9.08	.609730	12
49	.378063	8.56	.987248	.52	.390815	9.07	.609185	11
50	9.378577	8.55	9.987217	.52	9.391360	9.06	10.608640	10
51	.379089	8.53	.987186	.52	.391903	9.05	.608097	9
52	.379601	8.52	.987155	.52	.392447	9.04	.607553	8
53	.380113	8.51	.987124	.52	.392989	9.03	.607011	7
54	.380624	8.50	.987092	.52	.393531	9.02	.606469	6
55	.381134	8.49	.987061	.52	.394073	9.01	.605927	5
56	.381643	8.48	.987030	.52	.394614	9.00	.605386	4
57	.382152	8.47	.986998	.52	.395154	8.99	.604846	3
58	.382661	8.46	.986967	.52	.395694	8.98	.604306	2
59	.383168	8.45	.986936	.52	.396233	8.97	.603767	1
60	.383675		.986904		.396771		.603229	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

103° **76°**

14° 165°

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.383675	8.44	9.986904	.53	9.396771	8.96	10.603229	60
1	.384182	8.43	.986873	.53	.397309	8.96	.602691	59
2	.384687	8.42	.986841	.53	.397846	8.95	.602154	58
3	.385192	8.41	.986809	.53	.398383	8.94	.601617	57
4	.385697	8.40	.986778	.53	.398919	8.93	.601081	56
5	.386201	8.39	.986746	.53	.399455	8.92	.600545	55
6	.386704	8.38	.986714	.53	.399990	8.91	.600010	54
7	.387207	8.37	.986683	.53	.400524	8.90	.599476	53
8	.387709	8.36	.986651	.53	.401058	8.89	.598942	52
9	.388210	8.35	.986619	.53	.401591	8.88	.598409	51
10	9.388711	8.34	9.986587	.53	9.402124	8.87	10.597876	50
11	.389211	8.33	.986555	.53	.402656	8.86	.597344	49
12	.389711	8.32	.986523	.53	.403187	8.85	.596813	48
13	.390210	8.31	.986491	.53	.403718	8.84	.596282	47
14	.390708	8.30	.986459	.53	.404249	8.83	.595751	46
15	.391206	8.28	.986427	.53	.404778	8.82	.595222	45
16	.391703	8.27	.986395	.53	.405308	8.81	.594692	44
17	.392199	8.26	.986363	.54	.405836	8.80	.594164	43
18	.392695	8.25	.986331	.54	.406364	8.79	.593636	42
19	.393191	8.24	.986299	.54	.406892	8.78	.593108	41
20	9.393685	8.23	9.986266	.54	9.407419	8.77	10.592581	40
21	.394179	8.22	.986234	.54	.407945	8.76	.592055	39
22	.394673	8.21	.986202	.54	.408471	8.75	.591529	38
23	.395166	8.20	.986169	.54	.408997	8.74	.591003	37
24	.395658	8.19	.986137	.54	.409521	8.74	.590479	36
25	.396150	8.18	.986104	.54	.410045	8.73	.589955	35
26	.396641	8.17	.986072	.54	.410569	8.72	.589431	34
27	.397132	8.17	.986039	.54	.411092	8.71	.588908	33
28	.397621	8.16	.986007	.54	.411615	8.70	.588385	32
29	.398111	8.15	.985974	.54	.412137	8.69	.587863	31
30	9.398600	8.14	9.985942	.54	9.412658	8.68	10.587342	30
31	.399088	8.13	.985909	.55	.413179	8.67	.586821	29
32	.399575	8.12	.985876	.55	.413699	8.66	.586301	28
33	.400062	8.11	.985843	.55	.414219	8.65	.585781	27
34	.400549	8.10	.985811	.55	.414738	8.65	.585262	26
35	.401035	8.09	.985778	.55	.415257	8.64	.584743	25
36	.401520	8.08	.985745	.55	.415775	8.63	.584225	24
37	.402005	8.07	.985712	.55	.416293	8.62	.583707	23
38	.402489	8.06	.985679	.55	.416810	8.61	.583190	22
39	.402972	8.05	.985646	.55	.417326	8.60	.582674	21
40	9.403455	8.04	9.985613	.55	9.417842	8.59	10.582158	20
41	.403938	8.03	.985580	.55	.418358	8.58	.581642	19
42	.404420	8.02	.985547	.55	.418873	8.57	.581127	18
43	.404901	8.01	.985514	.55	.419387	8.56	.580613	17
44	.405382	8.00	.985480	.55	.419901	8.56	.580099	16
45	.405862	7.99	.985447	.55	.420415	8.55	.579585	15
46	.406341	7.98	.985414	.56	.420927	8.54	.579073	14
47	.406820	7.97	.985380	.56	.421440	8.53	.578560	13
48	.407299	7.96	.985347	.56	.421952	8.52	.578048	12
49	.407777	7.95	.985314	.56	.422463	8.51	.577537	11
50	9.408254	7.94	9.985280	.56	9.422974	8.50	10.577026	10
51	.408731	7.94	.985247	.56	.423484	8.49	.576516	9
52	.409207	7.93	.985213	.56	.423993	8.49	.576007	8
53	.409682	7.92	.985180	.56	.424503	8.48	.575497	7
54	.410157	7.91	.985146	.56	.425011	8.47	.574989	6
55	.410632	7.90	.985113	.56	.425519	8.46	.574481	5
56	.411106	7.89	.985079	.56	.426027	8.45	.573973	4
57	.411579	7.88	.985045	.56	.426534	8.44	.573466	3
58	.412052	7.87	.985011	.56	.427041	8.43	.572959	2
59	.412524	7.86	.984978	.56	.427547	8.43	.572453	1
60	.412996		.984944		.428052		.571948	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

104° 75°

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.412996	7.85	9.984944	.57	9.428052	8.42	10.571948	60
1	.413467	7.84	.984910	.57	.428558	8.41	.571442	59
2	.413938	7.84	.984876	.57	.429062	8.40	.570938	58
3	.414408	7.83	.984842	.57	.429566	8.39	.570434	57
4	.414878	7.82	.984808	.57	.430070	8.38	.569930	56
5	.415347	7.81	.984774	.57	.430573	8.38	.569427	55
6	.415815	7.80	.984740	.57	.431075	8.37	.568925	54
7	.416283	7.79	.984706	.57	.431577	8.36	.568423	53
8	.416751	7.78	.984672	.57	.432079	8.35	.567921	52
9	.417217	7.77	.984637	.57	.432580	8.34	.567420	51
10	9.417684	7.76	9.984603	.57	9.433080	8.33	10.566920	50
11	.418150	7.75	.984569	.57	.433580	8.33	.566420	49
12	.418615	7.75	.984535	.57	.434080	8.32	.565920	48
13	.419079	7.74	.984500	.57	.434579	8.31	.565421	47
14	.419544	7.73	.984466	.57	.435078	8.30	.564922	46
15	.420007	7.72	.984432	.58	.435576	8.29	.564424	45
16	.420470	7.71	.984397	.58	.436073	8.28	.563927	44
17	.420933	7.70	.984363	.58	.436570	8.28	.563430	43
18	.421395	7.69	.984328	.58	.437067	8.27	.562933	42
19	.421857	7.68	.984294	.58	.437563	8.26	.562437	41
20	9.422318	7.67	9.984259	.58	9.438059	8.25	10.561941	40
21	.422778	7.67	.984224	.58	.438554	8.24	.561446	39
22	.423238	7.66	.984190	.58	.439048	8.24	.560952	38
23	.423697	7.65	.984155	.58	.439543	8.23	.560457	37
24	.424156	7.64	.984120	.58	.440036	8.22	.559964	36
25	.424615	7.63	.984085	.58	.440529	8.21	.559471	35
26	.425073	7.62	.984050	.58	.441022	8.20	.558978	34
27	.425530	7.61	.984015	.58	.441514	8.20	.558486	33
28	.425987	7.61	.983981	.58	.442006	8.19	.557994	32
29	.426443	7.60	.983946	.58	.442497	8.18	.557503	31
30	9.426899	7.59	9.983911	.58	9.442988	8.17	10.557012	30
31	.427354	7.58	.983875	.58	.443479	8.16	.556521	29
32	.427809	7.57	.983840	.59	.443968	8.16	.556032	28
33	.428263	7.56	.983805	.59	.444458	8.15	.555542	27
34	.428717	7.55	.983770	.59	.444947	8.14	.555053	26
35	.429170	7.54	.983735	.59	.445435	8.13	.554565	25
36	.429623	7.53	.983700	.59	.445923	8.13	.554077	24
37	.430075	7.52	.983664	.59	.446411	8.12	.553589	23
38	.430527	7.52	.983629	.59	.446898	8.11	.553102	22
39	.430978	7.51	.983594	.59	.447384	8.10	.552616	21
40	9.431429	7.50	9.983558	.59	9.447870	8.09	10.552130	20
41	.431879	7.49	.983523	.59	.448356	8.09	.551644	19
42	.432329	7.49	.983487	.59	.448841	8.08	.551159	18
43	.432778	7.48	.983452	.59	.449326	8.07	.550674	17
44	.433226	7.47	.983416	.59	.449810	8.06	.550190	16
45	.433675	7.46	.983381	.59	.450294	8.06	.549706	15
46	.434122	7.45	.983345	.59	.450777	8.05	.549223	14
47	.434569	7.44	.983309	.59	.451260	8.04	.548740	13
48	.435016	7.44	.983273	.60	.451743	8.03	.548257	12
49	.435462	7.43	.983238	.60	.452225	8.03	.547775	11
50	9.435908	7.42	9.983202	.60	9.452706	8.02	10.547294	10
51	.436353	7.41	.983166	.60	.453187	8.01	.546813	9
52	.436798	7.40	.983130	.60	.453668	8.00	.546332	8
53	.437242	7.40	.983094	.60	.454148	8.00	.545852	7
54	.437686	7.39	.983058	.60	.454628	7.99	.545372	6
55	.438129	7.38	.983022	.60	.455107	7.98	.544893	5
56	.438572	7.37	.982986	.60	.455586	7.97	.544414	4
57	.439014	7.36	.982950	.60	.456064	7.97	.543936	3
58	.439456	7.36	.982914	.60	.456542	7.96	.543458	2
59	.439897	7.35	.982878	.60	.457019	7.95	.542981	1
60	.440338		.982842		.457496		.542504	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

16° **163°**

M.	Sine.	D.1''.	Cosine.	D.1''	Tang.	D.1''.	Cotang.	M.
0	9.440338	7.34	9.982842	.60	9.457496	7.94	10.542504	60
1	.440778	7.33	.982805	.60	.457973	7.94	.542027	59
2	.441218	7.32	.982769	.61	.458449	7.93	.541551	58
3	.441658	7.31	.982733	.61	.458925	7.92	.541075	57
4	.442096	7.31	.982696	.61	.459400	7.91	.540600	56
5	.442535	7.30	.982660	.61	.459875	7.91	.540125	55
6	.442973	7.29	.982624	.61	.460349	7.90	.539651	54
7	.443410	7.28	.982587	.61	.460823	7.89	.539177	53
8	.443847	7.27	.982551	.61	.461297	7.88	.538703	52
9	.444284	7.27	.982514	.61	.461770	7.88	.538230	51
10	9.444720	7.26	9.982477	.61	9.462242	7.87	10.537758	50
11	.445155	7.25	.982441	.61	.462714	7.86	.537286	49
12	.445590	7.24	.982404	.61	.463186	7.86	.536814	48
13	.446025	7.24	.982367	.61	.463658	7.85	.536342	47
14	.446459	7.23	.982331	.61	.464129	7.84	.535871	46
15	.446893	7.22	.982294	.61	.464599	7.83	.535401	45
16	.447326	7.21	.982257	.61	.465069	7.83	.534931	44
17	.447759	7.20	.982220	.62	.465539	7.82	.534461	43
18	.448191	7.20	.982183	.62	.466008	7.81	.533992	42
19	.448623	7.19	.982146	.62	.466476	7.81	.533524	41
20	9.449054	7.18	9.982109	.62	9.466945	7.80	10.533055	40
21	.449485	7.17	.982072	.62	.467413	7.79	.532587	39
22	.449915	7.17	.982035	.62	.467880	7.78	.532120	38
23	.450345	7.16	.981998	.62	.468347	7.78	.531653	37
24	.450775	7.15	.981961	.62	.468814	7.77	.531186	36
25	.451204	7.14	.981924	.62	.469280	7.76	.530720	35
26	.451632	7.13	.981886	.62	.469746	7.76	.530254	34
27	.452060	7.13	.981849	.62	.470211	7.75	.529789	33
28	.452488	7.12	.981812	.62	.470676	7.74	.529324	32
29	.452915	7.11	.981774	.62	.471141	7.74	.528859	31
30	9.453342	7.10	9.981737	.62	9.471605	7.73	10.528395	30
31	.453768	7.10	.981700	.62	.472068	7.72	.527932	29
32	.454194	7.09	.981662	.63	.472532	7.71	.527468	28
33	.454619	7.08	.981625	.63	.472995	7.71	.527005	27
34	.455044	7.07	.981587	.63	.473457	7.70	.526543	26
35	.455469	7.07	.981549	.63	.473919	7.69	.526081	25
36	.455893	7.06	.981512	.63	.474381	7.69	.525619	24
37	.456316	7.05	.981474	.63	.474842	7.68	.525158	23
38	.456739	7.04	.981436	.63	.475303	7.67	.524697	32
39	.457162	7.04	.981399	.63	.475763	7.67	.524237	21
40	9.457584	7.03	9.981361	.63	9.476223	7.66	10.523777	20
41	.458006	7.02	.981323	.63	.476683	7.65	.523317	19
42	.458427	7.01	.981285	.63	.477142	7.65	.522858	18
43	.458848	7.01	.981247	.63	.477601	7.64	.522399	17
44	.459268	7.00	.981209	.63	.478059	7.63	.521941	16
45	.459688	6.99	.981171	.63	.478517	7.63	.521483	15
46	.460108	6.98	.981133	.63	.478975	7.62	.521025	14
47	.460527	6.98	.981095	.64	.479432	7.61	.520568	13
48	.460946	6.97	.981057	.64	.479889	7.61	.520111	12
49	.461364	6.96	.981019	.64	.480345	7.60	.519655	11
50	9.461782	6.96	9.980981	.64	9.480801	7.59	10.519199	10
51	.462199	6.95	.980942	.64	.481257	7.59	.518743	9
52	.462616	6.94	.980904	.64	.481712	7.58	.518288	8
53	.463032	6.93	.980866	.64	.482167	7.57	.517833	7
54	.463448	6.93	.980827	.64	.482621	7.57	.517379	6
55	.463864	6.92	.980789	.64	.483075	7.56	.516925	5
56	.464279	6.91	.980750	.64	.483529	7.55	.516471	4
57	.464694	6.90	.980712	.64	.483982	7.55	.516018	3
58	.465108	6.90	.980673	.64	.484435	7.54	.515565	2
59	.465522	6.89	.980635	.64	.484887	7.53	.515113	1
60	.465935		.980596		.485339		.514661	0
M.	Cosine.	D.1''.	Sine.	D.1''.	Cotang.	D.1''.	Tang.	M.

106° **73°**

17° **162°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.465935	6.88	9.980596	.64	9.485339	7.53	10.514661	60
1	.466348	6.88	.980558	.64	.485791	7.52	.514209	59
2	.466761	6.87	.980519	.65	.486242	7.51	.513758	58
3	.467173	6.86	.980480	.65	.486693	7.51	.513307	57
4	.467585	6.85	.980442	.65	.487143	7.50	.512857	56
5	.467996	6.85	.980403	.65	.487593	7.50	.512407	55
6	.468407	6.84	.980364	.65	.488043	7.49	.511957	54
7	.468817	6.83	.980325	.65	.488492	7.48	.511508	53
8	.469227	6.83	.980286	.65	.488941	7.48	.511059	52
9	.469637	6.82	.980247	.65	.489390	7.47	.510610	51
10	9.470046	6.81	9.980208	.65	9.489838	7.46	10.510162	50
11	.470455	6.81	.980169	.65	.490286	7.46	.509714	49
12	.470863	6.80	.980130	.65	.490733	7.45	.509267	48
13	.471271	6.79	.980091	.65	.491180	7.44	.508820	47
14	.471679	6.78	.980052	.65	.491627	7.44	.508373	46
15	.472086	6.78	.980012	.65	.492073	7.43	.507927	45
16	.472492	6.77	.979973	.65	.492519	7.43	.507481	44
17	.472898	6.76	.979934	.66	.492965	7.42	.507035	43
18	.473304	6.76	.979895	.66	.493410	7.41	.506590	42
19	.473710	6.75	.979855	.66	.493854	7.41	.506146	41
20	9.474115	6.74	9.979816	.66	9.494299	7.40	10.505701	40
21	.474519	6.74	.979776	.66	.494743	7.39	.505257	39
22	.474923	6.73	.979737	.66	.495186	7.39	.504814	38
23	.475327	6.72	.979697	.66	.495630	7.38	.504370	37
24	.475730	6.72	.979658	.66	.496073	7.38	.503927	36
25	.476133	6.71	.979618	.66	.496515	7.37	.503485	35
26	.476536	6.70	.979579	.66	.496957	7.36	.503043	34
27	.476938	6.69	.979539	.66	.497399	7.36	.502601	33
28	.477340	6.69	.979499	.66	.497841	7.35	.502159	32
29	.477741	6.68	.979459	.66	.498282	7.34	.501718	31
30	9.478142	6.67	9.979420	.66	9.498722	7.34	10.501278	30
31	.478542	6.67	.979380	.66	.499163	7.33	.500837	29
32	.478942	6.66	.979340	.67	.499603	7.33	.500397	28
33	.479342	6.65	.979300	.67	.500042	7.32	.499958	27
34	.479741	6.65	.979260	.67	.500481	7.31	.499519	26
35	.480140	6.64	.979220	.67	.500920	7.31	.499080	25
36	.480539	6.63	.979180	.67	.501359	7.30	.498641	24
37	.480937	6.63	.979140	.67	.501797	7.30	.498203	23
38	.481334	6.62	.979100	.67	.502235	7.29	.497765	22
39	.481731	6.61	.979059	.67	.502672	7.28	.497328	21
40	9.482128	6.61	9.979019	.67	9.503109	7.28	10.496891	20
41	.482525	6.60	.978979	.67	.503546	7.27	.496454	19
42	.482921	6.59	.978939	.67	.503982	7.27	.496018	18
43	.483316	6.59	.978898	.67	.504418	7.26	.495582	17
44	.483712	6.58	.978858	.67	.504854	7.25	.495146	16
45	.484107	6.57	.978817	.67	.505289	7.25	.494711	15
46	.484501	6.57	.978777	.67	.505724	7.24	.494276	14
47	.484895	6.56	.978736	.68	.506159	7.24	.493841	13
48	.485289	6.55	.978696	.68	.506593	7.23	.493407	12
49	.485682	6.55	.978655	.68	.507027	7.23	.492973	11
50	9.486075	6.54	9.978615	.68	9.507460	7.22	10.492540	10
51	.486467	6.54	.978574	.68	.507893	7.21	.492107	9
52	.486860	6.53	.978533	.68	.508326	7.21	.491674	8
53	.487251	6.52	.978493	.68	.508759	7.20	.491241	7
54	.487643	6.52	.978452	.68	.509191	7.20	.490809	6
55	.488034	6.51	.978411	.68	.509622	7.19	.490378	5
56	.488424	6.50	.978370	.68	.510054	7.18	.489946	4
57	.488814	6.50	.978329	.68	.510485	7.18	.489515	3
58	.489204	6.49	.978288	.68	.510916	7.17	.489084	2
59	.489593	6.48	.978247	.68	.511346	7.17	.488654	1
60	.489982		.978206		.511776		.488224	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

107° E **72°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.489982	6.48	9.978206	.68	9.511776	7.16	10.488224	60
1	.490371	6.47	.978165	.69	.512206	7.16	.487794	59
2	.490759	6.46	.978124	.69	.512635	7.15	.487365	58
3	.491147	6.46	.978083	.69	.513064	7.14	.486936	57
4	.491535	6.45	.978042	.69	.513493	7.14	.486507	56
5	.491922	6.45	.978001	.69	.513921	7.13	.486079	55
6	.492308	6.44	.977959	.69	.514349	7.13	.485651	54
7	.492695	6.43	.977918	.69	.514777	7.12	.485223	53
8	.493081	6.43	.977877	.69	.515204	7.12	.484796	52
9	.493466	6.42	.977835	.69	.515631	7.11	.484369	51
10	9.493851	6.41	9.977794	.69	9.516057	7.10	10.483943	50
11	.494236	6.41	.977752	.69	.516484	7.10	.483516	49
12	.494621	6.40	.977711	.69	.516910	7.09	.483090	48
13	.495005	6.39	.977669	.69	.517335	7.09	.482665	47
14	.495388	6.39	.977628	.69	.517761	7.08	.482239	46
15	.495772	6.38	.977586	.69	.518185	7.08	.481815	45
16	.496154	6.38	.977544	.70	.518610	7.07	.481390	44
17	.496537	6.37	.977503	.70	.519034	7.07	.480966	43
18	.496919	6.36	.977461	.70	.519458	7.06	.480542	42
19	.497301	6.36	.977419	.70	.519882	7.05	.480118	41
20	9.497682	6.35	9.977377	.70	9.520305	7.05	10.479695	40
21	.498064	6.34	.977335	.70	.520728	7.04	.479272	39
22	.498444	6.34	.977293	.70	.521151	7.04	.478849	38
23	.498825	6.33	.977251	.70	.521573	7.03	.478427	37
24	.499204	6.33	.977209	.70	.521995	7.03	.478005	36
25	.499584	6.32	.977167	.70	.522417	7.02	.477583	35
26	.499963	6.31	.977125	.70	.522838	7.02	.477162	34
27	.500342	6.31	.977083	.70	.523259	7.01	.476741	33
28	.500721	6.30	.977041	.70	.523680	7.01	.476320	32
29	.501099	6.30	.976999	.70	.524100	7.00	.475900	31
30	9.501476	6.29	9.976957	.70	9.524520	6.99	10.475480	30
31	.501854	6.28	.976914	.71	.524939	6.99	.475061	29
32	.502231	6.28	.976872	.71	.525359	6.98	.474641	28
33	.502607	6.27	.976830	.71	.525778	6.98	.474222	27
34	.502984	6.27	.976787	.71	.526197	6.97	.473803	26
35	.503360	6.26	.976745	.71	.526615	6.97	.473385	25
36	.503735	6.25	.976702	.71	.527033	6.96	.472967	24
37	.504110	6.25	.976660	.71	.527451	6.96	.472549	23
38	.504485	6.24	.976617	.71	.527868	6.95	.472132	22
39	.504860	6.24	.976574	.71	.528285	6.95	.471715	21
40	9.505234	6.23	9.976532	.71	9.528702	6.94	10.471298	20
41	.505608	6.22	.976489	.71	.529119	6.94	.470881	19
42	.505981	6.22	.976446	.71	.529535	6.93	.470465	18
43	.506354	6.21	.976404	.71	.529950	6.93	.470050	17
44	.506727	6.21	.976361	.71	.530366	6.92	.469634	16
45	.507099	6.20	.976318	.72	.530781	6.91	.469219	15
46	.507471	6.19	.976275	.72	.531196	6.91	.468804	14
47	.507843	6.19	.976232	.72	.531611	6.90	.468389	13
48	.508214	6.18	.976189	.72	.532025	6.90	.467975	12
49	.508585	6.18	.976146	.72	.532439	6.89	.467561	11
50	9.508956	6.17	9.976103	.72	9.532853	6.89	10.467147	10
51	.509326	6.16	.976060	.72	.533266	6.88	.466734	9
52	.509696	6.16	.976017	.72	.533679	6.88	.466321	8
53	.510065	6.15	.975974	.72	.534092	6.87	.465908	7
54	.510434	6.15	.975930	.72	.534504	6.87	.465496	6
55	.510803	6.14	.975887	.72	.534916	6.86	.465084	5
56	.511172	6.14	.975844	.72	.535328	6.86	.464672	4
57	.511540	6.13	.975800	.72	.535739	6.85	.464261	3
58	.511907	6.12	.975757	.72	.536150	6.85	.463850	2
59	.512275	6.12	.975714	.72	.536561	6.84	.463439	1
60	.512642		.975670		.536972		.463028	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

19° **160°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.512642	6.11	9.975670	.73	9.536972	6.84	10.463028	60
1	.513009	6.11	.975627	.73	.537382	6.83	.462618	59
2	.513375	6.10	.975583	.73	.537791	6.83	.462209	58
3	.513741	6.09	.975539	.73	.538202	6.82	.461798	57
4	.514107	6.09	.975496	.73	.538611	6.82	.461389	56
5	.514472	6.08	.975452	.73	.539020	6.81	.460980	55
6	.514837	6.08	.975408	.73	.539429	6.81	.460571	54
7	.515202	6.07	.975365	.73	.539837	6.80	.460163	53
8	.515566	6.07	.975321	.73	.540245	6.80	.459755	52
9	.515930	6.06	.975277	.73	.540653	6.79	.459347	51
10	9.516294	6.05	9.975233	.73	9.541061	6.79	10.458939	50
11	.516657	6.05	.975189	.73	.541468	6.78	.458532	49
12	.517020	6.04	.975145	.73	.541875	6.78	.458125	48
13	.517382	6.04	.975101	.73	.542281	6.77	.457719	47
14	.517745	6.03	.975057	.73	.542688	6.77	.457312	46
15	.518107	6.03	.975013	.74	.543094	6.76	.456906	45
16	.518468	6.02	.974969	.74	.543499	6.76	.456501	44
17	.518829	6.02	.974925	.74	.543905	6.75	.456095	43
18	.519190	6.01	.974880	.74	.544310	6.75	.455690	42
19	.519551	6.00	.974836	.74	.544715	6.74	.455285	41
20	9.519911	6.00	9.974792	.74	9.545119	6.74	10.454881	40
21	.520271	5.99	.974748	.74	.545524	6.73	.454476	39
22	.520631	5.99	.974703	.74	.545928	6.73	.454072	38
23	.520990	5.98	.974659	.74	.546331	6.72	.453669	37
24	.521349	5.98	.974614	.74	.546735	6.72	.453265	36
25	.521707	5.97	.974570	.74	.547138	6.71	.452862	35
26	.522066	5.97	.974525	.74	.547540	6.71	.452460	34
27	.522424	5.96	.974481	.74	.547943	6.70	.452057	33
28	.522781	5.95	.974436	.74	.548345	6.70	.451655	32
29	.523138	5.95	.974391	.75	.548747	6.69	.451253	31
30	9.523495	5.94	9.974347	.75	9.549149	6.69	10.450851	30
31	.523852	5.94	.974302	.75	.549550	6.68	.450450	29
32	.524208	5.93	.974257	.75	.549951	6.68	.450049	28
33	.524564	5.93	.974212	.75	.550352	6.67	.449648	27
34	.524920	5.92	.974167	.75	.550752	6.67	.449248	26
35	.525275	5.92	.974122	.75	.551152	6.67	.448848	25
36	.525630	5.91	.974077	.75	.551552	6.66	.448448	24
37	.525984	5.90	.974032	.75	.551952	6.66	.448048	23
38	.526339	5.90	.973987	.75	.552351	6.65	.447649	22
39	.526693	5.89	.973942	.75	.552750	6.65	.447250	21
40	9.527046	5.89	9.973897	.75	9.553149	6.64	10.446851	20
41	.527400	5.88	.973852	.75	.553548	6.64	.446452	19
42	.527753	5.88	.973807	.75	.553946	6.63	.446054	18
43	.528105	5.87	.973761	.75	.554344	6.63	.445656	17
44	.528458	5.87	.973716	.76	.554741	6.62	.445259	16
45	.528810	5.86	.973671	.76	.555139	6.62	.444861	15
46	.529161	5.86	.973625	.76	.555536	6.61	.444464	14
47	.529513	5.85	.973580	.76	.555933	6.61	.444067	13
48	.529864	5.85	.973535	.76	.556329	6.60	.443671	12
49	.530215	5.84	.973489	.76	.556725	6.60	.443275	11
50	9.530565	5.83	9.973444	.76	9.557121	6.59	10.442879	10
51	.530915	5.83	.973398	.76	.557517	6.59	.442483	9
52	.531265	5.82	.973352	.76	.557913	6.59	.442087	8
53	.531614	5.82	.973307	.76	.558308	6.58	.441692	7
54	.531963	5.81	.973261	.76	.558702	6.58	.441298	6
55	.532312	5.81	.973215	.76	.559097	6.57	.440903	5
56	.532661	5.80	.973169	.76	.559491	6.57	.440509	4
57	.533009	5.80	.973124	.76	.559885	6.56	.440115	3
58	.533357	5.79	.973078	.77	.560279	6.56	.439721	2
59	.533704	5.79	.973032	.77	.560673	6.55	.439327	1
60	.534052		.972986		.561066		.438934	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

109° **70°**

20° **159°**

M.	Sine.	D.1''.	Cosine.	D.1''.	Tang.	D.1''.	Cotang.	M.
0	9.534052	5.78	9.972986	.77	9.561066	6.55	10.438934	60
1	.534399	5.78	.972940	.77	.561459	6.54	.438541	59
2	.534745	5.77	.972894	.77	.561851	6.54	.438149	58
3	.535092	5.77	.972848	.77	.562244	6.54	.437756	57
4	.535438	5.76	.972802	.77	.562636	6.53	.437364	56
5	.535783	5.76	.972755	.77	.563028	6.53	.436972	55
6	.536129	5.75	.972709	.77	.563419	6.52	.436581	54
7	.536474	5.75	.972663	.77	.563811	6.52	.436189	53
8	.536818	5.74	.972617	.77	.564202	6.51	.435798	52
9	.537163	5.74	.972570	.77	.564592	6.51	.435408	51
10	9.537507	5.73	9.972524	.77	9.564983	6.50	10.435017	50
11	.537851	5.73	.972478	.77	.565373	6.50	.434627	49
12	.538194	5.72	.972431	.78	.565763	6.50	.434237	48
13	.538538	5.71	.972385	.78	.566153	6.49	.433847	47
14	.538880	5.71	.972338	.78	.566542	6.49	.433458	46
15	.539223	5.70	.972291	.78	.566932	6.48	.433068	45
16	.539565	5.70	.972245	.78	.567320	6.48	.432680	44
17	.539907	5.69	.972198	.78	.567709	6.47	.432291	43
18	.540249	5.69	.972151	.78	.568098	6.47	.431902	42
19	.540590	5.68	.972105	.78	.568486	6.46	.431514	41
20	9.540931	5.68	9.972058	.78	9.568873	6.46	10.431127	40
21	.541272	5.67	.972011	.78	.569261	6.46	.430739	39
22	.541613	5.67	.971964	.78	.569648	6.45	.430352	38
23	.541953	5.66	.971917	.78	.570035	6.45	.429965	37
24	.542293	5.66	.971870	.78	.570422	6.44	.429578	36
25	.542632	5.65	.971823	.78	.570809	6.44	.429191	35
26	.542971	5.65	.971776	.78	.571195	6.43	.428805	34
27	.543310	5.64	.971729	.79	.571581	6.43	.428419	33
28	.543649	5.64	.971682	.79	.571967	6.43	.428033	32
29	.543987	5.63	.971635	.79	.572352	6.42	.427648	31
30	9.544325	5.63	9.971588	.79	9.572738	6.42	10.427262	30
31	.544663	5.62	.971540	.79	.573123	6.41	.426877	29
32	.545000	5.62	.971493	.79	.573507	6.41	.426493	28
33	.545338	5.61	.971446	.79	.573892	6.40	.426108	27
34	.545674	5.61	.971398	.79	.574276	6.40	.425724	26
35	.546011	5.60	.971351	.79	.574660	6.40	.425340	25
36	.546347	5.60	.971303	.79	.575044	6.39	.424956	24
37	.546683	5.59	.971256	.79	.575427	6.39	.424573	23
38	.547019	5.59	.971208	.79	.575810	6.38	.424190	22
39	.547354	5.58	.971161	.79	.576193	6.38	.423807	21
40	9.547689	5.58	9.971113	.79	9.576576	6.37	10.423424	20
41	.548024	5.57	.971066	.80	.576958	6.37	.423041	19
42	.548359	5.57	.971018	.80	.577341	6.37	.422659	18
43	.548693	5.56	.970970	.80	.577723	6.36	.422277	17
44	.549027	5.56	.970922	.80	.578104	6.36	.421896	16
45	.549360	5.55	.970874	.80	.578486	6.35	.421514	15
46	.549693	5.55	.970827	.80	.578867	6.35	.421133	14
47	.550026	5.55	.970779	.80	.579248	6.34	.420752	13
48	.550359	5.54	.970731	.80	.579629	6.34	.420371	12
49	.550692	5.54	.970683	.80	.580009	6.34	.419991	11
50	9.551024	5.53	9.970635	.80	9.580389	6.33	10.419611	10
51	.551356	5.53	.970586	.80	.580769	6.33	.419231	9
52	.551687	5.52	.970538	.80	.581149	6.32	.418851	8
53	.552018	5.52	.970490	.80	.581528	6.32	.418472	7
54	.552349	5.51	.970442	.80	.581907	6.32	.418093	6
55	.552680	5.51	.970394	.81	.582286	6.31	.417714	5
56	.553010	5.50	.970345	.81	.582665	6.31	.417335	4
57	.553341	5.50	.970297	.81	.583043	6.30	.416957	3
58	.553670	5.49	.970249	.81	.583422	6.30	.416578	2
59	.554000	5.49	.970200	.81	.583800	6.30	.416200	1
60	.554329		.970152		.584177		.415823	0
M.	Cosine.	D.1''.	Sine.	D.1''.	Cotang.	D.1''.	Tang.	M.

110° **69°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.554329	5.48	9.970152	.81	9.584177	6.29	10.415823	60
1	.554658	5.48	.970103	.81	.584555	6.29	.415445	59
2	.554987	5.47	.970055	.81	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.81	.585309	6.28	.414691	57
4	.555643	5.46	.969957	.81	.585686	6.28	.414314	56
5	.555971	5.46	.969909	.81	.586062	6.27	.413938	55
6	.556299	5.45	.969860	.81	.586439	6.27	.413561	54
7	.556626	5.45	.969811	.81	.586815	6.26	.413185	53
8	.556953	5.44	.969762	.81	.587190	6.26	.412810	52
9	.557280	5.44	.969714	.81	.587566	6.26	.412434	51
10	9.557606	5.44	9.969665	.82	9.587941	6.25	10.412059	50
11	.557932	5.43	.969616	.82	.588316	6.25	.411684	49
12	.558258	5.43	.969567	.82	.588691	6.24	.411309	48
13	.558583	5.42	.969518	.82	.589066	6.24	.410934	47
14	.558909	5.42	.969469	.82	.589440	6.24	.410560	46
15	.559234	5.41	.969420	.82	.589814	6.23	.410186	45
16	.559558	5.41	.969370	.82	.590188	6.23	.409812	44
17	.559883	5.40	.969321	.82	.590562	6.22	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.39	.969223	.82	.591308	6.22	.408692	41
20	9.560855	5.39	9.969173	.82	9.591681	6.21	10.408319	40
21	.561178	5.38	.969124	.82	.592054	6.21	.407946	39
22	.561501	5.38	.969075	.82	.592426	6.20	.407574	38
23	.561824	5.37	.969025	.82	.592798	6.20	.407202	37
24	.562146	5.37	.968976	.83	.593171	6.20	.406829	36
25	.562468	5.37	.968926	.83	.593542	6.19	.406458	35
26	.562790	5.36	.968877	.83	.593914	6.19	.406086	34
27	.563112	5.36	.968827	.83	.594285	6.18	.405715	33
28	.563433	5.35	.968777	.83	.594656	6.18	.405344	32
29	.563755	5.35	.968728	.83	.595027	6.18	.404973	31
30	9.564075	5.34	9.968678	.83	9.595398	6.17	10.404602	30
31	.564396	5.34	.968628	.83	.595768	6.17	.404232	29
32	.564716	5.33	.968578	.83	.596138	6.16	.403862	28
33	.565036	5.33	.968528	.83	.596508	6.16	.403492	27
34	.565356	5.32	.968479	.83	.596878	6.16	.403122	26
35	.565676	5.32	.968429	.83	.597247	6.15	.402753	25
36	.565995	5.32	.968379	.83	.597616	6.15	.402384	24
37	.566314	5.31	.968329	.83	.597985	6.15	.402015	23
38	.566632	5.31	.968278	.84	.598354	6.14	.401646	22
39	.566951	5.30	.968228	.84	.598722	6.14	.401278	21
40	9.567269	5.30	9.968178	.84	9.599091	6.13	10.400909	20
41	.567587	5.29	.968128	.84	.599459	6 13	.400541	19
42	.567904	5.29	.968078	.84	.599827	6.13	.400173	18
43	.568222	5.28	.968027	.84	.600194	6.12	.399806	17
44	.568539	5.28	.967977	.84	.600562	6.12	.399438	16
45	.568856	5.28	.967927	.84	.600929	6.12	.399071	15
46	.569172	5.27	.967876	.84	.601296	6.11	.398704	14
47	.569488	5.27	.967826	.84	.601662	6.11	.398338	13
48	.569804	5.26	.967775	.84	.602029	6.10	.397971	12
49	.570120	5.26	.967725	.84	.602395	6.10	.397605	11
50	9.570435	5.25	9 967674	.84	9.602761	6.10	10.397239	10
51	.570751	5.25	.967624	.84	.603127	6.09	.396873	9
52	.571066	5.24	.967573	.85	.603493	6.09	.396507	8
53	.571380	5.24	.967522	.85	.603858	6.09	.396142	7
54	.571695	5.24	.967471	.85	.604223	6.08	.395777	6
55	.572009	5.23	.967421	.85	.604588	6.08	.395412	5
56	.572323	5.23	.967370	.85	.604953	6.07	.395047	4
57	.572636	5.22	.967319	.85	.605317	6.07	.394683	3
58	.572950	5.22	.967268	.85	.605682	6.07	.394318	2
59	.573263	5.21	.967217	.85	.606046	6.06	.393954	1
60	.573575		.967166		.606410		.393590	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

22° **157°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.573575	5.21	9.967166	.85	9.606410	6.06	10.393590	60
1	.573888	5.20	.967115	.85	.606773	6.06	.393227	59
2	.574200	5.20	.967064	.85	.607137	6.05	.392863	58
3	.574512	5.20	.967013	.85	.607500	6.05	.392500	57
4	.574824	5.19	.966961	.85	.607863	6.05	.392137	56
5	.575136	5.19	.966910	.85	.608225	6.04	.391775	55
6	.575447	5.18	.966859	.86	.608588	6.04	.391412	54
7	.575758	5.18	.966808	.86	.608950	6.03	.391050	53
8	.576069	5.17	.966756	.86	.609312	6.03	.390688	52
9	.576379	5.17	.966705	.86	.609674	6.03	.390326	51
10	9.576689	5.17	9.966653	.86	9.610036	6.02	10.389964	50
11	.576999	5.16	.966602	.86	.610397	6.02	.389603	49
12	.577309	5.16	.966550	.86	.610759	6.02	.389241	48
13	.577618	5.15	.966499	.86	.611120	6.01	.388880	47
14	.577927	5.15	.966447	.86	.611480	6.01	.388520	46
15	.578236	5.14	.966395	.86	.611841	6.01	.388159	45
16	.578545	5.14	.966344	.86	.612201	6.00	.387799	44
17	.578853	5.14	.966292	.86	.612561	6.00	.387439	43
18	.579162	5.13	.966240	.86	.612921	6.00	.387079	42
19	.579470	5.13	.966188	.86	.613281	5.99	.386719	41
20	9.579777	5.12	9.966136	.87	9.613641	5.99	10.386359	40
21	.580085	5.12	.966085	.87	.614000	5.98	.386000	39
22	.580392	5.11	.966033	.87	.614359	5.98	.385641	38
23	.580699	5.11	.965981	.87	.614718	5.98	.385282	37
24	.581005	5.11	.965928	.87	.615077	5.97	.384923	36
25	.581312	5.10	.965876	.87	.615435	5.97	.384565	35
26	.581618	5.10	.965824	.87	.615793	5.97	.384207	34
27	.581924	5.09	.965772	.87	.616151	5.96	.383849	33
28	.582229	5.09	.965720	.87	.616509	5.96	.383491	32
29	.582535	5.09	.965668	.87	.616867	5.96	.383133	31
30	9.582840	5.08	9.965615	.87	9.617224	5.95	10.382776	30
31	.583145	5.08	.965563	.87	.617582	5.95	.382418	29
32	.583449	5.07	.965511	.87	.617939	5.95	.382061	28
33	.583754	5.07	.965458	.87	.618295	5.94	.381705	27
34	.584058	5.06	.965406	.88	.618652	5.94	.381348	26
35	.584361	5.06	.965353	.88	.619008	5.94	.380992	25
36	.584665	5.06	.965301	.88	.619364	5.93	.380636	24
37	.584968	5.05	.965248	.88	.619721	5.93	.380280	23
38	.585272	5.05	.965195	.88	.620076	5.93	.379924	22
39	.585574	5.04	.965143	.88	.620432	5.92	.379568	21
40	9.585877	5.04	9.965090	.88	9.620787	5.92	10.379213	20
41	.586179	5.04	.965037	.88	.621142	5.92	.378858	19
42	.586482	5.03	.964984	.88	.621497	5.91	.378503	18
43	.586783	5.03	.964931	.88	.621852	5.91	.378148	17
44	.587085	5.02	.964879	.88	.622207	5.91	.377793	16
45	.587386	5.02	.964826	.88	.622561	5.90	.377439	15
46	.587688	5.01	.964773	.88	.622915	5.90	.377085	14
47	.587989	5.01	.964720	.88	.623269	5.90	.376731	13
48	.588289	5.01	.964666	.89	.623623	5.89	.376377	12
49	.588590	5.00	.964613	.89	.623976	5.89	.376024	11
50	9.588890	5.00	9.964560	.89	9.624330	5.89	10.375670	10
51	.589190	4.99	.964507	.89	.624683	5.88	.375317	9
52	.589489	4.99	.964454	.89	.625036	5.88	.374964	8
53	.589789	4.99	.964400	.89	.625388	5.88	.374612	7
54	.590088	4.98	.964347	.89	.625741	5.87	.374259	6
55	.590387	4.98	.964294	.89	.626093	5.87	.373907	5
56	.590686	4.97	.964240	.89	.626445	5.87	.373555	4
57	.590984	4.97	.964187	.89	.626797	5.86	.373203	3
58	.591282	4.97	.964133	.89	.627149	5.86	.372851	2
59	.591580	4.96	.964080	.89	.627501	5.86	.372499	1
60	.591878		.964026		.627852		.372148	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

112° **67°**

23° **156°**

M.	Sine.	D.1″.	Cosine.	D.1″	Tang.	D.1″.	Cotang.	M.
0	9.591878	4.96	9.964026	.89	9.627852	5.85	10.372148	60
1	.592176	4.95	.963972	.89	.628203	5.85	.371797	59
2	.592473	4.95	.963919	.90	.628554	5.85	.371446	58
3	.592770	4.95	.963865	.90	.628905	5.84	.371095	57
4	.593067	4.94	.963811	.90	.629255	5.84	.370745	56
5	.593363	4.94	.963757	.90	.629606	5.84	.370394	55
6	.593659	4.93	.963704	.90	.629956	5.83	.370044	54
7	.593955	4.93	.963650	.90	.630306	5.83	.369694	53
8	.594251	4.93	.963596	.90	.630656	5.83	.369344	52
9	.594547	4.92	.963542	.90	.631005	5.82	.368995	51
10	9.594842	4.92	9.963488	.90	9.631355	5.82	10.368645	50
11	.595137	4.91	.963434	.90	.631704	5.82	.368296	49
12	.595432	4.91	.963379	.90	.632053	5.81	.367947	48
13	.595727	4.91	.963325	.90	.632401	5.81	.367599	47
14	.596021	4.90	.963271	.90	.632750	5.81	.367250	46
15	.596315	4.90	.963217	.90	.633098	5.80	.366902	45
16	.596609	4.89	.963163	.91	.633447	5.80	.366553	44
17	.596903	4.89	.963108	.91	.633795	5.80	.366205	43
18	.597196	4.89	.963054	.91	.634143	5.79	.365857	42
19	.597490	4.88	.962999	.91	.634490	5.79	.365510	41
20	9.597783	4.88	9.962945	.91	9.634838	5.79	10.365162	40
21	.598075	4.88	.962890	.91	.635185	5.78	.364815	39
22	.598368	4.87	.962836	.91	.635532	5.78	.364468	38
23	.598660	4.87	.962781	.91	.635879	5.78	.364121	37
24	.598952	4.86	.962727	.91	.636226	5.78	.363774	36
25	.599244	4.86	.962672	.91	.636572	5.77	.363428	35
26	.599536	4.86	.962617	.91	.636919	5.77	.363081	34
27	.599827	4.85	.962562	.91	.637265	5.77	.362735	33
28	.600118	4.85	.962508	.91	.637611	5.76	.362389	32
29	.600409	4.84	.962453	.92	.637956	5.76	.362044	31
30	9.600700	4.84	9.962398	.92	9.638302	5.76	10.361698	30
31	.600990	4.84	.962343	.92	.638647	5.75	.361353	29
32	.601280	4.83	.962288	.92	.638992	5.75	.361008	28
33	.601570	4.83	.962233	.92	.639337	5.75	.360663	27
34	.601860	4.83	.962178	.92	.639682	5.74	.360318	26
35	.602150	4.82	.962123	.92	.640027	5.74	.359973	25
36	.602439	4.82	.962067	.92	.640371	5.74	.359629	24
37	.602728	4.81	.962012	.92	.640716	5.73	.359284	23
38	.603017	4.81	.961957	.92	.641060	5.73	.358940	22
39	.603305	4.81	.961902	.92	.641404	5.73	.358596	21
40	9.603594	4.80	9.961846	.92	9.641747	5.73	10.358253	20
41	.603882	4.80	.961791	.92	.642091	5.72	.357909	19
42	.604170	4.79	.961735	.92	.642434	5.72	.357566	18
43	.604457	4.79	.961680	.93	.642777	5.72	.357223	17
44	.604745	4.79	.961624	.93	.643120	5.71	.356880	16
45	.605032	4.78	.961569	.93	.643463	5.71	.356537	15
46	.605319	4.78	.961513	.93	.643806	5.71	.356194	14
47	.605606	4.78	.961458	.93	.644148	5.70	.355852	13
48	.605892	4.77	.961402	.93	.644490	5.70	.355510	12
49	.606179	4.77	.961346	.93	.644832	5.70	.355168	11
50	9.606465	4.76	9.961290	.93	9.645174	5.69	10.354826	10
51	.606751	4.76	.961235	.93	.645516	5.69	.354484	9
52	.607036	4.76	.961179	.93	.645857	5.69	.354143	8
53	.607322	4.75	.961123	.93	.646199	5.69	.353801	7
54	.607607	4.75	.961067	.93	.646540	5.68	.353460	6
55	.607892	4.74	.961011	.93	.646881	5.68	.353119	5
56	.608177	4.74	.960955	.93	.647222	5.68	.352778	4
57	.608461	4.74	.960899	.94	.647562	5.67	.352438	3
58	.608745	4.73	.960843	.94	.647903	5.67	.352097	2
59	.609029	4.73	.960786	.94	.648243	5.67	.351757	1
60	.609313		.960730		.648583		.351417	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

113° **66°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.609313	4.73	9.960730	.94	9.648583	5.67	10.351417	60
1	.609597	4.72	.960674	.94	.648923	5.66	.351077	59
2	.609880	4.72	.960618	.94	.649263	5.66	.350737	58
3	.610164	4.72	.960561	.94	.649602	5.66	.350398	57
4	.610447	4.71	.960505	.94	.649942	5.65	.350058	56
5	.610729	4.71	.960448	.94	.650281	5.65	.349719	55
6	.611012	4.71	.960392	.94	.650620	5.65	.349380	54
7	.611294	4.70	.960335	.94	.650959	5.64	.349041	53
8	.611576	4.70	.960279	.94	.651297	5.64	.348703	52
9	.611858	4.69	.960222	.94	.651636	5.64	.348364	51
10	9.612140	4.69	9.960165	.95	9.651974	5.64	10.348026	50
11	.612421	4.69	.960109	.95	.652312	5.63	.347688	49
12	.612702	4.68	.960052	.95	.652650	5.63	.347350	48
13	.612983	4.68	.959995	.95	.652988	5.63	.347012	47
14	.613264	4.68	.959938	.95	.653326	5.62	.346674	46
15	.613545	4.67	.959882	.95	.653663	5.62	.346337	45
16	.613825	4.67	.959825	.95	.654000	5.62	.346000	44
17	.614105	4.67	.959768	.95	.654337	5.62	.345663	43
18	.614385	4.66	.959711	.95	.654674	5.61	.345326	42
19	.614665	4.66	.959654	.95	.655011	5.61	.344989	41
20	9.614944	4.65	9.959596	.95	9.655348	5.61	10.344652	40
21	.615223	4.65	.959539	.95	.655684	5.61	.344316	39
22	.615502	4.65	.959482	.95	.656020	5.60	.343980	38
23	.615781	4.64	.959425	.95	.656356	5.60	.343644	37
24	.616060	4.64	.959368	.96	.656692	5.60	.343308	36
25	.616338	4.64	.959310	.96	.657028	5.59	.342972	35
26	.616616	4.63	.959253	.96	.657364	5.59	.342636	34
27	.616894	4.63	.959195	.96	.657699	5.59	.342301	33
28	.617172	4.63	.959138	.96	.658034	5.58	.341966	32
29	.617450	4.62	.959081	.96	.658369	5.58	.341631	31
30	9.617727	4.62	9.959023	.96	9.658704	5.58	10.341296	30
31	.618004	4.61	.958965	.96	.659039	5.58	.340961	29
32	.618281	4.61	.958908	.96	.659373	5.57	.340627	28
33	.618558	4.61	.958850	.96	.659708	5.57	.340292	27
34	.618834	4.60	.958792	.96	.660042	5.57	.339958	26
35	.619110	4.60	.958734	.96	.660376	5.56	.339624	25
36	.619386	4.60	.958677	.96	.660710	5.56	.339290	24
37	.619662	4.59	.958619	.97	.661043	5.56	.338957	23
38	.619938	4.59	.958561	.97	.661377	5.56	.338623	22
39	.620213	4.59	.958503	.97	.661710	5.55	.338290	21
40	9.620488	4.58	9.958445	.97	9.662043	5.55	10.337957	20
41	.620763	4.58	.958387	.97	.662376	5.55	.337624	19
42	.621038	4.58	.958329	.97	.662709	5.54	.337291	18
43	.621313	4.57	.958271	.97	.663042	5.54	.336958	17
44	.621587	4.57	.958213	.97	.663375	5.54	.336625	16
45	.621861	4.57	.958154	.97	.663707	5.54	.336293	15
46	.622135	4.56	.958096	.97	.664039	5.53	.335961	14
47	.622409	4.56	.958038	.97	.664371	5.53	.335629	13
48	.622682	4.56	.957979	.97	.664703	5.53	.335297	12
49	.622956	4.55	.957921	.97	.665035	5.53	.334965	11
50	9.623229	4.55	9.957863	.97	9.665366	5.52	10.334634	10
51	.623502	4.54	.957804	.98	.665697	5.52	.334303	9
52	.623774	4.54	.957746	.98	.666029	5.52	.333971	8
53	.624047	4.54	.957687	.98	.666360	5.51	.333640	7
54	.624319	4.53	.957628	.98	.666691	5.51	.333309	6
55	.624591	4.53	.957570	.98	.667021	5.51	.332979	5
56	.624863	4.53	.957511	.98	.667352	5.51	.332648	4
57	.625135	4.52	.957452	.98	.667682	5.50	.332318	3
58	.625406	4.52	.957393	.98	.668013	5.50	.331987	2
59	.625677	4.52	.957335	.98	.668343	5.50	.331657	1
60	.625948		.957276		.668672		.331328	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

25° 154°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.625948	4.51	9.957276	.98	9.668673	5.50	10.331327	60
1	.626219	4.51	.957217	.98	.669002	5.49	.330998	59
2	.626490	4.51	.957158	.98	.669332	5.49	.330668	58
3	.626760	4.50	.957099	.98	.669661	5.49	.330339	57
4	.627030	4.50	.957040	.99	.669991	5.49	.330009	56
5	.627300	4.50	.956981	.99	.670320	5.48	.329680	55
6	.627570	4.49	.956921	.99	.670649	5.48	.329351	54
7	.627840	4.49	.956862	.99	.670977	5.48	.329023	53
8	.628109	4.49	.956803	.99	.671306	5.47	.328694	52
9	.628378	4.48	.956744	.99	.671634	5.47	.328366	51
10	9.628647	4.48	9.956684	.99	9.671963	5.47	10.328037	50
11	.628916	4.48	.956625	.99	.672291	5.47	.327709	49
12	.629185	4.47	.956566	.99	.672619	5.46	.327381	48
13	.629453	4.47	.956506	.99	.672947	5.46	.327053	47
14	.629721	4.47	.956447	.99	.673274	5.46	.326726	46
15	.629989	4.46	.956387	.99	.673602	5.46	.326398	45
16	.630257	4.46	.956327	.99	.673929	5.45	.326071	44
17	.630524	4.46	.956268	.99	.674257	5.45	.325743	43
18	.630792	4.45	.956208	1.00	.674584	5.45	.325416	42
19	.631059	4.45	.956148	1.00	.674910	5.45	.325090	41
20	9.631326	4.45	9.956089	1.00	9.675237	5.44	10.324763	40
21	.631593	4.44	.956029	1.00	.675564	5.44	.324436	39
22	.631859	4.44	.955969	1.00	.675890	5.44	.324110	38
23	.632125	4.44	.955909	1.00	.676216	5.44	.323784	37
24	.632392	4.43	.955849	1.00	.676543	5.43	.323457	36
25	.632658	4.43	.955789	1.00	.676869	5.43	.323131	35
26	.632923	4.43	.955729	1.00	.677194	5.43	.322806	34
27	.633189	4.42	.955669	1.00	.677520	5.42	.322480	33
28	.633454	4.42	955609	1.00	.677846	5.42	.322154	32
29	.633719	4.42	.955548	1.00	.678171	5.42	.321829	31
30	9.633984	4.41	9.955488	1.00	9.678496	5.42	10.321504	30
31	.634249	4.41	.955428	1.01	.678821	5 41	.321179	29
32	.634514	4.41	.955368	1.01	.679146	5.41	.320854	28
33	.634778	4.40	.955307	1.01	.679471	5.41	.320529	27
34	.635042	4.40	.955247	1.01	.679795	5.41	.320205	26
35	.635306	4.40	.955186	1.01	.680120	5.40	.319880	25
36	.635570	4.39	.955126	1.01	.680444	5.40	.319556	24
37	.635834	4.39	.955065	1.01	.680768	5.40	.319232	23
38	.636097	4.39	.955005	1.01	.681092	5.40	.318908	22
39	.636360	4.38	.954944	1.01	.681416	5.39	.318584	21
40	9.636623	4.38	9.954883	1.01	9.681740	5.39	10.318260	20
41	.636886	4.38	.954823	1.01	.682063	5.39	.317937	19
42	.637148	4.37	.954762	1.01	.682387	5.39	.317613	18
43	.637411	4.37	.954701	1.01	.682710	5.38	.317290	17
44	.637673	4.37	.954640	1.02	.683033	5.38	.316967	16
45	.637935	4.36	.954579	1.02	.683356	5.38	.316644	15
46	.638197	4.36	.954518	1.02	.683679	5.38	.316321	14
47	.638458	4.36	.954457	1.02	.684001	5.37	.315999	13
48	.638720	4.35	.954396	1.02	.684324	5.37	.315676	12
49	.638981	4.35	.954335	1.02	.684646	5.37	.315354	11
50	9.639242	4.35	9.954274	1.02	9.684968	5.37	10.315032	10
51	.639503	4.34	.954213	1.02	.685290	5.36	.314710	9
52	.639764	4.34	.954152	1.02	.685612	5.36	.314388	8
53	.640024	4.34	.954090	1.02	.685934	5.36	.314066	7
54	.640284	4.33	.954029	1.02	.686255	5.36	.313745	6
55	.640544	4.33	.953968	1.02	.686577	5.35	.313423	5
56	.640804	4.33	.953906	1.02	.686898	5.35	.313102	4
57	.641064	4.32	.953845	1.03	.687219	5.35	.312781	3
58	.641324	4.32	.953783	1.03	.687540	5.35	.312460	2
59	.641583	4.32	.953722	1.03	.687861	5.35	.312139	1
60	.641842		.953660		.688182		.311818	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

115° 64°

26° **153°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.641842	4.32	9.953660	1.03	9.688182	5.34	10.311818	60
1	.642101	4.31	.953599	1.03	.688502	5.34	.311498	59
2	.642360	4.31	.953537	1.03	.688823	5.34	.311177	58
3	.642618	4.31	.953475	1.03	.689143	5.34	.310857	57
4	.642877	4.30	.953413	1.03	.689463	5.33	.310537	56
5	.643135	4.30	.953352	1.03	.689783	5.33	.310217	55
6	.643393	4.30	.953290	1.03	.690103	5.33	.309897	54
7	.643650	4.29	.953228	1.03	.690423	5.33	.309577	53
8	.643908	4.29	.953166	1 03	.690742	5.32	.309258	52
9	.644165	4.29	.953104	1.03	.691062	5.32	.308938	51
10	9.644423	4.28	9.953042	1.03	9.691381	5.32	10.308619	50
11	.644680	4.28	.952980	1.04	.691700	5.32	.308300	49
12	.644936	4.28	.952918	1.04	.692019	5.31	.307981	48
13	.645193	4.27	.952855	1.04	.692338	5.31	.307662	47
14	.645450	4.27	.952793	1.04	.692656	5.31	.307344	46
15	.645706	4.27	.952731	1.04	.692975	5.31	.307025	45
16	.645962	4.26	.952669	1.04	.693293	5.30	.306707	44
17	.646218	4.26	.952606	1.04	.693612	5.30	.306388	43
18	.646474	4.26	.952544	1.04	.693930	5.30	.306070	42
19	.646729	4.26	.952481	1.04	.694248	5.30	.305752	41
20	9.646984	4.25	9.952419	1.04	9.694566	5.29	10.305434	40
21	.647240	4.25	.952356	1.04	.694883	5.29	.305117	39
22	.647494	4.25	.952294	1.04	.695201	5.29	.304799	38
23	.647749	4.24	.952231	1.04	.695518	5.29	.304482	37
24	.648004	4.24	.952168	1.05	.695836	5.29	.304164	36
25	.648258	4.24	.952106	1.05	.696153	5.28	.303847	35
26	.648512	4.23	.952043	1.05	.696470	5.28	.303530	34
27	.648766	4.23	.951980	1.05	.696787	5.28	.303213	33
28	.649020	4.23	.951917	1.05	.697103	5.28	.302897	32
29	.649274	4.22	.951854	1.05	.697420	5.27	.302580	31
30	9.649527	4.22	9.951791	1.05	9.697736	5.27	10.302264	30
31	.649781	4.22	.951728	1.05	.698053	5.27	.301947	29
32	.650034	4.22	.951665	1.05	.698369	5.27	.301631	28
33	.650287	4.21	.951602	1.05	.698685	5.26	.301315	27
34	.650539	4.21	.951539	1.05	.699001	5.26	.300999	26
35	.650792	4.21	.951476	1.05	.699316	5.26	.300684	25
36	.651044	4.20	.951412	1.05	.699632	5.26	.300368	24
37	.651297	4.20	.951349	1 06	.699947	5.26	.300053	23
38	.651549	4.20	.951286	1.06	.700263	5.25	.299737	22
39	.651800	4.19	.951222	1.06	.700578	5.25	299422	21
40	9.652052	4.19	9.951159	1.06	9.700893	5.25	10.299107	20
41	.652304	4.19	.951096	1.06	.701208	5.25	.298792	19
42	.652555	4.18	.951032	1.06	.701523	5.24	.298477	18
43	.652806	4.18	.950968	1.06	.701837	5.24	.298163	17
44	.653057	4.18	.950905	1.06	.702152	5.24	.297848	16
45	.653308	4.18	.950841	1.06	.702466	5.24	.297534	15
46	.653558	4.17	.950778	1.06	.702781	5.24	.297219	14
47	.653808	4.17	.950714	1.06	.703095	5.23	.296905	13
48	.654059	4.17	.950650	1.06	.703409	5.23	.296591	12
49	.654309	4.16	.950586	1.06	.703723	5.23	.296273	11
50	9.654558	4.16	9.950522	1.07	9.704036	5.23	10.295964	10
51	.654808	4.16	.950458	1.07	.704350	5.22	.295650	9
52	.655058	4.15	.950394	1.07	.704663	5.22	.295337	8
53	.655307	4.15	.950330	1.07	.704977	5.22	.295023	7
54	.655556	4.15	.950266	1.07	.705290	5.22	.294710	6
55	.655805	4.15	.950202	1.07	.705603	5.22	.294397	5
56	.656054	4.14	.950138	1.07	.705916	5.21	.294084	4
57	.656302	4.14	.950074	1.07	.706228	5.21	.293772	3
58	.656551	4.14	.950010	1.07	.706541	5.21	.293459	2
59	.656799	4.13	.949945	1.07	.706854	5.21	.293146	1
60	.657047		.949881		.707166		.292834	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

116° **63°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.657047	4.13	9.949881	1.07	9.707166	5.20	10.292834	60
1	.657295	4.13	.949816	1.07	.707478	5.20	.292522	59
2	.657542	4.12	.949752	1.07	.707790	5.20	.292210	58
3	.657790	4.12	.949688	1.08	.708102	5.20	.291898	57
4	.658037	4.12	.949623	1.08	.708414	5.20	.291586	56
5	.658284	4.12	.949558	1.08	.708726	5.19	.291274	55
6	.658531	4.11	.949494	1.08	.709037	5.19	.290963	54
7	.658778	4.11	.949429	1.08	.709349	5.19	.290651	53
8	.659025	4.11	.949364	1.08	.709660	5.19	.290340	52
9	.659271	4.10	.949300	1.08	.709971	5.18	.290029	51
10	9.659517	4.10	9.949235	1.08	9.710282	5.18	10.289718	50
11	.659763	4.10	.949170	1.08	.710593	5.18	.289407	49
12	.660009	4.10	.949105	1.08	.710904	5.18	.289096	48
13	.660255	4.09	.949040	1.08	.711215	5.18	.288785	47
14	.660501	4.09	.948975	1.08	.711525	5.17	.288475	46
15	.660746	4.09	.948910	1.08	.711836	5.17	.288164	45
16	.660991	4.08	.948845	1.09	.712146	5.17	.287854	44
17	.661236	4.08	.948780	1.09	.712456	5.17	.287544	43
18	.661481	4.08	.948715	1.09	.712766	5.17	.287234	42
19	.661726	4.08	.948650	1.09	.713076	5.16	.286924	41
20	9.661970	4.07	9.948584	1.09	9.713386	5.16	10.286614	40
21	.662214	4.07	.948519	1.09	.713696	5.16	.286304	39
22	.662459	4.07	.948454	1.09	.714005	5.16	.285995	38
23	.662703	4.06	.948388	1.09	.714314	5.15	.285686	37
24	.662946	4.06	.948323	1.09	.714624	5.15	.285376	36
25	.663190	4.06	.948257	1.09	.714933	5.15	.285067	35
26	.663433	4.05	.948192	1.09	.715242	5.15	.284758	34
27	.663677	4.05	.948126	1.09	.715551	5.15	.284449	33
28	.663920	4.05	.948060	1.09	.715860	5.14	.284140	32
29	.664163	4.05	.947995	1.10	.716168	5.14	.283832	31
30	9.664406	4.04	9.947929	1.10	9.716477	5.14	10.283523	30
31	.664648	4.04	.947863	1.10	.716785	5.14	.283215	29
32	.664891	4.04	.947797	1.10	.717093	5.14	.282907	28
33	.665133	4.03	.947731	1.10	.717401	5.13	.282599	27
34	.665375	4.03	.947665	1.10	.717709	5.13	.282291	26
35	.665617	4.03	.947600	1.10	.718017	5.13	.281983	25
36	.665859	4.03	.947533	1.10	.718325	5.13	.281675	24
37	.666100	4.02	.947467	1.10	.718633	5.13	.281367	23
38	.666342	4.02	.947401	1.10	.718940	5.12	.281060	22
39	.666583	4.02	.947335	1.10	.719248	5.12	.280752	21
40	9.666824	4.01	9.947269	1.10	9.719555	5.12	10.280445	20
41	.667065	4.01	.947203	1.11	.719862	5.12	.280138	19
42	.667305	4.01	.947136	1.11	.720169	5.11	.279831	18
43	.667546	4.01	.947070	1.11	.720476	5.11	.279524	17
44	.667786	4.00	.947004	1.11	.720783	5.11	.279217	16
45	.668027	4.00	.946937	1.11	.721089	5.11	.278911	15
46	.668267	4.00	.946871	1.11	.721396	5.11	.278604	14
47	.668506	3.99	.946804	1.11	.721702	5.10	.278298	13
48	.668746	3.99	.946738	1.11	.722009	5.10	.277991	12
49	.668986	3.99	.946671	1.11	.722315	5.10	.277685	11
50	9.669225	3.99	9.946604	1.11	9.722621	5.10	10.277379	10
51	.669464	3.98	.946538	1.11	.722927	5.10	.277073	9
52	.669703	3.98	.946471	1.11	.723232	5.09	.276768	8
53	.669942	3.98	.946404	1.11	.723538	5.09	.276462	7
54	.670181	3.98	.946337	1.12	.723844	5.09	.276156	6
55	.670419	3.97	.946270	1.12	.724149	5.09	.275851	5
56	.670658	3.97	.946203	1.12	.724454	5.09	.275546	4
57	.670896	3.97	.946136	1.12	.724760	5.08	.275240	3
58	.671134	3.96	.946069	1.12	.725065	5.08	.274935	2
59	.671372	3.96	.946002	1.12	.725370	5.08	.274630	1
60	.671609		.945935		.725674		.274326	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

28° **151°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.671609	3.96	9.945935	1.12	9.725674	5.08	10.274326	60
1	.671847	3.96	.945868	1.12	.725979	5.08	.274021	59
2	.672084	3.95	.945800	1.12	.726284	5.07	.273716	58
3	.672321	3.95	.945733	1.12	.726588	5.07	.273412	57
4	.672558	3.95	.945666	1.12	.726892	5.07	.273108	56
5	.672795	3.94	.945598	1.12	.727197	5.07	.272803	55
6	.673032	3.94	.945531	1.12	.727501	5.07	.272499	54
7	.673268	3.94	.945464	1.13	.727805	5.06	.272195	53
8	.673505	3.94	.945396	1.13	.728109	5.06	.271891	52
9	.673741	3.93	.945328	1.13	.728412	5.06	.271588	51
10	9.673977	3.93	9.945261	1.13	9.728716	5.06	10.271284	50
11	.674213	3.93	.945193	1.13	.729020	5.06	.270980	49
12	.674448	3.93	.945125	1.13	.729323	5.05	.270677	48
13	.674684	3.92	.945058	1.13	.729626	5.05	.270374	47
14	.674919	3.92	.944990	1.13	.729929	5.05	.270071	46
15	.675155	3.92	.944922	1.13	.730233	5.05	.269767	45
16	.675390	3.91	.944854	1.13	.730535	5.05	.269465	44
17	.675624	3.91	.944786	1.13	.730838	5.05	.269162	43
18	.675859	3.91	.944718	1.13	.731141	5.04	.268859	42
19	.676094	3.91	.944650	1.13	.731444	5.04	.268556	41
20	9.676328	3.90	9.944582	1.14	9.731746	5.04	10.268254	40
21	.676562	3.90	.944514	1.14	.732048	5.04	.267952	39
22	.676796	3.90	.944446	1.14	.732351	5.04	.267649	38
23	.677030	3.90	.944377	1.14	.732653	5.04	.267347	37
24	.677264	3.89	.944309	1.14	.732955	5.03	.267045	36
25	.677498	3.89	.944241	1.14	.733257	5.03	.266743	35
26	.677731	3.89	.944172	1.14	.733558	5.03	.266442	34
27	.677964	3.88	.944104	1.14	.733860	5.03	.266140	33
28	.678197	3.88	.944036	1.14	.734162	5.02	.265838	32
29	.678430	3.88	.943967	1.14	.734463	5.02	.265537	31
30	9.678663	3.88	9.943899	1.14	9.734764	5.02	10.265236	30
31	.678895	3.87	.943830	1.14	.735066	5.02	.264934	29
32	.679128	3.87	.943761	1.15	.735367	5.02	.264633	28
33	.679360	3.87	.943693	1.15	.735668	5.01	.264332	27
34	.679592	3.87	.943624	1.15	.735969	5.01	.264031	26
35	.679824	3.86	.943555	1.15	.736269	5.01	.263731	25
36	.680056	3.86	.943486	1.15	.736570	5.01	.263430	24
37	.680288	3.86	.943417	1.15	.736871	5.01	.263129	23
38	.680519	3.86	.943348	1.15	.737171	5.01	.262829	22
39	.680750	3.85	.943279	1.15	.737471	5.00	.262529	21
40	9.680982	3.85	9.943210	1.15	9.737771	5.00	10.262229	20
41	.681213	3.85	.943141	1.15	.738071	5.00	.261929	19
42	.681443	3.84	.943072	1.15	.738371	5.00	.261629	18
43	.681674	3.84	.943003	1.15	738671	5.00	.261329	17
44	.681905	3.84	.942934	1.15	.738971	4.99	.261029	16
45	.682135	3.84	.942864	1.16	.739271	4.99	.260729	15
46	.682365	3.83	.942795	1.16	.739570	4.99	.260430	14
47	.682595	3.83	.942726	1.16	.739870	4.99	.260130	13
48	.682825	3.83	.942656	1.16	.740169	4.99	.259831	12
49	.683055	3.83	.942587	1.16	.740468	4.98	.259532	11
50	9.683284	3.82	9.942517	1.16	9.740767	4.98	10.259233	10
51	.683514	3.82	.942448	1.16	.741066	4.98	.258934	9
52	.683743	3.82	.942378	1.16	.741365	4.98	.258635	8
53	.683972	3.82	.942308	1.16	.741664	4.98	.258336	7
54	.684201	3.81	.942239	1.16	.741962	4.98	.258038	6
55	.684430	3.81	.942169	1.16	.742261	4.97	.257739	5
56	.684658	3.81	.942099	1.16	.742559	4.97	.257441	4
57	.684887	3.80	.942029	1.17	.742858	4.97	.257142	3
58	.685115	3.80	.941959	1.17	.743156	4.97	.256844	2
59	.685343	3.80	.941889	1.17	.743454	4.97	.256546	1
60	.685571		.941819		.743752		.256248	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

118° **61°**

29° **150°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.685571	3.80	9.941819	1.17	9.743752	4.96	10.256248	60
1	.685799	3.79	.941749	1.17	.744050	4.96	.255950	59
2	.686027	3.79	.941679	1.17	.744348	4.96	.255652	58
3	.686254	3.79	.941609	1.17	.744645	4.96	.255355	57
4	.686482	3.79	.941539	1.17	.744943	4.96	.255057	56
5	.686709	3.78	.941469	1.17	.745240	4.96	.254760	55
6	.686936	3.78	.941398	1.17	.745538	4.95	.254462	54
7	.687163	3.78	.941328	1.17	.745835	4.95	.254165	53
8	.687389	3.78	.941258	1.17	.746132	4.95	.253868	52
9	.687616	3.77	.941187	1.17	.746429	4.95	.253571	51
10	9.687843	3.77	9.941117	1.18	9.746726	4.95	10.253274	50
11	.688069	3.77	.941046	1.18	.747023	4.95	.252977	49
12	.688295	3.77	.940975	1.18	.747319	4.94	.252681	48
13	.688521	3.76	.940905	1.18	.747616	4.94	.252384	47
14	.688747	3.76	.940834	1.18	.747913	4.94	.252087	46
15	.688972	3.76	.940763	1.18	.748209	4.94	.251791	45
16	.689198	3.76	.940693	1.18	.748505	4.94	.251495	44
17	.689423	3.75	.940622	1.18	.748801	4.93	.251199	43
18	.689648	3.75	.940551	1.18	.749097	4.93	.250903	42
19	.689873	3.75	.940480	1.18	.749393	4.93	.250607	41
20	9.690098	3.75	9.940409	1.18	9.749689	4.93	10.250311	40
21	.690323	3.74	.940338	1.18	.749985	4.93	.250015	39
22	.690548	3.74	.940267	1.19	.750281	4.93	.249719	38
23	.690772	3.74	.940196	1.19	.750576	4.92	.249424	37
24	.690996	3.74	.940125	1.19	.750872	4.92	.249128	36
25	.691220	3.73	.940054	1.19	.751167	4.92	.248833	35
26	.691444	3.73	.939982	1.19	.751462	4.92	.248538	34
27	.691668	3.73	.939911	1.19	.751757	4.92	.248243	33
28	.691892	3.73	.939840	1.19	.752052	4.92	.247948	32
29	.692115	3.72	.939768	1.19	.752347	4.91	.247653	31
30	9.692339	3.72	9.939697	1.19	9.752642	4.91	10.247358	30
31	.692562	3.72	.939625	1.19	.752937	4.91	.247063	29
32	.692785	3.72	.939554	1.19	.753231	4.91	.246769	28
33	.693008	3.71	.939482	1.19	.753526	4.91	.246474	27
34	.693231	3.71	.939410	1.19	.753820	4.91	.246180	26
35	.693453	3.71	.939339	1.20	.754115	4.90	.245885	25
36	.693676	3.71	.939267	1.20	.754409	4.90	.245591	24
37	.693898	3.70	.939195	1.20	.754703	4.90	.245297	23
38	.694120	3.70	.939123	1.20	.754997	4.90	.245003	22
39	.694342	3.70	.939052	1.20	.755291	4.90	.244709	21
40	9.694564	3.70	9.938980	1.20	9.755585	4.89	10.244415	20
41	.694786	3.69	.938908	1.20	.755878	4.89	.244122	19
42	.695007	3.69	.938836	1.20	.756172	4.89	.243828	18
43	.695229	3.69	.938763	1.20	.756465	4.89	.243535	17
44	.695450	3.69	.938691	1.20	.756759	4.89	.243241	16
45	.695671	3.68	.938619	1.20	.757052	4.89	.242948	15
46	.695892	3.68	.938547	1.20	.757345	4.88	.242655	14
47	.696113	3.68	.938475	1.21	.757638	4.88	.242362	13
48	.696334	3.68	.938402	1.21	.757931	4.88	.242069	12
49	.696554	3.67	.938330	1.21	.758224	4.88	.241776	11
50	9.696775	3.67	9.938258	1.21	9.758517	4.88	10.241483	10
51	.696995	3.67	.938185	1.21	.758810	4.88	.241190	9
52	.697215	3.67	.938113	1.21	.759102	4.87	.240898	8
53	.697435	3.66	.938040	1.21	.759395	4.87	.240605	7
54	.697654	3.66	.937967	1.21	.759687	4.87	.240313	6
55	.697874	3.66	.937895	1.21	.759979	4.87	.240021	5
56	.698094	3.66	.937822	1.21	.760272	4.87	.239728	4
57	.698313	3.65	.937749	1.21	.760564	4.87	.239436	3
58	.698532	3.65	.937676	1.21	.760856	4.86	.239144	2
59	.698751	3.65	.937604	1.22	.761148	4.86	.238852	1
60	.698970		.937531		.761439		.238561	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

119° **60°**

M.	Sine.	D. 1''.	Cosine.	D. 1''.	Tang.	D. 1''.	Cotang.	M.
0	9.698970	3.65	9.937531	1.22	9.761439	4.86	10.238561	60
1	.699189	3.64	.937458	1.22	.761731	4.86	.238269	59
2	.699407	3.64	.937385	1.22	.762023	4.86	.237977	58
3	.699626	3.64	.937312	1.22	.762314	4.86	.237686	57
4	.699844	3.64	.937238	1.22	.762606	4.86	.237394	56
5	.700062	3.63	.937165	1.22	.762897	4.85	.237103	55
6	.700280	3.63	.937092	1.22	.763188	4.85	.236812	54
7	.700498	3.63	.937019	1.22	.763479	4.85	.236521	53
8	.700716	3.63	.936946	1.22	.763770	4.85	.236230	52
9	.700933	3.62	.936872	1.22	.764061	4.85	.235939	51
10	9.701151	3.62	9.936799	1.22	9.764352	4.85	10.235648	50
11	.701368	3.62	.936725	1.23	.764643	4.84	.235357	49
12	.701585	3.62	.936652	1.23	.764933	4.84	.235067	48
13	.701802	3.61	.936578	1.23	.765224	4.84	.234776	47
14	.702019	3.61	.936505	1.23	.765514	4.84	.234486	46
15	.702236	3.61	.936431	1.23	.765805	4.84	.234195	45
16	.702452	3.61	.936357	1.23	.766095	4.84	.233905	44
17	.702669	3.60	.936284	1.23	.766385	4.83	.233615	43
18	.702885	3.60	.936210	1.23	.766675	4.83	.233325	42
19	.703101	3.60	.936136	1.23	.766965	4.83	.233035	41
20	9.703317	3.60	9.936062	1.23	9.767255	4.83	10.232745	40
21	.703533	3.59	.935988	1.23	.767545	4.83	.232455	39
22	.703749	3.59	.935914	1.23	.767834	4.83	.232166	38
23	.703964	3.59	.935840	1.23	.768124	4.82	.231876	37
24	.704179	3.59	.935766	1.24	.768413	4.82	.231587	36
25	.704395	3.59	.935692	1.24	.768703	4.82	.231297	35
26	.704610	3.58	.935618	1.24	.768992	4.82	.231008	34
27	.704825	3.58	.935543	1.24	.769281	4.82	.230719	33
28	.705040	3.58	.935469	1.24	.769571	4.82	.230429	32
29	.705254	3.58	.935395	1.24	.769860	4.82	.230140	31
30	9.705469	3.57	9.935320	1.24	9.770148	4.81	10.229852	30
31	.705683	3.57	.935246	1.24	.770437	4.81	.229563	29
32	.705898	3.57	.935171	1.24	.770726	4.81	.229274	28
33	.706112	3.57	.935097	1.24	.771015	4.81	.228985	27
34	.706326	3.56	.935022	1.24	.771303	4.81	.228697	26
35	.706539	3.56	.934948	1.24	.771592	4.81	.228408	25
36	.706753	3.56	.934873	1.25	.771880	4.80	.228120	24
37	.706967	3.56	.934798	1.25	.772168	4.80	.227832	23
38	.707180	3.55	.934723	1.25	.772457	4.80	.227543	22
39	.707393	3.55	.934649	1.25	.772745	4.80	.227255	21
40	9.707606	3.55	9.934574	1.25	9.773033	4.80	10.226967	20
41	.707819	3.55	.934499	1.25	.773321	4.80	.226679	19
42	.708032	3.54	.934424	1.25	.773608	4.80	.226392	18
43	.708245	3.54	.934349	1.25	.773896	4.79	.226104	17
44	.708458	3.54	.934274	1.25	.774184	4.79	.225816	16
45	.708670	3.54	.934199	1.25	.774471	4.79	.225529	15
46	.708882	3.54	.934123	1.25	.774759	4.79	.225241	14
47	.709094	3.53	.934048	1.25	.775046	4.79	.224954	13
48	.709306	3.53	.933973	1.26	.775333	4.79	.224667	12
49	.709518	3.53	.933898	1.26	.775621	4.78	.224379	11
50	9.709730	3.53	9.933822	1.26	9.775908	4.78	10.224092	10
51	.709941	3.52	.933747	1.26	.776195	4.78	.223805	9
52	.710153	3.52	.933671	1.26	.776482	4.78	.223518	8
53	.710364	3.52	.933596	1.26	.776769	4.78	.223231	7
54	.710575	3.52	.933520	1.26	.777055	4.78	.222945	6
55	.710786	3.51	.933445	1.26	.777342	4.78	.222658	5
56	.710997	3.51	.933369	1.26	.777628	4.77	.222372	4
57	.711208	3.51	.933293	1.26	.777915	4.77	.222085	3
58	.711419	3.51	.933217	1.26	.778201	4.77	.221799	2
59	.711629	3.51	.933141	1.26	.778488	4.77	.221512	1
60	.711839		.933066		.778774		.221226	0
M.	Cosine.	D. 1''.	Sine.	D. 1''.	Cotang.	D. 1''.	Tang.	M.

31° 148°

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.711839	3.50	9.933066	1.27	9.778774	4.77	10.221226	60
1	.712050	3.50	.932990	1.27	.779060	4.77	.220940	59
2	.712260	3.50	.932914	1.27	.779346	4.77	.220654	58
3	.712469	3.50	.932838	1.27	.779632	4.76	.220368	57
4	.712679	3.49	.932762	1.27	.779918	4.76	.220082	56
5	.712889	3.49	.932685	1.27	.780203	4.76	.219797	55
6	.713098	3.49	932609	1.27	.780489	4.76	.219511	54
7	.713308	3.49	.932533	1.27	.780775	4.76	.219225	53
8	.713517	3.48	.932457	1.27	.781060	4.76	.218940	52
9	.713726	3.48	.932380	1.27	.781346	4.76	.218654	51
10	9.713935	3.48	9.932304	1.27	9.781631	4.75	10.218369	50
11	.714144	3.48	.932228	1.27	.781916	4.75	.218084	49
12	.714352	3.48	.932151	1.28	.782201	4.75	.217799	48
13	.714561	3.47	.932075	1.28	.782486	4.75	.217514	47
14	.714769	3.47	.931998	1.28	.782771	4.75	.217229	46
15	.714978	3.47	.931921	1.28	.783056	4.75	.216944	45
16	.715186	3.47	.931845	1.28	.783341	4.75	.216659	44
17	.715394	3.46	.931768	1.28	.783626	4.74	.216374	43
18	.715602	3.46	.931691	1.28	.783910	4.74	.216090	42
19	.715809	3.46	.931614	1.28	.784195	4.74	.215805	41
20	9.716017	3.46	9.931537	1.28	9.784479	4.74	10.215521	40
21	.716224	3.46	.931460	1.28	.784764	4.74	.215236	39
22	.716432	3.45	.931383	1.28	.785048	4.74	.214952	38
23	.716639	3.45	.931306	1.28	.785332	4.74	.214668	37
24	.716846	3.45	.931229	1.29	.785616	4.73	.214384	36
25	.717053	3.45	.931152	1.29	.785900	4.73	.214100	35
26	.717259	3.44	.931075	1.29	.786184	4.73	.213816	34
27	.717466	3.44	.930998	1.29	.786468	4.73	.213532	33
28	.717673	3.44	.930921	1.29	.786752	4.73	.213248	32
29	.717879	3.44	.930843	1.29	.787036	4.73	.212964	31
30	9.718085	3.43	9.930766	1.29	9.787319	4.73	10.212681	30
31	.718291	3.43	.930688	1.29	.787603	4.72	.212397	29
32	.718497	3.43	.930611	1.29	.787886	4.72	.212114	28
33	.718703	3.43	.930533	1.29	.788170	4.72	.211830	27
34	.718909	3.43	.930456	1.29	.788453	4.72	.211547	26
35	.719114	3.42	.930378	1.29	.788736	4.72	.211264	25
36	.719320	3.42	.930300	1.30	.789019	4.72	.210981	24
37	.719525	3.42	.930223	1.30	.789302	4.72	.210698	23
38	.719730	3.42	.930145	1.30	.789585	4.71	.210415	22
39	.719935	3.41	.930067	1.30	.789868	4.71	.210132	21
40	9.720140	3.41	9.929989	1.30	9.790151	4.71	10.209849	20
41	720345	3.41	.929911	1.30	.790433	4.71	.209567	19
42	.720549	3.41	.929833	1.30	.790716	4.71	.209284	18
43	.720754	3.41	.929755	1.30	.790999	4.71	.209001	17
44	.720958	3.40	.929677	1.30	.791281	4.71	.208719	16
45	.721162	3.40	.929599	1.30	.791563	4.70	.208437	15
46	.721366	3.40	.929521	1.30	.791846	4.70	.208154	14
47	.721570	3.40	.929442	1.31	.792128	4.70	.207872	13
48	.721774	3.39	.929364	1.31	.792410	4.70	.207590	12
49	.721978	3.39	.929286	1.31	.792692	4.70	.207308	11
50	9.722181	3.39	9.929207	1.31	9.792974	4.70	10.207026	10
51	.722385	3.39	.929129	1.31	.793256	4.70	.206744	9
52	.722588	3.39	.929050	1.31	.793538	4.70	.206462	8
53	.722791	3.38	.928972	1.31	.793819	4.69	.206181	7
54	.722994	3.38	.928893	1.31	.794101	4.69	.205899	6
55	.723197	3.38	.928815	1.31	.794383	4.69	.205617	5
56	.723400	3.38	.928736	1.31	.794664	4.69	.205336	4
57	.723603	3.37	.928657	1.31	.794945	4.69	.205055	3
58	.723805	3.37	.928578	1.31	.795227	4.69	.204773	2
59	.724007	3.37	.928499	1.32	.795508	4.69	.204492	1
60	.724210		.928420		.795789		.204211	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

121° 58°

32° 147°

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.724210	3.37	9.928420	1.32	9.795789	4.68	10.204211	60
1	.724412	3.37	.928342	1.32	.796070	4.68	.203930	59
2	.724614	3.36	.928263	1.32	.796351	4.68	.203649	58
3	.724816	3.36	.928183	1.32	.796632	4.68	.203368	67
4	.725017	3.36	.928104	1.32	.796913	4.68	.203087	56
5	.725219	3.36	.928025	1.32	.797194	4.68	.202806	55
6	.725420	3.36	.927946	1.32	.797475	4.68	.202525	54
7	.725622	3.35	.927867	1.32	.797755	4.68	.202245	53
8	.725823	3.35	.927787	1.32	.798036	4.67	.201964	52
9	.726024	3.35	.927708	1.32	.798316	4.67	.201684	51
10	9.726225	3.35	9.927629	1.32	9.798596	4.67	10.201404	50
11	.726426	3.34	.927549	1.33	.798877	4.67	.201123	49
12	.726626	3.34	.927470	1.33	.799157	4.67	.200843	48
13	.726827	3.34	.927390	1.33	.799437	4.67	.200563	47
14	.727027	3.34	.927310	1.33	.799717	4.67	.200283	46
15	.727228	3.34	.927231	1.33	.799997	4.66	.200003	45
16	.727428	3.33	.927151	1.33	.800277	4.66	.199723	44
17	.727628	3.33	.927071	1.33	.800557	4.66	.199443	43
18	.727828	3.33	.926991	1.33	.800836	4.66	.199164	42
19	.728027	3.33	.926911	1.33	.801116	4.66	.198884	41
20	9.728227	3.33	9.926831	1.33	9.801396	4.66	10.198604	40
21	.728427	3.32	.926751	1.33	.801675	4.66	.198325	39
22	.728626	3.32	.926671	1.33	.801955	4.66	.198045	38
23	.728825	3.32	.926591	1.34	.802234	4 65	.197766	37
24	.729024	3.32	.926511	1.34	.802513	4.65	.197487	36
25	.729223	3.31	.926431	1.34	.802792	4.65	.197208	35
26	.729422	3.31	.926351	1.34	.803072	4.65	.196928	34
27	.729621	3.31	.926270	1.34	.803351	4.65	.196649	33
28	.729820	3.31	.926190	1.34	.803630	4.65	.196370	32
29	.730018	3.31	.926110	1.34	.803908	4.65	.196092	31
30	9.730216	3.30	9.926029	1.34	9.804187	4.65	10.195813	30
31	.730415	3.30	.925949	1.34	.804466	4.64	.195534	29
32	.730613	3.30	.925868	1.34	.804745	4.64	.195255	28
33	.730811	3.30	.925788	1.34	.805023	4.64	.194977	27
34	.731009	3.30	.925707	1.35	.805302	4.64	.194698	26
35	.731206	3.29	.925626	1.35	.805580	4.64	.194420	25
36	.731404	3.29	.925545	1.35	.805859	4.64	.194141	24
37	.731602	3.29	.925465	1.35	.806137	4.64	.193863	23
38	.731799	3.29	.925384	1.35	.806415	4.64	.193585	22
39	.731996	3.28	.925303	1.35	.806693	4.63	.193307	21
40	9.732193	3.28	9.925222	1.35	9.806971	4.63	10.193029	20
41	.732390	3.28	.925141	1.35	.807249	4.63	.192751	19
42	.732587	3.28	.925060	1.35	.807527	4.63	.192473	18
43	.732784	3.28	.924979	1.35	.807805	4.63	.192195	17
44	.732980	3.27	.924897	1.35	.808083	4.63	.191917	16
45	.733177	3.27	.924816	1.35	.808361	4.63	.191639	15
46	.733373	3.27	.924735	1.36	.808638	4.63	.191362	14
47	.733569	3.27	.924654	1.36	808916	4.62	.191084	13
48	.733765	3.27	.924572	1.36	.809193	4.62	.190807	12
49	.733961	3.26	.924491	1.36	.809471	4.62	.190529	11
50	9.734157	3 26	9.924409	1.36	9.809748	4.62	10.190252	10
51	.734353	3.26	.924328	1 36	.810025	4.62	.189975	9
52	.734519	3.26	.924246	1.36	.810302	4.62	.189698	8
53	.734744	3.26	.924164	1.36	.810580	4.62	.189420	7
54	.734939	3.25	.924083	1.36	.810857	4.62	.189143	6
55	.735135	3.25	.924001	1.36	.811134	4.61	.188866	5
56	.735330	3.25	.923919	1.36	.811410	4.61	.188590	4
57	.735525	3.25	.923837	1.37	.811687	4.61	.188313	3
58	.735719	3.25	.923755	1.37	.811964	4.61	.188036	2
59	.735914	3.24	.923673	1.37	.812241	4.61	.187759	1
60	.736109		.923591		.812517		.187483	0
M	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

122° 57°

33° **146°**

M.	Sine.	D.1″.	Cosine.	D.1″	Tang	D.1″.	Cotang.	M.
0	9.736109	3.24	9.923591	1.37	9.812517	4.61	10.187483	60
1	.736303	3.24	.923509	1.37	.812794	4.61	.187206	59
2	.736498	3.24	.923427	1.37	.813070	4.61	.186930	58
3	.736692	3.23	.923345	1.37	.813347	4.61	.186653	57
4	.736886	3.23	.923263	1.37	.813623	4.60	.186377	56
5	.737080	3.23	.923181	1.37	.813899	4.60	.186101	55
6	.737274	3.23	.923098	1.37	.814176	4.60	.185824	54
7	.737467	3.23	.923016	1.37	.814452	4.60	.185548	53
8	.737661	3.22	.922933	1.37	.814728	4.60	.185272	52
9	.737855	3.22	.922851	1.38	.815004	4.60	.184996	51
10	9.738048	3.22	9.922768	1.38	9.815280	4.60	10.184720	50
11	.738241	3.22	.922686	1.38	.815555	4.60	.184445	49
12	.738434	3.22	.922603	1.38	.815831	4.59	.184169	48
13	.738627	3.21	.922520	1.38	.816107	4.59	.183893	47
14	.738820	3.21	.922438	1.38	.816382	4.59	.183618	46
15	.739013	3.21	.922355	1.38	.816658	4.59	.183342	45
16	.739206	3.21	.922272	1.38	.816933	4.59	.183067	44
17	.739398	3.21	.922189	1.38	.817209	4.59	.182791	43
18	.739590	3.20	.922106	1.38	.817484	4.59	.182516	42
19	.739783	3.20	.922023	1.38	.817759	4.59	.182241	41
20	9.739975	3.20	9.921940	1.39	9.818035	4.59	10.181965	40
21	.740167	3.20	.921857	1.39	.818310	4.58	.181690	39
22	.740359	3.20	.921774	1.39	.818585	4.58	.181415	38
23	.740550	3.19	.921691	1.39	.818860	4.58	.181140	37
24	.740742	3.19	.921607	1.39	.819135	4.58	.180865	36
25	.740934	3.19	.921524	1.39	.819410	4.58	.180590	35
26	.741125	3.19	.921441	1.39	.819684	4.58	.180316	34
27	.741316	3.19	.921357	1.39	.819959	4.58	.180041	33
28	.741508	3.18	.921274	1.39	.820234	4.58	.179766	32
29	.741699	3.18	.921190	1.39	.820508	4.58	.179492	31
30	9.741889	3.18	9.921107	1.39	9.820783	4.57	10.179217	30
31	.742080	3.18	.921023	1.39	.821057	4.57	.178943	29
32	.742271	3.18	.920939	1.40	.821332	4.57	.178668	28
33	.742462	3.17	.920856	1.40	.821606	4.57	.178394	27
34	.742652	3.17	.920772	1.40	.821880	4.57	.178120	26
35	.742842	3.17	.920688	1.40	.822154	4.57	.177846	25
36	.743033	3.17	.920604	1.40	.822429	4.57	.177571	24
37	.743223	3.17	.920520	1.40	.822703	4.57	.177297	23
38	.743413	3.16	.920436	1.40	.822977	4.57	.177023	22
39	.743602	3.16	.920352	1.40	.823250	4.56	.176750	21
40	9.743792	3.16	9.920268	1.40	9.823524	4.56	10.176476	20
41	.743982	3.16	.920184	1.40	.823798	4.56	.176202	19
42	.744171	3.16	.920099	1.40	.824072	4.56	.175928	18
43	.744361	3.15	.920015	1.41	.824345	4.56	.175655	17
44	.744550	3.15	.919931	1.41	.824619	4.56	.175381	16
45	.744739	3.15	.919846	1.41	.824893	4.56	.175107	15
46	.744928	3.15	.919762	1.41	.825166	4.56	.174834	14
47	.745117	3.15	.919677	1.41	.825439	4.56	.174561	13
48	.745306	3.14	.919593	1.41	.825713	4.55	.174287	12
49	.745494	3.14	.919508	1.41	.825986	4.55	.174014	11
50	9.745683	3.14	9.919424	1.41	9.826259	4.55	10.173741	10
51	.745871	3.14	.919339	1.41	.826532	4.55	.173468	9
52	.746060	3.14	.919254	1.41	.826805	4.55	.173195	8
53	.746248	3.13	.919169	1.41	.827078	4.55	.172922	7
54	.746436	3.13	.919085	1.42	.827351	4.55	.172649	6
55	.746624	3.13	.919000	1.42	.827624	4.55	.172376	5
56	.746812	3.13	.918915	1.42	.827897	4.55	.172103	4
57	.746999	3.13	.918830	1.42	.828170	4.54	.171830	3
58	.747187	3.12	.918745	1.42	.828442	4.54	.171558	2
59	.747374	3.12	.918659	1.42	.828715	4.54	.171285	1
60	.747562		.918574		.828987		.171013	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

34° **145°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.747562	3.12	9.918574	1.42	9.828987	4.54	10.171013	60
1	.747749	3.12	.918489	1.42	.829260	4.54	.170740	59
2	.747936	3.12	.918404	1.42	.829532	4.54	.170468	58
3	.748123	3.11	.918318	1.42	.829805	4.54	.170195	57
4	.748310	3.11	.918233	1.42	.830077	4.54	.169923	56
5	.748497	3.11	.918147	1.43	.830349	4.54	.169651	55
6	.748683	3.11	.918062	1.43	.830621	4.53	.169379	54
7	.748870	3.11	.917976	1.43	.830893	4.53	.169107	53
8	.749056	3.10	.917891	1.43	.831165	4.53	.168835	52
9	.749243	3.10	.917805	1.43	.831437	4.53	.168563	51
10	9.749429	3.10	9.917719	1.43	9.831709	4.53	10.168291	50
11	.749615	3.10	.917634	1.43	.831981	4.53	.168019	49
12	.749801	3.10	.917548	1.43	.832253	4.53	.167747	48
13	.749987	3.10	.917462	1.43	.832525	4.53	.167475	47
14	.750172	3.09	.917376	1.43	.832796	4.53	.167204	46
15	.750358	3.09	.917290	1.43	.833068	4.53	.166932	45
16	.750543	3.09	.917204	1.43	.833339	4.52	.166661	44
17	.750729	3.09	.917118	1.44	.833611	4.52	.166389	43
18	.750914	3.09	.917032	1.44	.833882	4.52	.166118	42
19	.751099	3.08	.916946	1.44	.834154	4.52	.165846	41
20	9.751284	3.08	9.916859	1.44	9.834425	4.52	10.165575	40
21	.751469	3.08	.916773	1.44	.834696	4.52	.165304	39
22	.751654	3.08	.916687	1.44	.834967	4.52	.165033	38
23	.751839	3.08	.916600	1.44	.835238	4.52	.164762	37
24	.752023	3.07	.916514	1.44	.835509	4.52	.164491	36
25	.752208	3.07	.916427	1.44	.835780	4.52	.164220	35
26	.752392	3.07	.916341	1.44	.836051	4.51	.163949	34
27	.752576	3.07	.916254	1.44	.836322	4.51	.163678	33
28	.752760	3.07	.916167	1.45	.836593	4.51	.163407	32
29	.752944	3.06	.916081	1.45	.836864	4.51	.163136	31
30	9.753128	3.06	9.915994	1.45	9.837134	4.51	10.162866	30
31	.753312	3.06	.915907	1.45	.837405	4.51	.162595	29
32	.753495	3.06	.915820	1.45	.837675	4.51	.162325	28
33	.753679	3.06	.915733	1.45	.837946	4.51	.162054	27
34	.753862	3.05	.915646	1.45	.838216	4.51	.161784	26
35	.754046	3.05	.915559	1.45	.838487	4.51	.161513	25
36	.754229	3.05	.915472	1.45	.838757	4.50	.161243	24
37	.754412	3.05	.915385	1.45	.839027	4.50	.160973	23
38	.754595	3.05	.915297	1.45	.839297	4.50	.160703	22
39	.754778	3.05	.915210	1.46	.839568	4.50	.160432	21
40	9.754960	3.04	9.915123	1.46	9.839838	4.50	10.160162	20
41	.755143	3.04	.915035	1.46	.840108	4.50	.159892	19
42	.755326	3.04	.914948	1.46	.840378	4.50	.159622	18
43	.755508	3.04	.914860	1.46	.840648	4.50	.159352	17
44	.755690	3.04	.914773	1.46	.840917	4.50	.159083	16
45	.755872	3.03	.914685	1.46	.841187	4.49	.158813	15
46	.756054	3.03	.914598	1.46	.841457	4.49	.158543	14
47	.756236	3.03	.914510	1.46	.841727	4.49	.158273	13
48	.756418	3.03	.914422	1.46	.841996	4.49	.158004	12
49	.756600	3.03	.914334	1.46	.842266	4.49	.157734	11
50	9.756782	3.02	9.914246	1.47	9.842535	4.49	10.157465	10
51	.756963	3.02	.914158	1.47	.842805	4.49	.157195	9
52	.757144	3.02	.914070	1.47	.843074	4.49	.156926	8
53	.757326	3.02	.913982	1.47	.843343	4.49	.156657	7
54	.757507	3.02	.913894	1.47	.843612	4.49	.156388	6
55	.757688	3.02	.913806	1.47	.843882	4.49	.156118	5
56	.757869	3.01	.913718	1.47	.844151	4.48	.155849	4
57	.758050	3.01	.913630	1.47	.844420	4.48	.155580	3
58	.758230	3.01	.913541	1.47	.844689	4.48	.155211	2
59	.758411	3.01	.913453	1.47	.844958	4.48	.155652	1
60	.758591		.913365		.845227		.154773	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

124° **55°**

35° **144°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.758591	3.01	9.913365	1.47	9.845227	4.48	10.154773	60
1	.758772	3.00	.913276	1.48	.845496	4.48	.154504	59
2	.758952	3.00	.913187	1.48	.845764	4.48	.154236	58
3	.759132	3.00	.913099	1.48	.846033	4.48	.153967	57
4	.759312	3.00	.913010	1.48	.846302	4.48	.153698	56
5	.759492	3.00	.912922	1.48	.846570	4.48	.153430	55
6	.759672	2.99	.912833	1.48	.846839	4.48	.153161	54
7	.759852	2.99	.912744	1.48	.847108	4.47	.152892	53
8	.760031	2.99	.912655	1.48	.847376	4.47	.152624	52
9	.760211	2.99	.912566	1.48	.847644	4.47	.152356	51
10	9.760390	2.99	9.912477	1.48	9.847913	4.47	10.152087	50
11	.760569	2.99	.912388	1.48	.848181	4.47	.151819	49
12	.760748	2.98	.912299	1.49	.848449	4.47	.151551	48
13	.760927	2.98	.912210	1.49	.848717	4.47	.151283	47
14	.761106	2.98	.912121	1.49	.848986	4.47	.151014	46
15	.761285	2.98	.912031	1.49	.849254	4.47	.150746	45
16	.761464	2.98	.911942	1.49	.849522	4.47	.150478	44
17	.761642	2.97	.911853	1.49	.849790	4.46	.150210	43
18	.761821	2.97	.911763	1.49	.850057	4.46	.149943	42
19	.761999	2.97	.911674	1.49	.850325	4.46	.149675	41
20	9.762177	2.97	9.911584	1.49	9.850593	4.46	10.149407	40
21	.762356	2.97	.911495	1.49	.850861	4.46	.149139	39
22	.762534	2.97	.911405	1.49	.851129	4.46	.148871	38
23	.762712	2.96	.911315	1.50	.851396	4.46	.148604	37
24	.762889	2.96	.911226	1.50	.851664	4.46	.148336	36
25	.763067	2.96	.911136	1.50	.851931	4.46	.148069	35
26	.763245	2.96	.911046	1.50	.852199	4.46	.147801	34
27	.763422	2.96	.910956	1.50	.852466	4.46	.147534	33
28	.763600	2.95	.910866	1.50	.852733	4.46	.147267	32
29	.763777	2.95	.910776	1.50	.853001	4.45	.146999	31
30	9.763954	2.95	9.910686	1.50	9.853268	4.45	10.146732	30
31	.764131	2.95	.910596	1.50	.853535	4.45	.146465	29
32	.764308	2.95	.910506	1.50	.853802	4.45	.146198	28
33	.764485	2.95	.910415	1.51	.854069	4.45	.145931	27
34	.764662	2.94	.910325	1.51	.854336	4.45	.145664	26
35	.764838	2.94	.910235	1.51	.854603	4.45	.145397	25
36	.765015	2.94	.910144	1.51	.854870	4.45	.145130	24
37	.765191	2.94	.910054	1.51	.855137	4.45	.144863	23
38	.765367	2.94	.909963	1.51	.855404	4.45	.144596	22
39	.765544	2.93	.909873	1.51	.855671	4.44	.144329	21
40	9.765720	2.93	9.909782	1.51	9.855938	4.44	10.144062	20
41	.765896	2.93	.909691	1.51	.856204	4.44	.143796	19
42	.766072	2.93	.909601	1.51	.856471	4.44	.143529	18
43	.766247	2.93	.909510	1.51	.856737	4.44	.143263	17
44	.766423	2.93	.909419	1.52	.857004	4.44	.142996	16
45	.766598	2.92	.909328	1.52	.857270	4.44	.142730	15
46	.766774	2.92	.909237	1.52	.857537	4.44	.142463	14
47	.766949	2.92	.909146	1.52	.857803	4.44	.142197	13
48	.767124	2.92	.909055	1.52	.858069	4.44	.141931	12
49	.767300	2.92	.908964	1.52	.858336	4.44	.141664	11
50	9.767475	2.91	9.908873	1.52	9.858602	4.44	10.141398	10
51	.767649	2.91	.908781	1.52	.858868	4.43	.141132	9
52	.767824	2.91	.908690	1.52	.859134	4.43	.140866	8
53	.767999	2.91	.908599	1.52	.859400	4.43	.140600	7
54	.768173	2.91	.908507	1.52	.859666	4.43	.140334	6
55	.768348	2.91	.908416	1.53	.859932	4.43	.140068	5
56	.768522	2.90	.908324	1.53	.860198	4.43	.139802	4
57	.768697	2.90	.908233	1.53	.860464	4.43	.139536	3
58	.768871	2.90	.908141	1.53	.860730	4.43	.139270	2
59	.769045	2.90	.908049	1.53	.860995	4.43	.139005	1
60	.769219		.907958		.861261		.138739	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

125° **54°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.769219	2.90	9.907958	1.53	9.861261	4.43	10.138739	60
1	.769393	2.90	.907866	1.53	.861527	4.43	.138473	59
2	.769566	2.89	.907774	1.53	.861792	4.43	.138208	58
3	.769740	2.89	.907682	1.53	.862058	4.42	.137942	57
4	.769913	2.89	.907590	1.53	.862323	4.42	.137677	56
5	.770087	2.89	.907498	1.53	.862589	4.42	.137411	55
6	.770260	2.89	.907406	1.54	.862854	4.42	.137146	54
7	.770433	2.88	.907314	1.54	.863119	4.42	.136881	53
8	.770606	2.88	.907222	1.54	.863385	4.42	.136615	52
9	.770779	2.88	.907129	1.54	.863650	4.42	.136350	51
10	9.770952	2.88	9.907037	1.54	9.863915	4.42	10.136085	50
11	.771125	2.88	.906945	1.54	.864180	4.42	.135820	49
12	.771298	2.88	.906852	1.54	.864445	4.42	.135555	48
13	.771470	2.87	.906760	1.54	.864710	4.42	.135290	47
14	.771643	2.87	.906667	1.54	.864975	4.42	.135025	46
15	.771815	2.87	.906575	1.54	.865240	4.41	.134760	45
16	.771987	2.87	.906482	1.55	.865505	4.41	.134495	44
17	.772159	2.87	.906389	1.55	.865770	4.41	.134230	43
18	.772331	2.87	.906296	1.55	.866035	4.41	.133965	42
19	.772503	2.86	.906204	1.55	.866300	4.41	.133700	41
20	9.772675	2.86	9.906111	1.55	9.866564	4.41	10.133436	40
21	.772847	2.86	.906018	1.55	.866829	4.41	.133171	39
22	.773018	2.86	.905925	1.55	.867094	4.41	.132906	38
23	.773190	2.86	.905832	1.55	.867358	4.41	.132642	37
24	.773361	2.85	.905739	1.55	.867623	4.41	.132377	36
25	.773533	2.85	.905645	1.55	.867887	4.41	.132113	35
26	.773704	2.85	.905552	1.55	.868152	4.41	.131848	34
27	.773875	2.85	.905459	1.56	.868416	4.41	.131584	33
28	.774046	2.85	.905366	1.56	.868680	4.40	.131320	32
29	.774217	2.85	.905272	1.56	.868945	4.40	.131055	31
30	9.774388	2.84	9.905179	1.56	9.869209	4.40	10.130791	30
31	.774558	2.84	.905085	1.56	.869473	4.40	.130527	29
32	.774729	2.84	.904992	1.56	.869737	4.40	.130263	28
33	.774899	2.84	.904898	1.56	.870001	4.40	.129999	27
34	.775070	2.84	.904804	1.56	.870265	4.40	.129735	26
35	.775240	2.84	.904711	1.56	.870529	4.40	.129471	25
36	.775410	2.83	.904617	1.56	.870793	4.40	.129207	24
37	.775580	2.83	.904523	1.57	.871057	4.40	.128943	23
38	.775750	2.83	.904429	1.57	.871321	4.40	.128679	22
39	.775920	2.83	.904335	1.57	.871585	4.40	.128415	21
40	9.776090	2.83	9.904241	1.57	9.871849	4.40	10.128151	20
41	.776259	2.83	.904147	1.57	.872112	4.39	.127888	19
42	.776429	2.82	.904053	1.57	.872376	4.39	.127624	18
43	.776598	2.82	.903959	1.57	.872640	4.39	.127360	17
44	.776768	2.82	.903864	1.57	.872903	4.39	.127097	16
45	.776937	2.82	.903770	1.57	.873167	4.39	.126833	15
46	.777106	2.82	.903676	1.57	.873430	4.39	.126570	14
47	.777275	2.82	.903581	1.57	.873694	4.39	.126306	13
48	.777444	2.81	.903487	1.58	.873957	4.39	.126043	12
49	.777613	2.81	.903392	1.58	.874220	4.39	.125780	11
50	9.777781	2.81	9.903298	1.58	9.874484	4.39	10.125516	10
51	.777950	2.81	.903203	1.58	.874747	4.39	.125253	9
52	.778119	2.81	.903108	1.58	.875010	4.39	.124990	8
53	.778287	2.81	.903014	1.58	.875273	4.39	.124727	7
54	.778455	2.80	.902919	1.58	.875537	4.38	.124463	6
55	.778624	2.80	.902824	1.58	.875800	4.38	.124200	5
56	.778792	2.80	.902729	1.58	.876063	4.38	.123937	4
57	.778960	2.80	.902634	1.58	.876326	4.38	.123674	3
58	.779128	2.80	.902539	1.59	.876589	4.38	.123411	2
59	.779295	2.79	.902444	1.59	.876852	4.38	.123148	1
60	.779463		.902349		.877114		.122886	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M

37° **142°**

M.	Sine.	D.1″	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.779463	2.79	9.902349	1.59	9.877114	4.38	10.122886	60
1	.779631	2.79	.902253	1.59	.877377	4.38	.122623	59
2	.779798	2.79	.902158	1.59	.877640	4.38	.122360	58
3	.779966	2.79	.902063	1.59	.877903	4.38	.122097	57
4	.780133	2.79	.901967	1.59	.878165	4.38	.121835	56
5	.780300	2.78	.901872	1.59	.878428	4.38	.121572	55
6	.780467	2.78	.901776	1.59	.878691	4.38	.121309	54
7	.780634	2.78	.901681	1.59	.878953	4.38	.121047	53
8	.780801	2.78	.901585	1.59	.879216	4.37	.120784	52
9	.780968	2.78	.901490	1.60	.879478	4.37	.120522	51
10	9.781134	2.78	9.901394	1.60	9.879741	4.37	10.120259	50
11	.781301	2.77	.901298	1.60	.880003	4.37	.119997	49
12	.781468	2.77	.901202	1.60	.880265	4.37	.119735	48
13	.781634	2.77	.901106	1.60	.880528	4.37	.119472	47
14	.781800	2.77	.901010	1.60	.880790	4.37	.119210	46
15	.781966	2.77	.900914	1.60	.881052	4.37	.118948	45
16	.782132	2.77	.900818	1.60	.881314	4.37	.118686	44
17	.782298	2.76	.900722	1.60	.881577	4.37	.118423	43
18	.782464	2.76	.900626	1.60	.881839	4.37	.118161	42
19	.782630	2.76	.900529	1.61	.882101	4.37	.117899	41
20	9.782796	2.76	9.900433	1.61	9.882363	4.37	10.117637	40
21	.782961	2.76	.900337	1.61	.882625	4.37	.117375	39
22	.783127	2.76	.900240	1.61	.882887	4.36	.117113	38
23	.783292	2.75	.900144	1.61	.883148	4.36	.116852	37
24	.783458	2.75	.900047	1.61	.883410	4.36	.116590	36
25	.783623	2.75	.899951	1.61	.883672	4.36	.116328	35
26	.783788	2.75	.899854	1.61	.883934	4.36	.116066	34
27	.783953	2.75	.899757	1.61	.884196	4.36	.115804	33
28	.784118	2.75	.899660	1.61	.884457	4.36	.115543	32
29	.784282	2.74	.899564	1.62	.884719	4.36	.115281	31
30	9.784447	2.74	9.899467	1.62	9.884980	4.36	10.115020	30
31	.784612	2.74	.899370	1.62	.885242	4.36	.114758	29
32	.784776	2.74	.899273	1.62	.885504	4.36	.114496	28
33	.784941	2.74	.899176	1.62	.885765	4.36	.114235	27
34	.785105	2.74	.899078	1.62	.886026	4.36	.113974	26
35	.785269	2.73	.898981	1.62	.886288	4.36	.113712	25
36	.785433	2.73	.898884	1.62	.886549	4.36	.113451	24
37	.785597	2.73	.898787	1.62	.886811	4.35	.113189	23
38	.785761	2.73	.898689	1.62	.887072	4.35	.112928	22
39	.785925	2.73	.898592	1.62	.887333	4.35	.112667	21
40	9.786089	2.73	9.898494	1.63	9.887594	4.35	10.112406	20
41	.786252	2.73	.898397	1.63	.887855	4.35	.112145	19
42	.786416	2.72	.898299	1.63	.888116	4.35	.111884	18
43	.786579	2.72	.898202	1.63	.888378	4.35	.111622	17
44	.786742	2.72	.898104	1.63	.888639	4.35	.111361	16
45	.786906	2.72	.898006	1.63	.888900	4.35	.111100	15
46	.787069	2.72	.897908	1.63	.889161	4.35	.110839	14
47	.787232	2.72	.897810	1.63	.889421	4.35	.110579	13
48	.787395	2.71	.897712	1.63	.889682	4.35	.110318	12
49	.787557	2.71	.897614	1.63	.889943	4.35	.110057	11
50	9.787720	2.71	9.897516	1.64	9.890204	4.35	10.109796	10
51	.787883	2.71	.897418	1.64	.890465	4.35	.109535	9
52	.788045	2.71	.897320	1.64	.890725	4.34	.109275	8
53	.788208	2.71	.897222	1.64	.890986	4.34	.109014	7
54	.788370	2.70	.897123	1.64	.891247	4.34	.108753	6
55	.788532	2.70	.897025	1.64	.891507	4.34	.108493	5
56	.788694	2.70	.896926	1.64	.891768	4.34	.108232	4
57	.788856	2.70	.896828	1.64	.892028	4.34	.107972	3
58	.789018	2.70	.896729	1.64	.892289	4.34	.107711	2
59	.789180	2.70	.896631	1.64	.892549	4.34	.107451	1
60	.789342		.896532		.892810		.107190	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

127° **52°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.789342	2.69	9.896532	1.65	9.892810	4.34	10.107190	60
1	.789504	2.69	.896433	1.65	.893070	4.34	.106930	59
2	.789665	2.69	.896335	1.65	.893331	4.34	.106669	58
3	.789827	2.69	.896236	1.65	.893591	4.34	.106409	57
4	.789988	2.69	.896137	1.65	.893851	4.34	.106149	56
5	.790149	2.69	.896038	1.65	.894111	4.34	.105889	55
6	.790310	2 68	.895939	1.65	.894372	4.34	.105628	54
7	.790471	2.68	.895840	1.65	.894632	4.34	.105368	53
8	.790632	2.68	.895741	1.65	.894892	4.33	.105108	52
9	.790793	2.68	.895641	1.65	.895152	4.33	.104848	51
10	9.790954	2.68	9.895542	1.66	9.895412	4.33	10.104588	50
11	.791115	2.68	.895443	1.66	.895672	4.33	.104329	49
12	.791275	2.67	.895343	1.66	.895932	4.33	.104068	48
13	.791436	2.67	.895244	1.66	.896192	4.33	.103808	47
14	.791596	2.67	.895145	1.66	.896452	4.33	.103548	46
15	.791757	2.67	.895045	1.66	.896712	4.33	.103288	45
16	.791917	2.67	.894945	1.66	.896971	4.33	.103029	44
17	.792077	2.67	.894846	1.66	.897231	4.33	.102769	43
18	.792237	2.67	.894746	1.66	.897491	4.33	.102509	42
19	.792397	2.66	.894646	1.66	.897751	4.33	.102249	41
20	9.792557	2.66	9.894546	1.67	9.898010	4.33	10.101990	40
21	.792716	2.66	.894446	1.67	.898270	4.33	.101730	39
22	.792876	2.66	.894346	1.67	.898530	4 33	.101470	38
23	.793035	2.66	.894246	1.67	.898789	4.33	.101211	37
24	.793195	2.66	.894146	1.67	.899049	4.33	.100951	36
25	.793354	2.65	.894046	1.67	.899308	4.32	.100692	35
26	.793514	2.65	.893946	1.67	.899568	4.32	.100432	34
27	.793673	2.65	.893846	1.67	.899827	4.32	.100173	33
28	.793832	2.65	.893745	1.67	.900086	4.32	.099914	32
29	.793991	2.65	.893645	1.67	.900346	4.32	.099654	31
30	9.794150	2.65	9.893544	1.68	9.900605	4.32	10.099395	30
31	.794308	2.64	.893444	1.68	.900864	4.32	.099136	29
32	.794467	2.64	.893343	1.68	.901124	4.32	.098876	28
33	.794626	2.64	.893243	1.68	.901383	4.32	.098617	27
34	.794784	2.64	.893142	1.68	.901642	4.32	.098358	26
35	.794942	2.64	.893041	1.68	.901901	4.32	.098099	25
36	.795101	2.64	.892940	1.68	.902160	4.32	.097840	24
37	.795259	2.64	.892839	1.68	.902420	4.32	.097580	23
38	.795417	2.63	.892739	1.68	.902679	4.32	.097321	22
39	.795575	2.63	.892638	1.68	.902938	4.32	.097062	21
40	9.795733	2.63	9 892536	1.69	9.903197	4.32	10.096803	20
41	.795891	2.63	.892435	1.69	.903456	4.32	.096544	19
42	.796049	2.63	.892334	1.69	.903714	4.31	.096286	18
43	.796206	2.63	.892233	1.69	.903973	4.31	.096027	17
44	.796364	2.62	.892132	1.69	.904232	4.31	.095768	16
45	.796521	2.62	.892030	1.69	.904491	4.31	.095509	15
46	.796679	2.62	.891929	1.69	.904750	4.31	.095250	14
47	.796836	2.62	.891827	1 69	.905008	4.31	.094992	13
48	.796993	2.62	.891726	1 69	.905267	4.31	.094733	12
49	.797150	2.61	.891624	1.69	.905526	4.31	.094474	11
50	9.797307	2.61	9.891523	1.70	9.905785	4.31	10.094215	10
51	.797464	2.61	.891421	1.70	.906043	4.31	.093957	9
52	.797621	2.61	.891319	1.70	.906302	4.31	.093698	8
53	.797777	2.61	.891217	1.70	.906560	4.31	.093440	7
54	.797934	2.61	.891115	1.70	.906819	4.31	.093181	6
55	.798091	2.61	.891013	1.70	.907077	4.31	.092923	5
56	.798247	2.61	.890911	1.70	.907336	4.31	.092664	4
57	.798403	2.60	.890809	1.70	.907594	4.31	.092406	3
58	.798560	2.60	.890707	1.70	.907852	4.31	.092148	2
59	.798716	2.60	.890605	1.70	.908111	4.31	.091889	1
60	.798872		.890503		.908369		.091631	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

39° **140°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.798872	2.60	9.890503	1.71	9.908369	4.30	10.091631	60
1	.799028	2.60	.890400	1.71	.908628	4.30	.091372	59
2	.799184	2.60	.890298	1.71	.908886	4.30	.091114	58
3	.799339	2.59	.890195	1.71	.909144	4.30	.090856	57
4	.799495	2.59	.890093	1.71	.909402	4.30	.090598	56
5	.799651	2.59	.889990	1.71	.909660	4.30	.090340	55
6	.799806	2.59	.889888	1.71	.909918	4.30	.090082	54
7	.799962	2.59	.889785	1.71	.910177	4.30	.089823	53
8	.800117	2.59	.889682	1.71	.910435	4.30	.089565	52
9	.800272	2.59	.889579	1.71	.910693	4.30	.089307	51
10	9.800427	2.58	9.889477	1.72	9.910951	4.30	10.089049	50
11	.800582	2.58	.889374	1.72	.911209	4.30	.088791	49
12	.800737	2.58	.889271	1.72	.911467	4.30	.088533	48
13	.800892	2.58	.889168	1.72	.911724	4.30	.088276	47
14	.801047	2.58	.889064	1.72	.911982	4.30	.088018	46
15	.801201	2.58	.888961	1.72	.912240	4.30	.087760	45
16	.801356	2.57	.888858	1.72	.912498	4.30	.087502	44
17	.801511	2.57	.888755	1.72	.912756	4.30	.087244	43
18	.801665	2.57	.888651	1.72	.913014	4.30	.086986	42
19	.801819	2.57	.888548	1.72	.913271	4.30	.086729	41
20	9.801973	2.57	9.888444	1.73	9.913529	4.29	10.086471	40
21	.802128	2.57	.888341	1.73	.913787	4.29	.086213	39
22	.802282	2.57	.888237	1.73	.914044	4.29	.085956	38
23	.802436	2.56	.888134	1.73	.914302	4.29	.085698	37
24	.802589	2.56	.888030	1.73	.914560	4.29	.085440	36
25	.802743	2.56	.887926	1.73	.914817	4.29	.085183	35
26	.802897	2.56	.887822	1.73	.915075	4.29	.084925	34
27	.803050	2.56	.887718	1.73	.915332	4.29	.084668	33
28	.803204	2.56	.887614	1.73	.915590	4.29	.084410	32
29	.803357	2.55	.887510	1.74	.915847	4.29	.084153	31
30	9.803511	2.55	9.887406	1.74	9.916104	4.29	10.083896	30
31	.803664	2.55	.887302	1.74	.916362	4.29	.083638	29
32	.803817	2.55	.887198	1.74	.916619	4.29	.083381	28
33	.803970	2.55	.887093	1.74	.916877	4.29	.083123	27
34	.804123	2.55	.886989	1.74	.917134	4.29	.082866	26
35	.804276	2.55	.886885	1.74	.917391	4.29	.082609	25
36	.804428	2.54	.886780	1.74	.917648	4.29	.082352	24
37	.804581	2.54	.886676	1.74	.917906	4.29	.082094	23
38	.804734	2.54	.886571	1.74	.918162	4.29	.081838	22
39	.804886	2.54	.886466	1.75	.918420	4.29	.081580	21
40	9.805039	2.54	9.886362	1.75	9.918677	4.28	10.081323	20
41	.805191	2.54	.886257	1.75	.918934	4.28	.081066	19
42	.805343	2.54	.886152	1.75	.919191	4.28	.080809	18
43	.805495	2.53	.886047	1.75	.919448	4.28	.080552	17
44	.805647	2.53	.885942	1.75	.919705	4.28	.080295	16
45	.805799	2.53	.885837	1.75	.919962	4.28	.080038	15
46	.805951	2.53	.885732	1.75	.920219	4.28	.079781	14
47	.806103	2.53	.885627	1.75	.920476	4.28	.079524	13
48	.806254	2.53	.885522	1.75	.920733	4.28	.079267	12
49	.806406	2.52	.885416	1.76	.920990	4.28	.079010	11
50	9.806557	2.52	9.885311	1.76	9.921247	4.28	10.078753	10
51	.806709	2.52	.885205	1.76	.921503	4.28	.078497	9
52	.806860	2.52	.885100	1.76	.921760	4.28	.078240	8
53	.807011	2.52	.884994	1.76	.922017	4.28	.077983	7
54	.807163	2.52	.884889	1.76	.922274	4.28	.077726	6
55	.807314	2.52	.884783	1.76	.922530	4.28	.077470	5
56	.807465	2.51	.884677	1.76	.922787	4.28	.077213	4
57	.807615	2.51	.884572	1.76	.923044	4.28	.076956	3
58	.807766	2.51	.884466	1.77	.923300	4.28	.076700	2
59	.807917	2.51	.884360	1.77	.923557	4.28	.076443	1
60	.808067		.884254		.923813		.076187	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

129° **50°**

40° **139°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.808067	2.51	9.884254	1.77	9.923813	4.28	10.076187	60
1	.808218	2.51	.884148	1.77	.924070	4.28	.075930	59
2	.808368	2.51	.884042	1.77	.924327	4.27	.075673	58
3	.808519	2.50	.883936	1.77	.924583	4.27	.075417	57
4	.808669	2.50	.883829	1.77	.924840	4.27	.075160	56
5	.808819	2.50	.883723	1.77	.925096	4.27	.074904	55
6	.808969	2.50	.883617	1.77	.925352	4.27	.074648	54
7	.809119	2.50	.883510	1.77	.925609	4.27	.074391	53
8	.809269	2.50	.883404	1.78	.925865	4.27	.074135	52
9	.809419	2.50	.883297	1.78	.926122	4.27	.073878	51
10	9.809569	2.49	9.883191	1.78	9.926378	4.27	10.073622	50
11	.809718	2.49	.883084	1.78	.926634	4.27	.073366	49
12	.809868	2.49	.882977	1.78	.926890	4.27	.073110	48
13	.810017	2.49	.882871	1.78	.927147	4.27	.072853	47
14	.810167	2.49	.882764	1.78	.927403	4.27	.072597	46
15	.810316	2.49	.882657	1.78	.927659	4.27	.072341	45
16	.810465	2.48	.882550	1.78	.927915	4.27	.072085	44
17	.810614	2.48	.882443	1.79	.928171	4.27	.071829	43
18	.810763	2.48	.882336	1.79	.928427	4.27	.071573	42
19	.810912	2.48	.882229	1.79	.928683	4.27	.071317	41
20	9.811061	2.48	9.882121	1.79	9.928940	4.27	10.071060	40
21	.811210	2.48	.882014	1.79	.929196	4.27	.070804	39
22	.811358	2.48	.881907	1.79	.929452	4.27	.070548	38
23	.811507	2.47	.881799	1.79	.929708	4.27	.070292	37
24	.811655	2.47	.881692	1.79	.929964	4.27	.070036	36
25	.811804	2.47	.881584	1.79	.930220	4.27	.069780	35
26	.811952	2.47	.881477	1.79	.930475	4.26	.069525	34
27	.812100	2.47	.881369	1.80	.930731	4.26	.069269	33
28	.812248	2.47	.881261	1.80	.930987	4.26	.069013	32
29	.812396	2.47	.881153	1.80	.931243	4.26	.068757	31
30	9.812544	2.46	9.881046	1.80	9.931499	4.26	10.068501	30
31	.812692	2.46	.880938	1.80	.931755	4.26	.068245	29
32	.812840	2.46	.880830	1.80	.932010	4.26	.067990	28
33	.812988	2.46	.880722	1.80	.932266	4.26	.067734	27
34	.813135	2.46	.880613	1.80	.932522	4.26	.067478	26
35	.813283	2.46	.880505	1.80	.932778	4.26	.067222	25
36	.813430	2.46	.880397	1.81	.933033	4.26	.066967	24
37	.813578	2.45	.880289	1.81	.933289	4.26	.066711	23
38	.813725	2.45	.880180	1.81	.933545	4.26	.066455	22
39	.813872	2.45	.880072	1.81	.933800	4.26	.066200	21
40	9.814019	2.45	9.879963	1.81	9.934056	4.26	10.065944	20
41	.814166	2.45	.879855	1.81	.934311	4.26	.065689	19
42	.814313	2.45	.879746	1.81	.934567	4.26	.065433	18
43	.814460	2.45	.879637	1.81	.934823	4.26	.065177	17
44	.814607	2.44	.879529	1.81	.935078	4.26	.064922	16
45	.814753	2.44	.879420	1.81	.935333	4.26	.064667	15
46	.814900	2.44	.879311	1.82	.935589	4.26	.064411	14
47	.815046	2.44	.879202	1.82	.935844	4.26	.064156	13
48	.815193	2.44	.879093	1.82	.936100	4.26	.063900	12
49	.815339	2.44	.878984	1.82	.936355	4.26	.063645	11
50	9.815485	2.44	9.878875	1.82	9.936611	4.26	10.063389	10
51	.815631	2.43	.878766	1.82	.936866	4.26	.063134	9
52	.815778	2.43	.878656	1.82	.937121	4.26	.062879	8
53	.815924	2.43	.878547	1.82	.937376	4.25	.062624	7
54	.816069	2.43	.878438	1.82	.937632	4.25	.062368	6
55	.816215	2.43	.878328	1.83	.937887	4.25	.062113	5
56	.816361	2.43	.878219	1.83	.938142	4.25	.061858	4
57	.816507	2.43	.878109	1.83	.938398	4.25	.061602	3
58	.816652	2.42	.877999	1.83	.938653	4.25	.061347	2
59	.816798	2.42	.877890	1.83	.938908	4.25	.061092	1
60	.816943		.877780		.939163		.060837	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

130° **49°**

41° **138°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.816943	2.42	9.877780	1 83	9.939163	4.25	10.060837	60
1	.817088	2.42	.877670	1.83	.939418	4.25	.060582	59
2	.817233	2.42	.877560	1.83	.939673	4.25	.060327	58
3	.817379	2.42	.877450	1.83	.939928	4.25	.060072	57
4	.817524	2.42	.877340	1.84	.940183	4.25	.059817	56
5	.817668	2.41	.877230	1.84	.940438	4.25	.059562	55
6	.817813	2.41	.877120	1.84	.940694	4.25	.059306	54
7	.817958	2.41	.877010	1.84	.940949	4.25	.059051	53
8	.818103	2.41	.876899	1.84	.941204	4.25	.058796	52
9	.818247	2.41	.876789	1.84	.941458	4.25	.058542	51
10	9.818392	2.41	9.876678	1.84	9.941713	4.25	10.058287	50
11	.818536	2.41	.876568	1.84	.941968	4.25	.058032	49
12	.818681	2.40	.876457	1.84	.942223	4.25	.057777	48
13	.818825	2.40	.876347	1.84	.942478	4.25	.057522	47
14	.818969	2.40	.876236	1.85	.942733	4.25	.057267	46
15	.819113	2.40	.876125	1.85	.942988	4.25	.057012	45
16	.819257	2.40	.876014	1.85	.943243	4.25	.056757	44
17	.819401	2.40	.875904	1.85	.943498	4.25	.056502	43
18	.819545	2.40	.875793	1.85	.943752	4.25	.056248	42
19	.819689	2.39	.875682	1.85	.944007	4.25	.055993	41
20	9.819832	2.39	9.875571	1.85	9.944262	4.25	10.055738	40
21	.819976	2.39	.875459	1.85	.944517	4.25	.055483	39
22	.820120	2.39	.875348	1.85	.944771	4.24	.055229	38
23	.820263	2.39	.875237	1.86	.945026	4.24	.054974	37
24	.820406	2.39	.875126	1.86	.945281	4.24	.054719	36
25	.820550	2.39	.875014	1.86	.945535	4.24	.054465	35
26	.820693	2.38	.874903	1.86	.945790	4.24	.054210	34
27	.820836	2.38	.874791	1.86	.946045	4.24	.053955	33
28	.820979	2.38	.874680	1.86	.946299	4.24	.053701	32
29	.821122	2.38	.874568	1.86	.946554	4.24	.053446	31
30	9.821265	2.38	9.874456	1.86	9.946808	4.24	10.053192	30
31	.821407	2.38	.874344	1.86	.947063	4.24	.052937	29
32	.821550	2.38	.874232	1.87	.947318	4.24	.052682	28
33	.821693	2 37	.874121	1.87	.947572	4.24	.052428	27
34	.821835	2.37	.874009	1.87	.947826	4.24	.052174	26
35	.821977	2.37	.873896	1.87	.948081	4.24	.051919	25
36	.822120	2.37	.873784	1.87	.948336	4.24	.051664	24
37	.822262	2.37	.873672	1.87	.948590	4.24	.051410	23
38	.822404	2.37	.873560	1.87	.948844	4 24	.051156	22
39	.822546	2.37	.873448	1.87	.949099	4.24	.050901	21
40	9.822688	2.37	9.873335	1.87	9.949353	4.24	10.050647	20
41	.822830	2.36	.873223	1.88	.949608	4.24	.050392	19
42	.822972	2.36	.873110	1.88	.949862	4.24	.050138	18
43	.823114	2.36	.872998	1.88	.950116	4.24	.049884	17
44	.823255	2.36	.872885	1.88	.950371	4.24	.049629	16
45	.823397	2.36	.872772	1.88	.950625	4.24	.049375	15
46	.823539	2.36	.872659	1.88	950879	4.24	.049121	14
47	.823680	2.36	.872547	1.88	.951133	4.24	.048867	13
48	.823821	2.35	.872434	1.88	.951388	4.24	.048612	12
49	.823963	2.35	.872321	1.88	.951642	4.24	.048358	11
50	9.824104	2.35	9.872208	1.89	9.951896	4 24	10.048104	10
51	.824245	2.35	.872095	1.89	.952150	4.24	.047850	9
52	.824386	2.35	.871981	1.89	.952405	4.24	.047595	8
53	.824527	2.35	.871868	1.89	.952659	4.24	.047341	7
54	.824668	2.35	.871755	1.89	.952913	4.24	.047087	6
55	.824808	2.34	.871641	1.89	.953167	4.24	.046833	5
56	.824949	2.34	.871528	1.89	.953421	4.24	.046579	4
57	.825090	2.34	.871414	1.89	.953675	4.23	.046325	3
58	.825230	2.34	.871301	1.89	.953929	4.23	.046071	2
59	.825371	2.34	.871187	1.90	.954183	4.23	.045817	1
60	.825511		.871073		.954437		.045563	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

131° **48°**

42° **137°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.825511	2.34	9.871073	1.90	9.954437	4.23	10.045563	60
1	.825651	2.34	.870960	1.90	.954691	4.23	.045309	59
2	.825791	2.33	.870846	1.90	.954945	4.23	.045055	58
3	.825931	2.33	.870732	1.90	.955200	4.23	.044800	57
4	.826071	2.33	.870618	1.90	.955454	4.23	.044546	56
5	.826211	2.33	.870504	1.90	.955707	4.23	.044293	55
6	.826351	2.33	.870390	1.90	.955961	4.23	.044039	54
7	.826491	2.33	.870276	1.90	.956215	4.23	.043785	53
8	.826631	2.33	.870161	1.91	.956469	4.23	.043531	52
9	.826770	2.33	.870047	1.91	.956723	4.23	043277	51
10	9.826910	2.32	9.869933	1.91	9.956977	4.23	10.043023	50
11	.827049	2.32	.869818	1.91	.957231	4.23	.042769	49
12	.827189	2.32	.869704	1.91	.957485	4.23	.042515	48
13	.827328	2.32	.869589	1.91	.957739	4.23	.042261	47
14	.827467	2.32	.869474	1.91	.957993	4.23	.042007	46
15	.827606	2.32	.869360	1.91	.958246	4.23	.041754	45
16	.827745	2.32	.869245	1.91	.958500	4.23	.041500	44
17	.827884	2.31	.869130	1.92	.958754	4.23	.041246	43
18	.828023	2.31	.869015	1.92	.959008	4 23	.040992	42
19	.828162	2.31	.868900	1.92	.959262	4.23	.040738	41
20	9.828301	2.31	9.868785	1.92	9.959516	4.23	10.040484	40
21	.828439	2.31	.868670	1.92	.959769	4.23	.040231	39
22	.828578	2.31	.868555	1.92	.960023	4.23	.039977	38
23	.828716	2.31	.868440	1.92	.960277	4.23	.039723	37
24	.828855	2.31	.868324	1.92	.960530	4.23	.039470	36
25	.828993	2.30	.868209	1.92	.960784	4.23	.039216	35
26	.829131	2.30	.868093	1.93	.961038	4.23	.038962	34
27	.829269	2.30	.867978	1.93	.961292	4.23	.038708	33
28	.829407	2.30	.867862	1.93	.961545	4.23	.038455	32
29	.829545	2.30	.867747	1 93	.961799	4.23	.038201	31
30	9.829683	2.30	9.867631	1.93	9.962052	4.23	10.037948	30
31	.829821	2.30	.867515	1.93	.962306	4.23	.037694	29
32	.829959	2.29	.867399	1.93	.962560	4.23	.037440	28
33	.830097	2.29	.867283	1.93	.962813	4.23	.037187	27
34	.830234	2.29	.867167	1.93	.963067	4.23	.036933	26
35	.830372	2.29	.867051	1.94	.963320	4.23	.036680	25
36	.830509	2.29	.866935	1.94	.963574	4.23	.036426	24
37	.830646	2.29	.866819	1.94	.963828	4.23	.036172	23
38	.830784	2.29	.866703	1.94	.964081	4.23	.035919	22
39	.830921	2.29	.866586	1.94	.964335	4.23	.035665	21
40	9.831058	2.28	9.866470	1.94	9.964588	4.22	10.035412	20
41	.831195	2.28	.866353	1.94	.964842	4.22	.035158	19
42	.831332	2.28	.866237	1.94	.965095	4.22	.034905	18
43	.831469	2.28	.866120	1.94	.965349	4.22	.034651	17
44	.831606	2.28	.866004	1.95	.965602	4.22	.034398	16
45	.831742	2.28	.865887	1.95	.965855	4.22	.034145	15
46	.831879	2.28	.865770	1.95	.966109	4.22	.033891	14
47	.832015	2.27	.865653	1.95	.966362	4.22	.033638	13
48	.832152	2.27	.865536	1.95	.966616	4.22	.033384	12
49	.832288	2.27	.865419	1.95	.966869	4.22	.033131	11
50	9.832425	2.27	9.865302	1.95	9.967123	4.22	10.032877	10
51	.832561	2.27	.865185	1.95	.967376	4.22	.032624	9
52	.832697	2.27	.865068	1.95	.967629	4.22	.032371	8
53	.832833	2.27	.864950	1.96	.967883	4.22	.032117	7
54	.832969	2.27	.864833	1.96	.968136	4.22	.031864	6
55	.833105	2.26	.864716	1.96	.968389	4.22	.031611	5
56	.833241	2.26	.864598	1.96	.968643	4.22	.031357	4
57	.833377	2.26	.864481	1.96	.968896	4.22	.031104	3
58	.833512	2.26	.864363	1.96	.969149	4.22	.030851	2
59	.833648	2.26	.864245	1.96	.969403	4.22	.030597	1
60	.833783		.864127		.969656		.030344	0
M.	Cosine,	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

132° **47°**

43° 136°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.833783	2.26	9.864127	1.96	9.969656	4.22	10.030344	60
1	.833919	2.26	.864010	1.97	.969909	4.22	.030091	59
2	.834054	2.25	.863892	1.97	.970162	4.22	.029838	58
3	.834189	2.25	.863774	1.97	.970416	4.22	.029584	57
4	.834325	2.25	.863656	1.97	.970669	4.22	.029331	56
5	.834460	2.25	.863538	1.97	.970922	4.22	.029078	55
6	.834595	2.25	.863419	1.97	.971175	4.22	.028825	54
7	.834730	2.25	.863301	1.97	.971429	4.22	.028571	53
8	.834865	2.25	.863183	1.97	.971682	4.22	.028318	52
9	.834999	2.25	.863064	1.97	.971935	4.22	.028065	51
10	9.835134	2.24	9.862946	1.98	9.972188	4.22	10.027812	50
11	.835269	2.24	.862827	1.98	.972441	4.22	.027559	49
12	.835403	2.24	.862709	1.98	.972694	4.22	.027306	48
13	.835538	2.24	.862590	1.98	.972948	4.22	.027052	47
14	.835672	2.24	.862471	1.98	.973201	4.22	.026799	46
15	.835807	2.24	.862353	1.98	.973454	4.22	.026546	45
16	.835941	2.24	.862234	1.98	.973707	4.22	.026293	44
17	.836075	2.23	.862115	1.98	.973960	4.22	.026040	43
18	.836209	2.23	.861996	1.98	.974213	4.22	.025787	42
19	.836343	2.23	.861877	1.99	974466	4.22	.025534	41
20	9.836477	2.23	9.861758	1.99	9.974720	4.22	10.025280	40
21	.836611	2.23	.861638	1.99	.974973	4.22	.025027	39
22	.836745	2.23	.861519	1.99	.975226	4.22	.024774	38
23	.836878	2.23	.861400	1.99	.975479	4.22	.024521	37
24	.837012	2.23	.861280	1.99	.975732	4.22	.024268	36
25	.837146	2.22	.861161	1.99	.975985	4.22	.024015	35
26	.837279	2.22	.861041	1.99	.976238	4.22	.023762	34
27	.837412	2.22	.860922	2.00	.976491	4.22	.023509	33
28	.837546	2.22	.860802	2.00	.976744	4.22	.023256	32
29	.837679	2.22	.860682	2.00	.976997	4.22	.023003	31
30	9.837812	2.22	9.860562	2.00	9.977250	4.22	10.022750	30
31	.837945	2.22	.860442	2.00	.977503	4.22	.022497	29
32	.838078	2.22	.860322	2.00	.977756	4.22	.022244	28
33	.838211	2.21	.860202	2.00	.978009	4.22	.021991	27
34	.838344	2.21	.860082	2.00	.978262	4.22	.021738	26
35	.838477	2.21	.859962	2.00	.978515	4.22	.021485	25
36	.838610	2.21	.859842	2.01	.978768	4.22	.021232	24
37	.838742	2.21	.859721	2.01	.979021	4.22	.020979	23
38	.838875	2.21	.859601	2.01	.979274	4.22	.020726	22
39	.839007	2.21	.859480	2.01	.979527	4.22	.020473	21
40	9.839140	2.21	9 859360	2.01	9.979780	4.22	10.020220	20
41	.839272	2.20	.859239	2.01	.980033	4.22	.019967	19
42	.839404	2.20	.859119	2.01	.980286	4.22	.019714	18
43	.839536	2.20	.858998	2.01	.980538	4.22	.019462	17
44	.839668	2.20	.858877	2.02	.980791	4.22	.019209	16
45	.839800	2.20	.858756	2.02	.981044	4.21	.018956	15
46	.839932	2.20	.858635	2.02	.981297	4.21	.018703	14
47	.840064	2.20	.858514	2.02	.981550	4.21	.018450	13
48	.840196	2.19	.858393	2.02	.981803	4 21	.018197	12
49	.840328	2.19	.858272	2.02	.982056	4.21	.017944	11
50	9.840459	2.19	9.858151	2.02	9.982309	4.21	10.017691	10
51	.840591	2.19	.858029	2.02	.982562	4.21	.017438	9
52	.840722	2.19	.857908	2.02	.982814	4.21	.017186	8
53	.840854	2.19	.857786	2.03	.983067	4.21	.016933	7
54	.840985	2.19	.857665	2.03	.983320	4.21	.016680	6
55	.841116	2.19	.857543	2.03	.983573	4.21	.016427	5
56	.841247	2.18	.857422	2.03	.983826	4.21	.016174	4
57	.841378	2.18	.857300	2.03	984079	4.21	.015921	3
58	.841509	2.18	.857178	2.03	.984331	4.21	.015669	2
59	.841640	2.18	.857056	2.03	.984584	4.21	.015416	1
60	.841771		.856934		.984837		.015163	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

133° 46°

44° **135°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.841771	2.18	9.856934	2.03	9.984837	4.21	10.015163	60
1	.841902	2.18	.856812	2.04	.985090	4.21	.014910	59
2	.842033	2.18	.856690	2.04	.985343	4.21	.014657	58
3	.842163	2.18	.856568	2.04	.985596	4.21	.014404	57
4	.842294	2.17	.856446	2.04	.985848	4.21	.014152	56
5	.842424	2.17	.856323	2.04	.986101	4.21	.013899	55
6	.842555	2.17	.856201	2.04	.986354	4.21	.013646	54
7	.842685	2.17	.856078	2.04	.986607	4.21	.013393	53
8	.842815	2.17	.855956	2.04	.986860	4.21	.013140	52
9	.842946	2.17	.855833	2.04	.987112	4.21	.012888	51
10	9.843076	2.17	9.855711	2.05	9.987365	4.21	10.012635	50
11	.843206	2.17	.855588	2.05	.987618	4.21	.012382	49
12	.843336	2.16	.855465	2.05	.987871	4.21	.012129	48
13	.843466	2.16	.855342	2.05	.988123	4.21	.011877	47
14	.843595	2.16	.855219	2.05	.988376	4.21	.011624	46
15	.843725	2.16	.855096	2.05	.988629	4.21	.011371	45
16	.843855	2.16	.854973	2.05	.988882	4.21	.011118	44
17	.843984	2.16	.854850	2.05	.989134	4.21	.010866	43
18	.844114	2.16	.854727	2.06	.989387	4.21	.010613	42
19	.844243	2.16	.854603	2.06	.989640	4.21	.010360	41
20	9.844372	2.15	9.854480	2.06	9.989893	4.21	10.010107	40
21	.844502	2.15	.854356	2.06	.990145	4.21	.009855	39
22	.844631	2.15	.854233	2.06	.990398	4.21	.009602	38
23	.844760	2.15	.854109	2.06	.990651	4.21	.009349	37
24	.844889	2.15	.853986	2.06	.990903	4.21	.009097	36
25	.845018	2.15	.853862	2.06	.991156	4.21	.008844	35
26	.845147	2.15	.853738	2.06	.991409	4.21	.008591	34
27	.845276	2.15	.853614	2.07	.991662	4.21	.008338	33
28	.845405	2.14	.853490	2.07	.991914	4.21	.008086	32
29	.845533	2.14	.853366	2.07	.992167	4.21	.007833	31
30	9.845662	2.14	9.853242	2.07	9.992420	4.21	10.007580	30
31	.845790	2.14	.853118	2.07	.992672	4.21	.007328	29
32	.845919	2.14	.852994	2.07	.992925	4.21	.007075	28
33	.846047	2.14	.852869	2.07	.993178	4.21	.006822	27
34	.846175	2.14	.852745	2.07	.993430	4.21	.006570	26
35	.846304	2.14	.852620	2.08	.993683	4.21	.006317	25
36	.846432	2.13	.852496	2.08	.993936	4.21	.006064	24
37	.846560	2.13	.852371	2.08	.994189	4.21	.005811	23
38	.846688	2.13	.852247	2.08	.994441	4.21	.005559	22
39	.846816	2.13	.852122	2.08	.994694	4.21	.005306	21
40	9.846944	2.13	9.851997	2.08	9.994947	4.21	10.005053	20
41	.847071	2.13	.851872	2.08	.995199	4.21	.004801	19
42	.847199	2.13	.851747	2.08	.995452	4.21	.004548	18
43	.847327	2.13	.851622	2.09	.995705	4.21	.004295	17
44	.847454	2.12	.851497	2.09	.995957	4.21	.004043	16
45	.847582	2.12	.851372	2.09	.996210	4.21	.003790	15
46	.847709	2.12	.851246	2.09	.996463	4.21	.003537	14
47	.847836	2.12	.851121	2.09	.996715	4.21	.003285	13
48	.847964	2.12	.850996	2.09	.996968	4.21	.003032	12
49	.848091	2.12	.850870	2.09	.997221	4.21	.002779	11
50	9.848218	2.12	9.850745	2.09	9.997473	4.21	10.002527	10
51	.848345	2.12	.850619	2.10	.997726	4.21	.002274	9
52	.848472	2.11	.850493	2.10	.997979	4.21	.002021	8
53	.848599	2.11	.850368	2.10	.998231	4.21	.001769	7
54	.848726	2.11	.850242	2.10	.998484	4.21	.001516	6
55	.848852	2.11	.850116	2.10	.998737	4.21	.001263	5
56	.848979	2.11	.849990	2.10	.998989	4.21	.001011	4
57	.849106	2.11	.849864	2.10	.999242	4.21	.000758	3
58	.849232	2.11	.849738	2.10	.999495	4.21	.000505	2
59	.849359	2.11	.849611	2.11	.999748	4.21	.000252	1
60	.849485		.849485		10.000000		.000000	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

134° **45°**

TABLE V.

LATITUDES AND DEPARTURES,

OR

TRAVERSE TABLE.

B'ng	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		B'ng
° '	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° '
0 15	1.0000	0.0044	2.0000	0.0087	3.0000	0.0131	4.0000	0.0175	5.0000	0.0218	89 45
30	0000	0087	1.9999	0175	2.9999	0262	3.9998	0349	4.9998	0436	30
45	0.9999	0131	9998	0262	9997	0393	9997	0524	9996	0654	15
1 0	9998	0175	9997	0349	9995	0524	9994	0698	9992	0873	89 0
15	9998	0218	9995	0436	9993	0654	9990	0873	9988	1091	45
30	9997	0262	9993	0524	9990	0785	9986	1047	9983	1309	30
45	9995	0305	9991	0611	9986	0916	9981	1222	9977	1527	15
2 0	9994	0349	9988	0698	9982	1047	9976	1396	9970	1745	88 0
15	9992	0393	9985	0785	9977	1178	9969	1570	9961	1963	45
30	9990	0436	9981	0872	9971	1309	9962	1745	9952	2181	30
45	0.9988	0.0480	1.9977	0.0960	2.9965	0.1439	3.9954	0.1919	4.9942	0.2399	15
3 0	9986	0523	9973	1047	9959	1570	9945	2093	9931	2617	87 0
15	9984	0567	9968	1134	9952	1701	9936	2268	9920	2835	45
30	9981	0610	9963	1221	9944	1831	9925	2442	9907	3052	30
45	9979	0654	9957	1308	9936	1962	9914	2616	9893	3270	15
4 0	9976	0698	9951	1395	9927	2093	9903	2790	9878	3488	86 0
15	9973	0741	9945	1482	9918	2223	9890	2964	9863	3705	45
30	9969	0785	9938	1569	9908	2354	9877	3138	9846	3923	30
45	9966	0828	9931	1656	9897	2484	9863	3312	9828	4140	15
5 0	9962	0872	9924	1743	9886	2615	9848	3486	9810	4358	85 0
15	0.9958	0.0915	1.9916	0.1830	2.9874	0.2745	3.9832	0.3660	4.9790	0.4575	45
30	9954	0958	9908	1917	9862	2875	9816	3834	9770	4792	30
45	9950	1002	9899	2004	9849	3006	9799	4008	9748	5009	15
6 0	9945	1045	9890	2091	9836	3136	9781	4181	9726	5226	84 0
15	9941	1089	9881	2177	9822	3266	9762	4355	9703	5443	45
30	9936	1132	9871	2264	9807	3396	9743	4528	9679	5660	30
45	9931	1175	9861	2351	9792	3526	9723	4701	9653	5877	15
7 0	9925	1219	9851	2437	9776	3656	9702	4875	9627	6093	83 0
15	9920	1262	9840	2524	9760	3786	9680	5048	9600	6310	45
30	9914	1305	9829	2611	9743	3916	9658	5221	9572	6526	30
45	0.9909	0.1349	1.9817	0.2697	2.9726	0.4046	3.9635	0.5394	4.9543	0.6743	15
8 0	9903	1392	9805	2783	9708	4175	9611	5567	9513	6959	82 0
15	9897	1435	9793	2870	9690	4305	9586	5740	9483	7175	45
30	9890	1478	9780	2956	9670	4434	9561	5912	9451	7390	30
45	9884	1521	9767	3042	9651	4564	9534	6085	9418	7606	15
9 0	9877	1564	9754	3129	9631	4693	9508	6257	9384	7822	81 0
15	9870	1607	9740	3215	9610	4822	9480	6430	9350	8037	45
30	9863	1650	9726	3301	9589	4951	9451	6602	9314	8252	30
45	9856	1693	9711	3387	9567	5080	9422	6774	9278	8467	15
10 0	9848	1736	9696	3473	9544	5209	9392	6946	9240	8682	80 0
15	0.9840	0.1779	1.9681	0.3559	2.9521	0.5338	3.9362	0.7118	4.9202	0.8897	45
30	9833	1822	9665	3645	9498	5467	9330	7289	9163	9112	30
45	9825	1865	9649	3730	9474	5596	9298	7461	9123	9326	15
11 0	9816	1908	9633	3816	9449	5724	9265	7632	9081	9540	79 0
15	9808	1951	9616	3902	9424	5853	9231	7804	9039	9755	45
30	9799	1994	9598	3987	9398	5981	9197	7975	8996	9968	30
45	9790	2036	9581	4073	9371	6109	9162	8146	8952	1.0182	15
12 0	9781	2079	9563	4158	9344	6237	9126	8316	8907	0396	78 0
15	9772	2122	9545	4244	9317	6365	9089	8487	8862	0609	45
30	9763	2164	9526	4329	9289	6493	9052	8658	8815	0822	30
45	0.9753	0.2207	1.9507	0.4414	2.9260	0.6621	3.9014	0.8828	4.8767	1.1035	15
13 0	9744	2250	9487	4499	9231	6749	8975	8998	8719	1248	77 0
15	9734	2292	9468	4584	9201	6876	8935	9168	8669	1460	45
30	9724	2334	9447	4669	9171	7003	8895	9338	8618	1672	30
45	9713	2377	9427	4754	9140	7131	8854	9507	8567	1884	15
14 0	9703	2419	9406	4838	9109	7258	8812	9677	8515	2096	76 0
15	9692	2462	9385	4923	9077	7385	8769	9846	8462	2308	45
30	9681	2504	9363	5008	9044	7511	8726	1.0015	8407	2519	30
45	9670	2546	9341	5092	9011	7638	8682	0184	8352	2730	15
15 0	9659	2588	9319	5176	8978	7765	8637	0353	8296	2941	75 0
° '	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° '
B'ng	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		B'ng

B'ng	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		B'ng
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
0 15	5.9999	0.0262	6.9999	0.0305	7.9999	0.0349	8.9999	0.0393	9.9999	0.0436	89 45
30	9998	0524	9997	0611	9997	0698	9997	0785	9996	0873	30
45	9995	0785	9994	0916	9993	1047	9992	1178	9991	1309	15
1 0	9991	1047	9989	1222	9988	1396	9986	1571	9985	1745	89 0
15	9986	1309	9983	1527	9981	1745	9979	1963	9976	2181	45
30	9979	1571	9976	1832	9973	2094	9969	2356	9966	2618	30
45	9972	1832	9967	2138	9963	2443	9958	2748	9953	3054	15
2 0	9963	2094	9957	2443	9951	2792	9945	3141	9939	3490	88 0
15	9954	2356	9946	2748	9938	3141	9931	3533	9923	3926	45
30	9943	2617	9933	3053	9924	3490	9914	3926	9905	4362	30
45	5.9931	0.2879	6.9919	0.3358	7.9908	0.3838	8.9896	0.4318	9.9885	0.4798	15
3 0	9918	3140	9904	3664	9890	4187	9877	4710	9863	5234	87 0
15	9904	3402	9887	3968	9871	4535	9855	5102	9839	5669	45
30	9888	3663	9869	4273	9851	4884	9832	5494	9813	6105	30
45	9872	3924	9850	4578	9829	5232	9807	5886	9786	6540	15
4 0	9854	4185	9829	4883	9805	5581	9781	6278	9756	6976	86 0
15	9835	4447	9808	5188	9780	5929	9753	6670	9725	7411	45
30	9815	4708	9784	5492	9753	6277	9723	7061	9692	7846	30
45	9794	4968	9760	5797	9725	6625	9691	7453	9657	8281	15
5 0	9772	5229	9734	6101	9696	6972	9658	7844	9619	8716	85 0
15	5.9748	0.5490	6.9706	0.6405	7.9664	0.7320	8.9622	0 8235	9.9580	0.9150	45
30	9724	5751	9678	6709	9632	7668	9586	8626	9540	9585	30
45	9698	6011	9648	7013	9597	8015	9547	9017	9497	1.0019	15
6 0	9671	6272	9617	7317	9562	8362	9507	9408	9452	0453	84 0
15	9643	6532	9584	7621	9525	8709	9465	9798	9406	0887	45
30	9614	6792	9550	7924	9486	9056	9421	1.0188	9357	1320	30
45	9584	7052	9515	8228	9445	9403	9376	0578	9307	1754	15
7 0	9553	7312	9478	8531	9404	9750	9329	0968	9255	2187	83 0
15	9520	7572	9440	8834	9360	1.0096	9280	1358	9200	2620	45
30	9487	7832	9401	9137	9316	0442	9230	1747	9144	3053	30
45	5.9452	0.8091	6.9361	0.9440	7.9269	1.0788	8.9178	1.2137	9.9087	1.3485	15
8 0	9416	8350	9319	9742	9221	1134	9124	2526	9027	3917	82 0
15	9379	8610	9276	1.0044	9172	1479	9069	2914	8965	4349	45
30	9341	8869	9231	0347	9121	1825	9011	3303	8902	4781	30
45	9302	9127	9185	0649	9069	2170	8953	3691	8836	5212	15
9 0	9261	9386	9138	0950	9015	2515	8892	4079	8769	5643	81 0
15	9220	9645	9090	1252	8960	2859	8830	4467	8700	6074	45
30	9177	9903	9040	1553	8903	3204	8766	4854	8629	6505	30
45	9133	1.0161	8989	1854	8844	3548	8700	5241	8556	6935	15
10 0	9088	0419	8937	2155	8785	3892	8633	5628	8481	7365	80 0
15	5.9042	1.0677	6.8883	1.2456	7.8723	1.4235	8.8564	1.6015	9.8404	1.7794	45
30	8995	0934	8828	2756	8660	4579	8493	6101	8325	8224	30
45	8947	1191	8772	3057	8596	4922	8421	6787	8245	8652	15
11 0	8898	1449	8714	3357	8530	5265	8346	7173	8163	9081	79 0
15	8847	1705	8655	3656	8463	5607	8271	7558	8079	9509	45
30	8795	1962	8595	3956	8394	5949	8193	7943	7992	9937	30
45	8743	2219	8533	4255	8324	6291	8114	8328	7905	2.0364	15
12 0	8689	2475	8470	4554	8252	6633	8033	8712	7815	0791	78 0
15	8634	2731	8406	4852	8178	6974	7951	9096	7723	1218	45
30	8578	2986	8341	5151	8104	7315	7867	9480	7630	1644	30
45	5.8521	1.3242	6.8274	1.5449	7.8027	1.7656	8.7781	1.9863	9.7534	2.2070	15
13 0	8462	3497	8206	5747	7950	7996	7693	2.0246	7437	2495	77 0
15	8403	3752	8137	6044	7870	8336	7604	0628	7338	2920	45
30	8342	4007	8066	6341	7790	8676	7513	1010	7237	3345	30
45	8281	4261	7994	6638	7707	9015	7421	1392	7134	3769	15
14 0	8218	4515	7921	6935	7624	9354	7327	1773	7030	4192	76 0
15	8154	4769	7846	7231	7538	9692	7231	2154	6923	4615	45
30	8089	5023	7770	7527	7452	2.0030	7133	2534	6815	5038	30
45	8023	5276	7693	7822	7364	0368	7034	2914	6705	5460	15
15 0	7956	5529	7615	8117	7274	0706	6933	3294	6593	5882	75 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° ′
B'ng	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		B'ng

B'ng	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		B'ng
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
15 15	0.9648	0.2630	1.9296	0.5261	2.8944	0.7891	3.8591	1.0521	4.8239	1.3152	74 45
30	9636	2672	9273	5345	8909	8017	8545	0690	8182	3362	30
45	9625	2714	9249	5429	8874	8143	8498	0858	8123	3572	15
16 0	9613	2756	9225	5513	8838	8269	8450	1025	8063	3782	74 0
15	9600	2798	9201	5597	8801	8395	8402	1193	8002	3991	45
30	9588	2840	9176	5680	8765	8520	8353	1361	7941	4201	30
45	9576	2882	9151	5764	8727	8646	8303	1528	7879	4410	15
17 0	9563	2924	9126	5847	8689	8771	8252	1695	7815	4619	73 0
15	9550	2965	9100	5931	8651	8896	8201	1862	7751	4827	45
30	9537	3007	9074	6014	8612	9021	8149	2028	7686	5035	30
45	0.9524	0.3049	1.9048	0.6097	2.8572	0.9146	3.8096	1.2195	4.7620	1.5243	15
18 0	9511	3090	9021	6180	8532	9271	8042	2361	7553	5451	72 0
15	9497	3132	8994	6263	8491	9395	7988	2527	7485	5658	45
30	9483	3173	8966	6346	8450	9519	7933	2692	7416	5865	30
45	9469	3214	8939	6429	8408	9643	7877	2858	7347	6072	15
19 0	9455	3256	8910	6511	8366	9767	7821	3023	7276	6278	71 0
15	9441	3297	8882	6594	8323	9891	7764	3188	7204	6485	45
30	9426	3338	8853	6676	8279	1.0014	7706	3352	7132	6690	30
45	9412	3379	8824	6758	8235	0138	7647	3517	7059	6896	15
20 0	9397	3420	8794	6840	8191	0261	7588	3681	6985	7101	70 0
15	0.9382	0.3461	1.8764	0.6922	2.8146	1.0384	3.7528	1.3845	4.6910	1.7306	45
30	9367	3502	8733	7004	8100	0506	7467	4008	6834	7510	30
45	9351	3543	8703	7086	8054	0629	7405	4172	6757	7715	15
21 0	9336	3584	8672	7167	8007	0751	7343	4335	6679	7918	69 0
15	9320	3624	8640	7249	7960	0873	7280	4498	6600	8122	45
30	9304	3665	8608	7330	7913	0995	7217	4660	6521	8325	30
45	9288	3706	8576	7411	7864	1117	7152	4822	6440	8528	15
22 0	9272	3746	8544	7492	7816	1238	7087	4984	6359	8730	68 0
15	9255	3786	8511	7573	7766	1359	7022	5146	6277	8932	45
30	9239	3827	8478	7654	7716	1481	6955	5307	6194	9134	30
45	0.9222	0.3867	1.8444	0.7734	2.7666	1.1601	3.6888	1.5468	4.6110	1.9336	15
23 0	9205	3907	8410	7815	7615	1722	6820	5629	6025	9537	67 0
15	9188	3947	8376	7895	7564	1842	6752	5790	5940	9737	45
30	9171	3987	8341	7975	7512	1962	6682	5950	5853	9937	30
45	9153	4027	8306	8055	7459	2082	6612	6110	5766	2.0137	15
24 0	9135	4067	8271	8135	7406	2202	6542	6269	5677	0337	66 0
15	9118	4107	8235	8214	7353	2322	6470	6429	5588	0536	45
30	9100	4147	8199	8294	7299	2441	6398	6588	5498	0735	30
45	9081	4187	8163	8373	7244	2560	6326	6746	5407	0933	15
25 0	9063	4226	8126	8452	7189	2679	6252	6905	5315	1131	65 0
15	0.9045	0.4266	1.8089	0.8531	2.7134	1.2797	3.6178	1.7063	4.5223	2.1328	45
30	9026	4305	8052	8610	7078	2915	6103	7220	5129	1526	30
45	9007	4344	8014	8689	7021	3033	6028	7378	5035	1722	15
26 0	8988	4384	7976	8767	6964	3151	5952	7535	4940	1919	64 0
15	8969	4423	7937	8846	6906	3269	5875	7692	4844	2114	45
30	8949	4462	7899	8924	6848	3386	5797	7848	4747	2310	30
45	8930	4501	7860	9002	6789	3503	5719	8004	4649	2505	15
27 0	8910	4540	7820	9080	6730	3620	5640	8160	4550	2700	63 0
15	8890	4579	7780	9157	6671	3736	5561	8315	4451	2894	45
30	8870	4617	7740	9235	6610	3852	5480	8470	4351	3087	30
45	0.8850	0.4656	1.7700	0.9312	2.6550	1.3968	3.5400	1.8625	4.4249	2.3281	15
28 0	8829	4695	7659	9389	6488	4084	5318	8779	4147	3474	62 0
15	8809	4733	7618	9466	6427	4200	5236	8933	4045	3666	45
30	8788	4772	7576	9543	6365	4315	5153	9086	3941	3858	30
45	8767	4810	7535	9620	6302	4430	5069	9240	3836	4049	15
29 0	8746	4848	7492	9696	6239	4544	4985	9392	3731	4240	61 0
15	8725	4886	7450	9772	6175	4659	4900	9545	3625	4431	45
30	8704	4924	7407	9848	6111	4773	4814	9697	3518	4621	30
45	8682	4962	7364	9924	6046	4886	4728	9849	3410	4811	15
30 0	8660	5000	7321	1.0000	5981	5000	4641	2.0000	3301	5000	60 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	′ °
B'ng	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		B'ng

B'ng	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		B'ng
° '	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° '
15 15	5.7887	1.5782	6.7535	1.8412	7.7183	2.1042	8.6831	2.3673	9.6479	2.6303	74 45
30	7818	6034	7454	8707	7090	1379	6727	4051	6363	6724	30
45	7747	6286	7372	9001	6996	1715	6621	4430	6246	7144	15
16 0	7676	6538	7288	9295	6901	2051	6514	4807	6126	7564	74 0
15	7603	6790	7203	9588	6804	2386	6404	5185	6005	7983	45
30	7529	7041	7117	9881	6706	2721	6294	5561	5882	8402	30
45	7454	7292	7030	2.0174	6606	3056	6181	5938	5757	8820	15
17 0	7378	7542	6941	0466	6504	3390	6067	6313	5630	9237	73 0
15	7301	7792	6851	0758	6402	3723	5952	6689	5502	9654	45
30	7223	8042	6760	1049	6297	4056	5835	7064	5372	3.0071	30
45	5.7144	1.8292	6.6668	2.1341	7.6192	2.4389	8.5716	2.7438	9.5240	3.0486	15
18 0	7063	8541	6574	1631	6085	4721	5595	7812	5106	0902	72 0
15	6982	8790	6479	1921	5976	5053	5473	8185	4970	1316	45
30	6899	9038	6383	2211	5866	5384	5349	8557	4832	1730	30
45	6816	9286	6285	2501	5754	5715	5224	8930	4693	2144	15
19 0	6731	9534	6186	2790	5641	6045	5097	9301	4552	2557	71 0
15	6645	9781	6086	3078	5527	6375	4968	9672	4409	2969	45
30	6558	2.0028	5985	3366	5411	6705	4838	3.0043	4264	3381	30
45	6471	0275	5882	3654	5294	7033	4706	0413	4118	3792	15
20 0	6382	0521	5778	3941	5175	7362	4572	0782	3969	4202	70 0
15	5.6291	2.0767	6.5673	2.4228	7.5055	2.7689	8.4437	3.1151	9.3819	3.4612	45
30	6200	1012	5567	4515	4934	8017	4300	1519	3667	5021	30
45	6108	1257	5459	4800	4811	8343	4162	1886	3514	5429	15
21 0	6015	1502	5351	5086	4686	8669	4022	2253	3358	5837	69 0
15	5920	1746	5241	5371	4561	8995	3881	2619	3201	6244	45
30	5825	1990	5129	5655	4433	9320	3738	2985	3042	6650	30
45	5729	2233	5017	5939	4305	9645	3593	3350	2881	7056	15
22 0	5631	2476	4903	6222	4175	9969	3447	3715	2718	7461	68 0
15	5532	2719	4788	6505	4043	3.0292	3299	4078	2554	7865	45
30	5433	2961	4672	6788	3910	0615	3149	4442	2388	8268	30
45	5.5332	2.3203	6.4554	2.7070	7.3776	3.0937	8.2998	3.4804	9.2220	3.8671	15
23 0	5230	3444	4435	7351	3640	1258	2845	5166	2050	9073	67 0
15	5127	3685	4315	7632	3503	1580	2691	5527	1879	9474	45
30	5024	3925	4194	7912	3365	1900	2535	5887	1706	9875	30
45	4919	4165	4072	8192	3225	2220	2378	6247	1531	4.0275	15
24 0	4813	4404	3948	8472	3084	2539	2219	6606	1355	0674	66 0
15	4706	4643	3823	8750	2941	2858	2059	6965	1176	1072	45
30	4598	4882	3697	9029	2797	3175	1897	7322	0996	1469	30
45	4489	5120	3570	9306	2651	3493	1733	7679	0814	1866	15
25 0	4378	5357	3442	9583	2505	3809	1568	8036	0631	2262	65 0
15	5.4267	2.5594	6.3312	2.9860	7.2356	3.4125	8.1401	3.8391	9.0446	4.2657	45
30	4155	5831	3181	3.0136	2207	4441	1233	8746	0259	3051	30
45	4042	6067	3049	0411	2056	4756	1063	9100	0070	3445	15
26 0	3928	6302	2916	0686	1904	5070	0891	9453	8.9879	3837	64 0
15	3812	6537	2781	0960	1750	5383	0719	9806	9687	4229	45
30	3696	6772	2645	1234	1595	5696	0544	4.0158	9493	4620	30
45	3579	7006	2509	1507	1438	6008	0368	0509	9298	5010	15
27 0	3460	7239	2370	1779	1281	6319	0191	0859	9101	5399	63 0
15	3341	7472	2231	2051	1121	6630	0012	1209	8902	5787	45
30	3221	7705	2091	2322	0961	6940	7.9831	1557	8701	6175	30
45	5.3099	2.7937	6.1949	3.2593	7.0799	3.7249	7.9649	4.1905	8.8499	4.6561	15
28 0	2977	8168	1806	2863	0636	7558	9465	2252	8295	6947	62 0
15	2853	8399	1662	3132	0471	7866	9280	2599	8089	7332	45
30	2729	8630	1517	3401	0305	8173	9094	2944	7882	7716	30
45	2604	8859	1371	3669	0138	8479	8905	3289	7673	8099	15
29 0	2477	9089	1223	3937	6.9970	8785	8716	3633	7462	8481	61 0
15	2350	9317	1075	4203	9800	9090	8525	3976	7250	8862	45
30	2221	9545	0925	4470	9628	9394	8332	4318	7036	9242	30
45	2092	9773	0774	4735	9456	9697	8138	4659	6820	9622	15
30 0	1962	3.0000	0622	5000	9282	4.0000	7942	5000	6603	5.0000	60 0
° '	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep	Lat.	Dep.	Lat.	° '
B'ng	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist.10.		B'ng

B'ng		Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		B'ng	
°	′	Lat	Dep	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	°	′
30	15	0.8638	0.5038	1.7277	1.0075	2.5915	1.5113	3.4553	2.0151	4.3192	2.5189	59	45
	30	8616	5075	7233	0151	5849	5226	4465	0302	3081	5377		30
	45	8594	5113	7188	0226	5782	5339	4376	0452	2970	5565		15
31	0	8572	5150	7142	0301	5715	5451	4287	0602	2858	5752	59	0
	15	8549	5188	7098	0375	5647	5563	4196	0751	2746	5939		45
	30	8526	5225	7053	0450	5579	5675	4106	0900	2632	6125		30
	45	8504	5262	7007	0524	5511	5786	4014	1049	2518	6311		15
32	0	8480	5299	6961	0598	5441	5898	3922	1197	2402	6496	58	0
	15	8457	5336	6915	0672	5372	6008	3829	1345	2286	6681		45
	30	8434	5373	6868	0746	5302	6119	3736	1492	2170	6865		30
	45	0.8410	0.5410	1.6821	1.0819	2.5231	1.6229	3.3642	2.1639	4.2052	2.7049		15
33	0	8387	5446	6773	0893	5160	6339	3547	1786	1934	7232	57	0
	15	8363	5483	6726	0966	5089	6449	3451	1932	1814	7415		45
	30	8339	5519	6678	1039	5017	6558	3355	2077	1694	7597		30
	45	8315	5556	6629	1111	4944	6667	3259	2223	1573	7779		15
34	0	8290	5592	6581	1184	4871	6776	3162	2368	1452	7960	56	0
	15	8266	5628	6532	1256	4798	6884	3064	2512	1329	8140		45
	30	8241	5664	6483	1328	4724	6992	2965	2656	1206	8320		30
	45	8216	5700	6433	1400	4649	7100	2866	2800	1082	8500		15
35	0	8192	5736	6383	1472	4575	7207	2766	2943	0958	8679	55	0
	15	0.8166	0.5771	1.6333	1.1543	2.4499	1.7314	3.2666	2.3086	4.0832	2.8857		45
	30	8141	5807	6282	1614	4423	7421	2565	3228	0706	9035		30
	45	8116	5842	6231	1685	4347	7527	2463	3370	0579	9212		15
36	0	8090	5878	6180	1756	4271	7634	2361	3511	0451	9389	54	0
	15	8064	5913	6129	1826	4193	7739	2258	3652	0322	9565		45
	30	8039	5948	6077	1896	4116	7845	2154	3793	0193	9741		30
	45	8013	5983	6025	1966	4038	7950	2050	3933	0063	9916		15
37	0	7986	6018	5973	2036	3959	8054	1945	4073	3.9932	3.0091	53	0
	15	7960	6053	5920	2106	3880	8159	1840	4212	9800	0265		45
	30	7934	6088	5867	2175	3801	8263	1734	4350	9668	0438		30
	45	0.7907	0.6122	1.5814	1.2244	2.3721	1.8367	3.1628	2.4489	3.9534	3.0611		15
38	0	7880	6157	5760	2313	3640	8470	1520	4626	9400	0783	52	0
	15	7853	6191	5706	2382	3560	8573	1413	4764	9266	0955		45
	30	7826	6225	5652	2450	3478	8675	1304	4901	9130	1126		30
	45	7799	6259	5598	2518	3397	8778	1195	5037	8994	1296		15
39	0	7771	6293	5543	2586	3314	8880	1086	5173	8857	1466	51	0
	15	7744	6327	5488	2654	3232	8981	0976	5308	8720	1635		45
	30	7716	6361	5432	2722	3149	9082	0865	5443	8581	1804		30
	45	7688	6394	5377	2789	3065	9183	0754	5578	8442	1972		15
40	0	7660	6428	5321	2856	2981	9284	0642	5712	8302	2139	50	0
	15	0.7632	0.6461	1.5265	1.2922	2.2897	1.9384	3.0529	2.5845	3.8162	3.2306		45
	30	7604	6494	5208	2989	2812	9483	0416	5978	8020	2472		30
	45	7576	6528	5151	3055	2727	9583	0303	6110	7878	2638		15
41	0	7547	6561	5094	3121	2641	9682	0188	6242	7735	2803	49	0
	15	7518	6593	5037	3187	2555	9780	0074	6374	7592	2967		45
	30	7490	6626	4979	3252	2469	9879	2.9958	6505	7448	3131		30
	45	7461	6659	4921	3318	2382	9976	9842	6635	7303	3294		15
42	0	7431	6691	4863	3383	2294	2.0074	9726	6765	7157	3457	48	0
	15	7402	6724	4804	3447	2207	0171	9609	6895	7011	3618		45
	30	7373	6756	4746	3512	2118	0268	9491	7024	6864	3780		30
	45	0.7343	0.6788	1.4686	1.3576	2.2030	2.0364	2.9373	2.7152	3.6716	3.3940		15
43	0	7314	6820	4627	3640	1941	0460	9254	7280	6568	4100	47	0
	15	7284	6852	4567	3704	1851	0555	9135	7407	6419	4259		45
	30	7254	6884	4507	3767	1761	0651	9015	7534	6269	4418		30
	45	7224	6915	4447	3830	1671	0745	8895	7661	6118	4576		15
44	0	7193	6947	4387	3893	1580	0840	8774	7786	5967	4733	46	0
	15	7163	6978	4326	3956	1489	0934	8652	7912	5815	4890		45
	30	7133	7009	4265	4018	1398	1027	8530	8036	5663	5045		30
	45	7102	7040	4204	4080	1306	1120	8407	8161	5509	5201		15
45	0	7071	7071	4142	4142	1213	1213	8284	8284	5355	5355	45	0
°	′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	°	′
B'ng		Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		B'ng	

B'ng	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		B'ng
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
30 15	5.1830	3.0226	6.0468	3.5264	6.9107	4.0302	7.7745	4.5340	8.6384	5.0377	59 45
30	1698	0452	0314	5528	8930	0603	7547	5678	6163	0754	30
45	1564	0678	0158	5791	8753	0903	7347	6016	5941	1129	15
31 0	1430	0902	0002	6053	8573	1203	7145	6353	5717	1504	59 0
15	1295	1126	5.9844	6314	8393	1502	6942	6690	5491	1877	45
30	1158	1350	9685	6575	8211	1800	6738	7025	5264	2250	30
45	1021	1573	9525	6835	8028	2097	6532	7359	5035	2621	15
32 0	0883	1795	9363	7094	7844	2394	6324	7693	4805	2992	58 0
15	0744	2017	9201	7353	7658	2689	6116	8025	4573	3361	45
30	0603	2238	9037	7611	7471	2984	5905	8357	4339	3730	30
45	5.0462	3.2458	5.8873	3.7868	6.7283	4.3278	7.5694	4.8688	8.4104	5.4097	15
33 0	0320	2678	8707	8125	7094	3571	5480	9018	3867	4464	57 0
15	0177	2898	8540	8381	6903	3863	5266	9346	3629	4829	45
30	0033	3116	8372	8636	6711	4155	5050	9674	3389	5194	30
45	4.9888	3334	8203	8890	6518	4446	4832	5.0001	3147	5557	15
34 0	9742	3552	8033	9144	6323	4735	4613	0327	2904	5919	56 0
15	9595	3768	7861	9396	6127	5024	4393	0652	2659	6280	45
30	9448	3984	7689	9648	5930	5312	4171	0977	2413	6641	30
45	9299	4200	7515	9900	5732	5600	3948	1300	2165	7000	15
35 0	9149	4415	7341	4.0150	5532	5886	3724	1622	1915	7358	55 0
15	4.8998	3.4629	5.7165	4.0400	6.5331	4.6172	7.3498	5.1943	8.1664	5.7715	45
30	8847	4842	6988	0649	5129	6456	3270	2263	1412	8070	30
45	8694	5055	6810	0897	4926	6740	3042	2582	1157	8425	15
36 0	8541	5267	6631	1145	4721	7023	2812	2901	0902	8779	54 0
15	8387	5479	6451	1392	4516	7305	2580	3218	0644	9131	45
30	8231	5689	6270	1638	4309	7586	2347	3534	0386	9482	30
45	8075	5899	6088	1883	4100	7866	2113	3849	0125	9832	15
37 0	7918	6109	5904	2127	3891	8145	1877	4163	7.9864	6.0182	53 0
15	7760	6318	5720	2371	3680	8424	1640	4476	9600	0529	45
30	7601	6526	5535	2613	3468	8701	1402	4789	9335	0876	30
45	4.7441	3.6733	5.5348	4.2855	6.3255	4.8977	7.1162	5.5100	7.9069	6.1222	15
38 0	7281	6940	5161	3096	3041	9253	0921	5410	8801	1566	52 0
15	7119	7146	4972	3337	2825	9528	0679	5718	8532	1909	45
30	6956	7351	4783	3576	2609	9801	0435	6026	8261	2251	30
45	6793	7555	4592	3815	2391	5.0074	0190	6333	7988	2592	15
39 0	6629	7759	4400	4052	2172	0346	6.9943	6639	7715	2932	51 0
15	6464	7962	4207	4289	1951	0616	9695	6943	7439	3271	45
30	6297	8165	4014	4525	1730	0886	9446	7247	7162	3608	30
45	6131	8366	3819	4761	1507	1155	9196	7550	6884	3944	15
40 0	5963	8567	3623	4995	1284	1423	8944	7851	6604	4279	50 0
15	4.5794	3.8767	5.3426	4.5229	6.1059	5.1690	6.8691	5.8151	7.6323	6.4612	45
30	5624	8967	3228	5461	0832	1956	8437	8450	6041	4945	30
45	5454	9166	3030	5693	0605	2221	8181	8748	5756	5276	15
41 0	5283	9364	2830	5924	0377	2485	7924	9045	5471	5606	49 0
15	5110	9561	2629	6154	0147	2748	7666	9341	5184	5935	45
30	4937	9757	2427	6383	5.9916	3010	7406	9636	4896	6262	30
45	4763	9953	2224	6612	9685	3271	7145	9929	4606	6588	15
42 0	4589	4.0148	2020	6839	9452	3530	6883	6.0222	4314	6913	48 0
15	4413	0342	1815	7066	9217	3789	6620	0513	4022	7237	45
30	4237	0535	1609	7291	8982	4047	6355	0803	3728	7559	30
45	4.4059	4.0728	5.1403	4.7516	5.8746	5.4304	6.6089	6.1092	7.3432	6.7880	15
43 0	3881	0920	1195	7740	8508	4560	5822	1380	3135	8200	47 0
15	3702	1111	0986	7963	8270	4815	5553	1666	2837	8518	45
30	3522	1301	0776	8185	8030	5068	5284	1952	2537	8835	30
45	3342	1491	0565	8406	7789	5321	5013	2236	2236	9151	15
44 0	3160	1680	0354	8626	7547	5573	4741	2519	1934	9466	46 0
15	2978	1867	0141	8845	7304	5823	4467	2801	1630	9779	45
30	2795	2055	4.9928	9064	7060	6073	4193	3082	1325	7.0091	30
45	2611	2241	9713	9281	6815	6321	3917	3361	1019	0401	15
45 0	2426	2426	9497	9497	6569	6569	3640	3640	0711	0711	45 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° ′
B'ng	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		B'ng

For Correction of Courses on Random Lines.

Minutes.	10 Chains.	20 Chains.	40 Chains.	80 Chains.	Minutes.
1	.003	.006	.012	.023	1
2	006	012	023	046	2
3	009	017	035	070	3
4	012	023	046	093	4
5	014	029	058	116	5
6	017	035	070	140	6
7	020	041	081	163	7
8	023	046	093	186	8
9	026	052	105	209	9
10	029	058	116	233	10
11	032	064	128	256	11
12	035	070	140	279	12
13	038	076	151	302	13
14	041	081	163	326	14
15	044	087	174	349	15
16	046	093	186	372	16
17	049	099	198	396	17
18	052	105	209	419	18
19	055	110	221	442	19
20	058	116	233	466	20
21	061	122	244	488	21
22	064	128	256	512	22
23	067	134	268	535	23
24	070	140	279	558	24
25	073	145	291	581	25
26	076	151	302	605	26
27	078	157	314	628	27
28	081	163	326	651	28
29	084	169	337	674	29
30	087	174	349	698	30
31	090	180	361	722	31
32	093	186	372	744	32
33	096	192	384	767	33
34	099	198	395	790	34
35	102	204	407	814	35
36	105	209	419	837	36
37	108	215	430	860	37
38	110	221	442	883	38
39	113	227	454	906	39
40	116	233	465	929	40
41	119	238	477	953	41
42	122	244	488	976	42
43	125	250	500	999	43
44	128	256	512	1.022	44
45	131	262	523	1.045	45
46	134	268	535	1.068	46
47	137	273	546	1.092	47
48	140	279	558	1.115	48
49	142	285	570	1.138	49
50	145	291	581	1.161	50
51	148	297	593	1.184	51
52	151	302	605	1.207	52
53	154	308	616	1.230	53
54	157	314	628	1.253	54
55	160	320	639	1.276	55
56	163	326	651	1.299	56
57	166	331	663	1.323	57
58	169	337	674	1.346	58
59	172	343	686	1.369	59
60	174	349	698	1.392	60

1° - - - - - - 11°			11° - - - - - - 21°			21° - - - - - - 31°			31° - - - - - - 41°		
Angle.		Secant.	Angle.		Secant.	Angle.		Secant.	Angle.		Secant.
°	′		°	′		°	′		°		
1		1.00015	11		1.01872	21		1.07115	31		1.16663
	10	1.00021		10	1.01930		10	1.07235		10	1.16868
	20	1.00027		20	1.01989		20	1.07356		20	1.17075
	30	1.00034		30	1.02049		30	1.07479		30	1.17283
	40	1.00042		40	1.02110		40	1.07602		40	1.17493
	50	1.00051		50	1.02171		50	1.07727		50	1.17704
2		1.00061	12		1.02234	22		1.07853	32		1.17918
	10	1.00072		10	1.02298		10	1.07981		10	1.18133
	20	1.00083		20	1.02362		20	1.08109		20	1.18350
	30	1.00095		30	1.02428		30	1.08239		30	1.18569
	40	1.00108		40	1.02494		40	1.08370		40	1.18790
	50	1.00122		50	1.02562		50	1.08503		50	1.19012
3		1.00137	13		1.02630	23		1.08636	33		1.19236
	10	1.00153		10	1.02700		10	1.08771		10	1.19463
	20	1.00169		20	1.02770		20	1.08907		20	1.19691
	30	1.00187		30	1.02842		30	1.09044		30	1.19920
	40	1.00205		40	1.02914		40	1.09183		40	1.20152
	50	1.00224		50	1.02987		50	1.09323		50	1.20386
4		1.00244	14		1.03061	24		1.09464	34		1.20622
	10	1.00265		10	1.03137		10	1.09606		10	1.20859
	20	1.00287		20	1.03213		20	1.09750		20	1.21099
	30	1.00309		30	1.03290		30	1.09895		30	1.21341
	40	1.00333		40	1.03368		40	1.10041		40	1.21584
	50	1.00357		50	1.03447		50	1.10189		50	1.21830
5		1.00382	15		1.03528	25		1.10338	35		1.22070
	10	1.00408		10	1.03609		10	1.10488		10	1.22327
	20	1.00435		20	1.03691		20	1.10640		20	1.22579
	30	1.00463		30	1.03774		30	1.10793		30	1.22833
	40	1.00491		40	1.03858		40	1.10947		40	1.23089
	50	1.00521		50	1.03944		50	1.11103		50	1.23347
6		1.00551	16		1.04030	26		1.11260	36		1.23607
	10	1.00582		10	1.04117		10	1.11419		10	1.23869
	20	1.00614		20	1.04206		20	1.11579		20	1.24134
	30	1.00647		30	1.04295		30	1.11740		30	1.24400
	40	1.00681		40	1.04385		40	1.11903		40	1.24669
	50	1.00715		50	1.04477		50	1.12067		50	1.24940
7		1.00751	17		1.04569	27		1.12233	37		1.25214
	10	1.00787		10	1.04663		10	1.12400		10	1.25489
	20	1.00825		20	1.04757		20	1.12568		20	1.25767
	30	1.00863		30	1.04853		30	1.12738		30	1 26047
	40	1.00902		40	1.04950		40	1.12910		40	1 26330
	50	1.00942		50	1.05047		50	1.13083		50	1.26615
8		1.00983	18		1.05146	28		1.13257	38		1.26902
	10	1.01024		10	1.05246		10	1.13433		10	1.27191
	20	1.01067		20	1.05347		20	1.13610		20	1.27483
	30	1.01111		30	1.05449		30	1.13789		30	1.27778
	40	1.01155		40	1.05552		40	1.13970		40	1.28075
	50	1.01200		50	1.05657		50	1.14152		50	1.28374
9		1.01247	19		1.05762	29		1.14335	39		1.28676
	10	1.01294		10	1.05869		10	1.14521		10	1.28980
	20	1.01342		20	1.05976		20	1.14707		20	1.29287
	30	1.01391		30	1.06085		30	1.14896		30	1.29597
	40	1.01440		40	1.06195		40	1.15085		40	1.29909
	50	1.01491		50	1.06306		50	1.15277		50	1.30223
10		1.01543	20		1.06418	30		1.15470	40		1.30541
	10	1.01595		10	1.06531		10	1.15665		10	1.30831
	20	1.01649		20	1.06645		20	1.15861		20	1.31183
	30	1.01703		30	1.06761		30	1.16059		30	1.31509
	40	1.01758		40	1.06878		40	1.16259		40	1.31837
	50	1.01815		50	1.06995		50	1.16460		50	1.32168

41° - - - - 46°		46° - - - - 51°		51° - - - - 56°		56° - - - - 61°	
Angle.	Secant.	Angle.	Secant.	Angle.	Secant.	Angle.	Secant.
° ′		° ′		° ′		° ′	
41	1.32501	46	1.43956	51	1.58902	56	1.78829
10	1.32838	10	1.44391	10	1.59475	10	1.79604
20	1.33177	20	1.44831	20	1.60054	20	1.80388
30	1.33519	30	1.45274	30	1.60639	30	1.81180
40	1.33864	40	1.45721	40	1.61229	40	1.81981
50	1.34212	50	1.46173	50	1.61825	50	1.82790
42	1.34563	47	1.46628	52	1.62427	57	1.83608
10	1.34917	10	1.47087	10	1.63035	10	1.84435
20	1.35274	20	1.47551	20	1.63648	20	1.85271
30	1.35634	30	1.48019	30	1.64268	30	1.86116
40	1.35997	40	1.48491	40	1.64894	40	1.86990
50	1.36363	50	1.48967	50	1.65526	50	1.87834
43	1.36733	48	1.49448	53	1.66164	58	1.88708
10	1.37105	10	1.49933	10	1.66809	10	1.89591
20	1.37481	20	1.50422	20	1.67460	20	1.90485
30	1.37860	30	1.50916	30	1.68117	30	1.91388
40	1.38242	40	1.51415	40	1.68782	40	1.92302
50	1.38628	50	1.51918	50	1.69452	50	1.93226
44	1.39016	49	1.52425	54	1.70130	59	1.94160
10	1.39409	10	1.52938	10	1.70815	10	1.95106
20	1.39804	20	1.53455	20	1.71506	20	1.96062
30	1.40203	30	1.53977	30	1.72205	30	1.97029
40	1.40606	40	1.54504	40	1.72911	40	1.98008
50	1.41012	50	1.55036	50	1.73624	50	1.98998
45	1.41421	50	1.55572	55	1.74345	60	2.00000
10	1.41835	10	1.56114	10	1.75073	10	2.01014
20	1.42251	20	1.56661	20	1.75808	20	2.02039
30	1.42670	30	1.57213	30	1.76552	30	2.03077
40	1.43096	40	1.57771	40	1.77303	40	2.04128
50	1.43524	50	1.58333	50	1.78062	50	2.05191

JAN. 1. TABLE VIII. JAN. 1.

AZIMUTHS OF POLARIS AT EXTREME ELONGATIONS.

Lat. N.	1896	1897	1898	1899	1900	1901	1902	1903	1904	1905
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
25	1 22.5	1 22.2	1 21 8	1 21.5	1 21.1	1 20.8	1 20.5	1 20.1	1 19.8	1 19.4
26	23.2	22.9	22.5	22.2	21.8	21.5	21.1	20.8	20.4	20.1
27	23.9	23 6	23.2	22.9	22.5	22.2	21.8	21.5	21.1	20 8
28	24.7	24.4	24.0	23.6	23.3	22.9	22.6	22.2	21.9	21.5
29	25.5	25.2	24.8	24.4	24.1	23.7	23.4	23.0	22.6	22.3
30	26.4	26 0	25.6	25.3	24.9	24.6	24.2	23.8	23.5	23 1
31	27.3	26.9	26 5	26.2	25.8	25.4	25.1	24.7	24.3	24.0
32	28.2	27.8	27.5	27.1	26.7	26.4	26.0	25.6	25.2	24 9
33	29.2	28.8	28.4	28.1	27.7	27.3	26.9	26.6	26.2	25 8
34	30.2	29.8	29.5	29.1	28.7	28.3	27.2	27.6	27.2	26.8
35	31.3	30.9	30.5	30.2	29.8	29.4	29.0	28.6	28.2	27.8
36	32.5	32 1	31 7	31.3	30.9	30.5	30.1	29.7	29.3	29.0
37	33.7	33.3	32.9	32.5	32.1	31.7	31.3	30.9	30.5	30.1
38	34.9	34.5	34.1	33.7	33.3	32.9	32.5	32.1	31.7	31.3
39	36.2	35.8	35 4	35.0	34.6	34.2	33.8	33.4	33.0	32.6
40	37.6	37.2	36.8	36.4	36.0	35.6	35.2	34.8	34.4	34.0
41	39.1	38.7	38.3	37.9	37.4	37.0	36.6	36.2	35.8	35.4
42	40.6	40.2	39.8	39.4	39.0	38.5	38.1	37.7	37.3	36.9
43	42.3	41.8	41.4	41.0	40.6	40.1	39.7	39.3	38.8	38.4
44	44.0	43.5	43.1	42.7	42.2	41.8	41.4	40.9	40.5	40.1
45	45.8	45.3	44.9	44.4	44.0	43.6	43.1	42.7	42.2	41.8
46	47.7	47.2	46.8	46.3	45.9	45.4	45.0	44.5	44.0	43.6
47	49.7	49.2	48.8	48.3	47.8	47.4	46.9	46.5	46.0	45.5
48	51.8	51.3	50.8	50.4	49.9	49.4	49.0	48.5	48.0	47.6
49	54.0	53.5	53.1	52.6	52 1	51.6	51.2	50.7	50.2	49.7
50	1 56.4	1 55.9	1 55.4	1 54.9	1 54.4	1 53.9	1 53.4	1 53.0	1 52.5	1 52.0

For one revolution of Gradienter Screw, used in finding d′ *and* d. *Page 99.*

Elevation.	Multipliers of r.		Elevation.	Multipliers of r.		Elevat'n.	Multipliers of r.	
e.	Inc. Dist.	Hor. Dist.	e.	Inc. Dist.	Hor. Dist.	e.	Inc. Dist.	Hor. Dist.
° ′			° ′			° ′		
1 00	99.97	99.95	14	96.79	93.91	22 30	92.01	85.01
2	99.90	99.84	14 30	96.56	93.49	23	91.66	84.37
3	99.81	99.67	15	96.33	93.05	23 30	91.31	83.73
4	99.69	99.44	15 30	96.09	92.59	24	90.95	83.08
5	99.53	99.15	16	95.85	92.13	24 30	90.58	82.42
6	99.35	98.80	16 30	95.60	91.66	25	90.21	81.75
7	99.13	98.39	17	95.34	91.17	25 30	89.83	81.08
8	98.89	97.92	17 30	95.07	90.67	26	89.44	80.39
9	98.61	97.39	18	94.80	90.15	26 30	89.05	79.69
10	98.31	96.81	18 30	94.52	89.63	27	88.65	78.99
10 30	98.14	96.50	19	94.23	89.09	27 30	88.24	78.27
11	97.97	96.17	19 30	93.93	88.54	28	87.83	77.55
11 30	97.79	95.83	20	93.63	87.97	28 30	87.40	76.81
12	97.61	95.47	20 30	93.32	87.41	29	86.98	76.07
12 30	97.41	95.10	21	93.00	86.82	29 30	86.54	75.32
13	97.21	94.72	21 30	92.68	86.23	30	86.10	74.57
13 30	97.00	94.22	22	92.34	85.61	30 30	85.66	73.81

TABLE X. ANGLES OF ELEVATION,

Corresponding to numbers of Revolution of the Gradienter Screw.

Screw.	Angle.	Screw.	Angle.	Screw.	Angle.
Rev. Div.	° ′ ″	Rev. Div.	° ′ ″	Rev. Div.	° ′ ″
0 1	0 00 21	0 10	0 3 26	1 00	0 34 23
2	0 41	20	6 53	2	1 08 45
3	1 02	30	10 19	3	1 43 06
4	1 23	40	13 45	4	2 17 26
5	1 43	50	17 11	5	2 51 45
6	2 04	60	20 38	6	3 26 01
7	2 24	70	24 04	7	4 00 15
8	2 45	80	27 30	8	4 34 26
9	3 06	90	30 56	9	5 08 34
0 10	0 3 26	1 00	0 34 23	10 00	5 42 38

In Declination, for use with Solar Compass.

Hour Angle.	Declinations.								
	For Latitude 30°.								
	+20°	**+15°**	**+10°**	**+5°**	**0°**	**−5°**	**−10°**	**−15°**	**−20°**
0h.	10″	15″	21″	27″	33″	40″	48″	57″	1′08″
2	14	19	25	31	38	46	54	1′05	1 18
3	20	26	32	39	47	55	1′06	1 19	1 36
4	32	39	46	52	1′06	1′19	1 35	1 57	2 29
5	1′00	1′10	1′24	1′52	2 07	2 44	3 46	5 43	13 06
	For Latitude 32° 30′.								
0h.	13″	18″	24″	30″	36″	44″	52″	1′02″	1′14″
2	17	22	28	35	42	50	1′00	1 11	1 26
3	23	29	35	43	51	1′01	1 13	1 28	1 47
4	35	43	51	1′01	1′13	1 27	1 46	2 13	2 54
5	1′03	1′15	1′31	1 53	2 20	3 05	4 25	7 36	
	For Latitude 35°.								
0h.	15″	21″	27″	33″	40″	48″	57″	1′08″	1′21″
2	20	25	32	38	46	55	1′05	1 18	1 35
3	26	33	39	47	56	1′07	1 21	1 38	2 00
4	39	47	56	1′07	1′20	1 36	1 59	2 32	3 25
5	1′07	1′20	1′38	2 00	2 34	3 29	5 14	10 16	
	For Latitude 37° 30′.								
0h.	18″	24″	30″	36″	44″	52″	1′02″	1′14″	1′29″
2	22	28	35	42	50	1′00	1′12	1 26	1 45
3	29	36	43	52	1′02	1 14	1 29	1 49	2 16
4	43	51	1′01	1′13	1 27	1 49	2 14	2 54	4 05
5	1′11	1′26	1 54	2 10	2 49	3 55	6 15	14 58	
	For Latitude 40°.								
0h.	21″	27″	33″	40″	48″	57″	1′08″	1′21″	1′39″
2	25	32	39	46	52	1 06	1 19	1 35	1 57
3	33	40	48	57	1′08	1 21	1 38	2 02	2 36
4	47	55	1′06	1′19	1 36	1 58	2 30	3 21	4 59
5	1′15	1′31	1 51	2 20	3 05	4 25	7 34	25 18	
	For Latitude 42° 30′.								
0h.	24″	30″	36″	44″	52″	1′02″	1′14″	1′29″	1′49″
2	28	35	39	50	1′00	1 12	1 26	1 45	2 11
3	36	43	52	1′02	1 13	1 29	1 49	2 17	2 59
4	50	1′00	1′11	1 26	1 44	2 10	2 49	3 55	6 16
5	1′16	1 36	1 58	2 30	3 22	5 00	9 24		
	For Latitude 45°.								
0h.	27″	33″	40″	48″	57″	1′08″	1′21″	1′39″	2′02″
2	32	39	46	52	1′06	1 19	1 35	1 57	2 29
3	40	47	56	1′07	1 21	1 38	2 00	2 34	3 29
4	54	1′04	1′16	1 33	1 54	2 24	3 11	4 38	8 15
5	1′23	1 41	2 05	2 41	3 40	5 40	12 02		
	For Latitude 47° 30″								
0h.	30″	36″	44″	52″	1′02″	1′14″	1′29″	1′49″	2′18″
2	35	42	50	1′00	1 12	1 26	1 45	2 01	2 51
3	43	51	1′01	1 13	1 28	1 47	2 15	2 56	4 08
4	56	1′09	1 23	1 40	2 05	2 40	3 39	5 37	11 18
5	1′27	1 46	2 12	2 52	4 01	6 30	16 19		

Showing Number of Acres served by drains having bottom widths from 1 ft. to 10 ft., with side slopes of 1 to 1, on the supposition of 1 inch rainfall in 24 hours, one-half of which reaches the drain.

Computed by B. F. WELLES, C. E., *Marshall, Mich.*

Fall in feet per			Bottom Widths.					
			1 ft.		2 ft.		3 ft.	
1 mi.	100 ft.	8 rd.	2 ft. deep.	3 ft deep.	2 ft. deep.	3 ft. deep.	2 ft, deep.	3 ft. deep.
1.6	.030	.04	407	981	594	1311	780	1649
2.0	.038	.05	462	1105	665	1473	879	1861
2.4	.045	.06	508	1218	732	1622	968	2047
2.8	.053	.07	553	1319	797	1762	1053	2217
3.2	.060	.08	592	1416	853	1889	1128	2377
3 6	.070	.09	631	1505	939	2009	1198	2529
4.0	.076	.10	666	1590	959	2115	1264	2665
4.8	.091	.12	733	1748	1057	2333	1391	2935
5.6	.110	.14	794	1895	1143	2523	1499	3172
6.4	.120	.16	852	2030	1225	2700	1612	3401
7 2	.136	.18	905	2154	1300	2869	1715	3612
8.0	.150	.20	956	2273	1373	3031	1809	3815

Fall in Feet per			Bottom Widths.					
			4 ft.		5 ft.		6 ft.	
1 mi.	100 ft.	8 rd.	2 ft. deep.	3 ft. deep.	2 ft deep.	3 ft. deep.	2 ft. deep.	3 ft. deep.
1.6	.030	.04	976	2003	1171	2357	1368	2716
2.0	.038	.05	1094	2249	1316	2650	1541	3046
2.4	.045	.06	1206	2477	1448	2910	1699	3362
2.8	.053	.07	1308	2684	1572	3158	1835	3642
3.2	.060	.08	1404	2872	1684	3384	1970	3908
3.6	.070	.09	1494	3049	1790	3598	2097	4150
4.0	.076	.10	1579	3227	1894	3800	2211	4322
4.8	.091	.12	1731	3553	2089	4173	2436	4810
5.6	.110	.14	1878	3849	2257	4512	2632	5203
6.4	.120	.16	2013	4115	2415	4838	2820	5571
7 2	.136	.18	2137	4372	2566	5141	3001	5927
8.0	.150	.20	2256	4609	2705	5412	3165	6257

Fall in Feet per			Bottom Widths.					
			7 ft.		8 ft.		10 ft.	
1 mi.	100 ft.	8 rd.	2 ft. deep.	3 ft. deep.	2 ft. deep.	3 ft. deep.	2 ft. deep.	3 ft. deep.
1.6	.030	.04	1574	3074	1767	3458	2177	4179
2.0	.038	.05	1768	3469	1983	3877	2448	4710
2.4	.045	.06	1946	3807	2181	4265	2695	5169
2.8	.053	.07	2115	4131	2369	4622	2921	5609
3.2	.060	.08	2259	4427	2538	4948	3136	6014
3.6	.070	.09	2403	4695	2697	5258	3327	6378
4.0	.076	.10	2538	4963	2848	5552	3508	6745
4.8	.091	.12	2792	5443	3130	6094	3857	7405
5.6	.110	.14	3029	5894	3393	6591	4184	8010
6.4	.120	.16	3240	6317	3628	7057	4489	8578
7.2	.136	.18	3143	6715	3854	7507	4760	9110
8.0	.150	.20	3629	7078	4070	7910	5038	9623

FORMULAS: $v = \left\{ \frac{af \times 9000}{p} \right\}^{\frac{1}{2}} - 0.11.$ $Q = av.$ $A = Q \times 47.6 =$ Acreage.

Showing Number of Acres drained by different sizes of tile, the rainfall being considered as equal to one-half inch in depth each 24 hours. Computed by R. C. CARPENTER, *Lansing, Mich.*

Rate of Inclination.	Acres Drained.						
Feet to one of rise.	2-in Tile.	3-in. Tile.	4-in. Tile.	6-in. Tile.	8-in. Tile.	10-in. Tile.	12-in. Tile.
1 foot in 10 feet	6.6	18.9	....	....	....	....	....
1 " 20 "	4.7	13.0	26.8	....	....	....	....
1 " 25 "	4.2	11.4	24.0	66.2	....	....	....
1 " 30 "	3·9	10.9	21.9	61.5	126.4	....	....
1 " 40 "	3.4	9.4	19.0	53.3	109.6	190.5	....
1 " 50 "	3.0	8.4	17.0	47.7	98.0	170.4	269.0
1 " 60 "	2.7	7.6	15.6	43.4	90.0	156.0	246.0
1 " 70 "	2 5	6.9	14.5	39.9	83.0	144.4	228.1
1 " 80 "	2.3	6.5	13.4	37.2	77.0	135.0	213.0
1 " 90 "	2.2	6.1	12.6	35.0	72.5	127.0	200.5
1 " 100 "	2.0	5.7	11.9	33.1	69.2	120.6	190.5
1 " 150 "	1.6	4.5	9.5	26.6	56.0	97.3	154.4
1 " 200 "	....	3.9	8.2	22.8	48.0	83.9	132.5
1 " 250 "	....	3.5	7.5	20.4	43.4	74.4	117.0
1 " 300 "	....	....	6.9	18.4	38 2	65.5	107.0
1 " 400 "	....	....	5.9	16.5	34.6	60.3	90.7
1 " 500 "	....	....	....	14.8	30.1	54.0	81.6
1 " 600 "	....	....	....	13.3	28.0	48.6	74.0
1 " 800 "	....	....	....	....	24.0	41.9	65.0
1 " 1,000 "	....	....	....	....	21.2	37.2	56.0
1 " 1,500 "	....	....	....	....	....	30.8	47.0
1 " 2,000 "	....	....	....	....	....	....	40.8

NOTE.—Tile should not be laid to grades where numbers are replaced by dashes.

TABLE XIV. CAPACITY OF TILE.

Showing carrying capacity of different sizes of tile, in gallons. From Catalogue of the Bennett Sewer Pipe Co., Jackson, Mich.

Carrying Capacity—Gallons per Minute.								
Size of pipes.	1½ in. fall per 100 ft.	3 in. fall per 100 ft.	6 in. fall per 100 ft.	9 in. fall per 100 ft.	1 ft. fall per 100 ft.	1½ ft. fall per 100 ft.	2 ft. fall per 100 ft.	3 ft. fall per 100 ft.
2½ in.	14	20	28	34	40	49	55	68
3 "	21	30	42	52	60	74	85	104
4 "	36	52	76	92	108	132	148	184
5 "	54	78	111	134	159	192	219	269
6 "	84	120	169	206	240	294	338	414
8 "	144	208	304	368	432	528	592	736
9 "	232	330	470	570	660	810	930	1140
10 "	267	378	463	655	803	926	1340	1613
12 "	470	680	960	1160	1360	1670	1920	2350
15 "	830	1180	1680	2040	2370	2920	3340	4100
18 "	1300	1850	2630	3200	3740	4600	5270	6470
20 "	1760	2450	3450	4180	4860	5980	6850	8410
24 "	3000	4152	5871	7202	8303	10021	11743	14466

Latitude.	1 mile.			2 miles.			3 miles.			4 miles.		
°	°	′	″	°	′	″	°	′	″	°	′	″
30	89	59	30	89	58	59.9	89	58	29.9	89	57	59.9
31		59	28.8		58	57.5		58	26.3		57	55.0
32		59	27.5		58	55.0		58	22.5		57	50.0
33		59	26.2		58	52.5		58	18.7		57	44.9
34		59	24.9		58	49.9		58	14.8		57	39.7
35		59	23.6		58	47.2		58	10.8		57	34.4
36		59	22.2		58	44.4		58	06.8		57	28.9
37		59	20.8		58	41.6		58	02.5		57	23.3
38		59	19.4		58	38.8		57	58.2		57	17.5
39		59	17.9		58	35.8		57	53.7		57	11.6
40		59	16.4		58	32.8		57	49.2		57	05.5
41		59	14.8		58	29.6		57	44.4		56	59.3
42		59	13.2		58	26.4		57	39.6		56	52.8
43		59	11.5		58	23.1		57	34.6		56	46.2
44		59	09.8		58	19.6		57	29.5		56	39.3
45		59	08.0		58	16.1		57	24.1		56	32.1
46		59	06.2		58	12.4		57	18.6		56	24.8
47	89	59	04.3	89	58	08.6	89	57	12.9	89	56	17.1

Latitude.	5 miles.			6 miles.			7 miles.			8 miles.		
°	°	′	″	°	′	″	°	′	″	°		″
30	89	57	29.9	89	56	59.8	89	56	29.8	89	55	59.8
31		57	23.8		56	52.5		56	21.3		55	50.0
32		57	17.5		56	45.0		56	12.5		55	40.0
33		57	11.2		56	37.4		56	03.6		55	29.9
34		57	04.6		56	29.6		55	54.5		55	19.4
35		56	58.0		56	21.6		55	45.2		55	08.8
36		56	51.1		56	13.4		55	35.6		54	57.8
37		56	44.1		56	05.0		55	25.8		54	46.6
38		56	36.9		55	56.3		55	15.7		54	35.1
39		56	29.6		55	47.5		55	05.4		54	23 3
40		56	21.9		55	38.3		54	54.7		54	11.1
41		56	14.1		55	28.9		54	43.7		53	58.5
42		56	06.0		55	19.2		54	32.4		53	45.6
43		55	57.7		55	09.2		54	20.8		53	32.3
44		55	49.1		54	58.9		54	08.7		53	18.5
45		55	40.2		54	48.2		53	56.3		53	04.3
46		55	31.0		54	37.2		53	43.4		52	49.5
47	89	55	21.4	89	54	25.7	89	53	30.0	89	52	34.3

Latitude.	9 miles.			10 miles.			11 miles.			12 miles.		
°	°	′	″	°	′	″	°		″	°	′	″
30	89	55	29.8	89	54	59.7	89	54	29.7	89	53	59.7
31		55	18.8		54	47.6		54	16.3		53	45.1
32		55	07.6		54	35.1		54	02.6		53	30.1
33		54	56.1		54	22.3		53	48.5		53	14.8
34		54	44.4		54	09.3		53	34.2		52	59.1
35		54	32.3		53	55.9		53	19.5		52	43.1
36		54	20.0		53	42.3		53	04.5		52	26.7
37		54	07.4		53	28.2		52	49.1		52	09.9
38		53	54.5		53	13.9		52	33.2		51	52.6
39		53	41.2		52	59.1		52	17.0		51	34.9
40		53	27.5		52	43.8		52	00.2		51	16.6
41		53	13.4		52	28.2		51	43.0		50	57.8
42		52	58.8		52	12.0		51	25.2		50	38.4
43		52	43.8		51	55.4		51	06.9		50	18.5
44		52	28.4		51	38.2		50	48.0		49	57.8
45		52	12.3		51	20.4		50	28.4		49	36.4
46		51	55.7		51	01.9		50	08.1		49	14.3
47	89	51	38.6	89	50	42.9	89	49	47.2	89	48	51.4

Latitude.	1 mile.	2 miles.	3 miles.	4 miles.
°	Feet.	Feet.	Feet.	Feet.
30	0.39	1.54	3.47	6.17
31	0.40	1.60	3.61	6.42
32	0.42	1.67	3.76	6.67
33	0.43	1.73	3.90	6.93
34	0.45	1.80	4.05	7.20
35	0.47	1.87	4.20	7.47
36	0.48	1.94	4.36	7.75
37	0.50	2.01	4.52	8.04
38	0.52	2.08	4.69	8.33
39	0.54	2.16	4.86	8.63
40	0.56	2.24	5.03	8.95
41	0.58	2.32	5.21	9.27
42	0.60	2.40	5.40	9.59
43	0.62	2.48	5.59	9.93
44	0.64	2.57	5.79	10.29
45	0.67	2.66	5.99	10.65
46	0.69	2.76	6.20	11.02
47	0.71	2.85	6.42	11.41

Latitude.	5 miles.	6 miles.	7 miles.	8 miles.
°	Feet.	Feet.	Feet.	Feet.
30	9.64	13.88	18.89	24.67
31	10.03	14.44	19.66	25.68
32	10.42	15.02	20.44	26.69
33	10.82	15.60	21.23	27.74
34	11.25	16.20	22.05	28.80
35	11.68	16.81	22.89	29.89
36	12 11	17.41	23.74	31.01
37	12.57	18.09	24.62	32.16
38	13.02	18.75	25.52	33.33
39	13.49	19.43	26.44	34.54
40	13.98	20.11	27.40	35.78
41	14.48	20.85	28.37	37.06
42	14.99	21.59	29.38	38.38
43	15.52	22.35	30.42	39.74
44	16.07	23.14	31.50	41.14
45	16.64	23.96	32.61	42.59
46	17.21	24.80	33.76	44.10
47	17.83	25.68	34.95	45.65

Latitude.	9 miles.	10 miles.	11 miles.	12 miles.
°	Feet.	Feet.	Feet.	Feet.
30	31.23	38.55	46.65	55.52
31	32.49	40.12	48.54	57.77
32	33.78	41.71	50.47	60.06
33	35.10	43.34	52.44	62.41
34	36.45	45.00	54.45	64.80
35	37.83	46.71	56.62	67.26
36	39.25	48.45	58.63	69.77
37	40.70	50.24	60.79	72.35
38	42.19	52.08	63.02	75.00
39	43.71	53.97	65.30	77.71
40	45.29	55.91	67.65	80.51
41	46.90	57.91	70.07	83.39
42	48.57	59.97	72.56	86.35
43	50.29	62.09	75.13	89.41
44	52.07	64.28	77.78	92.57
45	53.91	66.55	80.53	95.84
46	55.81	68.90	83.37	99.22
47	57.78	71.34	86.32	102.72

1′	.0167	11′	.1833	21′	.3500	31′	.5167	41′	.6833	51′	.8500
2	.0333	12	.2000	22	.3667	32	.5333	42	.7000	52	.8667
3	.0500	13	.2167	23	.3833	33	.5500	43	.7167	53	.8833
4	.0667	14	.2333	24	.4000	34	.5667	44	.7333	54	.9000
5	.0833	15	.2500	25	.4167	35	.5833	45	.7500	55	.9167
6	.1000	16	.2667	26	.4333	36	.6000	46	.7667	56	.9333
7	.1167	17	.2833	27	.4500	37	.6167	47	.7833	57	.9500
8	.1333	18	.3000	28	.4667	38	.6333	48	.8000	58	.9667
9	.1500	19	.3167	29	.4833	39	.6500	49	.8167	59	.9833
10	.1667	20	.3333	30	.5000	40	.6667	50	.8333	60	1.0000

TABLE XVIII.—Inches in Decimals of a Foot.

1-16	3-32	⅛	3-16	¼	5-16	⅜	½	⅝	¾	⅞
.0052	.0078	.0104	.0156	.0208	.0260	.0313	.0417	.0521	.0625	.0729
1	2	3	4	5	6	7	8	9	10	11
.0833	.1667	.2500	.3333	.4167	.5000	.5833	.6667	.7500	.8333	.9167

TABLE XIX.—Radii, and Deflections.

Deg.	Radius	Tan. Def	Chd. Def.	Def. for 1 Foot	Deg.	Radius	Tan. Def.	Chd. Def.	Def. for 1 Foot
0° 10′	34377.	.145	.291	0.05′	7°	819.0	6.105	12.21	2.10′
20	17189.	.291	.582	0.10	20′	781.8	6.395	12.79	2.20
30	11459.	.436	.873	0.15	30	764.5	6.540	13.08	2.25
40	8594.4	.582	1.164	0.20	40	747 9	6.685	13.37	2.30
50	6875.5	.727	1.454	0.25	8	716.8	6.976	13.95	2.40
1	5729 6	.873	1 745	0.30	20	688.2	7.266	14.53	2.50
10	4911.2	1.018	2.036	0.35	30	674.7	7.411	14.82	2.55
20	4297.3	1.164	2.327	0.40	40	661.7	7.556	15.11	2 60
30	3819.8	1.309	2.618	0.45	9	637.3	7.846	15.69	2.70
40	3437.9	1.454	2.909	0.50	20	614.6	8.136	16.27	2.80
50	3125.4	1.600	3.200	0.55	30	603.8	8.281	16.56	2.85
2	2864.9	1.745	3.490	0.60	40	593.4	8.426	16.85	2.90
10	2644.6	1.891	3.781	0.65	10	573 7	8.716	17.43	3.00
20	2455.7	2.036	4 072	0.70	30	546.4	9.150	18.30	3.15
30	2292.0	2.181	4.363	0.75	11	521.7	9.585	19.16	3.30
40	2148.8	2.327	4.654	0.80	30	499.1	10.02	20.04	3.45
50	2022.4	2.472	4 945	0.85	12	478.3	10.45	20.91	3.60
3	1910 1	2.618	5.235	0.90	30	459.3	10.89	21.77	3.75
10	1809.6	2.763	5.526	0.95	13	441.7	11.32	22.64	3.90
20	1719.1	2.908	5.817	1.00	30	425.4	11.75	23.51	4.05
30	1637 3	3.054	6.108	1.05	14	410.3	12.18	24.37	4.20
40	1562.9	3.199	6.398	1.10	30	396.2	12.62	25.24	4.35
50	1495.0	3.345	6.689	1.15	15	383.1	13.05	26.11	4.50
4	1432.7	3 490	6.980	1.20	30	370.8	13.49	26.97	4.65
10	1375.4	3.635	7.271	1.25	16	359.3	13.92	27.84	4.80
20	1322.5	3.781	7.561	1.30	30	348.5	14.35	28.70	4.95
30	1273.6	3.926	7.852	1.35	17	338.3	14.78	29.56	5.10
40	1228.1	4.071	8.143	1.40	18	319.6	15.64	31.29	5.40
50	1185.8	4.217	8.433	1.45	19	302.9	16.51	33.01	5.70
5	1146.3	4.362	8.724	1.50	20	287.9	17.37	34.73	6.00
10	1109.3	4.507	9.014	1.55	21	274.4	18.22	36.44	6.30
20	1074.7	4.653	9.305	1.60	22	262.0	19.08	38.16	6.60
30	1042.1	4.798	9.596	1.65	23	250.8	19.94	39.87	6.90
40	1011.5	4 943	9.886	1.70	24	240.5	20.79	41.58	7.20
50	982.6	5.088	10.18	1.75	25	231.0	21.64	43.28	7.50
6	955.4	5.234	10.47	1.80	26	222.3	22.50	44.99	7.80
10	929.6	5.379	10.76	1.85	27	214.2	23.35	46.69	8.10
20	905.1	5.524	11.05	1.90	28	206.7	24.19	48.38	8.40
30	881.9	5.669	11.34	1.95	29	199.7	25.04	50.07	8.70
40	859.9	5.814	11.63	2.00	30	193.2	25.88	51.76	9.00

Angle.	Tangent.	Exter'l.	Angle.	Tangent.	External	Angle.	Tangent.	External
1°	50.00	.22	11°	551.70	26.50	21°	1061.9	97.57
10′	58.34	.30	10′	560.11	27.31	10′	1070.6	99.16
20	66.67	.39	20	568.53	28.14	20	1079 2	100.75
30	75.01	.49	30	576.95	28 97	30	1087.8	102.35
40	83.34	.61	40	585.36	29.82	40	1096.4	103 97
50	91.68	.73	50	593.79	30.68	50	1105.1	105.60
2	100.01	.87	12	602.21	31.56	22	1113.7	107 24
10	108.35	1.02	10	610.64	32.45	10	1122.4	108 90
20	116.68	1.19	20	619.07	33.35	20	1131.0	110.57
30	125.02	1.36	30	627.50	34.26	30	1139.7	112 25
40	133.36	1.55	40	635.93	35.18	40	1148.4	113 95
50	141.70	1.75	50	644.37	36.12	50	1157.0	115.66
3	150.04	1.96	13	652.81	37 07	23	1165.7	117.38
10	158.38	2.19	10	661.25	38.03	10	1174.4	119.12
20	166.72	2.43	20	669.70	39.01	20	1183.1	120 87
30	175.06	2.67	30	678.15	39.99	30	1191.8	122 63
40	183.40	2.93	40	686.60	40.99	40	1200.5	124 41
50	191.74	3.21	50	695.06	42.00	50	1209.2	126.20
4	200.08	3.49	14	703 51	43.03	24	1217.9	128 00
10	208.43	3.79	10	711.97	44.07	10	1226.6	129.82
20	216.77	4.10	20	720.44	45 12	20	1235.3	131.65
30	225.12	4.42	30	728.90	46.18	30	1244.0	133.50
40	233.47	4.76	40	737.37	47.25	40	1252.8	135.35
50	241 81	5.10	50	745.85	48.34	50	1261.5	137.23
5	250.16	5.46	15	754.32	49.44	25	1270.2	139 11
10	258.51	5.83	10	762.80	50.55	10	1279.0	141.01
20	266.86	6.21	20	771.29	51.68	20	1287 7	142.93
30	275.21	6.61	30	779.77	52.89	30	1296.5	144.85
40	283.57	7.01	40	788.26	53.97	40	1305.3	146.79
50	291.92	7.43	50	796.75	55.13	50	1314.0	148.75
6	300.28	7.86	16	805.25	56 31	26	1322.8	150 71
10	308.64	8.31	10	813.75	57.50	10	1331.6	152.69
20	316.99	8.76	20	822.25	58.70	20	1340.4	154.69
30	325.35	9.23	30	830.76	59.91	30	1349.2	156.70
40	333.71	9 71	40	839.27	61.14	40	1358.0	158.72
50	342.08	10.20	50	847.78	62.38	50	1366.8	160.76
7	350.44	10.71	17	856.30	63.63	27	1375.6	162.81
10	358.81	11.22	10	864.82	64.90	10	1384.4	164.86
20	367.17	11.75	20	873.35	66.18	20	1393.2	166.95
30	375.54	12.29	30	881.88	67.47	30	1402.0	169.04
40	383.91	12.85	40	890.41	68.77	40	1410.9	171.15
50	392.28	13.41	50	898.95	70.09	50	1419.7	173.27
8	400.66	13.99	18	907.49	71.42	28	1428.6	175.41
10	409.03	14.58	10	916.03	72.76	10	1437.4	177.55
20	417.41	15.18	20	924.58	74.12	20	1446 3	179.72
30	425.79	15.80	30	933.13	75.49	30	1455.1	181.89
40	434.17	16.43	40	941.69	76.86	40	1464 0	184.08
50	442.55	17.07	50	950.25	78.26	50	1472.9	186 29
9	450.93	17.72	19	958.81	79.67	29	1481.8	188.51
10	459 32	18.38	10	967.38	81.09	10	1490.7	190.74
20	467.71	19.06	20	975.96	82.53	20	1499.6	192.99
30	476.10	19.75	30	984.53	83.97	30	1508.5	195.25
40	484.49	20.45	40	993.12	85.43	40	1517.4	197.53
50	492.88	21.16	50	1001.7	86.90	50	1526.3	199.82
10	501.28	21.89	20	1010.3	88.39	30	1535.3	202 12
10	509.68	22.62	10	1018.9	89.89	10	1544.2	204.44
20	518.08	23.38	20	1027.5	91.40	20	1553.1	206.77
30	526.48	24.14	30	1036.1	92.92	30	1562.1	209.12
40	534.89	24.91	40	1044.7	94.46	40	1571.0	211.48
50	543.29	25.70	50	1053.3	96 01	50	1580.0	213.86

Angle.	Tangent.	Exter'l.	Angle.	Tangent.	External	Angle.	Tangent.	External
31°	1589.0	216.3	**41°**	2142.2	387.4	**51°**	2732.9	618.4
10′	1598.0	218.7	10′	2151.7	390.7	10′	2743.1	622.8
20	1606.9	221.1	20	2161.2	394.1	20	2753.4	627.2
30	1615.9	223.5	30	2170.8	397.4	30	2763.7	631.7
40	1624.9	226.0	40	2180.3	400.8	40	2773.9	636.2
50	1633.9	228.4	50	2189.9	404.2	50	2784.2	640.7
32	1643.0	230.9	**42**	2199.4	407.6	**52**	2794.5	645.2
10	1652.0	233.4	10	2209.0	411.1	10	2804.9	649.7
20	1661.0	235.9	20	2218.6	414.5	20	2815.2	654.3
30	1670.0	238.4	30	2228.1	418.0	30	2825.6	658.8
40	1679.1	241.0	40	2237.7	421.4	40	2835.9	663.4
50	1688.1	243.5	50	2247.3	425.0	50	2846.3	668.0
33	1697.2	246.1	**43**	2257.0	428.5	**53**	2856.7	672.7
10	1706.3	248.7	10	2266.6	432.0	10	2867.1	677.3
20	1715.3	251.3	20	2276.2	435.6	20	2877.5	682.0
30	1724.4	253.9	30	2285.9	439.2	30	2888.0	686.7
40	1733.5	256.5	40	2295.6	442.8	40	2898.4	691.4
50	1742.6	259.1	50	2305.2	446.4	50	2908.9	696.1
34	1751.7	261.8	**44**	2314.9	450.0	**54**	2919.4	700.9
10	1760.8	264.5	10	2324.6	453.6	10	2929.9	705.7
20	1770.0	267.2	20	2334.3	457.3	20	2940.4	710.5
30	1779.1	269.9	30	2344.1	461.0	30	2951.0	715.3
40	1788.2	272.6	40	2353.8	464.6	40	2961.5	720.1
50	1797.4	275.3	50	2363.5	468.4	50	2972.1	725.0
35	1806.6	278.1	**45**	2373.3	472.1	**55**	2982.7	729.9
10	1815.7	280.8	10	2383.1	475.8	10	2993.3	734.8
20	1824.9	283.6	20	2392.8	479.6	20	3003.9	739.7
30	1834.1	286.4	30	2402.6	483.8	30	3014.5	744.6
40	1843.3	289.2	40	2412.4	487.2	40	3025.2	749.6
50	1852.5	292.0	50	2422.3	491.0	50	3035.8	754.6
36	1861.7	294.9	**46**	2432.1	494.8	**56**	3046.5	759.6
10	1870.9	297.7	10	2441.9	498.7	10	3057.2	764.6
20	1880.1	300.6	20	2451.8	502.5	20	3067.9	769.7
30	1889.4	303.5	30	2461.7	506.4	30	3078.7	774.7
40	1898.6	306.4	40	2471.5	510.3	40	3089.4	779.8
50	1907.9	309.3	50	2481.4	514.3	50	3100.2	784.9
37	1917.1	312.2	**47**	2491.3	518.2	**57**	3110.9	790.1
10	1926.4	315.2	10	2501.2	522.2	10	3121.7	795.2
20	1935.7	318.1	20	2511.2	526.1	20	3132.6	800.4
30	1945.0	321.1	30	2521.1	530.1	30	3143.4	805.6
40	9454.3	324.1	40	2531.1	534.2	40	3154.2	810.9
50	1963.6	327.1	50	2541.0	538.2	50	3165.1	816.1
38	1972.9	330.2	**48**	2551.0	542.2	**58**	3176.0	821.4
10	1982.2	333.2	10	2561.0	546.3	10	3186.9	826.7
20	1991.5	336.3	20	2571.0	550.4	20	3197.8	832.0
30	2000.9	339.3	30	2581.0	554.5	30	3208.8	837.3
40	2010.2	342.4	40	2591.0	558.6	40	3219.7	842.7
50	2019.6	345.5	50	2601.1	562.8	50	3230.7	848.1
39	2029.0	348.6	**49**	2611.2	566.9	**59**	3241.7	853.5
10	2038.4	351.8	10	2621.2	571.1	10	3252.7	858.9
20	2047.8	354.9	20	2631.3	575.3	20	3263.7	864.3
30	2057.2	358.1	30	2641.4	579.5	30	3274.8	869.8
40	2066.6	361.3	40	2651.5	583.8	40	3285.8	875.3
50	2076.0	364.5	50	2661.6	588.0	50	3296.9	880.8
40	2085.4	367.7	**50**	2671.8	592.3	**60**	3308.0	886.4
10	2094.9	371.0	10	2681.9	596.6	10	3319.1	892.0
20	2104.3	374.2	20	2692.1	600.9	20	3330.3	897.5
30	2113.8	377.5	30	2702.3	605.3	30	3341.4	903.2
40	2123.3	380.8	40	2712.5	609.6	40	3352.6	908.8
50	2132.7	384.1	50	2722.7	614.0	50	3363.8	914.5

Table XX—Tangents and Externals to a 1° Curve.

Angle.	Tangent.	External	Angle.	Tangent.	External	Angle.	Tangent.	External
61°	3375.0	920.2	71°	4086.9	1308.2	81°	4893.6	1805.3
10′	3386.3	925.9	10′	4099.5	1315 6	10′	4908.0	1814.7
20	3397.5	931 6	20	4112.1	1322.9	20	4922.5	1824.1
30	3408.8	937.3	30	4124.8	1330.3	30	4937.0	1833 6
40	3420.1	943.1	40	4137.4	1337.7	40	4951.5	1843.1
50	3431.4	948.9	50	4150.1	1345.1	50	4966.1	1852.6
62	3442.7	954.8	72	4162.8	1352.6	82	4980.7	1862.2
10	3454.1	960.6	10	4175.6	1360.1	10	4995.4	1871.8
20	3465.4	966.5	20	4188.5	1367.6	20	5010 0	1881.5
30	3476 8	972.4	30	4201.2	1375 2	30	5024.8	1891.2
40	3488 3	978.3	40	4214 0	1382.8	40	5039.5	1900.9
50	3499.7	984.3	50	4226.8	1390.4	50	5054 3	1910.7
63	3511.1	990.2	73	4239.7	1398.0	83	5069.2	1920.5
10	3522.6	996.2	10	4252.6	1405.7	10	5084.0	1930.4
20	3534.1	1002.3	20	4265.6	1413.5	20	5099.0	1940.3
30	3545.6	1008.3	30	4278.5	1421.2	30	5113.9	1950.3
40	3557.2	1014.4	40	4291.5	1429.0	40	5128.9	1960.2
50	3568.7	1020.5	50	4304.6	1436.8	50	5143.9	1970.3
64	3580.3	1026.6	74	4317.6	1444.6	84	5159.0	1980.4
10	3591.9	1032.8	10	4330.7	1452.5	10	5174 1	1990.5
20	3603.5	1039 0	20	4343.8	1460 4	20	5189.3	2000.6
30	3615 1	1045.2	30	4356.9	1468.4	30	5204.4	2010.8
40	3626.8	1051.4	40	4370.1	1476.4	40	5219.7	2021.1
50	3638.5	1057.7	50	4383.3	1484.4	50	5234.9	2031.4
65	3650.2	1063.9	75	4396 5	1492.4	85	5250.3	2041.7
10	3661.9	1070.2	10	4409.8	1500.5	10	5265 6	2052.1
20	3673.7	1076.6	20	4423.1	1508 6	20	5281.0	2062.5
30	3685.4	1082.9	30	4436 4	1516.7	30	5296.4	2073.0
40	3697.2	1089.3	40	4449.7	1524.9	40	5311 9	2083.5
50	3709.0	1095.7	50	4463.1	1533.1	50	5327.4	2094.1
66	3720.9	1102.2	76	4476.5	1541.4	86	5343.0	2104.7
10	3732.7	1108.6	10	4489.9	1549.7	10	5358.6	2115.3
20	3744.6	1115.1	20	4503.4	1558.0	20	5374.2	2126 0
30	3756.5	1121.7	30	4516.9	1566.3	30	5389.9	2136.7
40	3768.5	1128.2	40	4530.4	1574.7	40	5405.6	2147.5
50	3780.4	1134.8	50	4544.0	1583.1	50	5421.4	2158.4
67	3792.4	1141.4	77	4557.6	1591 6	87	5437.2	2169.2
10	3804.4	1148 0	10	4571.2	1600.1	10	5453.1	2180.2
20	3816 4	1154.7	20	4584.3	1608 6	20	5469.0	2191.1
30	3828.4	1161.3	30	4598.5	1617.1	30	5484.9	2202.2
40	3840.5	1168.1	40	4612.2	1625.7	40	5500 9	2213.2
50	3852.6	1174.8	50	4626.0	1634.4	50	5517.0	2224.3
68	3864.7	1181.6	78	4639.8	1643.0	88	5533.1	2235.5
10	3876.8	1188.4	10	4653.6	1651.7	10	5549.2	2246.7
20	3889.0	1195.2	20	4667.4	1660.5	20	5565.4	2258.0
30	3901.2	1202.0	30	4681.3	1669.2	30	5581.6	2269.3
40	3913.4	1208.9	40	4695.2	1678.1	40	5597.8	2280.6
50	3925.6	1215.8	50	4709.2	1686.9	50	5614.2	2292.0
69	3937.9	1222.7	79	4723.2	1695.8	89	5630.5	2303.5
10	3950.2	1229.7	10	4737.2	1704.7	10	5646.9	2315.0
20	3962.5	1236.7	20	4751.2	1713.7	20	5663.4	2326.6
30	3974.8	1243.7	30	4765.3	1722.7	30	5679.9	2338.2
40	3987.2	1250.8	40	4779.4	1731.7	40	5696 4	2349.8
50	3999.5	1257.9	50	4793.6	1740.8	50	5713.0	2361.5
70	4011.9	1265.0	80	4807.7	1749.9	90	5729.7	2373.3
10	4024.4	1272.1	10	4822.0	1759.0	10	5746.3	2385.1
20	4036.8	1279.3	20	4836.2	1768.2	20	5763.1	2397.0
30	4049.3	1286.5	30	4850.5	1777.4	30	5779.9	2408.9
40	4061.8	1293.6	40	4864.8	1786.7	40	5796.7	2420.9
50	4074.4	1300.9	50	4879.2	1796.0	50	5813.6	2432.9

Angle.	Tangent	External	Angle.	Tangent	External	Angle.	Tangent	External
91°	5830.5	2444.9	**101°**	6950.6	3278.1	**111°**	8336.7	4386.1
10′	5847.5	2457.1	10′	6971.3	3294.1	10′	8362.7	4407.6
20	5864.6	2469.3	20	6992.0	3310.1	20	8388.9	4429.2
30	5881.7	2481.5	30	7012.7	3326.1	30	8415.1	4450.9
40	5898.8	2493.8	40	7033.6	3342.3	40	8441.5	4472.7
50	5916.0	2506.1	50	7054.5	3358.5	50	8468.0	4494.6
92	5933.2	2518.5	**102**	7075.5	3374.9	**112**	8494.6	4516.6
10	5950.5	2531.0	10	7096.6	3391.2	10	8521.3	4538.8
20	5967.9	2543.5	20	7117.8	3407.7	20	8548.1	4561.1
30	5985.3	2556.0	30	7139.0	3424.3	30	8575.0	4583.4
40	6002.7	2568.6	40	7160.3	3440.9	40	8602.1	4606.0
50	6020.2	2581.3	50	7181.7	3457.6	50	8629.3	4628.6
93	6037.8	2594.0	**103**	7203.2	3474.4	**113**	8656.6	4651.3
10	6055.4	2606.8	10	7224.7	3491.3	10	8684.0	4674.2
20	6073.1	2619.7	20	7246.3	3508.2	20	8711.5	4697.2
30	6090.8	2632.6	30	7268.0	3525.2	30	8739.2	4720.3
40	6108.6	2645.5	40	7289.8	3542.4	40	8767.0	4743.6
50	6126.3	2658.5	50	7311.7	3559.6	50	8794.9	4766.9
94	6144.3	2671.6	**104**	7333.6	3576.8	**114**	8822.9	4790.4
10	6162.6	2684.7	10	7355.6	3594.2	10	8851.0	4814.1
20	6180.2	2697.9	20	7377.8	3611.7	20	8879.3	4837.8
30	6198.3	2711.2	30	7399.9	3629.2	30	8907.7	4861.7
40	6216.4	2724.5	40	7422.2	3646.8	40	8936.3	4885.7
50	6234.6	2737.9	50	7444.6	3664.5	50	8965.0	4909.9
95	6252.8	2751.3	**105**	7467.0	3682.3	**115**	8993.8	4934.1
10	6271.1	2764.8	10	7489.6	3700.2	10	9022.7	4958.6
20	6289.4	2778.3	20	7512.2	3718.2	20	9051.7	4983.1
30	6307.9	2792.0	30	7534.9	3736.2	30	9080.9	5007.8
40	6326.3	2805.6	40	7557.7	3754.4	40	9110.3	5032.6
50	6344.8	2819.4	50	7580.5	3772.6	50	9139.8	5057.6
96	6363.4	2833.2	**106**	7603.5	3791.0	**116**	9169.4	5082.7
10	6382.1	2847.0	10	7626.6	3809.4	10	9199.1	5107.9
20	6400.8	2861.0	20	7649.7	3827.9	20	9229.0	5133.3
30	6419.5	2875.0	30	7672.9	3846.5	30	9259.0	5158.8
40	6438.4	2889.0	40	7696.3	3865.2	40	9289.2	5184.5
50	6457.3	2903.1	50	7719.7	3884.0	50	9319.5	5210.3
97	6476.2	2917.3	**107**	7743.2	3902.9	**117**	9349.9	5236.2
10	6495.2	2931.6	10	7766.8	3921.9	10	9380.5	5262.3
20	6514.3	2945.9	20	7790.5	3940.9	20	9411.3	5288.6
30	6533.4	2960.3	30	7814.3	3960.1	30	9442.2	5315.0
40	6552.6	2974.7	40	7838.1	3979.4	40	9473.2	5341.5
50	6571.9	2989.2	50	7862.1	3998.7	50	9504.4	5368.2
98	6591.2	3003.8	**108**	7886.2	4018.2	**118**	9535.7	5395.1
10	6610.6	3018.4	10	7910.4	4037.8	10	9567.2	5422.1
20	6630.1	3033.1	20	7934.6	4057.4	20	9598.9	5449.2
30	6649.6	3047.9	30	7950.0	4077.2	30	9630.7	5476.5
40	6669.2	3062.8	40	7983.5	4097.1	40	9662.6	5504.0
50	6688.8	3077.7	50	8008.0	4117.0	50	9694.7	5531.7
99	6708.6	3092.7	**109**	8032.7	4137.1	**119**	9727.0	5559.4
10	6728.4	3107.7	10	8057.4	4157.3	10	9759.4	5587.4
20	6748.2	3122.9	20	8082.3	4177.5	20	9792.0	5615.5
30	6768.1	3138.1	30	8107.3	4197.9	30	9824.8	5643.8
40	6788.1	3153.3	40	8132.3	4218.4	40	9857.7	5672.3
50	6808.2	3168.7	50	8157.5	4239.0	50	9890.8	5700.9
100	6828.3	3184.1	**110**	8182.8	4259.7	**120**	9924.0	5729.7
10	6848.5	3199.6	10	8208.2	4280.5	10	9957.5	5758.6
20	6868.8	3215.1	20	8233.7	4301.4	20	9901.0	5787.7
30	6889.2	3230.8	30	8259.3	4322.4	30	10025.0	5817.0
40	6909.6	3246.5	40	8285.0	4343.6	40	10059.0	5846.5
50	6930.1	3262.3	50	8310.8	4364.8	50	10093.0	5876.1

CURVE FORMULÆ.

$T = R \tan. \frac{1}{2} I$

$T = \frac{50 \tan. \frac{1}{2} I}{\text{Sin. } D}$

$\text{Sin. } D = \frac{50}{R}$

$\text{Sin. } D = \frac{50 \tan. \frac{1}{2} I}{T}$

$R = T \cot. \frac{1}{2} I$

$R = \frac{50}{\text{Sin. } D}$

$E = R \text{ ex. sec. } \frac{1}{2} I$

$E = T \tan. \frac{1}{4} I$

$\text{Chord def.} = \frac{\text{chord}^2}{R}$

$\text{No. chords} = \frac{\frac{1}{2} I}{D}$

$\text{Tan. def.} = \frac{1}{2} \text{ chord def.}$

The square of any distance, divided by twice the radius, will equal the distance from tangent to curve, very nearly.

Table XX contains Tangents and Externals to a 1° curve. Tan. and Ext. to any other radius may be found, nearly enough, by dividing the Tan. or Ext. opposite the given Central Angle by the given degree of curve.

To find Deg. of Curve, having the central Angle and Tangent: Divide Tan. opposite the given Central Angle by the given Tangent.

To find Deg. of Curve, having the Central Angle and External: Divide Ext. opposite the given Central Angle by the given External.

To find Nat. Tan. and Nat. Ex. Sec. for any angle by Table XX: Tan. or Ext. of twice the given angle divided by the radius of a 1° curve will be the Nat. Tan. or Ex. Sec.

To find angle for a given distance and deflection.

Rule 1. Multiply given distance by .01745 (def. for 1° for 1 ft.), and divide given deflection by the product.

Rule 2. Multiply given deflection by 57.3, and divide the product by the given distance.

To find deflection for a given angle and distance: Multiply the angle by .01745, and the product by the distance.

APPENDIX.

DETERMINATION OF THE AZIMUTH OF POLARIS AND TRUE MERIDIAN AT ANY HOUR, THE STAR BEING VISIBLE, AND THE CORRECT LOCAL MEAN TIME BEING KNOWN.

[From U. S. Surveying Instructions, 1894.]

IN this article it is proposed to present a method, with two new and compact tables adapted to common clock time, with such plain directions for use that any person of ordinary intelligence can understand and apply them.

As the surveyor should have a perfectly clear idea of what is meant by *Astronomical Time* (used to simplify computations), and the *Hour Angle of Polaris*, these terms will now be explained.

The *Civil Day*, according to the customs of society, commences at midnight and comprises twenty-four hours from one midnight to the next following. The hours are counted from 0 to 12 from midnight to noon, after which they are again reckoned from 0 to 12 from noon to midnight. Thus the day is divided into two periods of 12 hours each; the first of which is marked a. m., the last p. m.

The *Astronomical Day* commences at noon on the civil day of the same date. It also comprises twenty-four hours; but they are reckoned from 0 to 24, and from the noon of one day to that of the next following.

The civil day begins twelve hours before the astronomical day; therefore the first period of the civil day answers to the last part of the preceding astronomical day, and the last part of the civil day corresponds to the first part of the astronomical day. Thus, January 9, 2 o'clock p. m., civil time, is also January 9, 2^h, astronomical time; and January 9, 2 o'clock a. m., civil time, is January 8, 14^h, astronomical time.

The rule, then, for the transformation of civil time into astronomical time is this: *If the civil time is marked* *p. m., take away the designation p. m., and the astronomical time is had without further change; if the civil time is marked a. m., take one from the day and add twelve to the hours, remove the initials a. m., and the result is the astronomical time wanted.*

The substance of the above rule may be otherwise stated, as follows: When the surveyor takes an observation during p. m. hours, civil time. he can say: the *astronomical* time is the hours and minutes passed since the noon of *this* day, and when observing in the a. m. hours, he can say the *astronomical* time is the hours and minutes elapsed since the noon of *yesterday*, in either case omitting the designation a. m. or p. m., and writing for the day of the month, that civil date on which the noon falls, from which the time is reckoned. Finally, *the astronomical time may be called the hours and minutes elapsed since the* NOON LAST PASSED, *the astronomical* DATE *being that of the civil day to which the noon belongs.* Thus, April 23, 4:15 p. m., civil time, is April 23, $4^h\ 15^m$, astronomical time, and April 23, 4:15 a.m., civil time, is April 22, $16^h\ 15^m$, astronomical time.

The surveyor should thoroughly master this transformation of the civil time into astronomical time, as it will be the first duty he will have to perform after observing Polaris out of the meridian.

The change can always be made mentally, no written work being required. Table I might be easily altered to give the times by the civil count marked a. m. and p. m., but such an arrangement would greatly extend and complicate the following rules and examples, and correspondingly increase the chances for making mistakes.

Hour Angle of Polaris.—In Fig. 2, Plate I, the full vertical line represents a portion of the meridian passing through the zenith Z (the point directly overhead), and intersecting the northern horizon at the north point N, from which, for surveying purposes, the azimuths of Polaris are reckoned east or west. The meridian is pointed out by the plumb line when it is in the same plane with the eye of the observer and Polaris on the meridian, and a visual representation is also seen in the vertical wire of the transit, when it bisects the star on the meridian.

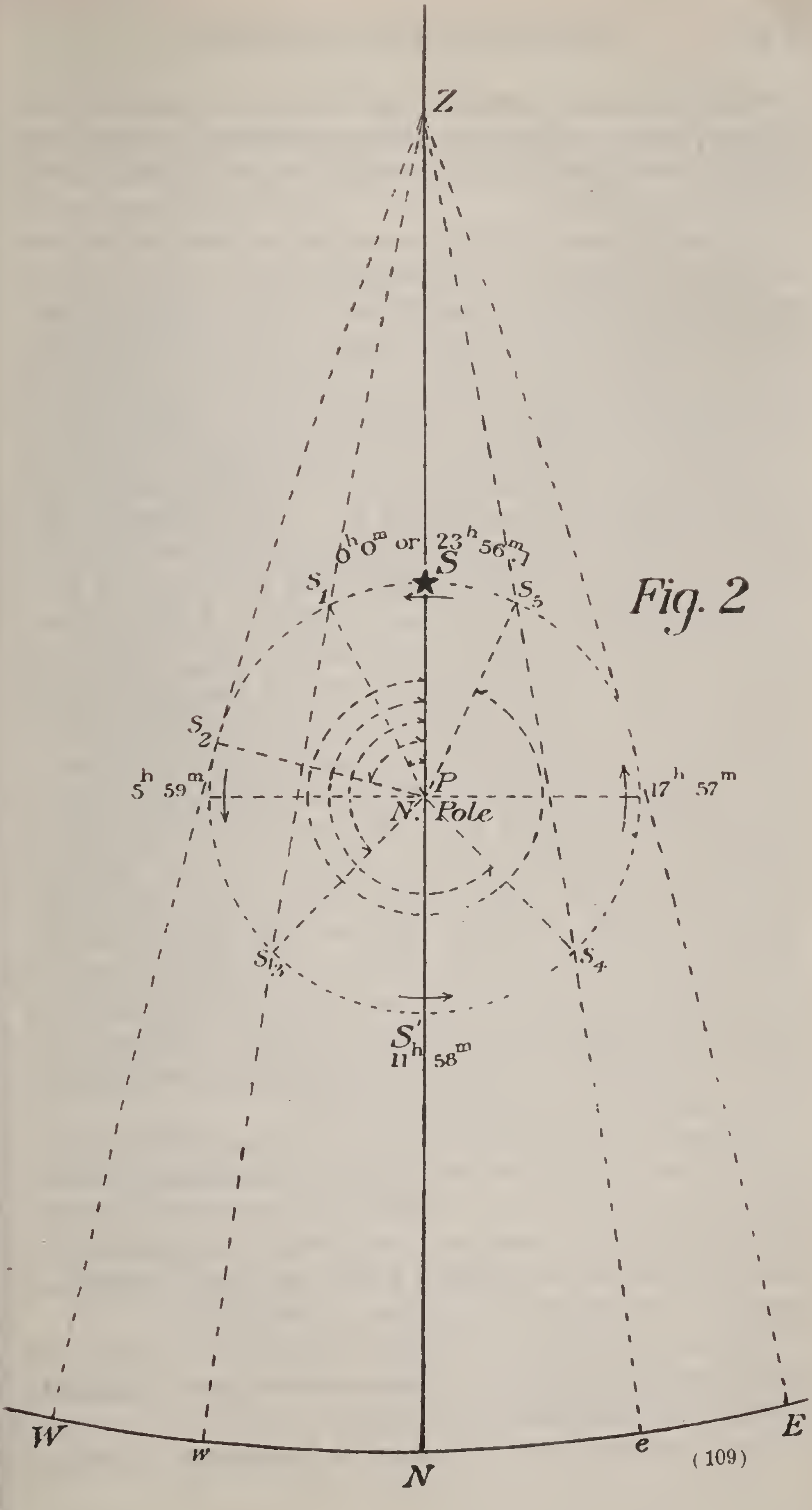
Z
$0^h 0^m$ or $23^h 56^m$
S
S_1
S_5
S_2
$5^h 59^m$
P
N. Pole
$17^h 57^m$
S_3
S_4
S'
$11^h 58^m$
W
w
N
e
E

Fig. 2

When Polaris crosses the meridian it is said to culminate; above the pole (at S), the passage is called the *Upper Culmination*, in contradistinction to the *Lower Culmination* (at S′).

In the diagram,—which the surveyor may better understand by holding it up perpendicular to the line of sight when he looks toward the pole,—Polaris is supposed to be on the meridian, where it will be about *noon* on April 10th of each year. The star appears to revolve around the pole in the direction of the arrows, once in every $23^h\ 56^m\ 4^s.09$ of *mean solar time;* it consequently comes to and crosses the meridian, or *culminates*, nearly four minutes *earlier* each successive day. The apparent motion of the star being uniform, one quarter of the circle will (omitting fractions) be described in $5^h\ 59^m$, one half in $11^h\ 58^m$, and three quarters in $17^h\ 57^m$. For the positions s_1, s_2, s_3, etc., the angles SPs_1, SPs_2, SPs_3, etc., are called *Hour Angles of Polaris* for the instant the star is at s_1, s_2, or s_3, etc., and they are measured by the arcs Ss_1, Ss_2, Ss_3, etc., expressed (in these instructions) in *mean solar* (common clock) time, and are always counted from the *upper* meridian (at S), to the *west*, around the circle from $0^h\ 0^m$ to $23^h\ 56^m.1$, and may have any value between the limits named. The hour angles, measured by the arcs Ss_1, Ss_2, Ss_3, Ss_4, Ss_5, and Ss_6, are approximately $1^h\ 8^m$, $5^h\ 55^m$, $9^h\ 4^m$, $14^h\ 52^m$, $18^h\ 01^m$, and $22^h\ 48^m$ respectively; their extent is also indicated, graphically, by broken fractional circles about the pole. The hour angle, $5^h\ 55^m$ and $18^h\ 01^m$ are those at west and east elongation, respectively, in latitude 40° N.

Suppose the star observed (*e. g.*) at the point S_3; the time it was at S (the time of upper culmination), taken from the whole circle, $23^h\ 56^m.1$, will leave the arc Ss_1, s_2, s_3, or the *hour angle* at the instant of observation; similar relations will obtain when the star is observed in any other position; therefore, in general:—

Subtract the time of Upper Culmination from the correct local mean time of observation; the remainder will be the Hour Angle of Polaris.

The *observation* will be made as heretofore directed, modified as follows: There will be no waiting for the star to reach elongation; the observation may be made at any instant when Polaris is visible, the exact time being carefully noted.

Table I.

This table gives, in "Part I," the local mean time of the upper culmination of Polaris, on the 1st and 15th of each month, for the years 1890 to 1900, inclusive. The times decrease, in each year, to April 10, when they become zero; then, commencing at $23^h\ 56^m.1$, the times again decrease until the following April, and so on, continuously. The quantity in the column marked "Diff. for 1 day" is the decrease *per day* during the interval of time against which it stands, and answers for *all the years* marked in the table. For any intermediate date, the "Diff. for 1 day" will be multiplied by the days elapsed since the preceding tabular *date*, and the product *subtracted* from the corresponding *time*, to obtain the required time of upper culmination for the date under consideration. The table answers directly for 90° west longitude. For places east or west of the assumed meridian, a small correction, dependent on the longitude, may be applied to the deduced time of culmination. The correction for longitude should *not* be used for dates subsequent to Dec. 31, 1896. This correction may be taken from Part III, and, with sufficient accuracy, for the longitude *nearest* that of the station. Use the correction according to the direction placed over it. A few examples will illustrate the use of the table.

1. Required, the time of upper culmination of Polaris for a station in longitude 116° west, for March 3, 1892.

		h. m.
Astron. time, U. C. of Polaris, 1892, March 1	....	2 37.8
Red. for 2 days is $3^m.94 \times 2 = 7^m.9$ (Part II) Cor. for 116° long. is........$0^m.3$ (Part III)	Subtract.........	8.2
Local mean time, U. C. of Polaris, 1892, March 3	....	2 29.6

The required time may also be obtained by using the table in the opposite direction; by taking the time for March 15, and *adding* the reduction, as follows:—

	h. m.
Astron. time, U. C. of Polaris, 1892, March 15	1 42.6
Red. for 12 days is $3^m.94 \times 12 = 47^m.3$, add	47.3
Sum	2 29.9
Correction for longitude 116° (Part III), subtract	0.3
Local mean time, U. C. of Polaris, 1892, March 3	2 29.6

In this case the two results are identical. If the computation is made both ways, the results will check each other.

Part II has been inserted to save the surveyor the little trouble of making multiplications; thus, for the above example, look in Part II, under the proper tabular difference, $3^m.94$, and opposite the day of the month in left hand column is the correction $7^m.9$; also in Part III is found the correction for 116° longitude, $0^m.3$, the sum being $8^m.2$. The work may be put down as follows:—

	h. m.
Astron. time. U. C. of Polaris, 1892, March 1 (Part I)	2 37.8
Red. (Part II), and correction for long. (Part III), subtract	8.2
Local mean time, U. C. of Polaris, 1892, March 3	2 29.6

The longitude correction being small, may generally be omitted; it will not be considered in the following examples.

Computing from a *preceding* date, for days between April 11 and 15 of any year, the reduction in Part II will be *greater* than the tabulated time of culmination, in which case $23^h\ 56^m.1$ will be *added*, to make the subtraction possible.

2. Required, for a station in long. 90° west, the time of U. C. of Polaris for April 14, 1891: —

	h. m.
Astron. time, U. C. of Polaris, 1891, April 1 (Part I)	0 38.4
Add	23 56.1
Sum	24 34.5
Reduction to April 14 (Part II), subtract	51.1
Local mean time, U. C. of Polaris, April 14	23 43.4

Working from a *following* date, for days between the 9th and 15th of April, the *sum* will exceed $23^h\ 56^m.1$, and when this occurs *subtract* $23^h\ 56^m.1$ from the sum, and the *remainder* will be the required time.

3. Required, for a station in long. 90° west, the time of U. C. of Polaris for April 10, 1892: —

	h.	m.
Astron. time, U. C. of Polaris, 1892, April 15 (Part I)........	23	36.8
Reduction for 5 days (Part II), add		19.6
Sum....................	23	56.4
Subtract..........................	23	56.1
Local mean time, U. C. of Polaris, 1892, April 10........	0	0.3

This example, worked like the last one, from the *preceding* date (April 1), will give precisely the result above written. (See example above.) If to the above time of culmination we add $23^h\ 56^m.1$, and then subtract $3^m.9$, we obtain $23^h\ 52^m.5$, the time of the *second* upper culmination on April 10, since both occur within 24 hours of noon and consequently on the *same day*. The upper culmination, to be used at any time, will always be the *last* one that occurs before the observation. In this instance it is, of course, the first one that takes place on the 10th. The *second* culmination occurs $7^m.5$ *before* noon of April 11, and consequently in broad daylight.

The surveyor should be careful to employ Part II, Table I, correctly. When the table is used in regular order, the "Reduction" may be taken from Part II with the argument,† "Day of the month" in *left* hand column, or, "Number of days elapsed" in *right* hand column, as may be preferred. In example 2, Part II, may be entered in with the argument 13 days elapsed (from 1st to 14th) in *right* hand column; then the reduction, $51^m.1$, results, as above written; but, when working from a *following* date (example 3), the day of the month in left hand column *cannot be used.*

Mistakes are often made by using the wrong column in Part I; as a matter of course, the time should *always* be taken out for the *current year.*

The foregoing examples embrace all cases which can occur in the use of Table I, and will be a sufficient guide for its application.

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† "Argument," the quantity on which another quantity in a table depends.

TABLE I.

Local mean (astronomical) time of the upper culmination of Polaris, computed for longitude 6 hours (90°) west of Greenwich.

[The time on line with any date in Part I is the hours and minutes elapsed (measured by a common clock or watch) since the preceding noon.]

Part I.

Date.		1895.	1896.	1897.	1898.	1899.	1900.	Diff. for 1 day.
		h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	m.
Jan.	1	6 34.7	6 36.1	6 33.0	6 34.1	6 35.2	6 36.3	3.95
	15	5 39.4	5 40.8	5 37.7	5 38.8	5 39.9	5 41.0	3.95
Feb.	1	4 32.3	4 33.7	4 30.6	4 31.7	4 32.8	4 33.9	3.95
	15	3 37.1	3 38.5	3 35.3	3 36.4	3 37 5	3 38.6	3.95
Mar.	1	2 41.8	2 39.3	2 40.1	2 41.2	2 42.3	2 43.4	3.94
	15	1 46.7	1 44.1	1 44.9	1 46.0	1 47.1	1 48.2	3.94
April	1	0 39 7	0 37.2	0 38.0	0 39.1	0 40.2	0 41.3	3.94
	15	23 40 8	23 38.3	23 39.1	23 40.2	23 41.3	23 42.4	3.93
May	1	22 38.0	22 35.5	22 36 2	22 37.3	22 38.4	22 39.5	3.93
	15	21 43.0	21 40.6	21 41.3	21 42.4	21 43.5	21 44.6	3.92
June	1	20 36.4	20 33.9	20 34.7	20 35.8	20 36.9	20 38 0	3 92
	15	19 41.6	19 39.1	19 39.9	19 41.0	19 42.1	19 43.2	3 92
July	1	18 38.9	18 36.5	18 37.2	18 38.3	18 39.4	18 40.5	3.92
	15	17 44.1	17 41.7	17 42.4	17 43.5	17 44.6	17 45.7	3.92
Aug.	1	16 37.6	16 35.1	16 35.8	16 36.9	16 38.0	16 39 1	3.91
	15	15 42.7	15 40.3	15 41.0	15 42.1	15 43.1	15 44.3	3.92
Sept.	1	14 36.1	14 33.7	14 34.3	14 35.4	14 46.5	14 37.6	3.92
	15	13 41.2	13 38.8	13 39.4	13 40.5	13 41.6	13 42.7	3.92
Oct.	1	12 38.4	12 36.0	12 36.6	12 37.7	12 38.8	12 39 9	3.93
	15	11 43 4	11 41.0	11 41.6	11 42.7	11 43.8	11 44.9	3.93
Nov.	1	10 36 6	10 34.1	10 34.8	10 35.9	10 37.0	10 38.1	3.93
	15	9 41.5	9 39.0	9 39.6	9 40.7	9 41.8	9 42 9	3.94
Dec.	1	8 38.4	8 35.9	8 36.6	8 37.7	8 38.8	8 39 9	3.94
	15	7 43.2	7 40.7	7 41.4	7 42.5	7 43.6	7 44.7	3.94
								3.95

Part II.—Reduction of tabular times to *intermediate dates.* Subtract the reduction when computing from a *preceding*, or add it when working from a *following* date.

Day of the month.	Reduction. Arg.—"Diff. for 1 day."					No. of days elaps'd.
	m. 3.91.	m. 3.92.	m. 3.93.	m. 3.94.	m. 3.95.	
	m.	m.	m.	m.	m.	
2 or 16	3.9	3.9	3.9	3.9	3.9	1
3 or 17	7.8	7.8	7.9	7.9	7.9	2
4 or 18	11.7	11.8	11.8	11.8	11.8	3
5 or 19	15.6	15.7	15.7	15.8	15.8	4
6 or 20	19.5	19.6	19.6	19.7	19.7	5
7 or 21	23 5	23.5	23.6	23.6	23.7	6
8 or 22	27.4	27.4	27.5	27.6	27.6	7
9 or 23	31.3	31.4	31.4	31.5	31.6	8
10 or 24	35.2	35.3	35.4	35.5	35.5	9
11 or 25	39.1	39.2	39.3	39.4	39.5	10
12 or 26	43.0	43.1	43.2	43.3	43.4	11
13 or 27	47.0	47.0	47.2	47.3	47.4	12
14 or 28	50.8	51.0	51.1	51.2	51.3	13
29	54.7	54.9	55.0	55.2	55.3	14
30	58.6	58.8	58.9	59.1	59.2	15
31	62.6	62.7	62.9	63.0	63.2	16

Applications of Tables I and II.

4. Required the *Hour Angle* and *Azimuth of Polaris*, for a station in latitude 46° N., longitude 90° W., at 8^h 24^m p. m., November 7, 1891.

			h.	m.
Astronomical time of observation, 1891, Nov. 7..................			8	24.0
	h.	m.		
Astron. time, U. C. Polaris, Nov. 1 (Table I, Part I),	10	35.1		
Reduction to Nov. 6* (Part II), subtract..........		†19.7		
Astron. time, U. C. Polaris, Nov. 6..............	10	15.4, subt.	‡10	15.4
Hour Angle of Polaris, at observation.........................			22	8.6
Subtract from...			§23	56.1
Time Argument for Table II................................			1	47.5
Azimuth of Polaris, at observation................................			1° 51′ E.	

PART III.—Correction of the tabular time for longitude.

Longitude.	63°	72°	81°	90°	99°	108°	117°	127°
	Add	Add	Add	Add	Subt.	Subt.	Subt.	Subt.
	m.	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*
Correction............	0.3	0.2	0.1	0.0	0.1	0.2	0.3	0.4

5. Required the *Hour Angle* and *Azimuth of Polaris*, for a station in latitude 41° 12′ N., longitude 94° W., at 6^h 16^m a. m., Nov. 19, 1898.

			h.	m.
Astronomical time of observation, 1898, Nov. 18..................			18	16.0
	h.	m.		
Astron. time, U. C. Polaris, Nov. 15 (Table I, Part I),	9	40.7		
Reduction to Nov. 19 (Part II), subtract...............		15.8		
Astron. time, U. C. Polaris, Nov. 19..................	9	24.9, subt.	9	24.9
Hour Angle of Polaris, at observation, and *Time Argument* for Table II..			‖8	51.1
Azimuth of Polaris, at observation (Table II)..			¶ 1° 11′ W.	

*By reference to the above table, the surveyor will observe that the times, between Nov. 1 and 15, are greater than 8^h 24^m; consequently, the culmination for one day earlier, Nov. 6, will be used; see directions on page 111; also, last clause of example 3, page 112.

† From Part II, Table I, opposite 6th day of month, and under "3 94^m."

‡ To subtract, take 1 day from Nov. 7, and add its equivalent, 24^h, to 8^h 24^m, making, Nov. 6, 32^h 24^m (which is the time expressed by Nov. 7, 8^h 24^m); then subtract in the usual manner.

§ See last clause of footnote, page 115.

‖ In case the *Hour Angle* comes out *greater* than 11^h 58^m, *subtract* it from 23^h 56.1^m; see example 4, on above.

¶ The *Hour Angle* being *less* than 11^h 58^m, the Azimuth is *west;* see precepts, top of Table II.

TABLE II.

This table gives, for various hour angles, expressed in *mean solar time*, and for even degrees of latitude from 30 to 50 degrees, the *Azimuths of Polaris* during the remainder of this century, computed for average values of the north polar distance of the star — the arguments (reference numbers), being the *hour angle* (or $23^h\ 56^m.1$, *minus* the hour angle, when the latter exceeds $11^h\ 58^m$), which is termed the *Time Argument;* and the *latitude* of the place of observation. The table is so extended that azimuths may be taken out by mere inspection, and all interpolation avoided, except such as can be performed mentally.

The vertical diameter SS′, Plate I, Fig, 2, divides the apparent path of Polaris into two equal parts, and for the star at any point s_6 on the *east* side, there is a corresponding point s_1, on the *west* side of the meridian, for which the azimuth Nw is equal to the azimuth Ne. The arc $Ss_1S's_6$, taken from the entire circle (or $23^h\ 56^m.1$), leaves the arc Ss_6, and its equal, Ss_1, expressed in time, may be used to find, from Table II, the azimuth Nw, which is equal to Ne.

The hour angles entered in Table II include only those of the *west half* of the circle ending at S′, and when an hour angle *greater* than $11^h\ 58^m$ results from observation, it will be *subtracted* from $23^h\ 56^m.1$, and the *remainder* will be used as the "time argument" for the table. The surveyor should not confound these two quantities. The *hour angle itself* always decides the *direction* of the azimuth and defines the place of the star with reference to the pole and meridian, as noted at top of Table II. See examples below Table I, page 114.

The *hours* of the "time arguments" are placed in the columns headed "Hours," on left of each page. The *minutes* of the time arguments will be found in the columns marked "m.," under the years for which they are computed, and they are included between the same heavy zigzag lines which inclose the hours to which they belong.

The time arguments are given to the nearest half minute; the occurrence of a period after the *minutes* of any one of them, indicates that its value is 0.5^m greater than printed, the table being so arranged to economize space.

The tables will be used as follows: Find the *hours* of the time argument in the left-hand column of either page; then, between the heavy lines which inclose the hours, find the *minutes* in the column marked at the top with the current year. On the same horizontal line with the *minutes*, the azimuth will be found under the given latitude, which is marked at the top of the right-hand half of each page. Thus, for 1892, time argument, $0^h\ 40^m$, latitude 42°; find 0^h on left-hand page and under 1892, find 40^m, on tenth line from the top, and on same line with the *minutes*, under latitude 42°, is the azimuth 0° 18′. For 1896, time argument $7^h\ 58^m$, lat. 36°, the azimuth is 1° 19′, found on the 9th line from bottom of right-hand page.

If the *exact* time argument is not found in the table, the azimuth should be proportioned to the difference between the given and tabular values of said argument. Thus, if the time argument in the first of the above examples (for 1892) was $0^h\ 42^m$, instead of $0^h\ 40^m$, the azimuth would be the mean between 0° 18′ and 0° 20′, or 0° 19′. In a similar manner, if the *latitude* is nearer an *odd* than an *even* degree, the mean of the azimuths for the next greater and next less latitude will be used; thus, in the above example for 1896, if the given latitude was 37°, the mean between 1° 19′ and 1° 21′, or 1° 20′, would be the corresponding azimuth. The table has been arranged to give the azimuths as exemplified above, by simple inspection. No written arithmetical work is required, all being performed mentally. It will always be sufficient to take the nearest *whole* degree of latitude and use it as above directed, except for a few values near the bottom of either page, where the difference of azimuths, for 2° difference of latitude, amounts to 4 or 5 minutes of arc; for example, 1890, time argument, $7^h\ 29^m$, lat. 46° 41′. In this case the latitude may be taken to the nearest *half* degree (46½°); the corresponding azimuth is 1° 42′.

3. The attention of the surveyor is directed to the fact that he should always use *one day of twenty-four hours* as the unit when he subtracts the time of *culmination*

from the time of *observation*. See example 4, page 114. In any case when the time of upper culmination, taken from Table I, for the given date, would be numerically *greater* than the *astronomical* time of observation, the former time will be taken out for a date *one day earlier* than the date of observation. The surveyor will decide when such condition exists by comparing the time given in the table with his *astronomical* time of observation. See example 4 and explanations in footnotes below Table I, page 113.

When an hour angle comes out within *one minute* of either $0^h\ 0^m$, or $23^h\ 56^m.1$, the observation may be regarded as having been taken with the star on the meridian, *above* the pole; if within one minute of $11^h\ 58^m$, Polaris may be considered on the meridian *below* the pole at the time of observation.

At *elongation* Polaris is nearly $5^h\ 55^m$ west (or east) of its position at upper culmination; consequently if the hour angle for *any* observation comes out within *five minutes* of $5^h\ 55^m$ or $18^h\ 1^m$, the star may be assumed to be *at elongation*, *west* for the first and *east* for the second hour angle, and its azimuth may be taken from a preceding table, which gives its value at elongation, from 1896 to 1905, inclusive.

Should the surveyor wish the time of *Lower Culmination*, for use with the plumb-line method, described on page 32, or for any other purpose, he will first determine the time of *upper* culmination for the date (Table I), and then *subtract* $11^h\ 58^m$ for the *preceding* lower culmination, or *add* $11^h\ 58^m$ for the lower culmination *following* the derived time for upper culmination, attending to the addition or subtraction of $23^h\ 56^m.1$, as directed on page 111.

The time to be used when making observations on Polaris off the meridian, should be as accurate as can be obtained. Looking at Table II, near the top of either page, the surveyor will observe, that for a difference of *four* minutes in the time argument, there is a change of about *two* minutes in azimuth; consequently, to obtain the azimuth to the *nearest whole minute of arc*, the *local* *mean time*, upon which all depends, should be known *within two minutes.* When the surveyor uses a solar instrument, he can readily determine the time for himself during the afternoon *before* observing Polaris, or in the morning *after* observation, and, without moving the hands of his watch, apply the necessary correction to his *observed* watch time. When the surveyor uses *standard railroad time*, he will correct the same for the difference of longitude between his station and the standard meridian for which the time is given, at the rate of *four minutes* of time *for each degree* of the difference in arc. Thus, if the difference of longitude is 6° 45′, the equivalent in time will be 27 minutes. The difference of longitude may be taken from a good map. The number of seconds taken from Table III, multiplied by the number of *ranges*, will give the correction for longitude in *seconds* of time. The correction will be *subtracted* from the standard railroad time of observation, when the surveyor's station is *west*, or *added* when *east* of the standard meridian, as the case may require, to obtain *local* time. It is immaterial *where* the surveyor obtains the standard time, provided he gets it right: a result which will be determined in the most satisfactory manner, by a direct comparison at telegraph office, personally conducted.

Generally, the surveyor will have only two or three simple additions or subtractions to make, and ten minutes will be ample time in which to make the observation and perform the little computation required.

NOTE.—The *azimuths* entered in the following table were calculated with the mean North Polar Distance of Polaris (1° 16′ 32″), the assumed latitudes of the table, and the stated *hour angles* for the year 1890. The resulting values having been tabulated, the process was reversed, and with the mean N. P. D. of the star, for the 1st of July of each of the remaining ten years of the series, the *latitudes* named, and *azimuths* already determined, the corresponding *hour angles* were found.

Table II.—*Azimuths of Polaris for the use of land surveyors.*

[The hour angles are expressed in *mean solar* time. The occurrence of a period after minutes of an hour angle indicates that its value is 0m.5 greater than printed.]

Star and Azimuth.

W. of N. when hour angle is *less* than 11h 58m.
E. of N. when hour angle is *greater* than 11h 58m.

Time argument, the star's hour angle (or 23h 56m.1 *minus* the star's hour angle), for the year—

Polaris *above* **the Pole.**

To determine the true meridian, the azimuth will be laid off to the *east* when the hour angle is *less* than 11h 58m, and to the *west* when *greater* than 11h 58m.

Azimuths for latitude—

Hours.	1895.	1896.	1897.	1898.	1899.	1900.	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
h.	m.	m.	m.	m.	m.	m.	° ′	° ′	°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	4	4	4	4	4	4	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2
	8	8	8	8.	8.	8.	3	3	3	3	3	4	4	4	4	4	4
	12	12.	12	12.	12.	12.	5	5	5	5	5	5	6	6	6	6	6
	16.	16.	16.	16.	16.	16.	6	6	6	7	7	7	7	8	8	8	8
	20.	20.	20.	20.	21	21	8	8	8	8	9	9	9	9	10	10	11
	24.	24.	24.	24.	25	25	9	10	10	10	10	11	11	11	12	12	13
	28.	28.	28.	29	29.	29	11	11	11	12	12	12	13	13	14	14	15
	32.	32.	33	33	33.	33.	12	13	13	13	14	14	15	15	16	16	17
	36.	37	37	37	37.	37.	14	14	15	15	15	16	16	17	18	18	19
	41	41	41	41.	41.	41.	15	16	16	17	17	18	18	19	20	20	21
	45	45	45.	45.	45.	46	17	17	18	18	19	19	20	21	21	23	23
	49	49	49.	49.	50	50	19	19	19	20	21	22	22	23	23	24	25
	53	53.	53	53.	54	54	20	21	21	22	22	23	24	24	25	26	27
0	57	57.	57.	58	58	58	22	22	23	23	24	25	25	26	27	28	29
1	1.	1.	2	2	2.	2.	23	24	24	25	26	26	27	28	29	30	32
	6.	6	7	7	7.	7.	25	26	26	27	28	28	29	30	31	33	34
	11.	11.	12	12	13	13	27	27	28	29	30	31	32	33	34	35	37
	16.	17	17.	17.	18	18	29	29	30	31	32	33	34	35	36	38	39
	21.	22	22.	22.	23	23.	31	31	32	33	34	35	36	37	38	40	42
	27	27	28	28.	28.	28.	32	33	34	35	36	37	38	39	41	42	44
	32	32.	33	33.	34	34	34	35	36	37	38	39	40	42	43	45	47
	37	37	38	38	38.	39	36	37	38	39	40	41	42	44	45	47	49
	42	42	43	43.	44	44.	38	39	40	41	42	43	44	46	47	49	51
	47	47.	48	48.	49	49.	39	40	41	42	44	45	46	48	50	52	54
	52.	53	53.	54	54.	55	41	42	43	44	46	47	48	50	52	54	56
1	57	58	58.	59	59.	0	43	44	45	46	47	49	50	52	54	56	0 59
2	2.	3	3.	4.	5	5.	45	46	47	48	49	51	52	54	56	0 58	1 1
	8	8.	9	9.	10	10.	46	47	49	50	51	53	54	56	0 58	1 1	3
	12.	13.	14	14.	15	16	48	49	50	51	53	54	56	0 58	1 0	3	5
	18	18.	19	20	20.	21.	50	51	52	53	55	56	0 58	1 0	2	5	8
	23	24	24.	25	26	26.	51	52	54	55	57	0 58	1 0	2	4	7	10
	28	29	30	30	31	32	53	54	55	57	0 58	1 0	2	4	6	9	12
	33.	34.	35	35.	36.	37	54	55	57	0 58	1 0	2	4	6	8	11	14
	38.	39	40	41	41.	42.	56	57	0 58	1 0	2	3	6	8	10	13	16
	43.	44.	45.	46.	47	48	57	0 59	1 0	2	3	5	7	9	12	15	18
	49	49.	50.	51.	53	53	0 59	1 0	2	3	5	7	9	11	14	17	20
	54	55	56	57	5	58.	1 0	2	3	5	7	8	11	13	16	19	22
2	59	0	1	2	3	4	2	3	5	6	8	10	12	15	17	20	24
3	4.	5.	6.	7.	8.	9.	3	4	6	8	10	12	14	16	19	22	26

Table II.— *Continued.*

Star and Azimuth.							Polaris *above* the Pole.										
							Azimuths for latitude —										
Hours.	1895.	1896.	1897.	1898.	1899.	1900.	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
h.	m.	m.	m.	m.	m.	m.	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
3	10.	12	13	14	15	16	5	6	8	10	12	13	16	18	21	24	28
	17	18	19.	20.	21.	23	6	8	9	11	13	15	18	20	23	27	30
	23	24.	25.	27	28	29	8	9	11	13	15	17	19	22	25	28	32
	29.	31	32.	33.	35	36	9	11	13	14	16	19	21	24	27	30	34
	36	37.	39	40	41.	43	11	13	14	16	18	20	23	25	29	32	36
	43.	45	46.	48	49	51	12	14	16	17	20	22	25	27	31	34	38
	51	52.	54	55	57	59	14	15	17	19	21	24	26	29	32	36	40
3	58.	0	2	3.	5	7	15	17	19	21	23	25	28	31	34	38	42
4	8	10	12	13.	15.	17.	17	19	21	23	25	27	30	33	36	40	44
	19	21.	23.	25.	28	30	19	21	23	24	27	29	32	35	39	43	47
	30.	33	35.	38	40.	43	20	22	24	26	29	31	34	37	41	45	49
4	42	45	48	50.	54	57.	22	24	26	28	30	33	36	39	42	47	51
5	0.	4.	8.	12.	17.	23	24	26	28	30	33	35	38	41	45	49	54
	20.	29	37	50.	....	....	26	27	30	32	34	37	40	43	47	51	55
	....	....	....	....	....	....	27	29	31	33	36	39	42	45	49	53	58
5	....	..	....	....	....	....	1 29	1 30	1 32	1 35	1 37	1 40	1 43	1 47	1 50	1 55	1 59
Star and Azimuth.							Polaris *below* the Pole.										
11	54	54	54	54	54	54	0 1	0 1	0 1	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2
	50	50	50	50	50	50	3	3	3	3	3	3	3	4	4	4	4
	46	46.	45.	45.	45.	45.	5	5	5	5	5	5	5	5	6	6	6
	42	41	41.	41.	41.	41.	6	6	6	6	7	7	7	7	8	8	8
	37.	37.	37.	37.	37.	37	8	8	8	8	8	8	9	9	9	10	10
	33.	33.	33.	33.	33	33	9	9	9	10	10	10	11	11	11	12	12
	29.	29.	29.	29	29	29	11	11	11	11	12	12	12	13	13	14	14
	25.	25	25	25	25	25	12	12	13	13	13	14	14	14	15	15	16
	21	21	21	21	21	21	14	14	14	15	15	15	16	16	17	17	18
	17.	17	17	17	16.	16.	15	15	16	16	17	17	18	18	19	19	20
	13.	13	13	12.	12.	12.	17	17	17	18	18	19	19	20	21	21	22
	9.	9	8.	8.	8.	8	18	18	19	19	20	20	21	22	22	23	25
	5.	5	4.	4.	4.	4	20	20	20	21	22	22	23	23	24	25	26
11	1	1	1	0.	0.	0	21	21	22	23	23	24	24	25	26	27	28
10	56.	56.	56.	56.	55.	55.	23	23	24	24	25	25	26	27	28	29	30
	51.	51.	51	51	50.	50.	24	25	25	26	27	27	28	29	30	31	32
	46.	46.	46	46	45.	45	26	27	27	28	29	29	30	31	32	34	35
	41.	41.	41	40.	40.	40	28	29	29	30	31	31	32	33	35	36	37
	36.	36	35.	35.	35	35	30	30	31	32	33	34	35	36	37	38	40
	31.	31	30.	30.	30	29.	32	32	33	34	35	36	37	38	39	41	42
	26	26	25.	25	24.	24.	33	34	35	36	37	38	39	40	41	43	44
	21	21	20.	20	19.	19	35	36	37	37	39	40	41	42	43	45	47
	16	15.	15	15	14.	14	37	38	39	39	40	41	43	41	46	47	49
10	11	10.	10.	9.	9	8.	39	39	40	41	42	43	45	46	48	49	51

I

TABLE II.— *Concluded.*

Star and Azimuth.							Polaris *below* the Pole.										
							Azimuths for latitude —										
Hours.	1895.	1896.	1897.	1898.	1899.	1900.	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
h.	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
10	6	5.	5	4.	4	3.	40	41	42	43	44	45	47	48	50	52	54
10	1	0	59.	59	58.	58	42	43	44	45	46	47	49	50	52	54	56
9	55.	55.	54.	54	53	53	44	45	46	47	48	49	51	52	54	56	0 58
	50.	50.	49	49	48	47.	45	46	47	48	50	51	53	54	56	0 58	1 0
	45.	44.	44	43.	43	42	47	48	49	50	51	53	54	56	0 58	1 0	2
	40.	39.	39	38	37.	37	49	50	51	52	53	55	56	0 58	1 0	2	5
	35	34.	33	33	32	31.	50	51	52	53	55	56	0 58	1 0	2	4	7
	30	29	28.	28	27	26.	51	53	54	55	57	0 58	1 0	2	4	6	9
	25	24	23	22.	21.	21	53	54	56	57	0 58	1 0	2	4	6	8	11
	19.	19	18	17.	16.	15.	55	56	57	0 58	1 0	2	4	5	8	10	13
	14.	13.	12.	12	11	10	56	57	0 59	1 0	2	3	5	7	10	12	15
	9.	8.	7.	6.	5.	5	58	0 59	1 0	2	3	5	7	9	11	14	17
9	4	3	2	1	0.	59.	0 59	1 0	2	3	5	7	9	11	13	16	19
8	58	58	57	56	55	54	1 0	2	3	5	6	8	10	12	15	18	21
	54	52.	52	51	49.	49	2	3	5	6	8	10	12	14	17	20	23
	47.	46	45.	44.	43	42.	3	5	6	8	10	12	14	16	19	22	25
	41.	40	39.	38	36.	35.	5	6	8	9	11	13	16	18	21	24	27
	35	33.	32.	31.	30	29	7	8	10	11	13	15	17	20	22	26	29
	28.	27	27	25	23.	23.	8	9	11	13	15	17	19	22	24	28	31
	21.	21	19.	17.	17	15.	10	11	13	14	16	18	21	23	26	29	33
	15	·13.	12	10.	9	7.	11	13	14	16	18	20	23	25	28	31	35
	7.	5.	4	2.	1	59.	13	14	16	18	20	22	25	27	30	33	37
8	0	58	56.	55	53	51.	14	16	18	19	21	24	26	29	32	35	39
7	50	48	46.	44.	42.	40.	16	18	19	21	24	26	28	31	34	38	42
	39	37	35	33	30.	28.	18	19	21	23	26	28	31	33	37	40	44
	28	25	23	20.	18	15.	20	21	23	25	27	30	33	35	39	43	47
7	16.	13.	10.	7	4.	1.	21	23	25	27	29	32	35	37	41	45	49
6	58	54	50	45.	41	35.	23	25	27	29	32	34	37	40	43	47	52
6	37.	30	22.	11	....	...	25	27	29	31	34	36	39	42	46	50	54
	....	...	....	....	....	.. .	27	29	31	33	36	38	41	44	48	52	57
	....	....	...	....	....	....	1 29	1 30	1 32	1 35	1 37	1 40	1 43	1 47	1 50	1 55	1 59

TABLE III.— *Difference of Longitude of Meridians six miles apart in Seconds of time.*

Lat.	Seconds.	Lat.	Seconds.	Lat.	Seconds.	Lat.	Seconds.
°		°		°		°	
30	24.02	35	25.40	40	27.14	45	29.39
31	24.27	36	25 71	41	27.55	46	29.92
32	24.53	37	26.04	42	27.97	47	30.47
33	24.80	38	26.39	43	28.42	48	31 05
34	25.09	39	26.76	44	28.90	49	31.67
						50	32.33

Second Method.

Charles W. Helmick of Helena, Montana, furnishes the following method of determining a meridian by an observation on Polaris at any time. It obviates the use of astronomical time, and is extensively used by deputy mineral surveyors in Montana.

1. Sight to Polaris and note the hour and minute of local mean solar time.

2. Take from the ephemeris the time of elongation nearest the time of observation. The difference between these times will be the hour angle east or west.

3. Reduce this hour angle to degrees and minutes.

4. Multiply the cosine of the resulting angle by the azimuth of Polaris for the latitude expressed in minutes. The result will be the approximate azimuth of Polaris expressed in minutes. If the time piece gives local time within 2 minutes, the azimuth will rarely vary 1′.

To Find a True Meridian or Other Line by the Sun.

We need for this work, first, the common engineer's transit fitted with vertical arc or circle. A piece of shade or dark glass will be useful though it is not imperative.

Second. The latitude of the place within a minute or two. This can readily be taken from a map.

Third. A table of mean refraction containing less than half a hundred figures.

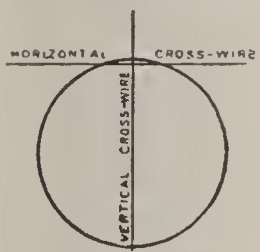
Fourth. A table of the declination of the sun. This table is furnished gratis by many of our instrument makers. A good form is a vest-pocket pamphlet sent out by George N. Saegmuller, of Washington, D. C. A glance at the spherical triangle involved will show how easily the sun may be used to determine azimuth.

In the figure, P represents the celestial pole of the earth, Z the zenith of the observer, and S the place of the sun at the time of observation. In the spherical triangle S P Z we know P Z, for it is the co-latitude of the place. Z S is the complement of A S, which latter is the altitude of the sun at the time of observation as measured by the transit. P S is the co-declination of the sun and is found from the table of declinations by subtracting the declination from 90°, paying heed to the sign of the declination. We then have the three sides of the triangle known and can readily compute the angle at Z by the formula

$$\sin \tfrac{1}{2}Z = \sqrt{\frac{\sin(\frac{1}{2}S-p)\ \sin(\frac{1}{2}S-s)}{\sin p \sin s}}$$

in which $p=$ co-altitude; $s=$ co-latitude, and $z=$ co-declination and $S=p+s+z$. This angle gives the azimuth of the sun at the instant of observation reckoned from the north point. The supplement of the angle would show its azimuth referred to the south point, the usual origin of azimuth.

Field Work.—About seven or eight o'clock in the morning or at five in the afternoon set the transit over a point commanding some prominent mark, as a church spire, flag pole or cupola to some building. Any time of the day will do except near noon, but the most favorable time except for uncertainties in refraction would be near sunrise or sunset. This appears, since we measure the altitude and from it compute the azimuth at the time the sun is changing rapidly in altitude and slowly in azimuth. Hence only that part of an error in altitude will affect azimuth which is shown by the small change in azimuth compared with the accompanying altitude change. Now bring the horizontal plates to zero and, being sure that the vertical circle reads zero for the collimation line horizontal (or note its index errors) and take a pointing at the sun. A better centering can be gained by bringing the horizontal cross wire very near the upper limb which gives a small segment to be bisected. Shown by the figure.



Having with the lower horizontal slow motion gained center, with the vertical motion quickly move the horizontal wire to the upper limb of the sun. Read the vertical circle and loose the alidade plate and sight the selected terrestial mark and read the horizontal circle. Should we care not to depend upon a single determination this operation may be repeated say four times, thus furnishing us not only a mean value twice as accurate as a single observation but by the agreement of the individual results among themselves we can judge much of their general reliability. By taking the second and fourth observations with the instrument in the reversed position—the full vertical circle permitting this—we eliminate both the error resulting from the horizontal axis of the instrument not being truly horizontal and the index error of the vertical circle.

Sample observations made Aug. 24, 1895, and the method of computation, are given on the following page.

AZIMUTH OBSERVATIONS.

Standard Time.	Altitude of upper limb.	Horizontal circle for center of sun.	Horizontal circle for Mark.
9.06.45	36° 01½′	0.0	141° 08½′

REMARKS.

Transit over stone post set in courthouse yard. Mark is cupola on public schoolhouse.

Latitude..44° 00′
Temperature....................................80°
Barometer......................................28.4
Center of sun is azimuth and upper limb observed upon.

COMPUTATION.

Declination, corrected for dif. of time	=	11° 04′
Co- " or z	=	78° 56′
Apparent altitude of upper limb	=	36° 01½′
Semi-diameter of sun	=	15½′
App. altitude of center of sun	=	35° 46′
Refraction	=	1′
True alt. of center of sun	=	35° 45′
Co-altitude of sun, or p	=	54° 15′

$$
\begin{aligned}
z &= 78^\circ\ 56' \\
p &= 54^\circ\ 15' \\
s &= 46^\circ\ 00' \\
\hline
S &= 179^\circ\ 11' \\
\tfrac{1}{2}S &= 89^\circ\ 35\tfrac{1}{2}'
\end{aligned}
$$

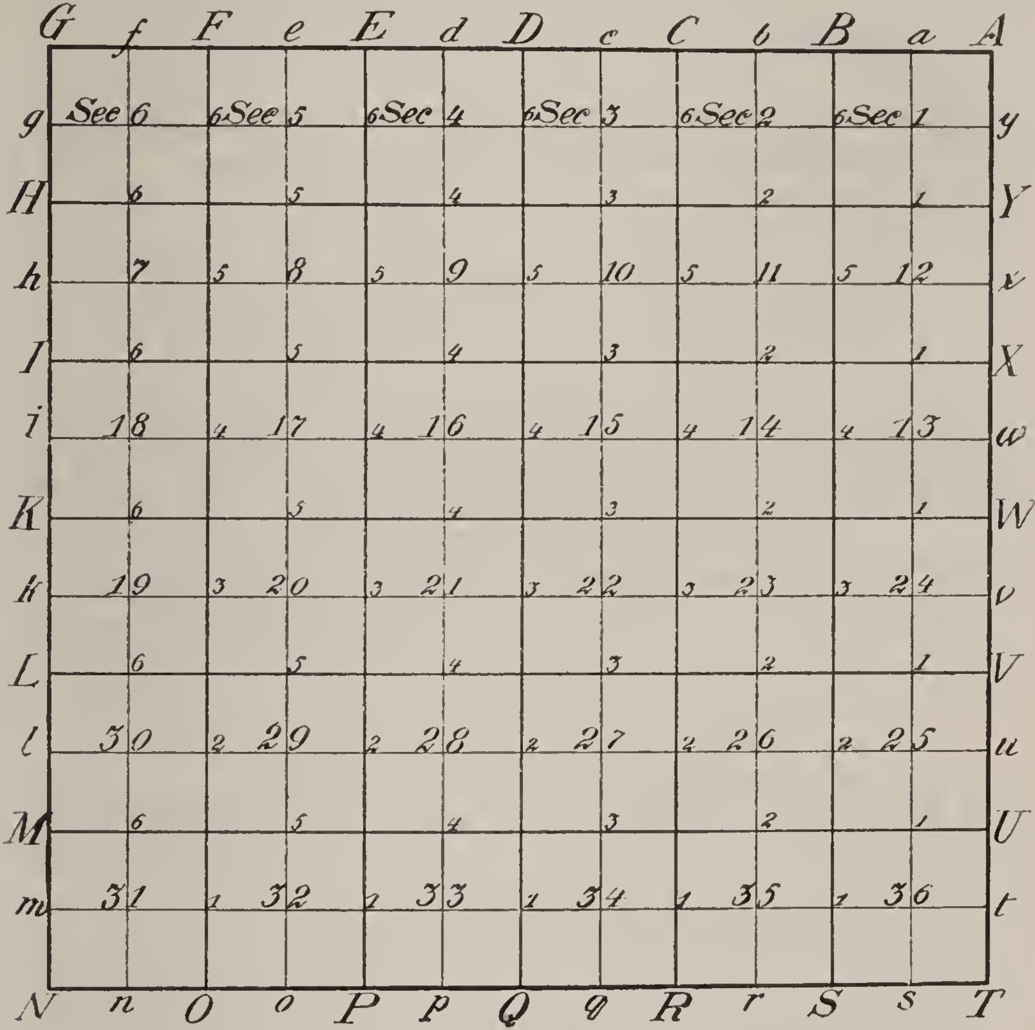
$\frac{1}{2}S - p$	=	35° 20½′	log sin	=	9.762267
$\frac{1}{2}S - s$	=	43° 35½′	"	=	9.838543
p	=	54° 15′	" A. C.	=	0.090672
s	=	46° 00′	" A. C.	=	.143066
					19.834548
$\frac{1}{2}Z$	=	55° 44½′	log sin. ½Z...........	=	9.917274

$Z = 111^\circ\ 29'$ reckoned from the north point.

Azimuth of sun $= 68^\circ\ 31'$ reckoned from the south point.

$141^\circ\ 08\frac{1}{2}' - 68^\circ\ 31' = 72^\circ\ 37\frac{1}{2}'$ = azimuth of cupola on Schoolhouse. A transit at the station with horizontal circle reading $72^\circ\ 37\frac{1}{2}'$ and pointing at the cupola when returned to zero will point in the true meridian.

The following method of abridging field notes is used by the land department of the United States. The plat of a township is lettered and numbered as shown in the diagram. Corners in the township boundary are



referred to by letter; *e. g.*, *B* or *k*. Interior section corners are referred to by the numbers of the sections; *e. g.*, corner of *9*, *10*, *15*, and *16*. Interior quarter section corners are referred to by their position on the lines, *e. g.*, *K* to *W* at *3* or *R* to *C* at *6*. The descriptions of corners thus referred to are written out in the margins of the plats, while all other matter contained in the field notes is, as far as possible, marked on the plats themselves. The letters along the margin of the diagram are arranged the same as in Plate IV, Instructions of 1894. A different arrangement has been used commencing in the upper left hand corner and passing around the plat in the opposite direction.

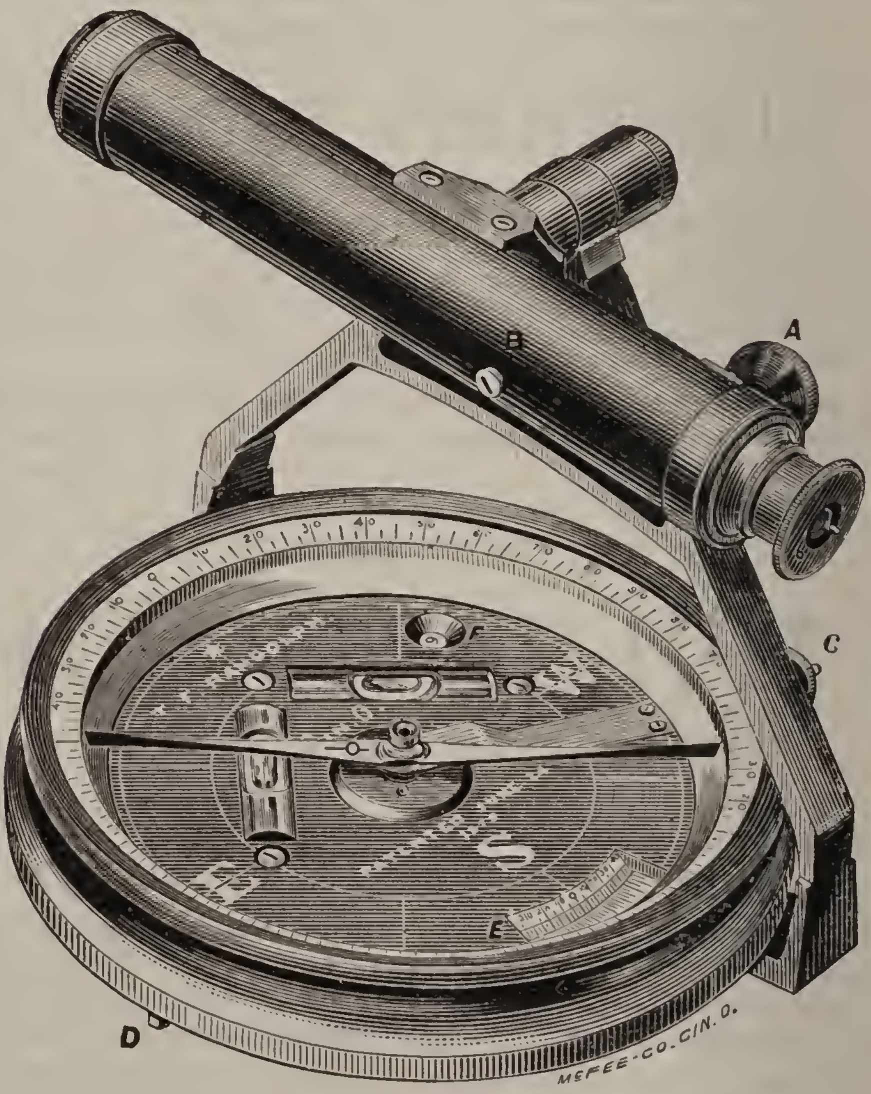
T. F. RANDOLPH,

MANUFACTURER, IMPORTER, AND DEALER IN

Surveyor's AND Engineer's Instruments,

AND SUPPLIES OF ALL KINDS.

ROE, CHESTERMAN, AND PAINE'S TAPES.



VERNIER TELESCOPE COMPASS,

PATENTED JUNE 24, 1879.

SOLE MANUFACTURER OF

Randolph's Patent Telescope Compass,
Patent Telescope Attachment for Common Compasses,
Patent Quick Leveling Tripod,
Patent Sole Leather Boxes,
Patent Daisy Level, Transit Level.

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Cincinnati, Ohio.

ESTABLISHED 1853. **WORKS RUN BY STEAM.**